Wake Me Up Before You CoCo

Implications of contingent convertible capital for financial regulation

Chan, S.

Citation for published version (APA):

General rights
It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations
If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: https://uba.uva.nl/en/contact, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.
Contingent convertible capital (CoCo) is designed to improve the loss absorption capacity of the issuing bank without resorting to new equity or taxpayer-funded bailouts. However attractive they might seem to the regulator, they may have undesirable and unexpected consequences. This dissertation examines the implications of issuing CoCos for the financial system. For instance, CoCo conversion may be construed as signal about the asset quality of the bank, which may lead to contagious bank runs in the system. Another is that if the CoCo is inappropriately designed, the bank may accelerate the conversion by choosing high levels of risk to increase the bank’s residual equity value. Finally, the regulator’s desire for a trigger that she can control is an invitation for regulatory forbearance, which is what she was trying to avoid in the first place.

Stephanie Chan (1984) holds a BSc in Economics and Accountancy from De La Salle University – Manila. She obtained a Master’s degree in Applied Economics from the same institution. In between, she was part of the Advisory group of PricewaterhouseCoopers Philippines. She obtained her MPhil from the Tinbergen Institute in 2013 and joined the UvA in the same year to write her PhD thesis on CoCos.
Wake Me Up Before You CoCo:
Implications of Contingent Convertible Capital for Financial Regulation
Wake Me Up Before You CoCo:

Implications of Contingent Convertible Capital for Financial Regulation

ACADEMISCH PROEFSCHRIFT

ter verkrijging van de graad van doctor

aan de Universiteit van Amsterdam

op gezag van de Rector Magnificus

prof. dr. ir. K. I. J. Maex

ten overstaan van een door het College voor Promoties ingestelde

commissie, in het openbaar te verdedigen in de Aula der Universiteit

op vrijdag 9 jun 2017, te 11:00 uur

doort

Stephanie Chan

egeboren te Manilla, Filipijnen
Promotiecommissie:

Promotor: Prof. dr. S. J. G van Wijnbergen Universiteit van Amsterdam
          Prof. dr. E. C. Perotti Universiteit van Amsterdam

Overige leden: Prof. dr. F. Allen Imperial College London
               Prof. dr. A. W. A. Boot Universiteit van Amsterdam
               Prof. dr. A. J. Menkveld Vrije Universiteit
               Dr. W. E. Romp Universiteit van Amsterdam
               Dr. T. Yorulmazer Universiteit van Amsterdam

Faculteit: Economie en Bedrijfskunde
Acknowledgements

My first day in Amsterdam was terrible: I took the train to the wrong direction at nighttime, all the shops were closed, I was growing weak from hunger, I had no internet access, and my mobile phone died! It foreshadowed the difficulties I would encounter while going through the MPhil and the PhD stages: I took quite a lot of wrong turns and ran into a lot of dead ends until I was finally able to find something that works. The process had not been easy, but one eventually finds a way (or more). I knew nothing about the world upon arriving in Amsterdam, but after six years in this lovely city, I can honestly say that I have grown up here. I want to thank mentors, colleagues, friends and family, who helped in this venture.

I would first like to thank Sweder van Wijnbergen, for having faith in my small idea when no one else did, and for taking me in for the PhD. He always challenged me to do my best, and goaded me to believe in myself. I am very happy to have been his student, and I hope that we can continue laughing and collaborating in the future. Enrico Perotti’s courses at the Tinbergen Institute and the UvA defined my research interests. His remarks about my MPhil thesis enabled me to reshape it into my first PhD essay. To the members of my dissertation committee, thank you for taking time to read my work. I especially thank Franklin Allen and Tanju Yorulmazer for their support during the job market period. I hope to make both of them proud within the foreseeable future. Adriaan Soetevent, in his capacity as the Tinbergen DGS, did not give up on me while I was floundering during the MPhil, and for that I am grateful.

The lovely ladies and gentlemen of MInt have been very supportive during the course of the PhD: Franc Klaassen, Massimo Giuliodori, Kostas Mavromatis, Ward Romp, Christian Stoltenberg, Dirk Veestraeten, Naomi Leefmans, Nicoleta Ciurila, Zina Lekniute, Damiaan Chen, Christiaan van der Kwaak, Pim Kastelein, Gabriele Ciminelli, Moutaz Altaghlibi, Jesper Hanson, Rui Zhuo, Jante Parlevliet and Julien Pinter.

I’ve had the pleasure of meeting members of the staff of Tinbergen and the UvA. Arianne de Jong, Ester van den Bragt, Judith van Kronenburg and Christina Månsson lit up the MPhil days between classes, and they were pivotal during the job market period. Robert Helmink and Wilma de Kruijf were very patient in answering a lot of my questions and requests. Without
José Kiss following up on my dissertation deadlines, I would not be able to make it in time!

I would also like to thank a number of friends that I have made in Amsterdam during these years, with whom I’ve had long talks, laughter, sunshine, tears and quite some beers. Without them, the journey would be neither meaningful nor enjoyable. Lin Zhao, you are the sister I never knew I had. One never knows where discussions with Erkki Silde will lead to, but they are always fun. Lucyna Górnicka taught me how to be brave and stand up for myself. Oana Furtuna, I hope we can have more lovely dinners in different parts of the world this time. Rutger Teulings, I will definitely miss your sharp but sweet Dutch directness. Eglė Jakučionytė, for leaving the office with me on that lovely (Saturday) summer day. Swapnil Singh, you understand the fragile but beautiful human condition. Alex Clymo and Gregor Boehl, for all the lovely singing, amazing pasta, and a song that I can never ever get out of my head. Ron van Maurik, your sage advice and yoga poses are always refreshing. Guilherme Vala Elias Pimentel de Oliveira, you are the best person to be stranded with on a cold dark Amsterdam night, and come to think of it, anywhere! Ieva Sakalauskaitė, for all the times we’ve happily sat by the sea, and to many more. Lennart Ziegler, one can learn the secret to happiness by observing you. Jindi Zheng probably saved my life during that very stressful day in Block 4 Year 2. Simin He is one of the few who understand my heart. David Smerdon always roped me into a number of goofy collaborations. Shawny Xiao and Zhiling Wang, for all the deep conversations about life and love. Sabina Albrecht’s gift of handwarmers also warmed my heart. Luca Pegorari hosted many wonderful feasts and movie marathons. Marius Zoican, for your advice and encouragement even during the MPhil. Stephan Jagau, equilibrium will never be the same without you. Margarita Leib, for teaching me all about sarcasm in the gentlest possible way. Andrej Woerner, you got me to dance when everyone else failed. Martin Wiegand, you’ve given me a wonderful new hobby. Elles Ouweleen, for teaching me to play the piano, and along with Antonie van den Berg, for welcoming me into your home every week for the past three and a half years.

I would like to thank my family for understanding and supporting my life decisions. My parents Cynthia Chan and Chan Ki Chi have provided me with an environment that eventually enabled me to pursue an academic career. My siblings Jennifer, Remington, Brandon and Cassandra have held up the fort wonderfully. My grandmother Luisa Loo and uncle Simon Loo bet on me during one of my most uncertain times.

Finally, I would like to thank my husband Andrew Adrian Pua. We have had a long journey together, and our quest for convergence has finally succeeded. Thank you for giving me the freedom to be myself. You are the anchor of my life.
# Contents

1 Introduction  
   1.1 A short primer on CoCos .............................................. 3  
   1.2 Thesis outline ...................................................... 6

2 CoCos, Contagion, and Systemic Risk  
   2.1 Introduction and literature review .................................... 9  
   2.2 Basic model .......................................................... 12  
      2.2.1 Setup .......................................................... 13  
      2.2.2 The regulator .................................................. 14  
      2.2.3 Timing .......................................................... 15  
      2.2.4 The probability of a bank run ................................ 18  
   2.3 Effect of CoCo conversion on the probability of a run $\theta^*$ ......... 19  
   2.4 CoCo design and run probabilities after conversion ..................... 22  
      2.4.1 Benchmark case: regulatory forbearance ......................... 23  
      2.4.2 Convert-to-equity (CE) CoCos ................................ 25  
      2.4.3 Principal writedown (PWD) CoCos ............................. 26  
      2.4.4 Principal writedown CoCos with cash outlays (CASH) ............ 28  
   2.5 Contagion and systemic risk .......................................... 30  
      2.5.1 Contagion ....................................................... 30  
      2.5.2 Systemic risk ................................................... 33  
   2.6 Conclusion .......................................................... 36

3 CoCo Design, Risk Shifting Incentives, and Financial Fragility ................. 45  
   3.1 Introduction ........................................................ 45  
   3.2 Related literature .................................................. 47  
   3.3 Revisiting the call options approach to residual equity valuation ........ 49  
      3.3.1 Setup .......................................................... 49
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3.2</td>
<td>The endogenous conversion probability</td>
<td>51</td>
</tr>
<tr>
<td>3.3.3</td>
<td>Residual equity valuation with CoCos in the capital structure</td>
<td>53</td>
</tr>
<tr>
<td>3.4</td>
<td>The risk-shifting incentives induced by CoCos</td>
<td>56</td>
</tr>
<tr>
<td>3.4.1</td>
<td>Risk-shifting incentives for given CoCo design</td>
<td>57</td>
</tr>
<tr>
<td>3.4.1.1</td>
<td>PWD CoCos</td>
<td>57</td>
</tr>
<tr>
<td>3.4.1.2</td>
<td>CE CoCos</td>
<td>59</td>
</tr>
<tr>
<td>3.4.2</td>
<td>Effect of other design features on risk-shifting incentives</td>
<td>60</td>
</tr>
<tr>
<td>3.4.2.1</td>
<td>Risk-shifting incentives as a function of the retention parameter ( \varphi )</td>
<td>61</td>
</tr>
<tr>
<td>3.4.2.2</td>
<td>Impact of ( \tau ) on the risk-shifting incentives</td>
<td>61</td>
</tr>
<tr>
<td>3.5</td>
<td>The bank’s optimization problem with CoCos</td>
<td>62</td>
</tr>
<tr>
<td>3.5.1</td>
<td>A bank’s objective function for given leverage ( D )</td>
<td>63</td>
</tr>
<tr>
<td>3.5.2</td>
<td>Subordinated debt vs. equity</td>
<td>64</td>
</tr>
<tr>
<td>3.5.3</td>
<td>Subordinated debt vs. PWD and CE CoCos</td>
<td>66</td>
</tr>
<tr>
<td>3.5.3.1</td>
<td>Optimal risk choices with PWD CoCos</td>
<td>67</td>
</tr>
<tr>
<td>3.5.3.2</td>
<td>Optimal risk choices with CE CoCos</td>
<td>68</td>
</tr>
<tr>
<td>3.5.3.3</td>
<td>Dilutive CE CoCos vs. equity</td>
<td>68</td>
</tr>
<tr>
<td>3.5.3.4</td>
<td>Interaction of ( \tau ) with probability of default</td>
<td>70</td>
</tr>
<tr>
<td>3.6</td>
<td>Interaction of CoCos with pre-existing financial regulation</td>
<td>71</td>
</tr>
<tr>
<td>3.6.1</td>
<td>Setup</td>
<td>72</td>
</tr>
<tr>
<td>3.6.2</td>
<td>Replacing subordinated debt with CoCos</td>
<td>74</td>
</tr>
<tr>
<td>3.7</td>
<td>Conclusion</td>
<td>77</td>
</tr>
<tr>
<td>4</td>
<td>Regulatory Forbearance in the Presence of CoCos</td>
<td>85</td>
</tr>
<tr>
<td>4.1</td>
<td>Introduction</td>
<td>85</td>
</tr>
<tr>
<td>4.2</td>
<td>Review of related literature</td>
<td>87</td>
</tr>
<tr>
<td>4.3</td>
<td>The model</td>
<td>89</td>
</tr>
<tr>
<td>4.3.1</td>
<td>Setup</td>
<td>91</td>
</tr>
<tr>
<td>4.3.1.1</td>
<td>Period ( t = 0 )</td>
<td>91</td>
</tr>
<tr>
<td>4.3.1.2</td>
<td>( t = 1 ): Conversion rules</td>
<td>93</td>
</tr>
<tr>
<td>4.3.1.3</td>
<td>( t = 1 ) and ( t = 2 ): Bank risk-taking and final payment</td>
<td>94</td>
</tr>
<tr>
<td>4.4</td>
<td>Backward induction at ( t = 1 )</td>
<td>94</td>
</tr>
<tr>
<td>4.4.1</td>
<td>The benchmark case</td>
<td>95</td>
</tr>
<tr>
<td>4.4.1.1</td>
<td>The bank’s choice between gambling and liquidation at ( t = 1 )</td>
<td>95</td>
</tr>
<tr>
<td>4.4.1.2</td>
<td>The regulator</td>
<td>96</td>
</tr>
<tr>
<td>4.4.2</td>
<td>Arrival of adverse information at ( t = 1 )</td>
<td>99</td>
</tr>
</tbody>
</table>
4.4.2.1 A $q$-shock: an increase in the proportion of bad loans within the risky asset class

4.4.2.2 A $p$-shock: a decrease in the probability of obtaining the return of the bad loans within the risky asset class

4.5 Endogenizing the cost faced by a regulator

4.5.1 Depositors’ beliefs

4.5.2 Taking depositors’ beliefs into account

4.6 $t = 0$ decisions when the regulator type is unknown

4.6.1 What drives the bank’s decision to choose the safe asset over the risky one?

4.6.2 High type regulator

4.6.3 Low type regulator

4.6.3.1 When the payoff of the safe asset exceeds that of the liquidated risky portfolio

4.6.3.2 When the payoff of the liquidated risky portfolio exceeds that of the safe asset

4.6.3.3 Does the size of the CoCo foreshadow expectations about shocks?

4.6.4 If the regulator’s type is unknown

4.6.4.1 Suppose there were relatively few CoCos

4.6.4.2 Suppose there were relatively many CoCos

4.7 Conclusion

Bibliography

Summary

Nederlandse Samenvatting
Chapter 1

Introduction

The creation of new banking regulation can hardly be described as a smooth process, because there are many conflicting elements that a regulator must accommodate. For instance, the regulator must be careful to create rules that will not stifle the industry. At the same time, the regulator must have sufficient political will to carry out its mandates, one of which is to protect the financial system from risks. The final form of any set of rules results from a protracted period of consultation between the industry and the regulator. The final outcome will only be as good as the foresight and intentions of the individuals involved. It is not inconceivable that the industry would put forward suggestions that maximize their own benefit, but impose negative externalities upon the financial system. As noted by Boyer and Ponce [2012] and Hardy [2006], regulatory capture is a common occurrence in banking supervision. Also, even if regulation was well-intentioned to start with, it is not always the case that all the possible consequences have been examined. I present two examples that by now have become textbook fodder: securitization and risk-weighted assets. The intention of allowing banks to securitize their loans was to allow them to dispose of nonperforming loans. However, since the type of loans wherein securitization was allowed was not specified, banks were able to game the system. The banks no longer cared about the quality of loans that they have been extending, since they were able to eliminate those loans off their balance sheets. As for the risk weights in Basel II, they were intended as a refinement of the crude risk weights in Basel I. However, it retained the relatively low weight assigned to mortgages, reflecting the belief that collateralized loans were safer than others. Without intending to, this led to an increase in the demand for housing, leading to a bubble, and eventually contributed to the financial crisis of

---

1For the consultative documents pertaining to Basel III, the maximum number of days between the release of the consultative document and the deadline for comments is 127 days. The number of nonanonymously comments ranged from 6 to 121. Most of the comments came from banks and banking associations, with the occasional academic in the mix.
There are many possible reasons for the incomplete analysis of the impact of regulation on the financial system. I present three of them here. One of them is the relatively short time frame wherein the said rules were prepared. This is not surprising, considering that regulation is usually formed as a reaction to adverse events.\(^2\) Around the period when Bear Stearns was making headlines, regulators were already making changes to the system. Proposed revisions to the market risk framework of Basel II had been set into motion in July 2008, and shortly after, the first proposals to move towards Basel III were made. The Dodd-Frank Act was proposed in July 2009 and finalized one year later. While new regulation has remedied the most obvious problems, they have not fixed the underlying ones. Acharya [2011] has criticized both the Dodd-Frank Act and Basel III for not being mindful of the impact of regulation on incentives for the agents in the financial system.

The second reason for the incomplete analysis is that most of the new rules are add-ons to existing regulation, in an effort to address problems that crop up. The problem with this approach is that there is a tendency to create kludges\(^3\) in the financial system. This is not to say that new regulation is completely harmful to the system. For instance, the increase in capital requirements brought about by Basel III and Dodd Frank has made the financial system relatively safer in the sense that the financial system is more able to withstand shocks.

Finally, financial innovations are being created at the same time as new rules are, leading to unexpected interactions. For instance, Blundell-Wignall and Atkinson [2010] note that credit default swaps arose at the same time that the refined risk weighted asset bucket weights did, which allowed the banks to undermine the fundamental idea of capital weights. It is because of such unexpected events that regulatory discretion is important. In particular, Pillar 2 of Basel II is meant to deal with bank-specific uncertainties. However, there must be a balance between rules and discretion. This does not seem to be the case, if one contrasts the voluminous requirements of Basel II’s Pillar 1, against the free rein that supervisors have in Basel II’s Pillar 2. This is important because there is a tendency for regulators to forbear on tough decisions, because of short term gains that may be foregone. Numerous papers have been written on regulatory forbearance (see Mailath and Mester [1994] and Shapiro and Skeie [2015] for instance). The problem with forbearing on tough decisions is that the system may survive in the short term, but may be more fragile in the long term.

With all these issues, it is not surprising that regulators are always on the lookout for

\(^2\)The time it takes to get from the proposal to the final document stage has high variance: for Basel II, it took 6 years, for Basel III and the Dodd-Frank Act, it took around one year each.

\(^3\)The Merriam Webster Dictionary defines kludge as "a system and especially a computer system made up of poorly matched components."
new tools that appear to address the problems they have encountered. For instance, shortly after the financial crisis of 1933 when bank failures were contagious, deposit insurance became prevalent (Federal Deposit Insurance Corporation [1998]). However, as is now widely known, deposit insurance alters the banking system’s incentives and encourages moral hazard. Most recently, regulators have encouraged the adoption of contingent convertible capital (CoCo), in light of the high cost of bailing out financial institutions during the financial crisis of 2007. CoCos are hybrid instruments that are issued by banks as debt, but convert to equity or are written down upon the occurrence of a trigger event. Upon the occurrence of a conversion, the issuer’s loss absorption capacity increases, without the involvement of the government or the taxpayers. Instead, the holder of the CoCo shoulders the losses. For this reason, CoCos have become very attractive to regulators, and their use has been passed into law in Europe. But if history is a guide, regulators should realize that there is always more to financial innovation than meets the eye. It is crucial that the properties of CoCos be deeply investigated before rolling it out on a wide basis. This dissertation is a step in that direction.

1.1 A short primer on CoCos

CoCos are hybrid instruments that are designed to improve the loss absorption capacity of the issuer without involving transfusions from new equity or taxpayer bailouts. These instruments were proposed by Flannery as early as 2002, but were thrust into the limelight after the financial crisis of 2007. Banks generally issue CoCos, though the insurance sector has already started looking into them as well. CoCos are issued as debt, but what happens after conversion depends on the type of CoCo issued. There are generally two types of CoCos based on design: principal writedown (PWD) CoCos are partially or fully written off the balance sheet, while convert-to-equity (CE) CoCos are converted to shares at a preset price.

There are two type of trigger events: automatic and discretionary. Automatic trigger events occur when the bank’s equity ratio falls below a preset amount. The calculation may be based on either market or book values, although all of the issued CoCos so far have calculations based on book value. Discretionary trigger events occur when the regulator deems the bank to be near or at the point of nonviability (PONV). Because of the nature of the trigger event, CoCos have also been known as reverse convertible bonds, because they convert when there is a negative event, rather than a positive one (as ordinary convertibles do). To qualify as part of

---

4The trigger event is when the issuer’s equity ratio falls below a preset threshold, or when the regulator assesses that the bank is close to the point of nonviability.

5More recently, the Financial Stability Board has created demand for CoCos by requiring globally systemic banks to increase their loss absorption capacity by CoCo issuance.
regulatory capital under Basel III, CoCos must have at least the discretionary trigger. Because of this, most of the issued CoCos possess both types of triggers.

Figure 1.1 presents the European issuances of CoCos by design.

**Figure 1.1: Annual CoCo Issuance of European Banks by Design**

<table>
<thead>
<tr>
<th>Year</th>
<th>PWD</th>
<th>CE</th>
<th>N/A</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>2.8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2011</td>
<td>2.8</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>2012</td>
<td>8.4</td>
<td>3.4</td>
<td>0</td>
</tr>
<tr>
<td>2013</td>
<td>16.3</td>
<td>7.2</td>
<td>0</td>
</tr>
<tr>
<td>2014</td>
<td>25.8</td>
<td>19.7</td>
<td>0</td>
</tr>
<tr>
<td>2015</td>
<td>4.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2016</td>
<td>12.9</td>
<td>2.6</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: Dealogic (through the Association for Financial Markets in Europe)

It is notable that PWD CoCo issuances have overtaken CE CoCo issuances since 2012. By the end of 2016, PWD CoCos amounted to 57% of total European issuances, while 43% were CE CoCos, and less than 1% were of an unspecified type. Most of the CoCo issuance is by European and Asian banks. US banks have not participated in the wave of CoCo issuances because CoCos are treated as equity under US GAAP and as such, do not have tax benefits.

Because of their loss absorption capacity, CoCos have made their way into formal regulation. In June 2011, the Basel Committee on Banking Supervision released the final version of Basel III, which addresses additional measures to ensure the stability of the banking system. One notable change from Basel II is the strengthening of the capital base by enforcing stronger requirements for regulatory capital: loss absorption capacity is now a necessary quality for instruments to be included as part of Additional Tier 1 (going concern) capital and Tier 2 (gone concern) capital. Existing instruments that no longer qualify as regulatory capital have been phased out beginning January 2013, and replaced by CoCos. The criteria for whether a CoCo falls under Additional Tier 1 or Tier 2 depends only on their trigger level: above 5.125% qualifies as Additional Tier 1, otherwise they qualify as Tier 2. Figure 1.2 shows the distribution of the European-issued CoCos by their trigger ratios.
While Basel III itself has no legal bite, it was translated into EU law in 2013 by the issuance of Directive 2013/36/EU, also known as the Capital Requirements Regulation and Directive (CRR/CRD-IV). This means that for EU banks, at most 3.5% of the 8% regulatory capital requirement will be filled in by CoCos. Moreover, there is no upper bound to the amount of CoCos they can issue. In addition, in November 2015, the Financial Stability Board (FSB) has released its Total Loss Absorption Capacity (TLAC) Standard for globally systemic financial institutions. The TLAC Standard mandates that for these institutions, minimum loss absorption capacity must be raised to 16% of risk weighted assets by January 2019, and to 18% by January 2022. The TLAC Standard’s description of the loss absorbing instruments fits CoCos. With this, one should see an increase in the CoCo issuances over the next few years.

As CoCos are new and not well-understood, steps have been taken to protect unwitting consumers. In October 2014, the U.K.’s Financial Conduct Authority has prohibited banks from issuing CoCos to ordinary retail investors. Moreover, the market has been shown to be sensitive to potential trigger events. In February 2016, the price of CoCos issued by Deutsche Bank fell from fears that the bank would not be able to meet its coupon payment obligations. However, the prices of other CoCos followed suit, despite the absence of adverse news regarding their issuers.
1.2 Thesis outline

Upon first glance, CoCos appear to be exactly the type of instrument that regulators wish for. Moreover, there are relatively few restrictions regarding the issuance of CoCos. One may argue that CoCo conversion is a straightforward task. However, one must look at how these rules will interact with the agents in the economy, as there may be undesirable and unexpected consequences. For instance, the conversion of the CoCo may be construed as signal about the asset quality of the bank, which may lead to contagious bank runs in the system, even for non-CoCo issuers. Another is that if the CoCo is of the principal writedown type, the bank may accelerate the conversion by choosing a high risk level, as doing so effectively increases the bank’s residual equity value. Finally, the regulator’s desire for a trigger that she can control is an invitation for regulatory forbearance, which is what she was trying to avoid in the first place.

Throughout this dissertation, we take the capital structure as a given. This is because banks are not able to instantaneously adjust their capital structures as the situation changes. Also, throughout this dissertation, we take the CoCo holders as passive agents. If CoCos are correctly priced, the CoCo holders are sufficiently compensated for the risk they bear, and therefore lose any incentive to monitor the bank. But also it is only once the CoCo is issued that all the other agents act. Banks may decide to risk-shift, depositors may run, and the regulator may decide never to convert CoCos at all. These events are not very obvious consequences of CoCo issuance. In each chapter, we consider different settings under which CoCos are issued, and examine how different agents behave under the settings, as well as the consequences. We do this with a variety of methods, but use three period models to make them as tractable as possible.

In Chapter 2, "CoCos, Contagion, and Systemic Risk," jointly written with Sweder van Wijnbergen, we examine the CoCos that convert upon regulatory discretion. We abstract away from bank decision-making, focusing instead on how the regulator’s decision to convert affects the depositors’ running decisions. We assume that the regulator has better information about the economic fundamentals compared to the rest of the agents in the economy. The regulator forces the conversion of CoCos when she obtains information that the bank is unlikely to remain viable given the economic state. Therefore, conversion is always interpreted as a negative signal, and results in a bank run. We apply global games in the spirit of Goldstein and Pauzner [2005] in order to obtain a measure of the probability of bank runs, as well as to eliminate the multiple equilibria issue that often arises in bank run models. The probability of bank runs also determines the measure of agents that run. The number of agents that run, in addition to the type of CoCo that is issued, have different impacts on a bank’s residual equity,
conditional on the bank surviving. Therefore, some CoCos are better than others ex post when converted. The tension that the regulator faces between facing a bank run and increasing the bank’s loss absorption capacity is highlighted, although in this chapter we do not take a stand on it. Instead, we focus on the impact of a conversion on the financial system. Assuming that asset returns are correlated, and increase in correlatedness during times of crisis, it becomes clear that the bank runs become contagious whenever CoCos are converted. For this reason, we argue that systemic risk increases upon CoCo conversion, which is surprising given that they were intended to act as shock absorbers in the first place.

In Chapter 3, "CoCos, Risk-Shifting, and Financial Fragility," jointly written with Sweder van Wijnbergen, we consider CoCos that convert automatically, without the intervention of the regulator. We abstract away from the depositors by assuming deposit insurance, and this allows us to assess whether banks would risk-shift more or less with CoCos in their capital structure. We argue that because banks are able to choose their own risk levels, and that the risk choice affects the return distribution, the probability that the CoCos convert is not exogenous. Depending on the design of the CoCo, the bank potentially gains a wealth transfer from the conversion. We argue that this motivates higher risk-shifting relative to standard instruments like debt and equity. In order to show this, we use the language of call options. Since CoCos contain elements of debt and equity, the valuation of the issuer’s residual equity must take these elements into account. In particular, the issuer’s residual equity can be expressed as residual equity with subordinated debt, plus an expected wealth transfer. However, we show that for certain CoCo designs, the expected wealth transfer is increasing in the risk level chosen by the bank. Therefore, whenever banks maximize their expected returns net of default costs, they would always choose higher risk levels under these types of CoCos than under the same amount of subordinated debt, or additional equity. The policy implication is that one cannot treat CoCos as true substitutes for equity, because while they have the same loss-absorption capacity, they induce different incentives. Finally, the use of CoCos as equity supplements distorts the true level of equity required, relative to the same amount of subordinated debt would, for the regulator to obtain a target probability of default that depends on the bank’s leverage and risk levels.

In Chapter 4, "Regulatory Forbearance in the Presence of Cocos," jointly written with Sweder van Wijnbergen, we again consider CoCos that convert upon regulatory discretion, and we model the bank’s asset choices as well. We do this in order to focus on the interaction between a regulator and a CoCo-issuing bank. In order to do this, we use a simple game-theoretic three-period model. We give the bank two opportunities to commit moral hazard upon obtaining its funds: the initial asset choice at \( t = 0 \), and whether to gamble for resurrection or
to liquidate bad loans when they occur, at $t = 1$. In this setting, the bank will only choose the socially optimal action (liquidate) if its skin in the game is high enough at $t = 1$. This is where CoCos would prove to be useful, as conversion reduces the bank’s outstanding liabilities. However, if the regulator faces sufficiently high costs of conversion, she will always forbear even if conversion improves a bank’s loss absorption capacity. We endogenize the regulator’s cost of conversion by embedding depositors’ beliefs regarding asset quality into it. Essentially, the cost of conversion is the marginal probability of a bank run based on depositors’ beliefs. The regulator’s action as anticipated by the bank then feeds back into its $t = 0$ decision: if the regulator is forbearing, the bank would choose the risky asset over the safe one as it provides a larger private benefit for the bank. If the regulator was tough, conversion would only be a sufficient risk deterrent at $t = 1$ if there were not too many CoCos issued to begin with. However, this again brings into the forefront the tradeoff between loss absorption capacity and risk shifting of banks.
Chapter 2

CoCos, Contagion, and Systemic Risk

2.1 Introduction and literature review

As early as 2002, Flannery proposed an early form of contingent convertible (CoCo) capital that he called reverse convertible debentures. The idea was simple: whenever the bank issuing such debentures reaches a market-based capital ratio which is below a pre-specified level (say, 8% of assets), a sufficient number of said debentures would automatically convert to equity at the prevailing market price of the bank’s shares. The automatic conversion feature frees the issuing bank from having to raise additional capital immediately when its capital ratio is lower than the minimum requirement. For larger shocks, conversion may not be enough to restore compliance with capital requirements, but it would make banks merely undercapitalized instead of bankrupt.

Flannery’s initial CoCo design proposal was attractive, as its automatic conversion feature had the potential to avoid socially costly bailouts. After the 2007 financial crisis, regulators realized that even though systemically important financial institutions (SIFIs) held Tier 2 Capital, that type of capital failed to be loss-absorbing during the time of distress. Instead, some of the SIFIs were bailed out while others were allowed to fail. Yet despite having Tier 2 status, many of the subordinated loans continued to be serviced. In response, the Basel Committee on Bank-
ing Supervision (BCBS) circulated a consultative document that was one of the precursors to what is now known as Basel III. Among the changes were the redefinition of "gone concern" to include potential bailout situations, and the inclusion of CoCo-like instruments as part of Additional Tier 1 Capital. Also, Basel III suggested that CoCos might play a role in ensuring that SIFIs would have higher loss absorption capacities than regular financial institutions.

The inclusion of CoCos as part of Additional Tier 1 Capital is a likely factor in the increase of CoCo issuance. European CoCo issuances totaled 23.5 billion Euros in 2016, up from only 3.2 billion Euros in 2010. Within the same period, the academic literature branched off in three different directions. Flannery [2005] and McDonald [2013] were among those that dealt with design features such as triggers and bases. Pennacchi [2010] dealt with the pricing and valuation of CoCos. Finally, Martynova and Perotti [2016], Hilscher and Raviv [2014] and Berg and Kaserer [2015] consider the effect of CoCos on risk-taking incentives of banks. Moreover, several survey articles have been written about CoCos. Maes and Schoutens [2012] provide an overview of CoCos and enumerate the potential downside of CoCo issuance such as contagion from the banking to the insurance sector, and the creation of a "death spiral" where CoCo holders short-sell the stock of the issuing bank in order to profit from potential conversion. Avdijev et al. [2013] discuss the features of the CoCo trend - from the reason why banks issue them to the main groups of investors that are interested in buying CoCos, as well as the pricing of CoCos. Wilkens and Bethke [2014] summarize and empirically assess some of the pricing models’ performance. There is disagreement in the literature on whether CoCo conversion should be triggered based on market prices or book values (e.g. capital ratios used under Basel III). On one side are authors like Sundaresan and Wang [2015], who argue that using market prices in calculating trigger values might lead to multiple equilibria problems and potentially destabilizing bear runs on bank stock. On the other side, Calomiris and Herring [2013] argue that this problem can be mitigated by using 90-day moving averages of what they call "quasi-market data", arguing that using book values creates room for creative accounting - for example pressure to delay recognition of losses. We do not take a position in this debate, our analysis applies to both types of triggers.

The effectiveness of CoCos hinges on bank failure being caused by banks having insufficient equity to absorb losses once they have occurred. However, the majority of bank assets is funded by demand deposits. One cannot ignore the possibility that a bank may fail because

---

3“Proposal to ensure the loss absorbency of regulatory capital at the point of non-viability”, Basel Committee on Banking Supervision [2010]
4To be counted as Additional Tier 1, the instruments must meet several requirements set forth in Basel III.
5Association for Financial Markets in Europe [2016d]
6Calomiris and Herring [2013] define quasi-market data as a ratio of market value of equity and book value of debt
depositors run before losses actually occur, as they anticipate what may happen once the losses do occur. Jacklin and Bhattacharya [1988] and Chari and Jagannathan [1988] build on the Diamond and Dybvig [1983] model of bank runs to show that depositors who are able to update their information about the realization of bank returns act accordingly. However, early bank run models have the disadvantage that runs are zero probability events, sunspot equilibria. That makes it impossible to assess the impact of fundamentals on the probability of runs and the associated bank collapse. Goldstein and Pauzner [2005] take the Diamond-Dybvig model a substantial step further by casting the standard banking problem into a global games framework, allowing them to obtain a measure for the probability of a bank run which can be linked to economic fundamentals.

In this paper, we argue that a CoCo conversion conveys information that will lead depositors to update their beliefs in a manner that increases the probability of bank runs. Furthermore we examine three major types of CoCos and show that some designs are better than others in terms of their effect on depositor run incentives. We make a second point that is crucial for the relation between CoCo conversions and systemic risk. If other banks hold assets with correlated returns, depositors of other banks will interpret the CoCo conversion as a negative signal on their asset returns too. This updates the beliefs of the depositors of the other banks, which raises the probability of runs on said banks, even if they were non-CoCo-issuing. This would not happen if conversion did not occur in the CoCo-issuing bank. In other words, through contagion effects conversion imposes an information externality on other banks, which raises systemic risk.

The large and growing literature on contagion has by and large highlighted three contagion types: Balance sheet contagion (through firesale effects, cf Diamond and Rajan [2011]), funding squeezes whereby distress in one bank causes liquidity to dry up for another bank (Luck and Schempp [2014]), and information contagion (Ahnert and Georg [2016]). The contagion channel that plays a role in our analysis falls in the third category: A CoCo conversion gives out a signal about asset quality that triggers a run not only in the bank concerned but also in banks with correlated assets.

This contagion channel is a second reason why we expect CoCos to raise rather than reduce systemic risk. This is worrisome also because CoCos are mentioned by Basel III as potentially useful for increasing the loss absorption capacity of SIFIs. While it is true that conversion may keep the issuing banks afloat in times of distress by immediately reducing their outstanding liability, it does not reduce the liability to depositors. As such, conversion increases the risk

7At the time of writing, there were three types of CoCos issued: principal writedown CoCos, convert-to-equity CoCos, and principal writedown CoCos that pay out cash to the CoCo holders upon conversion. Currently, only the first two types remain.
that the converting banks, and other banks to the extent that they have correlated assets, will face a run.\footnote{It is sometimes argued that the threat of dilution from CoCo conversion encourages existing equity holders to infuse at least enough additional capital to stave off conversion, rendering the signalling effect of conversion nonexistent. However, whether this incentive exists this depends on the CoCo design. In Chan and van Wijnbergen [2017a] we show in that in several currently popular CoCo designs, wealth transfers upon conversion actually go from junior creditors to equity holders. This reduces the incentives for equity holders to supply capital in times of distress, and may even reverse it. While by design equity holders cannot pull out capital, the existence of such such a perverse incentive might induce them to push for additional risk taking by bank management.}

While CoCos have different trigger points and conversion mechanisms, many of them have a "point of nonviability" clause which effectively gives regulators control over when CoCos convert. But regulators may end up having to make difficult choices in such circumstances. If conversion actually raises systemic risk, microprudential and macroprudential considerations may well be at odds, possibly leading to high pressure for regulatory forbearance.

### 2.2 Basic model

Bank runs arise due to asymmetric information regarding the need of depositors. Diamond and Dybvig [1983] show that when bank returns are certain, banks can write contracts that mimic the first best outcome that an omniscient social planner can implement, by taking the type of depositor into account. However, because the type of each depositor is unknown to the bank, a sequential service constraint is put in place. As there is nothing in their model that coordinates the beliefs of depositors, two equilibria emerge: either a bank run occurs, or it does not. Goldstein and Pauzner [2005] introduce the more realistic case of uncertain returns into the Diamond and Dybvig [1983] model, where the probability of obtaining positive returns depends on the economic fundamentals. In addition, they allow the depositors to have varying beliefs about the true state of the economy, as opposed to sharing a singular belief. In this way, the depositors are able to coordinate on something, thus eliminating the the multiple equilibria problem in Diamond and Dybvig [1983], as well as obtaining a measure of the probability of bank runs.

We extend Goldstein and Pauzner [2005] by adding CoCos and equity to the usual depositor-only setup. This is because we want to highlight the tension between the CoCos’ loss-absorption benefits and their potential impact on the probability of bank runs. This is impossible in standard bank run models, as they have no equity to speak of. Moreover, we add a zero-probability event where the positive return is lower than expected. This is because we want to model the situation where the regulator comes across more information than the rest of the agents, and has decided to act upon it. Goldstein and Pauzner [2005] show that the probability of a
run is increasing in the amount promised to the depositors who are impatient consumers. By the same token, any factor affecting the relative return of the patient consumers will alter the probability of a run. In a model where the lower returns will induce the regulator to convert the CoCos, conversion definitely signals bad news and informs the agents that the return that are worse than expected have materialized. This then leads to an increase in the probability of a bank run.

2.2.1 Setup

Our model has three periods ($t = 0, 1, 2$), a bank, a regulator, and three types of agents, each endowed with one unit of wealth: $\bar{n}$ depositors, $\bar{e} - \bar{n}$ CoCo holders, and $1 - \bar{e}$ equity holders. The notation was chosen such that the total measure of agents is 1. We assume that the measures are fixed, as we are not interested in optimal capital structure. Figure 2.1 illustrates the continuum of agents in this model.

![Figure 2.1: Agent Types and Measures](image)

At $t = 0$, the depositors, CoCo holders and equity holders essentially set up the bank by investing their funds into it. In order to induce the agents to invest their funds, they are promised returns: CoCo holders are promised some return $r_c$, while depositors that withdraw at $t = 1$ are promised some return $r_1 > 1$. Equity holders obtain any residual profit. We assume that in this model that the CoCo holders and the equity holders are risk-neutral and are long-term investors (or alternatively, have long-term liquidity needs). In contrast, the $\bar{n}$ depositors are risk averse with $-cu''(c)/u'(c) > 1$ for some $c > 0$. In addition, the depositors are uncertain of their own liquidity needs at $t = 0$: a fraction $\lambda$ will turn out to be impatient consumers who can only consume at $t = 1$ with corresponding utility $u(c_1)$. The remaining $1 - \lambda$ will turn out to be patient consumers who may consume at either $t = 1$ or $t = 2$, with corresponding utility $u(c_1 + c_2)$. At $t = 0$, there is no aggregate uncertainty: the proportion of impatient and patient consumers is known. At $t = 1$, each depositor discovers his own type, and this remains private information. Because the type of each depositor is unknown to the bank, a sequential service constraint is put in place.
There exists an investment technology that the bank has access to. It delivers returns which vary with the economic state $\theta \sim U[0, 1]$. A higher realization of $\theta$ indicates better economic conditions. Specifically, one unit of investment at $t = 0$ yields $R$ with probability $p(\theta)$, and 0 with probability $1 - p(\theta)$ at $t = 2$. Also, we assume that $p(\theta)$ is increasing in $\theta$. Of course, $p(\theta) \in [0, 1]$. The investment may be liquidated at $t = 1$ without cost other than the foregone yield. At $t = 1$, there may be a shock to the economy that affects not the fundamental value $\theta$, but the return $R$. In particular, the return may be some $R_L < R$, but is assigned a zero probability at $t = 0$. This is because the shock is presumed to be unknown at $t = 0$, materializing only at $t = 1$. In addition, each depositor has a belief $\theta_i$ about the economic fundamental $\theta$ that is drawn uniformly from $[\theta - \epsilon, \theta + \epsilon]$. Depositors know the probability function $p(\cdot)$, but evaluate it at their own $\theta_i$.

Because the risky investment can be liquidated without cost, agents are better off investing their endowment into the asset. Also, we assume that $R$ is high enough so that $E_\theta p(\theta)u(R) > u(1)$, making it worthwhile for patient consumers to wait until $t = 2$. Without any pooling of risk, the best attainable utility levels are $u(1)$ for the early consumers and $p(\theta)u(R)$ for the late consumers (for a given state of nature $\theta$).

Even with long term funding without early withdrawal possibilities, runs are still possible as long as $\frac{1}{\bar{r}_1} < \bar{\pi}$. We assume this throughout the paper. We furthermore assume that the contracts offered by the banks are such that the incentive compatibility constraint $u(r_1) < p(\theta)u\left(\frac{1-\lambda r_1}{1-\lambda}R\right)$ in fact holds: late consumers prefer to wait. Finally, there is no deposit insurance in this model.9

2.2.2 The regulator

Even though we have introduced CoCos in the model, we abstract away from the utility of the CoCo holders. The value of CoCos in this model is in the signal that their conversion transmits to the rest of the agents, as well as by increasing the bank’s equity levels upon conversion. There are two ways that CoCos may be converted in practice: by accounting/market value triggers, or by regulatory discretion. In this paper we model the latter.

There is a regulator who is interested in preserving financial stability. In accordance with the structure of many of the issued CoCos, we assume conversion occurs when the regulator decides to trigger the conversion. We assume that the regulator when she decides to force conversion knows more than the other agents: that a regulator may discover at $t = 1$ that

9Alternatively, there may be deposit insurance for retail deposits but there are also large and substantial wholesale deposits which are not insured. Or one can think of other forms of short term funding with roll over risk exposure like REPOS or commercial paper.
asset returns at $t = 2$ will be lower than what is compatible with a capital ratio above the CoCos trigger value. In particular, we assume that the regulator finds out that the returns in the good state of nature will be $R_L < R$. Based on the finding, the regulator decides whether or not to convert the CoCos. The regulator’s decision to intervene (or for that matter collect the additional information about asset quality to begin with) is not modeled in this paper. But as will be shown later, her decision to convert CoCos introduces a negative signal about asset returns even though the economic fundamentals $\theta$ remain the same. We assume in this paper that this regulatory inspection and its outcome are not anticipated (not priced in) by investors, in particular by the deposit holders. In the wording of Gennaioli et al. [2013], the regulatory intervention is a neglected risk. Zhou and van Oordt [2016] in fact show that tail risk events are not priced in the case of similar securities (options).

2.2.3 Timing

First, let us consider the situation prior to conversion. By assumption, at $t = 0$, a fraction $\bar{e} - \bar{n}$ of agents has invested in CoCos and a fraction $1 - \bar{e}$ has invested in equity. The remaining $\bar{n}$ are depositors whose types are unknown at $t = 0$.

With these agents’ endowments, the bank has a total of 1 unit of wealth. The bank invests the entire amount in the risky asset. It also promises a fixed return $r_1 > 1$ to depositors who withdraw at $t = 1$, and a stochastic return that in the absence of runs by late consumers equals $\tilde{r}_2 = \max \left[ \frac{\pi - \lambda n}{\pi - \lambda R} R, 0 \right]$, depending on the state of nature that materializes at $t = 2$.\footnote{One way to think of our set up is in line with the costly audit literature where audits, or in our case more likely an on-site regulatory inspection, are performed only infrequently and possibly randomly.} Note that this is similar to the Diamond and Dybvig [1983] contract since $\frac{\pi - \lambda n}{\pi - \lambda R} R = \frac{1 - \lambda n}{1 - \lambda} R$. Henceforth we use $r_D$ for $\frac{1 - \lambda n}{1 - \lambda} R$. Define $n$ as the proportion of agents who withdraw at $t = 1$. Since early consumers always withdraw at $t = 1$, $n \geq \lambda \bar{n}$. And because the CoCo holders and equity holders cannot withdraw early, we also have $n \leq \bar{n}$.

At $t = 1$, before agents can act, the regulator comes in and decides whether to convert CoCos or not. If conversion occurs, the return in the good state must have been found to be some $R_L < R$. In such a case, depositors’ return will be scaled downwards accordingly (they will receive $\frac{1 - \lambda n}{1 - \lambda} R_L$ instead of $\frac{1 - \lambda n}{1 - \lambda} R$). Effectively, the depositors have a variable-rate contract with the bank. Without conversion, no information is revealed. This still preserves the risk-sharing feature of Diamond-Dybvig, which concerns not so much the interest rate risk as the type-related liquidity risk.

Also at $t = 1$, depositor types are revealed. The bank gives $r_1 > 1$ to depositors withdrawing $\bar{e} - \bar{n}$ of agents has invested in CoCos and a fraction $1 - \bar{e}$ has invested in equity. The remaining $\bar{n}$ are depositors whose types are unknown at $t = 0$.

With these agents’ endowments, the bank has a total of 1 unit of wealth. The bank invests the entire amount in the risky asset. It also promises a fixed return $r_1 > 1$ to depositors who withdraw at $t = 1$, and a stochastic return that in the absence of runs by late consumers equals $\tilde{r}_2 = \max \left[ \frac{\pi - \lambda n}{\pi - \lambda R} R, 0 \right]$, depending on the state of nature that materializes at $t = 2$.\footnote{Appendix 3.A.1 contains the calculations for this section.} Note that this is similar to the Diamond and Dybvig [1983] contract since $\frac{\pi - \lambda n}{\pi - \lambda R} R = \frac{1 - \lambda n}{1 - \lambda} R$. Henceforth we use $r_D$ for $\frac{1 - \lambda n}{1 - \lambda} R$. Define $n$ as the proportion of agents who withdraw at $t = 1$. Since early consumers always withdraw at $t = 1$, $n \geq \lambda \bar{n}$. And because the CoCo holders and equity holders cannot withdraw early, we also have $n \leq \bar{n}$.

At $t = 1$, before agents can act, the regulator comes in and decides whether to convert CoCos or not. If conversion occurs, the return in the good state must have been found to be some $R_L < R$. In such a case, depositors’ return will be scaled downwards accordingly (they will receive $\frac{1 - \lambda n}{1 - \lambda} R_L$ instead of $\frac{1 - \lambda n}{1 - \lambda} R$). Effectively, the depositors have a variable-rate contract with the bank. Without conversion, no information is revealed. This still preserves the risk-sharing feature of Diamond-Dybvig, which concerns not so much the interest rate risk as the type-related liquidity risk.

Also at $t = 1$, depositor types are revealed. The bank gives $r_1 > 1$ to depositors withdrawing
at this time as long as it is able to do so. To this end, the bank must liquidate part of the amount invested in the risky asset. This means that the bank can only serve at most \( n = \frac{1}{r_1} \) agents at \( t = 1 \). The \( t = 2 \) payoffs to the depositors in the no-conversion case are summarized in Table 2.1.

<table>
<thead>
<tr>
<th>Withdrawal in</th>
<th>if ( \lambda \bar{n} &lt; n &lt; \lambda \bar{n} + \frac{e}{r_1} )</th>
<th>if ( \lambda \bar{n} + \frac{e}{r_1} &lt; n &lt; \frac{1}{r_1} )</th>
<th>if ( n \geq \frac{1}{r_1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t = 1 )</td>
<td>( r_1 )</td>
<td>( r_1 )</td>
<td>( \begin{cases} r_1 &amp; \text{w.p. } \frac{1}{nr_1} \ 0 &amp; \text{w.p. } 1 - \frac{1}{nr_1} \end{cases} )</td>
</tr>
<tr>
<td>( t = 2 )</td>
<td>( r_D = \frac{1-\lambda \bar{n}}{1-\lambda} R )</td>
<td>( \begin{cases} \frac{1-nr_1}{1-(\lambda \bar{n} + \frac{e}{r_1})r_1} &amp; \text{w.p. } p(\theta) \ 0 &amp; \text{w.p. } 1 - p(\theta) \end{cases} )</td>
<td>0</td>
</tr>
</tbody>
</table>

Depositors who wait until \( t = 2 \) to withdraw will receive a return which depends on how many depositors ran at \( t = 1 \). CoCo holders and equity holders, being junior to depositors, will receive amounts only once all the depositors have been served. How that surplus is divided between them depends on CoCo pricing and corresponding CoCo returns. With probability \( 1 - p(\theta) \), the return (to creditors and equity holders alike) will be zero.

The bank can ensure that it pays out \( r_D \) to the late consumers as long as \( \lambda \bar{n} < n < \lambda \bar{n} + \frac{e}{r_1} \). This is because unlike Goldstein and Pauzner [2005], our model has additional sources of funding from CoCo holders \( (\bar{e} - \bar{n}) \) and equity holders \( (1 - \bar{e}) \). Because depositors are senior to all other agents, these proceeds (call them \( e' = (\bar{e} - \bar{n}) + (1 - \bar{e}) = 1 - \bar{n} \)) may be exhausted in order to pay out \( \frac{e'}{r_1} \) more depositors at \( t = 1 \), and still manage to pay out \( r_D \) to the remaining depositors at \( t = 2 \).\(^{12}\)

But for values of \( n \) between \( \lambda \bar{n} + \frac{e}{r_1} \) and \( \frac{1}{r_1} \), withdrawers at \( t = 2 \) may not obtain the entire \( r_D \). As the bank’s resources are finite at 1 unit, there can be only at most \( n = \frac{1}{r_1} \) depositors who may be served at \( t = 1 \). In this case, the asset is fully liquidated - nothing is left to earn \( R \) in case the good state of nature materializes at \( t = 2 \). Between \( n = \lambda \bar{n} + \frac{e}{r_1} \) and \( n = \frac{1}{r_1} \), each additional runner requires further liquidation of the long-term asset, leaving a smaller quantity of the asset to potentially earn \( R \). One can then determine the rate at which \( r_D \) erodes - it depends on \( n \). Figure 2.2 shows the payoffs to depositors who wait until \( t = 2 \) as a function of \( n \) under different levels of asset returns.

\(^{12}\)Note that the region where money paid out to early runners eats into equity returns (because the assets generating those returns have to be liquidated) is shorter than \( e' \) because runners get paid out \( r_1 > 1 \)
Depositors who are impatient consumers will always withdraw at $t = 1$. But those who are patient consumers realize the tradeoff between waiting until $t = 2$ or running at $t = 1$, and know that their payoffs depend on the number of people that are expected to run ($n$) at $t = 1$ as well. At this point it is useful to define $v(\theta, n)$ as the difference in utility of waiting versus running for given values of $\theta$ and $n$, for the patient consumer. This equation may be derived from Table 2.1.

$$v(\theta, n) = \begin{cases} 
p(\theta)u\left(\lambda \frac{1-n\bar{r}}{r_1} \right) - u(r_1) & \text{if } \lambda \bar{n} + \varepsilon' < n < \frac{1}{r_1} \\
0 - u(r_1) \frac{1}{n\bar{r}} & \text{if } \frac{1}{r_1} < n < \bar{n}
\end{cases} \quad (2.1)$$

(2.1) shows that the patient depositor’s payoffs are a function of $n$. Once $n \geq \frac{1}{r_1}$, one gets zero upon waiting. On the other hand, by running, one gets repaid with probability $\frac{1}{r_1}$, as there are at most that many depositors who may be accommodated by the sequential service constraint at $t = 1$.

(2.1) depends on $r_D$ as well, which in turn depends on $t = 2$ return $R$. So if at $t = 2$ the good state of nature return on the risky asset turns out to be lower, say $R_L = R - \Delta < R$, the payoff schedule to $t = 2$ withdrawers shifts down to the dashed line in Figure 2.2. Throughout we are assuming that $\bar{n} > \frac{1}{r_1}$. If there is a relatively small measure of depositors ($\bar{n} \leq \frac{1}{r_1}$), then depositors know that if they all stage a run, all of them will receive $r_1$. But since the incentive compatibility constraint $u(r_1) < p(\theta)u\left(\frac{1-\lambda r_1}{1-\Delta} R\right)$ holds, only the early consumers will withdraw at $t = 1$, and there will be no run (in the sense that late consumers also withdraw early). This simply says that adequately-capitalized banks ($\varepsilon' > 1 - \frac{1}{r_1}$) are in no danger of a run. We will not consider this case any further.
2.2.4 The probability of a bank run

Throughout the previous section, we have taken the state of the economy $\theta$ as a given. However, the agents’ beliefs about $\theta$ is fundamental to the equilibrium outcome. In particular, Goldstein and Pauzner [2005] endow the agents with a belief regarding the state of the economy: they use a global games framework. By doing so, they obtain unique Bayesian equilibria with well-defined probabilities tied to fundamentals. We follow their approach in this paper. In the global games framework, depositors obtain private and imprecise information about the economic indicator $\theta$. In particular, at $t = 1$, each depositor obtains a private signal $\theta_i$ uniformly distributed along $[\theta - \epsilon, \theta + \epsilon]$, where the distribution is known to all. Clearly $\theta_i$ depends on the realization of $\theta$. Thus depositors know that the true value of fundamentals is at most $\epsilon$ away from their own signal. Depositors’ decisions crucially depend on their draw of $\theta_i$ and on what they can deduce from that draw on the likely signals other depositors must have received and what they are therefore likely to do.

There are two extreme regions where depositors’ decisions do not depend on the actions of other agents. First one can define a $\theta = \underline{\theta}$ below which a late consumer always finds it optimal to run even if all other late consumers were to wait. Thus $\underline{\theta}$ solves the equation $u(r_1) = p(\theta) u \left( \frac{1-r_1}{1-\lambda R} R \right)$. Goldstein and Pauzner [2005] call the region $[0, \underline{\theta})$ the lower dominance region. There are always feasible values in the lower dominance region such that all signals will fall into that region if $\theta > 2\epsilon$; for this to obtain it is sufficient that $\theta(1) > 2\epsilon$ since $\theta(r_1)$ is increasing in $r_1$. In turn, $\theta(1) > 2\epsilon$ can be rewritten as $p^{-1} \left( \frac{u(1)}{u(R)} \right) > 2\epsilon$, which shows that $\epsilon$ can always be chosen small enough for the lower dominance region to be non-empty.

One can similarly define a $\bar{\theta}$ above which a patient depositor finds it optimal to wait even if all other patient agents were to run. Goldstein and Pauzner [2005] call this the upper dominance region. It is assumed that in the region $(\bar{\theta}, 1]$, the investment is certain to yield $R$ - that is, $p(\theta) = 1$ whenever $\theta > \bar{\theta}$. Then it is never optimal to run since $R > r_1$. Alternatively one can assume a Central Bank standing ready to provide liquidity in a run for high enough $\theta$ since in that case the bank is clearly solvent. Either way, we follow Goldstein and Pauzner [2005] in postulating the existence of such an upper dominance region. Since $\epsilon$ can be chosen arbitrarily small, we can also safely assume that it is possible that all draws fall into the upper dominance region, which requires $\bar{\theta} < 1 - 2\epsilon$.

Within the region $[\underline{\theta}, \bar{\theta})$, depositors must rely on equilibrium behavior of other depositors receiving nearby signals, which in turn depends on their nearby signals, and so on; continu-

---


14 This can be seen by differentiating the implicit equation defining $\underline{\theta}$. 

---
ity requires that behavior smoothly pastes to the behavior in the extreme regions. Following Goldstein and Pauzner [2005], one can prove that the unique equilibrium strategy is a switching strategy in which late consumers run if they receive a signal \( \theta_i \leq \theta^* \) and wait otherwise.\(^{15}\) \( \theta^* \) is defined such that a depositor receiving a signal \( \theta^* \) is indifferent between waiting and running at \( t = 1 \) over all possible outcomes of other depositors’ behavior:

\[
\int_{n=\lambda \bar{n}+\frac{\epsilon}{\lambda}}^{1} \left[ p(\theta = \theta^*) u \left( \frac{1 - nr_1}{1 - (\lambda \bar{n} + \frac{\epsilon}{\lambda})r_1} r_D \right) - u(r_1) \right] dn - \int_{n=\lambda \bar{n}}^{1} \frac{1}{nr_1} u(r_1)dn = 0 \tag{2.2}
\]

where \( r_D = \frac{1-r_1}{1-\lambda} R \). (2.2) defines \( \theta^* \) implicitly and is formed from the payoffs described in Table 2.1 and (2.1).\(^{16}\) Because the depositors obtain signals \( \theta_i \) from a uniform distribution around \( \theta \) and \( \theta \) is itself uniformly distributed over \([0, 1] \), a higher \( \theta^* \) means depositors run in a larger set of signals. For small \( \epsilon \), \( \theta^* \) can be interpreted as the probability of a bank run. Also, each \( \theta^* \) corresponds to an \( n \) which is the measure of the number of runners at \( t = 1 \) for given value of \( \theta \). This is\(^{17}\)

\[
n = \lambda \bar{n} + (1 - \lambda) \bar{n} \left[ \frac{1}{2} + \frac{\theta^* - \theta}{2\epsilon} \right] \tag{2.3}
\]

for \( \theta^* - \epsilon \leq \theta \leq \theta^* + \epsilon \). For \( \theta < \theta^* - \epsilon \), \( n = \bar{n} \) and for \( \theta > \theta^* + \epsilon \), \( n = \lambda \bar{n} \).

### 2.3 Effect of CoCo conversion on the probability of a run \( \theta^* \)

Consider now the case when the regulator finds out that the return will be low. While \( \theta^* \) depends on \( r_1 \), it also depends on \( R \) and \( \bar{n} \). As mentioned in Section 2.2.3, we introduce the regulator action at \( t = 1 \), before depositors can act. In the absence of CoCo conversion, depositors and other investors believe that the return of the risky asset is \( R \) with probability \( p(\theta) \) and 0 with probability \( 1 - p(\theta) \). But when the regulator forces CoCos to convert, a signal is given that the return of the risky asset is now some \( R_L < R \), without an accompanying change in the state of fundamentals \( \theta \). The impact of a lower \( R \) on period 2 payoffs can be seen in Figure 2.2 (the shift from the solid to the slotted line). Figure 2.3 recasts the payoffs described in Table 2.1 in terms of differential utility between waiting and early withdrawal for a given \( \theta \) as \( n \) changes. From the diagram it should be clear that once integrated over the entire range of

\(^{15}\)We present a short proof in Appendix 2.B.

\(^{16}\)(2.2) builds on the fact that \( \theta \) is uniformly distributed. Since \( n \) is linear in its arguments, \( n \) must also be uniformly distributed. The expression also assumes that \( p(\theta) \approx p(\theta^*) \) for \( \epsilon \) small enough, following Goldstein and Pauzner [2005].

\(^{17}\)This is similar to the Goldstein and Pauzner [2005] equilibrium \( n \) scaled down by \( \bar{n} \).
The utility differential shifts against waiting, so the indifference point in state space, $\theta^*$, will have to shift up to restore balance. So the threshold $\theta^*$ increases when the return of the risky asset is reduced to $R_L$.

Figure 2.3: Utility differential of waiting versus early withdrawal for different values of $R$

\[ v(\theta, n, R) \]
\[ v(\theta, n, R_L) \]

To prove this formally, we compute the threshold $\theta^*$ from the function that implicitly defines it. This function was introduced as (2.2). For convenience let us call this function as $\hat{f}(\theta^*, r_1, R)$:

\[
\hat{f}(\theta^*, r_1, R) = \int_{n=\lambda n + \frac{\epsilon}{r_1}}^{\hat{n}} p(\theta(\theta^*, n))\left(u\left(1 - nr_1 - \frac{nr_1}{1 - \lambda R}\right) - u(r_1)\right)dn - \int_{n=\lambda n + \frac{\epsilon}{r_1}}^{\hat{n}} \frac{1}{nr_1}u(r_1)dn
\]

where $\theta$ was written as a function of $n$'s intermediate value (away from $n = \lambda \hat{n}$ or $n = \hat{n}$), and $\theta$ is assumed to be within $\epsilon$-distance of $\theta^*$. That is, $\theta = \theta^* + \epsilon \left[1 - \frac{2}{1-\lambda} \left(\frac{n}{\hat{n}} - \lambda\right)\right]$ (see (2.3)). At $\theta = \theta^*$, a patient consumer is indifferent between waiting or running, by definition of $\theta^*$.

Note that since $\hat{f}(\cdot)$ is increasing in both $R$ and $\theta$, so in order to keep $\hat{f}(\cdot) = 0$, a decrease in $R$ must be compensated by an increase in $\theta^{18}$ Proposition 2.1 below follows:

**Proposition 2.1.** $\theta^*$ is decreasing in $R$: $\frac{\partial \theta^*}{\partial R} < 0$ for all values of $R$.

As a consequence, any negative signal about asset returns that is obtained by depositors will lead to a higher run probability. CoCo conversion delivers one such signal because the conversion in this model implies that the return in the good state of $t = 2$ is $R_L < R$. As a result, each depositor will expect a lower differential payoff than its value before conversion (see Figure 2.2). If for return $R$ a depositor is just indifferent between running and waiting for a given $\theta^*$, then for return $R_L < R$ it must be that the same depositor prefers to run for the same

---

18 Appendix 2.C shows the proof for this statement.
value of $\theta^\ast$. In order to restore the depositor’s indifference between running and waiting for return $R_L < R$, a higher signal about the fundamentals must be obtained, such that threshold value will go up to some $\theta^\ast_L > \theta^\ast$ at the point of indifference, which is what Proposition 2.1 says. But since depositors’ $\theta_i$ are uniformly distributed between $[\theta - \varepsilon, \theta + \varepsilon]$, a greater measure of them will have $\theta_i < \theta^\ast_L$, which implies a higher probability of a run. Note that the increase in $\theta^\ast$ also results in an increase in $n$ for given value of $\varepsilon$ and $\theta$, as can be seen from (2.3).

Proposition 2.1 also has an important corollary on the impact of the trigger level of a CoCo. Consider two trigger levels defined on a bank’s Common Equity Tier 1 ratio (CET1) $\tau_H$ and $\tau_L$ such that $\tau_H > \tau_L$, for otherwise identical CoCos. A CoCo with trigger level $\tau_L$ converts when the issuing bank’s CET1 falls below $\tau_L$. As $\tau_H > \tau_L$, the conversion of a $\tau_L$ CoCo implies the conversion of a $\tau_H$ CoCo. On the other hand, the conversion of a $\tau_H$ CoCo does not necessarily lead to the conversion of a $\tau_L$ CoCo. In other words, if the trigger level is low, the implied asset quality signal is more negative than the signal transmitted by a CoCo with a higher trigger level. Corollary 2.2 then follows immediately from Proposition 2.1:

**Corollary 2.2.** Conversion of a CoCo with a high trigger level will lead to a smaller increase in run probability than conversion of an otherwise identical CoCo but with a lower trigger level.

Formally, let $CET_0$ be the CET1 thought to apply before a regulator’s inspection reveals an equity shortfall. The term $CET_0 \cdot (\tau_H - \tau_L)$ represents the difference in asset quality that is given by the conversion of both the $\tau_H$ and the $\tau_L$ CoCos. Define $\theta^\ast_H (\theta^\ast_L)$ as the run probability that will obtain after conversion of a $\tau_H (\tau_L)$ CoCo. Direct application of Proposition 2.1 with the definitions just introduced shows that the following holds (exactly, since the derivative is positive for all $R$ so we can apply the mean value theorem):

$$\theta^\ast_L - \theta^\ast_H = -\left(\frac{\partial \theta^\ast}{\partial R}\right) \cdot (\tau_H - \tau_L) \cdot CET_0 > 0.$$  

This result suggests that the Bank for International Settlements (BIS) is right to require sufficiently high trigger levels before CoCos are accepted as part of Tier 1. According to the BIS, CoCos are either Tier 2 (T2) or Additional Tier 1 (AT1) capital, depending on their trigger ratio: a trigger above 5.125% satisfies the going concern requirement for AT1 and thus allows classification as AT1. Lower triggers lead to a classification as gone concern instruments and consequently to a T2 status. A conversion lowers the issuing bank’s leverage ratio, and increases its CET1 capitalization. If the CoCo design did not satisfy Tier 1 (T1) requirements (for example because of a trigger ratio that is too low to satisfy the going concern requirement), conversion will increase the bank’s overall T1 capital requirement also.\(^\text{19}\)

\(^{19}\)There is one possible exception to this observation. Under some some CoCo designs, the CoCo does not
It is also worth noting that a change from $R$ to $R_L$ alters the dominance regions. Because the supremum for the lower dominance region is determined by the equation $u(r_1) = \rho(\theta)u\left(\frac{1-\lambda r_1}{1-\lambda}\right)$, a change from $R$ to $R_L$ necessarily increases $\theta$. Also, the infimum of the upper dominance region should not increase but may decline because if a minimum of $\tilde{\theta}$ ensures that $R$ will be obtained with certainty, then there must at least as many $\theta$-values that will ensure $R_L$ will be obtained with certainty. This means that the post-conversion $\tilde{\theta}$ must be no lower than the pre-conversion one. Figure 2.4 shows the shift in the dominance regions and the effect on the upper and lower bounds of $n$.

Figure 2.4: Change in the dominance regions due to a change in $R$

2.4 CoCo design and run probabilities after conversion

Until now we have left unspecified what specifically happens after conversion. What happens after the issuing bank’s capital falls below the trigger value depends on the type of CoCo issued. For CoCos to qualify as capital at all, they need to include a so called “point of nonviability” trigger, i.e. the possibility for the regulator to enforce conversion if the regulator decided that viability is threatened. Currently used CoCo designs fall into three distinct types. First are convert-to-equity (CE) CoCos. These CoCos completely convert to equity at some conversion rate $\psi$ or, equivalently, at a price $P = \psi^{-1}$. Most commentators and academics have this type of CoCo design in mind when discussing CoCos in general. Next are principal writedown (PWD) CoCos. Upon breaching the trigger value, these CoCos are partially or fully written down. In case of partial writedown, the remaining part effectively turns into subordinated debt. Finally there are also principal writedown CoCos with cash outlays (CASH). Similar to the PWD CoCos with partial write off, CASH CoCos are also partially written off upon the bank’s breach of the convert into equity; instead the principal is partially written off with the remainder converting into unsecured debt. If such a partial write down CoCo is converted, the T1 capital asset ratio actually falls.

20 Association for Financial Markets in Europe [2016d] contains a list of the CoCos recently issued in Europe.
trigger value. The remaining value is paid out in cash. Notably, Rabobank of the Netherlands has issued this type of CoCo.21

In general, since depositors are senior claimants, none of all these consequences of conversion (aside from asset value changes) matters for them, as conversion merely redistributes between junior claimants. The one exception is the CASH CoCo, because there conversion implies less cash available for distribution to depositors in distress situations. In the remainder of this section, we examine the impact of CoCo design on the probability of a bank run after conversion, and on the equity position of the bank if partial runs do occur.

2.4.1 Benchmark case: regulatory forbearance

As a benchmark, we consider the case where the regulator finds out that returns will be low but decides not to publicize this finding such that the CoCos do not convert. Depositors base their behavior on the belief that in the good state of nature returns are \( R \), not knowing that in fact they will be \( R_L \). Table 2.2 shows the payoffs to depositors.

<table>
<thead>
<tr>
<th>Decision (t)</th>
<th>If ( \lambda \bar{n} + \frac{R}{r_1} &lt; n &lt; \frac{1}{r_1} )</th>
<th>If ( n \geq \frac{1}{r_1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( t = 1 )</td>
<td>( r_1 ) w.p. 1 ( \frac{n}{r_1} )</td>
<td>0 w.p. 1 - ( \frac{n}{r_1} )</td>
</tr>
<tr>
<td>2 ( t = 2 )</td>
<td>( \left( \frac{1-nr_1}{1-(\lambda \bar{n}+\lambda r_1)} \right) \left( \frac{1-\lambda r_1}{1-\lambda} \right) R_L ) w.p. ( p(\theta) ) ( 0 ) w.p. 1 - ( p(\theta) )</td>
<td></td>
</tr>
</tbody>
</table>

If depositors do not know that returns will be low, the differential payoff function remains the same as in (2.5). We call it \( v_{fb} \) (for forbearance) here.

\[
v_{fb}(\theta) = \begin{cases} 
    p(\theta) u \left( \left[ \frac{1-nr_1}{1-(\lambda \bar{n}+\lambda r_1)} \right] \left( \frac{1-\lambda r_1}{1-\lambda} \right) R \right) - u(r_1) & \text{if } \lambda \bar{n} + \frac{R}{r_1} \leq n \leq \frac{1}{r_1} \\
    0 - \frac{u(r_1)}{n r_1} & \text{if } \frac{1}{r_1} \leq n \leq \bar{n}
\end{cases} \tag{2.5}
\]

Let \( \theta_{fb}^* \) denote the threshold probability of runs under regulatory forbearance. The correspond-

---

212 billion Euros worth of PWD CoCo were issued by Rabobank in January 2011 which had a cash payout to the CoCo holders in case of a trigger event. They have been redeemed by Rabobank in July 2016.
ing implicit function that determines $\theta^*_b$ is given by (2.6).

$$\hat{f}(\theta^*_b, r_1, R) = \int_{n=\lambda\hat{n} + \frac{e}{r_1}}^{\lambda\hat{n}} \left[ \frac{1 - nr_1}{1 - (\lambda\hat{n} + \frac{e}{r_1})r_1} \right] \left( 1 - \frac{\lambda r_1}{1 - \lambda} \right) R - u(r_1) \right] dn$$

$$- \int_{n=\frac{1}{r_1}}^{\lambda\hat{n}} \frac{1}{nr_1} u(r_1) dn = 0$$

Obviously, since the derivations of $\theta^*_b$ are based on the same set of beliefs as in our base case without bad news, $\theta^*_b = \theta^*$. Depositors do not know that $R$ has fallen to $R_L$, so the run probability $\theta^*$ is not affected. Let $n_{fb}$ denote the number of runners implied by the probability of bank run $\theta^*_b$. In the event that $n_{fb} < \frac{1}{r_1}$ at $t = 1$, then

$$\left( \hat{n} - n_{fb} \right) \left[ \frac{1 - n_{fb}r_1}{1 - (\lambda\hat{n} + \frac{e}{r_1})r_1} \right] \left( 1 - \frac{\lambda r_1}{1 - \lambda} \right) R_L$$

will be given to the remaining depositors who did not run at $t = 1$ (this amounts to $\hat{n} - n_{fb}$ depositors), while what remains of the asset base

$$\left[ \left( 1 - n_{fb}r_1 \right) - \frac{\left( \hat{n} - n_{fb} \right) \left( 1 - n_{fb}r_1 \right)}{1 - (\lambda\hat{n} + \frac{e}{r_1})r_1} \right] \left( 1 - \frac{\lambda r_1}{1 - \lambda} \right) R_L$$

will be used to first pay out the junior CoCo holders who collectively have $\hat{e} - \hat{n}$ worth of claims that earn a return $rc$ per unit. Finally, anything that remains after that will go to equity holders. The remaining equity base under regulatory forbearance ($E_{fb}$) will be

$$E_{fb} = \max \left\{ \left( 1 - n_{fb}r_1 \right) - \frac{\left( \hat{n} - n_{fb} \right) \left( 1 - n_{fb}r_1 \right)}{1 - (\lambda\hat{n} + \frac{e}{r_1})r_1} \right] \left( 1 - \frac{\lambda r_1}{1 - \lambda} \right) R_L - rc (\hat{e} - \hat{n}) , 0 \right\}.$$

Under regulatory forbearance, there is no way to reduce a bank’s liabilities, so any negative asset development is immediately absorbed by equity.

---

22Here $rc$ is an arbitrary return to CoCo holders. In this paper we are taking this return as a given, as we do not delve into the pricing of CoCos.
2.4.2 Convert-to-equity (CE) CoCos

Consider now the case where the regulator converts the CoCos. Upon the conversion of convert-to-equity (CE) CoCos, CoCo holders turn into equity holders and therefore, forfeit the right to receive the amount up to \( r_c (\bar{e} - \bar{n}) \) but become entitled to a share in any residual income. Table 2.3 shows the resulting payoffs to depositors.

Table 2.3: Depositor payoffs after CE CoCos conversion

<table>
<thead>
<tr>
<th></th>
<th>If ( \lambda \bar{n} + \frac{e'}{r_1} &lt; n &lt; \frac{1}{r_1} )</th>
<th>If ( n \geq \frac{1}{r_1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t = 1 )</td>
<td>( r_1 )</td>
<td>( \begin{cases} r_1 &amp; \text{w.p. } \frac{1}{nr_1} \ 0 &amp; \text{w.p. } 1 - \frac{1}{nr_1} \end{cases} )</td>
</tr>
<tr>
<td>( t = 2 )</td>
<td>( \begin{cases} \left( \frac{1 - nr_1}{1 - (\lambda \bar{n} + \frac{e'}{r_1})r_1} \right) \left( 1 - \frac{1}{1 - \lambda} \right) R_L &amp; \text{w.p. } p(\theta) \ 0 &amp; \text{w.p. } 1 - p(\theta) \end{cases} )</td>
<td>0</td>
</tr>
</tbody>
</table>

The differential payoff function used by depositors is different now, since depositors receive the negative signal about the asset return that is associated with CoCo conversion. As such, (2.10) has \( R_L \) rather than \( R \).

\[
v_{ce} = \begin{cases} p(\theta)u\left( \left( \frac{1 - nr_1}{1 - (\lambda \bar{n} + \frac{e'}{r_1})r_1} \right) \left( 1 - \frac{1}{1 - \lambda} \right) R_L \right) - u(r_1) & \text{if } \lambda \bar{n} + \frac{e'}{r_1} \leq n \leq \frac{1}{r_1} \\ 0 - \frac{u(r_1)}{nr_1} & \text{if } \frac{1}{r_1} \leq n \leq \bar{n} \end{cases} \quad (2.10)
\]

As before, we can compute the threshold run value of the economic fundamental for a CE CoCo implicitly. Denote by \( \theta_{ce}^* \) the probability of a run for the CE case. As before, the equation that implicitly defines \( \theta_{ce}^* \) is given by (2.11).

\[
\hat{f}_{ce}(\theta_{ce}^*, r_1, R_L) = \int_{n=\lambda \bar{n} + \frac{e'}{r_1}}^{1} p(\theta(\theta_{ce}^*, n))u\left( \left( \frac{1 - nr_1}{1 - (\lambda \bar{n} + \frac{e'}{r_1})r_1} \right) \left( 1 - \frac{1}{1 - \lambda} \right) R_L \right) - u(r_1) \ dn
\]

Then application of Proposition 2.1 immediately shows that \( \theta_{ce}^* > \theta_{fb}^* \). This highlights the bind regulators are in when they must convert the CoCos. The negative signal that conveys to depositors actually increases financial fragility through the probability of runs. On the other hand, converting CE CoCos increases the equity at \( t = 2 \) relative to the forbearance case. This is clear because when CE CoCos are converted, the CoCo holders no longer have to be paid at
Let $n_{ce}$ denote the number of runners implied by the probability of bank run $\theta_{ce}^*$. We can actually see the beneficial effect of a CoCo conversion, because provided that $\theta_{ce}^*$ yields $n_{ce} < \frac{1}{r_1}$, the bank survives until $t = 2$ with more capital as CoCo holders are no longer creditors. We can denote by $E_{ce}$ the resulting equity upon conversion of the CE CoCos.

$$E_{ce} = \max \left\{ \left( 1 - n_{ce} r_1 \right) - \frac{\bar{n} - n_{ce} \left( 1 - n_{ce} r_1 \right)}{\left( 1 - (\lambda \bar{n} + \frac{\bar{e}}{r_1}) r_1 \right)} \left( \frac{1 - \lambda r_1}{1 - \lambda} \right) R_L, \ 0 \right\}$$  \tag{2.12}

From Section 2.4.1, since $\theta_{fb}^* < \theta_{ce}^*$, it must also be true that $n_{fb} < n_{ce}$. $E_{ce}$ differs from $E_{fb}$ by the difference between $n_{fb}$ and $n_{ce}$, and also by the amount that must be paid to the CoCo holders $r_c (\bar{e} - \bar{n})$. We may write:\footnote{Calculations are in Appendix 2.D.}

$$E_{ce} - E_{fb} = r_c (\bar{e} - \bar{n}) + R_L \left( n_{ce} - n_{fb} \right) \left( \Gamma (\bar{n} r_1 + 1) - 1 \right) + \left( n_{fb}^2 r_1 - n_{ce}^2 r_1 \right) \Gamma R_L$$  \tag{2.13}

where $\Gamma = \frac{1 - \lambda r_1}{1 - (\lambda \bar{n} + \frac{\bar{e}}{r_1}) r_1 (1 - \lambda)} > 0$. We have $n_{ce} - n_{fb} > 0$, and $\Gamma (1 + \bar{n} r_1) - 1 > 0$ so up to a first-order approximation (ignoring the quadratic terms in $n$), the conversion indeed improves the equity base of the bank if it survives into the good state of nature.

**Proposition 2.3.** If $n_{ce} < \frac{1}{r_1}$ (i.e. the bank survives period 1), conversion of CE CoCos improves the bank’s equity position at $t = 2$ relative to regulatory forbearance, as the bank is able to eliminate up to $r_c (\bar{e} - \bar{n})$ worth of liabilities.

This result points to an incentive for regulators to actually force conversion once they find out about lower returns $R_L$. The regulator faces conflicting incentives upon the discovery of $R_L$. On the one hand, conversion increases the probability of a run because it conveys a negative signal about asset returns. On the other hand, conversion also ensures that if runs occur, there is a possibility that there will be a surviving equity base, and that it will be higher than when the regulator is forbearing. Regulators thus are forced to choose between keeping fragility low at the expense of widening the consequences of a run if it does occur, and increasing the likelihood of a run but leaving the bank better equipped to deal with the aftermath of one.

### 2.4.3 Principal writedown (PWD) CoCos

We have previously described PWD CoCos as having a fraction written down upon conversion. Let $1 - \varphi$ denote the fraction of CoCos that is written off when conversion occurs, so $\varphi$ is the...
fraction that is left, where \(0 \leq \varphi \leq 1\). Table 2.4 describes the payoffs to depositors in the PWD case after conversion.

**Table 2.4: Depositor payoffs after PWD CoCos conversion**

<table>
<thead>
<tr>
<th>(t = 1)</th>
<th>If (\lambda \bar{n} + \frac{\xi}{r_1} &lt; n \leq \frac{1}{r_1})</th>
<th>If (n \geq \frac{1}{r_1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r_1)</td>
<td>(r_1) w.p. (\frac{1}{nr_1})</td>
<td>0 w.p. (1 - \frac{1}{nr_1})</td>
</tr>
</tbody>
</table>

As in the CE CoCo case, the amount that each depositor would obtain is the same as that under no conversion because depositors are senior to remaining CoCo holders. Therefore the differential payoff function used by depositors here (2.4) is identical to what it is in the case of CE CoCos.

\[ f_{\text{pwd}}(\theta^{*}_{\text{pwd}}, \theta^{*}_{\text{pwd}}, r_1, R_L) = \int_{n=\lambda \bar{n} + \frac{\xi}{r_1}}^{\frac{1}{r_1}} \left[ p(\theta^{*}_{\text{pwd}}, n) u \left( \frac{1 - nr_1}{1 - (\lambda \bar{n} + \frac{\xi}{r_1})r_1} \left( \frac{1 - \lambda r_1}{1 - \lambda} \right) R_L \right) - u(r_1) \right] dn \]

Let \(\theta^{*}_{\text{pwd}}\) denote the threshold level of \(\theta\) for the PWD case. We can again find \(\theta^{*}_{\text{pwd}}\) from the implicit function in (2.15).

Since the differential payoff function is the same, it follows that \(\theta^{*}_{\text{pwd}} = \theta^{*}_{\text{ce}}\). This means that PWD CoCos are not an improvement over CE CoCos if evaluated solely for their impact on probability of runs, because neither type of CoCo changes the incentives for depositors. The explanation is straightforward: while PWD and CE CoCos imply different wealth transfers between CoCo holders and equity holders, depositors are senior to both groups of claimants, so depositors do not care how losses are allocated between the other types of agents.

**Proposition 2.4.** PWD CoCos have the same impact on the probability of bank runs as CE CoCos: \(\theta^{*}_{\text{pwd}} = \theta^{*}_{\text{ce}} > \theta^{*}_{fb}\).
2.4.4 Principal writedown CoCos with cash outlays (CASH)

CASH CoCos are a variant of PWD where in addition to writing off a fraction of CoCo claims, the remaining fraction is paid out in cash to the CoCo holders upon conversion. This effectively means that the seniority of depositors is partially negated by promising a cash payment to CoCo holders. Letting $\delta r_1$ represent this cash payment, it can be seen that there will only be $1 - \delta r_1$ funds available for depositors at $t = 1$. It also only means that only $\frac{1}{r_1} - \delta$ running depositors at $t = 1$ can be accommodated, rather than $\frac{1}{r_1}$. Therefore, rather than there being $n = \lambda \hat{n} + \frac{c'}{r_1}$ running depositors at $t = 1$, there can only be at most $\lambda \hat{n} + \frac{c'}{r_1} - \delta$ runners until the asset runs out. Figure 2.5 shows what happens under that case, where the maximum number of running depositors that may be accommodated at $t = 1$ is reduced by $\delta$.

![Figure 2.5: Depositor returns at $t = 2$ under a cash payout to CoCo holders](image)

Because of the $\delta r_1$ payout to the CoCo holders, the remaining assets of the firm will be $1 - \delta r_1 - nr_1$, as opposed to just $1 - nr_1$. As a result, each waiting depositor will receive less compared to any of the other CoCo designs: only $\left[\frac{1-\delta r_1 - nr_1}{1-(\lambda \hat{n} + \frac{c'}{r_1}) r_1}\right] \left(\frac{1-\lambda r_1}{1-\lambda}\right) R_L$ instead of the larger amount $\left[\frac{1-nr_1}{1-(\lambda \hat{n} + \frac{c'}{r_1}) r_1}\right] \left(\frac{1-\lambda r_1}{1-\lambda}\right) R_L$. Notice the impact of the cash payout $\delta r_1$ on the amounts that the depositors receive. Table 2.5 shows the depositor payoffs under the CASH design. Notice also the change in the thresholds of $n$.

![Table 2.5: payoff to depositors after CASH CoCos conversion](image)
Even though equity holders absorb $\delta r_1$, depositors will still be affected: if $\delta r_1$ is paid out in cash upon conversion, there is correspondingly less cash available to pay out in case of early withdrawals. This will affect the differential payoff function, and therefore $\theta^*$ and the corresponding expected number of runners $n$. Consider first the impact of paying out cash on $n$. The cash payoff decreases the maximum value of $n$ from $\frac{1}{r_1}$ to $\frac{1}{r_1} - \delta$. However, because we let the equity holders and the CoCo holders absorb first losses, this also means that the value of $n$ where the amount $r_D$ is scaled by the number of runners is also pushed back by $\delta$ (from $\bar{n} + \frac{c'}{r_1}$ to $\bar{n} + \frac{c'}{r_1} - \delta$). This means that the bounds of $n$ change. (2.16) shows the differential payoff function for the CASH case.

$$v_{\text{cash}} = \begin{cases} p(\theta)u \left( \frac{1-\delta r_1 - r_1 n}{1-(\bar{n} + \frac{c'}{r_1})r_1} \right) \left( 1 - \lambda r_1 \frac{1}{1 - \lambda} \right) R_L - u(r_1) & \text{if } \lambda \bar{n} + \frac{c'}{r_1} - \delta \leq n \leq \frac{1}{r_1} - \delta \\ 0 - \frac{1}{n} \left( \frac{1}{r_1} - \delta \right) u(r_1) & \text{if } \frac{1}{r_1} - \delta \leq n \leq \bar{n} \end{cases}$$ (2.16)

The equation that implicitly defines $\theta^*_{\text{cash}}$ can be formed from the differential payoff equation. This is given by (2.17).

$$\hat{f}_{\text{cash}}(\theta^*_{\text{cash}}, r_1, R_L) = \int_{n=\lambda \bar{n} + \frac{c'}{r_1} - \delta}^{\frac{1}{r_1} - \delta} p(\theta(\theta^*_{\text{cash}}, n)) u \left( \frac{1-\delta r_1 - r_1 n}{1-(\lambda \bar{n} + \frac{c'}{r_1})r_1} \right) \left( 1 - \lambda r_1 \frac{1}{1 - \lambda} \right) R_L - u(r_1) \right) \right) dn$$

$$- \int_{n=\frac{1}{r_1} - \delta}^{\bar{n}} \frac{1}{n} \left( \frac{1}{r_1} - \delta \right) u(r_1) dn = 0$$

We can see that as $\delta \to 0$, $\theta^*_{\text{cash}} \to \theta^*_{\text{pwd}} = \theta^*_{\text{ce}}$. However, because the bounds of the integral change along with the expression within the utility function, it is difficult to be precise unless we look at the derivative of $\theta^*_{\text{cash}}$ with respect to $\delta$. A cash payout $\delta r_1$ reduces the amount that is available to depositors who wait until $t = 2$ (see the reduction in the numerator of $u(\cdot)$). However, by choosing to wait, depositors forgo receiving $r_1$ at $t = 1$. If $n$ falls into the range $\frac{1}{r_1} - \delta \leq n \leq \bar{n}$, a depositor’s “expected opportunity loss” is only $-\frac{1}{n} \left( \frac{1}{r_1} - \delta \right) u(r_1)$ rather than $-\frac{1}{n} u(r_1)$. As such, there is less to lose by waiting if $n$ happens to be large, but one must note as well that the range $\left[ \frac{1}{r_1} - \delta, \bar{n} \right]$ rises with $\delta$. The ambiguity arises because both the gain from waiting and the loss from waiting fall at the same time. Figure 2.6 illustrates the differential payoff functions for different values of $\delta$. 
In this section, we follow the earlier procedures and calculate the derivatives of $\theta^*_{\text{cash}}$ with respect to $\delta$ explicitly using the implicit function theorem. The expressions are laborious and so relegated to the Appendix, but we can unambiguously sign the derivative: $\frac{\partial \theta^*_{\text{cash}}}{\partial \delta} > 0$. The impact of $\delta$ on the gain from waiting is higher than its impact on the expected opportunity loss from waiting. Thus, a higher $\theta^*_{\text{cash}}$ is needed to compensate for the impact of an increase in the cash component $\delta$.

**Proposition 2.5.** $\theta^*_{\text{cash}}$ is increasing in $\delta$: $\frac{\partial \theta^*_{\text{cash}}}{\partial \delta} > 0$

Combining Proposition 2.5 with our earlier results allows us to give a definitive ranking of the types of CoCos in terms of impact on probability of bank runs:

**Corollary 2.6.** For $\delta > 0$, $\theta^*_{fb} < \theta^*_{ce} = \theta^*_{pwd} < \theta^*_{cash}$

## 2.5 Contagion and systemic risk

### 2.5.1 Contagion

Banks may have correlated asset returns for several reasons. The most obvious one is that banks often have cross-holdings of deposits (Allen and Gale [2000]). Another is when banks invest in the same set of industries, either by intentionally herding (like in Acharya and Yorulmazer [2008, 2007], Farhi and Tirole [2012]) or as a result of their individual diversification policies as in Wagner [2010]. Banks also tend to invest in similar assets as a result of conforming to regulatory requirements by institutions such as BIS (as in Iannotta and Pennacchi [2014]). Thus, negative information about one bank may have an adverse impact on other financial institutions. This information contagion effect has been well-documented empirically in the literature and is not confined to the banking sector (see Aharony and Swary [1983, 1996], Lang and Stulz [1992]). Thus, when CoCos of one bank convert, they impose an information
externality on the other banks that hold assets with returns correlated to those of the converting bank. In this section we show how this could happen.

To do so we consider a two-bank system. Let Bank 1 be a CoCo-issuing bank (as discussed in Sections 2.4.2, 2.4.3 and 2.4.4) (at this stage, the type of CoCo does not matter - only the conversion does) and without loss of generality, let Bank 2 be an ordinary bank without CoCos. Similar to Bank 1, Bank 2 also has a continuum of depositors who obtain private signals \( \theta_2 \sim U[\theta_2 - \varepsilon, \theta_2 + \varepsilon] \), and investments in risky technology with stochastic return \( R_2 \), and with equity but without CoCos. Table 2.6 summarizes the setup for the two-bank case.

Table 2.6: Summary of Bank Features: Two-Bank System

<table>
<thead>
<tr>
<th></th>
<th>Bank 1</th>
<th>Bank 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank type</td>
<td>CoCo-issuing</td>
<td>ordinary</td>
</tr>
<tr>
<td>agents</td>
<td>continuum from ([0, 1])</td>
<td>continuum from ([0, 1])</td>
</tr>
<tr>
<td>CoCo holders</td>
<td>( \bar{e} - \bar{n} )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>equity holders</td>
<td>( 1 - \bar{e} )</td>
<td>( 1 - \bar{e} )</td>
</tr>
<tr>
<td>early consumers</td>
<td>( \lambda \bar{n} )</td>
<td>( \lambda \bar{e} )</td>
</tr>
<tr>
<td>late consumers</td>
<td>( (1 - \lambda) \bar{n} )</td>
<td>( (1 - \lambda) \bar{e} )</td>
</tr>
<tr>
<td>probability of run</td>
<td>( \theta_1^r )</td>
<td>( \theta_2^r )</td>
</tr>
<tr>
<td>potential returns</td>
<td>0 or ( R_1 \in {R_L, R} )</td>
<td>0 or ( R_2 = {R_L, R} )</td>
</tr>
</tbody>
</table>

As Bank 2 depositors also obtain private signals \( \theta_2 \), its late consumers also decide whether to wait or to run at \( t = 1 \) depending on their posterior assessment of \( \theta_2 \). The decision is made by using the differential payoff function for depositors of Bank 2, shown in (2.18):

\[
v_2(\theta, n) = \begin{cases} 
    p(\theta)u \left( \frac{1 - nr_1}{1 - (\lambda \bar{e} + \bar{e}'') r_1} \right) \left( \frac{1 - \lambda r_1}{1 - \lambda} \right) R_2 - u(r_1) & \text{if } \lambda \bar{e} + \bar{e}'' \leq n \leq \frac{1}{r_1}, \\
    0 - \frac{u(r_1)}{nr_1} & \text{if } \frac{1}{r_1} \leq n \leq \bar{e}
\end{cases}, \tag{2.18}
\]

where \( \bar{e}'' = 1 - \bar{e} \). Figure 2.7 illustrates this case.

Figure 2.7: Depositor payoffs at \( t = 2 \) for a non-CoCo bank

Return to depositors who withdraw at \( t = 2 \)

Measure of agents

\( e'' = 1 - \bar{e} \)
As before, there is only one value of $\theta$ which makes them indifferent between waiting and running. Call this value $\theta^*_2$. As before, this can be interpreted as the probability of a run in Bank 2, and is defined implicitly by its differential payoff function where now $\lambda \bar{e} \leq n \leq \bar{e}$ because Bank 2 did not issue CoCos.

The function that implicitly defines Bank 2’s probability of a run is given by

$$\Phi_2(\theta^*_2, r_1, R)$$

$$= \int \frac{1}{r_1} \left[p(\theta(\theta^*_2, n)) u \left(\frac{1 - nr_1}{1 - (\lambda \bar{e} + e''_{r_1}) r_1} \right) \left(1 - \frac{\lambda r_1}{1 - \lambda} \right) R_2 - u(r_1)\right] dn$$

$$- \int_{n = \frac{1}{r_1}}^{\bar{e}} \frac{1}{nr_1} u(r_1) dn = 0$$

We now want to determine the impact of Bank 1’s CoCo conversion on Bank 2’s probability of a bank run. Formally, we want to determine the sign of the derivative $\frac{\partial \theta^*_2}{\partial R_1}$ at $t = 1$. This can be written as

$$\frac{\partial \theta^*_2}{\partial R_1} = \frac{\partial \theta^*_2}{\partial R_2} \frac{\partial R_2}{\partial R_1}$$

where the first term is the impact of a change in Bank 2’s returns on Bank 2’s run probability. From Proposition 2.1, it is clear that $\frac{\partial \theta^*_2}{\partial R_2} < 0$. The sign of $\frac{\partial R_2}{\partial R_1}$ of course depends on the correlation of $R_2$ and $R_1$. If they are positively correlated, $\frac{\partial R_2}{\partial R_1} > 0$. If not, then $\frac{\partial R_2}{\partial R_1} = 0$. Any information about $R_1$ (and therefore $R_2$) is revealed only when CoCos convert. Otherwise, no information is revealed. Thus, we have that in the event of a CoCo conversion and correlated asset returns, $\frac{\partial \theta^*_2}{\partial R_1} < 0$.

We have mentioned in Section 2.2 that when $\bar{n}$ is small $(\bar{n} < \frac{1}{r_1})$, depositors know that they will all be served at $t = 1$ if they all withdraw. In this case only the early consumers withdraw, and all the late consumers wait until $t = 2$. However, this small $\bar{n}$ does not preclude the possibility that the regulator finds it necessary to force conversion of CoCos.

From Proposition 2.1, the knowledge of Bank 1’s conversion leads Bank 2 depositors to have a higher required indifference threshold $\theta^*_2 > \theta^*_2$. This increases the proportion of depositors who obtain signals that are lower than the new threshold. Thus while conversion in Bank 1 may not cause a run in Bank 1, it raises the probability of runs in Bank 2. Moreover, it may even cause full runs in Bank 2 because $n \in [\lambda \bar{e}, \bar{e}] \supset [\lambda \bar{n}, \bar{n}]$ such that when $\theta^*_2$ is high enough, the associated $n_2$ exceeds $\frac{1}{r_1}$. Proposition 2.7 then follows.

**Proposition 2.7.** If bank returns are correlated, CoCo conversion of Bank 1 leads to a higher
probability of runs in Bank 2. This is true regardless of the type of CoCo issued by Bank 1, and even if Bank 1 has small $\bar{n}$.

2.5.2 Systemic risk

From the above discussion, it is only a small step to show that CoCo conversion raises systemic risk. In general, systemic risk can be described as a situation where the banks fail at the same time, or if the failure of one bank spreads to other banks. While banks are not compelled under Basel III to issue CoCos, an increasing number of banks have been issuing them. It is therefore natural to examine the impact on systemic risk of having CoCos in the banking system. There are several ways of measuring systemic risk which are reminiscent of the CoVaR measure proposed by Adrian and Brunnermeier [2016]. Segoviano and Goodhart [2009] use PAO (Probability that at Least One Bank becomes Distressed) which is the conditional probability of having at least one extra bank failure given that a particular bank fails. However, PAO does not measure the systemic impact of a bank failure, other than providing insights on the inter-linkages of the banks within the system. Zhou [2010] proposes two measures that improve on the PAO in this respect: SII (Systemic Impact Index) which measures the number of expected failures in the system given that a particular bank fails, and VI (Vulnerability Index), the conditional probability that a particular bank fails given that at least some other bank fails. The VI is closely related to the PAO measure.

Our measure of systemic risk is in the spirit of the VI measure proposed by Zhou [2010], except that bank failures are defined not in terms of the probability that its losses exceed a threshold VaR, but in terms of the probability that a bank will experience a run. The VI measure can only be used in the context of a nontrivial financial system (with at least two banks). Therefore as a starting point, we consider a two-bank system and move up to three banks. We define systemic risk to be the impact of one event on the other elements of the financial system. This measure is additive over the number of banks. Also, systemic risk depends on whether the initial event has a simultaneous effect on all the other elements of the system, or a sequential, cascading effect. These represent two extreme measures of systemic risk: the actual measure is somewhere in between.

We show first the two-bank case. From Section 2.5.1, we have shown that conversion of CoCos not only has an impact on the probability of runs in the converting bank, but also raises the probability that a second bank in the system would also have bank runs. This is due to the assumption of correlated asset returns. We have then that $\theta_{n}^{*}$ (the probability of a bank run in Bank 2 conditional on a conversion of CoCos in Bank 1) is analogous to VI. In this model, the only source of information from the point of view of the bank agents is the decision of the
regulator. Therefore, the measure of systemic risk in a two bank system is always sequential: any change in Bank 2’s run probability is entirely conditional on whether CoCos in Bank 1 convert or not. Systemic risk in the two bank case conditional on Bank 1’s CoCo conversion can then be measured as

\[ S_{2}^{seq} = \theta_{\text{convert}}^{*} + \theta_{2}^{**}. \]  

(2.21)

The situation changes when we increase the number of banks from 2 to 3. This is because the impact of Bank 1’s CoCo conversion can be either simultaneously transmitted to Banks 2 and 3, or may cascade down to Bank 3 from effects on Bank 2. The simultaneous case is easy to see, as conversion is a signal that is almost instantaneously seen by everyone once it has been revealed. The cascading case requires that beliefs of Bank 3 depositors be updated by inferring low returns from both Banks 1 and 2.

Let us set up Bank 3 first. Suppose for simplicity that Bank 3 does not issue CoCos. Then, as in Section 2.5.1, the function that implicitly defines Bank 3’s probability of a run is given by

\[ \hat{f}(\theta_{3}^{*}, r_{1}, R) = \int_{\frac{1}{r_{1}}}^{\hat{e}} \frac{1}{nr_{1}}u(r_{1})dn - \int_{n=\frac{1}{r_{1}}}^{\hat{e}} \frac{1}{nr_{1}}u(r_{1})dn = 0 \]

where all the symbols’ definitions carry forward from Section 2.5.1. \( \theta_{3}^{*} \) can be treated as the unconditional probability of a run in Bank 3.

If the effect of the conversion of Bank 1’s CoCos is transmitted simultaneously to Banks 2 and 3, then as in the case of Bank 2, Bank 3’s run probability will rise from \( \theta_{3}^{*} \) to \( \theta_{3}^{**} \) because conversion of Bank 1’s CoCos lead Bank 3 depositors to infer that the returns are lower than \( R_{3} \). This leads to an increase in the threshold signal from \( \theta_{3}^{*} \) to \( \theta_{3}^{**} \). Systemic risk in the three bank case conditional on Bank 1’s CoCo conversion is then represented by

\[ S_{3}^{sim} = \theta_{\text{convert}}^{*} + \theta_{2}^{**} + \theta_{3}^{**}. \]  

(2.23)

Note that here, the inference of depositors about lower \( R_{3} \) is only based from the implied fall of \( R_{1} \) to \( R_{L} \).

If the effect of the conversion of Bank 1’s CoCos cascades sequentially from Bank 2 to Bank 3, systemic risk will be higher, because the belief of Bank 3’s depositors about low \( R_{3} \) is
reinforced by seeing low returns from Bank 2 after seeing low returns from Bank 1. This is easy to demonstrate when Bank 2 is a CoCo-issuing bank. Let’s suppose that is the case. Then, the function that implicitly defines Bank 2’s probability of a run is given by

\[
\hat{f}(\theta^*_{2,\text{coco}}, r_1, R) = \int_{n = \frac{1}{r_1}}^{\frac{1}{r_1}} \left[ p(\theta(\theta^*_{2,\text{coco}}, n))u\left(\left(\frac{1 - nr_1}{1 - (\bar{\epsilon} + \frac{\epsilon'}{r_1}) r_1}\right)\left(\frac{1 - \lambda r_1}{1 - \lambda}\right) R_2 - u(r_1)\right) \right] dn
\]

\[
- \int_{n = \frac{1}{r_1}}^{\frac{1}{r_1}} \frac{1}{nr_1} u(r_1)dn = 0
\]

where \(\epsilon' = (1 - \bar{\epsilon}) + (\bar{\epsilon} - \bar{n})\) is the measure of CoCo holders and equity holders, as in Section 2.2.4. As described in Section 2.5.1, a conversion from Bank 1 will lead to a fall in depositors’ belief about \(R_2\), such that the threshold \(\theta\) for Bank 2 depositors rises from \(\theta^*_{2,\text{coco}}\) to some \(\theta^{**}_{2,\text{coco}} > \theta^*_{2,\text{coco}}\). Suppose at this time, the regulator decides to convert Bank 2’s CoCos. This action confirms the Bank 2 depositors’ beliefs about the low \(R_2\). Thus in the sequential case, Bank 3 depositors get definite signals that not only has Bank 1’s returns fallen, but Bank 2’s returns as well. Bank 3’s depositors will update their beliefs about \(R_3\): seeing low returns from two banks makes it more likely that their own bank’s returns are low as well such that \(R_3\) is low. Let us call the associated \(\theta\) threshold as \(\theta^{***}_{3}\), where the \(R_3\) value under the sequential case is lower than the \(R_3\) value under the simultaneous case. Systemic risk in the sequential transmission case is then

\[
S^\text{seq}_3 = \theta^*_{\text{convert}} + \theta^{**}_{2,\text{coco}} + \theta^{***}_{3}\]

which is larger than \(S^\text{sim}_3\) for the sole reason that the run probabilities of Banks 2 and 3 are amplified by the conversion of Bank 2’s CoCos.

Note that if the regulator exercises forbearance and does not convert CoCos of Bank 1 despite knowing that returns will be low, no signal will ever be transmitted to either Banks 2 and 3. Systemic risk under forbearance would be

\[
S^\text{fb}_3 = \theta^*_{\text{fb}} + \theta^*_{2} + \theta^*_{3}.
\]

Clearly \(S^\text{fb}_3 < S^\text{sim}_3 < S^\text{seq}_3\).

**Proposition 2.8.** When the regulator is forbearing, systemic risk due to bank runs at \(t = 1\) remains low. On the other hand, when the regulator is not forbearing and forces CoCo conversion in one bank, systemic risk rises either through a simultaneous effect or a sequential one.
The example also suggests that widespread use of CoCos by many banks threatens larger increases in systemic risk as waves of sequential conversions can be triggered, bringing the actual systemic risk closer to the sequential measure than to the simultaneous measure.

2.6 Conclusion

We have written this paper in an effort to explore the effect of CoCo conversion on systemic risk. We have done this by adding CoCo holders and equity holders to the agent types of an otherwise standard Diamond and Dybvig [1983] setup recast in a global games framework as in Goldstein and Pauzner [2005]. Using this framework, we were able to show the impact of CoCo conversion on depositors, as well as on CoCo holders and equity holders. First we have shown that when an unanticipated decline in asset returns leads to a CoCo conversion, that has the immediate effect of raising the probability of a bank run. This is true regardless of the type of CoCos that are converted because they all send the same kind of signal (lowering of returns) which affects depositor incentives in the same manner. However, this is only true provided that the CoCo conversion does not alter the order of creditor seniority. Therefore, CoCos which provide a cash payment to CoCo holders before writing them off (like the RABO CoCo does) are actually worse than straight principal writedown (PWD) CoCos or convert-to-equity (CE) CoCos in terms of raising the likelihood of a run. This is so because by paying out cash in a distress situation they reduce the amount that may be distributed to the remaining creditors of the bank after conversion occurs.

Therefore one of the main consequences of our analysis is that a regulator faces conflicting incentives when finding out about lower asset returns than expected ($R_L < R$). On the one hand, conversion increases the probability of a run because of the negative signal on asset returns that conversion conveys. But on the other hand, conversion also ensures that if runs occur, there is a higher probability that there will be a surviving equity base. Regulators thus are forced to choose between keeping fragility low at the expense of making the consequences of a run if it does occur worse, or increasing the likelihood of a run but leaving the bank better equipped to deal with the aftermath of one.

We then extend the analysis to a multibank framework to analyze the impact of CoCo conversion on systemic risk. When different banks hold assets that have correlated returns, a signal indicating one bank’s asset quality deterioration has negative consequences for the other banks to the extent that the other banks’ assets are positively correlated to those of the bank whose CoCo has been forced into conversion: conversion carries an information externality giving rise to contagion across banks. There are many reasons to expect positive correlation
between asset returns of different banks. A very direct link leading to asset correlation establishing a channel of contagion occurs when banks hold each others’ CoCos. Given the obvious dangers of contagion such cross holdings give rise to, it is disturbing to see that about 50% of all CoCos issued so far is in fact held by banks (Avdjiev et al. [2013]). Other mechanisms leading to asset correlation may be the predominance of a few large banks in a relatively small country, industry specialization of several banks into the same industry, or herding behavior, for example to increase the pressure on regulators to bail out banks in distress if that situation arises. We show unambiguously that in an environment of correlated risks, CoCo conversion, even in a single bank, leads to higher systemic risk, defined as the joint probability of failure of banks. We show that as long as bank assets are positively correlated, a CoCo conversion in one bank leads to an increase in the probability of a run in the other bank, regardless of CoCo type. This implies that systemic risk will increase when CoCos convert. So when regulators consider CoCo conversion, microprudential and macroprudential objectives are likely to be in direct conflict.

Appendix for Chapter 2

2.A Calculations involving the payoffs of the agents

2.A.1 Calculation of $\tilde{r}_2$

We obtain the expression for $\tilde{r}_2$ by paying out the funds by seniority. While the total assets of the bank amount to 1, $\tilde{n}$ of that is from the depositors. $\lambda \tilde{n}$ of those are early depositors, which means that in the absence of a run, $\lambda \tilde{n} r_1$ will certainly be paid out at $t = 1$. This leaves, from the allocation from the depositors, $\tilde{n} - \lambda \tilde{n} r_1$ in total that will be left to earn $R$ at $t = 2$. As there are $\tilde{n} - \lambda \tilde{n}$ remaining depositors, each late consumer obtains $\frac{n - \lambda \tilde{n} r_1}{\tilde{n} - \lambda r_1} R$ with probability $p(\theta)$ and 0 otherwise. By factoring out $\tilde{n}$, we obtain $\frac{1 - \lambda r_1}{1 - \lambda}$, as in Diamond and Dybvig [1983].

2.A.2 Calculations for Table 1 depositor payoffs at $t = 1$ and $t = 2$

In this section, we justify our statement that as long as $\lambda \tilde{n} < n < \lambda \tilde{n} + \frac{\tilde{r}_1}{\tilde{n}}$, depositors who wait until $t = 2$ are able to obtain the full $r_D$ promised to them, in the event of good returns at $t = 2$. There are two parts to this discussion: how much is left after $n$ depositors withdraw that may earn $R$, and how many depositors must share the profits. If $\lambda \tilde{n} \leq n \leq \lambda \tilde{n} + \frac{\tilde{r}_1}{\tilde{n}}$, the bank must liquidate $\lambda \tilde{n} r_1$ at least (to be shared among $\tilde{n} - \lambda \tilde{n}$ agents, leaving $\tilde{n} - \lambda \tilde{n} r_1$ to potentially
earn \( R \) at \( t = 2 \) and \((\lambda \hat{n} + \frac{e'}{r_1}) r_1\) at most (to be shared among \( \hat{n} - (\lambda \hat{n} + \frac{e'}{r_1}) \) agents, leaving \( 1 - (\lambda \hat{n}r_1 + 1 - \hat{n}) \) to potentially earn \( R \) at \( t = 2 \). We consider each one in turn.

For \( \lambda \hat{n} \leq n \), this means that each patient consumer obtains

\[
\frac{\hat{n} - \lambda \hat{n} r_1}{n - \lambda \hat{n}} = \frac{1 - \lambda r_1}{1 - \lambda} = r_D,
\]

as stated in Appendix 2.A.1. For \( n \leq \lambda \hat{n} + \frac{e'}{r_1} \), notice that \( 1 - (\lambda \hat{n}r_1 + 1 - \hat{n}) \) simplifies to \( \hat{n} (1 - \lambda r_1) \), which means that the late depositors can consume at most

\[
\frac{\hat{n} (1 - \lambda r_1)}{\hat{n} - \left( \lambda \hat{n} + \frac{e'}{r_1} \right)} > \frac{\hat{n} - \lambda \hat{n} r_1}{\hat{n} - \lambda \hat{n}} = r_D,
\]

implying that the amount \( r_D \) should be obtainable for this amount of runners.

### 2.B Threshold value of the probability of bank runs

#### 2.B.1 The unique equilibrium strategy

Goldstein and Pauzner [2005] show that the unique equilibrium strategy is a switching strategy in which patient agents run if \( \theta \leq \theta^* \) and wait otherwise. We provide a short sketch of the proof here, tailored to fit our model specification. Consider the differential payoff function \( v(\theta, n) \) defined in (2.1) and reproduced here.

\[
v(\theta, n) = \begin{cases} 
  p(\theta)u \left( \frac{1 - nr_1}{1 - (\lambda \hat{n} + \frac{e'}{r_1}) r_1} r_D \right) - u(r_1) & \text{if } \lambda \hat{n} + \frac{e'}{r_1} < n < \frac{1}{r_1} \\
  0 - u(r_1) \frac{1}{nr_1} & \text{if } \frac{1}{r_1} < n < \hat{n}
\end{cases}
\]

A patient agent is indifferent from waiting or running at \( t = 1 \) if in expectation, the payoffs from waiting or running are the same regardless of what the other agents do (in other words, for whatever value \( n \) may take). This happens when \( \theta = \theta^* \), as in (2.2) and reproduced here.

\[
\int_{n=\lambda \hat{n} + \frac{e'}{r_1}}^{\frac{1}{r_1}} \left[ p(\theta = \theta^*) u \left( \frac{1 - nr_1}{1 - (\lambda \hat{n} + \frac{e'}{r_1}) r_1} r_D \right) - u(r_1) \right] dn - \int_{n=\frac{1}{r_1}}^{\hat{n}} \frac{1}{nr_1} u(r_1) dn = 0
\]

where \( r_D = \frac{1 - \lambda \hat{n}}{1 - \lambda} \).

However, an agent does not observe \( \theta \) but instead observes \( \theta_i \sim U[\theta - \epsilon, \theta + \epsilon] \). Because an agent’s posterior distribution of \( \theta \) depends on his realization of \( \theta_i \), he calculates the expected
differential payoff over the range of \( \theta \) that is compatible with his signal \( \theta_i \), for some \( n \):

\[
\Delta r^i (\theta_i, n) = \frac{1}{2\epsilon} \int_{\theta_i-\epsilon}^{\theta_i+\epsilon} v (\theta, n) \, d\theta.
\]

\( \Delta r^i (\theta_i, n) \) is linear, continuous, and strictly increasing in \( \theta \), such that there is only one value of \( \theta \) that makes \( \Delta r^i (\theta_i, n) = 0 \), and that is \( \theta^* \).

2.B.2 \( \theta^* \) as the probability of a bank run

If all agents follow the same strategy, then \( n \) is deterministic. Specifically, because \( \theta_i \sim U [\theta - \epsilon, \theta + \epsilon] \), the proportion of agents who run for a given \( \theta \) is given by

\[
n (\theta, \theta^*) = \bar{n} \left[ \lambda + (1 - \lambda) \cdot \text{Prob} (\epsilon_i < \theta^* - \theta) \right].
\]

We can then write

\[
n (\theta, \theta^*) = \begin{cases} 
\bar{n} & \text{if } \theta \leq \theta^* - \epsilon \\
\lambda \bar{n} + (1 - \lambda) \bar{n} \left[ \frac{1}{2} + \frac{\theta^* - \theta}{2\epsilon} \right] & \text{if } \theta^* - \epsilon \leq \theta < \theta^* + \epsilon \\
\lambda \bar{n} & \text{if } \theta > \theta^* + \epsilon.
\end{cases}
\]

Thus, in a threshold strategy, the measure of withdrawing agents \( n \) at \( t = 1 \) is completely determined by the threshold value \( \theta^* \). Because the agents obtain signals \( \theta_i \) from a uniform distribution, a fraction \( \theta^* \) of the agent’s signals are below \( \theta^* \), such that as in Goldstein and Pauzner [2005], \( \theta^* \) can be defined as the probability of a bank run.

2.C Proof for Proposition 2.1

Proposition 2.1 states that \( \theta^* \) is decreasing in \( R \): \( \frac{\partial \theta^*}{\partial R} < 0 \) for all values of \( R \). We show this by using the implicit function theorem. Consider the function \( \hat{f} (\theta^*, r_1, R) \) that is in terms of \( \theta^* \) and \( R \), reproduced here:

\[
\hat{f} (\theta^*, r_1, R) = \int_{\frac{1}{n} = \lambda \bar{n} + \frac{\epsilon}{r_1}}^{\frac{1}{n}} \left\{ p(\theta^*, n))u\left( \frac{1 - nr_1}{1 - (\lambda \bar{n} + \frac{\epsilon}{r_1})r_1} \frac{1 - \lambda r_1}{1 - \lambda} - u(r_1) \right) \right\} \, dn - \int_{\frac{1}{n} = \frac{1}{r_1}}^{\frac{1}{n}} \frac{1}{nr_1}u(r_1) \, dn = 0.
\]

It is easy to see that \( \frac{\partial \hat{f}}{\partial \theta^*} > 0 : \hat{f} \) is increasing in \( \theta^* \) because \( p(\cdot) \)is increasing in \( \theta^* \). Next, the function \( \theta(n, \theta^*) \) rises in \( \theta^* \). Now \( \hat{f} \) is also rising in \( R \), as can be seen by taking its derivative.
We have that
\[
\frac{\partial \hat{f}}{\partial R} = \int_{n=\lambda \bar{n} + \frac{e_1}{r_1}}^{1} \left[ p(\theta(\theta^*, n)) \frac{\partial u \left( \frac{1-nr_1}{1-(\lambda \bar{n} + \frac{e_1}{r_1})r_1} \frac{1-\lambda r_1}{1-\lambda} \right)}{\partial R} \right] \, dn
\]
\[
= \int_{n=\lambda \bar{n} + \frac{e_1}{r_1}}^{1} \left[ p(\theta(\theta^*, n)) u' \left( \frac{1-nr_1}{1-(\lambda \bar{n} + \frac{e_1}{r_1})r_1} \frac{1-\lambda r_1}{1-\lambda} \right) \right] \, dn
\]
\[
> 0
\]
since \[ \left( \frac{1-nr_1}{1-(\lambda \bar{n} + \frac{e_1}{r_1})r_1} \right) \left( \frac{1-\lambda r_1}{1-\lambda} \right) \] is positive over the entire interval of integration. Finally, with the implicit function theorem, we have that
\[
\frac{\partial \theta^*}{\partial R} = - \frac{\partial \hat{f}/\partial R}{\partial \hat{f}/\partial \theta^*} < 0,
\]
and Proposition 2.1 follows.

2.D Proof for Proposition 2.3

Proposition 2.3 states that the conversion of CE CoCos improves the bank’s equity position relative to regulatory forbearance. The difference stems not only from the elimination of the CoCo liabilities, but also from the change in the number of runners \( n_{ce} \) and \( n_{fb} \). Let us take the difference between \( E_{ce} \) and \( E_{fb} \). Write first the term \( \left( \frac{1-\lambda r_1}{1-\lambda} \right) \left( \frac{1}{1-(\lambda \bar{n} + \frac{e_1}{r_1})r_1} \right) \) as \( \Gamma \). We can then rewrite
\[
\frac{E_{ce}}{R_L} = (1 - n_{ce} r_1) - (\bar{n} - n_{ce}) (1 - n_{ce} r_1) \Gamma
\]
and
\[
\frac{E_{fb}}{R_L} = (1 - n_{fb} r_1) - (\bar{n} - n_{fb}) (1 - n_{fb} r_1) \Gamma - \frac{r_c (\bar{e} - \bar{n})}{R_L}
\]
such that the difference is

\[
\frac{E_{ce}}{R_L} - \frac{E_{fb}}{R_L} = (1 - n_{ce} r_1) - (\bar{n} - n_{ce}) (1 - n_{ce} r_1) \Gamma - (1 - n_{fb} r_1) + \left( \bar{n} - n_{fb} \right) \left( 1 - n_{fb} r_1 \right) \Gamma + \frac{r_c (\bar{e} - \bar{n})}{R_L}
\]

\[
= \frac{r_c (\bar{e} - \bar{n})}{R_L} + \Gamma \left[ \left( \bar{n} - n_{fb} \right) \left( 1 - n_{fb} r_1 \right) - (\bar{n} - n_{ce}) \left( 1 - n_{ce} r_1 \right) \right] + (1 - n_{ce} r_1) - (1 - n_{fb} r_1)
\]

\[
= \frac{r_c (\bar{e} - \bar{n})}{R_L} + \Gamma \left[ \left( 1 - n_{fb} r_1 \right) - n_{fb} \left( 1 - n_{fb} r_1 \right) - (\bar{n} - n_{ce}) \left( 1 - n_{ce} r_1 \right) \right] + (1 - n_{ce} r_1) - (1 - n_{fb} r_1)
\]

Note that we may expand \( \Gamma (\bar{n} r_1 + 1) - 1 \) as follows:

\[
\Gamma (\bar{n} r_1 + 1) - 1 = \frac{(1 - \lambda r_1) (1 + \bar{n} r_1)}{\left[ 1 - \left( \lambda \bar{n} + \frac{e'}{r_1} \right) r_1 \right] (1 - \lambda)} - 1 = \frac{(1 - \lambda r_1) (1 + \bar{n} r_1)}{\bar{n} (1 - \lambda r_1) (1 - \lambda)} - 1 = \frac{(1 + \bar{n} r_1)}{\bar{n} (1 - \lambda)} - 1 > 0,
\]

using the definition of \( e' = 1 - \bar{n} \). Therefore, up to a first-order approximation (ignoring the quadratic terms in \( n \)), \( E_{ce} - E_{fb} > 0 \).

### 2.E Proof for Proposition 2.5

Proposition 2.5 states that \( \theta^* \) is increasing in \( \delta \): \( \frac{\partial \theta^*}{\partial \delta} > 0 \). We show this here. By the implicit function theorem, we can show \( \frac{\partial \theta^*}{\partial \delta} > 0 \) if \( \partial \hat{f} / \partial \theta^* > 0 \), and \( \partial \hat{f} / \partial \delta < 0 \). We already know that \( \partial \hat{f} / \partial \theta^* > 0 \) from Proposition 2.1. We now only look at whether \( \partial \hat{f} / \partial \delta > 0 \) (here we drop the
subscript \( \text{cash} \) for ease of exposition)

\[
\hat{f}(\theta, r_1 \delta) = \int_{n=\lambda\delta}^{1-\lambda\delta} \left[ p(\theta(\theta^*, n))u\left( \frac{1 - \delta r_1 - nr_1}{1 - (\lambda\delta + \frac{\ell'}{r_1}) r_1} r_D^L \right) - u(r_1) \right] dn
\]

\[
- \int_{n=\lambda\delta}^{1-\lambda\delta} u(r_1) \left( \frac{1}{r_1} - \delta \right) \left( \frac{1}{n} \right)
\]

\[
= \int_{n=\lambda\delta}^{1-\lambda\delta} p(\theta(\theta^*, n)) u\left( \frac{1 - \delta r_1 - nr_1}{1 - (\lambda\delta + \frac{\ell'}{r_1}) r_1} r_D^L \right) dn - \int_{n=\lambda\delta}^{1-\lambda\delta} u(r_1) \left( \frac{1}{r_1} - \delta \right) \left( \frac{1}{n} \right)
\]

\[
= \int_{n=\lambda\delta}^{1-\lambda\delta} p(\theta(\theta^*, n)) u\left( \frac{1 - \delta r_1 - nr_1}{1 - (\lambda\delta + \frac{\ell'}{r_1}) r_1} r_D^L \right) dn
\]

\[-u(r_1) \left[ \frac{1}{r_1 - \lambda\delta - \ell'}{r_1} - u(r_1) \left( \frac{1}{r_1} - \delta \right) \left[ \ln \frac{1}{1 - (r_D^L \delta)} \right] \]

where \( r_D^L = \frac{1 - \lambda r_1}{1 - \lambda} R_L \). The derivative of \( \hat{f} \) with respect to \( \delta \) is

\[
\frac{\partial \hat{f}}{\partial \delta} = \frac{\partial}{\partial \delta} \left[ \int_{n=\lambda\delta + \frac{\ell'}{r_1} - \delta}^{1-\lambda\delta} p(\theta(\theta^*, n)) u(A) \left( \frac{1 - \delta r_1 - nr_1}{1 - (\lambda\delta + \frac{\ell'}{r_1}) r_1} r_D^L \right) dn \right] - u(r_1) \left[ 1 - (\ln \frac{1}{1 - (r_D^L \delta)}) \right]
\]

where the last term is negative for as long as \( 0 < \ln \frac{1}{1 - (r_D^L \delta)} < 1 \). Now consider the first term:

\[
\frac{\partial}{\partial \delta} \left[ \int_{n=\lambda\delta + \frac{\ell'}{r_1} - \delta}^{1-\lambda\delta} p(\theta(\theta^*, n)) u(A) \right]
\]

\[
= \int_{n=\lambda\delta + \frac{\ell'}{r_1} - \delta}^{1-\lambda\delta} p(\theta(\theta^*, n)) \frac{\partial u(A)}{\partial \delta} dn + p(\theta(\theta^*, \lambda\delta + \frac{\ell'}{r_1} - \delta)) \left( \frac{1 - \delta r_1 - (\frac{1}{r_1} - \delta) \ell'}{1 - (\lambda\delta + \frac{\ell'}{r_1}) r_1} r_D^L \right)(-1)
\]

\[
- p\left( \theta(\theta^*, \lambda\delta + \frac{\ell'}{r_1} - \delta) \right) \left( \frac{1 - (\lambda\delta + \frac{\ell'}{r_1} - \delta) \ell'}{1 - (\lambda\delta + \frac{\ell'}{r_1}) r_1} r_D^L \right)(-1)
\]

\[
= \int_{n=\lambda\delta + \frac{\ell'}{r_1} - \delta}^{1-\lambda\delta} p(\theta(\theta^*, n)) \frac{\partial u(A)}{\partial \delta} dn + p\left( \theta(\theta^*, \lambda\delta + \frac{\ell'}{r_1} - \delta) \right) \left( \frac{1 - (\lambda\delta + \frac{\ell'}{r_1} - \delta) \ell'}{1 - (\lambda\delta + \frac{\ell'}{r_1}) r_1} r_D^L \right)
\]

\[
42
\]
where \( A = \frac{1-\delta r_1-nr_1}{1-(\lambda n+\epsilon \frac{r'}{r_1})r_D} \). But since \( \frac{\partial u(A)}{\partial \delta} = \frac{\partial u(A)}{\partial n} \), we can write
\[
\frac{\partial}{\partial \delta} \left[ \int_{n=\lambda n+\epsilon \frac{r'}{r_1} - \delta}^{\frac{1}{\lambda} - \delta} p (\theta (\theta^*, n)) u (A) \, dn \right]
\]
in terms of \( \frac{\partial u(A)}{\partial n} \):
\[
\frac{\partial}{\partial \delta} \left[ \int_{n=\lambda n+\epsilon \frac{r'}{r_1} - \delta}^{\frac{1}{\lambda} - \delta} p (\theta (\theta^*, n)) u (A) \, dn \right]
= \int_{n=\lambda n+\epsilon \frac{r'}{r_1} - \delta}^{\frac{1}{\lambda} - \delta} p (\theta (\theta^*, n)) \frac{\partial u}{\partial n} \, dn + p \left( \theta (\theta^*, \lambda n + \epsilon \frac{r'}{r_1} - \delta) \right) u \left( \frac{1-\delta r_1-nr_1}{1-(\lambda n+\epsilon \frac{r'}{r_1})r_D} \frac{p' (\theta (\theta^*, n))}{\bar{n} (1-\lambda)} \right) \, dn
\]
where we used integration by parts. The first term is clearly negative. The second term can be made arbitrarily small by letting \( \epsilon \to 0 \). Thus the derivative of \( \hat{f} \) with respect to \( \delta \) is completely given by
\[
\frac{\partial \hat{f}}{\partial \delta} = -p \left( \theta (\theta^*, \lambda n + \epsilon \frac{r'}{r_1} - \delta) \right) u \left( r_D^L \right) - u \left( 1 - \frac{\delta r_1}{1-(\lambda n+\epsilon \frac{r'}{r_1})r_D} \right) \left( \frac{p' (\theta (\theta^*, n))}{\bar{n} (1-\lambda)} \right) \, dn
\]
\[
+ 2\epsilon \int_{n=\lambda n+\epsilon \frac{r'}{r_1} - \delta}^{\frac{1}{\lambda} - \delta} u \left( \frac{1-\delta r_1-nr_1}{1-(\lambda n+\epsilon \frac{r'}{r_1})r_D} \frac{p' (\theta (\theta^*, n))}{\bar{n} (1-\lambda)} \right) \, dn - u \left( r_1 \right) \left[ 1 - \left( \ln \frac{\bar{n}}{1-\delta} \right) \right]
\]
\[
< 0
\]
Chapter 3

CoCo Design, Risk Shifting Incentives, and Financial Fragility

3.1 Introduction

This chapter aims to show the risk-shifting incentives that arise from letting banks issue contingent convertible capital (CoCo) in order to fulfill capital requirements set by regulators. CoCos are hybrid instruments that are issued as debt but convert to equity or written off if the issuing bank fails to meet a distress threshold. The threshold may be contractual, as when the bank fails to meet a preset equity ratio, or discretionary, as when regulators deem the bank to be close to the point of nonviability. CoCos are designed this way in order to relieve the issuer of the burden of raising capital in situations of financial distress (Flannery [2005]). As a result, CoCos have become favored by regulators because of their enhanced loss absorption capacity relative to subordinated debt.

While CoCo conversion increases the loss absorption capacity of banks, it also potentially changes the order of seniority. If CoCos are written off, CoCo holders absorb the first losses, instead of the original shareholders. This implies that at the moment of conversion, there is a wealth transfer in favor of the shareholders. If CoCos are converted to equity, CoCo holders absorb the losses together with the existing shareholders. In this case, the wealth transfer may be in favor of either the CoCo holder or the existing shareholder, depending on the terms of the conversion. The wealth transfers are defined relative to when the bank has issued subordinated

\footnote{This chapter is based on Chan and van Wijnbergen [2017a], which has been awarded the Best Paper Prize at the European Capital Markets Institute (ECMI) Annual Conference 2016. We thank Florencio Lopez de Silanes, Andrei Kirilenko, Enrico Perotti, Tanju Yorulmazer, and participants from the Tinbergen PhD Seminar series, 2016 ECMI Conference, and the 2016 Paris Financial Management Conference for numerous helpful discussions and comments. Financial support from ECMI is gratefully acknowledged.}
debt in place of the CoCos, and always from the point of view of the original shareholder. Because of these wealth transfers, the bank may find it beneficial to engage in risk-shifting by choosing a riskier class of assets.

Our contribution to the literature is to provide a simple theoretical model of risk-shifting in the presence of CoCos, when the conversion is based on a breach of a preset equity ratio. The simplicity buys us a complete analytical solution, without much loss of generality. Using a call options framework, we show that risk-shifting incentives arise from two forces: an increase in the conversion probability of a given CoCo, and a decrease in the wealth transfer relative to issuing subordinated debt.

We define wealth transfers from the existing (i.e. prior to conversion) shareholders’ point of view - that is, as the change in residual equity that results from a conversion-induced reduction in leverage. Within the call options framework, subordinated debt and unconverted CoCos of the same amount are equivalent because both are senior to equity. In the same way, there is no difference between equity and converted CoCos of the same amount, at least to the extent that the newly created equity value accrues to the old equity holders. This fact enables us to write the ex ante residual value of a CoCo-issuing bank as a weighted average of the respective residual values with subordinated debt, and with additional equity, with the conversion probability as the weight on the latter, and one minus that probability as the weight on the former. This approach allowed us to decompose this value as the residual value with subordinated debt, plus an expected wealth transfer term. The expected wealth transfer is the product of the conversion probability and the wealth transfer term. Our analysis differs from the existing literature in that we pay explicit attention to that probability of conversion, rather than treating it as a given term.

We apply our framework to the full range of CoCos issued so far: principal writedown (PWD) CoCos, which are not well-covered in the academic literature but widely issued, and convert-to-equity (CE) CoCos with dilutive and nondilutive conversion ratios. We show that for equal loss absorption capacity, all PWD and nondilutive CE CoCos each have substantially worse risk-shifting incentives than requiring additional equity would lead to. Moreover, we show that all PWD CoCos and nondilutive CE CoCos have worse risk-shifting incentives compared to the same amount of subordinated debt. This is because the wealth transfer is always away from the CoCo holders towards the existing shareholders.

But when the CoCos are of the dilutive CE variety, we show that the risk-shifting incentive turns negative. This is because the wealth transfer itself becomes negative - while shareholders in aggregate obtain a higher residual equity upon conversion, the old shareholders must share the total residual value (i.e. old and new claims) with the new shareholders created upon
conversion. The sharing of residual equity, while not strictly skin in the game ex ante, is a credible threat such that the shareholders can be expected to choose risk levels that make the conversion probability smaller. As a result, the risk level chosen under dilutive CE CoCos will be lower than the risk level chosen under the same amount of subordinated debt.

Therefore, the risk-shifting incentives arising from the expected wealth transfers can be viewed as a wedge that affects a bank’s optimal risk choices relative to when the bank has issued subordinated debt in place of CoCos. While there is no question about the superiority of additional equity over subordinated debt, the wedge brought about by the risk-shifting incentives matters in determining whether CoCos are superior to subordinated debt. We find that PWD and nondilutive CE CoCos encourage banks to take riskier choices relative to subordinated debt, while dilutive CE CoCos discourage them. However, as 57% of the CoCos issued to date are of the PWD kind, it is important to recognize the possibility that CoCos might contribute to, rather than mitigate the buildup of risk in the banking system.

Recent regulation has encouraged the use of CoCos in order to meet regulatory capital or loss absorption capacity requirements. However, regulation neither distinguishes between these two CoCo designs for the purpose of meeting capital requirements, nor considers the interaction of CoCo issuance with existing frameworks. We show that even though CoCos and equity provide equal loss absorption capacity ex post, replacing subordinated debt with CoCos changes the interaction of the regulator and the bank ex ante, because of the risk-shifting incentive wedge. The regulatory bodies would seem to be well advised to pay more attention to the risk incentives brought about by the design of CoCos.

3.2 Related literature

There is a small but growing body of research on the impact of CoCos on the risk-shifting incentives of banks. Koziol and Lawrenz [2012] only consider CE CoCos, and argue that risk-shifting incentives always increase relative to ordinary bonds, as long as the old equity holder gets to keep some shares after conversion. This strong result depends critically on their assumption that the conversion trigger coincides with the default trigger: If asset values decline enough to trigger default at a particular leverage ratio, replacing some of the debt by CoCos will leave shareholders better off: with an equal decline in asset values they are left with some claims and default is staved off, while in the straight debt case they would have lost everything. Berg and Kaserer [2015] numerically simulate the value of equity given an exogenously set mixture of debt and equity converter CoCos for four specific conversion ratios as a function of asset return variance. They argue that risk-shifting rises as wealth transfers from CoCo
holders to equity holders increase, and observe that the price at which conversion takes place has a direct impact on the magnitude and even sign of these wealth transfers. They also show that several of the existing CoCos such as those issued by Lloyds and Rabobank have prices that fall with changes in implied asset volatility, inferring that the market recognizes the risk taken by the banks. This finding points at very clear risk-taking incentives inherent in the CoCo designs issued by those two banks. Hilscher and Raviv [2014] argue that risk-taking incentives of banks may be mitigated by choosing the conversion ratio properly. For a capital structure containing CoCos, they found conversion ratios such that the resulting equity vega is equal to zero. This is akin to the suggestion of Calomiris and Herring [2013] on having CoCos which are sufficiently dilutive. On the other hand, Martynova and Perotti [2016] claim that both CE and PWD CoCos can mitigate risk-shifting if the trigger level is set properly. In their paper, risk-shifting takes the form of not exerting sufficient effort in monitoring the assets of the bank. However, they do not consider the possibility that the bank’s risk choice affects both wealth transfers and the probability of conversion. Accounting for the latter link is at the core of the analysis presented in this paper.

Chen et al. [2017] endogenize the conversion in an asset pricing setup similar to Koziol and Lawrenz [2012] and like them, only consider equity conversion CoCos. Although they derive closed form solutions, they use numerical procedures to obtain their results, which necessarily depend on chosen parameter values. They chose parameter values such that at least some dilution of old shareholders is taking place. As a consequence, conversion in the cases they analyze always imply a loss to old shareholders. But of the more than 150 billion Euro face value CoCos issued by European banks as of 2016, substantially more than half are issued on terms that imply a wealth transfer towards equity holders once conversion takes place, a possibility that plays a substantial role in our paper. In their set up, banks need to continuously roll over debt. This gives rise to rollover costs whenever the market value of the issued debt is lower than the par value of the newly issued debt. The possibility of this happening leads to lower risk-shifting by banks, because higher risk increases rollover costs.

---

2Vega is the sensitivity of the option value with respect to the volatility of its underlying assets.
3In their continuous time framework, endogenizing conversion comes down to endogenously determining the timing of conversion.
3.3 Revisiting the call options approach to residual equity valuation

Black and Scholes [1973] and Merton [1974] have noted that the shareholders of a firm effectively hold a call option on their company’s assets. While it is true that the creditors of the firm have claim over the assets to the extent of the outstanding liability, the shareholders can obtain the full claim to the assets upon paying off all outstanding liabilities. Therefore, the residual claim held by the shareholders can be thought of as a call option on the firm’s asset, with the outstanding liability as the strike price.

For a bank that has issued hybrid instruments such as CoCos, the valuation of its residual equity is slightly more involved. This is because the change in the hybrid’s "state" necessarily changes the bank’s capital structure. This implies a corresponding change in the valuation of the residual equity. Therefore, the valuation of residual equity involving hybrids must take the various "states" into account.

If the probability of conversion was exogenous, valuation is straightforward: the residual equity value of a CoCo-issuing bank can simply be expressed as a linear combination of the residual equity values before conversion (when the CoCo is treated as debt) and after conversion (when the CoCo is either written off or is converted to equity), with the conversion probability as the weighting factor. However, CoCos convert whenever the bank encounters either an automatic or a discretionary trigger. The bank’s ability to choose risk levels affects the shape of the return distribution, which in turn affects the bank’s ability to meet either type of trigger. Therefore, we cannot assume that the probability of conversion is exogenous.

By expressing the bank’s residual equity as a call option, and by recognizing that the probability of CoCo conversion is affected by risk levels chosen by the bank, we are able to examine the risk-taking incentives of a CoCo-issuing bank. Moreover, using the method outlined above, we can examine each type of CoCo design and determine which of them provides the best and the worst incentives for risk-taking.

3.3.1 Setup

Issued CoCos have two kinds of trigger: an automatic one which occurs whenever the bank fails to meet a preset equity ratio, and a discretionary one which occurs whenever the regulator believes the bank has reached the point of non-viability. In this paper, we focus on the automatic type.

A model with CoCos must have at least three dates because the risk choice, the conversion itself, and the final payoffs happen at distinct dates. However, if one wants to determine the ex
ante risk-shifting incentives induced by a CoCo, it is enough to know the impact of risk on the expected realizations of the asset value at the time of conversion. Therefore, while we refer to $t = 1$ and $t = 2$ events (for the sake of exposition), our analysis focuses only on the $t = 0$ actions.

Consider a CoCo-issuing bank. At $t = 0$, its capital structure is composed of $D_d$ deposits, $D_s$ CoCo, and $E$ initial equity. We assume that the CoCo does not convert at $t = 0$. At this stage, the CoCo-issuing bank is indistinguishable from an ordinary bank with $D_s$ subordinated debt in place of CoCos. We normalize the amounts such that $D_d + D_s + E = 1$. We take these amounts as given, because we are interested in seeing how banks choose risk for a given capital structure. Since banks face capital regulation, the bank is constrained in choosing its capital structure in the first place.

Upon obtaining these funds, the bank invests them in an asset that gives return $R_t$ at $t > 0$. We assume that $R_t$ follows a lognormal distribution with parameters $(\mu, \sigma^2)$ for the corresponding normal distribution of $\ln(R_t)$. The bank can choose the risk level $\sigma$ of the assets at $t = 0$. However, once the bank has chosen $\sigma$, it cannot make changes at any further time. Because we analyze at $t = 0$, we assume that the bank only knows and works with expectations about future returns. In particular, the bank works with expected return $R = \mathbb{E}_0 (R_1) = \mathbb{E}_0 (\mathbb{E}_1 (R_2))$. Also, to ensure that we analyze a pure risk effect not confounded with increases in wealth, we structure the increase in risk in such a way that $\mathbb{E} (R_t) = R$ stays unchanged (i.e. a mean-preserving spread in variance).

The setup described above allows us to write the equity holder’s claim as a call option on the asset return, as in Black and Scholes [1973] and Merton [1974]. For ease, we assume there is only one share, and the bank does not issue any new shares aside from those that may arise from CoCo conversion. Denote the value of the share at $t = 0$ as $e_0$. Thus, before conversion, the bank’s residual equity may be expressed as

$$e_0 = C [R, D_d + D_s]$$

(3.1)

where $C [R, D]$ is a call option on an asset with gross return $R$ and strike price $D$. Henceforth, we use “liability”, “leverage”, and “strike price” interchangeably, to refer to a bank’s outstanding liability. In all subsequent calculations, we use $D$ to refer to a general strike price, but specify the actual level of debt (e.g. $D_d$ or $D_d + D_s$) when appropriate. As the unconverted CoCo is indistinguishable from subordinated debt, we also refer to the amount $e_0$ as the bank’s residual equity value with subordinated debt.

At $t = 1$, the asset return realization is observed to be $R_1$. Provided that $R_1$ exceeds the

---

4 Appendix 3.A contains the mathematical foundations of the call options framework.
total liability $D_d + D_s$, the bank remains solvent, otherwise, the bank is in default. Of course it is possible for the realization $R_1$ to be low enough to cause default even at $t = 1$. In that case, the bank is assumed to be closed down. However, we only consider cases when conversion precedes default. Henceforth, we assume that the bank’s $t = 0$ expectation about the $t = 1$ return is larger than $D_d + D_s$: $E_0 (R_1) > D_d + D_s$.

CoCos convert at $t = 1$ when $R_1$ is lower than what is consistent with a preset trigger equity ratio $\tau$. At $t = 2$ (provided that the bank has survived $t = 1$ events) when $R_2$ materializes, the creditors of the bank are paid, and anything left accrues to the residual claimant, which is the equity holder of the bank. We assume there is no risk of depositor runs (for example because of deposit insurance) in order to focus entirely on the risk-shifting implications of various CoCo designs.\footnote{In principle it is also possible to draw conclusions from those risk choices for run probabilities: for such an analysis in a global games framework, see Chan and van Wijnbergen [2014].}

3.3.2 The endogenous conversion probability

We have shown in the previous subsection that it is straightforward to value residual equity when $D_s$ is subordinated debt. When CoCos are involved, we need to consider both the change in the value of the residual equity arising from the change in the outstanding liability, as well as the probability that the CoCo converts. A number of papers (for instance, Martynova and Perotti [2016]) treat this probability as exogenous. However, since the bank’s choice of risk affects the distribution of the asset returns, the probability of CoCo conversion cannot be exogenous.

In this section, we define this probability endogenously by using the concept of distance-to-default and modifying it accordingly.

As the name suggests, distance-to-default is a measure of the closeness of the asset return and the value of the outstanding liability. For lognormally distributed asset returns $R$ and total face value of debt $D$, distance-to-default $d_d$ at $t = 0$ can be written as

$$d_d = \frac{1}{\sigma} \left[ \ln \frac{R}{D} + r - \frac{\sigma^2}{2} \right]$$

(3.2)

where $r$ is the risk-free rate.\footnote{The standard form for distance-to-default is $d_d = \left[ \ln \frac{R}{D} + (r - \frac{1}{2} \sigma^2) T \right] / \left[ \sigma \sqrt{T} \right]$ for $T$ periods ahead. Since in our model, conversion only occurs at $t = 1$, $T$ takes the value of 1. Moreover, since we are performing the analysis at $t = 0$, we use the expected asset return $R$ at $t = 0$ instead of the actual realization at $t = 1$ which is $R_1$.} It is implicit from the use of this measure that the default event occurs when the equity ratio of the bank is 0. However, with CoCos, the relevant event is not default, but conversion. For CoCos with automatic conversion, the trigger event is when the bank’s equity ratio falls short of the trigger level $\tau > 0$. We therefore introduce a measure simi-
lar to distance-to-default by incorporating the trigger level \( \tau \), and call it distance-to-conversion \( d_c \). Formally, automatic conversion occurs whenever

\[
\frac{R - D}{R} \leq \tau \iff R(1 - \tau) \leq D,
\]

allowing us to write the distance-to-conversion \( d_c \) as

\[
d_c = \frac{1}{\sigma} \left( \ln \frac{R(1 - \tau)}{D} + r - \frac{\sigma^2}{2} \right).
\]

With the assumption of lognormally distributed returns, the conversion probability is then simply

\[
p_c = \Phi(-d_c)
\]

where \( \Phi(\cdot) \) is the cumulative standard normal distribution. With the conversion probability now well-defined, we are now able to value the equity of a bank that has issued CoCos within our framework, as a linear combination of values of residual equity with differing amounts of outstanding liability.

As \( d_c \) is a function of both \( \tau \) and \( \sigma \), the probability of conversion \( p^c \) must be as well. We have

\[
\frac{\partial p^c}{\partial \tau} = -\phi(-d_c) \frac{\partial d_c}{\partial \tau} = \phi(-d_c) \times \left( -\frac{1}{\sigma (1 - \tau)} \right) > 0
\]

and

\[
\frac{\partial p^c}{\partial \sigma} = -\phi(-d_c) \frac{\partial d_c}{\partial \sigma} = \phi(-d_c) \times \left( 1 + \frac{d_c}{\sigma} \right) > 0
\]

where \( \phi(\cdot) \) is the standard normal distribution. This leads to the following lemma:

**Lemma 3.1.** The conversion probability is increasing in the risk \( \sigma \) taken, as well as in the trigger ratio \( \tau \) that is given.

The intuition behind this result lies in the distance-to-conversion expression. \( d_c \) is a standardized variable that is affected by the trigger ratio \( \tau \) and the risk level \( \sigma \). \( d_c \) falls in \( \tau \) because ceteris paribus, the equity ratio of a bank is closer to a higher value of \( \tau \) than to a lower one. On the other hand, an increase in \( \sigma \) always decreases the value of a variable that it standardizes. The fall in the distance-to-conversion induced by both of these factors, combined with the derivative of the cumulative standard normal distribution with respect to its parameter,
deliver this lemma.

From Lemma 3.1, one can see that the trigger ratio $\tau$ and the risk level $\sigma$ are substitutes to an extent, as they affect the conversion probability in the same direction. If one takes the cross partial derivative of (3.7) with respect to $\tau$, one obtains

$$\frac{\partial^2 p_c}{\partial \tau \partial \sigma} = \frac{\phi (-d_c) (1 - \tau) \left[ \sigma d_c \frac{\partial d}{\partial \sigma} - 1 \right]}{\sigma^2 (1 - \tau)^2} < 0,$$

which shows that the marginal conversion probability with respect to risk $\sigma$ falls as the trigger ratio $\tau$ rises. By Young’s theorem, the marginal conversion probability with respect to the trigger ratio $\tau$ also falls as the risk level $\sigma$ rises. This leads to following corollary:

**Corollary 3.2.** The risk level $\sigma$ and the trigger ratio $\tau$ are substitutes in terms of their effect on the conversion probability.

Corollary 3.2 suggests that if the bank has a target level of the probability of conversion, the bank can choose lower risk levels if the trigger ratio is high enough. Similarly, if the trigger ratio is low, the bank can achieve the target by choosing higher risk levels.

### 3.3.3 Residual equity valuation with CoCos in the capital structure

In this section, we consider the valuation of residual equity when CoCos are in the capital structure. The two states (pre- and post-conversion) must be considered in the valuation. To this end, we examine how conversion alters the issuing bank’s residual equity.

There are two types of CoCos that have been issued to date: principal writedown (PWD) CoCos and convert-to-equity (CE) CoCos. PWD CoCos are written off by the fraction $(1 - \phi) \in [0, 1]$ from the issuing bank’s balance sheet whenever the bank encounters an automatic trigger event. That is, provided that a bank has the capital structure described in Section 3.4.1, but with $D_s$ PWD CoCos instead of subordinated debt, conversion would change the bank’s residual equity from $C [R, D_d + D_s]$ to $C [R, D_d + \phi D_s]$, where $\phi$ represents the fraction of the CoCos that are retained on the balance sheet. We henceforth refer to $\phi$ as the retention parameter.

On the other hand, CE CoCos convert to equity at some conversion rate $\psi$ per unit of CoCo when the issuing bank encounters an automatic trigger event. That is, provided that a bank has the capital structure described in Section 3.4.1, but with $D_s$ CE CoCos instead of subordinated debt, conversion would change the bank’s residual equity from $C [R, D_d + D_s]$ to $C [R, D_d + \psi D_s]$, where $\psi$ represents the fraction of the CoCos that are retained on the balance sheet. We henceforth refer to $\psi$ as the conversion parameter.

---

8 Some papers refer to the conversion price, which is the inverse of the conversion rate. That is, for conversion rate $\psi$, the conversion price is $1/\psi$. 53
subordinated debt, conversion would change the bank’s residual equity from $C[R, D_d + D_s]$ to
\[ \frac{1}{1 + \psi D_s} (C[R, D_d]). \]

Both the writeoff and the equity conversion features can be accommodated by the expression
in (3.9) to represent a general CoCo-issuing bank’s residual equity after conversion.

\[ \frac{C[R, D_d + \varphi D_s]}{1 + \psi D_s} \]  

(3.9)

PWD CoCos can be represented by setting $\psi = 0$ in (3.9) and keeping $\varphi \in [0, 1]$. A PWD that
is fully written off has $\varphi = 0$. Similarly, CE CoCos can be represented by setting $\varphi = 0$ in
(3.9) and keeping $\psi \in [0, \infty)$. A full PWD CoCo ($\varphi = 0$) is equivalent to a CE CoCo with zero
dilution ($\psi = 0$). At the time of writing, there does not exist an issued CoCo which has both
writedown and equity conversion features.

Denote by $e_{coco}$ the value of a general CoCo-issuing bank’s residual equity at $t = 0$. As
previously mentioned, the value of residual equity of a bank with CoCos in the capital structure
can be written as a linear combination of the pre-conversion state and the post-conversion
state, with the probability of conversion $p^e$ as the weighting factor. With this, we may write
the CoCo-issuing bank’s residual equity as

\[ e_{coco} = p^e \frac{C[R, D_d + \varphi D_s]}{1 + \psi D_s} + (1 - p^e) \ C[R, D_d + D_s] \]

\[ = C[R, D_d + D_s] + p^e \left( \frac{C[R, D_d + \varphi D_s]}{1 + \psi D_s} - C[R, D_d + D_s] \right) \]

\[ = e_0 + p^e W, \]  

(3.10)

where the wealth transfer is

\[ W = \frac{C[R, D_d + \varphi D_s]}{1 + \psi D_s} - C[R, D_d + D_s] \]  

(3.11)

Thus, the ex ante value of residual equity of a CoCo-issuing bank can be expressed as the
value of a bank’s residual equity if it has issued subordinated debt $e_0$, plus an expected wealth
transfer term $p^e W$.

The expected wealth transfer may be positive or negative, depending on the values of $\psi$
and $\varphi$. A PWD CoCo’s expected wealth transfer $p^e W_{pwd}$ is

\[ p^e W_{pwd} = p^e (C[R, D_d + \varphi D_s] - C[R, D_d + D_s]), \]  

(3.12)

which is always positive because the lower implied strike price after conversion ($D_d + \varphi D_s$)
increases the value of the call option held by the bank’s shareholder. Thus, the difference between $C[R, D_d + \varphi D_s] - C[R, D_d + D_s]$ is always larger than 0, and increases as $\varphi$ moves from 1 to 0. Figure 3.1 illustrates the change in the wealth transfer from the point of view of the bank shareholder. At Point A in the Figure, when $\varphi = 0$, the wealth transfer from the CoCo holder to the existing shareholder is at its highest value. This is because nothing is left for the CoCo holder.

Figure 3.1: Wealth transfers from CoCo holders to equity holders for various levels of $\varphi$

On the other hand, a CE CoCo’s expected wealth transfer $p^C W_{ce}$ is

$$p^C W_{ce} = p^C \left( \frac{C[R, D_d]}{1 + \psi D_s} - C[R, D_d + D_s] \right), \quad (3.13)$$

which may be positive or negative over the range of $\psi$, which is $\mathbb{R}^+$. Figure 3.2 illustrates the wealth transfer, again from the point of view of the original equity holder.

Figure 3.2: Wealth transfers from CoCo holders to equity holders for various levels of $\psi$
Point B of Figure 3.2 shows that wealth transfer is highest when $\psi = 0$. At this value of $\psi$, the CE CoCo is equivalent to a full PWD CoCo. However, as $\psi \to \infty$, the CoCo holder completely dilutes the original shareholder such that the claim of the original shareholder disappears. Hence, the wealth transfer is from the original shareholder to the CoCo holder. As the wealth transfer term $W_{ce}$ is continuous in $\psi$, there exists a value of $\psi$ that sets the wealth transfer of a CE CoCo exactly equal to 0, and it is found by setting $W_{ce} = 0$. Call this value $\bar{\psi}$.

We have that

$$\bar{\psi} = \frac{1}{D_s} \left( \frac{C[R, D_d]}{C[R, D_d + D_s]} - 1 \right).$$ (3.14)

At $\bar{\psi}$, the number of new shares $\tilde{\psi} D_s$ valued at the pre-conversion value of $C[R, D_d + D_s]$ is just equal to the difference in the values of residual equity pre- and post-conversion: $C[R, D_d] - C[R, D_d + D_s]$.9 Because a wealth transfer from the CoCo holder to the shareholder is observationally equivalent to the dilution of the shareholder, we also refer to $\psi$ as the dilution parameter. Any value of $\psi < \bar{\psi}$ leads to a wealth transfer from the CoCo holder to the shareholder (nondilutive CoCos). Any value of $\psi > \bar{\psi}$ leads to a wealth transfer from the shareholder to the CoCo holder (dilutive CoCos) Only at $\psi = \bar{\psi}$ is there a neutral conversion in the sense of not causing any wealth transfers in either direction.

### 3.4 The risk-shifting incentives induced by CoCos

In the previous section, we have shown that PWD CoCos always have positive wealth transfers upon conversion, but the direction of CE CoCo wealth transfers vary with the dilution parameter $\psi$. To examine the risk-shifting incentives of each type of CoCo, we take the derivative of the expected wealth transfers with respect to $\sigma$. This is because the expected wealth transfer measures the impact of replacing a given amount of subordinated debt with an equivalent amount of CoCos. In effect, we are looking at the differential effect of CoCos on a bank’s risk-making decisions, with subordinated debt as the benchmark. As previously mentioned, we assume that changes in $\sigma$ do not change the expected return $R$ - that is, we assume a mean-preserving spread in variance, in order to abstract away from wealth effects that are not brought about by changes in $\sigma$.

If one uses an exogenous probability of conversion in the expected wealth transfers, then CoCo conversion necessarily leads to lower risk-shifting. This is because wealth transfers shrink as $\sigma$ rises, ceteris paribus. However, we cannot ignore the impact of risk on the conver-

---

9Calomiris and Herring [2013] has a similar discussion and the recommendation to use a conversion price closely related to our definition of $\bar{\psi}$. Also, this price is critical according to Sundaresan and Wang [2015] if multiple equilibria are to be avoided in the case of market-based (share price) conversion triggers.
sion probability, as we have shown in Lemma 3.1 that the probability of conversion increases in risk. In this section, we find conditions for which the conversion probability effect dominates the wealth transfer effect. As PWD and CE CoCos have differing mechanisms, we discuss them separately.

3.4.1 Risk-shifting incentives for given CoCo design

3.4.1.1 PWD CoCos

The value of residual equity of a bank that has issued a PWD CoCo is

\[ e_{pwd} = e_0 + p^c (C[R, D_d + \phi D_s] - C[R, D_d + D_s]). \] (3.15)

The differential effect of using a PWD CoCo in place of the same amount of subordinated debt is given by the expected wealth transfer term \( p^c W_{pwd} \):

\[ p^c W_{pwd} = e_{pwd} - e_0 = p^c (C[R, D_d + \phi D_s] - C[R, D_d + D_s]). \] (3.16)

Define now the risk-shifting incentive of such a bank as \( RSI_{pwd} \). This term is the derivative of \( p^c W_{pwd} \) with respect to \( \sigma \), as shown in (3.17).

\[ RSI_{pwd} = \frac{\partial p^c}{\partial \sigma} \frac{(C[R, D_d + \phi D_s] - C[R, D_d + D_s])}{CF_{pwd}} + p^c \frac{\partial}{\partial \sigma} \frac{(C[R, D_d + \phi D_s] - C[R, D_d + D_s])}{WF_{pwd}} \] (3.17)

Two components of \( RSI_{pwd} \) arise from the differentiation: the conversion probability factor \( (CF_{pwd}) \) and the wealth transfer factor \( (WF_{pwd}) \). \( CF_{pwd} \) represents the increase in the probability of conversion as risk increases, holding the wealth transfer constant. On the other hand, \( WF_{pwd} \) represents the change in the wealth transfer as risk increases, holding the conversion probability constant.

Let us first consider the conversion probability factor \( CF_{pwd} \), reproduced in (3.18):

\[ CF_{pwd} = \frac{\partial p^c}{\partial \sigma} (C[R, D_d + \phi D_s] - C[R, D_d + D_s]). \] (3.18)

\( CF_{pwd} \) has two components, the derivative of the conversion probability with respect to \( \sigma \), and the wealth transfer itself. From Lemma 3.1, we know that \( \frac{\partial p^c}{\partial \sigma} > 0 \). The sign of \( CF_{pwd} \) then
depends on the sign of the wealth transfer: for the case of PWD CoCos, it is always positive. Therefore, an increase in risk raises the probability of conversion, makes it more likely for the wealth transfer to be obtained. Considering only an exogenous probability of conversion would ignore the impact arising from $CF_{pwd}$.

Consider now the wealth transfer factor $WF_{pwd}$, reproduced below as (3.19):

$$WF_{pwd} = p^c \frac{\partial}{\partial \sigma} (C[R, D_d + \varphi D_s] − C[R, D_d + D_s]).$$ (3.19)

$WF_{pwd}$ represents the impact of the increase in the risk level on the value of the wealth transfer itself, holding the probability of conversion constant. While the wealth transfer itself is positive, it is decreasing in the risk taken. The intuition behind this is that a conversion increases a bank’s skin in the game. Prior to conversion, the bank has less of its own capital. After conversion, the disappearance of $1 - \varphi$ of the CoCo implies that the bank has more of its own capital, making risk-shifting less attractive than in the previous case. To see this formally, note that (3.19) takes the derivative of the difference of two call option expressions with respect to $\sigma$. This can be written as the difference between the vegas\(^{10}\) of two call options that differ only in the strike price. That is,

$$WF_{pwd} = p^c (V[R, D_d + \varphi D_s] − V[R, D_d + D_s])$$ (3.20)

where $V[\cdot]$ is the call option vega. As $V[\cdot]$ is continuously differentiable, we may rewrite (3.20) using the mean value theorem. Denote by $V_D$ the derivative of vega with respect to the strike price $D$. Then, (3.20) may be rewritten as

$$WF_{pwd} = -p^c ((1 - \varphi) D_s V_D[R, D'])$$ (3.21)

where $D' \in [D_d + \varphi D_s, D_d + D_s]$.

$WF_{pwd}$ is negative given any value of risk and leverage. However, it consists of $V_D[\cdot]$, which is positive whenever $\sigma^2 > 2 \left( \ln \frac{R}{D} + r \right)$, and goes to zero as $\sigma$ outpaces $d_1$, where $d_1 = \frac{1}{\sigma} \left( \ln \frac{R}{D} + r + \frac{\sigma^2}{2} \right)$. Let us call these as the high fragility conditions. The high fragility conditions captures the substitutability of risk and leverage for banks: given a high leverage ratio $\frac{D}{R}$, the bank needs a smaller level of risk $\sigma$ to keep $V_D[\cdot]$ constant, as well as the diminishing marginal returns to risk: a higher level of $\sigma$ leads to lower values of $V_D[\cdot]$. The effect is more pronounced as $\sigma$ outpaces $d_1$. When the high fragility conditions are met, $WF_{pwd}$ goes to zero as well while

\(^{10}\) Vaga is the sensitivity of the option value with respect to the volatility of its underlying assets, represented by the derivative of a call option with respect to $\sigma$. 
$CF_{pwd}$ stays positive, such that $CF_{pwd}$ dominates $WF_{pwd}$.

**Proposition 3.3.** The risk-shifting incentive of a principal writedown CoCo is positive whenever the high fragility conditions hold.

### 3.4.1.2 CE CoCos

Consider now the value of residual equity when a firm has issued a CE CoCo:

$$
e_{ce} = e_0 + p^c \left( \frac{C[R, D_d]}{1 + \psi D_s} - C[R, D_d + D_s] \right).$$  \hspace{1cm} (3.22)

The differential effect of using a CE CoCo in place of the same amount of subordinated debt is given by the expected wealth transfer term $p^c W_{ce}$:

$$p^c W_{ce} = e_{ce} - e_0 = p^c \left( \frac{C[R, D_d]}{1 + \psi D_s} - C[R, D_d + D_s] \right).$$  \hspace{1cm} (3.23)

Define now the risk-shifting incentive of such a bank as $RSI_{ce}$. This term is the derivative of $p^c W_{ce}$ with respect to $\sigma$, as shown in (3.24):

$$RSI_{ce} = \frac{\partial p^c}{\partial \sigma} \left( \frac{C[R, D_d]}{1 + \psi D_s} - C[R, D_d + D_s] \right) + p^c \left( \frac{V[R, D_d]}{1 + \psi D_s} - V[R, D_d + D_s] \right).$$  \hspace{1cm} (3.24)

where we have used the vega notation to simplify matters. As with $RSI_{pwd}$, $RSI_{ce}$ also has two components, the conversion probability factor ($CF_{ce}$) and the wealth transfer factor ($WF_{ce}$). However, the expressions for CE CoCos involve the dilution parameter $\psi$, which causes changes in the direction of the wealth transfer. Analyzing the risk-shifting incentives must take the size of $\psi$ into consideration.

To begin, take the derivative of $RSI_{ce}$ with respect to $\psi$. We have that

$$\frac{\partial RSI_{ce}}{\partial \psi} = -\frac{D_s}{(1 + \psi D_s)^2} \left( \frac{\partial p^c}{\partial \sigma} C[R, D_d] + p^c V[R, D_d] \right),$$  \hspace{1cm} (3.25)

so the risk-shifting incentives fall as the dilution parameter increases. When $\psi = 0$, the CE CoCo is equivalent to a full PWD CoCo. Therefore, the risk-shifting incentives for this type of CE CoCo is positive, from the results of the previous section. On the other hand, when $\psi \to \infty$,
we would have, at the limit,

$$RSI_{ce} (\psi \to \infty) = \frac{\partial P^c}{\partial \sigma} (-C [R, D_d + D_s]) + P^c \left( -V [R, D_d + D_s] \right).$$

(3.26)

Conversion then allows the CoCo holder to completely dilute the original shareholder. This causes the wealth transfer to be negative, leading to a negative $CF_{ce}$ term. Similarly, a full dilution leads to a negative $WF_{ce}$ term because the shareholder compares the marginal risk incentive from having no share after conversion (0) with the marginal risk incentive from holding a call option value of $C [R, D_d + D_s]$. Thus, $RSI_{ce} (\psi \to \infty)$ has negative risk-shifting incentives.

The above analysis implies that there is a value of $\psi$ that just makes the CE CoCo deliver zero risk-shifting incentives. Since $RSI_{ce} (\psi = 0) > 0 > RSI_{ce} (\psi \to \infty)$, we get by continuity a crossing at zero for a positive $\psi$. Call this value $\tilde{\psi}$. We obtain this value by setting (3.24) to 0 and solving for $\psi$. The resulting expression for $\tilde{\psi}$ is

$$\tilde{\psi} = \frac{1}{D_s} \left( \frac{\partial P^c}{\partial \sigma} C [R, D_d] + P^c V [R, D_d] \right) - 1,$$

(3.27)

which we show to be less than $\bar{\psi}$ in Appendix 3.B.3. Thus, any $\psi \in [0, \tilde{\psi})$ will yield a positive risk-shifting incentive (i.e. worse than in the alternative capital structure with subordinated debt instead of CoCos). Any $\psi \in [\tilde{\psi}, \infty)$ makes the risk-shifting incentives negative, regardless of whether the high fragility conditions discussed in the previous section are met. This result is stronger than the one obtained for the case of PWD CoCos, because it holds for a nonlimiting value of $\psi$.

**Corollary 3.4.** For any risk level $\sigma$ and leverage $D$, the risk-shifting incentives of a convert-to-equity CoCo is negative if the dilution parameter $\psi$ is larger than $\tilde{\psi}$, and positive otherwise.

### 3.4.2 Effect of other design features on risk-shifting incentives

Thus far, we had considered the risk-shifting incentives brought about by having CoCos in a bank’s capital structure. These incentives were studied taking design parameters as given. However, certain aspects of CoCo design may mitigate the risk-shifting incentives. In the previous section, we have shown that the risk-shifting incentive for a CE CoCo falls when the

[11]The results are consistent with those of Hilscher and Raviv [2014], who find the conversion ratio that achieves zero vega. However, they only consider the wealth transfer and the leverage channels. Our calculations for the conversion ratio also takes the endogenous probability of conversion into account.
dilution parameter increases. In this section, we examine the impact of the retention parameter for a PWD CoCo, and the trigger ratio for both types of CoCo on the risk-shifting incentives. There are two channels where these operate: the probability of conversion, and the wealth transfer.

### 3.4.2.1 Risk-shifting incentives as a function of the retention parameter $\varphi$.

We have shown that the risk-shifting incentives for any PWD CoCo ($RSI_{pwd}$) are positive when the fragility condition is met, given the retention parameter $\varphi$. But the risk-shifting incentive changes with $\varphi$, because $\varphi$ affects the size of the wealth transfer $W_{pwd}$, even though the probability of conversion is unaffected. We have

$$\frac{\partial RSI_{pwd}}{\partial \varphi} = \frac{-\partial \rho^c}{\partial \sigma} \exp (-r) \Phi (d^*_2) D_s + p^c V^*_D \frac{D_s}{D_d + \varphi D_s},$$

where the notations $V^*_D$ and $d^*_2$ refer to $V_D$ and $d_2$ evaluated at liabilities $D_d + \varphi D_s$.\(^\text{12}\) The term $\partial CF_{pwd}/\partial \varphi$ is always negative: since $\varphi$ is the fraction of the debt retained, a higher retention rate (smaller writedown) leads to lower risk-shifting incentives because the actual wealth transfer is also smaller.

Consider now the term $\partial WF_{pwd}/\partial \varphi$ in (3.28). While this expression is always positive, we show in Appendix 3.A.4 that $V_D$ tends to zero whenever the high fragility conditions hold, so $\partial CF_{pwd}/\partial \varphi$ dominates $\partial WF_{pwd}/\partial \varphi$. Thus the higher the writedown fraction, the higher the risk-shifting incentives become.

**Corollary 3.5.** When the high fragility conditions hold, the risk-shifting incentive of a principal writedown CoCo is increasing in the fraction of the CoCo written off upon conversion.

### 3.4.2.2 Impact of $\tau$ on the risk-shifting incentives

In this section, we examine the impact of the trigger level $\tau$ on the risk-shifting incentives. The results from this section emanate from Lemma 3.1, which means the effect is solely through the probability of conversion, not the wealth transfer. To see this, we again use the residual equity of a bank that has issued a general CoCo, (3.9), introduced in Section 3.4.3 and reproduced here as (3.29):

\(^\text{12}\)We have that $d_2 = \frac{1}{\sigma} \left[ \ln \frac{D}{D_s} + r - \frac{1}{2} \sigma^2 \right]$ for strike price $D$. $d_2$ is the same as distance-to-default measure introduced in Section 3.4.2.
\[ e_{coco} = C [R, D_d + D_s] + p^c \left( \frac{C [R, D_d + \varphi D_s]}{1 + \psi D_s} - C [R, D_d + D_s] \right). \] (3.29)

The trigger level \( \tau \) does not appear in the wealth transfer component of (3.29), so we may use \( W \) to represent the wealth transfer without loss of information. As before, the risk-shifting incentive is calculated by taking the derivative of the expected wealth transfer \( p^c W \) with respect to \( \sigma \), as shown in (3.30).

\[ RSI = \frac{\partial p^c W}{\partial \sigma} = \frac{\partial p^c}{\partial \sigma} W + p^c \frac{\partial W}{\partial \sigma}. \] (3.30)

Differentiating the risk-shifting incentive with respect to \( \tau \) leads to the following expression:

\[ \frac{\partial RSI}{\partial \tau} = \frac{\partial^2 p^c}{\partial \sigma \partial \tau} W + \frac{\partial p^c}{\partial \tau} \frac{\partial W}{\partial \sigma}. \] (3.31)

Note that the effect of \( \tau \) is solely through the probability of conversion. From Lemma 3.1, \( \frac{\partial p^c}{\partial \tau} > 0 \) while \( \frac{\partial^2 p^c}{\partial \sigma \partial \tau} < 0 \) follows from Corollary 3.2. The net effect must take the wealth transfers into consideration. For PWD and nondilutive CE CoCos, the wealth transfer is always positive, while the marginal effect of risk on the wealth transfer is negative. So raising the trigger level \( \tau \) always reduces the risk-shifting incentives embedded in those CoCo designs.\(^\text{13}\) This is a possible way of mitigating the ill effects of CoCos that were designed to favor the original shareholders. As for dilutive CE CoCos, the fact that \( \frac{\partial^2 p^c}{\partial \sigma \partial \tau} < 0 \) interacts with the negativity of the wealth transfer, such that the net effect is more ambiguous.

**Corollary 3.6.** For PWD and nondilutive CE CoCos, the risk-shifting incentive is decreasing in the trigger ratio \( \tau \). For dilutive CE CoCos, the impact of \( \tau \) depends on the size of the wealth transfer.

This result supports the Basel III requirement of a trigger level of 5.125% or higher for a CoCo to qualify as Additional Tier 1 capital.

### 3.5 The bank’s optimization problem with CoCos

We have shown in the previous section that a bank’s risk-shifting incentives are affected by CoCo design. These incentives are related to, but distinct from a bank’s problem of maximizing

\(^{13}\)Martynova and Perotti [2016] also find that increasing the trigger level induces the banks to exert more effort in order to stave off conversion. This is consistent with our result that risk-shifting incentives decline as the trigger level rises.
the net value of residual equity. In this section, we show how a bank would choose its risk levels when faced with a constrained optimization problem. To this end, we introduce expected costs of default, and show how a bank’s risk decision changes for different roles of $D_s$: additional equity, subordinated debt, PWD CoCo and CE CoCo.

In the literature, imposing expected costs of default is usually associated with social objective functions, as in Kashyap and Stein [2004]. In our model, it is necessary even for the private objective function. This is because while the call option function necessarily accounts for the probability of default by construction, it does not account for the costs associated with default other than the foregone asset returns. Moreover, without these expected costs, the bank’s maximization problem would remain unbounded for the range of parameters that we are interested in.

The expected default costs we have in mind have two components: the actual costs of bankruptcy, and the probability of default. The bankruptcy costs may be reputational or legal in nature, and distinct from social costs such as contagion effects on other banks, or taxpayer-funded bailouts. We keep these costs exogenous to our analysis, as we use a partial equilibrium framework.

The probability of default is a function of both risk $\sigma$ and leverage $D$. For analytical convenience we use the first order Taylor approximation of this probability function in $\sigma^2$ and in $D$. The probability of default is distinct from the probability of conversion, although a sufficiently low draw of $R_1$ at $t = 1$ would make both events coincide. The literature on CoCos has paid more attention to probability of default than on the probability of conversion, perhaps due to the emphasis on the loss-absorption capacity of CoCos. In Chen et al. [2017] and Hilscher and Raviv [2014], the probability of default is influenced by the asset value that leads to default, which is chosen endogenously by shareholders in their analysis. However, the interaction of risk choices with the bank’s capital structure is not considered explicitly in these papers.

### 3.5.1 A bank’s objective function for given leverage $D$

Let $X$ represent the bank’s private costs of default, and let $p^d$ represent the bank’s probability of default. As stated above, we let $X$ be given, and we adopt a functional form for $p^d$ which is a linear approximation of the probability of default that is obtained from the Merton model: that is,

$$p^d = \Phi (-d_d),$$  \hspace{1cm} (3.32)

where $d_d$ is the distance-to-default introduced in Section 3.3. We may write $p^d$ as a linear approximation around values of $\sigma^2$ and $D$ away from zero, say $\bar{\sigma}^2$ and $\bar{D}$. This can be done as
we are interested in values of $\sigma^2$ and $D$ for which the high fragility conditions hold:

$$p^d(\sigma^2, D) \approx p^d(\bar{\sigma}^2, \bar{D}) + \frac{\partial p^d}{\partial \sigma^2}(\bar{\sigma}^2, \bar{D}) \sigma^2 + \frac{\partial p^d}{\partial D}(\bar{\sigma}^2, \bar{D}) D = \frac{1}{2} \sigma^2 b + cD. \quad (3.33)$$

The probability of default in (3.33) is then obtained by omitting the irrelevant constant term as well as the higher-order terms, and where $b$ and $c$ are positive constants. Thus, the expected costs of default of a given bank is

$$p^d(\sigma, D) X = \left( \frac{1}{2} \sigma^2 b + cD \right) X, \quad (3.34)$$

This parameterization reflects that a higher risk choice and a higher leverage level make default more likely.

The bank would like to maximize the value of its residual equity (represented by the call option function), subject to the expected default costs in (3.34). The objective function takes the following form for expected return $R$, given leverage $D$:

$$\max C[R, D] - p^d X = \max \left[ C[R, D] - \left( \frac{1}{2} \sigma^2 b + cD \right) X \right]. \quad (3.35)$$

The bank maximizes (3.35) by choosing $\sigma$. Similar to Kashyap and Stein [2004], we assume that the bank’s leverage $D$ cannot be adjusted at the time of choosing $\sigma$. Therefore, in the maximization process, the leverage term $D$ drops out. For a given $D$, the first-order conditions associated with (3.35) is

$$V[R, D]|_{\sigma^*} = \sigma^* b X, \quad (3.36)$$

where the notation $V[R, D]|_{\sigma^*}$ means that the function $V[R, D]$ is evaluated at $\sigma = \sigma^*$. The objective function in (3.35) is concave in $\sigma$ when $\sigma^2 > 2 \left( \ln \frac{R}{D} + r \right)$. Therefore, for this range of $\sigma$, we know that there exists a $\sigma$ that solves first-order conditions of the form (3.36). Since we are determining how CoCos would be effective in a crisis, we assume throughout this section and the next that the bank is operating when $\sigma^2 > 2 \left( \ln \frac{R}{D} + r \right)$ holds. The next subsections consider how the banks’ optimal $\sigma$ changes with the capital structure.

### 3.5.2 Subordinated debt vs. equity

Consider first the case where the bank’s capital structure has $D_d$ deposits, and $D_s + E$ initial equity at $t = 0$. Given this capital structure, the bank essentially holds a call option on the
asset return $R$ at a strike price of $D_d$, leading to an objective function of the form

$$\max C [R, D_d] - \left( \frac{1}{2} \sigma^2 b + c D_d \right) X$$

(3.37)

and the first-order condition

$$V [R, D_d] | \sigma^*_e = \sigma^*_b X,$$

(3.38)

where $\sigma^*_e$ represents the optimal risk level under the circumstances.

Consider now the case where the bank’s capital structure at $t = 0$ consists of $D_d$ deposits, $D_s$ subordinated debt, and $E$ initial equity. Valuation of the bank’s residual equity in this case requires that the strike price be $D_d + D_s$, leading to the objective function

$$\max C [R, D_d + D_s] - \left( \frac{1}{2} \sigma^2 b + c (D_d + D_s) \right) X$$

(3.39)

and the first-order condition

$$V [R, D_d + D_s] | \sigma^*_s = \sigma^*_b X,$$

(3.40)

where $\sigma^*_s$ represents the optimal risk level with $D_d + D_s$ leverage.

We show in Appendix 3.3.A that the vega is decreasing in $\sigma$ and increasing in $D$ whenever $\sigma^2 > \ln \left( \frac{D}{D_d} + r \right)$. Therefore, since $D_d < D_d + D_s$, the graph of $V [R, D_d + D_s]$ should lie above that of $V [R, D_d]$ for any given $\sigma$. Figure 3.3 illustrates the case:

Figure 3.3: Optimal Risk Choice of Banks when $D_s$ is Additional Equity/Subordinated Debt

![Graph](image)

Figure 3.3 shows that the vega of a bank with $D_s$ additional equity intersects the marginal cost line $\sigma b X$ at a smaller value of $\sigma$ compared to the vega of a bank with $D_s$ subordinated debt. That $\sigma^*_s$ is higher than $\sigma^*_e$ reflects the higher risk-shifting incentives from issuing $D_s$.
subordinated debt relative to issuing the same amount of additional equity. We may derive it more formally as follows: note that we may rewrite $V[R, D_d + D_s] | \sigma_s^*$ in terms of $\sigma_e^*$ by using the mean value theorem, resulting in the following first-order approximation:

$$V[R, D_d + D_s] | \sigma_s^* = V[R, D_d] | \sigma_e^* + (V_o | \sigma_e^* ) (\sigma_s^* - \sigma_e^*) + (V_D | D_d) D_s,$$  

(3.41)

where the notation $V_o | \beta$ refers to the derivative of $V[\cdot ]$ with respect to $\alpha$, with $\alpha$ evaluated at $\beta$. By writing $\sigma_s^* bX$ as $\sigma_e^* bX + (\sigma_s^* - \sigma_e^*) bX$, we may rewrite (3.40) as

$$V[R, D_d] | \sigma_e^* + (V_o | \sigma_e^*) (\sigma_s^* - \sigma_e^*) + (V_D | D_d) D_s = \sigma_e^* bX + (\sigma_s^* - \sigma_e^*) bX.$$  

(3.42)

Subtracting (3.38) from (3.42) lets us obtain an expression showing that $\sigma_s^* > \sigma_e^*$.

$$\sigma_s^* = \sigma_e^* + (V_D | D_d) D_s \frac{bX}{bX - (V_o | \sigma_e^*)} > \sigma_e^*$$  

(3.43)

As $V_o [\cdot ]$ is always negative whenever $\sigma^2 > 2 \left( \ln \frac{R}{D} + r \right)$, the denominator $bX - (V_o | \sigma_e^*)$ is always positive as well.

**Proposition 3.7.** The optimal amount of risk that a bank takes with $D_s$ subordinated debt is higher than the optimal amount of risk if the bank has issued $D_s$ additional equity.

This result is intuitive: as the bank has more skin-in-the-game when it has issued more equity, it would choose lower risk levels as well.

### 3.5.3 Subordinated debt vs. PWD and CE CoCos

When a bank issues $D_s$ CoCos in place of the same amount of subordinated debt, the bank’s objective function becomes

$$\max \ C[R, D_d + D_s] + p^* W - \left( \frac{1}{2} \sigma^2 b + c (D_d + D_s) \right) X$$  

(3.44)

which is similar to (3.39) but with the expected wealth transfer term $p^* W$. The accompanying first order condition is

$$V[R, D_d + D_s] | \sigma_{\text{coco}}^* + \text{RSI} = \sigma_{\text{coco}}^* bX,$$  

(3.45)

where RSI is the risk-shifting incentive arising from the expected wealth transfer $p^* W$. If RSI is zero, then (3.45) coincides with (3.40), because the strike price $(D_d + D_s)$ is the same regardless of whether $D_s$ was issued as subordinated debt or as a CoCo. Therefore, the sign and magnitude
of \( RSI \) determines how much the bank’s behavior would change relative to the subordinated debt case.

We have shown in Section 3.5 that PWD CoCos and nondilutive CE CoCos have positive risk-shifting incentives, while dilutive CE CoCos have negative risk-shifting incentives. Therefore, for PWD CoCos and nondilutive CE CoCos, \( V[R, D_d + D_s] + RSI \) must lie above that of \( V[R, D_d + D_s] \) for any given \( \sigma \) provided that \( \sigma^2 > 2 \left( \ln \frac{R}{D} + r \right) \). Similarly, \( V[R, D_d + D_s] + RSI \) must lie below \( V[R, D_d + D_s] \) for dilutive CE CoCos. Figure 3.4 illustrates the first order conditions associated with \( D_s \) CoCos and \( D_s \) subordinated debt, for different \( RSI \) values.

Figure 3.4: Optimal Risk Choice of Banks when \( D_s \) is Subordinated Debt/CoCo

As mentioned before, the forms in (3.44) and (3.45) accommodate both type of CoCos. We consider each type separately.

### 3.5.3.1 Optimal risk choices with PWD CoCos

To analyze PWD CoCos, we use (3.45) but use the subscript \( pwd \) to be more specific. Letting \( \sigma^*_{pwd} \) denote the solution to the bank’s maximization problem, we may write the first-order condition as

\[
V[R, D_d + D_s] \big|_{\sigma^*_{pwd}} + RSI_{pwd} = \sigma^*_{pwd} bX. \tag{3.46}
\]

Since (3.46) differs from (3.40) only by the risk-shifting incentive \( RSI_{pwd} \), we can attribute the excess of \( \sigma^*_{pwd} \) over \( \sigma^*_s \) to the positive risk-shifting incentive brought about by the expected wealth transfer. Formally, we have

\[
\sigma^*_{pwd} = \sigma^*_s + \frac{RSI_{pwd}}{bX - \langle V_b|V_s \rangle} > \sigma^*_s \tag{3.47}
\]
Proposition 3.8. The optimal amount of risk that a bank takes with $D_s$ principal writedown CoCos is higher than the optimal amount of risk if the bank has issued $D_s$ subordinated debt.

It is true that PWD CoCos improve loss absorption after conversion, and therefore meet the criteria for inclusion in Additional Tier 1 capital. However, as they elicit positive risk-shifting incentives before conversion, their use may make it more likely that the loss absorption capacity will be necessary in the future.

3.5.3.2 Optimal risk choices with CE CoCos

Similarly, to analyze CE CoCos, we use (3.45) but use the subscript $ce$ to be more specific. Letting $\sigma_{ce}^*$ denote the solution to the bank’s maximization problem, we may write the first-order condition (up to a first-order approximation) as

$$V[R, D_d + D_s]|\sigma_{ce}^* + RSI_{ce} = \sigma_{ce}^* b X.$$  \hspace{1cm} (3.48)

As with the PWD CoCos, we can express $\sigma_{ce}^*$ in terms of $\sigma_s^*$ in the following manner:

$$\sigma_{ce}^* = \sigma_s^* + \frac{RSI_{ce}}{b X - (V_s|\sigma_s^*)}.$$  \hspace{1cm} (3.49)

The sign of $RSI_{ce}$ determines whether $\sigma_{ce}^*$ exceeds $\sigma_s^*$ or not. We have shown in Section 3.5.1.2 that the dilution parameter $\psi$ completely determines the sign of $RSI_{ce}$: a $\psi < \tilde{\psi}$ (nondilutive) leads to $RSI_{ce} > 0$, while $\psi > \tilde{\psi}$ (dilutive) leads to $RSI_{ce} < 0$.

Proposition 3.9. The optimal amount of risk that a bank takes with $D_s$ nondilutive CE CoCos is higher than the optimal amount of risk if the bank has issued $D_s$ subordinated debt, but the opposite is true if the bank has issued the same amount of dilutive CE CoCos.

It is then clear that dilutive CE CoCos induce better risk choices than the same amount of subordinated debt. As such, their inclusion as Additional Tier 1 capital is an improvement, but as they do not constitute skin in the game ex ante, they are still different from equity. Nonetheless, the threat of dilution effectively deters risk-shifting.

3.5.3.3 Dilutive CE CoCos vs. equity

Thus far we have proven two sets of results, $\sigma_{ce}^* < \sigma_s^*$ with dilutive CE CoCos, and $\sigma_{ce}^* > \sigma_s^*$ otherwise. But can we determine how CE CoCos compare with straight equity in terms of risk choice? Post-conversion, dilutive CoCos and straight equity provide the same loss absorption capacity. But before conversion, it is the threat of a forthcoming dilution that leads to lower risk
choices for dilutive CE CoCos. In contrast, it is higher skin in the game which leads to lower risk choices before conversion for the same amount of additional equity. It is worth examining whether there exists a dilution parameter that leads to better risk-shifting incentives for CE CoCos relative to additional equity.

Recall from (3.38) that when $D_s$ is equity, the strike price is $D_d$, so the first order condition is $V[R, D_d] | \sigma^* = \sigma_e b X$. From (3.48), for the case when $D_s$ is a convert-to-equity CoCo, the first order condition is $V[R, D_d + D_s] | \sigma^* + RS_{ce} = \sigma_{ce}^* b X$.

If we decompose $V[R, D_d + D_s] | \sigma^*_{ce}$ in terms of $\sigma_e$ and $V[R, D_d]$, we can rewrite the first order condition of a CE CoCo as

$$V[R, D_d] | \sigma^*_{ce} + V \sigma(\sigma^*_{ce} - \sigma^*_{e}) + (V_D | D_d) D_s + RS_{ce} = (\sigma^*_{ce} - \sigma^*_{e}) b + \sigma^*_{e} b$$

Thus, any $\psi$ that sets $(V_D | D_d) D_s + RS_{ce} \geq 0$ makes the risk-shifting incentive of $D_s$ CE CoCo smaller than or equal to the risk-shifting incentive for $D_s$ additional equity, for equal loss absorption capacity after conversion. In particular, it is

$$\psi \geq \psi_{eq} = \frac{1}{D_s} \left( \frac{p^e V[R, D_d] + \frac{\partial p^e}{\partial \sigma} C[R, D_d]}{p^e V[R, D_d + D_s] + \frac{\partial p^e}{\partial \sigma} C[R, D_d + D_s] - \frac{D_s}{\sigma} \phi(d_1) D_s} - 1 \right).$$

Note that $\psi_{eq}$ resembles $\tilde{\psi}$ in (3.27). However, $\psi_{eq} > \tilde{\psi}$ because

$$\frac{\partial p^e}{\partial \sigma} C[R, D_d + D_s] + p^e V[R, D_d + D_s] > \frac{\partial p^e}{\partial \sigma} C[R, D_d + D_s] + p^e V[R, D_d + D_s] - \frac{R d_1 \phi(d_1)}{D} D_s$$

whenever $\sigma^2 > 2 \left( \ln \frac{R}{b} + r \right)$. Also, $\psi_{eq} > \tilde{\psi}$ because at $\tilde{\psi}$, $RS_{ce} = 0$ and since $RS_{ce}$ is decreasing in $\psi$, it must be that $\psi_{eq} > \tilde{\psi}$.

This means that if the conversion ratio $\psi$ of CE CoCos are superdilutive (i.e. when $\psi \in [\psi_{eq}, \infty)$), they are better than straight equity in terms of risk-shifting incentives. Figure 3.5 illustrates the relationship between the risk-shifting line for equity and for CE CoCos with varying dilution parameters.
The following proposition holds:

**Proposition 3.10.** For $\psi \in [0, \tilde{\psi}]$, we have $\sigma^*_e < \sigma^*_s < \sigma^*_ce$ For $\psi \in [\tilde{\psi}, \psi_{eq}]$ we have $\sigma^*_e < \sigma^*_ce < \sigma^*_s < \sigma^*_{pwd}$. Finally, for $\psi \in [\psi_{eq}, \infty]$, we get a strong result: $\sigma^*_ce < \sigma^*_e < \sigma^*_s < \sigma^*_{pwd}$.

So when the CoCo is superdilutive (i.e. $\psi > \psi_{eq}$), $D_s$ CE CoCos provide lower risk-shifting incentive compared to straight equity, for equal loss absorption capacity. And even when they are not superdilutive but still provide at least a zero wealth transfer to the old shareholder, they still perform better than either subordinated debt or PWD CoCos, in that they provide less risk-shifting incentives for the same loss absorption capacity as subordinated debt would. But if the CoCos are not dilutive at all, they are worse than subordinated debt in that they provide even worse risk-shifting incentives for equal loss absorption capacity. In that case they clearly should not be part of Additional Tier 1 capital.

**3.5.3.4 Interaction of $\tau$ with probability of default**

In the previous sections, we have already seen that an increase in $\tau$ reduces the distance-to-conversion, thereby increasing the conversion probability. However, it does not play a role in the probability of default. To see this, consider again the first order condition for a general CoCo, as in (3.45) relative to the one for subordinated debt, as in (3.40). This results in the following optimal risk choice:

$$\sigma^*_{coco} = \sigma^*_s + \frac{RSI}{bX - (V\sigma^*_s)} > \sigma^*_s$$

(3.52)
\( \tau \) only plays a role in \( RSI \). Therefore, taking the derivative of \( \sigma_{coco}^* \) with respect to \( \tau \) is equivalent to looking at the sign of \( RSI \)'s derivative with respect to \( \tau \):

\[
\frac{\partial \sigma_{coco}^*}{\partial \tau} = \frac{1}{bX - (V_{\sigma|\tau})} \frac{\partial RSI}{\partial \tau}.
\]

(3.53)

We already know from Corollary 3.6 that \( \frac{\partial RSI}{\partial \tau} < 0 \) for PWD and nondilutive CE CoCos, while the sign is ambiguous for dilutive CE CoCos. Therefore, holding everything else constant, an increase in the trigger ratio causes a decrease in the risk taking incentives of a bank that has issued either PWD or nondilutive CE CoCos.

**Corollary 3.11.** Taking the probability of default into consideration, a bank that has issued PWD or nondilutive CE CoCos will lower its risk-taking in response to a higher trigger ratio.

### 3.6 Interaction of CoCos with pre-existing financial regulation

The goal of banking regulation is to protect the system from default externalities, and by extension, prevent the use of taxpayer money for bailout purposes. We consider the capital requirement aspect of banking regulation in this section.\(^\text{14}\) There are two sides to capital requirements: a target probability of default, and the capital requirement itself. When the regulator sets a target probability of default, she does so taking the bank’s leverage as an input, among other factors. The bank must choose a risk level which is compatible with its leverage, and complies with the target probability of default at the same time. When the regulator sets capital requirements, she forces the bank to change its capital structure in such a way that the bank’s skin in the game increases. This increase leads to less risky behavior by banks. Both actions discourage banks from making risk choices that may adversely affect the financial system.

Recent regulatory changes pushed CoCos to the frontline. From Basel III, CoCos now form part of Additional Tier 1 and Tier 2 capital for bank. This means that CoCos will comprise at most 3.5% out of the 8.0% minimum total capital required based on risk-weighted assets. Moreover, in November 2015, the Financial Stability Board has mandated that an additional 8% of capital requirements (based on risk-weighted assets) be filled in by CoCos for globally systemic financial institutions. These regulations imply that CoCos will form a substantial portion of a bank’s balance sheet in the near future, replacing subordinated debt to a large extent. However, as we have seen in the previous section, the replacement of subordinated

\(^{14}\)See VanHoose [2007] for a very informative survey on bank behavior and capital regulation.
debt with CoCos have implications on a bank’s risk choices because of the expected wealth transfers.

In this section, we examine how replacing subordinated debt with CoCos affects bank risk choices, given that the regulator has imposed both capital requirements and a target probability of default. In order to do this, we build on the bank’s maximization problem from the previous section. We have previously mentioned that the bank’s expected costs of default are a function of both risk \( \sigma \) and leverage \( D \), as in (3.34). This implies that for a target probability of default \( \bar{p}_d \), there is a tradeoff between risk and leverage. Because the regulator is assured that the bank will comply with its mandates, we can model the situation as a Stackelberg game: the regulator sets the target probability level knowing the bank’s objective function, letting the bank react to the requirements.

### 3.6.1 Setup

The expected costs of default were defined in (3.34), where the probability of default was

\[
p^d = \frac{1}{2} \sigma^2 b + cD.
\]  

(3.54)

The regulator sets a target level of this probability to be a constant equal to \( \bar{p}^d \), similar to what is set out under Basel II and III. From (3.54), there is a tradeoff between risk \( \sigma \) and leverage \( D \) for a constant \( p^d \). For a bank to comply with \( \bar{p}^d \), any increase in \( \sigma \) must be compensated by a decrease in \( D \) and vice versa. By totally differentiating (3.54) and setting it to 0, we obtain the following negative slope:

\[
0 = \sigma b \frac{d \sigma}{dD} + c \frac{dD}{dD} = - \frac{c}{\sigma b X}.
\]  

(3.55)

The downward sloping line labeled \( \bar{p}^d \) in Figure 3.6 illustrates the tradeoff between risk and leverage that this choice of a given default probability implies. Given \( \bar{p}^d \), a bank may choose higher \( \sigma \) if leverage \( D \) is lower. A higher (lower) target default probability corresponds to an

---

15The regulator’s imposition of a target probability of default is a simple way of capturing bank-regulator interactions in the context of capital requirements, as in Boyson et al. [2016]. This target may be derived from the regulator wanting to impose socially optimal risk levels, rather than privately optimal ones, as in Kashyap and Stein [2004].

16The internal ratings-based approach set forth in Basel II and III links capital requirements to the credit losses (and the probability that these losses occur) that regulators are willing to accept. This probability can be construed as the probability of default of a bank.
upward (downward) shift in the downward sloping line in Figure 3.6.

Figure 3.6: Bank’s risk curve against regulator’s chosen probability of default

We turn now to the bank’s reaction function. In Section 3.6.1, we have shown that there is a positive relationship between a bank’s leverage and choice of risk levels, because the bank’s risk-shifting incentives increase with leverage. We can draw a reaction curve \( RC \) that shows the bank’s best risk choice as leverage changes. \( RC \) can be interpreted as the reaction of the Stackelberg follower. As a benchmark, we first derive the bank’s \( RC \) for a given leverage \( D \). By totally differentiating the bank’s first-order condition in (3.36), we obtain the condition that the bank must obey if it wants to maximize the value of its residual equity:

\[
0 = V_D [R, D] \ dD + (V_\sigma [R, D] - bX) \ d\sigma
\]

\[
\left. \frac{d\sigma}{dD} \right|_{RC} = \frac{V_D [R, D]}{bX - V_\sigma [R, D]}, \quad (3.56)
\]

which is positive. \( RC \) is also illustrated in Figure 3.6. The representation is very much simplified: we draw the curves as linear, but it is only the slopes of the curves that are important.

The regulator can also set capital requirements (leverage) \( D \) in addition to \( p^d \), which when combined with the bank’s reaction curve, forces a bank to choose a particular level of \( \sigma \). At issue then is how the Stackelberg leader (regulator) picks the right point off that curve by imposing capital requirements or equivalently in our setup, the maximum amount of leverage \( D \). To a regulator, there is a tradeoff between risk and leverage. Imposing a maximum leverage \( D_3 \) will allow the regulator to accept leverage of at most \( \sigma_2 \), if the target is \( p^d \). However, to a bank, risk and leverage reinforce each other, as reflected in the slope of the reaction curve. Therefore, it will choose a low level of risk, say \( \sigma_3 \), meaning that the bank takes too little risk relative to that which is considered optimal by the regulator, as Point 3 lies on \( p^d < \overline{p^d} \).
Similarly, if the regulator imposes a maximum leverage \( D_2 \), the optimal risk from her viewpoint
is $\sigma_3$. The bank’s reaction curve implies that it will choose $\sigma_2$, which is now too much risk compared to what the regulator deems optimal, as Point 2 lies on $p^d < \overline{p}^d$. Only if the regulator imposes leverage $D_1$ will the bank choose a risk level $\sigma_1$ that is compatible with the $\overline{p}^d$ specified by the regulator, at the intersection of the $\overline{p}^d$ and $RC$ lines: Point 1 is the equilibrium solution to the Stackelberg game between the regulator and the bank. This example shows that the regulator must keep a bank’s reaction curve in mind when setting capital requirements.

### 3.6.2 Replacing subordinated debt with CoCos

While a bank is always able to meet a leverage requirement with both deposits $D_d$ and subordinated debt $D_s$, the regulator can only force a bank to choose her desired risk level when $D_s$ cannot be bailed in, written down, or converted to equity. This is because the ability to eliminate all or part of $D_s$ changes a bank’s reaction curve, meriting further attention. Consider now what happens when, possibly in response to the recent change in capital standards, subordinated debt is replaced by CoCos. In Section 3.6.2, we have shown that CoCos have risk-shifting incentives which differ from subordinated debt, because of the expected wealth transfers. Therefore, a CoCo-issuing bank’s first order condition for a given debt $D$ should take the risk-shifting incentives into account, as in (3.57):

$$V[R, D] + RSI = \sigma bX.$$  \hspace{1cm} (3.57)

This means that replacing subordinated debt by CoCos necessarily alters the reaction curve of a bank, because of the additional $RSI$ term, which involves both $\sigma$ and $D$ as well. If we totally differentiate $RSI$ with respect to both parameters, we obtain

$$\frac{0}{dD} = \frac{\partial RSI}{\partial \sigma} d\sigma + \frac{\partial RSI}{\partial D} dD$$

$$\frac{d\sigma}{dD} = -\frac{\partial RSI}{\partial \sigma}.$$  \hspace{1cm} (3.58)

For a CoCo with positive $RSI$ (such as PWD and nondilutive CE CoCos), (3.58) is positive, because the risk-shifting incentive is increasing in leverage (less skin in the game implies higher gambling incentives) and decreasing in risk (diminishing marginal returns). Of course, for a CoCo with negative $RSI$ (dilutive CE CoCos), (3.58) is negative.

Consider first PWD and nondilutive CE CoCos. Let $RC'$ denote the reaction curve drawn using (3.57). Since the risk-shifting incentive is positive, the reaction curve $RC'$ must lie above that of $RC$. Figure 3.7 represents the change simply as an upward twist in the slope.
So suppose that the regulator has chosen the probability of default $p^d$ and has imposed leverage $D_1$ on the banks, i.e. Point 1 in Figure 3.7, as in the benchmark case. Then, suppose for the sake of increasing loss absorption capacity, $D_3$ subordinated debt is completely replaced with either a PWD or a nondilutive CE CoCo. This change causes the reaction curve to twist up from $RC$ to $RC'$. As the bank did not change its leverage ratio, it still has $D_1$ leverage, but because of the potential wealth transfer brought about by the change from subordinated debt to equity, the risk incentives are higher: the bank’s position is now at Point 2, where leverage is at $D_1$ but risk choice is at $\sigma_2 > \sigma_1$. What should the regulator do in this situation? At Point 2, the risk level $\sigma_2$ and leverage $D_1$ combination implies a probability of default which is higher than $p^d$. To get back at $p^d$ for risk level $\sigma_2$, she should impose higher capital requirements (lower leverage) $D_2$, as indicated in Figure 3.7. But raising capital requirements by an additional $D_1 - D_2$ in turn leads to a lower risk choice of $\sigma_3$, which now implies a probability of default below $p^d$, and so on. The new set of equilibrium values is at Point 4, with a higher risk choice than at Point 1 but a correspondingly larger loss absorption capacity because of the associated higher capital requirement.

**Proposition 3.12.** When PWD and nondilutive CE CoCos are used by banks in their capital structure in place of subordinated debt, regulators should increase capital requirements if they want banks to choose risk levels that are consistent with the regulators’ own preference.

So given that subordinated debt only qualifies as Tier 2 capital under Basel III, it is arguable that PWD CoCos should not have been included as Additional Tier 1 equity regardless of the trigger level, because PWD CoCos lead to higher risk-shifting incentives. As conversion of a writedown CoCo wipes out a junior creditor, it allows the shareholder/manager to jump the
seniority ladder. Therefore, they will not act in a safer manner even when compared with the case where these instruments are subordinated debt instead. Much of the CoCos issued between 2013 to 2015 have done just that, replace expiring subordinated debt.

The situation is better when dilutive CE CoCos are considered, because the movement of the expected wealth transfer is away from the shareholder to the CoCo holder. Relative to subordinated debt, the same amount of CoCos have an additional term, $RSI$. The $RSI$ for CE CoCos fall as the dilution parameter $\psi$ increases, and are negative for $\psi < \tilde{\psi}$. Therefore, combining (3.57) and (3.58) for a negative value of $RSI$, the $RC$ twists downwards to some $RC''$ instead of upwards. Figure 3.8 shows this other case.

Figure 3.8: Downward shift of the risk curve due to replacing subordinated debt by dilutive CoCos

As with the other case, suppose that the regulator has chosen the probability of default $\hat{p}^d$ and has imposed leverage $D_1$ on the banks, i.e. Point 1 in Figure 3.8, as in the benchmark case. Then, suppose for the sake of increasing loss absorption capacity, $D_s$ subordinated debt is completely replaced with a dilutive CoCo. This change causes the reaction curve to twist down from $RC$ to $RC''$. The fall in the reaction curve for a given leverage $D_1$ actually causes the bank’s risk choice to fall from $\sigma_1$ to $\sigma_2$, in contrast to if the reaction curve twists upwards. To reach Point 4 in Figure 3.8, the regulator actually has to lower capital requirements to induce banks to take the optimal level of risk given $RC''$ and $\hat{p}^d$, which is $\sigma_4$. Seen this way, dilutive CoCos are a legitimate component of Additional Tier 1 capital, because they induce banks to choose lower risk levels for a given leverage $D$.

**Proposition 3.13.** When dilutive CE CoCos are used by banks in their capital structure in place of subordinated debt, regulators may decrease capital requirements if they want banks to choose risk levels that are consistent with their own preference.
3.7 Conclusion

CoCos have become popular among banks since the emergence of Basel III and the Total Loss Absorption Capacity (TLAC) Standard by the Financial Stability Board. The reason is that CoCo conversion enhances loss absorption capacity by reducing the bank’s leverage. However, an unintended consequence of this feature is that a wealth transfer occurs between the CoCo holders and the original shareholders when the conversion takes place. The wealth transfers may encourage the issuing bank to make conversion more likely. In this paper, we have looked at the implications of these wealth transfers on the issuing bank’s risk-shifting incentives, relative to the same amount of subordinated debt.

By writing the issuing bank’s residual equity as a linear combination of the pre-and post-conversion states, with the probability of conversion as the weighting factor, we were able to express the residual equity as one of a bank that has issued subordinated debt, plus an expected wealth transfer. The expected wealth transfer is the product of the wealth transfer and the conversion probability. While the literature has paid attention to the wealth transfer, it has largely taken the conversion probability as exogenous. We have endogenized this probability, as we recognize that this is influenced by a bank’s risk choices.

The expected wealth transfer is affected by risk in two conflicting ways. First, higher risk levels increase the probability of conversion, which increases the expected wealth transfer. Effectively, this allows the shareholder to make conversion more likely. Second, the gains from the wealth transfer decrease as risk increases. In short there is a diminishing marginal gain in wealth transfers as risk increases, as the bank’s skin in the game rises upon conversion. Unfortunately, the positive first effect dominates the negative second effect when initial risk levels or leverage ratios are sufficiently high, which are the circumstances that should give regulators cause for concern.

We have shown that the strength of the risk-shifting incentives is strongly influenced by CoCo design. As PWD CoCos and nondilutive CE CoCos always transfer wealth to equity holders upon conversion, the risk-shifting incentive is positive. On the other hand, dilutive CE CoCos transfer wealth from equity holders to CoCo holders. The threat of dilution results in negative risk-shifting incentives relative to subordinated debt. The risk-shifting incentives act as a wedge in a bank’s optimization problem, such that the optimal risk choice is different from that under the same amount of subordinated debt. For PWD CoCos and nondilutive CE CoCos, the risk choices are higher than under the same amount of subordinated debt, while for dilutive CE CoCos, it is lower.

These results naturally lead to further questions concerning capital requirements. A corollary of our results is that the interaction between capital requirements and asset-side portfolio
risk must be carefully considered whenever amendments are made to existing policies. If Co-Cos are to continue to play an important role in the capital structure of banks, the level of capital requirements should also depend on how they are met. In that vein we have shown that some of the disadvantages of nondilutive CoCos can be offset by raising the bar higher: if inappropriate CoCo design increases risk taking incentives, that effect can be counteracted by requiring more skin in the game, i.e. by setting the requirement ratios higher than they are set for the case of pure equity or sufficiently dilutive CoCos.

These results are important in setting regulations. Basel III and the TLAC Standard were written with the focus on increasing loss absorption capacity of the financial system. To a substantial extent, this loss absorption capacity is being filled by CoCos, in particular for meeting TLAC requirements. But to achieve a more robust financial system, it is not enough to only consider loss absorption capacity. We must also consider regulation that prevents banks from choosing excessively risky actions in the first place, as the designers of Basel II fully realized when introducing risk weights. Capital regulation is meant to force banks to put more skin in the game in order to reduce risk-shifting incentives, and not just to increase loss absorption capacity for given risk levels. While CoCos are hybrids of debt and equity, it doesn’t always mean that the risk levels they induce will be between those induced by debt and equity. As we have shown, not all CoCos are created equal - some have higher risk-shifting incentives than others. At the very least, the type of CoCo that is allowed to fill in Additional Tier 1 capital requirements should be restricted to equity converters, and among those only CE CoCos which are sufficiently dilutive. In this way, one minimizes the chance that the loss absorption capacity has to be used in the first place.

Appendix for Chapter 3

3.A Mathematical foundations: the call option function and its derivatives

Denote a call option with strike price $D$ and expected return $R$ as $C \left[ R, D \right]$. The full expression for $C \left[ R, D \right]$ is

\[
C \left[ R, D \right] = \exp \left( -r \right) \left[ R \exp \left( r \right) \Phi \left( d_1 \right) - D \Phi \left( d_2 \right) \right] \\
= R \Phi \left( d_1 \right) - \exp \left( -r \right) D \Phi \left( d_2 \right)
\]
where \( r \) is the risk-free rate, \( \Phi(\cdot) \) is the cumulative density function of the standard normal distribution, \( d_1 = \frac{1}{\sigma} \left[ \ln \frac{R}{D} + r + \frac{1}{2} \sigma^2 \right] \) and \( d_2 = \frac{1}{\sigma} \left[ \ln \frac{R}{D} + r - \frac{1}{2} \sigma^2 \right] \). We use the following first and second-order partial derivatives of \( C [R, D] \) in the chapter.

### 3.A.1 Vega

Vega is the sensitivity of the option value with respect to the volatility of its underlying assets. It is calculated by taking the derivative of the call option with respect to volatility \( \sigma \):

\[
V [R, D] = \frac{\partial C [R, D]}{\partial \sigma} = R \phi (d_1) > 0
\]

where \( \phi (\cdot) \) is the probability density function of the standard normal distribution.

### 3.A.2 \( C_D \): The derivative of the call option with respect to the strike price \( D \)

\[
C_D = \frac{\partial C [R, D]}{\partial D} = -\exp (-r) \Phi (d_2) < 0
\]

### 3.A.3 \( V_\sigma \): The second-order derivative of \( C [R, D] \) with respect to \( \sigma \)

The second-order derivative of \( C [R, D] \) with respect to \( \sigma \) is the first-order derivative of vega with respect to \( \sigma \). We refer to this shorthand as \( V_\sigma \) in the text.

\[
V_\sigma = \frac{\partial^2 C [R, D]}{\partial \sigma^2} = \frac{\partial V [R, D]}{\partial \sigma} = R \phi' (d_1) \frac{\partial d_1}{\partial \sigma} = -R \phi (d_1) d_1 \left( 1 - \frac{d_1}{\sigma} \right)
\]

which is negative for values of \( \sigma^2 > 2 \left( \ln \frac{R}{D} + r \right) \).

### 3.A.4 \( V_D \): The cross-order partial derivative of \( C [R, D] \) with respect to \( \sigma \) and \( D \)

The cross-order partial derivative of \( C [R, D] \) with respect to \( \sigma \) and \( D \) is also the first-order derivative of vega with respect to the strike price \( D \). We refer to this shorthand as \( V_D \) in the main text.

\[
V_D = \frac{\partial^2 C [R, D]}{\partial \sigma \partial D} = \frac{\partial V [R, D]}{\partial D} = R \phi' (d_1) \frac{\partial d_1}{\partial D} = -R \phi (d_1) d_1 \left( -\frac{1}{\sigma D} \right) = \frac{R}{D} \phi (d_1) \frac{d_1}{\sigma} > 0
\]
Note that $V_D$ can be written in terms of $V_\sigma$ as follows:

$$V_D = -\frac{V_\sigma}{D (\sigma - d_1)}$$

which is positive whenever $\sigma^2 > 2 (\ln \frac{R}{D} + r)$, precisely the same condition that keep $V_\sigma < 0$. Moreover, $V_D \to 0$ as the gap between $\sigma$ and $d_1$ widens: as $\sigma$ increases, $V_D$ shrinks to 0. For a given $\sigma$, $V_D$ goes to zero as $D$ rises. We refer to $\sigma^2 > 2 (\ln \frac{R}{D} + r)$ and the widening of the gap between $\sigma$ and $d_1$ as high fragility conditions: $\sigma^2 > 2 (\ln \frac{R}{D} + r)$ is necessary but not sufficient.

### 3.B Proofs for various results in the chapter

#### 3.B.1 Proof that $\frac{\partial^2 p^c}{\partial \tau \partial \sigma} < 0$

$$\frac{\partial^2 p^c}{\partial \tau \partial \sigma} = \frac{\partial}{\partial \sigma} \left( \frac{\partial p^c}{\partial \tau} \right) = \frac{\sigma (1 - \tau) \phi (-d_c) d_c \left( \frac{\partial d_c}{\partial \sigma} \right) - \phi (-d_c) (1 - \tau)}{\sigma^2 (1 - \tau)^2}$$

$$= \frac{\phi (-d_c) (1 - \tau) \left[ \sigma d_c \frac{\partial d_c}{\partial \sigma} - 1 \right]}{\sigma^2 (1 - \tau)^2}$$

$$= \frac{\phi (-d_c) (1 - \tau) \left[ -\sigma d_c \left( 1 + \frac{d_c}{\sigma} \right) - 1 \right]}{\sigma^2 (1 - \tau)^2}$$

$$< 0$$

#### 3.B.2 Proof that $WF_{pwd} \to 0$

The risk-shifting incentive for a PWD CoCo (3.17) has two terms: the conversion probability factor $CF_{pwd}$ and the wealth transfer factor $WF_{pwd}$. $WF_{pwd}$ can be rewritten as the difference between the vegas of two call options that differ only in the strike price. Using the definition of vega from A.1 and $V_D$ from A.4, we can use the mean value theorem to rewrite this difference as follows:

$$WF_{pwd} = p^c \left( V \left[ R, D_d + \varphi D_s \right] - V \left[ R, D_d + D_s \right] \right)$$

$$= -p^c \left( (1 - \varphi) D_s V_D \left[ R, D' \right] \right)$$
for some \( D' \in [D_d + \varphi D_s, D_d + D_s] \). In A.4, we have noted that when the high fragility condition holds, we have that \( V_D \) goes to zero, such that \( WF_{pwd} \) goes to zero as well.

### 3.B.3 Impact of \( \varphi \) on the risk-shifting incentives of PWD CoCos.

Since \( C [R, D_d + D_s] \) and \( V [R, D_d + D_s] \) are not functions of \( \varphi \), we may express (3.28) as

\[
\frac{\partial RSI_{pwd}}{\partial \varphi} = \frac{\partial p^c}{\partial \sigma} \frac{\partial [C [R, D_d + \varphi D_s]]}{\partial \varphi} + p^c \frac{\partial V [R, D_d + \varphi D_s]}{\partial \varphi}
\]

Line 2 follows from the fact that \( \varphi \) is a variant of \( D \), enabling us to use the chain rule to link \( D \) and \( \varphi \). A.2 and A.4 describe how to differentiate \( C [\cdot] \) and \( V [\cdot] \) with respect to \( D \). The notations \( d_1^* \) and \( d_2^* \) indicate that the functions \( d_1 \) and \( d_2 \) were evaluated at strike price \( D_d + \varphi D_s \) instead of a generic strike price \( D \).

### 3.B.4 Proof that \( \tilde{\psi} < \bar{\psi} \)

The equation for \( RSI_{ce} \) is

\[
RSI_{ce} = \frac{\partial p^c}{\partial \sigma} \left( C [R, D_d] - C [R, D_d + D_s] \right) + p^c \left( V [R, D_d] - V [R, D_d + D_s] \right).
\]

In Section 3.4.3 we have found that \( \psi = \bar{\psi} \) sets the wealth transfer to 0, implying that \( CF_{ce} = 0 \), while \( WF_{ce} \) remains negative. As \( CF_{ce} \) and \( WF_{ce} \) are generally of opposite signs, we need only choose a \( \psi \) that makes \( CF_{ce} \) positive and exactly offsets the negative value of \( WF_{ce} \). In other words, choose \( \psi \) such that \( p^c \left( \frac{V [R, D_d]}{1 + \psi D_s} - V [R, D_d + D_s] \right) = \frac{\partial p^c}{\partial \sigma} \left( \frac{C [R, D_d]}{1 + \psi D_s} - C [R, D_d + D_s] \right) \). Let us call this value \( \tilde{\psi} \). We claim that \( \tilde{\psi} < \bar{\psi} \). The expression for \( \tilde{\psi} \) is

\[
\tilde{\psi} = \frac{1}{D_s} \left( \frac{C [R, D_d]}{C [R, D_d + D_s]} - 1 \right).
\]
On the other hand, the expression for $\tilde{\psi}$ is

$$\tilde{\psi} = \frac{1}{D_s} \left( \frac{\partial \rho}{\partial \sigma} C[R, D_d] + \rho V[R, D_d] \right) \left( 1 - \frac{1}{D_s} \left( C[R, D_d] \left( \frac{\partial \rho}{\partial \sigma} + \rho \frac{V[R, D_d]}{C[R, D_d]} \right) \right) \right).$$

$\tilde{\psi} = \tilde{\psi}$ if and only if $V[R, D_d + D_s] = V[R, D_d + D_s]$. However, we can write $V[R, D_d + D_s] C[R, D_d + D_s]$ as follows:


where $V_D$ and $C_D$ are the derivatives of vega and the call option value with respect to the strike price, respectively. The inequality follows from $C_D < 0 < V_D$: the value of a call option falls when the strike price rises, while the vega of a call option rises when the strike price rises. Therefore we have shown that $\tilde{\psi} < \tilde{\psi}$, as claimed.

3.B.5 Derivation of $\sigma^*_s$ in terms of $\sigma^*_e$

We use the mean value theorem to write $V[R, D_d + D_s] |\sigma^*_s$ in terms of $V[R, D_d] |\sigma^*_e$, using $V_\sigma$ and $V_D$:

$$V[R, D_d + D_s] |\sigma^*_s = V[R, D_d] |\sigma^*_e + (V_\sigma |\sigma^*_e) \ (\sigma^*_s - \sigma^*_e) + (V_D |D_d) \ D_s,$$

enabling us to write the first order conditions as

$$V[R, D_d] |\sigma^*_e + (V_\sigma |\sigma^*_e) \ (\sigma^*_s - \sigma^*_e) + (V_D |D_d) \ D_s = (\sigma^*_s - \sigma^*_e) bX + \sigma^*_e bX. \ (3.60)$$

Subtracting (3.38) from (3.60) leads to

$$\sigma^*_s = \sigma^*_e + \frac{V_D |D_s}{bX - (V_\sigma |\sigma^*_e)} > \sigma_e.$$

We assume that the default coefficient $bX$ is large enough such that $bX - V_\sigma > 0$. Actually, from A.3, $V_\sigma < 0$ whenever $\sigma^2 > 2 \left( \ln \frac{R}{D} + r \right)$ holds, so the assumption that $bX > V_\sigma$ is always justified, as our analysis assumes it.
3.B.6 Derivation of $\sigma^*_pwd$ and $\sigma^*_ce$ in terms of $\sigma^*_s$

We may use the mean value theorem to rewrite (3.46), the first order condition of a PWD CoCo:

$$V[R, D_d + D_s] | \sigma^*_pwd + RSI_{pwd} = V[R, D_d + D_s] | \sigma^*_s + (V|\sigma^*_s) (\sigma^*_pwd - \sigma_s) + RSI_{pwd}.$$  

If we subtract (3.42) from it, we obtain

$$\sigma^*_pwd = \sigma^*_s + \frac{RSI_{pwd}}{bX - (V|\sigma^*_s)} > \sigma^*_s.$$  

Similarly, we may use the mean value theorem to write (3.48), the first order condition of a CE CoCo:

$$V[R, D_d + D_s] | \sigma^*_ce + RSI_{ce} = V[R, D_d + D_s] | \sigma^*_s + (V|\sigma^*_s) (\sigma^*_ce - \sigma^*_s) + RSI_{pwd}.$$  

If we subtract (3.42) from it, we obtain

$$\sigma^*_ce = \sigma^*_s + \frac{RSI_{ce}}{bX - (V|\sigma^*_s)} > \sigma^*_s.$$

83
Chapter 4

Regulatory Forbearance in the Presence of CoCos\(^1\)

4.1 Introduction

In the banking literature, regulatory forbearance shows up in many forms. There is always a tradeoff between toughness and softness, but the context differs. It may show up as keeping a bank open when it is better to close it down. It may also show up as regulators having to intervene in one form or another such as by lowering interest rates, rolling over loans, injecting capital, or bailing out banks whenever they are at the verge of insolvency in order to avoid a financial system meltdown. Banks know that by collective action, they may force the regulator to forbear on tough decisions. However, observable regulatory actions may have unintended consequences, such as sending signals when they are not wanted. As a result, many attempts have been made to find out how the regulator is able to commit to tough solutions. Reputation-saving is one driver, but rules-based regulation is another. Of particular note is the introduction of CoCos in order to commit the regulator to an intervention when it is necessary, by letting conversion be driven not by their own decisions, but by publicly observable market-based measures such as share prices.

One of the positive points of CoCos is to improve a bank’s loss absorption capacity in the event of a crisis. However, for this to happen, CoCos must first be converted. In general, CoCos convert in one of two ways: when the bank’s book-based or market-based equity ratio falls below a prespecified threshold, or when the regulator decides that the bank is close to the point of nonviability. Regardless of the design of the CoCo, the bank’s skin in the game upon conversion increases. But conversion is not without consequences. For instance, in Chan and

\(^1\)This chapter is based on Chan and van Wijnbergen [2017b].
van Wijnbergen [2014], we argued that conversion would lead to a possible increase in the probability of bank runs because conversion is a public matter and sends a signal about asset quality. For this reason, regulators may be hesitant to force conversion even when necessary, as they shoulder costs of conversion.

As it is not always true that CoCos convert automatically, it may fall short of being both a disciplining device for the banks and a commitment device for regulators. In this paper, we take the stance that CoCo conversion is also vulnerable to regulatory forbearance. This is because while a conversion improves a bank’s incentives by increasing its skin in the game as its loss absorption capacity increases, it also exposes the regulator to conversion costs. The regulator has to weigh these against the increased social welfare from improving a bank’s incentives. We show that if the regulator’s cost of conversion is high enough, then she will forbear on the conversion. Forbearing on conversion means that CoCos are not going to be useful for improving loss absorption. On the other hand, converting too readily may encourage ex ante risk shifting,\(^2\) unless there are very few CoCos, in which case they are also not going to significantly improve loss absorption capacity.

We illustrate these ideas with a three-period model, where we allow the bank to have actions at two points in time: the initial asset choice at \(t = 0\), and a choice between gambling for resurrection and liquidating assets at \(t = 1\). The regulator cannot directly control the initial choice of assets, but may be able to influence a bank’s decision towards the socially optimal one by causing CoCo conversion. This is possible because CoCos improve the issuing bank’s skin in the game. However, the circumstances when that occurs are very limited. In particular, there is a threshold level of skin in the game that the bank must exceed in order to be induced to make socially optimal choices. We find that there are times that even a conversion is not enough to bring the bank’s skin in the game up to the threshold. In such cases, one will observe regulatory forbearance, as conversion will only incur costs without bringing social benefits.

We endogenize the regulator’s cost of conversion by adding in a simple updating model of depositors’ beliefs. That is, since conversion is publicly observable, all the agents, including the depositors, are made aware of this information. The assumption that safe assets will never induce conversion means that conversion definitely indicates that the bank has invested in the risky assets at the very beginning, and increases doubt in the depositors’ minds as to the bank’s survival. This causes the threshold required belief in the return of the risky asset to increase.

The bank’s \(t = 0\) decision ultimately depends on its expectations regarding the level of conversion costs faced by the regulator. In a setting with imperfect information regarding the

\(^2\)This depends on the design of the CoCo. For instance, convert-to-equity CoCos may be sufficiently dilutive as to prevent this.
regulator’s costs of conversion, we show that the only way that safe assets would be chosen at \( t = 0 \) is if the CoCo issuance is sufficiently small. However, doing so negates the increase in loss absorption capacity that CoCos are intended to have. This highlights the tradeoff between inducing a safe choice ex ante, and increased loss absorption capacity ex post.

4.2 Review of related literature

Mailath and Mester [1994] is an early paper on the discretionary power of regulators. Central to their paper is that closure is deemed a major instrument of bank regulation, which may deter banks from making risky asset choices. The regulator’s decisions involve accounting for opportunity costs of the asset that are foregone if the bank is closed down. For a cost-minimizing regulator, this leads to conflicts between what is privately optimal for the regulator and what is socially optimal. In this sense, regulators may not always be welfare-enhancing, as their presence leads the banks to choose actions that are not first best.

The standard view is that regulatory discretion encourages moral hazard on the part of the banks but Cordella and Yeyati [2003] show that a regulator’s commitment to bailing out institutions during a crisis may reduce the possibility of moral hazard from banks. In their model, banks are maximizers of their charter value. If the bank’s charter value is sufficiently high, the shareholders will choose safer assets. A shock will naturally decrease a bank’s charter value, which may encourage risk-taking. In this situation, a regulator’s commitment to bail out the bank will automatically increase the bank’s probability of survival, which increases the charter value, and leads to safer choices after the shock. However, this will only hold if the bailout policy is contingent on the realization of the state of nature.

Another strand in the regulator discretion literature is that of forbearance being done to manage information. Morrison and White [2013] consider the case where regulatory forbearance may come about as a result of regulators trying to manage their reputation. In their model, the regulator can imperfectly screen the quality of banks, and grant banking licenses based on their findings. The proportion of sound banks is therefore a direct consequence of the regulator’s screening ability. Closure occurs only if auditing leads to evidence about the poor state of the bank. If the regulator decides to close down a bank, it causes the agents in the economy to infer that the regulator’s screening technology is bad. If the initial level of the regulator’s reputation is low enough, closure may lead to contagious bank failures, as the belief about the proportion of sound banks is affected as well. Therefore, even though closing down the bank

\(^3\)A bailout in their context is to provide the troubled bank with sufficient funds to repay the depositors and carry on operations.
may be a socially better option than keeping it open, regulators will choose to forbear because of the potential damage to their reputation as a good auditor, and the subsequent impact on the financial system.

Closure is not the only action that can be done by regulators. Shapiro and Skeie [2015] explore the impact of including bailouts as part of the regulatory toolkit. In their model, the regulator has three actions: liquidate, forbear, or bail out. Similar to Morrison and White [2013], the action that the regulator takes inadvertently gives a signal to the depositors about the quality of the bank. However, in the setup of Shapiro and Skeie [2015]), the preference of the regulator will depend upon her cost of injecting capital. A high cost regulator will never inject capital, and so will only choose between liquidation and forbearance. On the other hand, a low cost regulator will choose between bailouts and forbearance. In their model, the risk-shifting tendencies of banks will depend on their assessment of the type of the regulator they are dealing with. However, it turns out that regulators can take advantage of the uncertainty about their type to discourage bank misbehavior and manage depositor expectations at the same time.

In addition to liquidation, forbearance and bailouts, another regulatory tool is the bail in of outstanding liabilities. Walthar and White [2016] explore the regulator’s dilemma on choosing the optimal amount of bail-in, while abstracting from the bank’s risk-shifting incentives. In their model, the amount of bail-in triggered by the regulator is discretionary but publicly observable. This directly affects both the depositors’ decision to withdraw, and the outside buyers’ market valuation of the bank’s assets. They reason, like Chan and van Wijnbergen [2014], that runs occur upon the revelation of previously hidden negative information about the bank’s asset quality. As the regulator knows this, she has an incentive to hide bad information by bailing in an amount that is consistent with if she had obtained good information. However, not bailing in an sufficient amount is detrimental to social welfare. Walthar and White [2016] argue that if regulators were able to credibly commit to an optimal bail in rule based on public information, they can be tough without provoking runs, as public information does not necessarily reveal private information. They champion CoCos as a way to sidestep this pooling problem. However, CoCos do not necessarily convert automatically, as most of them have discretionary triggers within the control of the regulator, so that undermines their role as a commitment device.

Acharya and Yorulmazer [2007] examine the tradeoffs that a regulator faces when the banks in the system are able to herd by choosing to invest in similar industries. The banks are incentivized to choose low correlation of investment, if the surviving bank is allowed to purchase the failing bank. It is is still possible for both banks to fail even if they invested in different assets,
but if both banks fail, the regulator must choose between bailing out the two banks itself, or letting both banks be acquired by an outside investor. The problem of the regulator arises from the bailout option: if the costs of bailout is not very high, then the regulator’s declaration to choose the acquisition option is not credible, because by assumption, letting outside investors take over the bank lead to efficiency losses. They show a region of time inconsistency, where the regulator is able to credibly induce banks to choose low correlation ex ante, but upon the occurrence of the both-fail state, the regulator will choose to bail out the banks in the end. However, they do not look into effects on future reputation, they use a one-shot game setup.

Farhi and Tirole [2012] explore the time inconsistency problem in the context of a maturity mismatch by banks. The regulators bail out the banks in their model by setting very low interest rates. They highlight the potential cost faced by the regulator in losing credibility. However, doing so is always at the expense of the nonbank agents. Like Acharya and Yorulmazer [2007], Farhi and Tirole [2012] find that the lack of commitment by the central bank on interest rate setting creates moral hazard in banks depending on the expectation of banks regarding the stance of the central bank. If they expect that the central bank will adopt a tough stance, banks will choose to hoard liquidity, while if the central bank is expected to have a soft stance, the banks will collectively choose to incur maturity mismatch as the optimal strategy at that point of a regulator is to bail out all the banks at once. They show that imposing capital regulation is a means of curtailing the bank’s mismatch situation when the regulator has limited commitment.

4.3 The model

We are interested in the factors that affect a regulator’s decision on CoCos, when the CoCos have a discretionary trigger. In order to do so, we use a model where the bank and the regulator take turns in making decisions about where to invest and whether to forbear. Figure 4.1 illustrates the timeline of events of our model.
In this game, we take the bank’s capital structure as a given. We give the bank two consecutive chances to commit moral hazard after obtaining its funding: one on the choice between a safe and a risky asset, and another on the choice to gamble for resurrection or liquidate the bad fraction of the risky asset. The bank’s second decision depends upon whether its debt level would surpass a certain threshold. This is where CoCos are potentially useful, as when the regulator forces conversion, the bank may be able to surpass the threshold debt level. Therefore, even if there are two types of CoCos in practice, we only consider the type that is written off the issuing bank’s balance sheet, as the bank’s liability after a conversion will be the same regardless of the type of CoCo issued. In turn, the bank’s initial choice between the safe and the risky asset depends on the proceeds from the second decision.

CoCos convert when the bank’s equity ratio falls below a threshold ratio. To justify conversion, we introduce shocks on the probability of obtaining good returns of the risky asset into the model. We examine the no-shock case (henceforth referred to as the benchmark case) before cases that involve shocks in order to focus on the essential drivers of regulatory forbearance. Also, even though the bank raises its funds from both depositors and CoCo holders, we initially abstract away from the possibility of bank runs to focus on the interaction of the bank and the regulator. But since conversion is publicly observable, it may alter the beliefs held by the creditors of the bank and lead to runs. We address this issue in a later section. The regulator needs to take the changes in these beliefs into account in deciding whether to convert CoCos. At the same time, CoCo conversion alters the bank’s capital structure, which means that conversion may be used by the regulator to nudge banks into performing socially

---

4One of them is the principal writedown CoCo, where the CoCos are fully or partially written off upon the occurrence of a trigger event. The other is the equity-converting CoCo, where the CoCos are converted to equity at a prespecified ratio, and may be dilutive to the original shareholders.
optimal actions ex ante and ex post.

4.3.1 Setup

We develop a stylized three-period model, setting up a game between a regulator and a CoCo-issuing bank. The bank moves first, followed by the regulator, and then the bank again. During the regulator’s turn, she can decide whether to convert the CoCos or to forbear on conversion. We model it this way as we believe that CoCo conversion is at the discretion of the regulator, even in the presence of automatic conversion clauses, because of the discretion unavoidably embedded in accounting rules. This is the more relevant case because none of the CoCos issued to date have a market-based trigger. Figure 4.2 illustrates the game.

Figure 4.2: Interactive Game Between Bank and Regulator

- **Period $t = 0$**

At $t = 0$, the bank raises funds from a continuum of risk neutral creditors: (wholesale) depositors who collectively invest $D$, and CoCo holders who invest a total of $C$. In addition, the bank’s owner-manager invests $E$ of his own equity. The initial amounts are normalized such that $D + C + E = 1$. We do not delve into the optimal capital structure as our focus is on the interaction between the banker and the regulator for a given capital structure. Moreover, banks

---

5The model builds on Homar and van Wijnbergen [2016], but they do not put it in a game-theoretic context.

6All CoCos issued since the BIS published its new capital definition rules must have a point of nonviability clause under which the regulator can force conversion if the CoCo is to qualify as Additional Tier 1 capital.

7By wholesale depositors, we mean those that are not covered by deposit insurance. One may also think of them as holders of other forms of short term funding that are susceptible to rollover risk, such as commercial paper.
are subject to capital regulation, and as such, maybe unable to choose their capital structure optimally, at least, not instantly.

To entice the depositors and the CoCo holders to invest their money in the bank, they are promised a return \( r > 1 \) at \( t = 2 \).\(^8\) There is no deposit insurance, but we assume that depositors hold beliefs regarding the bank’s prospects such that the depositors’ participation constraints are assumed to be satisfied. That is, we assume that on average, depositors break even based on their own beliefs. As a result, the depositors are passive agents in the model.

Upon receiving funds from the aforementioned agents, the bank arrives at its first decision point: the choice of where to invest these funds at \( t = 0 \). For simplicity, there are only two available assets for the bank: a safe one and a risky one. The safe asset delivers a return \( R_s \) with certainty, and is enough to pay off the amounts promised to depositors and CoCo holders at \( t = 2 \). That is,

\[
R_s - r (D + C) > 0. \tag{4.1}
\]

The risky asset is a portfolio of \( 1 - q \) good loans, and \( q \) bad loans. We assume that the banks take the proportion \( q \) as given at \( t = 0 \). The good loans yield \( R_r > R_s \) with certainty, while the bad loans only yield \( R_r \) with probability \( p \) and 0 otherwise. Thus, the expected return of the risky asset at \( t = 0 \) is

\[
(1 - q) R_r + qpR_r = (1 - q + qp) R_r. \tag{4.2}
\]

Let \( s = 1 - q + qp \). Since \( s \in [0, 1] \), we can treat it as a composite probability, though artificially constructed. From this point onwards, we say that the risky asset delivers return \( R_r \) with probability \( s \) and 0 with probability \( 1 - s \).\(^9\) Furthermore we assume that the risky asset has negative expected net present value, in the sense that the expected return of the risky asset is less than the promised returns to the depositors and the CoCo holders.\(^10\) That is,

\[
sR_r - r (D + C) < 0. \tag{4.3}
\]

Therefore the risky asset is less socially desirable than the safe asset. But because the bank enjoys limited liability, the private returns of the risky portfolio exceeds its social value. The following relation holds between the private and public risky asset returns and safe asset returns:

\[\text{In principle, one could choose a different return for the depositors and the CoCo holders. However, doing so introduces cumbersome notation and yields no additional insights. It would become relevant in an analysis focused on asset pricing.}\]

\[\text{The artificially-constructed probability } s \text{ implies that the complement } 1 - s \text{ is also artificially constructed.}\]

\[\text{It is true that the risky asset still delivers a return } (1 - q) R_r \text{ if the bad loans yield 0. However, the assumption that } sR_r - r (D + C) < 0 \text{ still covers this situation, as } s > 1 - q. \text{ That is, if under } s, \text{ } sR_r - r (D + C) < 0, \text{ and } s = 1 - q + qp, \text{ then } (1 - q) R_r - r (D + C) < sR_r - r (D + C) < 0.\]
turns:

\[ s (R_r - r (D + C)) > R_s - r (D + C) > 0 > s R_r - r (D + C), \]  

(4.4)

because the social return counts depositor losses. Note that the private return of the risky project can be written as

\[ s (R_r - r (D + C)) = s R_r - r (D + C) + (1 - s) r (D + C), \]  

(4.5)

which is equal to the social return of the risky project, plus what amounts to the Merton put \((1 - s) r (D + C)\); this arises because of limited liability (Merton [1974]), limited liability implies a put option written by creditors to equity holders.

4.3.1.2 \( t = 1 \): Conversion rules

At \( t = 1 \), adverse information regarding the bank’s expected returns may arise, that comes to the attention of both the bank and the regulator. In this model, they come in the form of shocks to the composite probability \( s \) of the risky asset. If the size of the shock is such that the bank’s equity ratio falls below the trigger ratio associated with the CoCo, the CoCo should in principle be converted by the regulator. However, the regulator has discretion over the course of action: she has the ability to convert the CoCos even without new information, and she can forbear on conversion if she obtains new negative information regarding the bank. In this model, provided that the regulator refrains from forbearance, the regulator’s conversion decision is aligned with the automatic conversion rules. We discuss the conversion rules here.

Let \( \tau \) be the trigger ratio that the bank’s equity ratio must exceed in order for the CoCos to remain unconverted. The trigger ratio is independent of the amount of CoCos issued by the bank. The equity ratio equals net assets divided by total assets.\(^{11} \) At any time before \( t = 2 \), the expected value of the assets at \( t = 0 \) is used if no new information arrives. However, in the event of new information regarding the probability of obtaining returns by \( t = 1 \), the \( t = 1 \) expected value of the assets will be used. Both the bank and the regulator learn of the new information at the same time, although the regulator has discretion over the conversion of the CoCos. The parameter values at \( t = 0 \) are assumed to satisfy the trigger ratio \( \tau \) such that CoCo conversion will not be triggered at the start of the game, regardless of the bank’s initial choice.

This implies:

\[ \frac{R_s - r (D + C)}{R_s} \geq \tau \]  

(4.6)

\(^{11} \)To keep things simple, we have assigned the same risk weights to any asset chosen by the bank in our model. We may also choose to have different risk weights for the assets, but as risk weights are only constants, varying them would not materially affect the results.
if the bank had chosen the safe asset and
\[
\frac{sR_r - r (D + C)}{sR_r} \geq \tau \iff s \geq \frac{r (D + C)}{R_r (1 - \tau)}
\] (4.7)

if the bank had chosen the risky asset.

4.3.1.3 \( t = 1 \) and \( t = 2 \): Bank risk-taking and final payment

At \( t = 1 \), the regulator and the bank observe whether new adverse information regarding the economic conditions arrive. If there is bad news, the regulator chooses whether to convert the CoCos, or forbear on conversion. Without new information or after good news, the regulator has no reason to choose conversion, because the expected value of the assets at \( t = 1 \) would be the same or better than (after good news) that of \( t = 0 \), so that the trigger ratio remains satisfied. Upon conversion, the \( rC \) CoCos are written off.\(^{12}\)

After the regulator’s decision, the bank arrives at its second decision point regarding its bad loan: gamble for resurrection or liquidation. Gambling for resurrection does not change the probability of recovery that a bank faces. Instead, the bank retains the bad loans on its balance sheet. This is an attractive choice for the bank because it enjoys an implied Merton put that arises from limited liability. On the other hand, when a bank liquidates, we assume that it sells off the bad loans at a loss and relends the proceeds to a safe project, as in Homar and van Wijnbergen [2016]. We assume that liquidation is costly, that is, it always yields a return \( 0 < \lambda < 1 \) for every unit of asset. Therefore, safe assets are never liquidated because \( R_s > \lambda R_s \). The same is true for the \( 1 - q \) fraction of good loans of the risky asset, because \( R_r > \lambda R_r \). Henceforth, decisions on gambling or liquidation at \( t = 1 \) only ever pertain to the bad loans of the risky portfolio.

Finally at \( t = 2 \), if the bank survives, the creditors are paid in order of seniority, and the bank owner/manager receives any residual profits.

4.4 Backward induction at \( t = 1 \)

In order to find out the bank’s ultimate choice at \( t = 0 \), we must resolve the \( t = 1 \) events first. We therefore solve the game backwards from \( t = 1 \) as decisions are no longer made at \( t = 2 \).

\(^{12}\)The distinction between the two CoCo types is irrelevant at \( t = 1 \), as the bank’s \( t = 1 \) decision depends only on its outstanding liabilities, and not on the allocation over old and new shareholders. Of course, the type of CoCo influences a bank’s \( t = 0 \) decisions. If the CoCo was an equity converter, there is a conversion ratio that would lead the bank to choose the safe asset over the proceeds of the liquidated risky one. Calculations are presented in Appendix 4.A.
The bank can choose between gambling or liquidation but only after the regulator has decided between conversion and forbearance. Therefore, the regulator may be able to influence bank’s choice, as the regulator’s decision to convert the CoCos alters the level of the bank’s skin in the game.

The rationale of CoCo issuance is to improve the bank’s equity position in times of shocks. We discuss the benchmark case (without the shocks) before the cases with shocks. As previously mentioned, the benchmark case allows us to focus on the essential driver of regulatory forbearance. It also sheds light at \( t = 0 \), when decisions have to be made when the shocks are not known. Finally, the benchmark case allows us to examine how the bank anticipates the regulator’s action in the simplest setting, which feeds back into the \( t = 0 \) decision.

### 4.4.1 The benchmark case

#### 4.4.1.1 The bank’s choice between gambling and liquidation at \( t = 1 \)

Consider first the expected returns of a bank that has decided to gamble for resurrection at \( t = 1 \). As in (4.3), the expected return from the investment is

\[
(1 - q + qp) R_r = s R_r, \tag{4.8}
\]

where \( p \) is some low probability of recovering the \( q \) bad loans. Therefore, for given liability \( B \), the expected returns from gambling for resurrection is \( s (R_r - B) \). On the other hand, the expected returns of a bank that has decided to liquidate the bad loans is

\[
(1 - q) R_r + q \lambda R_r = (1 - q + q \lambda) R_r \equiv s^\lambda R_r, \tag{4.9}
\]

with \( s^\lambda \) is the recovery rate on the entire risky asset. We make the additional assumption that

\[
s^\lambda R_r > r (D + C), \tag{4.10}
\]

i.e. the bank is solvent in this case.\(^{13}\) We may then write, for liability \( B \), the bank’s expected returns from liquidation is \( s^\lambda R_r - B \). For the fraction \( q \) of bad loans in the risky portfolio, liquidation yields \( \lambda R_r \) with certainty, as opposed to obtaining \( R_r \) with some low probability \( p \). We assume that \( \lambda > p \) so that the regulator prefers that the bank choose liquidation over gambling. However, because of limited liability, the bank finds gambling for resurrection attractive. In particular, under gambling for resurrection, \( B \) only has to be paid with probability \( s \). That is,

\(^{13}\)In this model, the maximum amount of liabilities that the bank has at any given time is \( r (D + C) \), such that we can generalize to any \( B \) in the \([rD, r (D + C)]\) range.
the bank benefits from the Merton put implied by limited liability. On the other hand, under liquidation, $B$ is paid with certainty, because of the assumption in (4.10). Therefore, for some outstanding liability $B$, the bank will only choose liquidation over gambling if the following condition is satisfied:

$$s\lambda R_r - B \geq s (R_r - B)$$

(4.11)

We call (4.11) the liquidation incentive constraint (LIC). (4.11) further simplifies to

$$B \leq \left( \frac{\lambda - p}{1 - p} \right) R_r \equiv B^*,$$

(4.12)

where $B^*$ is the threshold amount of liability that the bank should not exceed in order for the bank to choose liquidation over gambling.

We can equivalently cast the LIC in terms of equity. By doing so, we can more clearly see the role of the bank’s level of skin in the game in the choice that it makes. We add the expected return of a bank’s asset conditional on the risky choice being taken ($sR_r$) to both sides of (4.12) in order to obtain the critical equity value $E^*$.

$$sR_r - B \geq sR_r - B^* \equiv E^*$$

(4.13)

$E^*$ is the equity level that corresponds to the maximum debt threshold $B^*$ defined in (4.12). Banks will gamble when their skin in the game falls short of $E^*$ and liquidate otherwise.

4.4.1.2 The regulator

A regulator may be classified according to different dimensions: either welfare-maximizing or cost-reducing as in Mailath and Mester [1994], with a good or bad audit technology as in Morrison and White [2013], or with a high or low cost of bailing out banks, as in Shapiro and Skeie [2015]. In this paper, we take the latter approach in the context of CoCos.

The regulator is interested in the total utility achieved in the system, irrespective of the distribution of the gains or losses over the various agents. One can see that the CoCo write-down is only a redistribution of wealth: conversion increases the equity value of the bank but deals an equivalent loss to the CoCo holders. As it does nothing to change the social value of the assets, the regulator is unaffected by conversion for its own sake.

However, conversion has consequences. Since it is publicly observable, it makes other agents aware of an adverse change in the system. On the other hand, since conversion increases the bank’s skin in the game, it may lead to socially better choices. But holding the bank’s choice constant, the regulator prefers forbearance, because forbearance does not transmit bad news
to the outside agents. We represent the costs of conversion by $\chi$. This cost is similar to those assumed by Mailath and Mester [1994] and Shapiro and Skeie [2015]. For the moment, the costs are exogenous, but in a later section, we endogenize $\chi$ by linking it to depositor runs. Table 4.1 illustrates the difference in regulator payoffs under conversion and forbearance.

### Table 4.1: Regulator Payoffs at $t = 2$: Benchmark

<table>
<thead>
<tr>
<th>Regulator Payoff</th>
<th>Conversion</th>
<th>Forbearance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safe asset</td>
<td>$R_s - \chi$</td>
<td>$R_s$</td>
</tr>
<tr>
<td>Risky asset, Gamble</td>
<td>$sR_r - \chi$</td>
<td>$sR_r$</td>
</tr>
<tr>
<td>Risky asset, Liquidate</td>
<td>$s^4R_r - \chi$</td>
<td>$s^4R_r$</td>
</tr>
</tbody>
</table>

The regulator has to balance the costs of conversion and the benefits of conversion if the latter induces the bank to choose liquidation over gambling. Therefore, the relevant comparison is between the (Conversion, Liquidate) and the (Forbearance, Gamble) cases in Table 4.1. That is,

$$s^4R_r - \chi > sR_r$$

and

$$\chi < q(\lambda - p)R_r = \tilde{\chi},$$

where $\tilde{\chi}$ is the threshold level of conversion costs. Whenever (4.14) holds, the regulator will choose conversion over forbearance, if the LIC holds after conversion.

We have determined the conditions under which a regulator and a bank would make their $t = 1$ decisions. Consider now their interactions. Table 4.2 shows the bank’s payoffs under its two possible strategies as a function of the regulator’s action. In all situations, holding the bank’s choice constant, the bank’s liabilities under forbearance is always $rC$ more than under conversion.

### Table 4.2: Bank Payoffs at $t = 2$: Benchmark

<table>
<thead>
<tr>
<th>Bank Payoff</th>
<th>Conversion</th>
<th>Forbearance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safe asset</td>
<td>$R_s - rD$</td>
<td>$R_s - r(D + C)$</td>
</tr>
<tr>
<td>Risky asset, Gamble</td>
<td>$s(R_r - rD)$</td>
<td>$s(R_r - r(D + C))$</td>
</tr>
<tr>
<td>Risky asset, Liquidate</td>
<td>$s^4R_r - rD$</td>
<td>$s^4R_r - r(D + C)$</td>
</tr>
</tbody>
</table>

Reading from Table 4.2, holding the regulator’s choice constant, the bank payoffs under gambling and liquidation reflects the LIC in (4.11), only for specific values of liability $B$. If the regulator chooses to forbear, the bank’s outstanding liability remains $r(D + C)$. But if the regulator chooses to convert, the outstanding liability is reduced to $rD$. The benefit of
conversion is that it changes the relevant bank payoffs from the LIC (same regulator decision) to a different one: \(s^3 R_r - rD\) against \(s (R_r - r(D + C))\). By defining the debt threshold in terms of equity, we are able to quickly assess whether conversion changes the bank’s \(t = 1\) decision or not.

Let \(E_{\text{forb}}\) denote the bank’s expected equity level under forbearance. The bank only chooses to liquidate rather than to gamble whenever the bank’s equity level exceeds the threshold defined in (4.13):

\[
E_{\text{forb}} = s R_r - r (D + C) \geq E^*.
\] (4.15)

If the regulator chooses conversion, then the bank’s liabilities would decrease from \(r (D + C)\) to \(rD\), since \(rC\) is written off. Let \(E_{\text{conv}}\) denote the bank’s expected equity level after conversion. The bank would choose liquidation over gambling if

\[
E_{\text{conv}} = s R_r - rD \geq E^*.
\] (4.16)

The regulator only prefers to convert if it makes the bank choose liquidation. So when is conversion enough to make the bank’s new equity exceed the threshold? If the bank was not able to satisfy (4.15), it may still be able to satisfy (4.16), provided that the shortfall is less than \(rC\). Table 4.3 summarizes the different cases that a bank’s capital structure may fall into, and the best response of the bank and the regulator given the cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>Situation</th>
<th>Action Chosen by Bank under Forbearance</th>
<th>Conversion</th>
<th>Regulator’s Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>(E_{\text{conv}} &gt; E_{\text{forb}} &gt; E^*)</td>
<td>Liquidate</td>
<td>Liquidate</td>
<td>Forbearance</td>
</tr>
<tr>
<td>II</td>
<td>(E_{\text{conv}} &gt; E^* &gt; E_{\text{forb}})</td>
<td>Gamble</td>
<td>Liquidate</td>
<td>Conversion, if (\chi &lt; \bar{\chi})</td>
</tr>
<tr>
<td>III</td>
<td>(E^* &gt; E_{\text{conv}} &gt; E_{\text{forb}})</td>
<td>Gamble</td>
<td>Gamble</td>
<td>Forbearance</td>
</tr>
</tbody>
</table>

Case I is when \(E_{\text{conv}} > E_{\text{forb}} > E^*\), or when the bank’s skin in the game already exceeds the threshold \(E^*\). Note that conversion only increases the bank’s skin in the game. Since the bank already satisfies the LIC without conversion, it will also satisfy the LIC with conversion, so in both cases, the bank will choose to liquidate. Because the conversion changes nothing for the bank, but incurs a cost to the regulator, the regulator will therefore forbear.

Case II is when \(E_{\text{conv}} > E^* > E_{\text{forb}}\), which implies that the bank’s skin in the game will only exceed the threshold \(E^*\) upon conversion. Therefore, conversion makes a difference.

Case III is when \(E^* > E_{\text{conv}} > E_{\text{forb}}\), i.e when the bank’s skin in the game falls short even after conversion. As a result, conversion also does nothing: the bank will choose to gamble.
even if the regulator converts the CoCos. Therefore, the regulator will also forbear in this case.

Proposition 4.1 summarizes the results.

**Proposition 4.1.** Given levels of deposits $D$ and CoCos $C$, and given that the bank has chosen risky assets at $t = 0$, if the bank’s equity level is within $rC$—distance of the liquidation incentive constraint, the regulator is able to force the bank to choose liquidation over gambling, by choosing conversion. However, the regulator will only choose conversion if the costs are sufficiently low.

### 4.4.2 Arrival of adverse information at $t = 1$

The arrival of adverse information occurs at $t = 1$. However, these events are completely unanticipated at $t = 0$, that is, we model them as zero-probability events. The new information takes the form of revised parameter values for the risky asset. In our setup, we consider two types of shocks: a shock on the fraction of bad loans $q$, and a shock on the probability of obtaining returns of the bad loans $p$. One may interpret the shock on $q$ as an aggregate shock that increases the volume of nonperforming loans, as in Homar and van Wijnbergen [2016]. On the other hand, the $p$ shock may be interpreted as a shock that only affects the existing nonperforming loans - that is, it makes their recovery more unlikely.\(^{14}\) We assume that both are large enough to cause the equity ratio to fall below the trigger $r$. Naturally, this implies that if the shocks are small, the regulator would forbear on conversion. These shocks alter both the bank’s liquidation incentive constraint, and the regulator’s threshold costs of conversion. We discuss each type of shock separately.

#### 4.4.2.1 A $q$-shock: an increase in the proportion of bad loans within the risky asset class

We first consider a shock to the proportion of bad loans within the risky asset class. That is, suppose at $t = 1$, the proportion of bad loans $q$ increase to some $q' > q$, holding the probability of obtaining the return from the bad loans $p$ constant. The effect of this is that the composite probability $s$ of obtaining the outlier return $R_r$ for the risky portfolio decreases. For ease of exposition, relabel by $s(q)$ the $s = 1 - q + qp$ defined in (4.2). We have

\[
\frac{\partial s(q)}{\partial q} = -(1 - p) < 0. \tag{4.17}
\]

\(^{14}\)The $p$ shock may be interpreted as an industry-specific or a demand-side shock shock that decreases the likelihood of obtaining returns from investments in a certain industry. An example is unexpected regulatory changes that that negatively affect the cash flow of a firm in a particular industry.
Denote the revised composite probability by \( s(q') \): \( s(q') = 1 - q' + q'p \). Consider a shock that is large enough to cause the equity ratio to fall below the trigger level \( \tau \). That is,

\[
\frac{s(q) R_r - r (D + C)}{s(q) R_r} \geq \tau > \frac{s(q') R_r - r (D + C)}{s(q') R_r}.
\] (4.18)

The shock to \( q \) also affects the bank’s liquidation payoffs. The liquidation value \( \lambda \) is larger than \( p \), but the liquidation value interacts with the proportion of bad loans \( q' \) in the risky portfolio. For ease of exposition, relabel by \( s^\lambda(q) \) the recovery value \( s^\lambda = 1 - q + q\lambda \) for the full risky asset defined in (4.9). We have

\[
\frac{\partial s^\lambda(q)}{\partial q} = -(1 - p) < 0,
\] (4.19)

which leads to the recovery value of the risky asset (given that the liquidation strategy is chosen), given a shock to \( q \) as

\[
(1 - q') R_r + q'\lambda R_r = s^\lambda(q') R_r.
\] (4.20)

Therefore, the liquidation incentive constraint changes after the shock to \( q \) at \( t = 1 \). The bank now considers the following inequality for some outstanding liability \( B \):

\[
s^\lambda(q') R_r - B \geq s(q') (R_r - B),
\] (4.21)

which leads to

\[
B \leq \left( \frac{\lambda - p}{1 - p} \right) R_r = B^*,
\] (4.22)

the same threshold that was obtained in the benchmark case. Intuitively, this is because the shock to \( q \) does not make gambling for resurrection any less attractive than in the benchmark case. If it did, it would show up in both the difference between \( \lambda \) and \( p \), as well as in the per-loan Merton put benefit \( (1 - p) \). One can see this upon closer inspection of the LIC. We can rewrite (4.21) as follows:

\[
s^\lambda(q') R_r - B \geq s(q') R_r - B + [1 - s(q')] B
q'\lambda R_r \geq q'pR_r + q'[1 - p] B.
\] (4.23)

Note that (4.23) shows that the simplified form of (4.21) has elements which all contain the factor \( q' \). This means that even if on aggregate, gambling for resurrection becomes more attractive, the increase in the Merton put implied by the bank’s limited liability exactly offsets
the attractiveness of gambling for resurrection.

As before, (4.22) may be transformed in terms of equity by adding the bank’s expected returns. But even if the composite probability \( s \) falls to some \( s(q') < s \), it affects both sides of (4.22) in the same way. In particular, if we let \( q' = q + \nu \), we have \( s(q') = s - \nu(1 - p) \) and the LIC (under regulatory forbearance) becomes

\[
[s - \nu(1 - p)] R_r - B > [s - \nu(1 - p)] R_r - B^*.
\] (4.24)

But this simplifies to

\[
sR_r - B > sR_r - B^* \equiv E^* ,
\] (4.25)

which is identical to (4.13) that we obtained in the benchmark case. This means that a shock to the quantity of bad loans, holding the probability constant does not change the bank’s threshold governing the choice of liquidation over gambling, relative to the benchmark case. The expected value of the asset falls as well, but it affects both sides of (4.22) in the same way. Therefore, the only thing that can possibly change a bank’s incentive is the conversion of the CoCo, as in the benchmark case.

While the bank is not affected by the increase in \( q \), the regulator is. This is because while the increase in \( q \) affects both the difference between \( \lambda \) and \( p \), and the Merton put in the same way and therefore cancel out, the regulator has no such mechanism. Instead, the regulator faces the social cost of a higher number of loan failures. As a result, the regulator’s threshold cost of conversion is also affected. As the shock to \( q \) changes both the composite probability \( s(q) \) to \( s(q') \), and the recovery value \( s^\lambda(q) \) to \( s^\lambda(q') \), the regulator’s payoff functions are also altered. Table 4.4 shows the payoffs after the arrival of new information.

Table 4.4: Regulator Payoffs at \( t = 2 \): \( q \)-shock

<table>
<thead>
<tr>
<th>Regulator Payoff</th>
<th>Conversion</th>
<th>Forbearance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safe asset</td>
<td>( R_s - \chi )</td>
<td>( R_s )</td>
</tr>
<tr>
<td>Risky asset, Gamble</td>
<td>( s(q') R_r - \chi )</td>
<td>( s(q') R_r )</td>
</tr>
<tr>
<td>Risky asset, Liquidate</td>
<td>( s^\lambda(q') R_r - \chi )</td>
<td>( s^\lambda(q') R_r )</td>
</tr>
</tbody>
</table>

The regulator will only choose conversion whenever

\[
s^\lambda(q') R_r - \chi > s(q') R_r
\]

\[
\chi < q'(\lambda - p) R_r = \overline{\chi q}
\] (4.26)

Since \( q' > q \), \( \overline{\chi q} > \overline{\chi} \). Therefore, the threshold of a regulator is higher after a bad \( q \)-shock than
without. This is because a $q$-shock means that a larger amount of loans could go bad, meaning that there are more opportunity losses for the system. Lemma 4.2 summarizes the results.

**Lemma 4.2.** Given $\chi$, a negative shock to $q$ makes the regulator less wary of conversion because the social cost of gambling for resurrection goes up (applies to more loans).

Because we were able to express the bank’s expected equity levels under a $q$-shock, in terms of those in the benchmark case, we may use the same notation as in the benchmark case. The analysis here is structurally similar to that of the benchmark case, with the exception that the regulator’s threshold is higher here. Table 4.5 summarizes the results.

Table 4.5: Bank and Regulator Interactions: $q$-shock

<table>
<thead>
<tr>
<th>Case</th>
<th>Situation</th>
<th>Action Chosen by Bank under</th>
<th>Regulator’s Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$E_{\text{conv}} &gt; E_{\text{forb}} &gt; E^*$</td>
<td>Liquidate</td>
<td>Liquidate</td>
</tr>
<tr>
<td>II</td>
<td>$E_{\text{conv}} &gt; E^* &gt; E_{\text{forb}}$</td>
<td>Gamble</td>
<td>Liquidate</td>
</tr>
<tr>
<td>III</td>
<td>$E^* &gt; E_{\text{conv}} &gt; E_{\text{forb}}$</td>
<td>Gamble</td>
<td>Gamble</td>
</tr>
</tbody>
</table>

**Proposition 4.3.** An increase in $q$ does not affect the bank’s decision rules such that the bank’s decisions under a $q$ shock are the same as under the benchmark case. However, a shock to $q$ raises social costs, and makes it “easier” to convince the regulator to convert the CoCos: the range of values for which the regulator chooses forbearance is smaller.

### 4.4.2.2 A $p$-shock: a decrease in the probability of obtaining the return of the bad loans within the risky asset class

Consider now a shock in the probability of obtaining the return from the bad loan, holding the proportion of bad loans constant. That is, at $t = 1$, suppose $p$ falls to some $p' < p$, holding $q$ constant. The effect of this is that the composite probability $s$ of obtaining return $R_r$ for the risky portfolio goes down. For ease of exposition, relabel by $s (p)$ the $s = 1 - q + qp$ defined in (4.2). We have

$$\frac{\partial s (p)}{\partial p} = q > 0.$$  \hspace{1cm} (4.27)

Denote the revised compound probability by $s (p')$: $s (p') = 1 - q + qp'$. Like the shock to $q$, we assume that the shock to $p$ is large enough to cause the equity ratio to fall below the trigger level $\tau$. That is,

$$\frac{r}{s (p) R_r} \geq \frac{\tau}{s (p') R_r} \geq \frac{r (D + C)}{s (p') R_r}.$$  \hspace{1cm} (4.28)
The shock to \( p \) does not affect the bank’s liquidation payoff \( s^\lambda \) at all. However, since the shock increases the gap between the liquidation value \( \lambda \) and the probability of obtaining positive returns for a given proportion \( q \) of bad loans, the \( p \)-shock makes gambling for resurrection less attractive compared to the benchmark case. The shock to \( p \) not only affects the relative gain of liquidation over gambling, it also affects the Merton put from limited liability. The effects do not cancel out, unlike that of the \( q \) shock case. For outstanding liability \( B \), the LIC that the bank faces in order to choose liquidation over gambling, for a given \( q \), becomes

\[
s^\lambda R_r - B \geq s \left( p' \right) \left( R_r - B \right)
\]

which further simplifies to

\[
B \leq \left( \frac{\lambda - p'}{1 - p'} \right) R_r = B^*_p,
\]

where \( B^*_p \) is the threshold level of liabilities that the bank must exceed for liquidation to be chosen. (4.30) is similar to (4.11) but with \( p' \) instead of \( p \). Because the derivative of \( B^* \) with respect to \( p \) is negative, a drop in \( p \) leads to an increase in \( B^* \), and so \( B^*_p > B^* \). This means that a shock to \( p \) may lead banks to choose liquidation, without having to be nudged by a CoCo conversion, because even if \( B > B^* \), it may be the case that \( B < B^*_p \). Of course, this also means that the corresponding equity threshold goes down.

Letting \( \delta \) be the size of the shock to \( p \), we may write \( p' = p - \delta \), and write \( B^*_p \) in terms of \( B^* \) as follows:

\[
B^*_p = \left( \frac{\lambda - p'}{1 - p'} \right) R_r + \left( \frac{\delta R_r}{1 - p + \delta} \right) \left( 1 - \frac{\lambda}{1 - p} \right) \]

\[
B^*_p = B^* + \Delta,
\]

such that (4.30) may be written in terms that appear in the benchmark case. We then have

\[
B \leq B^* + \Delta,
\]

which may be transformed in terms of equity by adding the bank’s expected returns \( s \left( p' \right) R_r \) to both sides. But even if the composite probability \( s \left( p \right) \) falls to some \( s \left( p' \right) < s \left( p \right) \), it affects both sides of (4.30) in the same way. More specifically, if we let \( p' = p - \delta \), we have \( s \left( p' \right) = s \left( p \right) - q\delta \) and the LIC (under regulatory forbearance) becomes, for outstanding liability \( B \),

\[
\left( s \left( p \right) - q\delta \right) R_r - B \geq \left( s \left( p \right) - q\delta \right) R_r - \left( B^* + \Delta \right).
\]

103
But this simplifies to

\[ s(p) R_r - B \geq s(p) R_r - (B^* + \Delta) = E^* - \Delta. \]

That is, the bank needs less skin in the game with a \( p \)-shock relative to the benchmark case in order to choose liquidation over gambling. For the \( p \)-shock, the fact that the equity threshold \( E^* \) is lower by \( \Delta \) means that if the liquidation incentive constraint was met in the benchmark case, it would definitely be met in the \( p \)-shock case.

**Lemma 4.4.** *Ceteris paribus, a \( p \)-shock increases the range of outcomes over which the bank will choose liquidation over gambling.*

This means that the bank acts in a more conservative manner when faced with a \( p \) shock, as opposed to a \( q \) shock.

The regulator’s conversion cost threshold is altered whenever there is a shock to \( p \). The shock to \( p \) changes the composite probability \( s(p) \) to \( s(p') \), but not the recovery value \( s^\lambda \). Table 4.6 shows the payoffs to the regulator after a shock to \( p \).

<table>
<thead>
<tr>
<th>Regulator Payoff</th>
<th>Conversion</th>
<th>Forbearance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safe asset</td>
<td>( R_s - \chi )</td>
<td>( R_s )</td>
</tr>
<tr>
<td>Risky asset, Gamble</td>
<td>( s(p') R_r - \chi )</td>
<td>( s(p') R_r - rD )</td>
</tr>
<tr>
<td>Risky asset, Liquidate</td>
<td>( s^\lambda R_r - \chi )</td>
<td>( s^\lambda R_r )</td>
</tr>
</tbody>
</table>

From this table, we can see that the regulator will only choose conversion whenever

\[
\begin{align*}
    s^\lambda R_r - \chi & > s(p) R_r \\
    \chi & < q (\lambda - p') R_r = \bar{\chi}_p.
\end{align*}
\]

This is structurally similar to the benchmark case, but with \( p' < p \), the threshold also rises. The regulator has higher tolerance for conversion when there is a shock to \( p \) because once again the social cost of allowing gambling for resurrection has gone up, and the forbearance region shrinks.

**Lemma 4.5.** *Given cost of conversion \( \chi \), a shock to \( p \) makes regulators less wary of conversion because of the higher social cost that comes from the shock.*

Because we were able to simplify the bank’s expected equity levels under a \( p \)-shock, in terms of those in the benchmark case, the analysis follows exactly as in the benchmark case,
with the exception that the bank’s equity threshold is lower by \( \Delta \). Table 4.7 and Proposition 4.6 summarizes the results.

**Table 4.7: Bank and Regulator Interactions: \( p \)-shock**

<table>
<thead>
<tr>
<th>Case</th>
<th>Situation</th>
<th>Action Chosen by Bank under Regulator’s Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Forbearance</td>
</tr>
<tr>
<td>I</td>
<td>( E_{\text{conv}} &gt; E_{\text{forb}} &gt; E^* - \Delta )</td>
<td>Liquidate</td>
</tr>
<tr>
<td>II</td>
<td>( E_{\text{conv}} &gt; E^* - \Delta &gt; E_{\text{forb}} )</td>
<td>Gamble</td>
</tr>
<tr>
<td>III</td>
<td>( E^* - \Delta &gt; E_{\text{conv}} &gt; E_{\text{forb}} )</td>
<td>Gamble</td>
</tr>
</tbody>
</table>

**Proposition 4.6.** A shock to \( p \) leads to a smaller equity threshold for the bank relative to the benchmark case, and thus, increases the range where the bank would choose liquidation over gambling. Structurally, all the results of the benchmark case do not change. However, the range of values for which the regulator chooses forbearance is smaller.

We have made the above analysis without specifying the size of the \( p \)-shock. However, we can let the size of the shock vary as well. Consider again Table 4.7. Given the amount of CoCos \( r_C \), if the bank’s capital structure falls within Cases I and II, an increase in the shock can only encourage the bank to choose liquidation over gambling. This is because the threshold \( E^* - \Delta \) becomes smaller as the shock increases. If the bank’s capital structure falls under Case III, the shock may move the bank from Case III to Case II or even Case I. In this narrow sense, the shock may be beneficial to the regulator in that it allows the conversion to be useful in changing the decision of the bank, given a particular amount of CoCos. Of course if the CoCos are too few, then a large shock would not change anything.

Figure 4.3 illustrates how the bank moves from Cases III to I as the shock \( \Delta \) changes.
The image labeled "small shock" illustrates Case III of Table 4.7. The crisis has caused the threshold to fall to some $E^* - \Delta_{small}$, but the bank’s equity after conversion is still smaller than the threshold, leading the bank to choose to gamble even if the regulator had chosen conversion. This illustrates the futility of conversion during a small shock, provided that the bank is in Case III to begin with. The image labeled "intermediate shock" illustrates the bank moving from Case III to Case II of Table 4.7. In this case, the threshold has fallen to $E^* - \Delta_{int}$. While $E_{forb} < E^* - \Delta_{int}$, a conversion leads to $E_{conv} > E^* - \Delta_{int}$. In this case, the regulator’s decision to convert the CoCos leads the bank to choose liquidation over gambling. Finally, the image labeled "large shock" illustrates the bank moving from Case III to Case I of Table (4.7). Consider the benchmark case. Even if the bank chose gambling in the benchmark case, the shock is large enough to cause the threshold to fall down to $E^* - \Delta_{large}$. This means that even without a conversion, the bank is already incentivized to choose liquidation. As such, conversion is also useless in this case, because the bank already chooses the right decision under forbearance.

**Proposition 4.7.** Provided that the LIC was not satisfied in the benchmark case, the regulator will only convert CoCos if the shocks are in the intermediate range and if the cost of conversion is not too high. CoCos will not be converted in the event of either a small shock or a large one: small shocks are not enough of a deterrent from gambling, and large shocks automatically cause the LIC to hold.

This result has significant policy implications, as it is precisely in the event of large shocks...
that CoCos are considered useful for the financial system. In the presence of large shocks, the banks will decide on liquidation over gambling for resurrection even without regulatory intervention. This is precisely why the regulator will tend to forbear, as she would like to avoid the conversion costs. However, her forbearance makes the financial system weaker than it should be, as it leaves the banks with less equity than it otherwise might have, as $E_{forb}$ is always smaller than $E_{conv}$.

4.5 Endogenizing the cost faced by a regulator

There are many reasons why the regulator would face conversion costs. One of them is the incompleteness of deposit insurance. In particular, wholesale deposits are not typically covered by deposit insurance, or are only marginally covered. Until now, we have abstracted from the issue of depositor behavior, focusing instead on the interaction between the regulator and the bank, and assuming an exogenous cost of conversion $\chi$. In this section, we endogenize the regulator’s $\chi$ by letting her conversion decision affect the threshold beliefs of the depositors. We assume that depositors have prior beliefs regarding the bank’s asset choice such that their participation constraint of depositors is satisfied. The belief level that causes the depositors’ participation constraint to bind may be interpreted as the probability of a bank run. That is, if the threshold belief is not satisfied, the depositors would run. We model depositors as uninformed in this section: they are only aware of the promised return $r$, but unaware of the asset that the bank had chosen at $t = 0$. Moreover, the depositors are unaware of the composite probability of obtaining a positive return from the risky asset. All they have are beliefs about these parameters. But as conversion is an observable event, and occurs only if the bank had invested in a risky asset, the conversion shifts the beliefs of the depositors. As a result, the probability of a bank run increases. The increase in the run probability after a conversion is a key component of the regulator’s endogenous cost of conversion.

4.5.1 Depositors’ beliefs

As mentioned in Section 4.3, the depositors are risk neutral. They will invest in a bank only if they at least break even in expectation. For simplicity, let the beliefs of all the depositors be the same, rather than being distributed along some interval. In principle, depositors may have beliefs that are drawn from a distribution. In particular, Goldstein and Pauzner [2005] have used this to model bank runs.
probability of obtaining the returns of the risky asset $s$. Instead, they have beliefs on these two dimensions.

Let the depositors’ belief that the bank invested in the safe asset be some $\theta \in [0, 1]$, such that the belief that the bank invested in the risky portfolio is $1 - \theta$. Safe assets ensure that $r(D + C)$ will be paid, which means that each of the depositors will obtain $r$. Risky assets only pay out $r$ with some probability $s = 1 - q + pq$, but we assume that the depositors do not know $s$. They do have a belief about the probability of obtaining returns from the risky asset, which is $\alpha$. The depositors can only perform this calculation if they hold beliefs about the bank’s initial asset choice, and the probability of obtaining the high return in the risky asset. Therefore, there are are two beliefs that the depositors must hold. In short, the depositors have a composite belief: about the bank’s investment in a safe or risky asset $(\theta, 1 - \theta)$, along with a belief $\alpha$ on how likely the risky asset pays off. For runs not to happen, the depositors’ $\alpha$ must satisfy (4.35):

$$\theta r + (1 - \theta)(\alpha r + (1 - \alpha)0) \geq 1$$

$$\alpha \geq \frac{1}{r \left( \frac{1 - \theta r}{1 - \theta} \right)} = \bar{\alpha},$$

where $\bar{\alpha}$ is the threshold belief of the depositor for which a run does not occur. Thus, the threshold belief of the depositors is a function of their belief regarding the bank’s $t = 0$ investment. We assume that in the absence of any information, the depositors’ $\alpha$ is exactly at $\bar{\alpha}$. Note as well that

$$\frac{\partial \bar{\alpha}}{\partial \theta} = -\frac{(r - 1)}{r (1 - \theta)^2} < 0,$$

which indicates that the higher the belief that the bank has invested in the safe asset, the lower $\bar{\alpha}$ will be. However, in our setup, the signals can only lower $\theta$, rather than raise it. This is because by construction, conversion will only occur if the bank has invested in the risky asset at $t = 0$. So depositors observing the conversion would interpret it adversely, and will therefore lead to an increase in $\bar{\alpha}$ to some $\bar{\alpha}'$. Thus, assuming that $\alpha = \bar{\alpha}$ implies that $\alpha < \bar{\alpha}'$. At the same time, even if the regulator knows that the bank has invested in a risky asset, the regulator would not convert the CoCo unless she comes across adverse information about the bank’s likelihood of obtaining positive returns. As such, forbearance is never informative about the bank’s asset choice.

Thus, if the regulator does not convert the CoCo, the threshold belief stays at $\bar{\alpha}$ for a given $\theta$. However, when the regulator converts CoCos, it is certain that the asset is a risky one. So the

\[16\] The modelling here is similar to Shapiro and Skeie [2015], except that instead of having a prior on whether the bank is good or bad, the prior is on whether the bank has invested in a safe or risky asset.
only time that the depositors can update their beliefs is when they observe a CoCo conversion. The belief that the bank has chosen a safe asset $\theta$ must be updated to 0, while the belief that the bank invested in the risky portfolio $1 - \theta$ must be updated to 1.\footnote{The updating mechanism here is similar to Morrison and White [2013] where they model the updating of the depositors’ perception of the regulator’s reputation based on information about the bank’s performance.} Therefore, the above equation simplifies to

$$\alpha r + (1 - \alpha) 0 \geq 1$$

$$\alpha \geq \frac{1}{r} = \bar{\alpha}'.$$

(4.37)

Since $\frac{\partial \bar{\alpha}}{\partial \theta} < 0$ from (4.35), it must be that $\bar{\alpha'} > \bar{\alpha}$. Therefore, if conversion is the only signal that depositors can obtain to update their beliefs, then a conversion definitely raises the threshold belief, as conversion would never happen with a safe asset. This means that the marginal probability of a bank run caused by a conversion is $\bar{\alpha'} - \bar{\alpha}$. This is summarized in the proposition below.

**Proposition 4.8.** For any belief that depositors hold regarding the bank’s initial choice of assets, a conversion updates those beliefs in such a way that the belief that the bank holds the risky asset is 1. This leads to the increase in the threshold belief for runs not to occur from $\alpha$ to $\bar{\alpha'} > \bar{\alpha}$.

In this section, we have only considered full conversions. In principle, partial conversions may be observed for small shocks. For whatever belief an individual attaches to the safe asset $\theta$, it will be at least lower in the event of a conversion, if not going completely to 0. This follows from (4.36), which means that the threshold for bank runs will still increase even with nonextreme beliefs about the safe and risky asset, as long as the belief regarding the safe asset goes down.

It is important to note that the conversion does not affect claims between depositors and CoCo holders. This is because depositors have seniority over all the other creditors. Conversion is merely a signal about the assets of the bank, and this is reflected in (4.35).

### 4.5.2 Taking depositors’ beliefs into account

By design, conversion (or forbearance) precedes both the depositors’ decision to run as well as the bank’s second decision point. This implies that by the time that bank has to choose between gambling and liquidation, the bank faces either run probability $\bar{\alpha'}$ if the regulator decided on conversion, or $\bar{\alpha}$ if the regulator decided on forbearance. Therefore, the bank’s choice between gambling and liquidation takes the run probabilities as given upon the regulator’s decision.
That is, when the regulator decides upon either conversion or forbearance, the bank faces the same probability of a run: upon conversion, it is $\bar{\alpha}'$ under both gambling and liquidation, which means that there is a $1 - \bar{\alpha}'$ probability of not being run on. Similarly, upon forbearance, the probability of a bank run is $\bar{\alpha}$ under both gambling and liquidation, which means that there is a $1 - \bar{\alpha}$ probability of not being run on. The bank survives conditional on a run not occurring, and receives nothing upon a run. Table 4.8 illustrates the bank payoffs for each choice given the regulator’s decision, while taking the run probabilities into account.

Table 4.8: Bank Payoffs Accounting for Runs

<table>
<thead>
<tr>
<th>Bank Payoff</th>
<th>Conversion</th>
<th>Forbearance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safe asset</td>
<td>$(1 - \bar{\alpha}') [R_s - rD]$</td>
<td>$(1 - \bar{\alpha}) [R_s - r(D + C)]$</td>
</tr>
<tr>
<td>Risky asset, Gamble</td>
<td>$(1 - \bar{\alpha}') [s (R_r - rD)]$</td>
<td>$(1 - \bar{\alpha}) [s (R_r - r(D + C))]$</td>
</tr>
<tr>
<td>Risky asset, Liquidate</td>
<td>$(1 - \bar{\alpha}') [s^2R_r - rD]$</td>
<td>$(1 - \bar{\alpha}) [s^2R_r - r(D + C)]$</td>
</tr>
</tbody>
</table>

If the bank takes the probability of runs into account, their decision between gambling and liquidation does not change relative to the case where we abstracted from runs. This is because the probability of surviving a run $(1 - \bar{\alpha})$ or $(1 - \bar{\alpha}')$ is a constant factor that affects the payoffs from gambling and liquidation in exactly the same way. As such, banks are not bothered by an increase in the threshold belief due to the conversion.

For the regulator, it is not as straightforward. In the event of a bank run without deposit insurance, the depositors will recover their funds at $t = 1$ but doing so interrupts the investment process. This means that when a run happens, the economy loses the potential profit of the expected return net of the initial investment of 1. To a welfare-maximizing regulator, this is an opportunity loss. The bank does not face this though, as it has limited liability, it calculates its gains conditional on surviving. Therefore, if we were to take this opportunity loss into account, the payoffs to the regulator upon conversion will always be less than the payoffs upon forbearance, if only because of the increase in the probability of bank runs. Table 4.9 shows the regulator payoffs if the threshold beliefs of depositors are taken into account:

Table 4.9: Regulator Payoffs Accounting for Runs

<table>
<thead>
<tr>
<th>Regulator Payoff</th>
<th>Conversion</th>
<th>Forbearance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safe asset</td>
<td>$(1 - \bar{\alpha}') R_s + \bar{\alpha}' (R_s - 1)$</td>
<td>$(1 - \bar{\alpha}) R_s + \bar{\alpha} (R_s - 1)$</td>
</tr>
<tr>
<td>Risky asset, Gamble</td>
<td>$(1 - \bar{\alpha}') sR_r + \bar{\alpha}' (sR_r - 1)$</td>
<td>$(1 - \bar{\alpha}) sR_r + \bar{\alpha} (sR_r - 1)$</td>
</tr>
<tr>
<td>Risky asset, Liquidate</td>
<td>$(1 - \bar{\alpha}') s^2R_r + \bar{\alpha}' (s^2R_r - 1)$</td>
<td>$(1 - \bar{\alpha}) s^2R_r + \bar{\alpha}' (s^2R_r - 1)$</td>
</tr>
</tbody>
</table>

18Bahaj and Malherbe [2016] refer to this phenomenon as the "internalization effect" in the context of capital regulation, where increased regulation decreases the marginal return of the bank, as higher requirements increase the bank’s skin in the game, so it internalizes the downside risk.
We may write the forbearance payoffs in terms of the conversion payoffs for all actions of the bank. We illustrate it for the safe asset, but it also works for the others. The difference between the payoffs of forbearance and conversion for the safe asset may be written as

\[
[(1 - \bar{\alpha}) R_s + \bar{\alpha} (R_s - 1)] - [(1 - \bar{\alpha}') R_s + \bar{\alpha}' (R_s - 1)] = \bar{\alpha}' - \bar{\alpha}
\] (4.38)

Thus, letting \( \chi = \bar{\alpha}' - \bar{\alpha} \), Table 4.9 may be simplified to Table 4.10.

Table 4.10: Regulator Payoffs Accounting for Runs: Simplified

<table>
<thead>
<tr>
<th>Regulator Payoff</th>
<th>Conversion</th>
<th>Forbearance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safe asset</td>
<td>((1 - \bar{\alpha}) R_s + \bar{\alpha} (R_s - 1) - \chi)</td>
<td>((1 - \bar{\alpha}) R_s + \bar{\alpha} (R_s - 1))</td>
</tr>
<tr>
<td>Risky asset, Gamble</td>
<td>((1 - \bar{\alpha}) sR_r + \bar{\alpha} (sR_r - 1) - \chi)</td>
<td>((1 - \bar{\alpha}) sR_r + \bar{\alpha} (sR_r - 1))</td>
</tr>
<tr>
<td>Risky asset, Liquidate</td>
<td>((1 - \bar{\alpha}) s^4R_r + \bar{\alpha} (s^4R_r - 1) - \chi)</td>
<td>((1 - \bar{\alpha}) s^4R_r + \bar{\alpha} (s^4R_r - 1))</td>
</tr>
</tbody>
</table>

But by rearranging terms, we arrive at Table 4.11, which, upon closer inspection, is essentially the same as Table 4.1 but shifted up by a constant \( \bar{\alpha} \).

Table 4.11: Regulator Payoffs Accounting for Runs: Even More Simplified

<table>
<thead>
<tr>
<th>Regulator Payoff</th>
<th>Conversion</th>
<th>Forbearance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safe asset</td>
<td>(R_s + \bar{\alpha} - \chi)</td>
<td>(R_s + \bar{\alpha})</td>
</tr>
<tr>
<td>Risky asset, Gamble</td>
<td>(sR_r + \bar{\alpha} - \chi)</td>
<td>(sR_r + \bar{\alpha})</td>
</tr>
<tr>
<td>Risky asset, Liquidate</td>
<td>(s^4R_r + \bar{\alpha} - \chi)</td>
<td>(s^4R_r + \bar{\alpha})</td>
</tr>
</tbody>
</table>

This endogenizes the cost of conversion as the increase in the probability of bank runs.

### 4.6 \( t = 0 \) decisions when the regulator type is unknown

A bank can choose any asset at \( t = 0 \) it wishes as long as it complies with regulation. However, its choice at \( t = 0 \) ultimately depends on what actions it expects the regulator would do at \( t = 1 \). We have illustrated these in the previous sections, for the benchmark case, and for two shock cases.

After a negative shock, the choice made may no longer be regulation-compliant. Ideally, this is when CoCos are useful. Whether they actually turn out to be useful depends on the type of regulator. A regulator may face high costs of conversion in times of greater financial fragility, as the beliefs of the depositors may not reach the threshold necessary for runs not to occur. As a result, a regulator that faces a high cost of conversion will forbear on tough
decisions, while a regulator that faces a lower cost of conversion will cause the conversion to happen.

If the type of regulator is known, then it is easy for the bank to make its \( t = 0 \) decision, as it can foresee what the regulator does in any situation. But if it is unknown, the bank must hold some beliefs regarding the type of regulator it is dealing with. In this section we will examine the more realistic (and interesting) case where the bank does not know the type of the regulator. This will also enable us to determine under which conditions does the game have a solution at \( t = 0 \). For this, we set up an extended game tree as shown in Figure 4.4.

Figure 4.4: Imperfect Information Game Tree

Figure 4.4 is composed of two copies of Figure 4.2 connected by a Nature node that determines the type of regulator that the bank is dealing with, and C, F, G, L stand for Conversion, Forbearance, Gambling, and Liquidation, respectively. On the Low Nature branch, the regulator’s cost of conversion is low enough to lead her to conversion, while on the High Nature branch, the cost of conversion is high enough to always lead the regulator to choose forbearance. As the bank does not know the type of regulator it is dealing with, it assigns a belief \( \beta \) that the regulator has a low cost of conversion, and \( 1 - \beta \) that the regulator has a high cost of conversion. We first consider the decisions made by the bank under perfect information - that is, if the bank knows the type of regulator it is dealing with. We then characterize the beliefs that the bank must have in order to rationalize its decisions in the imperfect information setting.
4.6.1 What drives the bank’s decision to choose the safe asset over the risky one?

CoCos are only effective moral hazard deterrants at \( t = 1 \) if they allow the liquidation incentive constraint to be met. There are two instances when they do not make a difference: if the LIC is met even without conversion (in which case the regulator forbears and the bank liquidates), and if the LIC is not met even with conversion (in which case the regulator forbears and the bank gambles). Given the \( t = 1 \) decisions, it is also interesting to see whether CoCos deter banks from choosing the risky asset at \( t = 0 \). In order to do so, we must take a closer look at the assumptions regarding the returns. Because this is a \( t = 0 \) assessment, we do not consider the shocks to either \( p \) or \( q \), because we have assumed that they are unexpected at \( t = 0 \). That is, neither the regulator nor the bank know that the shocks to either \( p \) or \( q \) are forthcoming. Instead, they assume that \( p \) and \( q \) at \( t = 0 \) are the true distributional parameters. (4.4) describes the relative returns, reproduced below for convenience.

\[
\begin{align*}
&\text{private return risky asset} \quad & R_r - r (D + C) > R_s - r (D + C) > sR_r - r (D + C) \\
&\text{return safe asset} & sR_r - r (D + C) \\
&\text{social return risky asset} & sR_r \\
\end{align*}
\]  

In the previous sections, we have not made any assumption about the size of the safe asset net return relative to that of the liquidated risky asset. We remedy that here. Consider again the risky portfolio. A fraction \( 1 - q \) yields \( R_r \) with certainty, and a fraction \( q \) yields \( R_r \) with probability \( p \). Therefore, only the fraction \( q \) of bad loans will be liquidated. We have denoted the recovery value from liquidating this portfolio as \( s\lambda R_r \).

We are now in a position to compare expected returns. First, note that given the assumption of risk neutrality of the bank, it does not make sense to assume that \( s\lambda R_r > R_s \), because if that were the case, no one would invest in safe assets in the first place. It would be more profitable to invest in the risky portfolio and then liquidate it with certain yield \( \lambda R_r \) per unit of bad loan \( q \). Therefore, for both asset types to play a role, we must have that \( R_s > s\lambda R_r \). Of course this means that

\[
R_s - r (D + C) > s\lambda R_r - r (D + C)
\]  

must also follow.

Setting (4.40) allows us to determine the \( t = 0 \) choices provided that the CoCo conversion does not make a difference in the bank’s actions. There are two such situations: when the bank is able to meet the liquidation incentive constraint even before conversion, and when the bank is not able to meet the liquidation incentive constraint even after conversion. We discuss them one at a time.
Consider when the bank is able to meet the LIC even without conversion. At $t = 0$, the bank compares the return from liquidating the risky asset $s^1 R_r - r (D + C)$, with that of the safe asset. However, (4.40) implies that in this case, the safe asset will always be chosen by the bank at $t = 0$, because the regulator forbears at $t = 1$, which means the bank has to pay $rC$ to the CoCo holders. The outcome will be (Safe, Forbear, Liquidate).

Next, if the bank was not able to meet the LIC requirements even with conversion, then the regulator will forbear and the bank will gamble. However, that means that the return faced by the bank from gambling will be $s (R_r - r (D + C))$, while that of the safe asset is $R_s - r (D + C)$. But by the assumption in (4.4), the bank will always choose the risky asset at $t = 0$. The outcome will be (Risky, Forbear, Gamble).

This leaves us with the cases where conversion makes a difference in the bank’s $t = 1$ decisions. We have seen in the previous section that this is only true for a limited number of situations: for a $q$ shock, when $E_{\text{conv}} > E^* > E_{\text{forb}}$, and for a $p$ shock, when $E_{\text{conv}} > E^* - \Delta > E_{\text{forb}}$. Whether these situations arise really depends on the initial values of $D$ and $C$ relative to the expected return for both the $q$ and the $p$ shocks. However, to be able to work back to $t = 0$ decisions, we must use the benchmark case, as the shocks are unexpected at $t = 0$. Essentially, we assume that whenever the bank falls into the $E_{\text{conv}} > E^* > E_{\text{forb}}$ case, the regulator who can bear the conversion costs will convert the CoCos.

Neither (4.4) nor (4.40) imply anything about the relative net return of the safe asset compared to the net return of having liquidated the risky asset combined with a CoCo conversion.

$$R_s - r (D + C) > s^1 R_r - rD.$$  

We have to assume either this, or the alternative. This will enable us to obtain more interesting results. We must also consider the payoffs of the bank under each regulator type, assuming that the regulator type is known. However, since the regulator makes the conversion decision after the shock occurs, her decision must take the shock into account, unlike the bank’s decision at $t = 0$.

### 4.6.2 High type regulator

Consider first the high-cost regulator, with costs of conversion $\chi_H$. To ensure that the high cost regulator will always forbear regardless of shock, we assume that

$$\chi_H > \max \left[ \bar{\chi}_q, \bar{\chi}_p \right].$$  

We have to assume either this, or the alternative. This will enable us to obtain more interesting results. We must also consider the payoffs of the bank under each regulator type, assuming that the regulator type is known. However, since the regulator makes the conversion decision after the shock occurs, her decision must take the shock into account, unlike the bank’s decision at $t = 0$.
where $\tilde{\chi}_q$ and $\tilde{\chi}_p$ were introduced in Section 4.2. By Lemmas 4.2 and 4.5, a high-type regulator will always forbear regardless of the type of shock, because the social benefit of conversion is lower than the cost, which is $\chi_H$. CoCos are only useful at $t = 1$ if for a $q$ shock, the bank faces $E_{\text{conv}} > E^* > E_{\text{forb}}$, and if for a $p$ shock, the bank faces $E_{\text{conv}} > E^* - \Delta > E_{\text{forb}}$. But since by assumption, the regulator faces too high costs of conversion, she will forbear, regardless. As a result, the bank will never have enough skin in the game to liquidate, therefore the bank will always gamble for resurrection at $t = 1$.

In choosing between the safe and the risky asset at $t = 0$ though, the bank gains $R_s - r(D + C)$ under the safe asset, and $s(R_r - r(D + C))$ under the risky asset while gambling for resurrection. Since at time $t = 0$, the bank is not aware of a forthcoming shock, he uses $s$ to calculate his expected returns from the risky portfolio. However since by assumption we had that $s(R_r - r(D + C)) > R_s - r(D + C)$, the bank will always choose the risky asset whenever the regulator is of the high type, as she will always be forbearing. Therefore, the outcome here, for both the $q$ and the $p$ shocks, is (Risky, Forbear, Gamble).

4.6.3 Low type regulator

Consider next the low-cost regulator, with costs of conversion $\chi_L$. We assume that

$$\chi_L < \tilde{\chi},$$

(4.43)

where $\tilde{\chi}$ was defined in Section 4.4.1. Therefore, whenever $E_{\text{forb}} < E^* < E_{\text{conv}}$ under a $q$ shock, or $E_{\text{forb}} < E^* - \Delta < E_{\text{conv}}$ under a $p$ shock, the regulator will always choose to convert the CoCos. Therefore, at $t = 1$, the bank will use the payoff that is consistent with the regulator’s choice to convert, which is $s\tilde{\chi}R_r - rD$. However, at $t = 0$, the shocks are unanticipated, so that only the benchmark case hold. In the following sections, we will consider both $R_s - r(D + C) > s\tilde{\chi}R_r - rD$ and $R_s - r(D + C) < s\tilde{\chi}R_r - rD$, and explore the resulting outcomes. However, to make the analysis meaningful, we restrict attention to only those cases where the LIC is satisfied after a conversion.

4.6.3.1 When the payoff of the safe asset exceeds that of the liquidated risky portfolio

Suppose that $R_s - r(D + C) > s\tilde{\chi}R_r - rD$. Then the bank will choose the safe asset at $t = 0$. If the equation holds, it must also be true that

$$rC < R_s - s\tilde{\chi}R_r.$$  

(4.44)
(4.44) is equivalent to stating that the bank will only choose the safe asset over the risky one if the amount of CoCos is less than the gap between the gross returns of the safe asset and the liquidated risky asset. If the gap is small, then the issued CoCos must be relatively few compared to the difference in the expected returns. At \( t = 0 \), because the shocks are unknown, then it is enough that (4.44) holds in order for (Safe, Convert, Liquidate) to be a credible outcome.

Other outcomes exist but they are not consistent ones. We list them here. (Safe, Convert, Gamble) is not an equilibrium because even if the regulator decided to convert the CoCo, the bank will still choose to gamble, which is not consistent with the regulator’s decision to convert. If this was the case, the regulator would deviate to Forbear, as it is costless. (Safe, Forbear, Gamble) is also not an equilibrium because this is inconsistent with the assumption on net present value. If the bank was going to gamble, then it could not pick the safe asset in the first place. Moreover, the low cost regulator will never forbear if conversion is useful. Finally, (Safe, Forbear, Liquidate) is also not an equilibrium because liquidation by the bank is not the best response to a forbearing regulator.

4.6.3.2 When the payoff of the liquidated risky portfolio exceeds that of the safe asset

Assume now that \( R_s - r(D + C) < s^t R_r - rD \). This means that the bank would choose the liquidated risky portfolio over the safe asset. It also means that

\[
rC > R_s - s^t R_r \tag{4.45}
\]

must hold. This means that whenever the bank issues a large enough amount of CoCos, and provided that the bank knew that the regulator faces a low cost of conversion, the bank will choose the risky asset at \( t = 0 \), because the low-cost regulator will certainly convert at \( t = 1 \) if necessary. The condition in (4.45) is sufficient at \( t = 0 \) because the shocks are unexpected at that time. Specifically, (4.45) is enough to let (Risky, Convert, Liquidate) be an equilibrium outcome.

As in the previous subsection, there exist outcomes that are not consistent. We present them here, along with a brief explanation of why they do not work. (Risky, Forbear, Gamble) is not an equilibrium for the low-cost regulator, because by assumption, she will always convert the CoCos when necessary. (Risky, Convert, Gamble) is not an equilibrium, because if the bank chooses to Gamble, then the regulator would deviate to Forbear. Finally, (Risky, Forbear, Liquidate) is not an equilibrium because liquidation is not the bank’s best response to a forbearing regulator.
4.6.3.3 Does the size of the CoCo foreshadow expectations about shocks?

One may argue that it is inconceivable that the regulator does not foresee a shock. It is conceivable that she miscalculates the amount of the shock though. Both the regulator and the bank foresee that some negative outcomes occur, otherwise they would not assign probability $p$ to the bad loans $q$. Even though the bad loans are anticipated, it may be the case there would be more bad loans than expected, or perhaps that the probability of recovering those bad loans fall even more. Therefore it is interesting to see how much (partial) knowledge of a shock will affect (4.44) and (4.45).

Recently, banks have been encouraged by regulators to issue some amount of CoCos. Under Basel III, banks may have 3.5% of the 8% regulatory capital requirement based on risk-weighted assets filled by CoCos. Under the Total Loss Absorption Capacity (TLAC) Standard issued by the Financial Stability Board (FSB), globally systemic financial institutions must have loss absorption capacity that is 8% above the Basel III requirement by 2019, and may be filled by CoCos. These figures are lower bounds, and banks are free to issue more than the prescribed amounts.

While in this paper, we have modelled the shocks to be unanticipated at $t = 0$, and while we take capital structure as a given, the actual amount of CoCos issued is a bank decision, which involves the bank’s expectations about the future. That is, knowing the minimum requirements for the benchmark case for certain outcomes to be equilibria, we may be able to infer something about shock expectation by the bank and the regulator by examining the capital structure chosen by the bank, as well as the limits on CoCo issuances imposed by the regulator.

Consider a shock to $q$ (the shock to $p$ is similar). From (4.19), we know that this causes the bank’s recovery value to fall from from $s^\lambda (q)$ to $s^\lambda (q')$. It also means that

$$R_s - r (D + C) > s^\lambda (q) R_r - rD > s^\lambda (q') R_r - rD,$$

which also implies that if the amount of CoCos are fewer than the difference between the gross returns of the safe and the liquidated risky assets, then

$$rC < R_s - s^\lambda (q) R_r < R_s - s^\lambda (q') R_r$$

is true as well. In other words, even if there was knowledge about the size of the $q$ shock, it will not be reflected in the amount of CoCos chosen at the start of the game, as long as (4.44) is true. However, if the amount of CoCos are greater than the difference between the gross
returns of the safe and the liquidated risky assets, then

$$r_C > R_s - s^\lambda (q') R_r > R_s - s^\lambda (q) R_r$$  \hspace{1cm} (4.48)$$
is true. We can rewrite this equation as follows: letting $$q' = q + \nu$$, we can write $$s^\lambda (q') = s^\lambda (q) - \nu (1 - p)$$, which leads to

$$r_C > (R_s - s^\lambda R_r) - \nu (1 - p).$$  \hspace{1cm} (4.49)$$

In other words, the size of the CoCo issuance whenever (4.45) holds is revealing about the size of the shock that the regulator expects. This is quite interesting as CoCo issuance is highly encouraged by the regulators, as seen from the recent regulation passed by Basel III and the Financial Stability Board.

**Corollary 4.9.** When $$r_C < R_s - s^\lambda (q) R_r$$, the regulator’s knowledge about the shock is not revealed, when $$r_C > R_s - s^\lambda (q) R_r$$ the size of the CoCo issuance may be indicative of the regulator’s belief about the size of the potential shock.

### 4.6.4 If the regulator’s type is unknown

From the previous section, we have obtained only two outcomes that are consistent with a low-cost regulator’s decision to convert a CoCo in the event of a shock. Both outcomes require that the liquidation incentive constraint is satisfied: either the safe asset return exceeds the return from the liquidated risky portfolio ($$r_C < R_s - s^\lambda R_r$$), or the returns from the liquidated risky portfolio exceeds the safe asset return ($$r_C > R_s - s^\lambda R_r$$).

However, it is not always the case that the bank knows exactly the type of regulator that he is dealing with. If the regulator’s type is unknown, the bank must make its $$t = 0$$ decisions based on its beliefs about the type of regulator. In this section, we characterize the beliefs of the bank. Note that given the regulator’s type, the regulator’s action is always known. Let $$\beta$$ represent the bank’s belief that the regulator is of the low-cost type, and $$1 - \beta$$ be the bank’s belief that the regulator is of the high-cost type. We already know that the high-type will always forbear, and the bank gains $$R_s - r (D + C)$$ under the safe asset, and $$s (R_r - r (D + C))$$ under the risky asset (while gambling for resurrection), because they calculate at $$t = 0$$, where the shock is not expected to happen.
4.6.4.1 Suppose there were relatively few CoCos \( r_C < R_s - s^3 R_r \)

We have seen that the only time that a safe asset will be chosen under the low-cost regulator is when \( r_C < R_s - s^3 R_r \). The payoff of the bank under \( t = 1 \) choice (Convert, Liquidate) is \( s^3 R_r - r D \), and the payoff of the bank under \( t = 1 \) choice (Forbear, Gamble) is \( s (R_r - r (D + C)) \). The shocks do not appear because ex ante the bank puts zero probability on the occurrence of a crisis. Therefore, the bank will only choose the safe asset if (4.50) holds:

\[
R_s - r (D + C) > \beta \left[ s^3 R_r - r D \right] + (1 - \beta) \left[ s (R_r - r (D + C)) \right]. \tag{4.50}
\]

\( \beta \) and \( 1 - \beta \) do not appear in the safe side because the safe asset pays exactly the same under any type of regulator.

By the assumption in (4.4), we have that \( s (R_r - r (D + C)) > R_s - r (D + C) \). If we assume that \( r_C < R_s - s^3 R_r \), it must be that \( s^3 R_r - r D < R_s - r (D + C) \). This means that \( s^3 R_r - r D < s (R_r - r (D + C)) \). Therefore there must exist a \( \beta \in [0, 1] \) that makes (4.50) hold exactly, as because we only need a linear combination of a high and a low outcome. Call this \( \tilde{\beta} \). If the bank’s belief is \( \beta = \tilde{\beta} \), the bank randomly chooses between the safe and the risky asset. On the other hand, if the bank’s beliefs about the low-cost regulator is such that \( \beta < \tilde{\beta} \), the bank will choose the risky asset, otherwise, the bank chooses the safe asset. We have the following proposition:

**Proposition 4.10.** With imperfect information about the regulator type, and when \( r_C < R_s - s^3 R_r \), there exists a threshold belief \( \tilde{\beta} \in [0, 1] \) that leads a bank to be indifferent between a safe and a risky asset at \( t = 0 \). If \( \beta \leq \tilde{\beta} \), the bank chooses the risky asset at \( t = 0 \) and eventually decides to liquidate, if \( \beta > \tilde{\beta} \), the bank will choose the safe asset at \( t = 0 \).

4.6.4.2 Suppose there were relatively many CoCos \( r_C > R_s - s^3 R_r \)

The only other case consistent with a low-cost regulator choosing to convert the CoCos is when \( r_C > (R_s - s^3 R_r) \). This means that even if the LIC holds in a crisis, the bank’s payoff under the liquidation of the risky asset is still higher than the payoff of the safe asset. Therefore, even if the regulator type is known to be low, the bank will choose the risky asset because the liquidation value after conversion is higher than the yield of the safe asset.

By the assumption in (4.4), we have that \( s (R_r - r (D + C)) > R_s - r (D + C) \). Also, the assumption \( r_C > (R_s - s^3 R_r) \) is equivalent to \( s^3 R_r - r D > R_s - r (D + C) \). However, the left hand side of (4.50) is \( R_s - r (D + C) \), while the right hand side is a linear combination of \( s^3 R_r - r D \) and \( s (R_r - r (D + C)) \), which are both larger than \( R_s - r (D + C) \). Therefore, no value of \( \beta \in [0, 1] \) will make (4.50) true. In short, we would have for any \( \beta \) then, it would always be...
true that

\[ R_s - r (D + C) < \beta \left[ s^3 R_r - rD \right] + (1 - \beta) \left[ s (R_r - r (D + C)) \right], \tag{4.51} \]

meaning the safe asset will never be chosen at \( t = 0 \). This leads to the following proposition:

**Proposition 4.11.** With imperfect information about the regulator type, and when \( rC > \left(R_s - s^3 R_r\right)\) holds, there is no belief that is consistent with the bank choosing a safe asset at \( t = 0 \).

From the preceding sections, it is clear that conversion is useful for letting the liquidation incentive constraint be satisfied after a crisis, but does not guarantee that a safe choice is induced ex ante. This is because the choice depends on on the relative gains of the occurrence of a conversion: whether \( rC < R_s - s^3 R_r \). That is, in order for CoCos to be effective deterrents ex ante, there must not be too many of them to begin with. *However, for CoCos to be useful at \( t = 1 \) in terms of loss absorption capacity, there must be sufficiently many of them.* It is alarming that safe choices are induced only when \( rC \) is small. Therefore CoCo issuance may actually be inviting risk shifting at \( t = 0 \). While CoCos undoubtedly increase loss absorption capacity ex post, they may encourage risk-shifting ex ante.

### 4.7 Conclusion

CoCos are perceived to be promising for increasing the loss-absorption capacity of banks. However, the manner of their conversion leaves room to be desired - in addition to the conversion based on the book value of the bank’s equity, there is also conversion based on regulatory discretion. While the literature has considered regulatory forbearance, it has not done so in the context of CoCos. Others in the literature have espoused that CoCos are very good as commitment devices. However, precisely because the conversion is not really automatic, in the sense that book values have a delay, and that regulators do have discretion and also bear some costs of conversion, we argue that CoCos will only be converted in a limited set of circumstances.

We have modeled a sequential three period game between the regulator and the bank. The bank can choose between safe and risky assets at the start, and are potentially subject to a shock at the next period. Based on the severity of the shock, the regulator can decide to convert the CoCos, or forbear on the conversion. The bank can then choose between liquidating the risky assets, and gambling for resurrection. However, the bank’s choice rests on whether its liquidation incentive constraint is met. It turns out that the type of shock matters: the constraint is loosened upon a shock on the probability of obtaining returns from the bad loans, but does not changes when instead it is a shock on the proportion of loans that turned out to be bad. It turns out that the regulator will only convert the CoCos if it makes a difference
in the bank’s choice between liquidation and gambling. In particular, this will only happen if the conversion is enough to make the bank’s skin in the game sufficiently high to do the right thing.

The regulator will only convert the CoCos if in addition to being able to change the bank’s decision, the regulator is able to face the costs of conversion. We have cast the cost of conversion in terms of an increase in the threshold belief of the depositors that are necessary to prevent the occurrence of bank runs. Risk neutral depositors are assumed to have beliefs regarding the bank’s initial choice, as well as on the likelihood of obtaining positive returns on the risky asset, that satisfy an incentive compatibility constraint. However, as conversion will never happen with a safe asset, observing a conversion can only mean that the bank had chosen a risky one instead. This leads the run probability to go up, providing a reason for the regulator to forbear on conversion.

If banks knew the conversion costs that the regulator is facing, the outcomes would be clear. A regulator who faces high conversion costs will never convert a CoCo, leading to the bank choosing a risky asset at the start, and to gamble for resurrection in the event of a shock, in the event that the liquidation incentive constraint is not met. A regulator who faces low conversion costs will always convert a CoCo, but as to whether or not this is sufficient to induce a safe choice at the start depends on whether the liquidation value of the asset ex ante exceeds that of the safe asset. This decision can be recast in terms of how much CoCos are issued at the beginning. One can think of the conversion as delivering a relative gain to the bank equal to the amount of the converted CoCo, and delivering a relative opportunity cost equal to the difference between the returns of the safe asset and the liquidated risky asset.

We find that only when the CoCos are sufficiently few will there be any incentive for the bank to choose the safe asset in the first place. When there are too many CoCos, in a sense we make precise in this paper, the bank anticipates regulatory forbearance, and therefore will find it more attractive to choose the risky asset at the start. This makes CoCos not very convincing in reducing ex ante risk. There is a clear tradeoff between mitigating risk ex ante and improving loss absorption capacity ex post. Only when the bank’s safety net is reduced, in the sense that there will be no significant changes in the equity ratio of the bank post conversion, will the regulator be able to hope that CoCo conversion will act as a deterrent in choosing risky assets to begin with.

The beliefs about the regulator’s type must be managed well. This is because learning about the regulator’s type will influence future decisions of the bank. While we do not model it here, it is a direction for future research. A regulator who cultivates a reputation for forbearance will encourage risky investments, while a regulator who cultivates a reputation for being tough will
encourage safe investments only if the banks gain sufficient skin in the game after a conversion. It becomes important to manage the regulator’s reputation in order to influence the beliefs of the bank as well.

Seen from a post-crisis perspective, the conversion of CoCos aligns the incentive of the bank with the incentive of the regulator. However, this treats the asset choices as a given. Moving forward, the banks are at liberty to rebalance their portfolio as they see fit, taking into account the type of regulator that they are dealing with. Therefore, there must be some merit in keeping the regulator type opaque in order to induce safer choices. However, given that there are limited circumstances in which the conversion will be useful, it seems that the CoCos were created with the forbearing regulator in mind.

Appendix for Chapter 4

4.A The impact of dilutive CE CoCos on a bank’s $t = 1$ and $t = 0$ decision

The bank’s $t = 1$ decision on whether to gamble for resurrection or liquidate bad assets depends on its skin in the game, which means that it depends on the bank’s outstanding liabilities at that time. When the CoCo is of the principal writedown type, the conversion of the CoCo immediately eliminates a part of the bank’s liability, without altering who owns the residual equity. This feature of the principal writedown CoCo allows us to use the liquidation incentive constraint described in (4.11) to determine whether CoCo conversion would be useful in changing a bank’s decision. This is because the liquidation incentive constraint is cast purely in terms of the threshold liability required in order to induce a certain decision.

When the CoCo is of the convert-to-equity variety, it is more complicated, because it alters the share held by the original equity holder. Such CoCos are dilutive in the sense that each unit of CoCo is transformed to some share of equity. However, since the regulator’s decision to convert precedes the bank’s second decision point, the degree of dilution does not matter. As such, the original equity holders make their decision based on the liquidation incentive constraint (4.11). On the other hand, dilution might matter for $t = 0$ decisions.

Let the conversion rate faced by the CoCo holders be $\psi$, which we call the dilution parameter. Conversion transforms the CoCos from $rC$ liabilities into $\psi rC$ equity. This means that the equity holders get rid of the CoCo liability, but must share with the CoCo holders-turned-equityholders. We normalize the equity held by the original shareholders to 1. Since the regulator only converts the CoCo to induce the bank to choose liquidation, the bank will
only liquidate when
\[
\frac{s^3 R_r - rD}{1 + \psi r C} \geq \frac{s (R_r - rD - rC)}{1 + \psi r C}.
\]  
(4.52)

Therefore, in assessing which asset to choose at \( t = 0 \), the values that will be carried over will be the diluted ones. Suppose that liquidation is more attractive than gambling for resurrection. Then the bank would only choose the safe asset if
\[
R_s - r (D + C) > \frac{s^3 R_r - rD}{1 + \psi r C}.
\]  
(4.53)

In Section 4.6.3, we have considered two possibilities about \( s^3 R_r - rD \) and \( R_s - r (D + C) \). If there were relatively few CoCos, it would be that \( R_s - r (D + C) > s^3 R_r - rD \) in which case, any nonnegative value of \( \psi \) can induce a safe choice ex ante, as can be seen from (4.52).

However, if there were relatively many CoCos, it follows that \( R_s - r (D + C) < s^3 R_r - rD \). We can find a value of \( \psi \) that is enough to cause the inequality in (4.53) to just bind, as in (4.54):
\[
\psi \geq \frac{1}{rC} \left[ \frac{s^3 R_r - rD}{R_s - r (D + C)} - 1 \right] = \bar{\psi}.
\]  
(4.54)

It is easier to convince a bank to choose the safe asset at \( t = 0 \) if it has to share the gains with the new shareholders if the regulator had to convert. One can also find a threshold that makes the safe asset more attractive than gambling for resurrection, by finding \( \psi \) that solves
\[
R_s - r (D + C) > \frac{s (R_r - rD - rC)}{1 + \psi r C}.
\]

Note that a high dilution parameter is not a solution for regulatory forbearance. It only works if the bank believes that the regulator is willing to convert the CoCo, which she will do only if her costs of conversion is low enough.

### 4.B Calculations for various results in the chapter

#### 4.B.1 Calculation for (4.32)

In the event of a \( p \) shock, the bank’s debt threshold is
\[
B \leq \left( \frac{\lambda - p'}{1 - p'} \right) R_r = B_p^*.
\]
Writing \( p' = p - \delta \), we may write \( B_p^* \) as follows.

\[
B_p^* = \left( \frac{\lambda - (p - \delta)}{1 - (p - \delta)} \right) R_r
= \left[ \left( \frac{\lambda - p}{1 - (p - \delta)} \right) + \left( \frac{\delta}{1 - (p - \delta)} \right) \right] R_r
= \left[ \left( \frac{\lambda - p}{1 - p} \right) \left( \frac{1 - p}{1 - \delta} \right) + \left( \frac{\delta}{1 - (p - \delta)} \right) \right] R_r
\]

Recall \( B^* \) as defined in (4.12). Since \( B^* \) is decreasing in \( p \), \( B_p^* > B^* \). Let \( \Delta = B_p^* - B^* \). We can then write

\[
\Delta = \left[ \left( \frac{\lambda - p}{1 - p} \right) \left( \frac{1 - p}{1 - \delta} \right) + \left( \frac{\delta}{1 - (p - \delta)} \right) \right] R_r - \frac{\lambda - p}{1 - p} R_r
\]

leading to the expression in (4.32).

4.B.2 Calculation for (4.38)

The difference between the payoffs of forbearance and conversion for the safe asset may be written as

\[
\frac{[(1 - \bar{\alpha}) R_s + \bar{\alpha} (R_s - 1)] - [(1 - \bar{\alpha}') R_s + \bar{\alpha}' (R_s - 1)]}{\text{forbearance}} - \frac{[(1 - \bar{\alpha}) R_s + \bar{\alpha} (R_s - 1)] - [(1 - \bar{\alpha}') R_s + \bar{\alpha}' (R_s - 1)]}{\text{conversion}}
\]

\[
= R_s \left[ 1 - \bar{\alpha} - 1 + \bar{\alpha}' \right] + (R_s - 1) (\bar{\alpha} - \bar{\alpha}')
\]

\[
= R_s (\bar{\alpha}' - \bar{\alpha}) - (R_s - 1) (\bar{\alpha}' - \bar{\alpha})
\]

\[
= (\bar{\alpha}' - \bar{\alpha}) [R_s - R_s + 1]
\]

\[
= \bar{\alpha}' - \bar{\alpha}.
\]
Bibliography


Basel Committee on Banking Supervision. Proposal to ensure the loss absorbency of regulatory capital at the point of non-viability. 2010.


Stephanie Chan and Sweder van Wijnbergen. Regulatory forbearance in the presence of cocos. mimeo, March 2017b.


Contingent convertible capital (CoCos) are hybrid instruments that are designed to improve the loss absorption capacity of the issuer without involving injections of new equity or taxpayer bailouts. Because they are relatively new, their properties must be critically examined, as there may be undesirable and unexpected consequences. This dissertation is composed of three chapters (all jointly written with Sweder van Wijnbergen), that explore the impact of issuing CoCos for the financial system, and for financial regulation as a whole.

In Chapter 2, "CoCos, Contagion, and Systemic Risk," we explore how the news of conversion triggered by a regulator affects the behavior of depositors in the banking system. In our model, the omnipotent regulator forces the conversion of CoCos when she obtains information that the bank is unlikely to remain viable given the economic state. Therefore, conversion never delivers good news, and results in a higher probability of a bank run. As bank runs are observable ex post, they are contagious if the banks in the system have highly correlated assets. Such a result leads one to wonder whether it is ever credible for a regulator to convert the CoCos, given that doing so may lead to higher financial fragility.

Chapter 3, "CoCos, Risk-Shifting and Financial Fragility," explores how CoCos potentially worsen financial fragility, as they encourage banks to choose higher risk levels than they otherwise would with regular debt instruments. This is because the CoCo-issuer’s residual equity has an expected wealth transfer component which is absent from the residual equity of non-CoCo-issuers. For certain CoCo designs, we find that the expected wealth transfer is increasing in the risk level chosen by the bank. Therefore, whenever banks maximize their expected returns net of default costs, they would always choose higher risk levels under these types of CoCos than under the same amount of subordinated debt, or additional equity. The policy implication is that one cannot treat CoCos as true substitutes for equity, because they induce different incentives despite having the same loss-absorption capacity.

Finally in Chapter 4, "Regulatory Forbearance in the Presence of Cocos," we explore whether it is ever credible that CoCos will be converted by the regulator in times of crises. CoCo conversion is essentially a tool used by the regulator to nudge the bank into choosing the socially
optimal choice after the occurrence of a negative shock. However, the bank's choice will only be aligned with the socially optimal one if its skin in the game is sufficiently high. Therefore, conversion is only effective if the amount of CoCos converted is sufficiently large. Otherwise, the threat of conversion may not lead the bank to make a safe choice at the beginning of the game. As a result of this limited effectivity, the regulator is likely to forbear on the conversion of the CoCos.
**Nederlandse Samenvatting**

Contingent Convertible Capital (CoCos) zijn hybride schuldinstrumenten, ontworpen om de verstrekker meer mogelijkheden te geven verliezen op te vangen zonder dat een injectie van nieuw kapitaal of een bail-out door de belastingbetaler nodig is. Omdat het relatief nieuw schuldinstrumenten zijn, moeten de eigenschappen kritisch worden onderzocht, daar er mogelijk onwenselijke en onverwachte consequenties zijn. Deze dissertatie bestaat uit drie artikelen (allen gezamenlijk geschreven met Sweder van Wijnbergen), die de impact van het uitvaardigen van CoCos op het financiële systeem en de financiële regelgeving in het geheel bestuderen.

In Hoofdstuk 2, "CoCos, Contagion, and Systemic Risk," onderzoeken wij hoe het nieuws van geïnitieerde conversie door de toezichthouder het gedrag van spaarders beïnvloed in het bancairesysteem. In ons model dwingt de almachtige toezichthouder de conversie van CoCos af als hij informatie krijgt dat de bank waarschijnlijk niet levensvatbaar blijft, gegeven het economische klimaat. Daarom betekent een conversie nooit goed nieuws en resulteert dit in een bankrun. Omdat een bankrun ex-post zichtbaar is, is een bankrun zeer besmettelijk als de banken in het systeem zwaar gecorreleerde activa hebben. Deze uitkomst doet afvragen of een conversie van de toezichthouder überhaupt geloofwaardig is, gegeven dat een conversie tot meer financiële kwetsbaarheid leidt.

Hoofdstuk 3, "CoCos, Risk-Shifting and Financial Fragility," onderzoekt hoe CoCos potentieel de financiële kwetsbaarheid vergroten, omdat ze banken aanmoedigen meer risico te nemen dan met standaard schuldinstrumenten. Dit is omdat het eigen vermogen van banken die CoCos uitgeven een verwachte waarde overdracht component heeft, terwijl die afwezig is bij het eigen vermogen van banken die geen CoCos uitgeven. Voor sommige CoCo-types vinden we dat de verwachte waarde overdracht toeneemt met het gekozen risico niveau van de bank. Daarom, als banken hun winst maximaliseren netto de kosten van het faillissement, zullen zij altijd kiezen voor een hoger risico niveaus met dit type CoCo dan wanneer zij dezelfde hoeveelheid achtergestelde schulden of extra vermogen hebben. De beleidsimplicatie is dat men CoCos niet als vervangers voor vermogen kan beschouwen, omdat zij tot andere prikkels
leiden ondanks dat ze dezelfde hoeveelheid verlies kunnen absorberen.

Tot slot, onderzoeken wij in Hoofdstuk 4, "Regulatory Forbearance in the Presence of Co-Cos," of het in tijden van crises ooit geloofwaardig is dat CoCos door de toezichthouder worden geconverteerd. De conversie van CoCos is eigenlijk een schuldfinancieringsinstrument dat door de toezichthouder gebruikt wordt om de banken aan te zetten voor het kiezen van het maatschappelijke optimum nadat een negatieve schok heeft plaats gevonden. Echter, de keuzen van een bank is alleen identiek aan die van het maatschappelijke optimum als de bank zelf genoeg te verliezen heeft. Daarom is conversie alleen effectief als de hoeveelheid geconverteerde CoCos voldoende hoog is. Anders zal de dreiging van een conversie er mogelijk niet toe leiden dat de bank een veilige keuze maakt aan het begin van het spel. Als gevolg van deze beperkte effectiviteit, zal de toezichthouder waarschijnlijk afzien van de conversie van CoCos.
The Tinbergen Institute is the Institute for Economic Research, which was founded in 1987 by the Faculties of Economics and Econometrics of the Erasmus University Rotterdam, University of Amsterdam and VU University Amsterdam. The Institute is named after the late Professor Jan Tinbergen, Dutch Nobel Prize laureate in economics in 1969. The Tinbergen Institute is located in Amsterdam and Rotterdam. The following books recently appeared in the Tinbergen Institute Research Series:

643 R.H. KLEIJN, Essays on Bayesian Model Averaging using Economic Time Series
644 S. MUNS, Essays on Systemic Risk
645 B.M. SADABA, Essays on the Empirics of International Financial Markets
646 H. KOC, Essays on Preventive Care and Health Behaviors
647 V.V.M. MISHEVA, The Long Run Effects of a Bad Start
648 W. LI, Essays on Empirical Monetary Policy
649 J.P. HUANG, Topics on Social and Economic Networks
650 K.A. RYSZKA, Resource Extraction and the Green Paradox: Accounting for Political Economy Issues and Climate Policies in a Heterogeneous World
651 J.R. ZWEERINK, Retirement Decisions, Job Loss and Mortality
654 D.A.R. BONAM, The Curse of Sovereign Debt and Implications for Fiscal Policy
655 Z. SHARIF, Essays on Strategic Communication
656 B. RAVESTEIJN, Measuring the Impact of Public Policies on Socioeconomic Disparities in Health
657 M. KOUDSTAAL, Common Wisdom versus Facts; How Entrepreneurs Differ in Their Behavioral Traits from Others
658 N. PETER, Essays in Empirical Microeconomics
659 Z. WANG, People on the Move: Barriers of Culture, Networks, and Language
660 Z. HUANG, Decision Making under Uncertainty-An Investigation from Economic and Psychological Perspective
661 J. CIZEL, Essays in Credit Risk, Banking, and Financial Regulation
662 I. MIKOLA lineNumber, Empirical Essays in International Economics
663 J. BAKENS, Economic Impacts of Immigrants and Ethnic Diversity on Cities
664 I. BARRA, Bayesian Analysis of Latent Variable Models in Finance
665 S. OZTURK, Price Discovery and Liquidity in the High Frequency World