Grothendieck inequalities, nonlocal games and optimization
Briët, J.

Citation for published version (APA):

General rights
It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations
If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: http://uba.uva.nl/en/contact, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.
# Contents

## Acknowledgments

## 1 Nonlocal Games and Optimization

1.1 Introduction ................................................. 1

1.2 Quantum information theory ................................. 3  
  1.2.1 States and quantum systems ............................ 4
  1.2.2 Measurements and observables ........................ 4
  1.2.3 Entangled states and local measurements ............. 5

1.3 Nonlocal games .............................................. 7  
  1.3.1 Classical strategies ................................... 7
  1.3.2 Entangled strategies .................................. 7

1.4 Two-player XOR games ....................................... 8  
  1.4.1 The CHSH game ....................................... 10

1.5 Tsirelson’s Theorem .......................................... 11

1.6 Multiplayer XOR games ...................................... 14  
  1.6.1 Mermin’s Game ....................................... 15
  1.6.2 Stabilizer states ..................................... 16

1.7 Semidefinite programs and relaxations ..................... 17  
  1.7.1 Approximation algorithms ............................. 17
  1.7.2 MAX CUT ............................................. 18
  1.7.3 The chromatic number and the Lovász theta number .. 22
  1.7.4 A little on the Unique Games Conjecture ............ 23
## Contents

2 Grothendieck inequalities  
2.1 Introduction ................................................. 25  
2.2 Grothendieck’s Inequality ................................. 26  
2.3 Generalizations of Grothendieck’s Inequality .......... 27  
2.3.1 The rank-\(r\) Grothendieck constant ................. 27  
2.3.2 The Grothendieck constant of a graph ................. 29  
2.3.3 The complex Grothendieck constant ................. 30  
2.3.4 Tonge’s Inequality ...................................... 30  

3 Nonlocal games that require high entanglement ........ 37  
3.1 Introduction ................................................. 37  
3.2 Grothendieck’s Inequality with operators ............ 39  
3.3 Lower bounds .............................................. 42  
3.4 Nonlocal games that require high entanglement .... 47  
3.5 Invariant operators and Grothendieck’s constant .... 50  
3.6 Open problems ............................................ 53  
3.7 Summary .................................................. 53  
3.8 Proof of the operator lemma ............................... 53  

4 The PSD Grothendieck problem ............................. 57  
4.1 Introduction ................................................. 57  
4.1.1 An optimal approximation algorithm? ............... 59  
4.1.2 Interpretations ......................................... 62  
4.1.3 More related work ...................................... 63  
4.2 The approximation ratio .................................. 64  
4.2.1 The expectation function ............................... 64  
4.2.2 Positive functions for spheres ....................... 66  
4.2.3 The Wishart distribution ............................... 67  
4.3 A refined, dimension-dependent analysis ............. 70  
4.4 Unique-Games hardness of approximation ............. 72  
4.5 The case of graphs ...................................... 72  
4.6 Summary .................................................. 74  

5 Grothendieck problems with rank constraint .......... 75  
5.1 Introduction ................................................. 75  
5.1.1 Applications .......................................... 77  
5.1.2 An efficient approximation algorithm for graphs with small chromatic number .................. 78  
5.2 A matrix version of Grothendieck’s Identity ........... 82
<table>
<thead>
<tr>
<th>Contents</th>
<th>ix</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.3 Convergence radius</td>
<td>85</td>
</tr>
<tr>
<td>5.4 Constructing new vectors</td>
<td>87</td>
</tr>
<tr>
<td>5.5 A refined, dimension-dependent analysis</td>
<td>90</td>
</tr>
<tr>
<td>5.6 Upper bounds for large chromatic numbers</td>
<td>95</td>
</tr>
<tr>
<td>5.7 Summary</td>
<td>99</td>
</tr>
<tr>
<td>6 Entanglement in multiplayer XOR games</td>
<td>101</td>
</tr>
<tr>
<td>6.1 Introduction</td>
<td>101</td>
</tr>
<tr>
<td>6.2 Bounded violations for a large class of states</td>
<td>103</td>
</tr>
<tr>
<td>6.2.1 Implications</td>
<td>105</td>
</tr>
<tr>
<td>6.3 Proof overview and techniques</td>
<td>107</td>
</tr>
<tr>
<td>6.3.1 First step: relating the entangled bias to the GIP bias</td>
<td>109</td>
</tr>
<tr>
<td>6.3.2 Second step: relating the GIP bias to the classical bias</td>
<td>110</td>
</tr>
<tr>
<td>6.4 Notation and definitions</td>
<td>110</td>
</tr>
<tr>
<td>6.5 Bounded violations for Schmidt states</td>
<td>111</td>
</tr>
<tr>
<td>6.5.1 Strategies with GHZ states.</td>
<td>111</td>
</tr>
<tr>
<td>6.5.2 Extension to Schmidt states</td>
<td>112</td>
</tr>
<tr>
<td>6.6 Bounded violations for clique-wise entanglement</td>
<td>114</td>
</tr>
<tr>
<td>6.6.1 Carne’s Theorem</td>
<td>115</td>
</tr>
<tr>
<td>6.6.2 Bounding the violations achievable by strategies with clique-</td>
<td>116</td>
</tr>
<tr>
<td>wise entanglement</td>
<td></td>
</tr>
<tr>
<td>6.7 Hardness of approximation of the entangled bias</td>
<td>119</td>
</tr>
<tr>
<td>6.8 Proof of Carne’s Theorem</td>
<td>120</td>
</tr>
<tr>
<td>6.9 Open questions</td>
<td>122</td>
</tr>
<tr>
<td>6.10 Summary</td>
<td>122</td>
</tr>
<tr>
<td>7 A problem of Varopoulos</td>
<td>123</td>
</tr>
<tr>
<td>7.1 Introduction</td>
<td>123</td>
</tr>
<tr>
<td>7.1.1 Banach algebras</td>
<td>124</td>
</tr>
<tr>
<td>7.1.2 Q-algebras</td>
<td>125</td>
</tr>
<tr>
<td>7.1.3 Schatten spaces and the Schur product</td>
<td>126</td>
</tr>
<tr>
<td>7.2 Varopoulos’s question and our part of the answer</td>
<td>127</td>
</tr>
<tr>
<td>7.2.1 The connection to the Schmidt states</td>
<td>133</td>
</tr>
<tr>
<td>7.3 The intermediate cases</td>
<td>134</td>
</tr>
<tr>
<td>A Some useful linear algebra and analysis</td>
<td>137</td>
</tr>
<tr>
<td>A.1 Vector spaces</td>
<td>137</td>
</tr>
<tr>
<td>A.2 Matrices</td>
<td>139</td>
</tr>
<tr>
<td>A.3 Tensor products</td>
<td>141</td>
</tr>
</tbody>
</table>