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Reconciling Stability and Growth: Smart Pacts and Structural Reforms: Unpublished Appendix

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Abstract

This provides the unpublished appendix for the article "Reconciling Stability and Growth: Smart Pacts and Structural Reforms"

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1 Numbered equations in the model

$$\mathrm{E}_0\left[u\left(c_1\right)+v(q_1)+u\left(c_2\right)+v(q_2)\right], \tag{1}$$

$$\mathrm{E}_0\left[u\left(c_1\right)+v(f_1)+u\left(c_2\right)+v(f_2)\right], \tag{2}$$

$$c_1=c_2=\tfrac{1}{2}\left[\left(1-\tau\right)\left(y_1+y_2+\alpha\Gamma\left(\gamma\right)\right)+\left(h-I\right)\gamma\right], \tag{3}$$

$$v'\left(\tau y_L-h\gamma+d\right)\left(1-k\right) \;=\; pv'\left[\tau y_2+\left(\tau\alpha+\beta\right)\Gamma\left(\gamma\right)-d\right], \tag{4}$$

$$v'\left(\tau y_H-h\gamma+d\right) \;=\; pv'\left[\tau y_2+\left(\tau\alpha+\beta\right)\Gamma\left(\gamma\right)-d\right], \tag{5}$$

$$\begin{aligned}&\mathrm{E}_0\left\{u'\left(c_1\right)\left[h+\left(1-\tau\right)\alpha\Gamma'\left(\gamma\right)-I\right]\right\}=\tfrac{1}{2}h\left[v'\left(f_{1L}\right)+v'\left(f_{1H}\right)\right]\\&-\tfrac{1}{2}p\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right)\left[v'\left(f_{2L}\right)+v'\left(f_{2H}\right)\right],\end{aligned} \tag{6}$$

$$u\left(x\right) \;=\; v\left(x\right)=-\left(\xi-1\right)x^2/2+\xi x,\;\xi>1, \tag{7}$$

$$\Gamma\left(\gamma\right) \;=\; \gamma, \tag{8}$$

$$-\left(\xi-1\right)\left(\tau y_1-h\gamma+d\right)+\xi \;>\; 0, \quad y_1=y_L,y_H, \tag{9}$$

$$-\left(\xi-1\right)\left[\tau y_2+\left(\tau\alpha+\beta\right)\gamma-d\right]+\xi \;>\; 0, \tag{10}$$

$$d_L \;=\; \left(\tfrac{\xi}{\xi-1}\right)\tfrac{1-p-k}{1+p-k}+\tfrac{\tau\left[py_2-(1-k)y_L\right]+\left[p(\tau\alpha+\beta)+(1-k)h\right]\gamma}{1+p-k}, \tag{11}$$

$$d_H \;=\; \left(\tfrac{\xi}{\xi-1}\right)\tfrac{1-p}{1+p}+\tfrac{\tau(py_2-y_H)+\left[p(\tau\alpha+\beta)+h\right]\gamma}{1+p}, \tag{12}$$

$$\begin{aligned}&\left[\xi-\tfrac{1}{2}\left(\xi-1\right)\left[\left(1-\tau\right)\left(\tilde{y}_1+y_2\right)-\varepsilon\gamma\right]\right]\left(-\varepsilon\right)=h\left[\xi-\left(\xi-1\right)\left(\tau\tilde{y}_1-h\gamma+\tilde{d}\right)\right]\\&-p\left(\tau\alpha+\beta\right)\left[\xi-\left(\xi-1\right)\left(\tau y_2+\left(\tau\alpha+\beta\right)\gamma-\tilde{d}\right)\right],\end{aligned} \tag{13}$$

$$\tfrac{1}{2}\left(\tfrac{1-p}{p}\right)\left[\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right)\left[v'(f_{1L})+v'(f_{1H})\right]\tfrac{\partial\gamma}{\partial k}-v'(f_{1L})\tfrac{\partial d_L}{\partial k}-v'(f_{1H})\tfrac{\partial d_H}{\partial k}\right]>0, \tag{14}$$

$$\bar{d}=\bar{d}^c+\delta\gamma,\;\delta>0, \tag{15}$$

$$\begin{aligned}&\mathrm{E}_0\left\{u'\left(c_1\right)\left[h+\left(1-\tau\right)\alpha\Gamma'\left(\gamma\right)-I\right]\right\}=\tfrac{1}{2}h\left[v'\left(f_{1L}\right)+v'\left(f_{1H}\right)\right]\\&-\tfrac{1}{2}p\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right)\left[v'\left(f_{2L}\right)+v'\left(f_{2H}\right)\right]-\tfrac{1}{2}\delta kv'\left(f_{1L}\right).\end{aligned} \tag{16}$$

Appendix

A Proofs and derivations

A.1 Derivation of (4) and (5)

Define $V(y_1, k, p; d, \gamma)$ as the value function for the (first-period) government (conditional on the realization of y_1) at the *end* of Stage 3, taking d and γ as given. That value function is a function of the punishment parameter k that characterizes the tightness of the stability pact and the probability p that the first-period government will be re-elected.

If $y_1 = y_L$, the government maximizes over $d > \bar{d}$ the function $V(y_L, k, p; d, \gamma)$, where:

$$V(y_L, k, p; d, \gamma) = v\left[\tau y_L - h\gamma + d - k\left(d - \bar{d}\right) + R\right] + pv\left[\tau y_2 + (\tau\alpha + \beta)\Gamma(\gamma) - d\right].$$

The first-order condition is:

$$v'\left[\tau y_L - h\gamma + d - k\left(d - \bar{d}\right) + R\right](1 - k) = pv'\left[\tau y_2 + (\tau\alpha + \beta)\Gamma(\gamma) - d\right]. \tag{17}$$

Given the properties of $v(\cdot)$, an increase in the deficit d reduces the left-hand side (the marginal benefit), but raises the right-hand side (the marginal cost), indicating that there exists at most one solution, which we denote by $d_L(\gamma)$. Further, notice that the optimal deficit $d_L(\gamma)$ increases with the magnitude of the negative shock (that is, the lower is y_L), so that $d_L(\gamma) > \bar{d}$ for a sufficiently large variance of the shock to period-1 resources.[1] Below (in Appendix A.1.1), we show that such a solution $d_L(\gamma)$ to (17) indeed yields the maximum of $V(y_L, k, p; d, \gamma)$. Finally, for notational convenience, we denote by V_L the value function at the *start* of Stage 3 when $y_1 = y_L$.

If $y_1 = y_H$, the government chooses $d \leq \bar{d}$ to maximize:

$$V(y_H, k, p; d, \gamma) = v(\tau y_H - h\gamma + d + R) + pv\left[\tau y_2 + (\tau\alpha + \beta)\Gamma(\gamma) - d\right],$$

which yields the first-order condition:

$$v'(\tau y_H - h\gamma + d + R) = pv'\left[\tau y_2 + (\tau\alpha + \beta)\Gamma(\gamma) - d\right], \tag{18}$$

where it can be checked ex-post that $d_H(\gamma) \leq \bar{d}$ for sufficiently large values of y_H. As above, we denote by V_H the value function at the *start* of Stage 3 when $y_1 = y_H$.

Because of the assumption of identical shocks hitting identical countries, $R = 0$ when $y_1 = y_H$ and $R = k\left(d - \bar{d}\right)$ when $y_1 = y_L$. Using this, we reduce (17) to (4) and (18) to (5).

[1] It is easy to check this numerically in the examples presented in Appendix B below.

A.1.1 Proof that, if $d_L > \bar{d}$, then d_L maximizes $V(y_L, k, p; d, \gamma)$

Denote by d_L^* the solution to (17) if $k = 0$. Then, by Lemma 3, $d_L^* > d_L > \bar{d}$. Combining this with strict concavity, we have that for all $d < \bar{d}$, $V(y_L, 0, p; d, \gamma) < V\left(y_L, 0, p; \bar{d}, \gamma\right)$. Hence, the only remaining candidate solution to the maximization problem would be $d = \bar{d}$. Finally, from (17), $V\left(y_L, 0, p; \bar{d}, \gamma\right) = V\left(y_L, k, p; \bar{d}, \gamma\right)$, while, again, by strict concavity and the assumption that $d_L > \bar{d}$, $V\left(y_L, k, p; \bar{d}, \gamma\right) < V(y_L, k, p; d_L, \gamma)$. Hence, $V\left(y_L, 0, p; \bar{d}, \gamma\right) < V(y_L, k, p; d_L, \gamma)$ and, hence, $V(y_L, k, p; d, \gamma)$ is maximized at $d = d_L$.

A.2 Proof of Lemma 1

Differentiating (4) and (5), respectively, with respect to γ, yields:

$$
\begin{aligned}
v''(f_{1L})(1-k)\left(\frac{\partial d_L}{\partial \gamma} - h\right) &= pv''(f_{2L})\left[(\tau\alpha + \beta)\Gamma'(\gamma) - \frac{\partial d_L}{\partial \gamma}\right], \\
v''(f_{1H})\left(\frac{\partial d_H}{\partial \gamma} - h\right) &= pv''(f_{2H})\left[(\tau\alpha + \beta)\Gamma'(\gamma) - \frac{\partial d_H}{\partial \gamma}\right].
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\frac{\partial d_L}{\partial \gamma} &= h\lambda_L + (\tau\alpha + \beta)\Gamma'(\gamma)(1 - \lambda_L) > 0, &(19)\\
\frac{\partial d_H}{\partial \gamma} &= h\lambda_H + (\tau\alpha + \beta)\Gamma'(\gamma)(1 - \lambda_H) > 0, &(20)
\end{aligned}
$$

where

$$
\lambda_L \equiv \frac{v''(f_{1L})(1-k)}{v''(f_{1L})(1-k) + pv''(f_{2L})}, \ \lambda_H \equiv \frac{v''(f_{1H})}{v''(f_{1H}) + pv''(f_{2H})}, \tag{21}
$$

and where

$$
f_{1j} \equiv \tau y_j - h\gamma + d_j, \ \ f_{2j} \equiv \tau y_2 + (\tau\alpha + \beta)\Gamma(\gamma) - d_j, \ \ j = L, H. \tag{22}
$$

A.3 Derivation of (6)

To determine the optimal reform package γ in Stage 1, the government solves the following program:

$$
\underset{\gamma}{\text{Max}} \left\{2\mathrm{E}_0\left[u(c_1)\right] + \tfrac{1}{2}\left[V_L + V_H\right]\right\}, \tag{23}
$$

taking k as given and where we have made use of the result that $c_2 = c_1$. Thanks to the fact that $\frac{\partial V_L}{\partial d_L} = 0$ and $\frac{\partial V_H}{\partial d_H} = 0$ (see (17) and (18), respectively) the first-order condition for optimal reforms can be written as:

$$
\mathrm{E}_0\left\{u'(c_1)\left[h + (1-\tau)\alpha\Gamma'(\gamma) - I\right]\right\} + \tfrac{1}{2}\left[\tfrac{\partial V_L}{\partial \gamma} + \tfrac{\partial V_H}{\partial \gamma}\right] = 0, \tag{24}
$$

where

$$\begin{aligned}\frac{\partial V_L}{\partial \gamma} &= -hv'\left(\tau y_L - h\gamma + d_L - k\left(d_L - \bar{d}\right) + R\right) + p\left(\tau\alpha + \beta\right)\Gamma'\left(\gamma\right)v'\left(f_{2L}\right), \\ \frac{\partial V_H}{\partial \gamma} &= -hv'\left(\tau y_H - h\gamma + d_H + R\right) + p\left(\tau\alpha + \beta\right)\Gamma'\left(\gamma\right)v'\left(f_{2H}\right).\end{aligned}$$

Using the cross-country symmetry of the equilibrium and after simplifying, we obtain (6).

A.4 Proof that (6) has at most one solution

The derivative of the left-hand side of (6) with respect to γ is:

$$\mathrm{E}_0\left\{\tfrac{1}{2}u''\left(c_1\right)\left[h + \left(1-\tau\right)\alpha\Gamma'\left(\gamma\right) - I\right]^2 + u'\left(c_1\right)\left(1-\tau\right)\alpha\Gamma''\left(\gamma\right)\right\} < 0.$$

Hence, an increase in γ reduces the left-hand side of (6). The derivative of the right-hand side of (6) with respect to γ is:

$$\begin{aligned}&\tfrac{1}{2}h\left[v''\left(f_{1L}\right)\left(\tfrac{\partial d_L}{\partial \gamma} - h\right) + v''\left(f_{1H}\right)\left(\tfrac{\partial d_H}{\partial \gamma} - h\right)\right] \\ &-\tfrac{1}{2}p\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right)\left\{\begin{array}{c} v''\left(f_{2L}\right)\left[\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right) - \frac{\partial d_L}{\partial \gamma}\right] + \\ v''\left(f_{2H}\right)\left[\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right) - \frac{\partial d_H}{\partial \gamma}\right]\end{array}\right\} \\ &-\tfrac{1}{2}p\left(\tau\alpha+\beta\right)\Gamma''\left(\gamma\right)\left[v'\left(f_{2L}\right) + v'\left(f_{2H}\right)\right].\end{aligned}$$

Using (19) and (20), this is written as:

$$\begin{aligned}&\tfrac{1}{2}h\left\{v''\left(f_{1L}\right)\left(1-\lambda_L\right)\left[\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right) - h\right] + v''\left(f_{1H}\right)\left(1-\lambda_H\right)\left[\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right) - h\right]\right\} \\ &-\tfrac{1}{2}p\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right)\left\{\begin{array}{c} v''\left(f_{2L}\right)\lambda_L\left[\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right) - h\right] \\ +v''\left(f_{2H}\right)\lambda_H\left[\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right) - h\right]\end{array}\right\} \\ &-\tfrac{1}{2}p\left(\tau\alpha+\beta\right)\Gamma''\left(\gamma\right)\left[v'\left(f_{2L}\right) + v'\left(f_{2H}\right)\right]\end{aligned}$$

$$\begin{aligned}= \;&\tfrac{1}{2}\left[\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right) - h\right]\left[\begin{array}{c} h\left[v''\left(f_{1L}\right)\left(1-\lambda_L\right) + v''\left(f_{1H}\right)\left(1-\lambda_H\right)\right] \\ -p\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right)\left[v''\left(f_{2L}\right)\lambda_L + v''\left(f_{2H}\right)\lambda_H\right]\end{array}\right] \\ &-\tfrac{1}{2}p\left(\tau\alpha+\beta\right)\Gamma''\left(\gamma\right)\left[v'\left(f_{2L}\right) + v'\left(f_{2H}\right)\right].\end{aligned}$$

Using (21) we can further write this as:

$$\begin{aligned}&\tfrac{1}{2}\left[\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right) - h\right]\left[\begin{array}{c} ph\left[\frac{v''(f_{1L})v''(f_{2L})}{v''(f_{1L})(1-k)+pv''(f_{2L})} + \frac{v''(f_{1H})v''(f_{2H})}{v''(f_{1H})+pv''(f_{2H})}\right] \\ -p\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right)\left[\frac{v''(f_{1L})v''(f_{2L})(1-k)}{v''(f_{1L})(1-k)+pv''(f_{2L})} + \frac{v''(f_{1H})v''(f_{2H})}{v''(f_{1H})+pv''(f_{2H})}\right]\end{array}\right] \\ &-\tfrac{1}{2}p\left(\tau\alpha+\beta\right)\Gamma''\left(\gamma\right)\left[v'\left(f_{2L}\right) + v'\left(f_{2H}\right)\right].\end{aligned}$$

If $k = 0$, this reduces to:

$$-\tfrac{1}{2}p\left[(\tau\alpha+\beta)\Gamma'(\gamma)-h\right]^2\left[\tfrac{v''(f_{1L})v''(f_{2L})}{v''(f_{1L})+pv''(f_{2L})}+\tfrac{v''(f_{1H})v''(f_{2H})}{v''(f_{1H})+pv''(f_{2H})}\right]$$
$$-\tfrac{1}{2}p(\tau\alpha+\beta)\Gamma''(\gamma)\left[v'(f_{2L})+v'(f_{2H})\right]>0,$$

because $(\tau\alpha+\beta)\Gamma'(\gamma)>h$. By continuity, the derivative of the right-hand side of (6) with respect to γ should be positive also when k is positive, but not too large. Hence, the right-hand side of (6) is increasing in γ when k is zero or positive, but not too large.

A.5 Conditions for an internal solution for γ

Using (4) and (5), we rewrite (6) further as:

$$\begin{aligned}
& \mathrm{E}_0\left\{u'(c_1)\left[h+(1-\tau)\alpha\Gamma'(\gamma)-I\right]\right\} \\
= \; & \tfrac{1}{2}\left[h-(\tau\alpha+\beta)\Gamma'(\gamma)(1-k)\right]v'(\tau y_L-h\gamma+d_L)+ \\
& \tfrac{1}{2}\left[h-(\tau\alpha+\beta)\Gamma'(\gamma)\right]v'(\tau y_H-h\gamma+d_H).
\end{aligned} \tag{25}$$

An interior solution $0<\gamma<\infty$, requires that both sides of this equation are of the same sign. For $k>0$ not too large, this, in turn, requires that either (i) $(\tau\alpha+\beta)\Gamma'(\gamma)>h$ *and* $I>h+(1-\tau)\alpha\Gamma'(\gamma)$ or (ii) $(\tau\alpha+\beta)\Gamma'(\gamma)<h$ *and* $I<h+(1-\tau)\alpha\Gamma'(\gamma)$.[2] In case (i), we find the intuitively plausible condition that the total (budgetary) benefit of structural reforms must be sufficiently large. The second condition in case (i) indicates that, to make the choice problem interesting, the individual costs of reforms must also be sufficiently large. Notice that an ambitious reform policy (that is, a higher γ when $\Gamma''<0$) makes the condition $(\tau\alpha+\beta)\Gamma'(\gamma)>h$ less likely to hold and the condition $I>h+(1-\tau)\alpha\Gamma'(\gamma)$ more likely to hold (and vice versa for a lower γ), indicating that if an optimal reform package exists, it will most probably reflect a middle-of-the-road approach. An appropriate choice of the function $\Gamma'(\gamma)$ allows to restrict the analysis to the intuitively plausible case (i), which we do in the main text. Under the conditions of case (i), the left-hand side of (25) is strictly decreasing in γ, while its right-hand side is strictly increasing in γ. Hence, if a solution $0<\gamma<\infty$ to (25) exists, it is unique. Appendix B.1 provides an explicit characterization of a solution $\gamma>0$ under weak parameter restrictions and the assumption that $u(\cdot)$ and $v(\cdot)$ are quadratic and $\Gamma(\gamma)$ is linear, while Appendix B.2 provides numerically computated solutions $\gamma>0$ when $\Gamma(\gamma)$ is nonlinear.

A.6 Proof of Lemma 2

Differentiate (4) and (5) with respect to p holding γ fixed. We find that $\partial d_L/\partial p<0$ and $\partial d_H/\partial p<0$. As the case of the planner corresponds to $p=1$ and $k=0$, the result follows for $k=0$. By continuity it follows also for k not too large.

[2] By continuity of all the functions involved, if a solution $\gamma>0$ exists for $k=0$, there also exists one for $k>0$, but not too large.

A.7 Proof of Proposition 1

Let $k = 0$ and consider (25) for both a partisan government and a social planner ($p = 1$, in which case the social planner's solutions d_L^S and d_H^S replace d_L and d_H, respectively). The left-hand side of (25) is decreasing in γ and in both cases equal for given γ. Taking account of the effect on the deficit, it is easy to show that the right-hand side of (25) is increasing in γ, while, for *given* γ, it is larger under a partisan government than under a planner, because $d_L(\gamma) > d_L^S(\gamma)$ and $d_H(\gamma) > d_H^S(\gamma)$ by Lemma 2 and $(\tau\alpha + \beta)\Gamma'(\gamma) > h$. Hence, the value of γ that solves (25) for the partisan government is smaller than the one that solves it for the planner. By continuity of all functions involved, the result also holds for $k > 0$, but not too large.

A.8 Proof of Lemma 3

Differentiate (4) with respect to k, to give:

$$v''(f_{1L})(1-k)\frac{\partial d_L}{\partial k} - v'(f_{1L}) = -pv''(f_{2L})\frac{\partial d_L}{\partial k}.$$

Hence,

$$\frac{\partial d_L}{\partial k} = \frac{v'(f_{1L})}{v''(f_{1L})(1-k) + pv''(f_{2L})} < 0.$$

A.9 Proof of Lemma 4

Differentiate (6) with respect to k, to give:

$$\begin{aligned}
& \mathrm{E}_0\left\{\tfrac{1}{2}u''(c_1)\left[h + (1-\tau)\alpha\Gamma'(\gamma) - I\right]^2 + u'(c_1)(1-\tau)\alpha\Gamma''(\gamma)\right\}\tfrac{\partial\gamma}{\partial k} \\
& -\tfrac{1}{2}hv''(f_{1L})\left[-h\tfrac{\partial\gamma}{\partial k} + \tfrac{\partial d_L}{\partial k}\right] - \tfrac{1}{2}hv''(f_{1H})\left[-h\tfrac{\partial\gamma}{\partial k} + \tfrac{\partial d_H}{\partial k}\right] \\
& +\tfrac{1}{2}p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2L})\left[(\tau\alpha+\beta)\Gamma'(\gamma)\tfrac{\partial\gamma}{\partial k} - \tfrac{\partial d_L}{\partial k}\right] \\
& +\tfrac{1}{2}p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2H})\left[(\tau\alpha+\beta)\Gamma'(\gamma)\tfrac{\partial\gamma}{\partial k} - \tfrac{\partial d_H}{\partial k}\right] \\
& +\tfrac{1}{2}p(\tau\alpha+\beta)\left[v'(f_{2L}) + v'(f_{2H})\right]\Gamma''(\gamma)\tfrac{\partial\gamma}{\partial k} \\
= \; & 0,
\end{aligned}$$

which is equivalent to

$$\begin{aligned}
& \begin{bmatrix}
\mathrm{E}_0\left\{\tfrac{1}{2}u''(c_1)\left[h + (1-\tau)\alpha\Gamma'(\gamma) - I\right]^2 + u'(c_1)(1-\tau)\alpha\Gamma''(\gamma)\right\} + \\
\tfrac{1}{2}p(\tau\alpha+\beta)^2(\Gamma'(\gamma))^2\left[v''(f_{2L}) + v''(f_{2H})\right] + \tfrac{1}{2}h^2\left[v''(f_{1L}) + v''(f_{1H})\right] \\
+\tfrac{1}{2}p(\tau\alpha+\beta)\left[v'(f_{2L}) + v'(f_{2H})\right]\Gamma''(\gamma)
\end{bmatrix}\tfrac{\partial\gamma}{\partial k} \\
= \; & \tfrac{1}{2}\left[hv''(f_{1L}) + p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2L})\right]\tfrac{\partial d_L}{\partial k} \\
& +\tfrac{1}{2}\left[hv''(f_{1H}) + p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2H})\right]\tfrac{\partial d_H}{\partial k}. \qquad (26)
\end{aligned}$$

The coefficients of $\frac{\partial\gamma}{\partial k}$, $\frac{\partial d_L}{\partial k}$ and $\frac{\partial d_H}{\partial k}$ in (26) are all negative. Hence, if $\frac{\partial d_L}{\partial k} < 0$ and $\frac{\partial d_H}{\partial k} < 0$, then $\frac{\partial\gamma}{\partial k} < 0$.

A.10 Proof of Proposition 2

Expression (26) allows us to write:

$$\frac{\partial \gamma}{\partial k} = C_L \frac{\partial d_L}{\partial k} + C_H \frac{\partial d_H}{\partial k}, \tag{27}$$

where

$$\begin{aligned}
C_L &= \frac{hv''(f_{1L}) + p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2L})}{p(\tau\alpha+\beta)^2(\Gamma'(\gamma))^2[v''(f_{2L})+v''(f_{2H})] + h^2[v''(f_{1L})+v''(f_{1H})] + F} \\
C_H &= \frac{hv''(f_{1H}) + p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2H})}{p(\tau\alpha+\beta)^2(\Gamma'(\gamma))^2[v''(f_{2L})+v''(f_{2H})] + h^2[v''(f_{1L})+v''(f_{1H})] + F},
\end{aligned}$$

and where

$$\begin{aligned}
F &= \mathrm{E}_0\left\{u''(c_1)[h+(1-\tau)\alpha\Gamma'(\gamma)-I]^2 + 2u'(c_1)(1-\tau)\alpha\Gamma''(\gamma)\right\} \\
&\quad + p(\tau\alpha+\beta)[v'(f_{2L})+v'(f_{2H})]\Gamma''(\gamma) < 0.
\end{aligned}$$

Hence, $C_L > 0$ and $C_H > 0$.

Further, by differentiating (4) and (5) we have that

$$\frac{\partial d_L}{\partial k} = A_L + B_L \frac{\partial \gamma}{\partial k}, \quad \frac{\partial d_H}{\partial k} = B_H \frac{\partial \gamma}{\partial k}, \tag{28}$$

where

$$\begin{aligned}
A_L &= \tfrac{v'(f_{1L})}{v''(f_{1L})(1-k)+pv''(f_{2L})} < 0, \quad B_L = \tfrac{hv''(f_{1L})(1-k)+p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2L})}{v''(f_{1L})(1-k)+pv''(f_{2L})} > 0, \\
B_H &= \tfrac{hv''(f_{1H})+p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2H})}{v''(f_{1H})+pv''(f_{2H})} > 0.
\end{aligned}$$

Combine (27) and (28) and solve to give:

$$\frac{\partial \gamma}{\partial k} = \frac{C_L A_L}{1 - C_L B_L - C_H B_H}, \quad \frac{\partial d_L}{\partial k} = \frac{A_L(1-C_H B_H)}{1 - C_L B_L - C_H B_H}, \quad \frac{\partial d_H}{\partial k} = \frac{C_L B_H A_L}{1 - C_L B_L - C_H B_H},$$

where all expressions are negative if $C_L B_L + C_H B_H < 1$.

We now show that $C_L B_L + C_H B_H < 1$ for $k \geq 0$ sufficiently close to zero. We bring the terms $C_L B_L$ and $C_H B_H$ both under the common denominator:

$$\begin{aligned}
&\left\{p(\tau\alpha+\beta)^2(\Gamma'(\gamma))^2[v''(f_{2L})+v''(f_{2H})] + h^2[v''(f_{1L})+v''(f_{1H})] + F\right\} * \\
&[v''(f_{1L})(1-k)+pv''(f_{2L})] * [v''(f_{1H})+pv''(f_{2H})]
\end{aligned}$$

The numerator of the term $C_L B_L$ is:

$$\begin{aligned}
& \left[p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2L})+hv''(f_{1L})\right]\left[hv''(f_{1L})(1-k)+p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2L})\right] \\
& *\left[v''(f_{1H})+pv''(f_{2H})\right] \\
= & \left[\begin{array}{c} h^2(1-k)\left[v''(f_{1L})\right]^2+p^2(\tau\alpha+\beta)^2\left[\Gamma'(\gamma)\right]^2\left[v''(f_{2L})\right]^2 \\ +hp(\tau\alpha+\beta)\Gamma'(\gamma)(2-k)v''(f_{1L})v''(f_{2L}) \end{array}\right]\left[v''(f_{1H})+pv''(f_{2H})\right]
\end{aligned}$$

$$\begin{aligned}
= \; & h^2(1-k)\left[v''(f_{1L})\right]^2 v''(f_{1H})+p^2(\tau\alpha+\beta)^2\left[\Gamma'(\gamma)\right]^2\left[v''(f_{2L})\right]^2 v''(f_{1H}) \\
& +hp(\tau\alpha+\beta)\Gamma'(\gamma)(2-k)v''(f_{1L})v''(f_{2L})v''(f_{1H})+ \\
& ph^2(1-k)\left[v''(f_{1L})\right]^2 v''(f_{2H})+p^3(\tau\alpha+\beta)^2\left[\Gamma'(\gamma)\right]^2\left[v''(f_{2L})\right]^2 v''(f_{2H}) \\
& +hp^2(\tau\alpha+\beta)\Gamma'(\gamma)(2-k)v''(f_{1L})v''(f_{2L})v''(f_{2H}).
\end{aligned}$$

The numerator of the term $C_H B_H$ is:

$$\begin{aligned}
& \left[p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2H})+hv''(f_{1H})\right]\left[hv''(f_{1H})+p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2H})\right] \\
& *\left[v''(f_{1L})(1-k)+pv''(f_{2L})\right] \\
= & \left[\begin{array}{c} h^2\left[v''(f_{1H})\right]^2+p^2(\tau\alpha+\beta)^2\left[\Gamma'(\gamma)\right]^2\left[v''(f_{2H})\right]^2 \\ +2hp(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{1H})v''(f_{2H}) \end{array}\right]\left[v''(f_{1L})(1-k)+pv''(f_{2L})\right]
\end{aligned}$$

$$\begin{aligned}
= \; & h^2(1-k)\left[v''(f_{1H})\right]^2 v''(f_{1L})+p^2(\tau\alpha+\beta)^2(1-k)\left[\Gamma'(\gamma)\right]^2\left[v''(f_{2H})\right]^2 v''(f_{1L}) \\
& +2hp(\tau\alpha+\beta)(1-k)\Gamma'(\gamma)v''(f_{1H})v''(f_{2H})v''(f_{1L})+ \\
& ph^2\left[v''(f_{1H})\right]^2 v''(f_{2L})+p^3(\tau\alpha+\beta)^2\left[\Gamma'(\gamma)\right]^2\left[v''(f_{2H})\right]^2 v''(f_{2L}) \\
& +2hp^2(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{1H})v''(f_{2H})v''(f_{2L}).
\end{aligned}$$

The denominator of both terms equals:

$$\begin{aligned}
& \left\{p(\tau\alpha+\beta)^2(\Gamma'(\gamma))^2\left[v''(f_{2L})+v''(f_{2H})\right]+h^2\left[v''(f_{1L})+v''(f_{1H})\right]+F\right\} \\
& *\left[v''(f_{1L})(1-k)+pv''(f_{2L})\right]\left[v''(f_{1H})+pv''(f_{2H})\right] \\
= & \left[\begin{array}{c} h^2(1-k)\left[v''(f_{1L})\right]^2+h^2(1-k)v''(f_{1L})v''(f_{1H})+ \\ h^2pv''(f_{1L})v''(f_{2L})+h^2pv''(f_{1H})v''(f_{2L})+ \\ p(\tau\alpha+\beta)^2(\Gamma'(\gamma))^2(1-k)\left[v''(f_{1L})v''(f_{2L})+v''(f_{1L})v''(f_{2H})\right] \\ +p^2(\tau\alpha+\beta)^2(\Gamma'(\gamma))^2\left[(v''(f_{2L}))^2+v''(f_{2L})v''(f_{2H})\right]+ \\ \left[v''(f_{1L})(1-k)+pv''(f_{2L})\right]F \end{array}\right]\left[v''(f_{1H})+pv''(f_{2H})\right]
\end{aligned}$$

$$
\begin{aligned}
= \; & h^2(1-k)\left[v''(f_{1L})\right]^2 v''(f_{1H}) + h^2(1-k)v''(f_{1L})\left[v''(f_{1H})\right]^2 + \\
& h^2 p v''(f_{1L})v''(f_{2L})v''(f_{1H}) + h^2 p\left[v''(f_{1H})\right]^2 v''(f_{2L}) + \\
& p(\tau\alpha+\beta)^2(\Gamma'(\gamma))^2(1-k)\left[v''(f_{1L})v''(f_{2L}) + v''(f_{1L})v''(f_{2H})\right]v''(f_{1H}) + \\
& p^2(\tau\alpha+\beta)^2(\Gamma'(\gamma))^2\left[(v''(f_{2L}))^2 + v''(f_{2L})v''(f_{2H})\right]v''(f_{1H}) + \\
& ph^2(1-k)\left[v''(f_{1L})\right]^2 v''(f_{2H}) + ph^2(1-k)v''(f_{1L})v''(f_{1H})v''(f_{2H}) + \\
& h^2p^2v''(f_{1L})v''(f_{2L})v''(f_{2H}) + h^2p^2v''(f_{1H})v''(f_{2L})v''(f_{2H}) + \\
& p^2(\tau\alpha+\beta)^2(\Gamma'(\gamma))^2(1-k)\left[v''(f_{1L})v''(f_{2L}) + v''(f_{1L})v''(f_{2H})\right]v''(f_{2H}) + \\
& p^3(\tau\alpha+\beta)^2(\Gamma'(\gamma))^2\left[(v''(f_{2L}))^2 + v''(f_{2L})v''(f_{2H})\right]v''(f_{2H}) + \\
& \left[v''(f_{1L})(1-k) + pv''(f_{2L})\right]\left[v''(f_{1H}) + pv''(f_{2H})\right]F.
\end{aligned}
$$

Hence, we need to show that (cancelling common terms):

$$
\begin{aligned}
= \; & h^2 p v''(f_{1L})v''(f_{2L})v''(f_{1H}) + ph^2(1-k)v''(f_{1L})v''(f_{1H})v''(f_{2H}) + \\
& h^2p^2v''(f_{1L})v''(f_{2L})v''(f_{2H}) + h^2p^2v''(f_{1H})v''(f_{2L})v''(f_{2H}) + \\
& \left[v''(f_{1L})(1-k) + pv''(f_{2L})\right]\left[v''(f_{1H}) + pv''(f_{2H})\right]F + \\
& p(\tau\alpha+\beta)^2(\Gamma'(\gamma))^2(1-k)\left[v''(f_{1L})v''(f_{2L}) + v''(f_{1L})v''(f_{2H})\right]v''(f_{1H}) + \\
& p^2(\tau\alpha+\beta)^2(\Gamma'(\gamma))^2 v''(f_{2L})v''(f_{2H})v''(f_{1H}) + \\
& p^2(\tau\alpha+\beta)^2(\Gamma'(\gamma))^2(1-k)v''(f_{1L})v''(f_{2L})v''(f_{2H}) + \\
& < \\
& hp(\tau\alpha+\beta)\Gamma'(\gamma)(2-k)v''(f_{1L})v''(f_{2L})v''(f_{1H}) + \\
& +hp^2(\tau\alpha+\beta)\Gamma'(\gamma)(2-k)v''(f_{1L})v''(f_{2L})v''(f_{2H}) + \\
& +2hp(\tau\alpha+\beta)(1-k)\Gamma'(\gamma)v''(f_{1H})v''(f_{2H})v''(f_{1L}) + \\
& +2hp^2(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{1H})v''(f_{2H})v''(f_{2L}).
\end{aligned}
$$

We need to show that this holds for k not too large. Hence, evaluating this inequality at $k = 0$ we have:

$$
\begin{aligned}
& \left[v''(f_{1L}) + pv''(f_{2L})\right]\left[v''(f_{1H}) + pv''(f_{2H})\right]F + \\
& h^2pv''(f_{1L})v''(f_{2L})v''(f_{1H}) + ph^2v''(f_{1L})v''(f_{1H})v''(f_{2H}) + \\
& h^2p^2v''(f_{1L})v''(f_{2L})v''(f_{2H}) + h^2p^2v''(f_{1H})v''(f_{2L})v''(f_{2H}) + \\
& p(\tau\alpha+\beta)^2(\Gamma'(\gamma))^2 v''(f_{1L})v''(f_{1H})\left[v''(f_{2L}) + v''(f_{2H})\right] + \\
& p^2(\tau\alpha+\beta)^2(\Gamma'(\gamma))^2 v''(f_{2L})v''(f_{2H})\left[v''(f_{1L}) + v''(f_{1H})\right] + \\
< \; & \\
& 2hp(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{1L})v''(f_{1H})\left[v''(f_{2L}) + v''(f_{2H})\right] + \\
& 2hp^2(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2L})v''(f_{2H})\left[v''(f_{1L}) + v''(f_{1H})\right],
\end{aligned}
$$

or

$$\begin{aligned}
& [v''(f_{1L}) + pv''(f_{2L})][v''(f_{1H}) + pv''(f_{2H})] F + \\
& p[h - (\tau\alpha + \beta)(\Gamma'(\gamma))]^2 v''(f_{1L}) v''(f_{1H}) v''(f_{2L}) + \\
& p[h - (\tau\alpha + \beta)(\Gamma'(\gamma))]^2 v''(f_{1L}) v''(f_{1H}) v''(f_{2H}) + \\
& p^2[h - (\tau\alpha + \beta)(\Gamma'(\gamma))]^2 v''(f_{1L}) v''(f_{2L}) v''(f_{2H}) + \\
& p^2[h - (\tau\alpha + \beta)(\Gamma'(\gamma))]^2 v''(f_{1H}) v''(f_{2L}) v''(f_{2H}) + \\
< \; & 0,
\end{aligned}$$

which holds, because all the terms on the left-hand side are negative. Hence, by continuity, also for k positive, but close enough to zero, $C_L B_L + C_H B_H < 1$.

A.11 Derivation and further evaluation of (14)

The derivative of (1) (realizing $c_1 = c_2$ is given by (3)) with respect to k is:

$$\begin{aligned}
& \mathrm{E}_0 \{u'(c_1)[h + (1-\tau)\alpha\Gamma'(\gamma) - I]\} \tfrac{\partial\gamma}{\partial k} \\
& + \tfrac{1}{2} v'(f_{1L}) \left[\tfrac{\partial d_L}{\partial k} - h\tfrac{\partial\gamma}{\partial k}\right] + \tfrac{1}{2} v'(f_{1H}) \left[\tfrac{\partial d_H}{\partial k} - h\tfrac{\partial\gamma}{\partial k}\right] \\
& + \tfrac{1}{2} v'(f_{2L}) \left[(\tau\alpha + \beta)\Gamma'(\gamma)\tfrac{\partial\gamma}{\partial k} - \tfrac{\partial d_L}{\partial k}\right] + \tfrac{1}{2} v'(f_{2H}) \left[(\tau\alpha + \beta)\Gamma'(\gamma)\tfrac{\partial\gamma}{\partial k} - \tfrac{\partial d_H}{\partial k}\right],
\end{aligned}$$

which, by (17) with $k = 0$ and (18), equals:

$$\begin{aligned}
& \mathrm{E}_0 \{u'(c_1)[h + (1-\tau)\alpha\Gamma'(\gamma) - I]\} \tfrac{\partial\gamma}{\partial k} \\
& + \tfrac{1}{2} v'(f_{1L}) \left[\left(\tfrac{p-1}{p}\right)\tfrac{\partial d_L}{\partial k} + \left[\tfrac{1}{p}(\tau\alpha + \beta)\Gamma'(\gamma) - h\right]\tfrac{\partial\gamma}{\partial k}\right] \\
& + \tfrac{1}{2} v'(f_{1H}) \left[\left(\tfrac{p-1}{p}\right)\tfrac{\partial d_H}{\partial k} + \left[\tfrac{1}{p}(\tau\alpha + \beta)\Gamma'(\gamma) - h\right]\tfrac{\partial\gamma}{\partial k}\right].
\end{aligned}$$

Furthermore, by (25) evaluated at $k = 0$, we have that

$$\begin{aligned}
& \mathrm{E}_0 \{u'(c_1)[h + (1-\tau)\alpha\Gamma'(\gamma) - I]\} \tfrac{\partial\gamma}{\partial k} \\
= \; & \tfrac{1}{2}[h - (\tau\alpha + \beta)\Gamma'(\gamma)][v'(f_{1L}) + v'(f_{1H})]\tfrac{\partial\gamma}{\partial k}.
\end{aligned}$$

Substituting this into the previous expression, we arrive at:

$$\begin{aligned}
& \tfrac{1}{2}[h - (\tau\alpha + \beta)\Gamma'(\gamma)][v'(f_{1L}) + v'(f_{1H})]\tfrac{\partial\gamma}{\partial k} \\
& + \tfrac{1}{2} v'(f_{1L}) \left[\left(\tfrac{p-1}{p}\right)\tfrac{\partial d_L}{\partial k} + \left[\tfrac{1}{p}(\tau\alpha + \beta)\Gamma'(\gamma) - h\right]\tfrac{\partial\gamma}{\partial k}\right] \\
& + \tfrac{1}{2} v'(f_{1H}) \left[\left(\tfrac{p-1}{p}\right)\tfrac{\partial d_H}{\partial k} + \left[\tfrac{1}{p}(\tau\alpha + \beta)\Gamma'(\gamma) - h\right]\tfrac{\partial\gamma}{\partial k}\right] \\
= \; & \tfrac{1}{2}\left(\tfrac{1-p}{p}\right)\left[\begin{array}{c} v'(f_{1L})\left[(\tau\alpha + \beta)\Gamma'(\gamma)\tfrac{\partial\gamma}{\partial k} - \tfrac{\partial d_L}{\partial k}\right] + \\ v'(f_{1H})\left[(\tau\alpha + \beta)\Gamma'(\gamma)\tfrac{\partial\gamma}{\partial k} - \tfrac{\partial d_H}{\partial k}\right] \end{array}\right] \\
= \; & \tfrac{1}{2}\left(\tfrac{1-p}{p}\right)\left[(\tau\alpha + \beta)\Gamma'(\gamma)[v'(f_{1L}) + v'(f_{1H})]\tfrac{\partial\gamma}{\partial k} - v'(f_{1L})\tfrac{\partial d_L}{\partial k} - v'(f_{1H})\tfrac{\partial d_H}{\partial k}\right],
\end{aligned}$$

which equals (14), which equals

$$\begin{array}{l}\frac{1}{2}\left(\frac{1-p}{p}\right)\dfrac{A_L}{1-C_LB_L-C_HB_H}*\\ \left[(\tau\alpha+\beta)\Gamma'(\gamma)\left[v'(f_{1L})+v'(f_{1H})\right]C_L-v'(f_{1L})(1-C_HB_H)-v'(f_{1H})C_LB_H\right]\end{array}$$

The first line of this expression is negative. Hence, the marginal welfare effect of introducing a pact is positive if the second line of this expression is also negative. Let us work out one-by-one the terms in the final line of this expression:

$$\begin{array}{l}(\tau\alpha+\beta)\Gamma'(\gamma)\left[v'(f_{1L})+v'(f_{1H})\right]C_L=\\ \frac{(\tau\alpha+\beta)\Gamma'(\gamma)[v'(f_{1L})+v'(f_{1H})][p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2L})+hv''(f_{1L})]}{p(\tau\alpha+\beta)^2(\Gamma'(\gamma))^2[v''(f_{2L})+v''(f_{2H})]+h^2[v''(f_{1L})+v''(f_{1H})]+F}\frac{v''(f_{1H})+pv''(f_{2H})}{v''(f_{1H})+pv''(f_{2H})}\end{array} \tag{29}$$

Let us work out the numerator of this term:

$$\begin{array}{rl} & (\tau\alpha+\beta)\Gamma'\left[v'(f_{1L})+v'(f_{1H})\right]\left[p(\tau\alpha+\beta)\Gamma'v''(f_{2L})+hv''(f_{1L})\right]\\ & *\left[v''(f_{1H})+pv''(f_{2H})\right]\\ = & (\tau\alpha+\beta)\Gamma'\left[\begin{array}{c}p(\tau\alpha+\beta)\Gamma'v'(f_{1L})v''(f_{2L})+hv'(f_{1L})v''(f_{1L})+\\ p(\tau\alpha+\beta)\Gamma'v'(f_{1H})v''(f_{2L})+hv'(f_{1H})v''(f_{1L})\end{array}\right]\\ & *\left[v''(f_{1H})+pv''(f_{2H})\right]\\ = & (\tau\alpha+\beta)\Gamma'*\\ & \left[\begin{array}{c}p(\tau\alpha+\beta)\Gamma'v'(f_{1L})v''(f_{2L})v''(f_{1H})+hv'(f_{1L})v''(f_{1L})v''(f_{1H})+\\ p(\tau\alpha+\beta)\Gamma'v'(f_{1H})v''(f_{2L})v''(f_{1H})+hv'(f_{1H})v''(f_{1L})v''(f_{1H})+\\ p^2(\tau\alpha+\beta)\Gamma'v'(f_{1L})v''(f_{2L})v''(f_{2H})+hpv'(f_{1L})v''(f_{1L})v''(f_{2H})+\\ p^2(\tau\alpha+\beta)\Gamma'v'(f_{1H})v''(f_{2L})v''(f_{2H})+hpv'(f_{1H})v''(f_{1L})v''(f_{2H})\end{array}\right]\end{array}$$

Now, turn to the next term:

$$\begin{array}{l}-v'(f_{1L})(1-C_HB_H)=\\ \dfrac{\begin{array}{c}-v'(f_{1L})\left\{p(\tau\alpha+\beta)^2(\Gamma')^2\left[v''(f_{2L})+v''(f_{2H})\right]+h^2\left[v''(f_{1L})+v''(f_{1H})\right]+F\right\}*\\ \left[v''(f_{1H})+pv''(f_{2H})\right]+\left[p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2H})+hv''(f_{1H})\right]^2v'(f_{1L})\end{array}}{\left\{p(\tau\alpha+\beta)^2(\Gamma')^2\left[v''(f_{2L})+v''(f_{2H})\right]+h^2\left[v''(f_{1L})+v''(f_{1H})\right]+F\right\}\left[v''(f_{1H})+pv''(f_{2H})\right]}\end{array} \tag{30}$$

Work out the numerator of this term to give:

$$\begin{array}{l}-p(\tau\alpha+\beta)^2(\Gamma')^2v'(f_{1L})v''(f_{2L})v''(f_{1H})-p(\tau\alpha+\beta)^2(\Gamma')^2v'(f_{1L})v''(f_{1H})v''(f_{2H})\\ -h^2v'(f_{1L})v''(f_{1L})v''(f_{1H})-h^2v'(f_{1L})\left[v''(f_{1H})\right]^2-Fv'(f_{1L})v''(f_{1H})\\ -p^2(\tau\alpha+\beta)^2(\Gamma')^2v'(f_{1L})v''(f_{2L})v''(f_{2H})-p^2(\tau\alpha+\beta)^2(\Gamma')^2v'(f_{1L})\left[v''(f_{2H})\right]^2\\ -ph^2v'(f_{1L})v''(f_{1L})v''(f_{2H})-ph^2v'(f_{1L})v''(f_{1H})v''(f_{2H})-pFv'(f_{1L})v''(f_{2H})\\ +\left[p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2H})+hv''(f_{1H})\right]^2v'(f_{1L})\end{array}$$

Now, turn to the final term:

$$-v'(f_{1H})C_LB_H = \frac{-v'(f_{1H})\left[p\left(\tau\alpha+\beta\right)\Gamma'v''\left(f_{2L}\right)+hv''\left(f_{1L}\right)\right]\left[p\left(\tau\alpha+\beta\right)\Gamma'v''\left(f_{2H}\right)+hv''\left(f_{1H}\right)\right]}{\left\{p\left(\tau\alpha+\beta\right)^2\left(\Gamma'\right)^2\left[v''\left(f_{2L}\right)+v''\left(f_{2H}\right)\right]+h^2\left[v''\left(f_{1L}\right)+v''\left(f_{1H}\right)\right]+F\right\}\left[v''\left(f_{1H}\right)+pv''\left(f_{2H}\right)\right]} \tag{31}$$

Work out the numerator of this term:

$$\begin{aligned}
&-\left[p\left(\tau\alpha+\beta\right)\Gamma'\right]^2 v'(f_{1H})v''\left(f_{2L}\right)v''\left(f_{2H}\right)-hp\left(\tau\alpha+\beta\right)\Gamma'v'(f_{1H})v''\left(f_{1L}\right)v''\left(f_{2H}\right)\\
&-hp\left(\tau\alpha+\beta\right)\Gamma'v'(f_{1H})v''\left(f_{2L}\right)v''\left(f_{1H}\right)-h^2v'(f_{1H})v''\left(f_{1L}\right)v''\left(f_{1H}\right)
\end{aligned}$$

Hence, summing the numerators we have:

$$\begin{aligned}
&p\left[\left(\tau\alpha+\beta\right)\Gamma'\right]^2 v'(f_{1L})v''\left(f_{2L}\right)v''\left(f_{1H}\right)+h\left(\tau\alpha+\beta\right)\Gamma'v'(f_{1L})v''\left(f_{1L}\right)v''\left(f_{1H}\right)+\\
&p\left[\left(\tau\alpha+\beta\right)\Gamma'\right]^2 v'(f_{1H})v''\left(f_{2L}\right)v''\left(f_{1H}\right)+h\left(\tau\alpha+\beta\right)\Gamma'v'(f_{1H})v''\left(f_{1L}\right)v''\left(f_{1H}\right)+\\
&p^2\left[\left(\tau\alpha+\beta\right)\Gamma'\right]^2 v'(f_{1L})v''\left(f_{2L}\right)v''\left(f_{2H}\right)+hp\left(\tau\alpha+\beta\right)\Gamma'v'(f_{1L})v''\left(f_{1L}\right)v''\left(f_{2H}\right)+\\
&p^2\left[\left(\tau\alpha+\beta\right)\Gamma'\right]^2 v'(f_{1H})v''\left(f_{2L}\right)v''\left(f_{2H}\right)+hp\left(\tau\alpha+\beta\right)\Gamma'v'(f_{1H})v''\left(f_{1L}\right)v''\left(f_{2H}\right)\\
&-p\left(\tau\alpha+\beta\right)^2\left(\Gamma'\right)^2 v'(f_{1L})v''\left(f_{2L}\right)v''\left(f_{1H}\right)-p\left(\tau\alpha+\beta\right)^2\left(\Gamma'\right)^2 v'(f_{1L})v''\left(f_{1H}\right)v''\left(f_{2H}\right)\\
&-h^2v'(f_{1L})v''\left(f_{1L}\right)v''\left(f_{1H}\right)-h^2v'(f_{1L})\left[v''\left(f_{1H}\right)\right]^2-Fv'(f_{1L})v''\left(f_{1H}\right)\\
&-p^2\left(\tau\alpha+\beta\right)^2\left(\Gamma'\right)^2 v'(f_{1L})v''\left(f_{2L}\right)v''\left(f_{2H}\right)-p^2\left(\tau\alpha+\beta\right)^2\left(\Gamma'\right)^2 v'(f_{1L})\left[v''\left(f_{2H}\right)\right]^2\\
&-ph^2v'(f_{1L})v''\left(f_{1L}\right)v''\left(f_{2H}\right)-ph^2v'(f_{1L})v''\left(f_{1H}\right)v''\left(f_{2H}\right)-pFv'(f_{1L})v''\left(f_{2H}\right)\\
&+\left[p\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right)v''\left(f_{2H}\right)+hv''\left(f_{1H}\right)\right]^2 v'(f_{1L})\\
&-\left[p\left(\tau\alpha+\beta\right)\Gamma'\right]^2 v'(f_{1H})v''\left(f_{2L}\right)v''\left(f_{2H}\right)-hp\left(\tau\alpha+\beta\right)\Gamma'v'(f_{1H})v''\left(f_{1L}\right)v''\left(f_{2H}\right)\\
&-hp\left(\tau\alpha+\beta\right)\Gamma'v'(f_{1H})v''\left(f_{2L}\right)v''\left(f_{1H}\right)-h^2v'(f_{1H})v''\left(f_{1L}\right)v''\left(f_{1H}\right)
\end{aligned}$$

$$\begin{aligned}
= \;& h\left[\left(\tau\alpha+\beta\right)\Gamma'-h\right]v'(f_{1L})v''\left(f_{1L}\right)v''\left(f_{1H}\right)\\
&+p\left[\left(\tau\alpha+\beta\right)\Gamma'\right]\left[\left(\tau\alpha+\beta\right)\Gamma'-h\right]v'(f_{1H})v''\left(f_{2L}\right)v''\left(f_{1H}\right)\\
&+h\left[\left(\tau\alpha+\beta\right)\Gamma'-h\right]v'(f_{1H})v''\left(f_{1L}\right)v''\left(f_{1H}\right)\\
&+hp\left[\left(\tau\alpha+\beta\right)\Gamma'-h\right]v'(f_{1L})v''\left(f_{1L}\right)v''\left(f_{2H}\right)\\
&+\left[2hp\left(\tau\alpha+\beta\right)\Gamma'-p\left(\tau\alpha+\beta\right)^2\left(\Gamma'\right)^2-ph^2\right]v'(f_{1L})v''\left(f_{1H}\right)v''\left(f_{2H}\right)\\
&-Fv'(f_{1L})\left[v''\left(f_{1H}\right)+pv''\left(f_{2H}\right)\right]
\end{aligned}$$

$$
\begin{aligned}
= \; & h\left[(\tau\alpha+\beta)\Gamma'-h\right]v'(f_{1L})v''(f_{1L})v''(f_{1H}) \\
& +p\left[(\tau\alpha+\beta)\Gamma'\right]\left[(\tau\alpha+\beta)\Gamma'-h\right]v'(f_{1H})v''(f_{2L})v''(f_{1H}) \\
& +h\left[(\tau\alpha+\beta)\Gamma'-h\right]v'(f_{1H})v''(f_{1L})v''(f_{1H}) \\
& +hp\left[(\tau\alpha+\beta)\Gamma'-h\right]v'(f_{1L})v''(f_{1L})v''(f_{2H}) \\
& -p\left[(\tau\alpha+\beta)\Gamma'-h\right]^2 v'(f_{1L})v''(f_{1H})v''(f_{2H}) \\
& -Fv'(f_{1L})\left[v''(f_{1H})+pv''(f_{2H})\right] \qquad (32)
\end{aligned}
$$

Because the (common) denominator in (29), (30) and (31) is positive, the marginal welfare effect of the introduction of a pact is positive if (32) is negative.

A.12 Computations for the smart stability pact

A.12.1 Proof that solution of (16) exceeds that of (6)

Assume that $k>0$ is not too large. We show that if an interior solution for γ exists for (6) and (16), then the solution of the latter exceeds the solution of the former when all parameters other than δ are held constant.

The left-hand sides of (6) and (16) are both decreasing and equal for given γ. The right-hand sides of (6) and (16) are both increasing in γ. Then, because for given γ the right-hand side of (16) is smaller than that of (6), reform under a stability pact $\left(k,\bar{d}(\gamma)\right)$ is higher than under a pact $\left(k,\bar{d}\right)$ with a fixed reference deficit level.

A.12.2 Effect of k on γ

Differentiate (16) with respect to k, to give:

$$
\begin{aligned}
& \mathrm{E}_0\left\{\tfrac{1}{2}u''(c_1)\left[h+(1-\tau)\alpha\Gamma'(\gamma)-I\right]^2+u'(c_1)(1-\tau)\alpha\Gamma''(\gamma)\right\}\tfrac{\partial\gamma}{\partial k} \\
& -\tfrac{1}{2}hv''(f_{1L})\left[-h\tfrac{\partial\gamma}{\partial k}+\tfrac{\partial d_L}{\partial k}\right]-\tfrac{1}{2}hv''(f_{1H})\left[-h\tfrac{\partial\gamma}{\partial k}+\tfrac{\partial d_H}{\partial k}\right] \\
& +\tfrac{1}{2}p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2L})\left[(\tau\alpha+\beta)\Gamma'(\gamma)\tfrac{\partial\gamma}{\partial k}-\tfrac{\partial d_L}{\partial k}\right] \\
& +\tfrac{1}{2}p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2H})\left[(\tau\alpha+\beta)\Gamma'(\gamma)\tfrac{\partial\gamma}{\partial k}-\tfrac{\partial d_H}{\partial k}\right] \\
& +\tfrac{1}{2}p(\tau\alpha+\beta)\left[v'(f_{2L})+v'(f_{2H})\right]\Gamma''(\gamma)\tfrac{\partial\gamma}{\partial k} \\
& +\tfrac{1}{2}\delta v'(f_{1L})+\tfrac{1}{2}\delta kv''(f_{1L})\left[-h\tfrac{\partial\gamma}{\partial k}+\tfrac{\partial d_L}{\partial k}\right]=0.
\end{aligned}
$$

When evaluated at $k=0$, this is equivalent to

$$
\begin{aligned}
& \left[\begin{array}{c}
\mathrm{E}_0\left\{\tfrac{1}{2}u''(c_1)\left[h+(1-\tau)\alpha\Gamma'(\gamma)-I\right]^2+u'(c_1)(1-\tau)\alpha\Gamma''(\gamma)\right\}+ \\
\tfrac{1}{2}p(\tau\alpha+\beta)^2\left(\Gamma'(\gamma)\right)^2\left[v''(f_{2L})+v''(f_{2H})\right]+\tfrac{1}{2}h^2\left[v''(f_{1L})+v''(f_{1H})\right] \\
+\tfrac{1}{2}p(\tau\alpha+\beta)\left[v'(f_{2L})+v'(f_{2H})\right]\Gamma''(\gamma)
\end{array}\right]\tfrac{\partial\gamma}{\partial k} \\
= \; & \tfrac{1}{2}\left[hv''(f_{1L})+p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2L})\right]\tfrac{\partial d_L}{\partial k}+ \\
& \tfrac{1}{2}\left[hv''(f_{1H})+p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2H})\right]\tfrac{\partial d_H}{\partial k}-\tfrac{1}{2}\delta v'(f_{1L}),
\end{aligned}
$$

Hence, we can write $\frac{\partial \gamma}{\partial k} = C_L \frac{\partial d_L}{\partial k} + C_H \frac{\partial d_H}{\partial k} + D$, where $C > 0$ and $D > 0$. Hence, the presence of the final term $-\frac{1}{2}\delta v'(f_{1L})$ in (16) implies an increase in $\frac{\partial \gamma}{\partial k}$. As before, $\frac{\partial d_L}{\partial k} = A_L + B_L \frac{\partial \gamma}{\partial k}$ and $\frac{\partial d_H}{\partial k} = B_H \frac{\partial \gamma}{\partial k}$.

A.12.3 The welfare effect of introducing a smart pact

Again, we differentiate (1) with respect to k, but now assuming (15), and evaluate the derivative at $k = 0$. Because the equilibrium is symmetric across countries (both in choices of γ and d), we again arrive at (14), which is simplified further as:

$$\tfrac{1}{2}\tfrac{1-p}{p}\left[(\tau\alpha+\beta)\,\Gamma'(\gamma)\,[v'(f_{1L})+v'(f_{1H})]\,\tfrac{\partial \gamma}{\partial k} - v'(f_{1L})\tfrac{\partial d_L}{\partial k} - v'(f_{1H})\tfrac{\partial d_H}{\partial k}\right].$$

Above, we showed that $\frac{\partial \gamma}{\partial k}_{|\delta>0} = \frac{\partial \gamma}{\partial k}_{|\delta=0} + D$, where $D > 0$ and linear in δ. The solutions for $\frac{\partial \gamma}{\partial k}$, $\frac{\partial d_L}{\partial k}$ and $\frac{\partial d_H}{\partial k}$ are now given by:

$$\frac{\partial \gamma}{\partial k} = \frac{C_L A_L + D}{1 - C_L B_L - C_H B_H}, \qquad \frac{\partial d_L}{\partial k} = \frac{A_L(1 - C_H B_H) + D B_L}{1 - C_L B_L - C_H B_H}, \qquad \frac{\partial d_H}{\partial k} = \frac{C_L B_H A_L + D B_H}{1 - C_L B_L - C_H B_H},$$

Hence, the marginal social welfare effect of introducing a pact equals the original marginal social welfare effect of introducing a pact (note that with $k = 0$ the evaluation takes place at the original solution for γ) *plus* a term

$$\tfrac{1}{2}\tfrac{1-p}{p}\tfrac{D}{1-C_L B_L - C_H B_H}\left[(v'(f_{1L})+v'(f_{1H}))\,(\tau\alpha+\beta)\,\Gamma'(\gamma) - v'(f_{1L})B_L - v'(f_{1H})B_H\right].$$

The term preceding the term in square brackets is positive. We can work out the term in square brackets (evaluated at $k = 0$) as:

$$\frac{\begin{array}{c}[v'(f_{1L})+v'(f_{1H})]\,(\tau\alpha+\beta)\,\Gamma'\,[v''(f_{1L})+pv''(f_{2L})]\,[v''(f_{1H})+pv''(f_{2H})] \\ -\,[hv''(f_{1L})+p\,(\tau\alpha+\beta)\,\Gamma' v''(f_{2L})]\,[v''(f_{1H})+pv''(f_{2H})]\,v'(f_{1L}) \\ -\,[hv''(f_{1H})+p\,(\tau\alpha+\beta)\,\Gamma' v''(f_{2H})]\,[v''(f_{1L})+pv''(f_{2L})]\,v'(f_{1H})\end{array}}{[v''(f_{1L})+pv''(f_{2L})][v''(f_{1H})+pv''(f_{2H})]},$$

$$= \frac{\begin{array}{c}(\tau\alpha+\beta)\,\Gamma'\,[v''(f_{1L})+pv''(f_{2L})]\,[v''(f_{1H})+pv''(f_{2H})]\,v'(f_{1L}) \\ -\,[hv''(f_{1L})+p\,(\tau\alpha+\beta)\,\Gamma' v''(f_{2L})]\,[v''(f_{1H})+pv''(f_{2H})]\,v'(f_{1L}) \\ +\,(\tau\alpha+\beta)\,\Gamma'\,[v''(f_{1H})+pv''(f_{2H})]\,[v''(f_{1L})+pv''(f_{2L})]\,v'(f_{1H}) \\ -\,[hv''(f_{1H})+p\,(\tau\alpha+\beta)\,\Gamma' v''(f_{2H})]\,[v''(f_{1L})+pv''(f_{2L})]\,v'(f_{1H})\end{array}}{[v''(f_{1L})+pv''(f_{2L})][v''(f_{1H})+pv''(f_{2H})]},$$

$$= \frac{\begin{array}{c}[(\tau\alpha+\beta)\,\Gamma' - h]\,v''(f_{1L})\,[v''(f_{1H})+pv''(f_{2H})]\,v'(f_{1L}) \\ +\,[(\tau\alpha+\beta)\,\Gamma' - h]\,v''(f_{1H})\,[v''(f_{1L})+pv''(f_{2L})]\,v'(f_{1H})\end{array}}{[v''(f_{1L})+pv''(f_{2L})][v''(f_{1H})+pv''(f_{2H})]} > 0,$$

Hence, the condition for the introduction of a stability pact to be social welfare improving is more easily fulfilled if the pact is made smart by setting $\delta > 0$. In fact, because D is linear, it is possible to pick δ large enough that the introduction of a stability pact can always be made social welfare improving.[3]

B (Numerical) examples

In this appendix we provide a (numerical) examples to show that well-defined solutions exist and that a stability pact may be welfare enhancing or welfare deteriorating, depending on the parameter values. We start with an example where $\Gamma(\gamma)$ is linear. Then, we turn to the case where $\Gamma(\gamma)$ is nonlinear.

B.1 Linear-quadratic example

The example is based on the following functional forms:

$$\begin{aligned} u(x) &= v(x) = -(\xi-1)\,x^2/2 + \xi x,\ \xi > 1, && (33)\\ \Gamma(\gamma) &= \gamma. && (34) \end{aligned}$$

Hence, $\Gamma' = 1$. Given that the first-order derivative is $-(\xi-1)\,x + \xi$, this requires that $x < \xi/(\xi-1)$, so that marginal utilities are always positive. Hence, in terms of our model, this requires that

$$\begin{aligned} -(\xi-1)(\tau y_1 - h\gamma + d) + \xi &> 0, \quad y_1 = y_L, y_H, && (35)\\ -(\xi-1)[\tau y_2 + (\tau\alpha+\beta)\gamma - d] + \xi &> 0. && (36) \end{aligned}$$

Further, define ε such that $I = h + (1-\tau)\,\alpha\Gamma' + \varepsilon$, where ε can be made arbitrarily small, if necessary.

The government's first-order condition (4) in **Stage 3**, when $y_1 = y_L$, is now written as:

$$[\xi - (\xi-1)(\tau y_L - h\gamma + d)](1-k) = p[\xi - (\xi-1)(\tau y_2 + \tau\alpha\gamma + \beta\gamma - d)] \Leftrightarrow$$

$$d_L = \left(\tfrac{\xi}{\xi-1}\right)\tfrac{1-p-k}{1+p-k} + \tfrac{\tau[py_2-(1-k)y_L]+[p(\tau\alpha+\beta)+(1-k)h]\gamma}{1+p-k}. \tag{37}$$

From (5) we find:

$$d_H = \left(\tfrac{\xi}{\xi-1}\right)\tfrac{1-p}{1+p} + \tfrac{\tau(py_2-y_H)+[p(\tau\alpha+\beta)+h]\gamma}{1+p}. \tag{38}$$

[3] If for $k = 0$, $d > \bar{d}$ when $y_1 = y_L$, then the introduction of a pact does not push the deficit into the range where sanctions are not applied, the reason being that we consider a marginal increase in k which, by continuity of all functions involved, has only a marginal impact on γ and, thus, on the threshold deficit level.

In the following, we denote by $\tilde{d}$ the expected deficit level, $\tilde{d} \equiv (d_L + d_H)/2$, and by $\tilde{y}_1$ the expected first-period income.

The government's first-order condition (6) in **Stage 1** is rewritten as:

$$\begin{aligned} & \left[\xi - \tfrac{1}{2}(\xi-1)\left[(1-\tau)(\tilde{y}_1 + y_2 + \alpha\gamma) + (h-I)\gamma\right]\right]\left[(h-I) + (1-\tau)\alpha\right] \\ = \;& h\left[\xi - (\xi-1)\left(\tau\tilde{y}_1 - h\gamma + \tilde{d}\right)\right] - p(\tau\alpha+\beta)\left[\xi - (\xi-1)\left(\tau y_2 + \tau\alpha\gamma + \beta\gamma - \tilde{d}\right)\right] \Leftrightarrow \end{aligned}$$

$$\begin{aligned} & \xi\left[(1-\tau)\alpha + p(\tau\alpha+\beta) - I\right] - \\ & \tfrac{1}{2}(\xi-1)\left\{\left[(h-I)+(1-\tau)\alpha\right](1-\tau)(\tilde{y}_1+y_2) + \left[(h-I)+(1-\tau)\alpha\right]^2\gamma\right\} \\ = \;& (\xi-1)\left[p(\tau\alpha+\beta)\left(\tau y_2 + \tau\alpha\gamma + \beta\gamma - \tilde{d}\right) - h\left(\tau\tilde{y}_1 - h\gamma + \tilde{d}\right)\right] \Leftrightarrow \end{aligned}$$

$$\begin{aligned} & \xi\left[(1-\tau)\alpha + p(\tau\alpha+\beta) - I\right] - \\ & \tfrac{1}{2}(\xi-1)\left\{\left[(h-I)+(1-\tau)\alpha\right](1-\tau)(\tilde{y}_1+y_2) + 2p(\tau\alpha+\beta)\tau y_2 - 2h\tau\tilde{y}_1\right\} \\ = \;& (\xi-1)\left[\begin{array}{c} \tfrac{1}{2}\left[\left((h-I)+(1-\tau)\alpha\right)^2 + 2p(\tau\alpha+\beta)^2 + 2h^2\right]\gamma \\ -\left[h + p(\tau\alpha+\beta)\right]\tilde{d} \end{array}\right] \Leftrightarrow \end{aligned}$$

$$\begin{aligned} & \xi\left[p(\tau\alpha+\beta) - h - \varepsilon\right] - \\ & \tfrac{1}{2}(\xi-1)\left\{-\varepsilon(1-\tau)(\tilde{y}_1+y_2) + 2p(\tau\alpha+\beta)\tau y_2 - 2h\tau\tilde{y}_1\right\} \\ = \;& (\xi-1)\left[\begin{array}{c} \tfrac{1}{2}\left[\varepsilon^2 + 2p(\tau\alpha+\beta)^2 + 2h^2\right]\gamma \\ -\left[h + p(\tau\alpha+\beta)\right]\tilde{d} \end{array}\right] \end{aligned} \tag{39}$$

Hence,

$$\gamma = 2\frac{\xi}{\xi-1}\frac{p(\tau\alpha+\beta)-(h+\varepsilon)}{\varepsilon^2+2p(\tau\alpha+\beta)^2+2h^2} + \frac{\varepsilon(1-\tau)(\tilde{y}_1+y_2)+2h\tau\tilde{y}_1-2p(\tau\alpha+\beta)\tau y_2+2[h+p(\tau\alpha+\beta)]\tilde{d}}{\varepsilon^2+2p(\tau\alpha+\beta)^2+2h^2}$$

We compute the solution for γ when we evaluate $\tilde{d}$ at $k = 0$. If an internal solution $\gamma > 0$ exists, then, by continuity, there also exists an internal solution $\gamma > 0$ when k is sufficiently small. When $k = 0$, then

$$\tilde{d} = \left(\frac{\xi}{\xi-1}\right)\frac{1-p}{1+p} + \frac{\tau(py_2-\tilde{y}_1)+[p(\tau\alpha+\beta)+h]\gamma}{1+p}.$$

Substitute this into (39), which then becomes:

$$\begin{aligned} & \xi\left[p(\tau\alpha+\beta) - h - \varepsilon\right] - \\ & \tfrac{1}{2}(\xi-1)\left\{-\varepsilon(1-\tau)(\tilde{y}_1+y_2) + 2p(\tau\alpha+\beta)\tau y_2 - 2h\tau\tilde{y}_1\right\} \\ = \;& (\xi-1)\left[\begin{array}{c} \tfrac{1}{2}\left[\varepsilon^2 + 2p(\tau\alpha+\beta)^2 + 2h^2\right]\gamma \\ -\left[h + p(\tau\alpha+\beta)\right]\left[\left(\frac{\xi}{\xi-1}\right)\left(\frac{1-p}{1+p}\right) + \frac{\tau(py_2-\tilde{y}_1)+[p(\tau\alpha+\beta)+h]\gamma}{1+p}\right] \end{array}\right] \Leftrightarrow \end{aligned}$$

$$
\begin{aligned}
& \xi\left[p(\tau\alpha+\beta)-h+(h+p(\tau\alpha+\beta))\left(\tfrac{1-p}{1+p}\right)-\varepsilon\right]+ \\
& \tfrac{1}{2}(\xi-1)\left\{2\tfrac{[h+p(\tau\alpha+\beta)]\tau(py_2-\tilde{y}_1)}{1+p}+\varepsilon(1-\tau)(\tilde{y}_1+y_2)-2p(\tau\alpha+\beta)\tau y_2+2h\tau\tilde{y}_1\right\} \\
= \; & \tfrac{1}{2}(\xi-1)\left[\varepsilon^2+2p(\tau\alpha+\beta)^2+2h^2-2\tfrac{[p(\tau\alpha+\beta)+h]^2}{1+p}\right]\gamma \Leftrightarrow
\end{aligned}
$$

$$
\begin{aligned}
& \xi\left[\tfrac{2p}{1+p}(\tau\alpha+\beta-h)-\varepsilon\right]+ \\
& \tfrac{1}{2}(\xi-1)\left\{2\tfrac{[h+p(\tau\alpha+\beta)]\tau(py_2-\tilde{y}_1)}{1+p}+\varepsilon(1-\tau)(\tilde{y}_1+y_2)-2p(\tau\alpha+\beta)\tau y_2+2h\tau\tilde{y}_1\right\} \\
= \; & \tfrac{1}{2}(\xi-1)\left[\varepsilon^2+\tfrac{2p}{1+p}(\tau\alpha+\beta-h)^2\right]\gamma.
\end{aligned}
$$

Some rewriting yields the following solution (for the case of $k=0$)

$$
\gamma=2\frac{2p(\tau\alpha+\beta-h)\left[\xi-\frac{1}{2}(\xi-1)\tau(\tilde{y}_1+y_2)\right]-\varepsilon(1+p)\left[\xi-\frac{1}{2}(\xi-1)(1-\tau)(\tilde{y}_1+y_2)\right]}{(\xi-1)\left[(1+p)\varepsilon^2+2p(\tau\alpha+\beta-h)^2\right]},
$$

which is positive if ε is not too large and $\frac{1}{2}\tau(\tilde{y}_1+y_2)<\xi/(\xi-1)$. This assumption seems reasonable, given that the marginal utilities of consumption and public spending become zero when they equal $\xi/(\xi-1)$.

Let us now check whether the introduction of a pact is welfare enhancing. For convenience, set $h=1$. Hence, (32) becomes:

$$
\begin{aligned}
& [(\tau\alpha+\beta)-1]v'(f_{1L})(\xi-1)^2+p(\tau\alpha+\beta)[(\tau\alpha+\beta)-1]v'(f_{1H})(\xi-1)^2 \\
& +[(\tau\alpha+\beta)-1]v'(f_{1H})(\xi-1)^2+p[(\tau\alpha+\beta)-1]v'(f_{1L})(\xi-1)^2 \\
& -p[(\tau\alpha+\beta)-1]^2v'(f_{1L})(\xi-1)^2+(1+p)Fv'(f_{1L})(\xi-1)
\end{aligned}
$$

The sign of this expression is equivalent to that of:

$$
\begin{aligned}
& \left[1+2p-p(\tau\alpha+\beta)-\frac{1+p}{(\tau\alpha+\beta)-1}\varepsilon^2\right]v'(f_{1L})+ \\
& [1+p(\tau\alpha+\beta)]v'(f_{1H})
\end{aligned}
$$

For $(\tau\alpha+\beta)\Gamma'=(\tau\alpha+\beta)$ sufficiently close to 1 and ε large enough, this expression is negative, in which case the introduction of a pact is welfare improving.

A proper solution also requires that the selected level of structural reform is positive and that the marginal utilities of the government under low and high income are positive. For example, setting $y_L=0.75$, $y_H=1.25$, $y_2=1$, $\tau=0.5$, $h=1$, $\alpha=\beta=0.75$, $p=0.5$, $\varepsilon=0.02$ and $\xi=3$, yields the outcomes $d_L=12.61$, $d_H=12.45$ and $\gamma=11.71$. A check on the marginal utilities from public and private consumption showed that these were all positive. Further, the conditions $(\tau\alpha+\beta)\Gamma'(\gamma)>h$ and $I>h+(1-\tau)\alpha\Gamma'(\gamma)$ are fulfilled for these parameters and this solution. Finally, the marginal social welfare effect from the introduction of a pact is negative in this case.

B.2 Non-linear - quadratic example

We retain specification (33) for $u(.)$ and $v(.)$ and assume that $\Gamma(.)$ is of the format:

$$\Gamma(\gamma) = \gamma^q, \quad 0 < q < 1. \tag{40}$$

Solving for (d_L, d_H, γ) is easy in this case, because the first-order conditions (4) and (5) for public debt again yield the explicit solutions (37) and (38) for d_L and d_H, respectively, which are linear in γ. Substituting these into the first-order condition (6), we are left with an equation that only needs to be solved for γ. We do this with a simple program written in Gauss (available upon request from the others).

From the linear-quadratic example we retain the parameter values $y_L = 0.75$, $y_H = 1.25$, $y_2 = 1$, $\tau = 0.5$, $\alpha = \beta = 0.75$, $p = 0.5$, $\xi = 3$. Assuming further that $h = 1.1$, $q = 0.5$, $I = 1.5$ and setting $k = 0$, we obtain the following outcomes: $d_L = 0.690$, $d_H = 0.523$, $\gamma = 0.247$. A check on the marginal utilities from public and private consumption show that these are all positive. Further, the conditions $(\tau\alpha + \beta)\Gamma'(\gamma) > h$ and $I > h + (1-\tau)\alpha\Gamma'(\gamma)$ are fulfilled for these parameters and this solution for γ. Finally, expression (32) is negative in this case, indicating that the marginal welfare effect of introducing a pact in this case is positive. Setting $q = 0.99$, yields $\gamma = 0.174$ and produces a negative marginal welfare effect from the introduction of a stability pact.

C Extension: second-period punishments and uncertain sanctions

The model is the same as before, except that we now allow for both first- and second-period sanctions (represented by punishment parameters k_1 and k_2, respectively), as well as uncertainty about whether sanctions will be applied (this uncertainty is only resolved *after* deficits have been selected). In the case of an excessive deficit, sanctions will be applied only with probability π. If they are applied, then they are applied for all countries. If they are not applied, then they are applied for none of the countries. We work backwards again, starting with the solution for public debt.

If $y_1 = y_L$, the government maximizes over $d > \bar{d}$ the function:

$$\begin{aligned}
&\pi v\left[\tau y_L - h\gamma + d - k_1\left(d - \bar{d}\right) + R_1\right] + (1-\pi)\, v\left(\tau y_L - h\gamma + d\right) \\
&+ p\pi v\left[\tau y_2 + (\tau\alpha + \beta)\Gamma(\gamma) - d - k_2\left(d - \bar{d}\right) + R_2\right] \\
&+ p(1-\pi)\, v\left[\tau y_2 + (\tau\alpha + \beta)\Gamma(\gamma) - d\right],
\end{aligned}$$

where $\psi \equiv (k_1, k_2, \pi)$ and $0 \leq k_1 \leq 1$, $0 \leq k_2 \leq 1$ and $0 < \pi < 1$. Further, R_1 and R_2 are the rebates in the first and second period, respectively. The first-order condition is:

$$\begin{aligned}
& \pi v'\left[\tau y_L - h\gamma + d - k_1\left(d-\bar{d}\right) + R_1\right](1-k_1) + (1-\pi)\, v'(\tau y_L - h\gamma + d) \\
= \; & p\pi v'\left[\tau y_2 + (\tau\alpha+\beta)\,\Gamma(\gamma) - d - k_2\left(d-\bar{d}\right) + R_2\right](1+k_2) \\
& + p(1-\pi)\, v'\left[\tau y_2 + (\tau\alpha+\beta)\,\Gamma(\gamma) - d\right].
\end{aligned} \tag{41}$$

If $y_1 = y_H$, the government maximizes over $d \leq \bar{d}$ the function:

$$v(\tau y_H - h\gamma + d + R_1) + pv\left[\tau y_2 + (\tau\alpha+\beta)\,\Gamma(\gamma) - d + R_2\right].$$

The first-order condition is:

$$v'(\tau y_H - h\gamma + d + R_1) = pv'\left[\tau y_2 + (\tau\alpha+\beta)\,\Gamma(\gamma) - d + R_2\right]. \tag{42}$$

Using that, as before, in equilibrium, fines and rebates cancel (both of them are zero when $y_1 = y_H$), the first-order conditions reduce to:

$$\begin{aligned}
& \pi v'(\tau y_L - h\gamma + d)(1-k_1) + (1-\pi)\, v'(\tau y_L - h\gamma + d) \\
= \; & p\pi v'\left[\tau y_2 + (\tau\alpha+\beta)\,\Gamma(\gamma) - d\right](1+k_2) \\
& + p(1-\pi)\, v'\left[\tau y_2 + (\tau\alpha+\beta)\,\Gamma(\gamma) - d\right],
\end{aligned} \tag{43}$$

$$v'(\tau y_H - h\gamma + d) = pv'\left[\tau y_2 + (\tau\alpha+\beta)\,\Gamma(\gamma) - d\right]. \tag{44}$$

Denote the value of the government's objective function at the *start* of Stage 3 when $y_1 = y_L$ (when $y_1 = y_H$), by V_L^ψ (by V_H^ψ).

By differentiating (43) and (44), we obtain the analogon of Lemmata 1 and 3:

Lemma 1'&3': *for a given value of γ, we have that*

$$\begin{aligned}
\frac{\partial d_L}{\partial k_1} &= \frac{\pi v'(f_{1L})}{v''(f_{1L})(1-\pi k_1) + pv''(f_{2L})(1+\pi k_2)} < 0; \; \frac{\partial d_L}{\partial k_2} = \frac{p\pi v'(f_{2L})}{v''(f_{1L})(1-\pi k_1) + pv''(f_{2L})(1+\pi k_2)} < 0, \\
\frac{\partial d_L}{\partial \pi} &= \frac{k_1 v'(f_{1L}) + pk_2 v'(f_{2L})}{v''(f_{1L})(1-\pi k_1) + pv''(f_{2L})(1+\pi k_2)} < 0,
\end{aligned}$$

while, trivially, d_H is not affected by any of the stability pact parameters. Further, $\frac{\partial d_L}{\partial \gamma} = h\lambda'_L + (\tau\alpha+\beta)\,\Gamma'(\gamma)(1-\lambda'_L) > 0$ *and* $\frac{\partial d_H}{\partial \gamma} = h\lambda'_H + (\tau\alpha+\beta)\,\Gamma'(\gamma)(1-\lambda'_H) > 0$. *Here,*

$$\lambda'_L \equiv \frac{v''(f_{1L})(1-\pi k_1)}{v''(f_{1L})(1-\pi k_1) + pv''(f_{2L})(1+\pi k_2)}, \; \lambda'_H \equiv \frac{v''(f_{1H})}{v''(f_{1H}) + pv''(f_{2H})}, \tag{45}$$

while f_{1L}, f_{2L}, f_{1H} and f_{2H} are again defined by (22).

Let us now turn to the choice of the optimal reform package. The government maximizes:

$$\underset{\gamma}{\mathrm{Max}} \left\{ 2\mathrm{E}_0 \left[u\left(c_1\right) \right] + \tfrac{1}{2} \left[V_L^{\psi} + V_H^{\psi} \right] \right\}, \tag{46}$$

Using the first-order conditions for public debt (43) and (44), we can write the first-order condition for γ again as (6). Once more using the first-order conditions for public debt, we can now rewrite (6) further as:

$$\begin{aligned} & \mathrm{E}_0 \left\{ u'\left(c_1\right) \left[h + (1-\tau)\, \alpha \Gamma'\left(\gamma\right) - I \right] \right\} \\ = \; & \tfrac{1}{2} \left[h - (\tau\alpha + \beta)\, \Gamma'\left(\gamma\right) \left(\tfrac{1-\pi k_1}{1+\pi k_2} \right) \right] v'\left(\tau y_L - h\gamma + d_L\right) + \\ & \tfrac{1}{2} \left[h - (\tau\alpha + \beta)\, \Gamma'\left(\gamma\right) \right] v'\left(\tau y_H - h\gamma + d_H\right). \end{aligned} \tag{47}$$

The lemma corresponding to Lemma 2 now becomes:

Lemma 2': *If* k_1 and k_2 *are not too large, then for a given level of structural reforms the deficit under a partisan government is always larger than the socially-optimal level.*

This lemma was obtained by differentiating (41) and (42) with respect to p and observing that the deficit is decreasing in p. The proposition corresponding to Proposition 1 becomes:

Proposition 1': *If the stability pact is not too tight (i.e.,* k_1 and k_2 *are not too large), then a partisan government provides a suboptimally low amount of structural reforms compared to the social optimum.*

Differentiating (6) with respect to $\theta = k_1$ and k_2, we can establish the analogon of Lemma 4:

Lemma 4': *If a stability pact is effective at reducing the deficit, it also leads to less ambitious structural reforms, as one can write* $\partial\gamma/\partial\theta = C_L\left(\partial d_L/\partial\theta\right) + C_H\left(\partial d_H/\partial\theta\right) < 0$, *where* $\theta = k_1$ or k_2, and where $C_L > 0$ and $C_H > 0$.

The expressions for C_L and C_H were already given above.

Differentiating (43) and (44), we obtain:

$$\begin{aligned} \frac{\partial d_L}{\partial k_1} &= \pi A_{L1} + B_L^* \frac{\partial\gamma}{\partial k_1}, \quad \frac{\partial d_L}{\partial k_2} = \pi A_{L2} + B_L^* \frac{\partial\gamma}{\partial k_2}, \\ \frac{\partial d_H}{\partial k_1} &= B_H \frac{\partial\gamma}{\partial k_1}, \quad \frac{\partial d_H}{\partial k_2} = B_H \frac{\partial\gamma}{\partial k_2}, \end{aligned}$$

where B_H was already defined above and where:

$$\begin{aligned} A_{L1} &= \tfrac{v'(f_{1L})}{v''(f_{1L})(1-\pi k_1)+p(1+\pi k_2)v''(f_{2L})} < 0, \quad A_{L2} = \tfrac{pv'(f_{2L})}{v''(f_{1L})(1-\pi k_1)+p(1+\pi k_2)v''(f_{2L})} < 0, \\ B_L^* &= \tfrac{hv''(f_{1L})(1-\pi k_1)+p(1+\pi k_2)(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2L})}{v''(f_{1L})(1-\pi k_1)+p(1+\pi k_2)v''(f_{2L})} > 0. \end{aligned}$$

Hence:

$$\begin{aligned}
\frac{\partial\gamma}{\partial k_1} &= \frac{\pi C_L A_{L1}}{1-C_L B_L^* - C_H B_H}, \quad \frac{\partial d_L}{\partial k_1} = \frac{\pi\left(1-C_H B_H\right) A_{L1}}{1-C_L B_L^* - C_H B_H}, \quad \frac{\partial d_H}{\partial k_1} = \frac{\pi C_L B_H A_{L1}}{1-C_L B_L^* - C_H B_H}, \\
\frac{\partial\gamma}{\partial k_2} &= \frac{\pi C_L A_{L2}}{1-C_L B_L^* - C_H B_H}, \quad \frac{\partial d_L}{\partial k_2} = \frac{\pi\left(1-C_H B_H\right) A_{L2}}{1-C_L B_L^* - C_H B_H}, \quad \frac{\partial d_H}{\partial k_2} = \frac{\pi C_L B_H A_{L2}}{1-C_L B_L^* - C_H B_H}.
\end{aligned}$$

If $k_1 = k_2 = 0$, then B_L^* is equal to B_L with $k = 0$. Hence, for k_1 and k_2 sufficiently small, $C_L B_L^* + C_H B_H < 1$, so that we can also state a proposition corresponding to Proposition 2:

Proposition 2': *Assuming that the pact is not too tight to start with (i.e., k_1 and k_2 are not too large), a further tightening of the pact (i.e., a higher k_1 or k_2) in equilibrium leads to a lower deficit as well as less structural reforms.*

Finally, the welfare effect of the introduction of a pact can be established in a manner analogous to the one we applied above. The derivative of (1) with respect to θ ($\theta = k_1, k_2$), when evaluated at $k_1 = k_2 = 0$ is:

$$\begin{aligned}
&\tfrac{1}{2}\left[h-\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right)\right]\left[v'\left(f_{1L}\right)+v'\left(f_{1H}\right)\right]\tfrac{\partial\gamma}{\partial\theta} \\
&+\tfrac{1}{2}v'(f_{1L})\left[\left(\tfrac{p-1}{p}\right)\tfrac{\partial d_L}{\partial\theta}+\left[\tfrac{1}{p}\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right)-h\right]\tfrac{\partial\gamma}{\partial\theta}\right] \\
&+\tfrac{1}{2}v'(f_{1H})\left[\left(\tfrac{p-1}{p}\right)\tfrac{\partial d_H}{\partial\theta}+\left[\tfrac{1}{p}\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right)-h\right]\tfrac{\partial\gamma}{\partial\theta}\right],
\end{aligned}$$

which is written further as:

$$\begin{aligned}
&\tfrac{1}{2}\left(\tfrac{1-p}{p}\right)\left[\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right)\left[v'(f_{1L})+v'(f_{1H})\right]\tfrac{\partial\gamma}{\partial\theta}-v'(f_{1L})\tfrac{\partial d_L}{\partial\theta}-v'(f_{1H})\tfrac{\partial d_H}{\partial\theta}\right] \\
=\ &\tfrac{1}{2}\left(\tfrac{1-p}{p}\right)\frac{\pi A_{L\theta}}{1-C_L B_L - C_H B_H} * \\
&\left[\left(\tau\alpha+\beta\right)\Gamma'\left(\gamma\right)\left[v'(f_{1L})+v'(f_{1H})\right]C_L - v'(f_{1L})\left(1-C_H B_H\right)-v'(f_{1H})C_L B_H\right],
\end{aligned}$$

where $A_{L\theta} = A_{L1}$ when $\theta = k_1$ and $A_{L\theta} = A_{L2}$ when $\theta = k_2$. The term in the final line of the above expression is the same as in the case analyzed in the main text. Hence, the conditions for the introduction of a pact to be social-welfare enhancing remain the same as before.

Let us now turn to the smart pact in this case. The first-order condition for γ now becomes:

$$\begin{aligned}
&\mathrm{E}_0\left\{u'\left(c_1\right)\left[h+\left(1-\tau\right)\alpha\Gamma'-I\right]\right\} = \tfrac{1}{2}h\left[v'\left(f_{1L}\right)+v'\left(f_{1H}\right)\right] \\
&-\tfrac{1}{2}p\left(\tau\alpha+\beta\right)\Gamma'\left[v'\left(f_{2L}\right)+v'\left(f_{2H}\right)\right]-\tfrac{1}{2}\pi\delta k_1 v'\left(f_{1L}\right)-\tfrac{1}{2}\pi\delta p k_2 v'\left(f_{2L}\right),
\end{aligned}$$

Differentiating this expression with respect to θ ($\theta = k_1, k_2$) and evaluating at $k_1 = k_2 = 0$.

$$\begin{aligned}
&\left[\begin{array}{c}
\mathrm{E}_0\left\{\frac{1}{2}u''(c_1)\left[h+(1-\tau)\alpha\Gamma'(\gamma)-I\right]^2+u'(c_1)(1-\tau)\alpha\Gamma''(\gamma)\right\}+\\
\frac{1}{2}p(\tau\alpha+\beta)^2(\Gamma'(\gamma))^2\left[v''(f_{2L})+v''(f_{2H})\right]+\frac{1}{2}h^2\left[v''(f_{1L})+v''(f_{1H})\right]\\
+\frac{1}{2}p(\tau\alpha+\beta)\left[v'(f_{2L})+v'(f_{2H})\right]\Gamma''(\gamma)
\end{array}\right]\frac{\partial\gamma}{\partial\theta}\\
=\;&\frac{1}{2}\left[hv''(f_{1L})+p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2L})\right]\frac{\partial d_L}{\partial\theta}+\\
&\frac{1}{2}\left[hv''(f_{1H})+p(\tau\alpha+\beta)\Gamma'(\gamma)v''(f_{2H})\right]\frac{\partial d_H}{\partial\theta}-P_\theta,
\end{aligned}$$

where $P_\theta \equiv \frac{1}{2}\pi\delta v'(f_{1L})$ when $\theta = k_1$ and $P_\theta \equiv \frac{1}{2}\pi\delta p v'(f_{2L})$ when $\theta = k_2$. Hence, $\frac{\partial\gamma}{\partial\theta}_{|\delta>0} = \frac{\partial\gamma}{\partial\theta}_{|\delta=0} + D_\theta$ ($\theta = k_1, k_2$). The solutions for $\frac{\partial\gamma}{\partial k_1}$, $\frac{\partial d_L}{\partial k_1}$, $\frac{\partial d_H}{\partial k_1}$, $\frac{\partial\gamma}{\partial k_2}$, $\frac{\partial d_L}{\partial k_2}$ and $\frac{\partial d_H}{\partial k_2}$ are now given by:

$$\begin{aligned}
\frac{\partial\gamma}{\partial k_1} &= \frac{\pi C_L A_{L1}+D_{k_1}}{1-C_L B_L^*-C_H B_H}, & \frac{\partial d_L}{\partial k_1} &= \frac{\pi A_{L1}(1-C_H B_H)+D_{k_1}B_L^*}{1-C_L B_L^*-C_H B_H}, & \frac{\partial d_H}{\partial k_1} &= \frac{\pi C_L B_H A_{L1}+D_{k_1}B_H}{1-C_L B_L^*-C_H B_H},\\
\frac{\partial\gamma}{\partial k_2} &= \frac{\pi C_L A_{L2}+D_{k_2}}{1-C_L B_L^*-C_H B_H}, & \frac{\partial d_L}{\partial k_2} &= \frac{\pi A_{L2}(1-C_H B_H)+D_{k_2}B_L^*}{1-C_L B_L^*-C_H B_H}, & \frac{\partial d_H}{\partial k_2} &= \frac{\pi C_L B_H A_{L2}+D_{k_2}B_H}{1-C_L B_L^*-C_H B_H}.
\end{aligned}$$

Hence, the marginal social welfare effect of introducing a pact (i.e., marginally increasing k_1 or k_2, while evaluating at $k_1 = k_2 = 0$) equals the marginal social welfare effect of introducing a non-smart pact *plus* a term

$$\frac{1}{2}\frac{1-p}{p}\frac{D_\theta}{1-C_L B_L-C_H B_H}\left[(v'(f_{1L})+v'(f_{1H}))(\tau\alpha+\beta)\Gamma'(\gamma)-v'(f_{1L})B_L-v'(f_{1H})B_H\right], \quad \theta = k_1, k_2,$$

which was earlier shown to be positive. Here, we have again used that B_L^* at $k_1 = k_2 = 0$ equals B_L at $k = 0$.