The Good, the Bad and the Missed Boom: Online Appendix

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OA 1. Endogenous opacity

In this Section we study the endogenous choice of balance sheet opacity or transparency by large banks. To this end we extend the baseline model with two new elements.

First, we assume that bank's perceived riskiness is associated with costs for the bank. We introduce this in a reduced form by assuming that when obtaining funding large banks have to pay additional funding costs equal to d_i, which increases in banks' perceived risk:

$$d_i = \chi \mathbb{E}(L_i | p, \tilde{w}_i) \tag{1}$$

where $\mathbb{E}(L_i|p, \tilde{w})$ represents the expected losses of bank i conditional on observed prices and the available information about bank's assets $\tilde{w}_i$. Parameter χ measures the sensitivity of bank funding costs to these expected losses. It may be positive because some of bank funding is uninsured. All types of governmental guarantees that protect banks' creditors can lower the level of χ.

Second, we maintain the assumption that banks' assets are difficult to value for outsiders, so that a bank making a statement about its asset quality, λ, and composition, x_i, y_i, z_i, is only credible if it has truth-telling incentives or if it pays an upfront verification cost, C. After paying the verification cost banks balance sheet is analyzed by an outside party and the value and composition of bank's assets is made transparent to the public (we refer to banks that pay verification costs as "transparent" and those that do not as "opaque"). The verification cost can be thought of as bank's expenses associated with hiring outside auditors who analyze the value of banks assets and make it publicly known.[1] Banks need to commit to transparency by paying the verification costs before making choices on their asset allocation.[2]

If the bank chooses not to pay the verification cost, its balance sheet remains opaque, $\mathbb{I}^O = 1$. It can still release unverifiable information about the quality of its legacy assets $\tilde{\theta}_i$ and the portfolio allocation $\tilde{x}_i, \tilde{y}_i, \tilde{z}_i$. Let $\tilde{w}$ be a vector representing the information released by the bank. If some of this information is credible, $\tilde{w}(m) = w(m)$ for some element m, it affects the additional funding costs d_i. Otherwise the additional funding cost is determined by the expectation of the cost of banks' default conditional only on observed price:

$$d_i(\mathbb{I}^O = 1, \tilde{w}_i, p) = \begin{cases} \chi\mathbb{E}(L|p) & \text{if } \tilde{w} \neq w \\ \chi\mathbb{E}(L_i|p, v_i) & \text{if } \exists m : \tilde{w}(m) = w(m) \end{cases}$$

Where v_i is a vector composed of all credibly communicated information $v_i = \{w_i(m)\}\forall m : \tilde{w}_i(m) = w_i(m)$.

If the bank pays the verification cost, $\mathbb{I}^O = 0$, its assets become transparent so the quality of its legacy assets as well as its portfolio allocation are observable to all agents, $\tilde{w} = w$. In this case, the additional funding cost reflects the updated expectations of the costs of bank's default:

$$d_i(\mathbb{I}^O = 0, \tilde{w}) = \chi\mathbb{E}(L_i|\lambda, x_i, y_i, z_i, p)$$

Denote as $\bar{d}$ the maximum additional funding cost, corresponding to the highest expected cost of default for a bank. We impose the following restrictions on parameters in relation to that maximum cost to ease models' exposition.

[1]It could also capture other costs associated with releasing information about bank's asset quality, that are outside the scope of the model- for instance competitive losses due to release of proprietary information.

[2]Alternatively, one could allow banks to inform the public about the realized productivity, in this case verification cost would represent the cost of certifying the information that large banks produce so that it can be trusted by outsiders. Since knowledge of realized productivity allows outsiders to correctly assess banks' risk, the logic of this problem is akin to the one that we study here.

Assumption OA 1. *The parameters satisfy:*

A. $\frac{\alpha^2}{4} > q(\alpha)R + \lambda + \bar{d} + C$

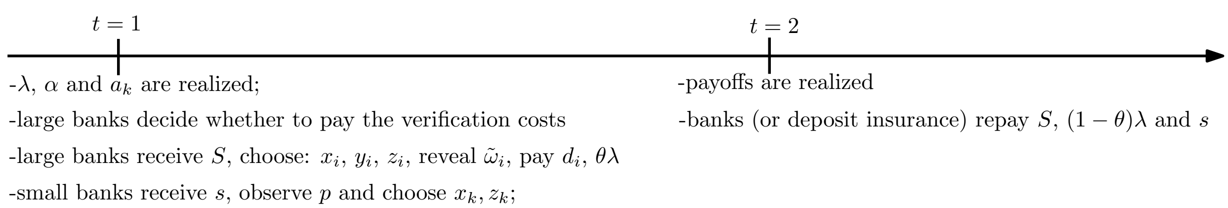
B. $S > \left(\frac{\alpha}{2}\right)^2 + q(\alpha)R + \lambda\theta + \bar{d}$

C. $(1 - q^2(\alpha))q(\alpha) < q'(\alpha)\left(\frac{\alpha}{2} - \lambda - \bar{d}\right)$

Assumption OA 1.A ensures that investment in the productive technology is more profitable than risk shifting even in the face of the extra costs: losses on past assets, additional funding costs and costs of verification. Assumption OA 1.B limits our attention to the cases when funding is always sufficient for the equilibrium price of the asset to be at least $p = q(\alpha)R$ (there are no missed booms). Assumption OA 1.C ensures that the risk shifting threshold under opacity decreases in productivity.

The timing is as follows:

Figure 1: Timeline with a choice between transparency and opacity



Large bank's problem can be represented as

$$\max_{\mathbb{I}^O, x_i, y_i, z_i, \tilde{w}_i} q(\alpha)\left(\alpha\sqrt{x_i} + Ry_i + z_i - S - (1-\theta)\lambda\right) + (1 - q(\alpha))\max\left[\alpha\sqrt{x_i} + z_i - S - (1-\theta)\lambda, 0\right] \tag{2}$$

subject to:

$$x_i + py_i + z_i + \lambda\theta + d_i(\mathbb{I}^O, \tilde{w}_i) + C\left(1 - \mathbb{I}^O\right) = S \qquad \text{(budget constraint)}$$

$$\tilde{w}(\mathbb{I}_i^O = 0) = w \qquad \text{(transparency condition)}$$

$$y_i \geq 0 \qquad \text{(no-short-selling constraint)}$$

We solve it by backward induction. First, we establish the optimal investment choice and information release given banks choice regarding the transparency of the balance sheet. Then, we discuss what determines banks' decision to pay the verification cost or remain opaque.

Optimal investment by large banks

Lemma OA 1. *There exists a threshold level of the available funding supply* $\hat{S}(\alpha, \lambda, p, d_i, \mathbb{I}_i^O, C)$ *(the individual risk shifting threshold), such that when funding is below that threshold,* $S < \hat{S}(\alpha, \lambda, p, d_i, \mathbb{I}_i^O, C)$ *the large bank follows a solvent strategy given by:*

$$x_i^{*s} = \begin{cases} \left(\frac{\alpha pn}{2q(\alpha)R}\right)^2 & \text{if } pn \leq q(\alpha)Rh \\ \left(\frac{\alpha h}{2}\right)^2 & \text{if } pn \geq q(\alpha)Rh \end{cases}$$

$$py_i^{*s} = \begin{cases} \min\left[S, \alpha\sqrt{x_i} - (1-\theta)\lambda\right] - x_i - z_i - d_i - \theta\lambda - C(1-\mathbb{I}^O), & \text{if } pn \leq q(\alpha)Rh \\ 0 & \text{if } pn \geq q(\alpha)Rh \end{cases}$$

$$z_i^{*s} = S - py_i - x_i - d - \lambda\theta - \left(1 - \mathbb{I}^O\right) C$$

where $pn(\mathbb{I}_i^O = 0) = \frac{p+d'_y}{1+d'_x}$ *and* $h(\mathbb{I}_i^O = 0) = \frac{1+d'_z}{1+d'_x}$ *if the bank is transparent, while* $pn(\mathbb{I}_i^O = 1) = 1$ *and* $h(\mathbb{I}_i^O = 1) = 1$ *if it is opaque. If the level of funding is above the threshold,* $S > \hat{S}(\alpha, \lambda, p, d_i, \mathbb{I}_i^O, C)$ *the large bank follows a risk shifting strategy, in which the investment levels are given by:*

$$x_i^{*r} = \left(\frac{\alpha pn}{2R}\right)^2 \qquad py_i^{*r} = S - x_i - d_i - \theta\lambda - \left(1 - \mathbb{I}^O\right) C \qquad z_i^{*r} = 0 \qquad \text{if } pn < Rh$$

Proof. In Appendix OA 1.A.1 □

As in the baseline model the optimal choice of productive lending reflects the marginal benefits of higher lending relative to the marginal costs, with the latter varying depending on whether the bank faces risk of default (in case of risk shifting) as well as the relative return of the storage and risky asset (in case of solvent investment). The choice of an opaque bank is analogous to that studied in the baseline, with the only change coming from a tighter budget constraint that now accounts for d_i. Productive lending under solvent and risk shifting strategies is the same as in the baseline, and the investment into storage or risky asset is lower due to additional funding costs.

The optimal productive lending of the transparent bank reflects also the sensitivity of funding costs to changes in lending, as well as to changes in storage or risky asset holdings (depending on which of the two is the best alternative investment). Lending falls as d'_x increases and rises as the sensitivity of funding costs to the size of the alternative investment increases d'_y or d'_z (which of the two is relevant is determined by the comparing the marginal returns on the asset and storage).

When banks are transparent other agents can observe their asset allocation and the quality of legacy assets. Based on bank's choices and the information regarding its leverage they are able to precisely infer the underlying productivity (by using bank's optimal investment rules stated in Lemma OA 1). When a large bank plays a solvent strategy its risk of default is zero. In this case changes in investment levels do not alter the funding costs $d'_x = d'_z = d'_y = 0$. The optimal lending of a transparent bank under the solvent strategy is thus the same as the one of the opaque bank. The level of investment in the best alternative (storage or risky asset) is higher than that of the opaque bank if and only if $d_i(\mathbb{I}^O = 1) - d_i(\mathbb{I}^O = 0) > C$.

If the portfolio choices and legacy assets of the transparent bank are such that it is clear that the large bank is playing a risk shifting strategy, changes in portfolio allocation affect the expected cost of bank's default. For a given level of deposits, higher investment in storage and higher productive lending both decrease losses in case of default, while larger exposure to the risky asset increases them. Consequently, when transparent bank shifts risk, $d'_x < 0$, $d'_z < 0$ and $d'_y > 0$. Thus, the optimal lending of the transparent bank playing the risk shifting strategy is higher than that of the opaque bank. The impact on the total investment in the risky asset depends on how the increase in lending compares to the impact of the funding costs and verification costs payment on the budget constraint. Intuitively, a sufficiently strong responsiveness of the funding cost to the expected cost of default (high χ) could eradicate the risk shifting incentives in transparent banks.

Information revelation The opaque bank chooses information that it reveals $\tilde{w}$ so that to minimize its additional funding costs

$$\tilde{w}^*(\mathbb{I}^O = 1) = \arg\min_{\tilde{w}} d_i(\mathbb{I}^O = 1, \tilde{w}) \tag{3}$$

Corollary OA 1. *The information released by an opaque bank differs from the true asset quality and composition whenever the bank risk shifts :*

$$S > \hat{S}(\alpha, \lambda, p, d_i, \mathbb{I}_i^O, C) \Rightarrow \tilde{w}^*(\mathbb{I}^O = 1) \neq \tilde{w}^*$$

Proof. In Appendix OA 1.A.2 □

Lacking the commitment to truth-telling associated with transparency, an opaque bank, prefers to release false information about its assets when it plays the risk shifting strategy. The information the bank chooses minimizes its perceived risk and thus its funding costs. Consequently, whenever the observed asset price is such that banks may be risk shifting, information about asset quality and choices released by an opaque bank is not credible. As a result the funding cost faced by an opaque bank is $d_i = \chi\mathbb{E}(L|p) = \chi\text{Prob.(bank is risk shifting}|p)\mathbb{E}\left((1 - q(\alpha))\left(\frac{\alpha^2 p}{4R} - (1-\theta)\lambda\right)|p\right)$.

Banks choose whether to pay the verification cost and become transparent by comparing profits that they can earn under each of the "regimes".

Lemma OA 2. *Large banks prefer to remain opaque if $\chi\mathbb{E}(L|p) < C$.*

Proof. In Appendix OA 1.A.2 □

The benefit of transparency is that the bank can lower its additional funding costs, by revealing information that decreases the expectations about its losses. This gain is the highest if the bank can credibly communicate that it is playing a solvent strategy and thus not facing any default risk. In this case funding costs is equal to 0 instead of $\chi\mathbb{E}(L|p)$ that it would be in case of opacity. If these maximum gains of transparency are not larger than the cost of verification, the bank always prefers to remain opaque.

Proposition OA 1. If the verification cost C is sufficiently high all banks choose to remain opaque and the *equilibrium risk shifting threshold* of funding supply is defined as:

$$\hat{\mathcal{S}}(\alpha,\lambda,d) = \hat{S}(\alpha,\lambda,q(\alpha)R,d,1,C) \tag{4}$$

The threshold is such that:

- if $S > \hat{\mathcal{S}}(\alpha,\lambda,d)$, fraction $\psi^*(\alpha,\lambda,d) = \frac{p^*}{S-\theta\lambda-d-\left(\frac{\alpha p^*}{2R}\right)^2}$ of banks play a risk shifting strategy, where $p^* = \hat{p}$ is the equilibrium price of the risky asset defined implicitly in $\hat{S}(\alpha,\lambda,\hat{p},d,1,C) = S$; the remaining $1-\psi^*(\alpha,\lambda,d)$ banks play the solvent strategy;

- if $S \leq \hat{\mathcal{S}}(\alpha,\lambda,d)$ all banks invest according to the solvent strategy, $1-\psi^*(\alpha,\lambda,d) = 1$; the equilibrium price of the risky asset is $p^* = q(\alpha)R$

where:

$$\begin{aligned} d = &-\chi Prob.(\lambda = 0|p^*)\psi^*(\tilde{\alpha}(p^*,0),0,d)(1-q(\alpha))\left[\frac{\tilde{\alpha}(p^*,0)^2 p}{2R} - S\right] \\ &-\chi Prob.(\lambda = \bar{\lambda}|p^*)\psi^*(\tilde{\alpha}(p,\bar{\lambda},d),\bar{\lambda})(1-q(\alpha))\left[\frac{\tilde{\alpha}(p^*,\bar{\lambda})^2 p}{2R} - S - (1-\theta)\bar{\lambda}\right] \end{aligned} \tag{5}$$

with $\tilde{\alpha}(p^*,0)$ and $\tilde{\alpha}(p^*,\bar{\lambda})$ corresponding to the productivity levels that result in the equilibrium price p^* under the respective realizations of losses on legacy assets.

Proof. Proof in Appendix OA 1.A.4 □

The intuition for Proposition OA 1 is similar as that for Proposition 1. The key difference is that now banks are faced with additional funding costs. Moreover, Proposition OA 1 shows that an equilibrium in which banks endogenously choose to remain opaque can emerge if costs of verification are sufficiently high.

Inference from prices works as in the baseline model. Precise inference of α and λ may occur if the equilibrium price can only emerge when all banks invest according to the solvent strategy $p = q(\alpha)R$. Alternatively precise inference is possible if the price can only emerge when some banks risk-shift and losses on legacy assets are high $p = p^*(\alpha,\bar{\lambda})$. If the equilibrium price is such that precise inference is not possible, two estimates of productivity emerge:

$$\tilde{\alpha} = \begin{cases} \tilde{\alpha}(p, 0) \text{ with probability } \kappa \\ \tilde{\alpha}(p, \bar{\lambda}) \text{ with probability } 1 - \kappa \end{cases} \tag{6}$$

Therefore if the verification costs are high enough for Proposition OA 1 to hold, the problem of small banks is equivalent to the one in the baseline model.

OA 1.A. Proofs for Online Appendix OA 1

OA 1.A.1 Optimal investment: Proof of Lemma OA 1

Opaque bank $\mathbb{I}_i^O = 1$

The FOCs are the same as in the benchmark model. The solvent strategy's investment in the risky asset is now given by:

$$py_i = S - x_i - d_i - \lambda\theta \qquad \text{if } p \leq q(\alpha)R \tag{7}$$

$$py_i = 0 \qquad \text{if } p \geq q(\alpha)R \tag{8}$$

The risky investment under the risk-shifting is: $py_i = S - x_i - d_i - \lambda\theta \ \forall \ p < R$. The optimal investment in storage is $z_i = S - py_i - x_i - d_i - \lambda\theta$. Comparing profits under the two strategies, yields the following risk shifting threshold:

$$\hat{S}^O(\alpha, \lambda, p, d_i) = \begin{cases} \frac{\alpha^2 p}{4q(\alpha)R}\left(1 + q(\alpha)\right) - \lambda(1-\theta) & \text{if } p < q(\alpha)R \\ \frac{\alpha^2}{4}\frac{1-\frac{q(\alpha)p}{R}}{q(\alpha)\left(\frac{R}{p}-1\right)} - \lambda\frac{1-q(\alpha)-q(\alpha)\theta\left(\frac{R}{p}-1\right)}{q(\alpha)\left(\frac{R}{p}-1\right)} - d_i\frac{1-\frac{q(\alpha)R}{p}}{q(\alpha)\left(\frac{R}{p}-1\right)} & \text{if } p \geq q(\alpha)R \end{cases} \tag{9}$$

Following the same logic as in the baseline, we can show that this is the relevant threshold by comparing it to the "feasibility" levels of funding for each strategy. We get that $\hat{s}_r^O < \hat{S} < \hat{s}_s^O$ whenever $\frac{\alpha^2}{4} - \lambda - d_i > 0$.

Transparent bank $\mathbb{I}_i^O = 0$

The FOCs of a bank problem if it *remains solvent* in the bad state yield:

$$\alpha \frac{1}{2} x_i^{-\frac{1}{2}} - \frac{q(\alpha) R(1+d'_x)}{p+d'_y} = 0 \qquad \text{if } p+d'_y < q(\alpha)R(1+d'_z) \tag{10}$$

$$\alpha \frac{1}{2} x_i^{\frac{-1}{2}} - \frac{1+d'_x}{1+d'_z} = 0 \qquad \text{if } p+d'_y \geq q(\alpha)R(1+d'_z) \tag{11}$$

where $d'_w = \frac{\partial d_i}{\partial w_i}$. Thus, the investment in the risky asset is

$$py_i = \begin{cases} S - x_i - d_i - \lambda\theta - C & \text{if } p+d'_y \leq q(\alpha)R(1+d'_z) \\ 0 & \text{if } p+d'_y > q(\alpha)R(1+d'_z) \end{cases} \tag{12}$$

The investment in storage is $z_i = S - py_i - x_i - d_i - \lambda\theta - C$. This strategy is feasible if it does not lead to default, that is when:

- $\left(\frac{\alpha(1+d'_z)}{2(1+d'_x)}\right)^2 - \lambda - d_i - C > 0$ if $p+d'_y > q(\alpha)R(1+d'_z)$, or
- $S < \left(\frac{\alpha(p+d'_y)}{2q(\alpha)R(1+d'_x)}\right)^2 - \lambda(1-\theta) = \hat{s}_s$ if $p+d'_y \leq q(\alpha)R(1+d'_z)$

If $S > \left(\frac{\alpha(p+d'_y)}{2q(\alpha)R(1+d'_x)}\right)^2 - (1-\theta)\lambda$ and $p+d'_y \leq q(\alpha)R(1+d'_z)$, a bank playing a solvent strategy needs to cap its investment into risky asset to $py_i = \alpha\sqrt{x_i} - x_i - \lambda - d_i - C$, where $x_i = \left(\frac{\alpha(1+d'_z)}{2(1+d'_x)}\right)^2$. If the *bank defaults* in the bad state, the FOCs yield:

$$\alpha \frac{1}{2} x^{\frac{-1}{2}} - \frac{R(1+d'_x)}{p+d'_y} = 0 \qquad \text{if } p+d'_y < R(1+d'_z) \tag{13}$$

The investment in the risky asset is $py_i = S - x_i - d_i - \lambda\theta - C$ and into storage is $z_i = 0$. The strategy is feasible if it results in a risk of default, that is if $S > \left(\frac{\alpha(p+d'_y)}{2R(1+d'_x)}\right)^2 - \lambda(1-\theta) = \hat{s}_r^T$. Strategies of a transparent bank can be expressed as in the Lemma using:

$$pn = \begin{cases} p & \text{if } \mathbb{I}_i^O = 1 \\ \frac{p+d'_y}{1+d'_x} & \text{if } \mathbb{I}_i^O = 0 \end{cases} \qquad h = \begin{cases} 1 & \text{if } \mathbb{I}_i^O = 1 \\ \frac{1+d'_z}{1+d'_x} & \text{if } \mathbb{I}_i^O = 0 \end{cases}$$

Since $\hat{s}_s^T > \hat{s}_r^T$, three scenarios are possible:

- If $\hat{s}_s^T > S > \hat{s}_r^T$ both solvent and risk shifting strategy are feasible. Banks are indifferent if both strategies offer the same profit.

  If $\frac{pn}{h} \leq q(\alpha)R$:

$$\Pi_r = \Pi_s \;\; \Rightarrow \hat{s}_0 = \frac{\alpha^2 p}{4q(\alpha)R} \frac{n_s(2-n_s) - q^2(\alpha)n_r(2-n_r)}{1-q(\alpha)} - \lambda(1-\theta)$$

  If $\frac{pn}{h} \geq q(\alpha)R$:

$$\Pi_s = \frac{\alpha^2 h_s(2-h_s)}{4} - d_i - C - \lambda$$
$$\Pi_r = q(\alpha)\frac{\alpha^2 p n_r(2-n_r)}{4R} + Sq(\alpha)\left(\frac{R}{p}-1\right) - q(\alpha)\frac{R}{p}(d_i+C) - \lambda q(\alpha)(1-\theta+\frac{\theta R}{p})$$
$$\Pi_r = \Pi_s \;\; \Rightarrow \hat{s}_0 = \frac{\frac{\alpha^2}{4}(h_s(2-h_s) - q(\alpha)\frac{p}{R}n_r(2-n_r)) - (d_i+C)(1-q(\alpha)\frac{R}{p})}{q(\alpha)\left(\frac{R}{p}-1\right)} - \lambda\frac{1-q(\alpha)(1-\theta+\frac{\theta R}{p})}{q(\alpha)(\frac{R}{p}-1)}$$

- If $S > \hat{s}_s$ & $S > \hat{s}_r$ risk-shifting is feasible and the solvent strategy involves a risky asset exposure capped at $py_i = \alpha\sqrt{x_i} - x_i - \lambda - d_i - C$ and investment in storage equal to $z_i = S + (1-\theta)\lambda - \alpha\sqrt{x_i}$. Banks are indifferent if both strategies offer the same profit.

  If $\frac{pn}{h} \leq q(\alpha)R$:

$$\Pi_s^C = \frac{\alpha^2 h_s}{2} + q(\alpha)\frac{R}{p}\left(\frac{\alpha^2 h_s}{2} - \frac{\alpha^2 h_s^2}{4} - d_i - C - \lambda\right) + S + (1-\theta)\lambda - \frac{\alpha^2 h_s}{2} - S - (1-\theta)\lambda$$
$$\Pi_r = \Pi_s^C \;\; \Rightarrow \hat{s}_1 = \frac{\frac{\alpha^2}{4}\left(\frac{q(\alpha)R}{p}h_s(2-h_s) - \frac{q(\alpha)pn_r(2-n_r)}{R}\right) - \lambda(1-\theta)(1-q(\alpha))}{q(\alpha)(\frac{R}{p}-1)}$$

  If $\frac{pn}{h} > q(\alpha)R$: banks do not invest in the risky asset under the solvent strategy so this strategy is feasible whenever $\left(\frac{\alpha(1+d'_z)}{2(1+d'_x)}\right)^2 - \lambda - d_i - C > 0$, in which case the risk shifting threshold is:

$$\hat{s}_1 = \hat{s}_0 = \frac{\frac{\alpha^2}{4}(h_s(2-h_s) - q(\alpha)\frac{p}{R}n_r(2-n_r)) - (d_i+C)(1-q(\alpha)\frac{R}{p})}{q(\alpha)(\frac{R}{p}-1)} - \lambda\frac{1-q(\alpha)(1-\theta+\frac{\theta R}{p})}{q(\alpha)(\frac{R}{p}-1)}$$

  As we discuss in Appendix OA 1, when banks play solvent strategy and are transparent

$d_z' = d_y' = d_x' = 0$, so this condition simplifies to: $\left(\frac{\alpha}{2}\right)^2 - \lambda - C > 0$, which is satisfied under Assumption OA 2.A

- If $\hat{s}_s > S$ & $\hat{s}_r > S$ banks always play the solvent strategy

$\frac{\partial \Pi_s}{\partial S} > 0$ and $\frac{\partial \Pi_s^C}{\partial S} = 0$, so bank profits under solvent strategy increase in S until $S = \hat{s}_s$ and are constant thereafter. Since $\frac{\partial \Pi_r}{\partial S} > 0$, the profits under the two strategies are equated either at $\hat{s}_0 < \hat{s}_s$ (in which case $\hat{s}_0 < \hat{s}_1$) or at $\hat{s}_1 > \hat{s}_s$ (in which case $\hat{s}_1 < \hat{s}_0$). Thus, a solvent strategy is optimal if $S < \max[\hat{s}_r, \min[\hat{s}_1, \hat{s}_0]] = \hat{S}^T$. The risk shifting strategy is optimal if $S > \max[\hat{s}_r, \min[\hat{s}_1, \hat{s}_0]] = \hat{S}^T$.

OA 1.A.2 Choice of opacity: Proof of Corollary OA 1 and Lemma OA 2

Since $\mathbb{E}(L_i|\lambda, x_r, y_r, z_r, p) > \mathbb{E}(L_i|\lambda, x_s, y_s, z_s, p)$, a risk-shifting bank prefers to mimic a solvent investment strategy in its information release.

Profits of a solvent bank can be represented as:

$$\Pi_s(\mathbb{I}^O) = \underbrace{(\alpha\sqrt{x_i} + q(\alpha)Ry_i - x_i - py_i - \theta)}_{\text{operating profit}} \underbrace{- \left[d_i(\mathbb{I}^O) + C\left(1 - \mathbb{I}^O\right)\right]}_{\text{information related costs}} \tag{14}$$

Since $w_s^*(\mathbb{I}^O = 1) = w_s^*(\mathbb{I}^O = 0)$ for $w = x, y, z$, $\Pi_s(\mathbb{I}^O = 1) - \Pi_s(\mathbb{I}^O = 0) = -d_i(\mathbb{I}^O = 1) + d_i(\mathbb{I}^O = 0) + C$. Since, $d_i(\mathbb{I}^O = 0) = 0$, this simplifies to: $\Pi_s(\mathbb{I}^O = 1) - \Pi_s(\mathbb{I}^O = 0) = C - \chi\mathbb{E}(L|p)$. so opacity is preferred if $C > \chi\mathbb{E}(L|p)$.

Profits of the risk shifting bank are:

$$\underbrace{q(\alpha)\left(\alpha\sqrt{x_i} + Ry_i - x_i - py_i - \theta\right)}_{\text{operating profit}=O\Pi_r} \underbrace{-q(\alpha)\left[d_i + C\left(1 - \mathbb{I}^O\right)\right]}_{\text{information related costs}} \tag{15}$$

Note that $x_r^*(\mathbb{I}^O = 1), y_r^*(\mathbb{I}^O = 1), z_r^*(\mathbb{I}^O = 1)$ maximize operating profit ($O\Pi_r$), while $x_r^*(\mathbb{I}^O = 0), y_r^*(\mathbb{I}^O = 0), z_r^*(\mathbb{I}^O = 0)$ accounts for impact of investment on d_i. Thus, $O\Pi_r(\mathbb{I}^O = 1) \geq O\Pi_r(\mathbb{I}^O = 0)$. The opaque bank also faces lower information related costs if:

$$\chi\mathbb{E}(L|p) < \chi\mathbb{E}(L_i|\theta, x_i, y_i, z_i, p) + C$$

Therefore, if $C > \chi\mathbb{E}(L|p)$ a risk-shifting bank can earn higher profits by remaining opaque than by investing in transparency for any $\mathbb{E}(L_i|\theta, x_i, y_i, z_i, p) \geq 0$.

OA 1.A.3 Comparative statics with respect to p

If $p < q(\alpha)R$:

$$\frac{\partial \hat{s}}{\partial p} = \left(\frac{\alpha}{2}\right)^2 \frac{1 + q(\alpha)}{q(\alpha)} > 0 \tag{16}$$

If $p \geq q(\alpha)R$:

$$\frac{\partial \hat{s}}{\partial p} = \frac{1}{q^2(\alpha)\left(\frac{R}{p} - 1\right)^2} \frac{q(\alpha)}{p^2 R} \left[\frac{\alpha^2}{4}\left(-2q(\alpha)pR + q(\alpha)p^2 + q(\alpha)R^2\right) - (\lambda + d_i)R^2(1 - q(\alpha))\right] \tag{17}$$

The expression $w(p) = q(\alpha)p^2 - 2q(\alpha)pR + R^2$ decreases in p for all $p < R$. Evaluating it at the highest feasible p: $w(p = R) = q(\alpha)R^2 - 2q(\alpha)R^2 + R^2 = R^2(1 - q(\alpha))$. Therefore the derivative is positive if $\frac{\alpha^2}{4} > (\lambda + d_i)$.

OA 1.A.4 Equilibrium: Proof of Proposition OA 1

First, we assume that d is defined as in (5) and cost of verification is high so that $C > d$ so that banks prefer opacity. We show that in this case the equilibrium is characterized as in Proposition OA 1. The proof follows the same steps as in the case of Proposition 1.

1. Assumption OA 1.B ensures that funding is always sufficient for the price to be at least $p = q(\alpha)R$. Thus, if $S < \hat{S}(\alpha, \lambda, q(\alpha)R, d, 1, C)$ all banks play the solvent strategy, $p = q(\alpha)R$ and no bank has an incentive to deviate. A positive share of risk shifting banks cannot be sustained as equilibrium because it would result in $p' > q(\alpha)R$, in which case all banks prefer solvent strategy since $S < \hat{S}(\alpha, \lambda, q(\alpha)R, d, 1, C) < \hat{S}(\alpha, \lambda, p', d, 1, C)$.

2. If $S > \hat{S}(\alpha, \lambda, q(\alpha)R, d, 1, C)$ all banks playing the solvent strategy is not an equilibrium as banks have incentive to deviate. In equilibrium fraction of risk shifting banks is $\psi^*(\alpha, \lambda, d) =$

$\frac{\hat{p}(\alpha,\lambda,d)}{y_r^*}$. It is such that the price of the asset makes banks indifferent between the two strategies. The asset is overpriced at $p^* = \hat{p}(\alpha, \lambda, d) > q(\alpha)R$. This is a unique equilibrium because all banks risk-shifting would result in a price that is too high to sustain the risky strategy. To see this note that $\left(\frac{\alpha p}{2R}\right)^2 + p + \theta\lambda + d < \hat{S}(\alpha, \lambda, d, p)$ can be represented as:

$$M(p) = q(\alpha)(R - p) + (\lambda + d)(1 - q(\alpha)) < \frac{\alpha^2}{4}\left[1 - 2\frac{q(\alpha)p}{R} + \frac{q(\alpha)p^2}{R^2}\right] = N(p)$$

We have that $\arg\min_p N(p) = R$ and $\frac{\partial M}{\partial p} < 0$. Thus, $N(p) > M(p)$ for all $p \in (q(\alpha)R, R)$ if $N(p = R) > M(p = q(\alpha)R)$, which holds if:

$$\frac{\alpha^2}{4} - q(\alpha)R > \lambda + d$$

This is satisfied underr Assumption OA 1.A. Thus, fraction $\psi^*(\alpha, \lambda, d)$ of banks risk shifting is the unique equilibrium when $S > \hat{S}(\alpha, \lambda, p, d, 1, C)$

Second, we show that under these investment choices d is indeed defined by (5). If the inference is precise and $p = q(\alpha)R$, then $\mathbb{E}(L|p) = 0$. If inference is precise and $p = p^*(\alpha, \bar{\lambda}, d)$, then

$$\mathbb{E}(L|p) = \psi^*(\alpha, \bar{\lambda}, d)(1 - q(\alpha))\left[\frac{\alpha^2 p}{2R} - S - \bar{\lambda}(1 - \theta)\right] \tag{18}$$

If the inference is imprecise, two productivity estimates emerge: $\tilde{\alpha} = \tilde{\alpha}(p, 0)$ with probability κ or $\tilde{\alpha} = \tilde{\alpha}(p, \bar{\lambda})$ otherwise. In this case, the expected losses of large banks conditional on observed prices are:

$$\begin{aligned}\mathbb{E}(L|p) = & - \kappa\psi^*(\tilde{\alpha}(p, 0), 0)(1 - q(\alpha))\left[\frac{\tilde{\alpha}(p, 0)^2 p}{2R} - S\right] \\ & - (1 - \kappa)\psi^*(\tilde{\alpha}(p, \bar{\lambda}), \bar{\lambda})(1 - q(\alpha))\left[\frac{\tilde{\alpha}(p, \bar{\lambda})^2 p}{2R} - S - \bar{\lambda}(1 - \theta)\right]\end{aligned} \tag{19}$$

Third, as d, $\hat{S}$ and investment choices do not depend on the cost of verification C, there exists C high such that $\chi\mathbb{E}(L|p) < C$ in which case, according to Lemma OA 2, opacity is preferred.

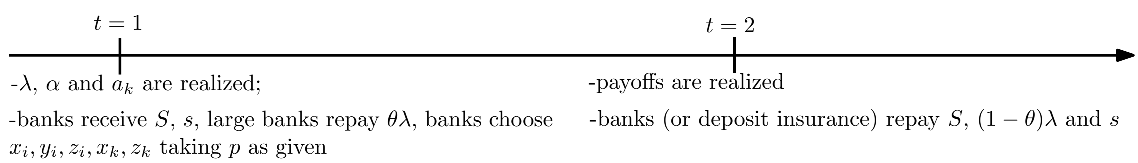
OA 2. Extensions

OA 2.1 Investment into the risky asset by small banks

In this section we allow for investment into the risky asset by small banks, by assuming both large and small banks have access to that market. Let y_k denote the investment in the asset by an small bank k. For tractability we focus on the case when $\gamma = 0$ and $\sigma = 0$ so that only aggregate productivity matters for all small banks.

The timing is as follows:

Figure 2: Timeline with small banks investing into the risky asset



When small banks have technology to operate the asset and earn its proceeds, the first best allocation must satisfy $\int y_i di + \int y_k dk = 1$, but does not specify which of the bank types should be holding the asset.

Strategies of small banks Since small banks do not observe α, they make their purchase of the risky asset conditional on the equilibrium price and the information about productivity that it contains. The demand by large banks is characterized in Lemma 3. Demand by small banks depends on the posterior beliefs about productivity associated with a given price. Because all small banks have the same information they are able to form a posterior from the price that they affect as a group, by backing out their demand. Let $f(\alpha|p)$ denote the posterior beliefs about the productivity given p, then small banks' problem is.

$$
\begin{aligned}
max_{x_k,y_k,z_k} & \int_\alpha \max\left[q(\alpha)\left(\alpha\sqrt{x_k} + Ry_k - x_k - y_k\right), 0\right] f(\alpha|p) d\alpha \\
& + \int_\alpha \max\left[(1-q(\alpha))\left(\alpha\sqrt{x_k} - x_k - y_k\right), 0\right] f(\alpha|p) d\alpha
\end{aligned} \tag{20}
$$

Investment under precise inference If there is only one productivity estimate corresponding to the given price then the small bank may be playing a solvent or a risk shifting strategy. The solvent strategy is characterized by $x_k^s = \frac{\tilde{\alpha}^2}{4}$ if $p > q(\tilde{\alpha})R$ and $x_k^s = \frac{\tilde{\alpha}^2 p^2}{4q^2(\tilde{\alpha})R^2}$ otherwise. The demand for risky asset follows from:

$$y_k^s(\tilde{\alpha}, q(\alpha)) = \begin{cases} 0 & \text{if } p > q(\tilde{\alpha})R \\ y \in (0, s - \frac{\tilde{\alpha}^2}{4}) & \text{if } p = q(\tilde{\alpha})R \\ s - \frac{\tilde{\alpha}^2}{4} & \text{if } p < q(\tilde{\alpha})R \end{cases} \tag{21}$$

In the risk shifting strategy the small bank lends $x_k^r = \frac{\tilde{\alpha}^2 p^2}{4R^2}$ and invests all the remaining funding into the risky asset $y_k^r = s - \frac{\tilde{\alpha}^2 p^2}{4R^2}$. These strategies are thus equivalent to those of a large bank.

The risk shifting threshold of small banks corresponds to the one faced by large banks in case of no losses $\lambda = 0$. Thus if $s < S$ then small banks do not have risk shifting incentives when large banks prefer to play a solvent strategy.

Moreover, as the risk shifting thresholds of large banks increases in price less steeply when $\lambda > 0$ (as can be seen in proof of Lemma 3 in Appendix A.1), we have that for any given α large banks are indifferent between risk shifting and solvent strategy at a weakly higher price level than small banks. Consequently, if risk shifting occurs, it is by large banks. Their individual risk shifting threshold pins down the price, which is too high for small banks to have incentives to play the risk shifting strategy.

When large banks are constrained, so that inference is precise because it is clear that they are in the missed boom, the extra demand for the risky asset by small banks relaxes the cash in the market constraint and inflates price. This allows large banks to lend more, moving their lending closer to the first best.

Investment under imprecise inference Now consider the case when equilibrium price results in imprecise inference with two productivity estimates:

$$\alpha = \begin{cases} \tilde{\alpha} & \text{with probability } \kappa \\ \tilde{\alpha}' & \text{with probability } 1-\kappa \end{cases} \tag{22}$$

Where $\tilde{\alpha} > \tilde{\alpha}'$ and $\mathbb{E}(\alpha|p) = \kappa\tilde{\alpha} + (1-\kappa)\tilde{\alpha}'$.

The problem can be represented as:

$$max_{x_k,y_k,z_k}\kappa\left[\underbrace{\max\left[q(\tilde{\alpha})\left(\tilde{\alpha}\sqrt{x_k}+Ry_k-x_k-y_k\right),0\right]}_{=W}+\underbrace{\max\left[(1-q(\tilde{\alpha}))\left(\tilde{\alpha}\sqrt{x_k}-x_k-y_k\right),0\right]}_{=V}\right]+$$
$$(1-\kappa)\left[\underbrace{\max\left[q(\tilde{\alpha}')\left(\tilde{\alpha}'\sqrt{x_k}+Ry_k-x_k-y_k\right),0\right]}_{=M}+\underbrace{\max\left[(1-q(\tilde{\alpha}'))\left(\tilde{\alpha}'\sqrt{x_k}-x_k-y_k\right),0\right]}_{=N}\right] \tag{23}$$

Uncertainty about the true value of productivity leads to a richer set of strategies available to small banks:

1. If $W>0, V>0, M>0, N>0$, solvent strategy: $x_k^s = \frac{\mathbb{E}(\alpha|p)^2}{4}$ if $p \geq (q(\tilde{\alpha})\kappa + q(\tilde{\alpha}')(1-\kappa))R = \tilde{q}R$ or $x_k^s = \frac{\mathbb{E}(\alpha|p)^2p^2}{4\tilde{q}^2R^2}$ and $y_k^s(\mathbb{E}(\alpha|p),\tilde{q})$ defined as in (21); feasible if $\tilde{\alpha}'\sqrt{x_k^s} - x_k^s - y_k^s > 0$ (small banks do not risk default in any state)

2. If $W>0, V>0, M>0, N=0$, risk shifting due to the losses in the case of zero asset payoff and low realization of productivity: $x_k^{ral} = \frac{(\kappa\tilde{\alpha}+(1-\kappa)q(\tilde{\alpha}')\tilde{\alpha}')p^2}{4\tilde{q}^2R^2}$ and $py_k^{ral} = s - x_k^{ral}$ if $p < \tilde{q}R$; feasible if $\tilde{\alpha}'\sqrt{x_k^{ral}} - s < 0$ (small banks have incentive to over invest in the risky asset if productivity is low), $\tilde{\alpha}'\sqrt{x_k^{ral}} + Ry_k^{ral} - s > 0$ (small banks do not have incentives to over-lend) and $\tilde{\alpha}\sqrt{x_k^{ral}} - s > 0$ (small banks do not have incentives to over invest in risky asset if productivity is high)

3. If $W>0, V=0, M>0, N=0$, risk shifting due to losses in the zero asset payoff state under both possible realizations of productivity ("pure asset-based risk shifting", akin to the one in the case of perfect inference): $x_k^{ra} = \frac{\mathbb{E}(\alpha|p)^2p^2}{4R^2}$ and $py_k^{ra} = s - x_k^{ra}$ if $p < R$; feasible if $\tilde{\alpha}\sqrt{x_k^{ra}} - s < 0$ (small banks have incentives to over invest in the risky asset) and $\tilde{\alpha}'\sqrt{x_k^{ra}} + Ry_k^{ra} - s > 0$ (small

banks do not have incentives to over-lend)

4. If $W > 0, V > 0, M = 0, N = 0$, risk shifting due to losses under low productivity realization ("over-lending", akin to the induced risk shifting in baseline) : $x_k^{rh} = \frac{\tilde{\alpha}^2}{4}$ if $p < q(\tilde{\alpha})R$ or $x_k^{rh} = \frac{\tilde{\alpha}^2 p^2}{4q^2(\tilde{\alpha})R^2}$ if $p \geq q(\tilde{\alpha})R$; and $y_k^{rh} = y_k^*(\tilde{\alpha}, q(\tilde{\alpha}))$; feasible if $\tilde{\alpha}'\sqrt{x_k^{rh}} + Ry_k^{rh} - s < 0$ (small banks have incentives to over-lend) and $\tilde{\alpha}\sqrt{x_k^{rh}} - s > 0$ (small banks do not have incentives to over-invest in the risky asset if productivity is high)

5. If $W > 0, V = 0, M = 0, N = 0$, risk shifting due to losses under low realization of productivity and under zero asset payoff: $x_k^{rah} = \frac{\tilde{\alpha}^2 p^2}{4R^2}$ and $y_k^{rah} = s - x_k^{rah}$ if $p < R$; feasible if $\tilde{\alpha}'\sqrt{x_k^{rah}} + Ry_k^{rah} - s < 0$ (small banks have incentive to over lend) and $\tilde{\alpha}'\sqrt{x_k^{rah}} - s < 0$ (small banks have incentives to over invest in risky asset if productivity is high)

The choice between these strategies depends whether they are feasible and more profitable. This depends on the size of the estimates of productivity $\tilde{\alpha}, \tilde{\alpha}'$, as well as their respective likelihoods, $\kappa, 1-\kappa$, the level of funding supply available to small banks, s, and the persistence of productivity, $q(\alpha)$. Critically, small banks face no uncertainty regarding the realizations of these parameters and hence know what are the optimal actions of other small banks. Hence, imperfect inference from prices is driven solely by uncertainty regarding the incentives and actions of large banks.

The key difference between the strategies available to small banks when inference is imprecise, relative to the case of the precise inference is the proliferation of risk shifting strategies, corresponding to different types and degrees of risk.

Small banks can risk shift by investing in the risky asset, in which case they under-invest in productive lending and take a risk of default by taking on high exposure in the asset. The degree of their under-lending depends on their perceived likelihood of default. If banks risk default only when asset payoff is low and the true productivity is low (strategy 2.) they under-invest less than when they risk default whenever asset payoff is low (strategy 3.). Relative to the case of precise inference small banks now face more risk and thus may have higher incentives to risk shift.

Small banks can also risk shift by lending excessively, as in the baseline model (strategy 4.). Possibility to invest in a risky asset may push up the level of uncertainty needed to make this type of risk shifting optimal whenever, $p < \tilde{q}R$, in which case small banks can earn a positive return on the asset (which dampens their risk incentives).

Access to the risky asset may also give rise to risk shifting that bets on both high realization of productivity and the high payoff of the asset (strategy 5.). This strategy is characterized by lower level of over-lending, relative to taking a gamble only on productivity in strategy 4. At the same time this strategy involves a lower exposure to the risky asset than taking risk only through the asset (strategy 3.).

As in the case of precise inference, demand for the risky asset by small banks can help ameliorate the severity of a "missed boom" by inflating asset prices. This decreases the opportunity cost of lending for large banks leading them to lend more efficiently. On the other hand, if the true underlying outcome of the large banks' subgame is a "good boom" demand for the risky asset by small banks can result in prices that exceed the expected payoff of the asset. In this case risky asset holding would be concentrated in the hands of small banks.

Thus, relative to the baseline model, when small banks can also invest in the risky asset, imprecise inference may continue to distort their investment. It can result in over- or under-lending when small banks play a solvent strategy, but can also induce them to engage in one of the large variety of risk shifting strategies.

OA 2.2 Correlation between productivity and losses

In the model aggregate productivity and losses on past assets are assumed to be independent of one another. Allowing for these shocks to be correlated does not alter the basic result of the paper as long as the support of the distribution of productivity conditional on a high level of losses overlaps sufficiently with the support conditional on the low level of realized losses .

Let $F_{\alpha|\lambda}(\alpha|\lambda)$ and $f_{\alpha|\lambda}(\alpha|\lambda)$ be the CDF and PDF of productivity conditional on λ. In this case an equilibrium price $\tilde{p}$ belongs to an the imprecise inference set if $\tilde{p} = p^*(\tilde{\alpha}, \bar{\lambda})$, $\tilde{p} = p^*(\tilde{\alpha}', 0)$ and $f_{\alpha|\lambda}(\tilde{\alpha}|\bar{\lambda}) > 0$,$f_{\alpha|\lambda}(\tilde{\alpha}'|0) > 0$.

In this case small banks form the following posterior about the productivity distribution:

$$\alpha = \begin{cases} \tilde{\alpha}' & \text{with prob.} \frac{f_{\alpha|\lambda}(\tilde{\alpha}'|0)\kappa}{f_{\alpha|\lambda}(\tilde{\alpha}'|0)\kappa + f_{\alpha|\lambda}(\tilde{\alpha}|\bar{\lambda})(1-\kappa)} \\ \tilde{\alpha} & \text{with prob.} \frac{f_{\alpha|\lambda}(\tilde{\alpha}|\bar{\lambda})(1-\kappa)}{f_{\alpha|\lambda}(\tilde{\alpha}'|0)\kappa + f_{\alpha|\lambda}(\tilde{\alpha}|\bar{\lambda})(1-\kappa)} \end{cases} \tag{24}$$

Correlation between productivity and the size of losses only affects the probability of the productivity estimates through its impact on the conditional probabilities.

Consider the case of negative correlation, i.e. low legacy losses are more likely when productivity is high and high legacy losses are more likely when productivity is low. Since missed booms occur when productivity is high, $\alpha > \overline{\alpha}(\lambda)$, missed booms characterized by high losses are less likely than in the case of independent shocks. As bad boom occur when productivity is low $\alpha < \hat{\alpha}(\lambda)$, bad booms characterized by high losses are more likely.

To illustrate the implications for the inference problem consider the case when the price falls in the imprecise inference set driven by the bad boom. Due to correlation the probability of the productivity estimate that corresponds to high losses (i.e. the low productivity) is higher than in the case of independent shocks. This lowers the overall lending if the estimates are not too dispersed, so that small banks lending is based on the expected productivity. It also makes makes the risk-shifting by small banks less likely as the probability that high estimate is the true productivity is lower.

OA 2.3 Endogenous funding level

In this exercise we endogenize large banks' choice of funding level by assuming that they face a price-elastic supply from savers. We show that the three types of booms may emerge also in this setting and discuss the conditions under which inference by small banks may be imprecise.

We assume that large banks face an aggregate funding supply schedule from depositors given by: $D^S(r)$, where r is the interest rate on deposits, with $\frac{\partial D^S}{\partial r} > 0$. Let $S_i(r)$ denote the amount of deposits that bank i receives in equilibrium at $t = 0$ when the interest rate is r. Banks need to repay $rS(r)$ at $t = 1$. Define $w = \max\left[r, \max\left[\frac{q(\alpha)R}{p}, 1\right]\right]$, then banks' strategies can be represented as below.

- Solvent strategy:

$$x_s = \left(\frac{\alpha}{2w}\right)^2$$

$$py_s = \begin{cases} S_i(r) - x_s - \theta\lambda & \text{if } w = \frac{q(\alpha)R}{p} \wedge w \neq 1 \\ py \in (S_i(r) - x_s - \theta\lambda, 0) & \text{if } w = \frac{q(\alpha)R}{p} \wedge w = 1 \\ 0 & \text{if } w \neq \frac{q(\alpha)R}{p} \end{cases}$$

$$z_s = \begin{cases} S_i(r) - x_s - \theta\lambda & \text{if } w = 1 \wedge w \neq \frac{q(\alpha)R}{p} \\ z \in (S_i(r) - x_s - \theta\lambda, 0) & \text{if } w = 1 \wedge w = \frac{q(\alpha)R}{p} \\ 0 & \text{if } w \neq 1 \end{cases}$$

$$D_s^D = \begin{cases} \infty & \text{if } w \neq r \\ D \in (x_s + \theta\lambda, \infty) & \text{if } w = r \wedge (w = \frac{q(\alpha)R}{p} \vee w = 1) \\ x_s + \theta\lambda & \text{if } w = r \end{cases}$$

- Risk shifting strategy :

$$x_r = \left(\frac{\alpha p}{2R}\right)^2 \qquad y_r^* = \frac{S_i(r) - x_r - \theta\lambda}{p} \qquad z_r = 0 \qquad \text{if } r < \frac{R}{p}$$

$$D_r^D = \infty \qquad \text{if } r < \frac{R}{p}$$

Banks compare the cost of funding to the return that they can earn on storage and the return on the risky asset to determine the opportunity cost of lending in the solvent strategy, the amount of deposits that they demand and how they allocate the residual funds. The risk shifting strategy is the same as in the case of exogenous funding, just that now it can only exist when cost of funding is lower than the upside return on risky asset.

Banks playing the solvent strategy are indifferent between any amount of funding $D \in (x_s + \theta\lambda, \infty)$ as long as $r = \frac{q(\alpha)R}{p} \geq 1$ or $r = 1 \geq \frac{q(\alpha)R}{p}$. In the former case they are indifferent between more funding and less risky asset holding, in the latter between more funding and less investment in storage. When interest rate is $r < \max\left[\frac{q(\alpha)R}{p}, 1\right]$, banks playing the solvent strategy have an infinite demand for funding. If interest rate is higher ($r > \max\left[\frac{q(\alpha)R}{p}, 1\right]$), they choose to not allocate any funding to storage or asset and demand $D^D(r) = \frac{\alpha^2}{4r^2} + \theta\lambda$. Notice that $D^S(r) = \frac{\alpha^2}{4r^2} + \theta\lambda$ can never be an equilibrium as null demand for the asset would lead to price equal to zero, so that $\frac{q(\alpha)R}{p} > r$.

The profits that a bank can earn by playing the solvent strategy is higher than those under the risk shifting strategy if $S_i(r)$ is higher than the "individual risk shifting threshold" given by:

$$\hat{S}(\alpha, \lambda, p, r) = \begin{cases} \frac{\alpha^2 p}{4q(\alpha)R}\frac{(1+q(\alpha))}{r} - \frac{\lambda(1-\theta)}{r} & \text{if } w = \frac{q(\alpha)R}{p} \\ \frac{\frac{\alpha^2}{4}\left(1-\frac{q(\alpha)p}{R}\right)-\lambda(1-\theta)(1-q(\alpha))+\theta\lambda\left(\frac{q(\alpha)R}{p}-1\right)}{q(\alpha)\left(\frac{R}{p}-r\right)-(1-r)} & \text{if } w = 1 \\ \frac{\frac{\alpha^2}{4}\left(\frac{1}{r}-\frac{q(\alpha)p}{R}\right)-(1-\theta)\lambda(1-q(\alpha))+\theta\lambda\left(\frac{q(\alpha)R}{p}-r\right)}{q(\alpha)\left(\frac{R}{p}-r\right)} & \text{if } w = r \end{cases} \tag{25}$$

We can also define the level of funding at which resources available to large banks are sufficient to achieve the fair price of the risky asset $p = q(\alpha)R$ under the solvent strategy as the "balanced funding threshold", given by $\underline{S}(\alpha, \lambda) = \frac{\alpha^2}{4} + \theta\lambda + q(\alpha)R$.

In equilibrium we must have that both asset and deposit markets clear. The latter implies that $D^D(r) = \int_i D_i^D(r) = D^S(r)$.Using the two thresholds together with the characterization of bank's strategies we can plot the demand and supply of funding for a given level of price. With the help of these plots we show how the three scenarios discussed in the baseline model (good, bad and missed boom) can emerge and the conditions under which each of these constitutes an equilibrium.

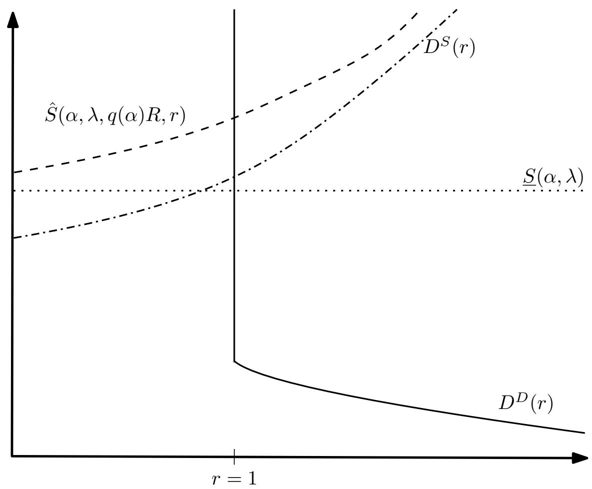
Good boom First, consider the case when $p = q(\alpha)R$ and all banks play the solvent strategy. Market clearing in deposits implies that funding available to banks is $S = D^S(1)$. This constitutes an equilibrium if:

- At $D^S(1)$ funding is sufficient to achieve the fair price $p = q(\alpha)R$: $D^S(1) > \underline{S}(\alpha, \lambda)$, so that asset and deposit markets clear at postulated prices

- At $D^S(1)$ funding is not too high, so that banks have no incentives to deviate to the risk shifting strategy: $D^S(1) < \hat{S}(\alpha, \lambda, q(\alpha)R, 1)$

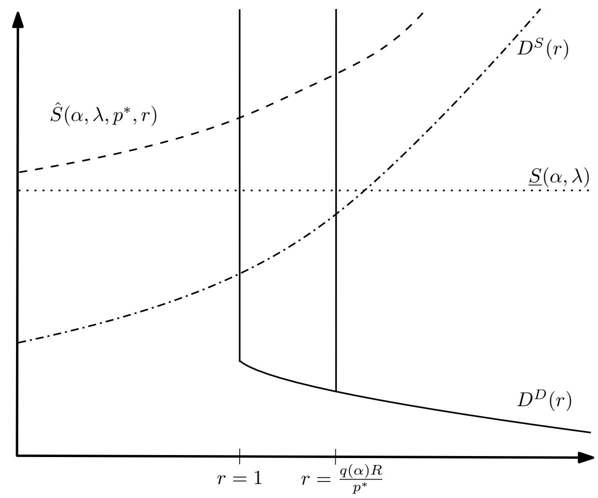
If these conditions are satisfied a "good boom", with bank lending equal to $x = \frac{\alpha^2}{4}$, and fair asset price, $p = q(\alpha)R$ constitutes an equilibrium. This situation is illustrated in a figure below.

Figure 3: Demand and supply for funding in the good boom case. Funding demand under the solvent strategy is represented by the solid curve, funding supply is given by the dash-dotted curve, the dotted line represents the "balanced funding threshold" and the dashed curve is the "individual risk shifting threshold".



Missed boom Next, consider an alternative setting, in which the level of funding supply available at any level of interest rate is lower than in the good boom scenario. We illustrate this in a figure below.

Figure 4: Demand and supply for funding in the missed boom case. Funding demand under the solvent strategy is represented by the solid curve, funding supply is given by the dash-dotted curve, the dotted line represents the "balanced funding threshold" and the dashed curve is the "individual risk shifting threshold"



Assume now that all banks play the solvent strategy and the prevailing asset price is such that $p = q(\alpha)R$. In this case the corresponding deposit market clearing would imply that amount of funds available to banks is $S = D^S(1)$. However, since $D^S(1) < \underline{S}(\alpha, \lambda)$ this level of funding is too low for asset demand by solvent-playing banks to achieve $p = q(\alpha)R$. This is a contradiction so $p = q(\alpha)R$ and $r = 1$ cannot be equilibrium for these parameter values.

Consider then a case when $p = p^* < q(\alpha)R$. At this price level the demand for funding is infinite if $r = 1$, and banks are indifferent between any amount of funding, $D \in (x_s + \theta\lambda, \infty)$ if $r = \frac{q(\alpha)R}{p^*} > 1$ (represented by the right-most vertical line in the plot). Thus, the deposit market clears if $S = D^S\left(\frac{q(\alpha)R}{p^*}\right)$. This constitutes an equilibrium if:

- At $D^S\left(\frac{q(\alpha)R}{p^*}\right)$ the demand for the asset results in the postulated price p^*, that is it solves: $p^* = D^S\left(\frac{q(\alpha)R}{p^*}\right) - \left(\frac{\alpha p^*}{2q(\alpha)R}\right)^2 - \theta\lambda$.

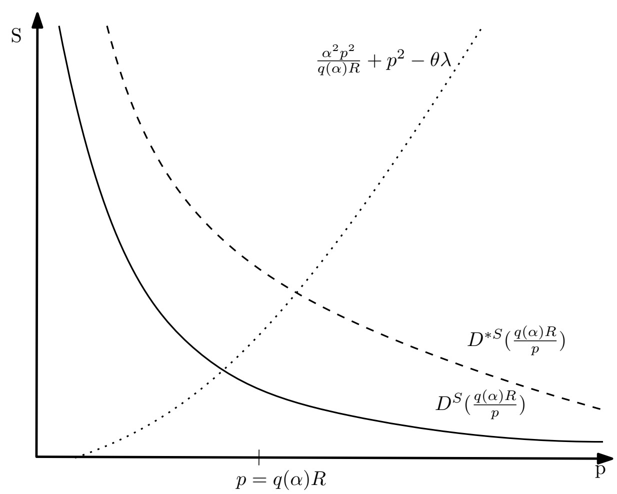
- The resulting price is lower than the expected payoff of the asset, $p < p^*$, which is the case if $D^S\left(\frac{q(\alpha)R}{p^*}\right) < \underline{S}(\alpha, \lambda)$, which ensures that the cash in the market pricing equation is applied only when it is relevant;

- At $D^S\left(\frac{q(\alpha)R}{p^*}\right)$ the funding supply available to banks is too low for the deviation to the risk shifting strategy to be profitable: $D^S\left(\frac{q(\alpha)R}{p^*}\right) > \hat{S}\left(\alpha, \lambda, p^*, \frac{q(\alpha)R}{p^*}\right)$

If these conditions are satisfied, a "missed boom", characterized by low lending, $x = \frac{\alpha^2(p^*)^2}{4q^2(\alpha)R^2}$, and undervalued risky asset price, $p^* < q(\alpha)R$, constitutes an equilibrium.

The equilibrium price that emerges from the missed-boom pricing equation increases as the sensitivity of funding supply to interest rate increases (while keeping the level of supply at $D^S(0)$ fixed). By rearranging the pricing equation we get: $p^* + \left(\frac{\alpha p^*}{2q(\alpha)R}\right)^2 + \theta\lambda = D^S(\frac{q(\alpha)R}{p^*})$. An increase in $\frac{\partial D^S}{\partial r}$, that keeps the value of $D^S(0)$ unchanged implies that $D^S(r)$ now takes higher values for every $r > 0$. This means that the solution of the market clearing equation yields a lower value of r^* and thus a higher level of price p^*. We illustrate this in a figure below, where schedule $D^S(r)$ is characterized by lower sensitivity to changes in interest rate than $D^{*S}(r)$, $\frac{\partial D^S}{\partial r} < \frac{\partial D^{*S}}{\partial r}$.

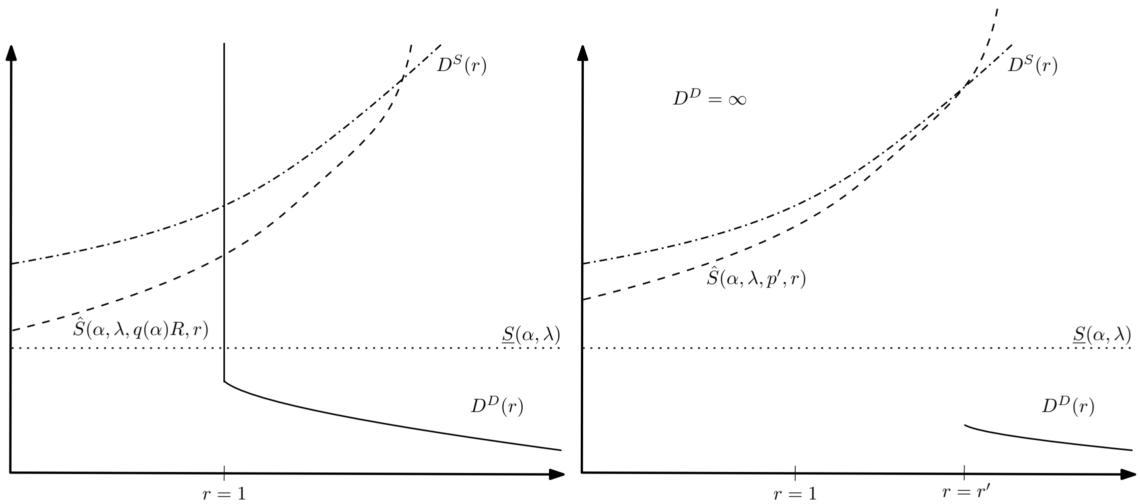
Figure 5: Shift in funding supply under a "missed boom". The dotted line represents the funding demand, the solid curve corresponds to the fudning supply with a low sensitivity to change in interest rates and the dashed curve represents funding supply with a high sensitivity to the change in interest rates.



As a consequence a missed boom may cease to exist when funding supply is too responsive to changes in interest rates, as in this case the market clearing can imply a price $p > q(\alpha)R$.

Bad boom Finally, consider the case when funding supply is high for all levels of the interest rate, illustrated in the left and right panel of a figure below.

Figure 6: Demand and supply for funding, bad boom case. Funding demand under the solvent strategy is represented by the solid curve, funding supply is given by the dash-dotted curve, the dotted line represents the "balanced funding threshold" and the dashed curve is the "individual risk shifting threshold". Funding demand under the risk shifting boom is infinite.



First, let's suppose that all banks play the solvent strategy and the price is $p = q(\alpha)R$, so that the demand for funding is given by the red curve in the left panel of the figure above. In this case, the market clearing level of funding available to banks is higher than the risk shifting threshold $S = D^S(1) > \hat{S}(\alpha, \lambda, q(\alpha)R, 1)$. This means that banks have incentives to deviate to play a risk shifting strategy and all banks playing the solvent strategy is not an equilibrium.

Under the risk shifting strategy banks demand infinite funding whenever $S(r) > \hat{S}(\alpha, \lambda, p, r)$. Now assume that the asset market clears at $p = p' > q(\alpha)R$. If that's the case, the deposit market can be in equilibrium only if the interest rate is such that $\hat{S}(\alpha, \lambda, p', r') = D^S(r')$. At this level depositors have no incentives to deviate from $r = r'$. They would earn less under lower interest rates. Meanwhile charging a higher rate would result in a discontinuous drop in demand to $D^D(r) = \frac{\alpha^2}{4} + \theta\lambda$ (as banks would lose their risk shifting incentives). The asset market is in equilibrium if at $S = D^S(r')$ large banks' demand is such that postulated price p' is achieved. This is the case if: $p' = \hat{S}(\alpha, \lambda, p', r') - \frac{\alpha^2 p'^2}{4R^2} - \theta\lambda$. We illustrate this example in the right panel of the figure above. If these solutions exist, a bad boom may constitutes an equilibrium. It is characterized by low level of lending, $x = \frac{\alpha^2 (p')^2}{4R^2}$, and overpriced risky asset, $p > q(\alpha)R$.

Thus, as in the case of exogenous and inelastic supply of deposits studied in the baseline model, the three types of booms can emerge in equilibrium. Whether each of these booms is feasible depends on the shape of the deposit supply schedule in relation to the two thresholds $\underline{S}$ and $\hat{S}$ and the asset demand function under the solvent and risk shifting strategy.

If small banks, which source their funding in the local market, do not observe the interest rate and the equilibrium quantity of funding in the large market their inference from asset price remains imprecise. This because asset price is still affected jointly by productivity and the level of legacy losses, which shape the two funding thresholds and thus determine feasibility of each of the three booms.

Impact of aggregate shocks and changes in funding supply In the remaining part of this section we discuss how changes in the shock realizations and in the shape of the funding supply schedule affect the likelihood of the three types of equilibrium discussed above. Let's assume that the starting point is a funding supply schedule and realizations of productivity and losses such that large banks sub-game is in the good boom.

An increase in productivity does not affect the deposit market clearing, however it pushes up the balanced funding threshold. If the increase is sufficiently high it may be that $D^S(1) < \underline{S}(\alpha, \lambda)$ and the good boom is no longer a feasible equilibrium. In this case, the missed boom may emerge. A decrease in productivity shifts downwards the risk shifting threshold. With a sufficiently large fall it may be that $D^S(1) > \hat{S}(\alpha, \lambda, q(\alpha)R, 1)$, so the good boom is no longer a feasible equilibrium and the bad boom may emerge.

If funding supply is positively correlated with changes in productivity, then an increase in productivity is less likely to generate a missed boom. Likewise a fall in productivity is less likely to lead to the emergence of the bad boom. This is because funding is more balanced relative to the quality of productive opportunities.

An increase in losses on legacy assets shifts the balanced funding threshold upwards and the risk shifting threshold downward. This makes the good boom feasible for a smaller set of productivity values.

An upward shift in the funding schedule results in a higher level of funding available at $r = 1$. The consequence can be that the good boom may no longer be sustained due to banks risk shifting incentives. A bad boom may emerge in this case. A downward shift in the funding schedule may imply that at $r = 1$ the supply of funding is too low to achieve the fair price of the asset, so the only feasible outcome is a missed boom.