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The Good, the Bad, and the Missed Boom

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Some credit booms result in financial crises. While excessive risk-taking could plausibly explain the boom-to-bust cycle, many investors do not anticipate increasing risk. We show that credit booms may be misunderstood as being driven by high productivity because opaque bank assets disguise risk incentives. Balanced funding relative to productive prospects can sustain prudent lending (good boom), whereas funding imbalances may induce high risk exposure and boost asset prices (bad boom) or lead to asset underpricing and insufficient lending (missed boom). Rational agents drawing inference from prices make mistakes that can amplify the effect of funding imbalances and propagate risk. (*JEL* G01, G21, D83)

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In the classic view of business cycles, real shocks drive the volume of credit and financial intermediation affects the cycle only via contractual frictions. However, recent evidence points to an independent role of credit supply in explaining output fluctuations (Krishnamurthy and Vissing-Jorgensen 2012; Mian, Sufi, and Verner 2017). Credit booms with weak productivity are more likely to end in crisis (Gorton and Ordoñez 2016; Mian, Sufi, and Verner 2019). This suggests that instability may stem from excess lending relative to productive demand and raises the question of why market participants condone such an unbalance.

A common argument is that the recent financial crisis was caused by deliberate risk-taking, while an alternative cause for excess credit may have

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been investors' inability to assess the accumulation of risk. During the 2002–2007 credit boom, financial prices, such as bank equity returns and credit spreads, suggested that investors did not anticipate rising losses (Baron and Xiong 2017; Krishnamurthy and Muir 2016). More generally, historical studies of credit expansions have documented that financial instability tends to come as a surprise (Reinhart and Rogoff 2009; Richter, Schularick, and Wachtel 2018).

Why would most investors and banks not recognize increasing risk? Behavioral biases are likely to play a role (Gennaioli, Shleifer, and Vishny 2015). Yet even rational agents may be unable to correctly assess the state of the economy from distorted signals. This paper offers a rational explanation for episodes of excess credit, where deliberate risk-taking by opaque financial institutions may induce further amplification due to imprecise inference by other players. It thus reconciles the risk-shifting view with the evidence of market participants underestimating rising risk during booms and identifies a complementary possibility of underinvestment due to impaired inference.

Our analysis points to a novel implication of bank opacity for financial stability. While existing literature recognizes its potential in undermining the monitoring of bank's risk-taking (Boot and Thakor 1997; Demirgüç-Kunt, Detragiache, and Tressel 2008; Danisewicz et al. 2018), our framework shows that bank opacity can also amplify investment inefficiencies and propagate risk incentives to other players by impairing inference about aggregate prospects from prices. Critically, opacity plays this role only if banks' incentives are potentially distorted by funding imbalances and initial leverage. This is a new mechanism by which information conveyed in asset prices can lead to inefficient decisions by less-informed agents. Relative to the previous work, it does not rely on feedback from learning (Bond, Edmans, and Goldstein 2012; Bond and Goldstein 2015) and instead is based solely on the incentives and informational advantage of well-informed players.

The setup comprises of large and small banks funded by a fixed amount of insured deposits.[1] Large banks represent intermediaries that operate broadly across a national or global market, a role allowing them to gauge a precise signal on aggregate productivity. They can lend to a diversified pool of firms at a decreasing marginal return, invest into a safe storage or a risky asset, whose expected payoff is correlated with aggregate productivity (such as long-term claims on commercial real estate or large corporations). Small banks specialize in issuing loans to local firms affected by aggregate and idiosyncratic productivity shocks. They can also invest in the safe storage technology. Their local expertise allows them to observe the location-specific idiosyncratic information, but leaves them uncertain about the aggregate productivity.

[1] While in reality not all bank liabilities are insured, banks enjoy implicit guarantees that reduce the sensitivity of funding costs to risk.

The key assumption is that banks' asets are opaque, so outsiders cannot precisely observe their leverage or portfolio allocation. Actual bank leverage is difficult to assess due to hidden contingent liabilities, indirect risk exposures via off balance sheet operations or unrecognized losses (Huizinga and Laeven 2012).[2] Opacity implies that small banks are unable to learn about the aggregate productivity by directly observing the investment choices of large intermediaries and must instead draw inference from the observable price of the risky asset. However, learning from the price can be complicated by the fact that large banks enter the game with an opaque share of legacy credit losses, which affects their initial leverage and consequently risk incentives. Their strategy choice depends also on the balance between the available funding and the quality of new investment opportunities, as excessive funding enables banks to take more leverage. Through its impact on incentives, the combination of losses and productivity gives rise to three equilibrium regions in the large banks' subgame.

If losses are low, and aggregate prospects are strong, large banks choose a solvent strategy. It is characterized by the efficient level of productive lending and a limited scale of risky investment, both of which ensure that large banks do not default (*good boom*). Under such balanced circumstances the risky asset price reflects its fundamental value. If initial leverage is high and funding is excessive relative to prospects, large banks have an incentive to risk shift by taking a large exposure to the risky asset at the expense of productive lending. A mixed equilibrium emerges in which some large banks risk shift, whereas others invest prudently. The intuition is that the price of the asset increases in the share of risk-shifting banks, undermining the relative attractiveness of this strategy (similar to Allen and Gale 2004). Risk shifting results in inefficient lending by large banks and inflates asset price above its expected value (*bad boom*).[3] If both legacy losses and the aggregate prospects are very high, large intermediaries have insufficient funding to use all valuable opportunities. This results in cash-in-the-market pricing of the long-term asset, which induces insufficient productive lending (*missed boom*).

Critically, asset prices in the bad and the missed boom depend on the realizations of both productivity and legacy losses, laying foundation for imprecise inference. Because of the opacity of banking assets, outsiders cannot distinguish between the impact of these two shocks on the price and thereby draw an impaired inference about aggregate prospects. This occurs because the same asset price may reflect either strong economic fundamentals and moderate initial leverage of large banks (good boom) or weak prospects,

[2] During the 2002–2007 credit boom, banks were able to shift assets to shadow banks while retaining exposure. Large banks even hid assets outright, as in the case of Lehman's Repo 105, a U.K. legal entity that held around 10% of Lehman's risky assets and leverage. In Japan, so-called "tobashi" schemes hide losses by shifting assets across accounts before reporting dates.

[3] Moral hazard justifies large banks' investment in the asset even when it is overpriced as in Allen and Gorton (1993).

coupled with high initial leverage (bad boom). Similarly, for other price levels small banks may be unable to distinguish between a good boom and a missed boom with high productivity but insufficient funding due to past losses.

Imprecise inference by small banks results in lending mistakes. Small banks lend excessively during the bad boom, when aggregate productivity is low and large banks risk shift, because they cannot exclude the possibility of a good boom. Likewise, they lend too little if the aggregate productivity is high and large banks have insufficient funding to make use of all opportunities, because they are unsure if large banks are indeed in the missed boom. The result is an amplification of investment inefficiencies associated with the bad and the missed boom, more excessive investment in the case of the former and larger insufficiency of credit in the case of the latter.

Here, uninformed banks are rational, so their inference mistakes are zero in expectation. However, this does not imply that their average lending is generally unbiased. When inference is sufficiently imprecise, small banks' incentives are distorted. High uncertainty about productivity may lead them into a form of *induced risk shifting*, where they bet on the higher productivity estimate in their lending decision. This way, the imprecise inference may propagate risk incentives from large to small intermediaries.

In our model, three key elements interact to cause the imprecise inference and the resultant amplification of investment and propagation of risk: limited liability, cash in the market pricing and the opacity of bank assets. The opacity obscures large banks' leverage. This can impair the inference only if prices are affected by both initial leverage and aggregate prospects, as is the case in a bad or a missed boom. These two types of outcomes arise because, in the presence of funding imbalances, limited liability and cash in the market pricing distort banks' incentives. The interaction of these elements gives rise to a new mechanism through which learning from prices may lead to inefficient decisions.

The problem of imprecise inference could be resolved by investment in better information by small banks or stress testing of the opaque banks by a regulator. However, these solutions are suboptimal if funding imbalances are viewed as rare or the cost of producing information is high. This makes the results particularly relevant to periods of perceived moderation, when large shocks are seen as relatively unlikely, or to markets characterized by high complexity, which obstructs precise assessment of the aggregate prospects and quality of banks' assets.

Our analysis applies to institutions covered by deposit insurance with expertise in issuing assets that may be difficult to value for outsiders, such as commercial or universal banks with large portfolios of opaque loans. It is especially relevant for banks with substantial off-balance sheet exposures, which further obscure the true value of leverage and risk.[4] On the other hand, the approach is not suited to study learning from assets in more transparent markets. For instance, the mechanism that we discuss is unlikely to matter in the context of institutions, such as mutual funds with a transparent portfolio of investments in tradable assets and risk-sensitive funding.

1. Related Literature

The opacity of banks' assets is a centerpiece of our analysis. Evidence suggests that banks indeed exalt the value of their assets (Huizinga and Laeven 2012) and outsiders have difficulty assessing banks' riskiness (Morgan 2002). Our baseline model takes opacity of banks' assets as given.[5] This view can be motivated by the nature of bank lending. Active monitoring allows banks to lend to opaque borrowers (Diamond 1984; James 1987), making loans difficult to value for outsiders (Greenspan 1996). Information on credit risk is a key intangible asset for banks, and its disclosure would allow competitors to free ride and compete away the return to monitoring (Rajan 1992). The promise to maintain confidentiality of information also makes bank loans more attractive for borrowers (Campbell 1979).

Equally importantly, opacity has a key role in enhancing the liquidity of bank liabilities. Information insensitivity can be seen as a key feature of banking throughout its history, boosting the scale of intermediation and improving risk sharing (Dang et al. 2017). Transparency can reduce adverse selection in periods of distress but may be suboptimal in normal times (Alvarez and Barlevy 2021; Bouvard, Chaigneau, and Motta 2015; Faria-e Castro, Martinez, and Philippon 2017; Goldstein and Leitner 2018; Gorton and Ordoñez 2020). A well-recognized concern is that opacity may weaken market and supervisory discipline and enable risk-taking (Boot and Thakor 1997; Demirgüç-Kunt, Detragiache, and Tressel 2008; Fosu et al. 2017; Danisewicz et al. 2018). We contribute by identifying a novel channel through which opacity may hamper financial stability: by impairing the inference from prices it can lead to amplification of investment inefficiencies and propagation of risk incentives.

In studying the capacity of intermediaries to make mistakes in their assessment of economic conditions, we relate to the work of Thakor (2016), who finds that uncertainty over banker quality may result in under- or overestimation of risk. Also related is Lee (2018), who finds that investors cannot interpret bank lending by observing bank funding demand alone. In our setting less-informed

[4] As we show in Online Appendix OA1 our mechanism remains at play when funding costs are somewhat sensitive to risk. Thus, the analysis may apply also to shadow banks that rely on loan securitization, as long as their cost of funding does not fully reflect their risk. In practice, this may be the case due to implicit government guarantees, such as those extended to shadow banks during the financial crisis of 2007–2008 (Pozsar et al. 2010).

[5] In an extension, we show how banks may indeed prefer to keep their assets opaque if their funding costs are somewhat sensitive to asset risk and verification is costly (see Online Appendix OA1).

agents seek to learn about the quality of aggregate prospects from the actions of the better-informed large banks, as in McKinnon and Pill (1997). They draw inference from the asset price, which offers an imprecise signal about banks' choices.

The literature has long recognized the role of information contained in prices in generating feedback between financial markets and real decisions (Fishman and Hagerty 1989; Holmström and Tirole 1993). That prices can aggregate dispersed information may help imperfectly informed managers gain insight into the quality of opportunities and improve their investment choices (Boot and Thakor 1997; Subrahmanyam and Titman 1999; Foucault and Frésard 2012).

We contribute to the branch of this literature, which studies the "revelatory price efficiency," that is, the ability of prices to reveal information needed for efficient decision-making (Bond, Edmans, and Goldstein 2012). Revelatory price efficiency may fail due to the interaction between the use of information contained in prices and speculators' incentives to collect information or trade (Dow and Gorton 1997; Bond and Goldstein 2015). Another reason may be that the equilibrium price is nonmonotonic in the state variable, for instance, because payoff relevant intervention is informed by the observed price (Bond, Goldstein, and Prescott 2010). We provide a novel foundation for this nonmonotonicity, that does not depend on the feedback of learning from prices. Instead, in our framework, the nonmonotonicity of price in productivity is generated by the risk-taking and funding constraints of informed players.

Our approach is also related to the work on how excess confidence and asset prices may arise under rational learning (Pástor and Veronesi 2003; Biais, Rochet, and Woolley 2015). An alternative view holds that rising risk is not fully anticipated as agents do not form rational beliefs. Even a small fraction of overconfident agents can lead to excessive asset prices thanks to rising leverage (Geanakoplos 2010). Representativeness heuristics recognizes that the structure of the economy evolves, so that agents are likely to miss rising risk signals (Gennaioli, Shleifer, and Vishny 2015; Bordalo, Gennaioli, and Shleifer 2018). Our contribution to the literature is a rational benchmark that explains why public inference may be confused, without denying the role of limited rationality.

Empirically speaking, credit expansions accompanied by a more sustained productivity increase tend to produce stable growth (Gorton and Ordoñez 2016). Meanwhile, credit booms funded by foreign inflows or wholesale funding often lead to surging house prices and are more likely to end in crisis (Richter, Schularick, and Wachtel 2018). This suggests a role of imbalances between productive investment and funding volumes in driving instability. Trends in the 2002–2007 credit expansion in the United States also point to excess credit supply as a factor contributing to the build-up of risk (Justiniano, Primiceri, and Tambalotti 2019; Mian and Sufi 2009; Mian, Sufi, and Verner 2019). This is consistent with our view that risk incentives may be acute at the time of imbalance between bank funding and productive prospects.[6] We show how the interaction between such imbalances and uncertainty about banks leverage can complicate the inference from prices and propagate risk.

2. The Model

There are two dates ($t=1,2$) and two types of active agents: a unit mass of large banks indexed by $i \in \mathbb{G}$ and a unit mass of small banks indexed by $k \in \mathbb{L}$. Large banks represent well-diversified intermediaries with a broad scope of operations, for example at a national or global level. In contrast, small banks operate locally or specialize in a specific industry, so their payoffs are affected by both idiosyncratic and aggregate shocks. Additionally, large banks have access to better information about aggregate productivity, while each small bank observes only its idiosyncratic realization. At $t=1$ banks make investment choices that pay off at $t=2$. They are protected by limited liability and derive linear utility from positive profits.

There are three investment opportunities in the economy: productive lending and storage available to all banks, and a risky asset in fixed supply, for simplicity assumed to be available only to large banks.[7]

2.1 Productive lending and storage

Each large and small bank has access to a bank-specific pool of productive loans. Lending offers decreasing marginal returns at the bank level, reflecting a scarcity of good quality projects that require bank credit. Specifically, lending x_j by bank j yields a payoff of $f_j(x_j)=\alpha_j\sqrt{x_j}$ at $t=2$, where α_j is a measure of productivity of loans issued by that bank.

Each large bank, $j=i$, lends to a diversified pool of firms from across the economy, so its loans depend on aggregate productivity with $\alpha_i=\alpha$. Lending by a small bank, $j=k$, is concentrated in one local market and so its payoff is given by $\alpha_k=\gamma a_k+(1-\gamma)\alpha$, where a_k measures the idiosyncratic local productivity and γ captures its relative importance. Realizations of the aggregate and local productivity are drawn at $t=1$ from

$$\alpha \sim U\left[\alpha^L, \alpha^H\right]$$

$$a_k \sim U[-\sigma, +\sigma]$$

with $\gamma\sigma<(1-\gamma)\alpha^L$, so small banks always have access to productive lending opportunities.

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[6] Imbalances in the funding supply relative to productive opportunities may emerge because of demographic shifts, safety-seeking capital inflows (Caballero and Krishnamurthy 2008; Eichengreen 2015), or financial deregulation (Favara and Imbs 2015; Mian, Sufi, and Verner 2019).

[7] In Online Appendix OA 2.1, we relax this assumption and show that, as in the baseline, the inference remains imprecise, and it can result in investment mistakes by small banks and the propagation of risk (with a broader and more complex spectrum of risk choices available to small banks).

Large banks observe the realization of α at the beginning of $t=1$, while other agents only know its distribution. This assumption reflects large banks' broad scope of operations, which allows them to receive more signals about the aggregate economic conditions. Moreover, large banks may have the capacity to develop specialized research and forecasting departments, which allow them to more accurately assess the aggregate prospects. Each small bank privately observes the relevant realization of the local component of loan productivity, a_k. This reflects their informational advantage in the local markets that allows them to generate profit from dealing with their local clients.

Banks can also invest into a storage technology, whose payoff is normalized to one. This corresponds to banks' holdings of safe government bonds or reserves. The investment by a large and a small bank is denoted by z_i and z_k, respectively.

2.2 Risky asset

Large banks have access to a risky asset whose payoff at $t=2$ depends on the future state of the economy. A good future state yields a high payoff and is correlated with the current aggregate productivity. An interpretation is that the final payoff reflects the long-term value of the asset and depends on future prospects, which are more likely to be high when current fundamentals are strong. The asset represents long-term financial claims, for instance, on commercial real estate or large corporations. Its payoff, $g(y)$, is drawn from the following binomial distribution:

$$g(y)=\begin{cases} Ry & \text{with probability } q(\alpha) \\ 0 & \text{with probability } 1-q(\alpha) \end{cases}$$

The probability of the good state is $q(\alpha)\in(0,1)$, with $q'(\alpha)>0$, which can be viewed as a proxy for the persistence of the productivity shock. When the persistence is large, a high draw of productivity today implies a high likelihood of strong fundamentals in the future. This increases the probability of the good state and the high asset payoff.

The initial asset owners are uninformed and derive linear utility only from $t=1$ consumption, so they accept any positive price. Thus, the asset price p is determined by large banks' demand. Small banks can observe the equilibrium price of the asset.

2.3 Legacy assets

Large banks enter the game with assets and liabilities from the previous period. Legacy assets may either pay off or yield losses. We denote by λ losses incurred on these assets relative to outstanding liabilities. The size of losses is subject to aggregate risk. Its realization is drawn from a binomial distribution at the beginning of $t=1$:

$$\lambda=\begin{cases} 0 & \text{with probability } \kappa, \\ \bar{\lambda} & \text{with probability } 1-\kappa. \end{cases}$$

The maturity structure of large banks' outstanding liabilities is such that fraction θ needs to be repaid at $t=1$, while the remaining $(1-\theta)$ is due at $t=2$. As a consequence, losses affect both banks' initial leverage and their investment capacity.

We assume that losses on legacy assets are independent of the aggregate productivity. As we show in Online Appendix OA 2.2, the main results of the model are unchanged when the two shocks are correlated, as long as the support of the distribution of productivity conditional on a high level of losses overlaps sufficiently with the support conditional on low legacy losses.

2.4 Funding supply

At time $t=1$ large and small banks receive deposits equal to $S > \left(\frac{\alpha^H}{2}\right)^2 + \theta\lambda$ and $s > \left(\frac{(1-\gamma)\alpha^H + \gamma\sigma}{2}\right)^2$, respectively, high enough for the first-best level of lending to always be an interior solution. We assume that deposit markets are segmented so that each bank faces an exogenous, price-inelastic supply.[8] Deposits need to be repaid at $t=2$ and are assumed to be insured, so that the funding cost equals the return of storage.

2.5 Opacity

The key feature of our framework is that banks' assets are opaque, so that their value and composition is private information. Specifically, we assume that the investment choices, x_i, y_i, z_i and x_k, z_k, as well as the value of legacy assets, λ, are not observable to other agents. The assumption of opacity reflects the difficulty in valuing banking assets by outsiders and banks' capacity to manipulate the reported value of their assets (Huizinga and Laeven, 2012).

Here, opacity is exogenous. In an extension studied in Online Appendix OA 1, we show that banks may indeed have incentives to not disclose their asset allocation and value truthfully if funding costs are somewhat sensitive to risk. Banks prefer to let their assets remain opaque whenever verification costs are high and sensitivity of funding costs to risk is not too large. These are plausible characteristics of large banks, with their complex and opaque assets making outside verification costly and protection by implicit and explicit government guarantees implying a low risk sensitivity of funding costs.

2.6 Timing

Figure 1 illustrates the timing.

[8] In Online Appendix OA 2.3, we allow for the level of deposits to be determined endogenously by banks facing a price-elastic funding supply. We show that main mechanism of our model is at play also in this case.

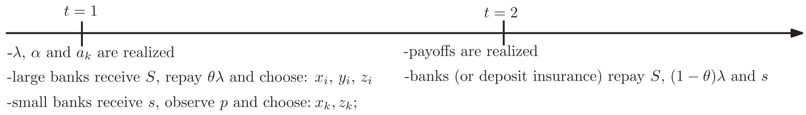

Figure 1
Timeline

2.7 Parameters

To streamline the exposition of the model, we introduce the following restrictions on parameters.

Assumption 1. The parameters satisfy:

A. $(1+q(\alpha))q(\alpha) < q'(\alpha)\left(\frac{\alpha}{2}-\lambda\right)$
B. $\lambda < q(\alpha)(\frac{\alpha^2}{4}-R)$
C. $\frac{1+q(\alpha)}{2}\left(\frac{\alpha^2}{4}(1+q(\alpha))-q(\alpha)R\right)-(1-\theta)\lambda < S$

Assumption 1.A requires that the persistence of the productivity shock is not too low. It ensures that a rise in productivity increases the attractiveness of the risky asset, so its equilibrium price always increases in productivity. While not necessary for our basic result, it is a sufficient condition for amplification of investment inefficiencies through distorted inference. Assumption 1.B limits the size of losses on legacy assets and the payoff of the risky asset relative to the return on productive lending. Assumption 1.C introduces a floor on the funding supply available to large banks. These two assumptions ensure a unique equilibrium cutoff for bank strategies.

2.8 First best

Given that all agents' utilities are linear, the first best can be viewed as the benchmark in which a social planner allocates resources to the available investment opportunities to maximize their net return. Since large banks are the only productive users of the risky asset, efficiency requires that they hold all of its stock, $\int_i y_i di = 1$. The planner chooses productive lending to maximize:

$$\max_{x_i x_k} \int_i \left[\alpha\sqrt{x_i}-x_i\right]di + \int_k \left[(\gamma a_k+(1-\gamma)\alpha)\sqrt{x_k}-x_k\right]dk \tag{1}$$

and allocates the residual resources to storage, so that to satisfy: $S+s=\int_i(x_i+z_i)di+\int_k(x_k+z_k)dk+\theta\lambda$.

Lemma 1. In the first best, lending by large and small banks are given by $x_i^{FB}=x^{FB}=\frac{\alpha^2}{4}$ and $x_k^{FB}=\frac{(\gamma\alpha+(1-\gamma)a_k)^2}{4}$, respectively.

Proof. Follows directly from first order conditions of (1). ■

The efficient level of productive lending by each bank equalizes its marginal productivity to the marginal cost of investment. The remaining resources are allocated to storage.

In the following sections, we solve the game in two steps. We first study the subgame of large banks and then explore how it affects the optimal behavior of small banks.

3. Subgame of Large Banks

In this section, we solve the individual optimization of the large banks, followed by the discussion of the equilibrium of their subgame.

3.1 Large bank's problem

After observing aggregate productivity and losses on legacy assets, each large bank chooses its portfolio to maximize expected profits subject to budget constraint and no-short-selling constraint.

$$\max_{x_i,y_i,z_i} q(\alpha)\left(\alpha\sqrt{x_i}+Ry_i+z_i-(1-\theta)\lambda-S\right)+ \\ (1-q(\alpha))\max\left[\alpha\sqrt{x_i}+z_i-(1-\theta)\lambda-S,0\right], \tag{2}$$

subject to

$$x_i+py_i+z_i+\theta\lambda=S \qquad \text{(budget constraint)}$$

$$y_i\geq 0\ z_i\geq 0 \qquad \text{(no-short-selling constraint).}$$

The solution of the problem is either a solvent strategy or a risk-shifting strategy. A solvent strategy requires a low amount of risky investment to ensure deposit repayment. A risk-shifting strategy involves default in the bad state. Our basic result is that the optimal strategy depends on the balance of funding relative to the quality of investment opportunities, as well as on banks initial leverage.

Lemma 2. There exists an *individual risk-shifting threshold* of funding supply, given by

$$\hat{S}(\alpha,\lambda,p)=\begin{cases}\frac{\alpha^2 p}{4q(\alpha)R}(1+q(\alpha))-\lambda(1-\theta) & \text{if } p\leq q(\alpha)R\\ \frac{\alpha^2}{4}\frac{1-\frac{q(\alpha)p}{R}}{q(\alpha)\left(\frac{R}{p}-1\right)}-\lambda\frac{1-q(\alpha)-q(\alpha)\theta\left(\frac{R}{p}-1\right)}{q(\alpha)\left(\frac{R}{p}-1\right)} & \text{if } p>q(\alpha)R\end{cases} \tag{3}$$

- If $S<\hat{S}(\alpha,\lambda,p)$ the bank prefers a solvent strategy, whose interior solution is

$$x_s^*=\begin{cases}\left(\frac{\alpha p}{2q(\alpha)R}\right)^2 & \text{if } p\le q(\alpha)R\\ \left(\frac{\alpha}{2}\right)^2 & \text{if } p>q(\alpha)R\end{cases}\qquad y_s^*=\begin{cases}\frac{S-x_s^*-\theta\lambda}{p} & \text{if } p<q(\alpha)R\\ y\in\left[0,\frac{S-x_s^*-\theta\lambda}{p}\right] & \text{if } p=q(\alpha)R\\ 0 & \text{if } p>q(\alpha)R\end{cases}$$

$$z_s^*=S-x_s^*-py_s^*-\theta\lambda$$

- If $S>\hat{S}(\alpha,\lambda,p)$ the bank prefers a risk-shifting strategy given by

$$x_r^*=\left(\frac{\alpha p}{2R}\right)^2 \qquad y_r^*=\frac{S-x_r^*-\theta\lambda}{p} \qquad z_r^*=0$$

- If $S=\hat{S}(\alpha,\lambda,p)$, the bank is indifferent between the two strategies.

Proof. In Appendix A.1. ■

Under the solvent strategy the bank does not risk default. It sets productive lending so that to equalize its marginal return with the expected return of the best alternative investment: storage or the risky asset. The residual funding is invested in storage if $p>(\alpha)R$ or into the risky asset if $p<(\alpha)R$. The bank is indifferent between investment in these two if their expected returns are the same, $p=q(\alpha)R$.

In a risk-shifting strategy the bank defaults in the bad state, with probability $1-q(\alpha)$, so it maximizes profits conditional on the good future state. The opportunity cost is the good state return of the asset, $\frac{R}{p}>1$, so a risk-shifting bank issues fewer productive loans, compared to a solvent bank. The bank invests all of the residual funding into the risky asset.

The key insight of Lemma 2 is that the strategic shift occurs when banks face an abundant supply of funding relative to the quality of available opportunities. A large funding supply enables banks to take on a high enough leverage to incentivize risk shifting. The intuition is that, since good lending opportunities are scarce, an increase in the availability of funds raises debt relative to the net value of safe investment. This increases leverage and due to bank's limited liability makes risk shifting more attractive. With high leverage banks can achieve a sufficient exposure to the risky asset, so that the gains from a large gamble on the asset offset the costs of forgoing some productive lending.

Lemma 3 shows that bank's risk-shifting incentives are also affected by the losses from legacy assets. Losses increase bank's initial leverage, which because of limited liability makes the gamble on the risky asset relatively more attractive. Consequently, if the bank faces higher losses, it is willing to play the risky strategy even at lower levels of funding.

Thus, bank's risk-shifting incentives are driven by two distinct forces: (1) initial leverage determined by the losses on legacy assets and (2) the new leverage available to large banks that depends on the level of funding relative to the payoff of investment opportunities.

Lemma 3. The individual risk-shifting threshold:

- decreases in the losses on legacy assets, $\frac{\partial \hat{S}}{\partial \lambda} < 0$,
- increases in the price of the risky asset, $\frac{\partial \hat{S}}{\partial p} > 0$.

Proof. In Appendix A.1. ■

The relative attractiveness of risk shifting depends also on the price of the risky asset. Intuitively, higher price decreases profits under the risky strategy, so a higher scale of investment (enabled by higher funding) is necessary to incentivize the bank to gamble on the asset.

3.2 Equilibrium of large bank's subgame

In equilibrium each large bank plays its optimal strategy taking asset prices and choices of the other large banks as given. Market clearing determines the price of the risky asset.

Proposition 1. There exist a unique *equilibrium risk-shifting threshold of funding supply* given by $\hat{\mathcal{S}}(\alpha,\lambda) = \hat{S}(\alpha,\lambda,q(\alpha)R) = \left(\frac{\alpha}{2}\right)^2(1+q(\alpha)) - (1-\theta)\lambda$ and a *balanced funding threshold* given by $\underline{\mathcal{S}}(\alpha,\lambda) = \left(\frac{\alpha}{2}\right)^2 + q(\alpha)R + \theta\lambda$, such that

- if $S < \underline{\mathcal{S}}(\alpha,\lambda)$ all banks play the solvent strategy; the equilibrium price of the asset is $p^* = \underline{p}(\alpha,\lambda) < q(\alpha)R$, implicitly defined in: $S = \theta\lambda + \underline{p} + \left(\frac{\alpha \underline{p}}{2q(\alpha)R}\right)^2$; Productive lending is lower than the first best, $x^* = \left(\frac{\alpha p^*}{2q(\alpha)R}\right)^2 < x^{FB}$;
- if $\underline{\mathcal{S}}(\alpha,\lambda) \leq S \leq \hat{\mathcal{S}}(\alpha,\lambda)$ all banks play the solvent strategy; the equilibrium price of the asset is $p^* = q(\alpha)R$; Productive lending is equal to the first best, $x^* = \left(\frac{\alpha}{2}\right)^2 = x^{FB}$;
- if $S > \hat{\mathcal{S}}(\alpha,\lambda)$, fraction $\psi^*(\alpha,\lambda) = \frac{p^*}{S - \theta\lambda - \left(\frac{\alpha p^*}{2R}\right)^2}$ of banks play a risk-shifting strategy, where $p^* = \hat{p}(\alpha,\lambda)$ is the equilibrium price of the asset defined implicitly in $\hat{S}\left(\alpha,\lambda,\hat{p}\right) = S$; the remaining $1-\psi^*(\alpha,\lambda)$ banks play the solvent strategy; Productive lending by risk-shifting banks is lower than the first best, $x_r^* = \left(\frac{\alpha p}{2R}\right)^2 < x^{FB}$, and lending by solvent banks is at the first best, $x_s^* = x^{FB}$.

Proof. In Appendix A.2. ■

If bank funding is low relative to the aggregate productivity, $S < \underline{\mathcal{S}}(\alpha,\lambda)$, it is optimal for all banks to play the solvent strategy as funding is insufficient to incentivize risk shifting. In this case funding is so scarce that the demand by large banks is too weak to push the asset price to its fundamental value. The resultant low asset price drives up the opportunity cost of issuing loans, resulting in an inefficiently low level of lending. This inefficiency is driven by the cash in the market pricing of the asset in the context of low funding. To reflect the fact that large banks underutilize productive opportunities, we refer to this outcome as a "missed boom."

If bank funding exceeds the balanced funding threshold, $S > \underline{\mathcal{S}}(\alpha,\lambda)$, the demand by banks playing the solvent strategy is sufficient for the price to equal asset's expected return, $p = q(\alpha)R$. If all banks play the solvent strategy the price cannot increase above this level, as banks wish to hold no asset when $p > q(\alpha)R$. All banks play the solvent strategy, and $p = q(\alpha)R$ is the equilibrium outcome if banks have no incentives to deviate, so when funding is not too high $S \leq \hat{\mathcal{S}}(\alpha,\lambda)$. In this range large banks lend efficiently because marginal returns of all opportunities are equal to the marginal cost of lending. Since the level of lending is aligned with the quality of economic fundamentals and large banks face no risk of default, we refer to this as a "good boom."

If funding is excessive relative to the aggregate productivity, $S > \hat{\mathcal{S}}(\alpha,\lambda)$, banks have risk-shifting incentives at the fair asset price, so all playing the solvent strategy cannot be an equilibrium. Since the asset demand of risk-shifting banks is high, the price increases as more of them gamble. However, as follows from Lemma 3, rising prices reduce banks' risk-shifting incentives, which makes playing solvent and risk-shifting strategic substitutes when $p > q(\alpha)R$. If Assumption 1.B is satisfied, profitability of lending is sufficiently high relative to the risky asset payoff so that all banks risk shifting cannot be sustained as an equilibrium: all banks risk shifting would result in the asset price being so high that banks prefer to deviate to the solvent strategy. Thus, in equilibrium only a fraction $\psi^*(\alpha,\lambda)$ of banks engages in risk shifting. Their investment inflates the price until it is such that banks are indifferent between the two strategies, $S = \hat{S}(\alpha,\lambda,p)$.

When funding is excessive, risk-shifting banks expose themselves to the risk of default by overinvesting in the asset. Underlending emerges because their opportunity cost of lending reflects the upside return on their gamble and, therefore, is inflated. Because of the inefficient lending level, high overall investment, and the risk of banks' default, we refer to this outcome as a "bad boom."

Remarks. In the bad boom equilibrium, ex ante identical banks choose to play different strategies. As in Allen and Gale (2004), this occurs because of the interaction between cash in the market pricing and banks' incentives. On one hand, the asset price is directly determined by the demand of large banks, and demand depends on their choice of strategy. On the other hand, an increase in the price makes the strategy associated with the high asset demand relatively less attractive. A consequence is that the asset market can only be in equilibrium when banks play different strategies, so that the price is not too extreme, and none of the strategies dominates the other.[9]

A key result in Proposition 1 is that the type of equilibrium that emerges depends on the relative funding imbalance, as measured by the level of bank funding compared to aggregate productivity. The two thresholds defined are monotonically increasing in α, allowing us to restate them as thresholds of productivity realizations.

- The *risk-shifting threshold of productivity* is denoted by $\hat{\alpha}(\lambda)$ and defined in $\hat{\mathcal{S}}(\hat{\alpha},\lambda)=S$. It is such that $S>\hat{\mathcal{S}}(\hat{\alpha},\lambda) \Longleftrightarrow \alpha<\hat{\alpha}(\lambda)$ (a bad boom occurs if and only if $\alpha<\hat{\alpha}$).
- The *balanced level of productivity* is denoted by $\overline{\alpha}(\lambda)$ and defined in $\underline{\mathcal{S}}(\overline{\alpha},\lambda)=S$. It is such that, $S<\underline{\mathcal{S}}(\overline{\alpha},\lambda) \Longleftrightarrow \alpha>\overline{\alpha}(\lambda)$ (a missed boom occurs if and only if $\alpha>\overline{\alpha}$).

The likelihood of the emergence of each boom depends on the level of funding and losses on legacy assets through the impact that these have on the risk-shifting threshold and the balanced level of productivity.

Corollary 1. The risk-shifting threshold of productivity increases in the losses on legacy assets, $\frac{\partial\hat{\alpha}}{\partial\lambda}>0$, and in the level of funding, $\frac{\partial\hat{\alpha}}{\partial S}>0$. The balanced level of productivity decreases in the losses on legacy assets, $\frac{\partial\overline{\alpha}}{\partial\lambda}<0$, and increases in the level of funding, $\frac{\partial\overline{\alpha}}{\partial S}>0$.

Proof. In Appendix A.2. ∎

An increase in the losses on legacy assets rises banks leverage and decreases their capacity to invest. The former moves down the equilibrium risk-shifting threshold, making the bad boom more likely. The latter implies that funding is insufficient already at a lower level of productivity, which increases the probability of a missed boom. Periods during which banks face higher losses are thus more likely to be plagued by inefficiencies of the missed or bad boom.

A higher level of funding means higher available leverage. This incentivizes risk shifting and makes the bad boom more likely. More funding also implies that solvent demand by banks can achieve the fair price even at higher productivity realizations. This shifts the balanced level of productivity upward, making the missed boom less likely. Therefore, economies whose banking

[9] In our setting, the relative attractiveness of the strategies, represented by the risk-shifting threshold, depends on the payoff of the risky asset and lending. If the return on the asset is very high, that is, when Assumption 1.B is not satisfied, the relative attractiveness of the risk-shifting strategy is high enough for the pure risk-shifting equilibrium to be a feasible outcome.

sector receives abundant funding are more likely to experience bad booms, while economies with low levels of savings available to banks (for instance, developing economies) are more prone to missed booms.

3.2.1 Equilibrium asset price. The price of the risky asset plays a key role in our framework as it is used by small banks to make inference about the aggregate productivity. Below, we discuss how the two aggregate shocks affect the equilibrium price.

Lemma 4. The price of the risky asset:

- decreases in losses on legacy assets in the missed boom, $\alpha > \overline{\alpha}(\lambda) \Rightarrow \frac{\partial p}{\partial \lambda} < 0$
- is unaffected by losses on legacy assets in the good boom, $\hat{\alpha}(\lambda) \geq \alpha \geq \overline{\alpha}(\lambda) \Rightarrow \frac{\partial p}{\partial \lambda} = 0$
- increases in losses on legacy assets in the bad boom, $\hat{\alpha}(\lambda) > \alpha \Rightarrow \frac{\partial p}{\partial \lambda} > 0$

If Assumption 1.A is satisfied, the price of the risky asset increases in productivity in the good, bad and missed boom, $\frac{\partial p}{\partial \alpha} > 0 \; \forall \alpha, \lambda$.

Proof. In Appendix A.2. ■

In the missed boom the equilibrium price reflects the available "cash in the market" relative to productive opportunities. As legacy losses decrease banks' investment capacity, their demand is depressed, lowering the price. The impact of changes in productivity depends on its effect on profitability of lending and the risky asset. Under Assumption 1.A the persistence of the productivity shock, $q'(\alpha)$, is high, so an increase in α makes the risky asset attractive enough to drive up its demand and price in the missed boom.

In the good boom, the equilibrium price equals to the fair value of the risky asset, $p = q(\alpha)R$, so the price increases in productivity. The price is not affected by an increase in losses as long as it is small enough for the economy to remain in the good boom.

In the bad boom, the equilibrium price reflects the degree of overinvestment by risk-shifting banks. An increase in losses on legacy assets raises banks' initial leverage, making risk shifting attractive even at higher prices. Thus, more overinvestment and a higher asset price is needed to make banks indifferent between the two strategies. Assumption 1.A guarantees that the persistence of the productivity is high, so that a rise in α makes risk shifting sufficiently more attractive to inflate the equilibrium price.

Figure 2 plots the equilibrium asset price. The critical insight is that both productivity and losses determine asset prices in the bad and missed booms. The role of losses in shaping the price implies that asset opacity may impair the inference.

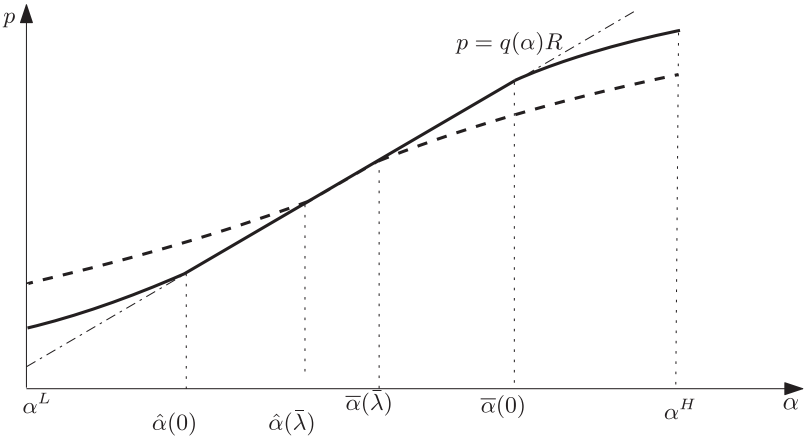

Figure 2
Asset price
The figure plots asset price as a function of productivity if $\lambda = 0$ (solid curve) and if $\lambda = \bar{\lambda}$ (dashed curve). The fair price of the risky asset, $p = q(\alpha)R$, is plotted using a dot-dashed line for reference.

4. Subgame of Small Banks

In this section, we first study the inference problem of small banks. Next, we solve their investment problem and explore the implications for the aggregate lending and risk. Finally, we discuss how this problem could be resolved by investment in information acquisition by small banks or regulatory intervention.

4.1 Inference from prices

Small banks use asset prices to infer the aggregate productivity. They understand how funding, productivity and losses shape the equilibrium outcomes. However, asset opacity introduces uncertainty about large banks' initial leverage that may impair the inference.

Proposition 2. Whenever a missed or a bad boom can occur in equilibrium, $\hat{\alpha}(\bar{\lambda}) > \alpha^L$ or $\overline{\alpha}(\bar{\lambda}) < \alpha^H$, there exists an *imprecise inference set* of prices, $\mathcal{I}$, such that

- when $p \in \mathcal{I}$ small banks infer two feasible productivity values
- when $p \notin \mathcal{I}$ small banks infer the correct value of productivity

Proof. In Appendix A.3. ■

Inference is precise if the asset price is such that it can only be supported by one combination of productivity and losses. This is the case when funding is balanced relative to productivity, so that a good boom is the only outcome. Precise inference is possible also if the price can only be achieved under a bad boom with low losses or a missed boom with high losses.

In missed and bad booms, the asset price is determined jointly by the funding imbalance relative to productivity and the initial leverage. As a result, the same price can be supported by different levels of productivity, depending on the realization of losses. Because asset opacity obscures the value of banks' initial leverage, in this case inference is distorted. The imprecise inference set is a union of two subsets:

- **Bad boom range:** If the equilibrium price is such that $\hat{p}(\alpha^L,\bar{\lambda}) \leq p < q(\hat{\alpha}(\bar{\lambda}))R$, inference is obstructed by the uncertainty regarding risk incentives of large banks.
  - If $p \in \left[\hat{p}(\alpha^L,\bar{\lambda}), q(\hat{\alpha}(0))R\right]$, the price can result from a bad boom with high losses or a bad boom with low losses
  - If $p \in \left[q(\hat{\alpha}(0))R, q(\hat{\alpha}(\bar{\lambda}))R\right]$, the price can result from a good boom with low losses or a bad boom with high losses.
- **Missed boom range:** If the equilibrium price is such that $q(\overline{\alpha}(\bar{\lambda})) < p \leq \underline{p}(\alpha^H,\bar{\lambda})$, inference is impaired by uncertainty regarding investment capacity of large banks.
  - If $p \in \left[q(\overline{\alpha}(\bar{\lambda}))R, q(\overline{\alpha}(0))R\right]$ the price can result from a good boom with low losses or a missed boom with high losses.
  - If $p \in \left[q(\overline{\alpha}(0))R, \underline{p}(\alpha^H,\bar{\lambda})\right]$ the price can result from a missed boom with high losses or a missed boom with low losses.

In the face of the opacity of banking assets, limited liability and cash in the market pricing may give rise to imprecise inference from prices. The relationship between the productivity and the price in the bad boom depends on the strength of the risk-shifting incentives, which arise because of limited liability. In the missed boom, this relationship is determined by large banks' investment capacity through cash in the market pricing. Since both the risk-shifting incentives and investment capacity are affected by the size of losses on legacy assets, opacity implies that the price is imperfectly informative.

As in models in the past literature, in our framework the price fails to reveal the true underlying economic conditions because it is nonmonotonic in the state variable (Bond, Edmans, and Goldstein, 2012). Interestingly, unlike the existing mechanisms (for instance, Bond, Goldstein, and Prescott, 2010), in our setting this nonmonotonic relationship between the productivity and the equilibrium price emerges solely because of incentives of the informed players (shaped by the limited liability and cash in the market pricing) and is not affected by the feedback effect of learning from prices.

Observing a price within the imprecise inference set yields the following estimates:

$$\tilde{\alpha} = \begin{cases} \tilde{\alpha}(p,0) \text{ with probability } \kappa \\ \tilde{\alpha}(p,\bar{\lambda}) \text{ with probability } 1-\kappa \end{cases} \tag{4}$$

The insights about sensitivity of price to changes in productivity and legacy losses from Lemma 4 imply the relative size of the productivity estimates corresponding to a high and low loss scenario.

Corollary 2. If Assumption 1.A is satisfied:

- When price is in the bad boom range, $\hat{p}(\alpha^L,0) \leq p < q(\hat{\alpha}(\bar{\lambda}))R$, the productivity estimate corresponding to the low legacy losses is high, $\tilde{\alpha}(p,0) > \tilde{\alpha}(p,\bar{\lambda})$
- When price is in the missed boom range, $q(\overline{\alpha}(\bar{\lambda}))R < p \leq \underline{p}(\alpha^H,\bar{\lambda})$, the productivity estimate corresponding to the low legacy losses is low, $\tilde{\alpha}(p,0) < \tilde{\alpha}(p,\bar{\lambda})$

Proof. In Appendix A.3. ■

Because losses increase the price under the bad boom and decrease the price in the missed boom, the ordering of the productivity estimates differs across these imprecise inference regions. High productivity corresponds to low losses if inference is distorted due to bad booms. It corresponds to the high losses if inference is distorted due to missed booms.

An implication of Corollary 1 discussed in Section 3.2 is that an increase in funding relative to productivity expands the bad boom range of imprecise inference. Meanwhile, a low level of funding is associated with a large missed boom range. Combining this insight with Corollary 2 implies that the types of imprecise inference problems faced by small banks differ across economies, depending on the degree of uncertainty about economic prospects and the relative supply of large banks' funding.

4.2 Lending by small banks

In this section, we study the lending decision by small banks. Each chooses its productive lending and investment in storage so that to maximize profits subject to budget constraints, while accounting for its limited liability.

To ease the notation, we define $\tilde{\alpha}^H = \max\left[\tilde{\alpha}(p,0),\tilde{\alpha}(p,\bar{\lambda})\right]$ and $\tilde{\alpha}^L = \min\left[\tilde{\alpha}(p,0),\tilde{\alpha}(p,\bar{\lambda})\right]$, as the high and the low estimate of productivity obtained through inference. Let ρ represent the probability that the high estimate is the true aggregate productivity, Prob$(\alpha = \tilde{\alpha}^H) = \rho$, so $\rho = \kappa$ if $\tilde{\alpha}(p,0) > \tilde{\alpha}(p,\bar{\lambda})$ and $\rho = 1-\kappa$ otherwise. With this notation the problem of the small bank can be represented as:

$$\max_{x_k,z_k} \rho\left[\left(\gamma a_k + (1-\gamma)\tilde{\alpha}^H\right)\sqrt{x_k} + z_k - s\right] +$$

$$(1-\rho)\max\left[\left(\gamma a_k + (1-\gamma)\tilde{\alpha}^L\right)\sqrt{x_k} + z_k - s, 0\right] \qquad (5)$$

subject to the budget constraint: $x_k + z_k \leq s$.

The optimal lending choice of a small bank depends on the precision of the inference from prices and the idiosyncratic realization of local productivity.

Proposition 3. If inference is precise, $p \notin \mathcal{I}$, then lending by a small bank is given by $x_k^* = \left(\frac{(1-\gamma)\alpha+\gamma a_k}{2}\right)^2 = x_k^{FB}$ and is equal to the first best. If inference is imprecise, $p \in \mathcal{I}$, and the dispersion of estimates is

- not too large: $\tilde{\alpha}^L > \frac{\sqrt{\rho}-\rho}{1-\rho}\tilde{\alpha}^H - \gamma a_k \frac{1-\sqrt{\rho}}{(1-\gamma)(1-\rho)} = A^L(\tilde{\alpha}^H, \rho, a_k)$, then lending by a small bank is given by $x_k^* = \left(\frac{(1-\gamma)\mathbb{E}(\alpha|p)+\gamma a_k}{2}\right)^2$; It is higher than the first best, $x_k^* > x_k^{FB}$, if $\alpha = \tilde{\alpha}^L$, and lower than the first best, $x_k^* < x_k^{FB}$, if $\alpha = \tilde{\alpha}^H$;
- large, $\tilde{\alpha}^L \leq A^L(\tilde{\alpha}^H, \rho, a_k)$, then lending by a small bank is given by $x_k^* = \left(\frac{(1-\gamma)\tilde{\alpha}^H+\gamma a_k}{2}\right)^2$; It is higher than the first best, $x_k^* > x_k^{FB}$, and results in default if $\alpha = \tilde{\alpha}^L$ and is equal to the first best, $x_k^* = x_k^{FB}$, if $\alpha = \tilde{\alpha}^H$.

Proof. In Appendix A.3. ■

If small banks are able to precisely infer the aggregate productivity, they face no uncertainty. In this case, they maximize profits given their correct inference of aggregate productivity and precise information about local productivity. Their lending corresponds to the first-best allocation.

If the equilibrium price is such that precise inference is impossible, $p \in \mathcal{I}$, lending is risky from the perspective of the uninformed. The choice of the small bank depends on how much uncertainty it faces.

If the risk that the small bank faces is not too high (i.e., the two productivity estimates are not too dispersed, $\tilde{\alpha}^L > A^L(\tilde{\alpha}^H, \rho, a_k)$), its problem boils down to maximizing expected profits, which depend on the expected productivity of loans, $\gamma a_k + (1-\gamma)\mathbb{E}(\alpha|p)$. This level of lending implies over- or underlending relative to the first best. The direction of misinvestment depends on whether inference is obstructed by the bad or the missed boom and on the underlying realization of losses, as follows from Corollary 2.

If imprecise inference is driven by a bad boom, small banks lend excessively whenever large banks face high losses on legacy assets (they underlend if losses are low). The excessive lending by small banks coincides with the larger risk-shifting incentives and more overinvestment in the risky asset by large banks. If imprecise inference is driven by a missed boom, small banks underlend whenever large banks face high losses on legacy assets (they lend excessively if losses are low). Thus, small banks lend too little precisely at the time when lending by large banks is more severely restricted by the cash in the market constraint.

Taken together, these dynamics imply that the optimal investment by banks facing imprecise inference amplifies the investment inefficiencies of large banks' subgame. It results in more aggregate overinvestment during the more severe bad booms and in larger underinvestment in the more constrained missed booms.

A different set of problems emerges when the dispersion of productivity estimates is high. In this case, limited liability of the small bank implies that it prefers to bet on the high productivity estimate when deciding how much to lend. This exposes the bank to a risk of default if the true underlying fundamentals are weak.

To gain further insight, we compare this choice to the level of lending that would emerge in the absence of limited liability, given by $x_k^{NL} = \left(\frac{(1-\gamma)\mathbb{E}(\alpha|p)+\gamma a_k}{2}\right)^2$. Since in equilibrium the small bank does not internalize the losses in case of default, it chooses a level of lending that is higher than this "no limited liability" benchmark: $\left(\frac{(1-\gamma)\tilde{\alpha}^H+\gamma a_k}{2}\right)^2 > x_k^{NL}$. Thus, small banks may engage in a form of risk-shifting induced by imprecise inference.

Small banks risk shift because high uncertainty about the payoff from lending induces them to lend excessively, relative to the "no-limited-liability" benchmark. Interestingly, this lending choice may either result in a large inefficiency and default or correspond to the first best. If the true productivity is low, lending by the small bank is much higher than the first-best level, the bank generates losses and defaults at $t=2$. If the true productivity is high, the bet by the small bank pays off and its lending is at the first-best level.

Consequently, in the missed boom range, lending by small banks counterbalances the inefficiencies that arise in the large banks' subgame. The induced risk-shifting results in high payoff to small banks when large banks face high losses and are thus most constrained in their investment. It increases the aggregate output at the time when large banks underinvest most severely.

An opposite pattern emerges when imprecise inference is driven by bad booms. In this case induced risk shifting by small banks exacerbates the inefficiencies from large banks' subgame. Specifically, small banks default precisely when a high share of large banks risk-shift and are highly exposed to losses on the asset. Thus, induced risk shifting by small banks results in propagation of risk from large to small banks. The outcome can be that both small and some large banks default at $t=2$ (this occurs if the future state is bad so that the asset payoff is zero).

Note that in the model small and large banks interact indirectly only, as the former learn from signals that are affected by the actions of the latter. The goal of our paper is to study how risk incentives and asset opacity may affect this learning. To zoom in on this question and provide a tractable answer, we abstract from other strategic interactions between the two groups, such as cross-exposures or competition. Cross-exposures could result in negative externalities of bank failures, giving large banks incentives to improve small banks' inference. Competition might imply that failure of one intermediary is an opportunity to expand the market share for the others. This may make imprecise inference by small banks desirable for large intermediaries.

In the presence of such interactions, if the expected cost of negative externalities dominates the potential gains from the expansion opportunities, large banks might have incentives to reveal information about their assets in order to improve inference. However, the opaque nature of banks' assets implies that providing credible information about their value can be prohibitively costly.[10] Moreover, in the presence of some heterogeneity among large banks, transparency of a single intermediary may not be sufficient to resolve the inference problem about aggregate information. In such an environment a coordination among large banks may be needed to alleviate negative externalities due to the distorted investment by small banks.

4.2.1 Preconditions for induced risk-shifting. Small bank's risk-shifting incentives depend on the level of the low productivity estimate relative to $A^L(\tilde{\alpha}^H, \rho, a_k)$, which we define as the *small bank's risk-shifting threshold.*

Lemma 5. The small bank's risk-shifting threshold:

- increases in the probability of the high productivity realization, $\frac{\partial A^L}{\partial \rho} > 0$
- decreases in the local productivity, $\frac{\partial A^L}{\partial a_k} < 0$ and its relative importance, $\frac{\partial A^L}{\partial \gamma} < 0$

Proof. In Appendix A.3. ■

A higher likelihood of the high productivity estimate implies that a risk-shifting small bank is more likely to generate high profits, increasing the attractiveness of this choice for any level of productivity estimates. This lowers the minimum dispersion between the productivity estimates that is necessary to induce risk shifting, making the risky outcome more likely.

A higher realization of the local productivity increases the payoff that the small bank can earn under both the high and the low productivity estimate. This increases the expected profits of a bank that targets expected productivity relatively more than profits of a risk-shifting bank. As a consequence, risk shifting is only optimal if the uncertainty that the bank faces is higher. Likewise, an increase in the relative importance of the local shock makes the uncertainty about aggregate productivity less relevant. Small banks facing high local productivity and those for which local shocks are more important are less likely to risk shift.

[10] This is the case in the extension studied in Online Appendix OA 1, where high verification costs imply that opacity is preferred by large banks. While the potential gains from transparency studied are of a different nature (transparency can result in lower cost of funding rather than lower externalities), the role of verification costs would be similar in an environment with strategic interactions.

4.3 Resolving the information friction

In this section, we discuss two solutions to the problem of imprecise inference: acquisition of additional signals about productivity by small banks and regulatory intervention through a stress test.

4.3.1 Additional signals. Imprecise inference about the aggregate productivity undermines the profitability of small banks, giving them an incentive to acquire additional signals about productivity. Their decision depends on the timing and the cost of acquiring additional signals, as well as on the likelihood of imprecise inference and the dispersion of productivity estimates.

If small banks can decide to acquire additional signals after they observe the asset price, their choice depends on the dispersion of productivity estimates. At low levels of dispersion (when small banks lend based on expected productivity) an increase in the dispersion makes signal acquisition more attractive. That's because rising dispersion implies larger ex post investment mistakes in either state of the world, so value of acquiring a signal is high.

However, at high levels of estimate dispersion (when small banks are induced to risk shift) further increases in the dispersion may decrease banks' incentives for information acquisition. In this case banks are betting on the highest productivity estimate. Acquiring additional information would allow them to validate their bet or realize that productivity is low, in which case they would choose a lower level of lending. The change in expected profit due to switching the strategy, given by $(1-\rho)\frac{((1-\gamma)\tilde{\alpha}^L+\gamma a_k)^2}{4}$, increases in the low productivity estimate. Consequently, an increase in dispersion driven by a fall in the low productivity estimate makes information acquisition less attractive.

If the decision to acquire the signal must be taken before the asset price is observed, then the expected costs associated with imprecise inference matter for small bank's decision. These costs depend on the expected dispersion of estimates. If imprecise inference is sufficiently likely, because uncertainty about funding imbalances or bank leverage is high, small banks prefer to incur the additional costs and acquire the signal in order to minimize their expected mistakes in investment. If the likelihood of bad or missed booms (and hence of imprecise inference) is sufficiently low or the cost of acquiring additional signals is high, small banks may prefer to forgo this opportunity and continue to use inference from prices to guide their lending.

4.3.2 Stress testing. Since opacity of banks' assets and the resultant uncertainty about their incentives causes the imprecise inference, an alternative solution to the problem is to seek better information about banks' incentives. A regulator, who evaluates banks' balance sheets in order to assess the losses on their legacy assets and then discloses the information to the rest of the economy, could resolve the problem.

Such a "stress test" can improve aggregate welfare by ensuring that lending by small banks is always efficient. However, because of the inherent opacity of banking assets, acquiring information about their losses is costly. It requires the regulator to invest resources into analyzing banks' data. Moreover, the process can be lengthy, so that the regulator may need to commit to this policy before asset prices are realized.

As in the case of signal acquisition by small banks, the regulator has higher incentives to commit to stress test large banks at the beginning of $t=1$ if the probability that bad or missed booms emerge is high. Since the regulator accounts also for the cost of repaying depositors of the small banks, the benefits of the stress test strictly increase in the dispersion of productivity estimates. Consequently, the regulator performs a stress test and discloses the results only if the uncertainty about large banks' initial leverage and the degree of funding imbalances is high or the cost of analyzing banks' assets is not too large.

In the discussion so far, we have abstracted from the social costs of bank transparency. The literature recognizes that bank opacity may be beneficial as it allows banks to issue information-insensitive debt (Dang et al., 2017) and prevent disruptions associated with bank runs (Faria-e Castro, Martinez, and Philippon, 2017, Gorton and Ordoñez, 2020). While we do not model these in our setting, we can think of benefits of opacity as increasing the cost of the disclosure of stress test's results. The stress test without disclosure still may be useful, if the regulator obtains the results in time to intervene in banks' asset allocation. However, even following the stress test, the regulator lacks information about the aggregate productivity. Consequently, any intervention would have to weigh the expected benefit of limiting banks' investment in the bad boom against the potential cost of further constraining banks' choices in the missed boom. The gains of such policy may be relatively limited.

This discussion implies that the mechanism that we analyze in this paper is more likely to be relevant in settings in which smaller players find it particularly difficult to learn about current prospects and when the social benefits of opacity or the complexity of banks' portfolios are high enough to discourage a regulatory response.

5. Conclusions

We argue that in the presence of funding imbalances, the opacity of banking assets may add noise to asset prices and lead rational market participants to overestimate prospects in risk-driven booms or to underestimate it when bank funding is scarce. The key mechanism is that limited liability and cash in the market pricing distort incentives when funding and productivity are unbalanced.

Scarcity of bank funding limits intermediaries' ability to utilize their productive opportunities leading to asset underpricing due to limited cash in the market (missed boom). Excessive funding relative to productive prospects gives rise to risk-shifting incentives and results in high asset price (bad boom).

Losses on legacy assets exacerbate both of these problems, and, thus, together with the level of current productivity, shape asset prices. Our key insight is that, since opacity obstructs the assessment of banks' incentives and investment capacity, outsiders cannot disentangle the effects of the quality of productive opportunities and leverage and consequently, form imprecise inference of productivity. This leads them to overinvest when large intermediaries take on excessive risk or to underinvest if large players are constrained. Moreover, large inference errors may induce less-informed, smaller banks to risk shift. Distorted inference may thus amplify the inefficiencies associated with bad and missed booms and propagate risk to less-informed players.

Stress testing can help resolve the problem. However, even in the absence of social benefits of bank opacity, regulators may choose to forgo that intervention if they view the probability of funding imbalances as low or if bank assets are highly opaque.

The analysis offers a rational benchmark to interpret the evidence on risk perception in credit booms, while introducing the symmetric possibility of missed and bad booms. The key role of bank opacity in our framework contributes to the discussion on its consequences for financial stability.

A. Appendix: Proofs

A.1 Optimal Strategy of a Large Bank

Proof of Lemma 2

Proof. If the bank remains solvent, the first-order conditions (FOCs) of (2) with respect to x_i are

$$\alpha\frac{1}{2}x_i^{-\frac{1}{2}}-\frac{q(\alpha)R}{p}=0 \qquad \text{if } p<q(\alpha)R, \tag{A1}$$

$$\alpha\frac{1}{2}x_i^{\frac{-1}{2}}-1=0 \qquad \text{if } p\geq q(\alpha)R. \tag{A2}$$

The FOC with respect to y_i combined with the budget constraint yield:

$$py_i=S-x_i-\theta\lambda \qquad \text{if } p<q(\alpha)R \tag{A3}$$

$$py_i\in(0,S-x_i-\theta\lambda) \qquad \text{if } p=q(\alpha)R \tag{A4}$$

$$py_i=0 \qquad \text{if } p\geq q(\alpha)R. \tag{A5}$$

The optimal investment in the storage technology is $z_i=S-x_i-py_i-\theta\lambda$. The strategy is feasible only if the bank remains solvent in the bad state, that is, if $\frac{\alpha^2}{4}-\lambda>0$ when $p>q(\alpha)R$ (which is satisfied under Assumption 1.B) or if $S<\frac{\alpha^2 p}{2q(\alpha)R}-(1-\theta)\lambda=\hat{s}_s(\alpha,\lambda,p)$ when $p\leq q(\alpha)R$.

The FOC of (2) with respect to x_i if the bank defaults in the bad state is

$$\alpha\frac{1}{2}x^{\frac{-1}{2}}-\frac{R}{p}=0 \qquad \text{if } p<R. \tag{A6}$$

The optimal investment in the risky asset is $py_i=S-x_i-\theta\lambda$ and in the storage is $z_i=0$. This strategy is feasible only if it results in a risk of default: $S>\frac{\alpha^2 p}{2R}-(1-\theta)\lambda=\hat{s}_r(\alpha,\lambda,p)$. It is straightforward that $\hat{s}_s(\alpha,\lambda,p)>\hat{s}_r(\alpha,\lambda,p)$.

If both solvent and risk-shifting strategies are feasible $\hat{s}_s(\alpha,\lambda,p) > S > \hat{s}_r(\alpha,\lambda,p)$, the bank chooses a strategy that generates higher profits: If $p \le q(\alpha)R$,

$$\Pi_s = \alpha\left(\frac{\alpha p}{2q(\alpha)R}\right) + q(\alpha)\frac{R}{p}\left[S-\theta\lambda-\left(\frac{\alpha p}{2q(\alpha)R}\right)^2\right] - S-(1-\theta)\lambda$$

$$\Pi_r = q(\alpha)\left[\alpha\left(\frac{\alpha p}{2R}\right)+\frac{R}{p}\left[S-\theta\lambda-\left(\frac{\alpha p}{2R}\right)^2\right]-S-(1-\theta)\lambda\right]$$

$$\Pi_r > \Pi_s \iff S > \frac{\alpha^2 p}{4q(\alpha)R}(1+q(\alpha)) - \lambda(1-\theta) = \hat{S}(\alpha,\lambda,p).$$

If $p > q(\alpha)R$,

$$\Pi_s = \alpha\left(\frac{\alpha}{2}\right)+S-\theta\lambda-\left(\frac{\alpha}{2}\right)^2 - S-(1-\theta)\lambda$$

$$\Pi_r = \frac{\alpha^2 p}{4R}q(\alpha)+Sq(\alpha)\left(\frac{R}{p}-1\right)-\lambda q(\alpha)\left(\frac{R}{p}\theta+(1-\theta)\right)$$

$$\Pi_r > \Pi_s \iff S > \frac{\alpha^2}{4}\frac{1-\frac{q(\alpha)p}{R}}{q(\alpha)\left(\frac{R}{p}-1\right)} - \lambda\frac{1-q(\alpha)-q(\alpha)\theta\left(\frac{R}{p}-1\right)}{q(\alpha)\left(\frac{R}{p}-1\right)} = \hat{S}(\alpha,\lambda,p).$$

If $p \le q(\alpha)R$, then $\hat{S}(\alpha,\lambda,p) < \hat{s}_s(\alpha,\lambda,p)$, because $2 > 1+q(\alpha)$. Consequently, as long as $\frac{\alpha^2}{4} > \lambda$, solvent strategy is feasible whenever it is preferred. If $p \le q(\alpha)R$, then $\hat{S}(\alpha,\lambda,p) > \hat{s}_r(\alpha,\lambda,p)$ because $q(\alpha) < 1$. If $p > q(\alpha)R$, then $\hat{S}(\alpha,\lambda,p) > \hat{s}_r(\alpha,\lambda,p)$ whenever $\frac{\alpha^2}{4}\left(1+\frac{q(\alpha)p}{R}-2q(\alpha)\right) > \lambda\left(1-\frac{q(\alpha)R}{p}\right)$, which is the case if $\frac{\alpha^2}{4} > \lambda$. Thus, $\hat{s}_s(\alpha,\lambda,p) > \hat{S}(\alpha,\lambda,p) > \hat{s}_r(\alpha,\lambda,p)$. So if $\frac{\alpha^2}{4} > \lambda$, banks prefer the solvent strategy whenever $S < \hat{S}(\alpha,\lambda,p)$ and prefer a risk-shifting strategy whenever $S > \hat{S}(\alpha,\lambda,p)$. They are indifferent between the two if $S = \hat{S}(\alpha,\lambda,p)$. ■

Proof of Lemma 3

Proof.

$$\frac{\partial \hat{S}}{\partial p} = \begin{cases} \left(\frac{\alpha}{2}\right)^2 \frac{1+q(\alpha)}{q(\alpha)} & \text{if } p \le q(\alpha)R \\ \frac{\frac{\alpha^2}{4}\left(-2q(\alpha)pR+q(\alpha)p^2+q(\alpha)R^2\right)-\lambda R^2(1-q(\alpha))}{q^2(\alpha)\left(\frac{R}{p}-1\right)^2}\frac{q(\alpha)}{p^2 R} & \text{if } p > q(\alpha)R \end{cases} \tag{A7}$$

Let $-2q(\alpha)pR+q(\alpha)p^2+q(\alpha)R^2 = w(p)$. Since $\frac{\partial w(p)}{\partial p} < 0$ and $w(p=R) = q(\alpha)R^2 - 2q(\alpha)R^2 + R^2 = R^2(1-q(\alpha))$, we have that $\frac{\partial \hat{S}}{\partial p} > 0$ for all $p < R$ whenever $\frac{\alpha^2}{4} > \lambda$.

$$\frac{\partial \hat{S}}{\partial \lambda} = \begin{cases} -(1-\theta) & \text{if } p \le q(\alpha)R \\ -\frac{1-q(\alpha)-q(\alpha)\theta\left(\frac{R}{p}-1\right)}{q(\alpha)\left(\frac{R}{p}-1\right)} & \text{if } p > q(\alpha)R \end{cases} \tag{A8}$$

It follows that $\frac{\partial \hat{S}}{\partial \lambda} < 0$, because for $p > q(\alpha)R$: $\theta(1-q(\alpha)) > q(\alpha)\theta\left(\frac{R}{p}-1\right) > 0$, so $1-q(\alpha)-q(\alpha)\theta\left(\frac{R}{p}-1\right) > 1-q(\alpha)-\theta(1-q(\alpha)) > 0$. ■

A.2 Equilibrium of Large Bank's Game
Proof of Proposition 1

Proof. We show that

1. if $S < \hat{\mathcal{S}}(\alpha,\lambda)$ all banks following the solvent strategy is the unique equilibrium
2. if $S > \hat{\mathcal{S}}(\alpha,\lambda)$ some banks risk shifting and some investing according to the solvent strategy is the unique equilibrium

1. Define the balanced funding threshold as $\underline{S}(\alpha,\lambda) = \left(\frac{\alpha}{2}\right)^2 + q(\alpha)R + \theta\lambda$ and notice that $\underline{S}(\alpha,\lambda) < \hat{\mathcal{S}}(\alpha,\lambda) = \hat{S}(\alpha,\lambda,q(\alpha)R)$ whenever $\left(\frac{\alpha}{2}\right)^2 - R > \lambda$, which is the case when Assumption 1.B is satisfied.

If $\hat{\mathcal{S}}(\alpha,\lambda) > S > \underline{S}(\alpha,\lambda)$ and all banks play the solvent strategy the price is $p = q(\alpha)R$. Banks have no incentives to deviate to risk-shifting since $S < \hat{\mathcal{S}}(\alpha,\lambda)$.

If $\underline{S}(\alpha,\lambda) > S$ and all banks play the solvent strategy the price is given by $\underline{p}$, which solves $\left(\frac{\alpha \underline{p}}{2q(\alpha)R}\right)^2 + \underline{p} + \theta\lambda = S$. That price is such that banks have no risk-shifting incentives if

$$\hat{S}(\alpha,\lambda,p) > \left(\frac{\alpha p}{2q(\alpha)R}\right)^2 + p + \theta\lambda \iff$$

$$-\left(\frac{\alpha p}{2q(\alpha)R}\right)^2 + p\left(\frac{\alpha^2}{4q(\alpha)R}(1+q(\alpha)) - 1\right) - \lambda > 0 \iff p \in (p_0^*, p_1^*),$$

where

$$p_0^* = \frac{\frac{\alpha^2}{4q(\alpha)R}(1+q(\alpha)) - 1 - \sqrt{\left(\frac{\alpha^2}{4q(\alpha)R}(1+q(\alpha)) - 1\right)^2 - \lambda\left(\frac{\alpha}{q(\alpha)R}\right)^2}}{2\left(\frac{\alpha}{2q(\alpha)R}\right)^2} \tag{A9}$$

$$p_1^* = \frac{\frac{\alpha^2}{4q(\alpha)R}(1+q(\alpha)) - 1 + \sqrt{\left(\frac{\alpha^2}{4q(\alpha)R}(1+q(\alpha)) - 1\right)^2 - \lambda\left(\frac{\alpha}{q(\alpha)R}\right)^2}}{2\left(\frac{\alpha}{2q(\alpha)R}\right)^2}. \tag{A10}$$

With some algebra it follows that $p_1^* > q(\alpha)R$ if $q(\alpha)\left(\frac{\alpha^4}{4} - R\right) > \lambda$. Next, we define $p_a = \frac{\frac{\alpha^2}{4q(\alpha)R}(1+q(\alpha)) - 1}{2\left(\frac{\alpha}{2q(\alpha)R}\right)^2}$, such that $p_1^* > p_a > p_0^*$. By Assumption 1.C, we restrict our focus to values of S such that p_a is the lowest feasible price: $S > S_a = \hat{S}(\alpha,\lambda,p_a) > \left(\frac{\alpha p_a}{2q(\alpha)R}\right)^2 + p_a + \theta\lambda$.

$$S_a = \frac{1+q(\alpha)}{2}\left(\frac{\alpha^2}{4}(1+q(\alpha) - q(\alpha)R)\right) - (1-\theta)\lambda \tag{A11}$$

Thus, under Assumptions 1.B and 1.C, banks do not have incentives to deviate from playing the solvent strategy also when $\underline{S}(\alpha,\lambda) > S$.[11]

[11] If Assumption 1.C is relaxed, $\hat{S}(\alpha,\lambda,p=p_0^*)$ would be the second equilibrium risk-shifting threshold. In this case risk shifting would emerge for $S < \hat{S}(\alpha,\lambda,p=p_0^*)$ and result in the underpriced asset (we abstract from this case in the model to streamline our discussion).

Therefore, when $S<\hat{\mathcal{S}}(\alpha,\lambda)$ all banks playing the solvent strategy is an equilibrium. It is unique because any outcome with a positive share of risk-shifting banks cannot be an equilibrium when $S<\hat{\mathcal{S}}(\alpha,\lambda)$: it would result in a higher price than in the all solvent equilibrium and since $\frac{\partial \hat{S}}{\partial p}>0$ risk-shifting banks would prefer to deviate to the solvent strategy.
2. If $S>\hat{S}(\alpha,\lambda,q(\alpha)R)$ all banks playing a solvent strategy cannot be an equilibrium as banks have incentives to deviate to the risk-shifting strategy. Let ψ denote the share of risk-shifting banks, then the price solves $p^*=\psi\left(S-\theta\lambda-\left(\frac{\alpha p^*}{2R}\right)^2\right)$, so $\frac{\partial p^*}{\partial \psi}>0$.

Banks deviate to risk shifting until $p^*=\hat{p}(\alpha,S)>q(\alpha)R$ such that $S=\hat{S}(\alpha,\lambda,\hat{p})$, at which point they are indifferent between risk shifting and solvent investment. The equilibrium fraction of risk-shifting banks is $\psi^*(\alpha,\lambda)y_r^*=1$, the remaining banks choose the solvent strategy.
If all banks play the risk-shifting strategy the price solves $S=\left(\frac{\alpha p}{2R}\right)^2+p+\theta\lambda$. This price is too high for the risk shifting to be individually optimal whenever

$$\hat{S}(\alpha,\lambda,p)>\left(\frac{\alpha p}{2R}\right)^2+p+\theta\lambda \iff \underbrace{\frac{\alpha^2}{4}\left[1-2\frac{q(\alpha)p}{R}+\frac{q(\alpha)p^2}{R^2}\right]}_{K(p)} > \underbrace{q(\alpha)(R-p)+\lambda(1-q(\alpha))}_{L(p)}.$$

Notice that $\underset{p}{\operatorname{argmin}} K(p)=R$, while $\frac{\partial L(p)}{\partial p}<0$. Thus, we have $K(p)>L(p)$ for all $p\in(q(\alpha)R,R)$ if $K(p=R)>L(p=q(\alpha)R)$. This is the case if $\frac{\alpha^2}{4}-q(\alpha)R>\lambda$, which is the case is Assumption 1.B is satisfied. Thus, neither all banks playing solvent nor all banks risk shifting can be an equilibrium, so the mixed equilibrium is unique. ■

Proof of Lemma 4

Proof. Comparative statics with respect to price follow from

- For $\alpha>\overline{\alpha}(\lambda)$: $\frac{\partial p}{\partial \alpha}=\frac{2\alpha p^2\left(\alpha q'(\alpha)-q(\alpha)\right)}{q(\alpha)\left[(2q(\alpha)R)^2+2\alpha^2 p\right]}>0$ since under Assumption 1.A $\alpha q'(\alpha)>q(\alpha)$
- For $\hat{\alpha}(\lambda)<\alpha<\overline{\alpha}(\lambda)$: $\frac{\partial p}{\partial \alpha}=q'(\alpha)R>0$.
- For $\alpha<\hat{\alpha}(\lambda)$: $\frac{\partial p}{\partial \alpha}=\frac{-1}{\frac{\partial \hat{S}}{\partial p}}\left[\frac{\frac{\alpha}{2}q(\alpha)\left(1-q(\alpha)\frac{p}{R}\right)-q'(\alpha)\frac{\alpha^2}{4}+\lambda q'(\alpha)}{\left(\frac{R}{p}-1\right)q^2(\alpha)}\right]$. Let $Z=\frac{\alpha}{2}q(\alpha)\left(1-q(\alpha)\frac{p}{R}\right)-q'(\alpha)\frac{\alpha^2}{4}+\lambda q'(\alpha)$. Since $\frac{\partial Z}{\partial p}<0$ and under Assumption 1.A $Z(p=q(\alpha)R)=(1-q^2(\alpha))q(\alpha)-q'(\alpha)\frac{\alpha}{2}+\lambda q'(\alpha)<0$, we have that $Z<0\ \forall p\in(q(\alpha)R,R)$ so $\frac{\partial p}{\partial \alpha}>0$.

Comparative statics with respect to losses on legacy assets follow from

- For $\alpha>\overline{\alpha}(\lambda)$: $\frac{\partial p}{\partial \lambda}=\frac{-\theta}{\frac{\alpha^2 p}{2q^2(\alpha)R^2}+1}<0$
- For $\hat{\alpha}(\lambda)<\alpha<\overline{\alpha}(\lambda)$: $\frac{\partial p}{\partial \lambda}=0$
- For $\alpha<\hat{\alpha}(\lambda)$: $\frac{\partial p}{\partial \lambda}=-\frac{\frac{\partial \hat{S}}{\partial \lambda}}{\frac{\partial \hat{S}}{\partial p}}>0$, since $\frac{\partial \hat{S}}{\partial \lambda}<0$ and $\frac{\partial \hat{S}}{\partial p}>0$ ■

Proof of Corollary 1

Proof. The risk-shifting threshold of productivity is defined in $\hat{\mathcal{S}}(\hat{\alpha},\lambda)=S$, by implicit differentiation:

$$\frac{\partial \hat{\alpha}}{\partial \lambda}=-\frac{\frac{\partial \hat{\mathcal{S}}}{\partial \lambda}}{\frac{\partial \hat{\mathcal{S}}}{\partial \hat{\alpha}}}=\frac{(1-\theta)}{\frac{\alpha}{2}(1+q(\alpha))+q'(\alpha)\left(\frac{\alpha}{2}\right)^2}>0. \tag{A12}$$

The balanced level of productivity is defined in $\underline{\mathcal{S}}(\underline{\alpha},\lambda)=S$, implicit differentiation yields:

$$\frac{\partial\underline{\alpha}}{\partial\lambda}=-\frac{\frac{\partial\underline{\mathcal{S}}}{\partial\lambda}}{\frac{\partial\underline{\mathcal{S}}}{\partial\underline{\alpha}}}=\frac{-\theta}{\frac{\alpha}{2}+q'(\alpha)R}<0. \tag{A13}$$

■

A.3 Imprecise Inference and Small Bank's Investment

Proof of Proposition 2 and Corollary 2

Proof. Consider the setting when $\alpha^L<\hat{\alpha}(0)<\overline{\alpha}(0)<\alpha^H$, we have that

- $\hat{p}(\alpha^L,\bar{\lambda})>\hat{p}(\alpha^L,0)$, since $\frac{\partial p}{\partial\lambda}>0$ if $\alpha<\hat{\alpha}(\lambda)$
- $\underline{p}(\alpha^H,0)>\underline{p}(\alpha^H,\bar{\lambda})$, since $\frac{\partial p}{\partial\lambda}<0$ if $\alpha>\overline{\alpha}(\lambda)$.

Precise inference

- Notice that $\underline{p}(\alpha^H,\bar{\lambda})$ and $\hat{p}(\alpha^L,\bar{\lambda})$ are, respectively, the minimum and maximum price that can be achieved under $\lambda=\bar{\lambda}$, so when $p>\underline{p}(\alpha^H,\bar{\lambda})$ or $p<\hat{p}(\alpha^L,\bar{\lambda})$ inference is precise since it must be that $\lambda=0$.
- If $\alpha\in\left[\hat{\alpha}(\bar{\lambda}),\overline{\alpha}(\bar{\lambda})\right]$ then $p(\alpha,\bar{\lambda})=p(\alpha,0)=q(\alpha)R$. Thus, if $p\in\left[q\left(\hat{\alpha}(\bar{\lambda})\right)R,q\left(\overline{\alpha}(\bar{\lambda})\right)R\right]$ then inference is precise.

Imprecise inference

Since $\frac{\partial p}{\partial\lambda}>0$ and $\frac{\partial p}{\partial\alpha}>0$ for $\alpha<\hat{\alpha}(\lambda)$:

- if $p\in\left[\hat{p}(\alpha^L,\bar{\lambda}),q\left(\hat{\alpha}(0)\right)R\right)$, then $p=\hat{p}(\alpha,0)$ & $\lambda=0$ or $p=\hat{p}(\alpha',\bar{\lambda})$ & $\lambda=\bar{\lambda}$, where $\alpha>\alpha'$
- if $p\in\left[q\left(\hat{\alpha}(0)\right)R,q\left(\hat{\alpha}(\bar{\lambda})\right)R\right)$, then $p=q(\alpha)R$ & $\lambda=0$ or $p=\hat{p}(\alpha',\bar{\lambda})$ & $\lambda=\bar{\lambda}$; where $\alpha>\alpha'$

Since $\frac{\partial p}{\partial\lambda}<0$ and $\frac{\partial p}{\partial\alpha}>0$ for $\alpha>\overline{\alpha}(\lambda)$:

- if $p\in\left(q\left(\overline{\alpha}(\bar{\lambda})\right)R,q(\overline{\alpha}(0))R\right]$, then $p=q(\alpha)R$ & $\lambda=0$ or $p=\underline{p}(\alpha',\bar{\lambda})$ & $\lambda=\bar{\lambda}$, where $\alpha<\alpha'$
- if $p\in\left(q(\overline{\alpha}(0)),\underline{p}(\alpha^H,\bar{\lambda})\right]$, then $p=\underline{p}(\alpha,0)$ & $\lambda=0$ or $p=\underline{p}(\alpha',\bar{\lambda})$ & $\lambda=\bar{\lambda}$, where $\alpha<\alpha'$

Thus, whenever $p\in\left[\hat{p}(\alpha^L,\bar{\lambda}),q\left(\hat{\alpha}(\bar{\lambda})\right)R\right)=\mathcal{I}^b$ or $p\in\left(q\left(\overline{\alpha}(\bar{\lambda})\right)R,\underline{p}(\alpha^H,\bar{\lambda})\right]=\mathcal{I}^m$ small banks infer two productivity values: $\alpha=\tilde{\alpha}(p,0)$ and $\alpha'=\tilde{\alpha}(p,\bar{\lambda})$. The imprecise inference set is $\mathcal{I}=\left\{\mathcal{I}^b\cup\mathcal{I}^m\right\}$, it is nonempty if $\alpha^L<\hat{\alpha}(\bar{\lambda})$ or $\overline{\alpha}(\bar{\lambda})<\alpha^H$. ■

Proof Proposition 3

Proof. If a small bank does not face the risk of default the FOCs imply that the interior solution for optimal lending is

$$x_S^*=\left(\frac{(1-\gamma)\mathbb{E}(\alpha|p)+\gamma a_k}{2}\right)^2. \tag{A14}$$

The solution is feasible if the bank does not risk default, that is, when $\tilde{\alpha}^L>\frac{(1-\gamma)\rho\tilde{\alpha}^H-\gamma a_k}{(1+\rho)(1-\gamma)}=v_s(\rho,\tilde{\alpha}^H,\gamma,a_k)$. If the small bank faces default under low productivity realization, the FOCs imply that the interior solution for optimal lending is

$$x_R^*=\left(\frac{(1-\gamma)\tilde{\alpha}^H+\gamma a_k}{2}\right)^2. \tag{A15}$$

The solution is feasible if the bank risks default, that is, when $\tilde{\alpha}^L < \frac{(1-\gamma)\tilde{\alpha}^H - \gamma a_k}{2(1-\gamma)} = v_r(\rho, \tilde{\alpha}^H, \gamma, a_k)$.

Both strategies are feasible if $v_s(\rho, \tilde{\alpha}^H, \gamma, a_k) < v_r(\rho, \tilde{\alpha}^H, \gamma, a_k)$, which holds since $\gamma\sigma < (1-\gamma)\alpha^L$, ensures that $\tilde{\alpha}^H(1-\gamma) > -a_k\gamma$. Thus, if $v_s(\rho, \tilde{\alpha}^H, \gamma, a_k) < \tilde{\alpha}^L < v_r(\rho, \tilde{\alpha}^H, \gamma, a_k)$ the small bank chooses the strategy that yields higher profits:

$$\Pi_k^R > \Pi_k^S \iff \rho\left(\frac{(1-\gamma)\tilde{\alpha}^H + \gamma a_k}{2}\right)^2 > \left(\frac{(1-\gamma)\mathbb{E}(\alpha|p) + \gamma a_k}{2}\right)^2$$

$$\iff \tilde{\alpha}^L < \frac{\sqrt{\rho}-\rho}{1-\rho}\tilde{\alpha}^H - \gamma a_k \frac{1-\sqrt{\rho}}{(1-\gamma)(1-\rho)} = \hat{v}(\rho, \tilde{\alpha}^H, \gamma, a_k)$$

Note that $v_s(\rho, \tilde{\alpha}^H, \gamma, a_k) > \hat{v}(\rho, \tilde{\alpha}^H, \gamma, a_k) > v_r(\rho, \tilde{\alpha}^H, \gamma, a_k)$ whenever $(1-\gamma)\tilde{\alpha}^H > -\gamma a_k$. Thus, the interior solution of the small bank k is to lend $x_k = x_R$ if $\tilde{\alpha}^L < \hat{v}(\rho, \tilde{\alpha}^H, \gamma, a_k)$ and lend $x_k = x_S$ if $\tilde{\alpha}^L \geq \hat{v}(\rho, \tilde{\alpha}^H, \gamma, a_k)$. ■

Proof of Lemma 5

Proof. The derivatives follow directly:

- $\frac{\partial A^L}{\partial \rho} = \frac{1-2\sqrt{\rho}+\rho}{2\sqrt{\rho}(1-\rho)^2}(\tilde{\alpha}^H + \frac{\gamma}{1-\gamma}a_k) > 0$, since $(1-\gamma)\alpha^L > \gamma\sigma$,
- $\frac{\partial A^L}{\partial a_k} = \frac{\gamma}{1-\gamma}\frac{-1+\sqrt{\rho}}{1-\rho} < 0$,
- $\frac{\partial A^L}{\partial \gamma} = \frac{-1}{(1-\gamma)^2}a_k\frac{1-\sqrt{\rho}}{(1-\rho)} < 0$. ■

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