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Dirk Damsma, 15 October, 2009

3. *Marx's Systematic Dialectics and Mathematics and their Articulation in his
'Schemes of Reproduction'*

Introduction

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Introduction

Can mathematics or mathematical modeling play a role within a systematic dialectical presentation? The answer to such a question depends first of all on the ontological nature of the subject under investigation and the premises from which the investigation starts (in systematic dialectics the two are intimately intertwined). Section 1 shows that Hegel and Marx's views were diametrically opposed regarding the latter. The ultimate abstraction of civil society to Hegel was 'Free Will', whereas Marx's analysis of capitalism starts with 'capitalist production' (*dissociation*). Marx's starting point immediately relegates 'Free Will' to the margins of the capitalist system and as a result, the abstractions pertaining to capitalism are more amenable to quantification and hence more suitable for mathematical treatment, than those pertaining to Hegel's (reconstruction of) civil society.

Secondly, the answer depends on the epistemological prowess one ascribes to mathematics as a means of investigation. Chapter 2 showed that Hegel did not think highly of mathematics in this respect: mathematical concepts and techniques to him are one-sided devices that need to be reconciled with the qualitative before real headway towards the actual truth can be made. The previous literature on Marx and mathematics discussed in section 2, by contrast, shows not only that Marx towards the end of his life had become quite conversant with the university textbooks on mathematics of his day, but also that he endeavored to reform the basis for mathematics (especially the calculus) dialectically and toyed with a lot of ideas for the application of mathematical and formal methods to his main studies in political economy. Thus his attitude in this respect is strikingly different from Hegel's who contended that mathematics cannot adopt dialectical moments at all.

Having established Marx's views on the nature of capitalism and the abstractions appropriate to it, as well as his attitude towards the dialectics of mathematics and the use of mathematical techniques within a systematic dialectical presentation of capitalism, the question becomes how these ideas could

be articulated together. In order to answer this second main question for this chapter, section 3 first of all tracks the outline of Marx's dialectics throughout the three volumes of *Capital*, so as to position Marx's 'schemes of reproduction' within Marx's overall framework and illustrate the nature of Marx's abstractions. Next, section 4 discusses what assumptions and formal expressions in these models can be considered dialectically motivated and which cannot. Thus it is shown that the inspiration for them can be conceived of as dialectically informed and that their results also illuminate how further concretization could proceed. Section 5 concludes.

1. Marx's Subject of Investigation and his Systematic Dialectics

This section will place Marx's subject of investigation in the context of Hegel's *Encyclopädie* (1830³, 1817¹).¹ This will give us a handle on the nature of Marx's abstractions and the dialectic appropriate to them. This, in turn, is relevant for interpretations of Marx's method discussed later on.

The *Encyclopädie* is divided first into parts I, II and III and then into subdivisions ('Abteilungen') 1, 2 and 3. These in turn are subdivided first into sections A, B and C and usually next into subsections a, b, and c. Finally, some of the subsections are subdivided into α , β and γ . The parts, subdivisions, sections and subsections relate to each other in very much the same way as α , β and γ do. Thus, as we saw in chapter 2, part I, the Logic ('the science of the Idea in and for itself' (Hegel 1830³, 1817¹: §18, Geraets, Suchting and Harris' 1991 translation)), relates to thoughts and concepts considered in isolation of whatever might be out there in the world, whereas part II, the philosophy of nature ('the science of the Idea in its otherness' (Hegel 1830³, 1817¹: §18, Geraets, Suchting and Harris' 1991 translation)) relates to the world out there only. The tension between the

¹ Superscripts behind a publication year denote editions. The edition that was actually used is always cited first. Thus (1830³, 1817¹) means that the current text relies on the third edition of the *Encyclopädie* and that the first edition of that work was published in 1817.

inherent freedom of thought and the material restrictions of nature is resolved in part III, the philosophy of mind, or, in Hegelian terms, the science ‘of the idea that returns into itself out of its otherness’ (Hegel 1830³, 1817¹: §18, Geraets, Suchting and Harris’ 1991 translation).

If we turn to the subdivisions of part I, we find it consists of 1) the doctrine of Being (‘die Lehre vom Sein’), 2) the doctrine of Essence (‘die Lehre vom Wesen’) and 3) the doctrine of the Notion (‘die Lehre vom Begriff’) (Hegel 1830³, 1817¹: §83).² The conceptual progression in the first of these doctrines is a result of the intellect’s failed attempt to get to grips with everything at once. It comprises A) Quality (Hegel 1830³, 1817¹: §86-98), B) Quantity (Hegel 1830³, 1817¹: §99-106) and C) Measure (Hegel 1830³, 1817¹: §107-111).³ All we can say at such an abstract level about the Quality of Being is that it consists of a manifold of indeterminate Ones upon which we can only externally and arbitrarily reflect, turning it into Quantity. To get rid of the arbitrariness, a Qualitative Quantum is required: Measure (see Chapter 2 for a lengthy elaboration of this argument).

Exactly what type of concepts one needs to get to grips with more determinate qualities is the subject of the doctrine of Essence. Not that any specific qualities

² All of these translations are a bit tricky. Although the German ‘Lehre’ is always translated as doctrine (e.g. Geraets, Suchting and Harris 1991; Wallace 1873; Carlson 2003: 8; Inwood 1992: 268), the German term is much more neutral, for it does not carry the connotation of ‘indoctrination’ with it at all. Instead, it is derived from ‘lernen’, the German for ‘learning’ or – sometimes – ‘teaching’. The German ‘Wesen’ (ibid.) refers to what you might call ‘essential Being’, the nature of something, as well as to unidentified bodies and beings, such as the building trade (das Bauwesen) or a God the speaker does not believe in (das Überwesen). Thus ‘Wesen’ necessarily implies some elusiveness. This is not the case with ‘Essenz’. This important distinction is lost in English. Finally, ‘Begriff’ is derived from the German for understanding: ‘begreifen’ (‘to grasp’ literally). In its various translations as ‘concept’ (e.g. Arthur 1993: 64; Geraets, Suchting and Harris 1991; Inwood 1992: 58; Smith 1993: 29) or ‘notion’ (e.g. Wallace 1873; Arthur (!) 2002: 47) this connotation, if not lost, is at least severely downplayed, for noting or conceptualizing implies more of a dim *awareness*, than an *understanding* of the matter at hand. In what follows, I will follow Arthur (2002) in using ‘concept’ to denote concepts in general and ‘notion’ when referring to Hegel’s ‘Begriff’ (2002: 45-47).

³ In this chapter, concepts that are dialectically important to Hegel will always be written with a capital letter, enabling the reader to see whether a word is used dialectically or not. In German, all nouns are written with a capital letter. So, this practice (although common among native English speaking Hegelians) has no warrant in German (Inwood 1992: 6). However, since this linguistically questionable convention usually clarifies dialectical presentations significantly, I will adopt it here. To avoid confusion between Hegel’s moments and Marx’s, Marx’s will be stressed by *italicizing* them.

can be invoked at such an abstract level yet, but the kind of concepts required to allow for a reentrance of qualitative distinctions are identified and systematized at this level. In overview this doctrine is concerned with A) elusive, hidden Essence (how things are) (Hegel 1830³, 1817¹: §115-130), B) Appearance (Hegel 1830³, 1817¹: §131-141) and C) Actuality (Hegel 1830³, 1817¹: §142-159). Whilst at many occasions appearance may very well be all we got, it is only when it is mediated by some theory on Essence, that we understand the laws of self-development of the actual. So while Essence categories are applicable to objects, Essence is fundamentally elusive at the same time.

When objective, but elusive Essence is mediated by subjective thoughts on Being as a whole, in principle we have concretely applicable Notions. Again, at the level of the Logic, the language refers to the type of concept, not to any concrete embodiment of it. This final subdivision of the logic consists of: A) Subjective Understanding (Hegel 1830³, 1817¹: §162, §163-193), B) the Object (Hegel 1830³, 1817¹: §162, §194-212) and C) the Idea (Hegel 1830³, 1817¹: §162, §213-244).⁴ As such it indicates how our Ideas are mediated and constrained by the objective givens of the world as well as by the subjective potential for thought.

The general conceptual distinctions of the Logic are applicable to Hegel's philosophical system as a whole as well as to its subfields considered in themselves. Since the doctrine of Being relates to subjective thought only, the type of concepts found in it best describe the Logic relative to the other two principal spheres. Considered in itself, the Logic of course displays a dialectic of 1) Being, 2) Essence and 3) Notion. Similarly, since the Philosophy of Nature relates to objective material things out there that exist independently of our thoughts (and hence may be fundamentally misrepresented in thought – as amply illustrated in the history of science (cf. e.g. Bryson 2003)), it is best described in

⁴ Based on Hegel's more elaborate exposition of the Logic in his *Wissenschaft der Logik*, Smith writes: "This part of the Logic is divided into the subjective idea, the objective idea, and the absolute idea" (1993: 29). I do not know where he got this idea (no pun intended), for at the level of the Notion as a whole, I-3, we find: A) Subjectivity, B) Objectivity, and C) the Idea (Hegel 1812, 1813, 1816: II.1-3). So the Idea only enters the stage at level I-3-C. Perhaps Smith's account relates to this level, then? Well, no, because here we find: a) Life, b) the Idea of Cognition, and c) the Absolute Idea (Hegel 1812, 1813, 1816: 82-83, II.3).

terms of Essence categories. When considered in themselves, these categories again display a dialectic of 1) essential Being (comprising Space and Time, Matter and Movement and Absolute Mechanics) (Hegel 1830³, 1817¹: §252, §253-271), 2) Essence (physics) (Hegel 1830³, 1817¹: §252, §272-336) and 3) essential Notions (organic physics or biology) (Hegel 1830³, 1817¹: §252, §337-376). Finally, and in the same vein, the Philosophy of Mind is supposed to resemble a structure of Notions. As will be expected by now, these in turn relate to 1) subjective Being (comprising Anthropology, Phenomenology of the Mind and Psychology) (Hegel 1830³, 1817¹: §385, §387-482), 2) Spirit objectified as Essence (society) (Hegel 1830³, 1817¹: §385, §483-552) and 3) absolute spirit (with philosophy as its ultimate notion) (Hegel 1830³, 1817¹: §385, §553-577). Figure 1 summarizes and schematizes the above. As such it is essentially a condensed version of the table of contents of the *Encyclopädie*.

I.	<u>The Logic</u>
1.	<i>The Doctrine of Being</i>
	A. Quality
	B. Quantity
	C. Measure
2.	<i>The Doctrine of Essence</i>
	A. Essence (as ground of Existence)
	B. Appearance
	C. Actuality
3.	<i>The Doctrine of the Notion</i>
	A. Subjective Understanding
	B. The Object
	C. The Idea
II.	<u>The Philosophy of Nature</u>
1.	<i>Mechanics</i>
2.	<i>Physics</i>
3.	<i>Biology</i>
III.	<u>The Philosophy of Mind</u>
1.	<i>Subjective Spirit</i>
2.	<i>Objective Spirit</i>
3.	<i>Absolute Spirit</i>

Figure 1. An overview of Hegel's Encyclopädie

Representations of Marx's systematic dialectics draw on the categories in the Logic in various ways. Arthur holds that Marx's representation of Capitalism roughly parallels Hegel's Logic. He writes: "The movement from exchange to value parallels his Doctrine of Being; the doubling of money and commodities parallels the Doctrine of Essence; and capital, positing its actualization in labor and industry, as absolute form claims all the characteristics of Hegel's Concept [i.e. Notion, in my and Arthur's later (2002: 45-47) terminology]" (Arthur 1993: 65). Thus, Arthur views the *exchangeability* of commodities as their Quality, the ratio of exchange determined in the *bargain* as their Quantity and *value in exchange* as their Measure (Arthur 1993: 73-77, 87).

Value is an Essential condition for commodities' exchangeability, but since it is not an inherent property of commodities, it Appears only fleetingly in the act of *exchange* unless *money* Actualizes it (which it must in the face of generalized exchange) (Arthur 1993: 78-82, 87). Given *money* the (Notion of) *price* can be Subjectively determined quite independently of any individual bargain. Thus, money first and foremost functions as a *measure of value*. However, this value is only Objectively realized when sales actually commence, thus allowing the seller to buy the commodity s/he actually desired in the first place. This gives us the circuit of Commodity (C) – Money (M) – different Commodity (C'). Secondly then, money is a *means of circulation*. But since both C and C' are usually consumed they cannot reenter circulation. Thus, this circuit is at a constant risk of breaking down. Therefore the Idea of money can only be fully developed if it actualizes itself as the *end of exchange*, so that we end up with the circuit M – C – M' which determines *capital* as 'money which begets money' (Marx 1890⁴, 1867¹: Ch. 4: 170; 1867F: 256). Thus, in its abstraction *capital* is posited as self-valorizing (Arthur 1993: 82-84, 87).

It takes Marx only two subdivisions in a space comprising less than a fifth of *Capital I* to make the move from *exchange* to *capital* outlined above. Arthur

therefore concludes that the dialectical progression in the rest of volume I and volume II and III is best represented as a dialectic of Notions (Arthur 2002: 47). More specifically, since according to Arthur capital can already be considered as the Idea of money, most of the dialectical progression in *Capital* must be concerned with Ideas in the Hegelian sense. These, he writes, are best represented in terms of the contrast between Universality ('Allgemeinheit') and Particularity ('Besonderheit') and its resolution in Individuality ('Einzelheit') (Arthur 2002: 47). These terms supposedly represent how the three volumes of *Capital* relate to each other as well as how each volume is organized (Arthur 2002: 48-49). Thus, the distinctions between Universality, Particularity and Individuality are applicable to *Capital* as a whole as well as to each of its subfields considered in themselves in much the same way as the general conceptual distinctions of the Logic are applicable to Hegel's philosophical system as a whole as well as to its subfields.

However, Arthur is very critical of Hegel. He accuses him of thinking that 'the Idea creates Nature' (Arthur 2003: 195). Clearly, if this is taken to mean that the world will automatically conform to whatever we think about it, there is no need whatsoever to do any empirical research adjusting our ideas to the world. This type of upside-down ontology *may* have *some* relevance for 'thinking about thinking', that is at the level of the Logic, but it is unlikely to be applicable to Nature (at level II) or the Mind (at level III) (Arthur 2003: 195-196). But Capitalism is an unlikely system that allows pure abstract thought (*value*) to gain material reality (as *money*). Hegel's Logic, then, is applicable to Capitalism, only because capitalism creates an inverted reality in which thought can indeed be said to preside over matter (Arthur 1993: 64). As long as this inverted reality is considered in itself, the Idea of *capital* in general can become self-subsistent relative to *many capitals*. But as soon as the inverted reality of capital is left and one tries to incorporate concrete people - and not just their value-expressions (such as wages and productivity) - in the system, problems arise, because people may not want to be treated as another means of production. So they may rebel.

These problems are similar to the problems Hegel runs into when he wants to make the transition from the Logic to the Philosophy of Nature. That is, Nature is independent of thought and hence it may 'rebel' against our classifications, just like labor may rebel against its treatment as a determinant of value only (Arthur 2003: 196-197). In short, Arthur argues that Hegel's confusion led Marx to his correct presentation of Capitalism as an inverted reality. Thus, by staying very close to Hegel regarding his *method* of presentation, Marx actually dismissed the *content* of Hegel's philosophical system.

Smith's reading of *Capital* is much more favorable to Hegel. He does not grant that Hegel was unconcerned with empirical reality. On the contrary, he claims that Hegel and Hegelians as well as Marx and Marxists must appropriate their concepts from elaborate empirical studies before dialectical representation can (re)commence. Marx and Marxists refer to this preliminary empirical kind of research as exploration ('Forschung') (Reuten 2000: 143). Hegel and Hegelians speak of the method of the understanding ('die Methode des Verstandes') (cf. Hegel, 1817¹, 1830³: §259). So, first there is the world, second our preliminary partial categorization of it, and only when our empirical studies of a field are exhausted (a process that may take up the better part of one's academic life or at least his undergraduate years) can we flesh out how these provisional categorizations are interrelated using systematic dialectics (Smith 1990: 3-8). But even then, the systematic dialectician, when stuck, might need to revert to exploratory types of research in order to gain a better understanding of his categories, and concomitantly their systematic dialectical interrelationships. So Hegel did not simply think the world would eventually conform to our ideas, but rather that, when we have done all we can to bring our ideas in agreement with reality, the ideas that can be shown to be systematically interrelated stand a greater chance of approximating the truth than those that resist efforts at systematization. Consequentially, Smith does not entirely reject Hegel's Philosophies of Nature and the Mind, although he is critical of a lot of its content. But, other than Arthur, he does not dismiss these philosophies for being

constructed to fit a normal reality on the basis of an upside-down ontology applicable to the inverted reality of capitalism only.

At the same time, Smith agrees with Arthur that capitalist abstractions are rather quaint in that they are both real and ideal (Smith 1990: 40-41, 93-94), but in his opinion this is not the only type of abstraction susceptible to a dialectical treatment. Thus, whereas Arthur thinks that Hegel's Logic is useful only as a guide to the presentation of capitalism, Smith argues that the whole of Hegel's system (as laid out in his *Encyclopädie*) has some merits of its own, for example with respect to properly positioning Marx's *Capital* vis-à-vis other scientific fields. As a social theory, Marx's *Capital* presupposes subjective thought and malleable but essentially unchanging Nature and thus falls entirely on level III, the Philosophy of Mind. So, relative to science as a whole, Notion categories are applicable here (Smith 1990: 18). Within this Philosophy however, the study of society belongs to level III-2. So, relative to other fields that study humans, be it the human mind (III-1) or human expression (III-3), Smith argues that Essence categories are most applicable. 'However within the realm of objective spirit [III-2] "civil society" is a determination on the level of ethical life (*Sittlichkeit*) [III-2-C] as opposed to abstract right [III-2-A] and morality [III-2-B]. As such it is a structure to which notion categories are applicable' (Smith 1990: 18). But at the level of ethical life itself, civil society (III-2-C-b) stands over and against the family (III-2-C-a), and it is not out to harmonize the two factions if conflicts were to arise. In Hegel's opinion, the latter is the task and the *raison d'être* of the State (III-2-C-c). So, in the last instance, capitalism, as a form of civil society, is best described in terms of Essence categories (Smith 1990: 18).

As far as his *mode* of representation is concerned, Smith opts for a much more general scheme than Arthur. Instead of looking for parallels between the categories in Hegel's doctrine of Essence and Marx's *Capital*, he presents the dialectic of *Capital* in terms of a movement from unity to difference to unity-in-difference. An abstract category unifies a multitude of particulars. Some stress what the particulars have in common (their unity), some what sets them apart

(their difference) and some explicate both together (unity-in-difference). As one moves from unity to difference and on to unity-in-difference the structure becomes more complex and the categories employed more concrete (Smith 1990: 5-6). Thus, these headings describe the general characteristics of *every* abstraction, not just of abstractions that belong to a certain Doctrine in the Logic. By implication, when a scheme like Smith's is adopted, whether there are clear parallels between Hegel's Logic and Marx's Capital or not, is immaterial to the mode of presentation.

Although Hegel's Philosophy of society is on the same plane as Marx's (from Smith's point of view at least), their content is very different. In contrast to Marx, Hegel is hardly concerned with the material conditions of production, but rather with the articulation of α) an individual's Free Will, given that its expression is limited by β) other people's Free Will and thus is γ) a Possibility only (Hegel 1821: §4-7; cf. Hegel 1830³, 1817¹: §382, §487; cf. Chapter 2). In overview, this possibility is actualized as A) universal abstract Right (of which property right is the most prominent constituent), B) individual Morality and C) Ethical Life ('Sittlichkeit'). Hence it analyzes the political and ethical dimensions of a truly liberal society, rather than the extent to which the society we actually live in lives up to this ideal. Marx's starting point, by contrast, is his observation that capitalist specialized production, predicated on a historically given division of labor, can only work when inputs and outputs are generally exchanged in the economic domain.⁵ Hence, on close inspection, the *exchange relation* appears to be the true starting point for his systematic dialectical presentation of capitalism (Arthur 1993: 72; Smith 1990: 67-68). Though this starting point appears to be much more historically specific than free Will, both Hegel and Marx contend that 'philosophy is its own time apprehended in thoughts' (Hegel 1821: 15; cf. Smith 1990: 4; cf. Smith 2003: 187). To Marx and Marxists, it is the task of historical materialism to

⁵ I purposely use the term 'capitalist specialized production' (or specialization for short) where most people would just speak of division of labor in a Smithian sense, because theoretically there can be division of labor, predicated upon specialized laborers, without the units of production also specializing in specific products. Thus, division of labor is only a necessary, but not a sufficient condition of existence of generalized exchange.

distinguish between ‘transhistorical notions’ that belong to *all* times, and ‘historically specific categories’ that belong to the *theoretician’s* time. Systematic dialectics should appropriate the latter if it is to represent ‘its own time’ adequately (Reuten 2000: 141). Though Hegel and Hegelians are less outspoken about this distinction and consequentially less adamant about the proper type of concepts to use, they too make use of both types of categories.

Given specialization, one’s produce is bound to differ from one’s means of subsistence, so people *must* enter into exchange relations and there is no guarantee that they would have done so on their own accord anyhow. This is why Marx’s starting point allows for negative results, like exploitation, and Hegel’s does not.⁶ Either way the philosophies at this level (III-2) investigate aspects of society that stand over and above individuals and potentially curtail their Freedom. With Hegel, individual’s actions are curtailed by the need to be at least a little considerate of other people’s freedoms and rights (in order to protect your own), whereas with Marx (i.e. in capitalism) individual freedom is thwarted by the imperative to engage in exchange in order to stay alive.

On close inspection, Smith and Arthur seem to concur that Capitalist societies are characterized by structures out there that individuals are both powerless against and dependent upon. But their convictions are based on a very different reading of both Hegel and Marx. For Smith the powerlessness results from the fact that capitalism in the last instance is best represented as an Essence structure, while Arthur claims that *Capital* – and systematic dialectics generally – can only grasp the interrelations between the materialized abstractions characteristic of capitalism, but is otherwise incapable of making sense of real things and people. So capitalism can only work to the extent that it succeeds in materializing the

⁶ This is what distinguishes Marx not only from Hegel, but also from the economic mainstream. Both Hegel and mainstream economists contend that individuals enter into a bargain, because they feel that the goods they will have after the exchange will make them happier than the goods they originally possessed. The possibility that some enter the exchange relation with nothing to exchange but themselves or may only have command of inedible commodities is thus abstracted from. However, if either predicament is yours, you must exchange at any cost or die from starvation while trying. Thus, pretty much anyone who does not produce or otherwise commands food can only afford desire and happiness after the necessary exchanges have been made. This inexorable logic is missing from Hegel’s and mainstream accounts alike.

abstractions it is predicated upon in the world. Because people are not materialized abstractions, the part they play in this process is capitalism's Achilles heel.

Smith's argument points to a strong parallel to the Philosophy of Nature (level II), because nature is the Essence structure *pur sang*. This parallel is relevant for the mere possibility of quantification. One of the reasons quantitative methods are successful in the natural sciences is that volition and subjectivity are neither present nor assumed; so that behaviors are law-like and subsuming a particular phenomenon under a law is considered satisfactory as an explanation. Since in Marx's system individual humans are powerless against capitalist relations and the concomitant imperatives for survival, quantitative methods are potentially just as adequate for the study of this particular mode of production as they are for the study of the natural world. According to Arthur of course systematic dialectics cannot deal with real people anyhow and must *therefore* distance itself from volition and subjectivity. So although he views capitalism mostly as a notion structure, Arthur's account implies a similar potential for the use of quantitative methods in the study of capitalism as Smith's.

It must be stressed that this powerlessness holds for 'individual individuals' only. If enough people are aware of the nature and institutional basis of capitalism and for whatever reason would want to overthrow it, they have that power. But of course the beneficiaries of capitalism – be it consciously or unconsciously – prevent such awareness from arising and throughout the history of the economics of capitalism, mainstream theory has helped their cause by claiming, like Smith and Ricardo, that the system is 'natural' and therefore inevitable or, as neoclassicals do, that capitalism's self-equilibrating tendencies automatically result in optimally efficient solutions.⁷ Thus, the hidden ideology is that

⁷ Cooptation is one of the tactics consciously adopted to prevent such awareness from arising. It refers 'to the tactic of neutralizing or winning over a minority by assimilating them into the established group or culture' (<http://en.wikipedia.org/wiki/Co-optation>). Many instances of cooptation can be found in pop music (among other fields). Meja's *it is all about the money*, for instance, was produced to become a mega-commercial hit, but yet its lyrics were written to appeal to those feeling uneasiness and discomfort at the commercialization of it all. Thus, the cultural

overthrowing capitalism is either impossible or detrimental to society's wellbeing and ultimately our own. Such powerlessness – be it real or perceived – is absent from Hegel's system for it is entirely predicated upon the very freedom of humans that Marx immediately relegates to the margins of his system. In conclusion, survival – by subsumption to the exchange relation – comes first for Marx, whereas Hegel seems to contend that when Free Will is secured (in the last instance by the State), survival is immanent.

But there is more. Although numbers and mathematical formulae can describe a lot of processes in the natural world, they are externally imposed on it: they do not *constitute* nature. I am aware that such a statement flies directly in the face of scientists that claim that 'mathematics is the grammar of the book of nature' and conclude that nature is inherently (i.e. ontologically) mathematical. To me, this is just as ludicrous as to conclude that a scientific field is constituted by language from the fact that the use of language adds to the field's intelligibility. Of course it does, for language has been developed as a tool to understanding the world and chapter 2 already showed that the concepts on which set theory is based have a qualitative basis in language.⁸ Since set theoretical propositions form the basis of the foundational systems of mathematics, it would be very strange indeed if these structures would have no applications in the world. After all, the world informed

establishment seamlessly incorporates dissident voices and renders them harmless, for someone that makes millions singing that a world that is 'all about the money' has 'got it all wrong anyway' is about as incorruptible as McDonald's advertising vegetarianism. The result of successful cooptation tactics therefore is apathy in the mildly rebellious as well as marginalization of full swing cultural rebels.

But it may very well be that even beneficiaries of capitalism never gave the system *qua system* a second thought or if they did, had not enough historical baggage to be able to imagine any alternative. Finally, they may have lulled themselves into thinking that this is the best of all thinkable worlds, even if deep down they know it is not. Such arguments carry a long way even to convince those who do not benefit from the capitalist status quo, because it is hard to live your life thinking "we would all be better off, if only..." It is much more comforting to count your blessings.

⁸ By saying that set theoretical concepts have a qualitative basis in language, I do not mean to imply that mathematics is just another type of language. For one thing, it is much more rigorous than any other language and since its subject matter is 'external reflection on a multitude of distinguishable yet arbitrarily divisible elements' (Chapter 2: 34), it is entirely free of qualitative considerations in a way that ordinary language can never be. As a result, it can traverse universes way beyond the reach of our imagination (such as the number of elements in $P(R)$, i.e. the power set of R), simply by consistently applying definitions and logical operations.

language and language informed mathematical concepts. So the fact that mathematical structures are applicable to the study of nature is a result of the way these structures came about. It has nothing to do with how nature is constituted.

In capitalism, by contrast, *value* must actualize itself as a certain Quantity of *money* for the mode of production to be viable. This universal monetary value permeates all entities and concepts in the economic domain. Consequentially, all concrete capitalist entities and concepts, like commodity, price, cost, profit, value, etcetera, can also be understood abstractly, as shares or elements in the produce of the system at large (Arthur 1993: 64; Arthur 2004: 79; Smith 1990: 83-94; Smith 1993: 22-23; Reuten and Williams 1989: 60-65). So, this is another reason why quantitative methods have potential in the study of capitalism. Although Hegel acknowledges the necessity of money as the quantitative measure of value, he holds that man imposes this social form on things, instead of the other way round as Marx claims.⁹ So it is human volition that in the last instance determines exchange value (Arthur 1988: 27, 35). Therefore Hegel's view of the matter leaves less scope for mathematically formulated inescapable laws of motion than Marx's.

In short, whereas Chapter 2 showed that Hegel sees qualitative and quantitative reflection as reconcilable ways of *thinking* about the world around us, Marx sees the capitalist world *itself* as being both qualitatively and quantitatively constituted. So quantities are an integral part of the capitalist economy, rather than being externally imposed upon it (cf. Arthur 1993: 64; Smith 1990: 93-94; Smith 1993: 22-23; Reuten and Williams 1989: 65). It is this characteristic of capitalism that enables (mathematical) modeling methodology to be integrated with systematic dialectics all the way through, albeit with regard to the study of capitalism only (that is, amongst the systems that Marx knew of).

⁹ As Marx wrote: 'Men make their own history, but not of their own free will; not under circumstances they themselves have chosen but under the given and inherited circumstances with which they are directly confronted' (Marx 1852: 146 #check original#). So the individual is confronted with a material reality imposing its social form (money) on them, but historically mankind has brought this reality into existence and has the power to overturn it if a powerful enough group of people wishes to do so. As long as that does not happen, however, each individual has to yield to the value imperatives sketched out above (cf. p. 13 above).

In section 3 of this chapter Marx's systematic dialectical presentation is tracked in order to substantiate the mentioned differences between Marx's conceptual abstractions and Hegel's and the associated potential for quantification and the use of quantitative methods. Despite the differences between Smith and Arthur regarding their respective modes of presentation, the categorial *content* of both accounts is very similar, so regarding the relevant moments in *Capital*, I will draw on both accounts. However, they will be presented in accordance with the α - β - γ) format introduced in chapter 2, which, for its general applicability, bears more resemblance to Smith's unity, difference and unity-in-difference than to Arthur's parallelization with Hegel's Logic.

2. Previous Literature on Marx and Mathematics

This section discusses Marx's acquaintance with, views on and technical skill in mathematics. This will serve as a background to his use of mathematics within the systematic dialectical presentation of *Capital* elaborated on in section 3 and 4.

When Marx graduated from the gymnasium of Trier in 1835 'his knowledge of mathematics was considered adequate' (Struik 1948, 1997: 173; cf. Kennedy 1977: 305), but he showed no specific interest in it until after the completion of the manuscript for the *Grundrisse* in 1857-58, when he wrote:

During the elaboration of the economic principles I have been so damned delayed by computational errors that out of despair I undertook again a quick scanning of the algebra. Arithmetic was always alien to me. Via the algebraic detour, however, I catch up quickly. (Marx #1858, 1930: 273, cited in Struik 1948, 1997: 174; and in Kennedy 1977: 305; cf. Matthews 2002: 6-7; cf. Smolinski 1973: 1193).

From then on, Marx kept 'returning to [the study of mathematics] as a diversion during his many days of illness' (Struik 1948, 1997: 174), turning from algebra to analytical geometry and the calculus (Struik 1948, 1997: 174). Despite his

original intent, ‘one finds surprisingly few actual applications of mathematical methods [...] to *any* practical problems’ (Smolinski 1973: 1193) in Marx’s notes on mathematics. Thus one may conclude that his mathematical interests increasingly shifted away from their direct practical relevance for ‘the elaboration of the economic principles’ and towards the study of mathematics for its own sake (Smolinski 1973: 1193). Marx, like Hegel (cf. section 2.2), was particularly interested in (infinitesimals in) the differential calculus (Matthews 2002: 11). ‘[I]n 1878-83 [i.e. the last five years of his life], his main objectives became reformulating its theoretical and philosophical foundations, by showing its development from elementary algebra, to represent the operation of differentiation as a particular case of his dialectical law of “the negation of a negation”’ (Smolinski 1973: 1194). While studying calculus, Marx had remarked that he found it ‘a much easier part of mathematics (as far as the purely technical side is concerned) than for instance the higher parts of algebra’ (Marx #1858, 1930: 149, cited in Struik 1948, 1997: 174). Thus, it seems that Marx found ‘the calculus easier than algebra’ and ‘algebra easier than arithmetic’ (Struik 1948, 1997: 174; cf. Smolinski 1973: 1197).

Marx classified all previous methods of developing the conception of the differential that he knew about as: the mystical method of Newton-Leibnitz, the rational one of D’Alembert and the algebraic one of Lagrange. He criticized all these because they all involved the derivation of the expression for *change*, dy/dx , from *neglecting* some infinitesimally small but essentially *static* difference h (as in D’Alembert and Lagrange) or dx (as in Leibnitz) between x and $x + h$ (or dx), instead of from the dynamic *variation* of x (and concomitantly y) *itself* (Struik 1948, 1997: 179-182; Kol’man 1983: 228-231).¹⁰ Like Hegel, he considered this procedure dialectically incorrect for it did not truly resolve Zeno’s paradox of Achilles and the tortoise. It still allowed dynamic laws of motion to be derived from a reflection on static differences and thus glossed over the fact that a

¹⁰ Ernst Kol’man is also referred to as Kolman or Colman. Since he was a Russian mathematician, his name – which would otherwise be written in Cyrillic letters - is usually translated (or rather transcribed) along with the rest of his text, leading to the variations mentioned.

‘sequence of positions of [a] point at rest [...] will never produce motion’ (Struik 1948, 1997: 184-185; cf. Hegel 1812, 1813, 1816: 295, 2.2Bc; cf. Fleischhacker 1982: 148; cf. section 2.2).

But, as section 1 indicated, Marx had a much more positive attitude concerning the scope of mathematical formalisms than Hegel did and set out to produce his own alternative method of developing the conception of the differential that he thought lived up to his and Hegel’s criticisms of infinitesimals. Hegel, by contrast, had only formulated his criticisms verbally and never bothered to rework mathematics on the basis of his dialectically derived insights, for, as Kol’man and Yanovskaya put it:¹¹

According to Hegel these dialectical moments, which are alien to the elementary mathematics of constant magnitudes, cannot be adopted by mathematics at all. All the attempts by mathematics to assimilate them are in vain, for since mathematics is not a science of 'concept' [i.e. notion], therefore naturally no dialectical development, no movement of its concepts and operations on its own ground is possible (1931, 1983: 246).

Marx’s method can be summed up as follows: starting from, say, $y = f(x) = x^3$, and postulating an x_1 that differs from x by some entirely arbitrary (as opposed to a small or infinitesimal) amount, we may write:

$$f(x_1) - f(x) = y_1 - y = x_1^3 - x^3 = (x_1 - x)(x_1^2 + xx_1 + x^2) \text{ so that:}$$

$$(f(x_1) - f(x)) / (x_1 - x) = (y_1 - y) / (x_1 - x) = x_1^2 + xx_1 + x^2$$

When $x_1 = x$, or $x_1 - x = 0$, we obtain:

¹¹ By the same token as Kol’man (see footnote 11) Sofya Yanovskaya is also referred to as Janovskaja or Ianovskaia.

$$0/0 = dy/dx = x^2 + xx + x^2 = 3x^2 \text{ (Struik 1948, 1997: 183).}^{12}$$

What is dialectical about this method is, first, that ‘the derivative only appears when both dy and dx are absolutely zero’ (Struik 1948, 1997: 185) and second, that x is allowed to change into *any* value x_1 in its domain and not just those infinitesimally close to it (Carchedi 2008: 423). The first characteristic does away with the annihilation of infinitesimal *static* differences in order to obtain an expression for a *dynamic* relationship. Since these two are qualitatively different, the dynamic expression can only spring forth from the real disappearance or negation (‘aufhebung’) of the difference between x_1 and x and not from neglecting a static difference h or dx at some point. In dialectical jargon, what happens in Marx’s method is that the negation of a static expression leads to a qualitatively different dynamic expression: the negation of the negation (C. Smith 1983: 265). As such, the derivative is *developed* (‘entwickelt’) from the original expression in Marx’s method and not *separated* (‘losgewickelt’) from some approximate expression (Kennedy 1977: 310-311). Thus, according to Carchedi, Marx shows that the potential for change is already inherent in x , even when no change whatsoever actually occurs (2008: 423). It is therefore ‘the theorization of a temporal, real process’ (2008: 423), in which the realized state of things is articulated alongside, and inseparable from, their potential for change (2008: 423-424).¹³ Moreover, the second characteristic shows this change to affect all of

¹² For some reason that eludes me, Struik first writes: $f(x_1) - f(x) = y_1 - y = x_1^3 - x^3 = (x_1 - x)(x_1^2 + xx_1 + x^2)$ without defining this ‘ S ’ or explaining where it comes from or why it is introduced. It does not resurface in the subsequent expressions, leaving the reader with the distinct impression that it was never supposed to be there in the first place.

¹³ In Carchedi’s view, the concept pair of realized versus potential is crucial to Marx’s dialectics, the gist of which must not be sought in Hegel, but rather ‘we should extract it from Marx’s own work’ (Carchedi 2008: 416). In short, Carchedi’s view boils down to the articulation at each stage of the presentation of a realized phenomenon and its (sometimes contradictory) potential(s). Next, the presentation is driven towards concreteness by introducing time, thus showing how the realized and the potential are interlinked, i.e. by what mechanism the two change into each other (Carchedi 2008: 416). Although Carchedi seems to contend otherwise, it seems to me that his position can easily be reconciled with that of most Hegelian Marxists.

With the notable exception of Arthur, who argues that the outline of Marx’s *Capital* is homologous to (the outline of) Hegel’s *Logic*, most Hegelian Marxists would readily admit that Marx’s method, although inspired by Hegel, differs from Hegel’s in many respects. This is why

reality, whereas working with infinitesimals points to a static view of reality ‘to which change is only added as an appendix’ (Carchedi 2008: 17-18). Marx’s, by contrast, is rooted in a dynamic ontology with respect to every element in all of reality.

So far for Marx’s views on mathematics as such. As for the application of mathematical techniques to the study of economics, Marx had at least one noteworthy intuition that did not make it into his economic texts. On May 31, 1873, Marx wrote to Engels:

[Y]ou know tables in which prices, calculated by percent etc. etc. are represented in their growth in the course of a year etc. showing the increases and decreases by zig-zag lines. I have repeatedly attempted, for the analysis of crises, to compute these “ups and downs” as fictional curves, and I thought (and even now I still think this possible with sufficient empirical material) to infer mathematically from this an important law of crises. Moore [...] considers the problem rather impractical, and I have decided for the time being to give it up. (Marx #1873, 1966: 82, cited in Kol’man 1983: 220; cf. Smolinski 1973: 1200)

Samuel Moore was Marx and Engels’ advisor in mathematics and they both usually (albeit sometimes reluctantly) accepted his judgment on issues like these as the last word (Matthews 2002: 8-9). According to Kol’man, however, Moore was mistaken in this case. Had he been more conversant with ‘Fourier analysis,

writers like Smith and I, for example, opt for general formats like unity, difference and unity-in-difference or α - β - γ respectively, to be applied to Marx’s work in order to track his dialectics, rather than look for specific homologies or matching details. The end result of the application of such a format to Capital need not be very different from Carchedi’s view of that work.

Take, for instance, the dialectics at the beginning of Capital as I have tracked them in section 3 below. The account runs: α) in conceptual isolation, to be a society requires *sociation*, which encompasses social production. Hence, any *realized* society is sociate by definition. β) Yet capitalist production is privately undertaken, in *dissociation*, and capitalist societies therefore run the *potential* risk of disintegration. γ) The tension between α) and β) is resolved when private produce is allowed to move (i.e. by introducing time) from the site of production to that of consumption through the associative moment of the *exchange relation*. Thus, these approaches are clearly reconcilable. That is, whether one progresses by viewing each concept from the oppositional angles α) and β) before moving on to the opposition’s resolution under γ) or by looking for a concept’s realization, its potential and the movement between the two, may very well amount to the same thing.

that branch of applied mathematics which deals with the detection of latent periodicities in complex oscillatory processes', he would probably have been more supportive of Marx's attempts at finding those 'fictional curves' (1983: 220). Smolinski, by contrast, asserts that 'even though both data and analytical methods of the study of the business cycle have greatly improved since 1873, Moore's skepticism with respect to the applicability of Marx's proposal appears to be well taken even from the vantage point of the 1970s' (1973: 1200).

All in all, Marx studied at least five textbooks on calculus and two texts on algebra (Struik 1948, 1997: 176-177) and explicitly intended to use the insights he gained from these to further his 'elaboration of the economic principles'. So we can safely conclude that Marx was neither ignorant of mathematics, nor considered it inapplicable to the field of political economy generally, or socioeconomic relations specifically (Smolinski 1973: 1191 – 1193, 1201). This being said and given that he found calculus easier than algebra and algebra easier than arithmetic, it is startling that he usually, if not always, sticks to numerical examples in *Capital* even when elementary algebraic techniques, like dividing the numerator and denominator by the same symbol, could have given him a direct and, moreover, perfectly general result (Smolinski 1973: 1197).

One possible explanation could be that Marx intended *Capital* for an audience of educated laborers (among others), and assumed that algebraic operations would be slightly over their heads. But if this were the case, one would expect Marx's notes to be for the most part written down in algebraic form even when in print he reverted to numerical examples for the sake of accessibility. Moreover, had Marx algebraically determined the outcome he was after in advance of his computations, one would not expect his published works to engage in algebraic mistakes or circular reasoning because of e.g. impractically chosen numerical values, nor for him to abandon promising lines of inquiry because of computational errors. But he does all of these things both in the works that were published during his lifetime and in the draft texts first worked up for publication

by Engels and later by others (Smolinski 1196-1197).¹⁴ So a more likely explanation for Marx's predilection for numerical computations is that he 'learned the wrong methods at the wrong time', that is 'his economic system [...] was already virtually completed by the time when, at the age of 40, he began studying mathematics' (Smolinski 1973: 1198-1199). By way of illustration, I have amended Reuten's 2003 table of the publication and manuscript dates of some of Marx's major works (2003: 150), with those of Marx's most important mathematical works in table 1.

¹⁴ In writing these lines I have greatly benefited from discussions with Geert Reuten, Harro Maas and Murat Kotan.

Table 1. Marx's mathematical texts in the context of the dates of publication of some major works

1 <i>First publication in German</i>	2 <i>years 1–3</i>	3 <i>First English translation</i>	4 <i>Date of manuscript</i>	5 <i>Years 1–4 Ms–German</i>	6 <i>Years 3–4 Ms–English</i>
1867 <i>Das Kapital I</i>	19	1886 <i>Capital I</i>	1861, 1863, 1865-7 ^a	0	19
1885 <i>Das Kapital II</i>	22	1907 <i>Capital II</i>	1865-1870, 1877-8	7	29
1894 <i>Das Kapital III</i>	15	1909 <i>Capital III</i>	1864-5	29	44
1905-10 <i>Theorien über den Mehrwert</i> (3 vols)	48-61	1952-71 <i>Theories of Surplus Value</i> ^b	1862-3	47	108
1932 <i>Pariser Manuskripte</i>	31	1963 <i>Economic-philosophical manuscripts</i>	1844	88	119
1932 <i>Die deutsche Ideologie</i> ^c	6	1938 <i>The German Ideology</i> (Parts I & III)	1845-6	86	92
1953 <i>Grundrisse</i> ^d	20	1973 <i>Grundrisse</i> ^e	1857-8	95	115
1968 <i>Mathematische Manuskripte</i> ^f	15	1983 <i>Marx's Mathematical Manuscripts</i>	1881-3	83-5	98-100
1992 <i>Ökonomische Manuskripte 1863-7</i> [including the <i>Capital III</i> manuscripts]	-	- -	1863-7	125-9	-
2003 <i>Mehrwertrate und Profitrate mathematisch behandelt</i>	-	- -	1875	128	-

- a Work on second German edition, 1867-72; and on French edition 1872-75.
- b First Volume 1952, *A History of Economic Theories*; extracts 1951.
- c Extracts 1902-3, 1921 and 1927.
- d Extracts 1964 and 1971.
- e Earlier scarcely available edition 1939-41; its Introduction was published in 1903.
- f Parts of these manuscripts appeared in Russian translation in 1933 as *Mathematischeskie Rukopsii*.

The argument above may also explain why Marx's 1875 manuscript (finally published in 2003) entitled 'The mathematical treatment of the Rate of Surplus Value and the Rate of Profit' ('Mehrwertsrate und Profitrate Mathematisch behandelt'), which is essentially a mathematical adaptation and reformulation of chapter 3 of volume III of *Capital*, never ventures beyond elementary algebra (Smolinski 1973: 1195). It is one thing to devise a theory on the basis of a model or a model and theory at the same time, but quite another to devise a model for a 17 year old theory on the basis of techniques that were alien to you at the time you conceived of the theory. In Smolinski's words: 'It would be a difficult task for Marx and, at the early stage of development of mathematical economics at the time, a pioneering venture to reformulate his economic system as a mathematical model using the tools most appropriate for that purpose, such as linear algebra, matrix algebra, and methods of finite mathematics' (Smolinski 1973: 1199), no matter how conversant Marx was with these when writing 'the mathematical treatment'. In short, mastery of mathematical techniques does not imply mastery of their applications to practical problems. Furthermore, Marx's interest in the foundation of the calculus so characteristic of dialectical thinkers was of no avail in his crisis-ridden conception of the economic system, for the calculus deals with continuous gradual changes and breaks down when leaps and discontinuities occur (Smolinski 1973: 1199).

All in all, it may be concluded that Marx wanted mathematics to reflect the systematic dialectical origins of its foundations and aspired to using such mathematical techniques as could withstand dialectical criticism to elaborate economic principles when appropriate. Seeing that his mathematical sophistication came a little late for the latter purpose, there is ample room to improve both on his technique as well as on the way mathematical formalisms are embedded in his dialectical presentation. After section 3 has elaborated in detail on Marx's systematic dialectical presentation of capitalism, section 4 will discuss his reproduction schemes as an example of Marx's attempts at the use of mathematics that is both technically crude and only partially embedded in the overall dialectic of *Capital*.

3. Marx's Presentation of Capitalism as a System: The Systematic-Dialectical Position.

This section presents Marx's systematic dialectical determination of Capitalism. Contentwise, it is based on Smith (1990 and 1993) and Arthur (1993 and 2002), but the mode of presentation is my own, that is the α - β - γ) format introduced in chapter 2.

3.1. Sociation

The first paragraph of *Capital* runs: 'The wealth of those societies in which the capitalist mode of production prevails, presents itself as "an immense accumulation of commodities", its unit being a single commodity. Our investigation must therefore begin with the analysis of a commodity' (Marx 1890⁴, 1867¹: 49). So it seems that the commodity is Marx's starting point. However, both Smith and Arthur argue that Marx's presentation proper actually has a different starting point.

According to Smith, one may also read the quotation above as saying that in order to analyze *capitalism* as a specific mode of *social production* we must start with the commodity. So on this reading 'the fundamental purpose of the first section of Chapter One of *Capital* is to explicate the relationships connecting a) the general realm of social production; b) a specific mode of social production; and c) the category that is the first determination of that specific mode' (Smith 1990: 62). This general realm of social production is termed '*sociation*' by Reuten and Williams (1989: 56) and Arthur adopts this terminology (1993: 71).

α) *Sociation* is the one word answer to the question what a viable society is when conceptually isolated from other concepts and processes. To be viable it must socialize production somehow, ensure that women conceive and that children receive sufficient care not to die (Reuten & Williams 1989: 56; cf. Arthur 1993: 71).¹⁵ *Sociation* refers to any and all situations in which these requirements

¹⁵ In abstraction, whether a society is viable of course has nothing to do with whether its members are happy. Since we have abstracted away from human volition and freedom for now, what

are somehow met, but it makes just as much sense to talk about a sociate society as to talk about a Being that *is*. The very existence of a society implies that it has somehow fulfilled the mentioned requirements to a sufficient degree. So *sociation* is a universal ahistorical characteristic of any and all societies.

3.2. Dissociation

β) Having established the general requirements for any and all societies, we must move on to the specific capitalist mode of production. The paradox is that although each society must somehow socialize its produce, capitalist production is undertaken in private units that are separated from the site of consumption. So capitalist production is expressed in the world as the antithesis of social production: *dissociation*.

Arthur (1993: 71) has again adopted this terminology from Reuten and Williams (1989: 56-57). Smith seems to have a similar idea in mind when he describes the capitalist mode of production as being characterized by indirect and unrestricted sociality. Capitalism is indirectly social in that production serves private instead of social goals and unrestrictedly so in that e.g. the market may be unboundedly extended (Smith 1990: 63-65).

3.3. Association: the Exchange Relation

γ) Given that society can only survive if it socializes its produce to be used (3.1) and that capitalist production is a private affair (3.2), there must be some third mechanism that articulates dissociate production as social: the *association*. In Capitalism *association* takes the form of γ) the *exchange relation*. If private produce is exchanged it is taken from the site of the producer to that of the consumer, thus bridging the gap between the two (Marx 1867: Ch. 2: 99-108).

Just like *sociation* and *dissociation*, the term *association* was first coined in this context by Reuten and Williams (1989: 59) and was subsequently adopted by

matters are the needs of the system instead of the needs of its members. From this point of view, mass raping women, taking care of them till they give birth and state raising the offspring will do just as nicely as falling in love, courtship rituals and marriage, culminating in a significant probability that the couple will think it cute to 'take' children and provide loving care for them.

Arthur (1993: 71-72). Since the exchange relation is the first condition of existence of the capitalist dissociate mode of production, Arthur claims that ‘the presentation proper [...] starts with exchange’ (1993: 72). Indeed, although dissociation is capitalism’s defining characteristic, this mode of production only stands a chance when mediated (‘aufgehoben’) by the exchange relation.

3.4. The Commodity, Exchangeability and the Bargain

If the site of consumption is generally separated from that of production, goods are not produced for one’s own use, but to be exchanged. γ) As such, they are more properly called *commodities*, because this type of good has some peculiar characteristics that goods generally do not necessarily have. Thus, *commodities* are a subspecies of goods in general.

α) First of all, *commodities* must be *exchangeable*. This requires a degree of perceived usefulness on the part of the consumer. Furthermore it must be possible to own and part with a *commodity* in delimited amounts. That is, exchange of them will only commence when the consumer assesses such and such an amount of it as potentially useful and is able to strike a β) *bargain*, exchanging this amount of x for that amount of y. The unit (e.g. weight, volume, hours, or whatever) in which these amounts are delimited is immaterial to the *bargain* struck: the ratio of exchange appears as a pure number (Arthur 1993: 74-75).

In short, γ) *commodities* are inherently α) *exchangeable* because they embody *use values* and can be owned and sold in discrete quantitatively delimited units, but when they are β) *bargained* over in the *exchange relation*, they present themselves as *exchange values* (Marx 1867: Ch. 1: 49-55). Arthur mainly emphasizes the opposition between *exchangeability* and the *bargain* in this respect (1993: 74). Marx discusses these characteristics of commodities in terms of *use value* and *exchange value* respectively and Reuten and Williams seem to have followed his lead in this respect (1989: 62-63).

3.5. Value in Exchange

In itself, a *commodity* is *exchangeable* if it is sufficiently divisible and perceived as useful. But only when it is confronted with at least one other *commodity* a *bargain* can be struck. So *bargaining* expresses two or more qualitative *use values* in a quantitative ratio of exchange. γ) In the ratio of exchange qualitatively distinct - and as use values incomparable - commodities are commensurated as *values in exchange* (Arthur 1993: 75-76; Marx 1867: Ch. 1: 62).

3.6. The Simple, Expanded and General Commodity Form and the Money Form of Value

α) In one-off, barter exchanges *value in exchange* appears only fleetingly during the exchange itself. It is alien to the exchanged commodities and as soon as the transaction is complete and the bargain fulfilled, it is gone. This is the *simple commodity form of value* (Arthur 1993: 79-80; Smith 1990: 80; Marx 1867: Ch. 1: 63-76).

β) But since the site of consumption is institutionally structurally separated from that of production in capitalism, exchanges are an integral part of social life in capitalism. Hence, each (conceptually) isolated act of exchange can be expanded to all other commodities. Thus, some n (meters) linen does not only exchange for z (kilograms) corn and the other way round (as in the *simple commodity form*), but also mediately or immediately for k (kilograms) iron, m (sacks) potatoes, c (liters) milk, p (hours) Internet access, q (hours) escort services, etc. etc. This is the *expanded commodity form of value* (Arthur 1993: 80; Smith 1990: 80-81; Marx 1867: Ch. 1: 77-78).

γ) Since the value of any *commodity* can thus be expressed in terms of each and every other *commodity* and the other way round, each and any *commodity* can in principle be singled out to serve as a general equivalent of value. This single commodity then serves as the *general commodity form of value* (Arthur 1993: 80; Smith 1990: 81-82; Marx 1867: Ch. 1: 79-83).

γ) However, as soon as a significant part of a community singles out the same commodity to serve as general equivalent (gold being the most likely candidate in Marx's time), its character changes for then it is no longer a commodity amongst

other commodities but *the* commodity: *money*. As soon as *money* is established, *value* is no longer just a relative thing to be *bargained* for. It now has a tangible pendant in the world (Marx 1867: Ch. 1: 84). Thus at this point in the presentation, *value* is established through *money* as an abstraction-in-practice.

Marx illustrates all these forms of value with numerical examples. Thus the *simple commodity form* is illustrated by the formula: ‘20 yards of linen = 1 coat’ (Marx 1890⁴, 1867¹: 63; 1867F: 141), which is next *expanded* by ‘...or = 10 lb. tea or = 40 lb. coffee or = 1 quarter of corn or = 2 ounces of gold or = ½ ton of iron or = etc’ (Marx 1890⁴, 1867¹: 77; 1867F: 155).¹⁶ He also gives us these first two formulas in a more algebraic form: ‘z commodity A = u commodity B or = ...’ (Marx 1890⁴, 1867¹: 77; 1867F: 154). But when he illustrates the *general commodity* and the *money form*, Marx reverts to the numerical examples of amounts of units of goods cited above, adds ‘x commodity A [etc.]’ (Marx 1890⁴, 1867¹F: 79, 84) and equates them to 20 yards of linen and 2 ounces of gold respectively (Marx 1890⁴, 1867¹: 79, 84; 1867F: 157, 162).¹⁷ As said, all this is purely illustrative. Marx’s numerical examples at this point play no part in driving his systematic dialectical presentation onwards, nor do they help him to form a better understanding of his categories. So they play no part in Marx’s conceptual explorations (‘Forschung’) either.

3.7. Money as Measure of Value, Means of Circulation and End of Exchange

¹⁶ As is the case with many authors (like e.g. Hegel) Marx’s major works come in various editions and translations. But what complicates things in Marx’s case is that only the first **and second edition** of *Das Kapital I* (denoted here as Marx 1867¹) was published during his lifetime and under his own supervision. The other two volumes of *Das Kapital* {**ik stel voor om dit weg te laten, omdat de 3e en 4e editie van Kap I niet uitgebreid geredigeerd werden door Engels = vervolg van de zin**} were posthumously published after thorough editing by Engels. This fact has long fueled controversies regarding whether Engels had done Marx justice. The last word in these disputes might be provided by the editorial teams of the Marx-Engels Gesamtausgabe (MEGA), that from the 1970’s onwards have been working on deciphering Marx’s (and sometimes Engels’) original manuscripts and rendering them into readable and meticulously annotated form. So as to make it easier for the reader to see the wood for the trees, I will refer to all variations by citing the year of the first publication in German, suffixing this with F for the Fowkes (translator of volume I) or Fernbach (translator of volume II **and III**) translation, M for Marx’s manuscripts and E for Engels’ editorial manuscripts (the latter two were both published in the MEGA-series).

¹⁷ Surprisingly, the abbreviation ‘etc.’ does feature in Marx’s illustration of the *general commodity form of value*, but is absent from his illustration of the supposedly even more general *money form*.

α) With *money* serving as a general equivalent, *value* is no longer just a relative thing to be determined through ever so many *bargains* as there are *commodities*. Instead ‘[e]ach separate commodity [now] has its unique value expressed in monetary terms *prior* to coming into contact with any other commodity’ (Smith 1990: 87, my emphasis). So when there is money, the seller just has to name his price and the buyer can take it, leave it or haggle over it. At any rate the bargain is between 1 unit of commodity A and the monetary expression of its exchange value, i.e. its unit price. Thus, *money* first and foremost serves as *measure of value* (Smith 1990: 87; Arthur 1993: 81-82; Marx 1867: Ch. 3: 109-118; Reuten & Williams 1989: 65).

β) But *money*, like all essential categories in capitalism, ultimately sprang forth from the necessity of *exchange* predicated upon the institutionalized separation of the sites of production and consumption given in *dissociation*. Thus, although considered in itself it is a static *measure of value*, it must mediate *exchange* between *commodities* in the capitalist world out there. So it just as much serves as *means of circulation*. The end of circulation is reestablishing dissociated production as socially useful, sociate. Thus in the first instance *money* appears to mediate between *commodities*, giving us the circuit of commodity (C) → money (M) → different commodity (C’) (Marx 1867: Ch. 3: 118-143; cf. Smith 1990: 87-88; cf. Arthur 1993: 82-83).

γ) However as soon as *money* is determined as the general equivalent of all commodities, it can be immediately exchanged for any commodity, while commodities can do so only mediately via money. So from the introduction of money in the systematic dialectical presentation onwards, it makes more sense to have a stock of money than a stock of commodities. This means that C – M – C’, breaks down into M – C – M and acquiring *money* instead of qualitatively different *commodities*, becomes the *end of exchange* (Smith 1990: 89-91; Arthur 1993: 83-84; Marx 1867: Ch. 4: 163-164).

3.8. Capital

Institutionalized *dissociation* means that privately undertaken production is only socialized when it is *exchanged*, but the aim of this *exchange* in turn is not to acquire another useful product but to acquire *money*. But of course no one would be bothered to take products to market just to end up with the same amount of *money* one started out with. People would only take this trouble if on average their *money* holdings grow in the process. Thus the aim of production is *exchange* and the aim of *exchange* is acquiring a higher sum of *money*. The first of these goals must be reached to ensure social recognition of capitalist production, whilst the attainment of the second is required to ensure continuity of production. Thus it seems that capitalist units of production are continuously throwing off value, more precisely surplus-value, ΔM , over and above the sum of *money*, M , originally laid out. So the circuit $M \rightarrow C \rightarrow M$ is now superseded ('aufgehoben') by the expanding circuit (or spiral of valorization) $M \rightarrow C \rightarrow M'$ (more *money*) (Marx 1867: Ch. 4: 165).

γ) With this, *capital* can be introduced as 'money which begets money' (Marx 1890⁴, 1867¹: Ch. 4: 170; 1867F: 256) or self-valorizing value (Marx 1890⁴, 1867¹: Ch. 4: 169; 1867F: 256; Arthur 1993: 82-84, 87; Arthur 2002: 51). *Capital* finally endows the capitalist mode of production with a structural ground for ongoing production. One can find unlimited usages for more *money*, but only limited ones for the *commodities* one happens to be able to *bargain* for at any one time, so only production for *money* is potentially continuous (Smith 1990: 98-99; Arthur 1993: 83-84). Note that, with the introduction of *capital*, our starting point, the necessity of *exchange* to socialize *dissociated* production, has all but vanished from the scope of the systematic dialectical presentation, whereas *value*, first introduced as a necessary facilitator of *exchange*, has now become the overriding motive of *exchange* as *money*. From this point onwards, the quality of what is *exchanged* is therefore subordinate to the quantity of *money* it *exchanges* for.¹⁸

¹⁸ The weirdness of this situation and the kind of inverted reality it can lead to, is aptly illustrated in the Cree Indian prophecy: 'Only after the last tree has been cut down, only after the last river has been poisoned, only after the last fish has been caught, only then you will find that money can not be eaten.'

3.9. Constant and Variable Capital and Accumulation

From the perspective of *capital* itself the realization of surplus value is the end of exchange ($M \rightarrow C \rightarrow M'$). From the point of view of society however, *money as a means of circulation* mediates between different *commodities* ($C \rightarrow M \rightarrow C'$). So whilst *capital* requires surplus-value to survive, society merely requires resocialization of dissociated produce to take place (through $C \rightarrow M \rightarrow C'$). Hence capital can only be capital (i.e. self-valorize) if it also aims for a qualitative difference between its inputs, C (bought for M), and outputs, C' (sold for M'). With this, the focus clearly shifts towards production.

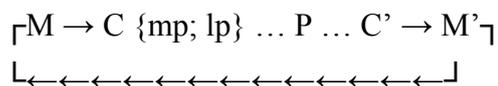
α) As self-valorizing value *capital* first appears as a material process: a sum of money buying certain commodities that undergo, as worked up by labour, a qualitative transformation in production so they can be *exchanged* for a higher sum of *money* than was originally laid out. As soon as commodities enter production this way, they function as *constant capital*. Tools and raw materials (i.e. means of production) thus take the (historically specific) form of *constant capital*. However, the reverse does not necessarily hold: tools and raw materials not engaged in potentially self valorizing circuits (i.e. in capitalist production with a view to exchange at a premium) are no form of capital but just (dormant) means of production.

β) This begs the question of how such qualitative transformation could come about. The picture we have drawn so far is that commodities bought to enter production as *constant capital* somehow get transformed into qualitatively different commodities that can then be sold at a premium. But to understand how this transformation comes about, the self-possessed and conceptually isolated material process must once again call upon an external and intangible input: labor power. As soon as labor power is employed to further the aims of *capital*, it functions as *variable capital*. So, *variable capital* relates to labor power in the same way that *constant capital* relates to means of production: all *variable capital* consists of labor power but not all labor power usually or necessarily functions as *variable capital* (Marx 1867: Ch. 6: 214-225).

In short, the fact that capitalist production is undertaken in private *dissociate* units together with the private ownership of *constant capital*, necessitates a form of production in which incremental growth of *money* is the driving force, but this incremental growth requires a qualitative transformation to be realized in production. This transformation comes about by applying labor power to the production process as *variable capital*. γ) Thus, all the requirements for an ongoing spiral of valorization have now been determined, laying the basis for *accumulation*, the production of commodities on an ever-expanding scale (Reuten & Williams 1989: 78; Arthur 2002: 51-52).¹⁹

3.10. The Money Capital, Production Capital and Commodity Capital Circuits

α) In conceptual isolation, *accumulation* appears as an influx of *money* into a *dissociate* unit of production, followed by an efflux and reflux of more *money* into production and so on. That is, since dissociate production is motivated first and foremost by the acquisition of ever higher sums of money, accumulation considered in itself appears as the *money capital circuit*, schematically summarized as:



(Marx 1885: Ch. 1: 56, 67; adapted from Reuten 2002a: 466).

This is to be read as: a sum of money M buys means of production (constant capital) and labor power (variable capital) {mp; lp}. In the phase of production P, variable capital transforms the means of production to create qualitatively

¹⁹ According to Arthur, *capital's* self-valorization 'is materially grounded in exploitation' (Arthur 2002: 51). Also he says that labor 'is a difference from capital that remains in *contradictory* unity with it' (Arthur 2002: 51) and he regards constant and variable capital as 'two distinct fractions that play particular roles in the process of valorization' (Arthur 2002: 51). Supposedly these are the roles of instruments of exploitation and exploited respectively. Since exploitation can only result in valorization if the exploited can be made to part with something not originally owned by the exploiters, at least the something the exploited are made to part with must originate outside the valorization circuits. However, Arthur never makes this argument. In effect, he only mentions the chief categories involved with self-valorization, exploitation and accumulation, but never explains how these processes come about.

different commodities C' , which sell for a higher monetary value M' than was required for the inputs (i.e. M). This increased sum of *money* is reinvested into production only to *accumulate* an even bigger money capital. So the *money capital circuit* represents accumulation as a relation of *money capital* to itself. Thus from this point of view, labor power, means of production, production itself and commodities appear only as instruments of monetary accumulation (cf. Arthur 2002: 53-54).

β) How much *money capital* can be accumulated however, crucially depends upon the effective control of *money capital* over its instruments, in the first place its interaction with *production capital*. Thus, if we stress how capital relates to the world, the circuit looks more like this:

$P \dots C' \rightarrow M'. M' \rightarrow C' \{mp'; lp'\} \dots P'$ (adapted from Marx 1885: Ch. 2: 90)

Marx calls this the *production capital circuit* (Marx 1885: 90). What this stresses is that, although a bigger *money capital* (M') is still the end goal (hence the full stop behind M'), this *money* must be reinvested if accumulation is to continue. In particular, it must engage in a new round of purchasing more *variable capital* (labor power) to transform more *constant capital* (means of production) into more qualitatively different *commodities*. Thus ongoing monetary *accumulation* requires an ongoing expansion of *production capital*. *Production capital* is a dynamic determination because in the process of production *commodities* undergo qualitative change. Thus, as *production capital*, labor power and means of production appear mainly as *use values*. As *constant capital* confronts *variable capital* during this process, this is where *exploitation* occurs and surplus value is created (cf. Arthur 2002: 53-54).

γ) Whether this surplus value can be realized, however, depends on successful valorization of *commodity capital*. Thus, next to the *money* and *production capital circuits*, we also have a *commodity capital circuit*:

$C' \rightarrow M'. M' \rightarrow C' \{mp'; lp'\} \dots P' \dots C''$

(critically adapted from Marx 1885: Ch. 3: 91, 99)

Whereas the *money capital circuit* relates *money* only to *money* as the tangible embodiment of *exchange value*, the *production capital circuit* relates it to the *use values* of labor power and means of production only. This tension is resolved in the *commodity capital circuit*, because as means of production commodities potentially associate *capital's* produce with *capital*, but as means of subsistence they associate it with labor. So the *commodity capital circuit* unifies the overtly self-possessed *money capital circuit* with the overtly outward oriented (that is, from the point of view of capital's overriding motive) *production capital circuit* (Marx 1885: 100-103; cf. Arthur 2002: 53-54).

3.11. Fixed and Circulating Capital

Now that we know that all *capital* must in one way or another be engaged in a circuit, our attention is drawn to turnover times. On the basis of their respective turnover times *capital* can be classified as either *fixed* or *circulating capital* (Marx 1885: Ch. 8: 158-162).

α) Means of production may either be used up within a single *capital circuit* or last several circuits. While the longer lasting means of production are used in the productive process their *exchange values* diminish in proportion to their use, and when their *use value* is exhausted, so is their *exchange value*. All of their value is then transferred to the *commodities* they helped produce. But until that happens, they perform the same function over and over. As such, they are *fixed capital* in the form of *use value*. Like all *capital*, *fixed capital* is socially validated only through the circuits it enters, but it enters those circuits only mediately by means of piecemeal transfers of its value to *commodities* (Marx 1885: Ch. 8: 158-159; cf. Arthur 2002: 54).

β) Labor power, and those means of production that are used up in a single process, circulate in their entirety (or not at all). They are *circulating capital* in that all of their *use values* and all of their *exchange values* are transferred to the produced *commodity* in one go, so they need to be replaced in every new

productive cycle. This part of *capital* therefore circulates at the same pace as the *commodities* it helps produce (Marx 1885: Ch. 8: 159-160; cf. Arthur 2002: 54).

One might object that different turnover times, though extremely likely, are not dialectically necessary conditions of existence of the capital circuit. Moreover, Marx himself usually abstracts from *fixed capital*. In the total absence of *fixed capital* however, a company would disappear from the face of the earth as soon as it temporarily seized production. The minimum requirement for a unit of production's continued existence therefore, is that it commands some *fixed capital*, albeit in the form of e.g. the physical space it occupies only.

3.12. Simple Reproduction, Means of Production, Consumption Goods, Total Social Capital and Expanded Reproduction

To stay in business all capital is necessarily engaged in circuits. These circuits can only be renewed if the efflux of *money* from the *money capital circuit* M' is at least equal to the original influx M, so that *production capital* can be maintained and *commodities* produced at the same scale as before. To achieve this γ) *simple reproduction* of the circuits, the valorization of *commodity capital* must at least cover its cost of production (cf. Arthur 2002: 54).

Commodity capital may either be valorized by selling commodities to consumers as β) *consumption goods*, i.e. as subsistence for labor, or by selling them to producers as α) *means of production*. In the former case they disappear from the macroeconomic circuit, whereas in the latter they reenter the circuit as inputs at the same time that they leave it as outputs (Arthur 2002: 54). From the point of view of *capital's reproduction*, therefore, replenishment of *means of production* is an internal affair (hence classified as α), while selling *consumption goods* requires *capital* to bridge the institutional separation of the site of production from that of consumption and thus requires *capital* to engage in external relations (hence the classification of *consumption goods* as β).

Marx schematizes (nowadays we would say models) the internal and external interactions between the production department of *means of production* and that of *consumer goods* and shows how wages, prices and quantities must be exactly

proportionate to enable *simple reproduction*, i.e. ongoing production on the same scale. These proportionality requirements show that the two major departments mentioned are systematically dependent upon one another and can therefore be treated as one organic whole: γ) *total social capital* (Marx 1885: Ch. 20: 391-465). However, given the necessity of *accumulation*, *simple reproduction* is not enough for capital to stay in business. Thus we require γ) *expanded reproduction* (of *total social capital*), the proportionality requirements for which Marx's schemes (or models) show to be even more forbidding (Marx 1885: Ch. 21: 485-518; cf. Arthur 2002: 54). Both Marx's *simple* and his *expanded reproduction* schemes are elaborated upon in detail in the next section.

3.13. General Rate of Profit, Many Capitals, Competition and Minimum Prices of Production

α) *Expanded reproduction* (of *total social capital*) means that there is a *general rate of profit* (Marx 1894: Ch. 9: 164-181). This *rate of profit* is essentially an expression of *capital's* self-valorization. Since this was the first (preliminary) definition of *capital* given, the *rate of profit* is essentially an expression of *capital's* relation to itself, an almost tautological definition of what it means to be *capital* (Arthur 2002: 54-55).

β) Yet the very existence of specialization, foreshadowed in *dissociation*, means that there must be *many capitals* differing in all of the respects categorially presented thus far. So, for example their ratios between e.g. *variable* and *constant capital* may differ, they may produce on different scales, they may sell to consumers or to other *capitals*, etc. (cf. Arthur 2002: 55).

γ) However, all these *capitals* are engaged in *competition* for the same funds (i.e. macroeconomic wages or companies' funds for replacement and accumulation of *means of production*). (They may even produce the same or similar products, but this is contingent from the present theoretical perspective). *Capital* responds to *competition* by trying to drive its cost of production down (e.g. by expulsion of labor and accumulation, i.e. production on an ever increasing scale (so that more profit can be made at a given price)) and its prices down (so as

to secure a higher share of the market). γ) Thus, *minimum prices of production* tend to emerge (Marx 1894: Ch. 10: 182-209; Arthur 2002: 55).

From the categorial interactions of all categories presented so far, we can derive the laws of motion of capital (such as the falling rate of profit and periodic crises) (Marx 1894: Ch. 13-15: 221-277; Arthur 2002: 55). But this is a slightly different ballpark, for they deal with how the institutions and their motives interact as a whole, rather than with the systematic dialectical determination of that whole. Therefore, I will not go into them in any more detail.

4. The role of mathematics in Marx's investigation and presentation in Capital: the case of Marx's 'schemes of reproduction'

Karl Marx was both extremely ambitious and very perfectionist. 'By 1858 he planned to write six books' together containing 'a complete systematic analysis of society: economic, social, political and historic' (Reuten 2003: 149). By the end of his life in 1883, he had written enough material for the first of these books, which by then had grown to 'the three volumes of Capital that we now have', but he still thought of most of it as rough drafts unfit for publication. In his opinion only *Capital I* really came of age during his life: its first edition was published in 1867 and a second one in 1873. Luckily for us, however, Marx did recognize that his rough incomplete drafts were too valuable to disappear along with him and thus he asked his friend and publisher Friedrich Engels 'to do something with them' after his death. Engels got to work and *Capital II* was published in 1885, followed by *Capital III* in 1894. Notwithstanding Engels' intentions to refrain from interpretation while editing Marx's drafts, he hardly left any of Marx's sentences untouched (but given Marx's perfectionism, this may have been exactly what Marx would have wanted).

It has long been impossible to flesh out exactly where Marx's text stops and Engels' editing begins. This situation started changing when work began on the Marx/Engels Gesamtausgabe (MEGA) in the 1970's. In essence the MEGA is a

letter-by-letter and word-by-word transcription of all of Marx's almost illegible original manuscripts into readable form. The only editing that the editorial teams of the MEGA have undertaken is the completion of certain abbreviated words. However, if a certain abbreviation allows for several possible completions, the several possibilities are meticulously acknowledged and the arguments contributing to the choice finally made spelled out in 'das Apparat' (the apparatus) that goes along with every volume in the MEGA.²⁰

Engels edited Marx's schemes of reproduction into the last chapters of part 3 of *Capital II*. In these schemes Marx has endeavored to model the conditions for reproduction and accumulation pertaining to sectors (Marx speaks of departments) producing production and consumption goods respectively and was thus able to establish some necessary relationships between the two. In doing so he felt he had to call upon a number of assumptions that he usually defends by appealing to some sort of negligibility clause (i.e. along the lines: 'removal of this assumption will not change anything in the problematic or its solution'). Although (as section 1 and 2 showed) Marx thought that dialectical and formal (e.g. mathematical) forms of reasoning could peacefully coexist and even strengthen each other, he hardly ever calls upon his dialectical presentation of the capitalist system as laid out in *Capital I* and *II* (and section 3.1-3.12 above) to defend his choice of assumptions. Thus, the exact dialectical status of the modeling choices Marx makes is still to be evaluated, as I intend to do in this section.

The ground material for Marx's 'schemes of reproduction' is to be found in his manuscript II, written between 1868 and 1870 (Mega 2008: 907), and VIII, conceived of between 1877 and 1881 (Mega 2008: 1606). Both of these were, if not unfinished contentwise, at least hardly organized when Marx bequeathed all his writings to Engels. Between the two, manuscript II was more rigorously structured and in this sense 'complete' than VIII. Towards the end of it however,

²⁰ As such, I consider the apparatus ('Apparat') as secondary literature in its own right and not as a variation or edition of Marx's or Engels' works. I therefore refer to pages in the apparatus to the MEGA by citing 'Mega' as author and the publication year - 2005 for Engels' editorial manuscript (i.e. 1885E) or 2008 for Marx's (1885M) manuscripts to *Capital II* - as date.

Marx starts rambling off in all directions, discusses ever more other authors, and seems to lose track of his main topic (Mega 2005: 506).

Manuscript VIII (conceived of almost ten years later), by contrast, is much more focused, but according to Engels it 'too is only a provisional treatment of the subject, the main point being to set down and develop the new perspectives arrived at since manuscript II, ignoring those points on which there was nothing new to say' (Engels 1885F: 12). Thus, VIII provides a more detailed, adequate and thought through account of the reproduction of total social capital than II, but is less structured than II and consciously omits all topics on which Marx had 'nothing new to say'.

Understandably, then, Engels, tasking himself 'to prepare the second volume of *Capital* for publication [...] in such a way that it appeared not only as an integrated work, as complete as possible, but also as the exclusive work of its author, and not its editor' (Engels 1885F: 7), based most of this part of *Capital II* on manuscript VIII and only inserted passages from manuscript II when VIII was silent on that specific topic (Mega 2005: 542).

All in all, although Engels sometimes seems to have seriously misrepresented Marx's position with regard to systematic dialectics (see chapter 1; cf. Fraser & Burns 2000: 1-23; Rockmore 2000: 95-105; McCarney 1999: 117-138), he has tried to abstain from interpretation while organizing the two mentioned manuscripts. He has however ventured to make Marx's manuscripts more accessible by replacing some idiosyncratic jargon with more everyday terminology and by (sometimes poorly) translating foreign language quotes into German (Mega 2005: 511, 519-520, 522). The terminological changes especially, often downplay Marx's frequent use of unmistakably dialectical jargon. Hence, I have based this section's appraisal of Marx's 'schemes of reproduction' on Marx's text, while adopting Engels' restructuring.

4.1. Simple Reproduction

Marx's systematic dialectical presentation leading up to the introduction of *simple reproduction* as described in section 3.1 to 3.12 has shown, among other

things, that γ) *capital* (section 3.8) can only throw off surplus value (as it must) if it employs labor power as β) *variable capital* to transform certain means of production, functioning as α) *constant capital* (section 3.9), into more valuable commodities. Furthermore, section 3.10 showed that the various forms of capital can only exist when they are engaged in *circuits* comprising both production and exchange. By articulating production and exchange together, these circuits *reproduce* the *capital* that built them.

Since *constant capital* necessarily takes the form of commodities that must be produced and both laborers and capitalists require commodities to sustain their livelihoods, the question becomes how reproduction of constant capital (more specifically means of production) is mediated by the requirement to present capitalists and laborers with an ongoing stream of commodities for their private use (more specifically means of consumption). This is the problem that Marx sets out to investigate in his models (1885E: 312-317; 1885M: 340-343). Thus, the problematic depicted in his models is clearly inspired by his dialectics. That is, the presentation thus far seems to be insufficient because the necessity of *capital's* appropriation of means of production and labor power to function as *constant* and *variable capital* respectively has been established, whereas the respective origin or sustenance of these component parts, and hence their fundamental relations, are not, leading to apparent contradictions that Marx hopes to resolve through his models.

Since we are primarily interested here in the function of the commodities (i.e. their use value), we evidently have to analyze the *commodity capital circuit* (see section 3.10 above), Marx says (1885M: 368; 1885E: 356). This also was the last of the capital circuits to turn up in our presentation and now it too appears to be lacking in concreteness. Hence, closer scrutiny is warranted. Thus, with these remarks, Marx is off to a dialectically defensible start.²¹

²¹ The word 'start', or even 'order' for that matter, is subject to numerous qualifications in the case of these unfinished manuscripts of Marx's. Apart from the matter of organizing materials pointed out in the introduction to section 4, there is the related matter of which materials to include in the editorial manuscript and which to dismiss. For instance, the part in manuscript II entitled *the societal circulation of constant capital, variable capital and surplus value* (1885M: 348-368, my translation) is largely missing in Engels's editorial manuscript. Most of it discusses and criticizes

Since the two passages referred to above are more than 20 pages apart (and even more in Engels' editorial manuscript), I must probably explain why they nevertheless can be taken together so as to constitute a starting point for Marx's elaboration of his models. As indicated, the very problem Marx wants to resolve is identified in a dialectical manner in his introduction. Then follows an excursus on the topic of 'money capital as a component part of total social capital' (1885M: 343-347)²² that, by Marx's own admission, should actually be addressed later on. Following that, Marx discusses other political economists' take on the matter (1885M: 350-368 (from II), 698-728 (from VIII); compounded by Engels into one chapter: 1885E: 322-355)²³, thus once more departing from his main line of argument. He is back on track when he begins the presentation of his model proper by pointing out the (dialectically) correct starting point for the analysis, viz. the *commodity capital circuit*. In effect then, when speaking of 'Marx's dialectically defensible start' I have focused on Marx's main argument and skipped the rest.

The first assumption that Marx introduces to delineate his model is still dialectically defensible:

(or ridicules) Adam Smith's take on the matter (1885M: 350-358), just like the opening section of manuscript VIII (1885M: 698-726) and apparently this led Engels to supplant the mentioned passages in II with those in VIII.

This may be a missed chance, for Marx casually introduces no less than three of his assumptions (respectively labeled assumption **f**, **b** and **c** in this section) at the very beginning of this part of manuscript II: 'Were [...] the rate of surplus value = 100% [assumption **f**] [...]. Fixed prices of the elements of production [assumption **b**] and a fixed scale of production [assumption **c**] assumed, [...]' (1885M: 348, my translation). Considering the context of these assertions and the casual way they are uttered, it seems likely that, when Marx wrote these lines, it had not yet occurred to him that he would retain these assumptions throughout. At any rate, these assumptions are more formally introduced in several distinct passages later on, so perhaps Engels' judgment that this part of manuscript II could be dispensed with, is sound after all.

²² The ground material for this chapter, namely the splendidly meticulous rendering of Marx's and Engels' original manuscripts into readable form by the editorial teams of the Marx/Engels Gesamtausgabe, the Mega, is only available in German. Hence, all translations are my own unless stated otherwise.

²³ Please bear in mind that what Marx labeled 'chapters' Engels christened 'parts'. Marx inserted some captions here and there in his manuscripts, but there is no indication of a structural attempt at organizing the material within his chapters. It follows that all of Engels's chapters are his own inventions, albeit that he paid close attention to Marx's casual captions in deciding where to break off one chapter and start the next one.

- a. ‘Furthermore it is not only assumed that products are exchanged according to their values, [...]’ (1885M: 369)

That is, the level of abstraction that Marx’s model pertains to is below the level of *capital* in general (that at this level turns out to be departmentalized by necessity), but above the level of *many capitals*. Hence, there is no *competition* between *many capitals* yet that could induce a divergence of prices from values. However, this is not Marx’s argument. Instead he defends this assumption by pointing out that a divergence between prices and values cannot influence ‘the movement of social capital’ (1885M: 369) in that the mass of products to be produced and exchanged is not altered by such divergence. In effect then, Marx considers it to be a negligibility assumption.

The next assumption is introduced in the same sentence as the first:

- b. ‘[...], but also that no revolution in values takes place in the component parts of the productive capital’ (1885M: 369).

At first sight, this assumption does not seem to be dialectically defensible. Since accumulation has been identified as the driving force for capital, we must expect the one (departmentalized) capital to take every measure available to accelerate the process. Clearly, the appropriation of new and improved means of production is among the most prominent of these measures. Hence, technological innovation geared towards accelerating accumulation is essential to capitalism. Thus, technological advances will inevitably increase the mass of products that can be produced with a given combination of constant and variable capital and their occurrence will therefore always correspond to a revolution in value.

On the other hand, the mentioned assumption was introduced in the context of *simple reproduction*. Hence, accumulation is absent (a requirement captured in assumption **c** below) and perhaps therefore the drive to accelerate that process is absent as well at this level of abstraction. Moreover, Marx explicitly assumes technical change away in assumption **g** (discussed below). Therefore, bearing in

mind that our model pertains to departmentalized *capital* in general only and thus abstracts from *many capitals* involved in *competition*, all factors that could induce ‘a revolution in values’ are absent here and justifiably so. All in all, assumption **b** is dialectically defensible as long as assumption **g** is upheld and we are considering simple reproduction (assumption **c**).

Considering Marx’s justification for the adoption of this assumption, however, it may even be upheld in the case of expanded reproduction. He writes:

[A]s far as revolutions in value are concerned, they change nothing in the relationships between the component parts of the annual social capital, as long as they are general and evenly distributed.²⁴ In as far as they are, by contrast, partial and unevenly distributed, they represent disturbances, which *firstly* can only be understood as such in as far as they are regarded as divergences from fixed value relations; *secondly* however, given proof of the law that one part of the value of the annual product, for instance constant capital, replaces another variable capital, then a revolution in values [...] would alter only the relative magnitudes of the portions of value that perform the one or the other function. (1885M: 369)

From this quote one may infer that the intended emphasis in assumption **b** is on ‘component parts’ rather than ‘values’. Thus, although value revolutions due to technological innovation are part and parcel of capitalist production, these value revolutions leave ‘the component parts of the productive capital’ unaffected. On this interpretation there is nothing undialectical about the assumption. After all, *constant* and *variable capital* (section 3.9) and the necessity to realize surplus value have all been dialectically determined prior to the presentation of the moment of *simple reproduction* (in section 3.12). Since one of the central tenets of systematic dialectics is that what (immediately) holds for the system considered in the abstract must also mediate hold at more concrete levels, these component

²⁴ Where I have said ‘relationships’, the original German speaks of ‘Verhältnisse’. The German term is more adequate, for it refers both to ratio and to relationship. In this case that double meaning is particularly fitting, because – as the reproduction schemes will show – the *ratios* concerned are unaffected by general and unevenly distributed revolutions in value, *because* the *relations* between the component parts of capital remain unaffected.

parts must always be present in capitalism, no matter what revolutions the system undergoes.

In his next assumption Marx defines simple reproduction:

- c. *Simple reproduction* implies that capitalists consume all surplus value. That is, it is assumed they just replace the constant and variable capital they started out with and buy consumption goods with whatever surplus value is left (1885M: 728).

Dialectically, this seems to be a strange assumption to (re)introduce here. We have long since identified *accumulation* as an essential characteristic of capitalist production, so by ignoring it, Marx's model separates itself from the capitalist basis. However, *accumulation* is of course only possible if simple reproduction is secured (Marx 1885M: 728). Hence, it makes sense to study the maintenance of capital (i.e. its reproduction on the same scale) before moving on to full-fledged accumulation. One might say that Marx first investigates a stationary system so as to make sure his analysis of capitalism's essentially dynamic reality starts from a correct vantage point.

This procedure is perfectly reconcilable with the general systematic dialectical appraisal of *Capital* developed in this chapter. That is, if one asks α) what something is in conceptual isolation, the answer points to a situation at rest that can only be given room to move through the means or medium stipulated in answering β) (how is α expressed in the world?).²⁵ So, as long as capital is able to renew itself, its conceptually isolated nature is unaffected (and the moment α secured) and only then can it find room to express its nature in the world through

²⁵ This procedure is also slightly reminiscent of the way Presence ('Dasein') was introduced in chapter 2. After establishing that everything is necessarily engaged in a process of Becoming, we could only regain our footing by insisting that a static snapshot could nevertheless be taken. Only our careful analysis of this snapshot and the others that followed it eventually allowed us to give a somewhat comprehensive analysis of a system that in essence is dynamic through and through. So such a procedure is basically legitimate as long as it is only used to 'set the stage'. A static picture must be the first and not the last word in any serious account of actuality.

accumulation. In effect then, assumption c) is a dialectically defensible heuristic assumption.

Next, Marx introduces an analytical distinction between society's two main production departments:

- d. 'Society's total yearly product breaks down into *two great departments*' –
I) '*means of production, commodities that possess a form in which they either have to enter productive consumption, or at least can enter this*', and
II) '*means of consumption, commodities that possess a form in which they enter the individual consumption of the capitalist and working classes*'
(1885M: 370; cf. 1885E: 359).²⁶

This departmentalization follows almost immediately from the locus of the schemes of reproduction within Marx's overall systematic dialectical presentation. Marx presents his 'schemes of reproduction' at the end of Part Three of *Capital II*, i.e. after the several *capital circuits* and the categories of *fixed* and *circulating capital* have been introduced into his general systematic dialectical framework in respectively Part One and Two of that volume. Thus, it has been established that at any point in time, a part of *capital* is invested in means of production (i.e. as *constant capital*) and another part in labor power (i.e. as *variable capital*) while the rest represents potential (as P or C') or temporal (as M') surplus value.²⁷

Furthermore, the fact that means of production do not cross the institutional divide

²⁶ Textually, all this is translated from Marx's Manuscript II. Throughout this manuscript however, Marx labeled the means of consumption department *I* and the means of production department *II*. Since he does this the other way round in his manuscript VIII and since Engels based his editorial activities on the latter manuscript as much as possible, it seems prudent to follow Engels' lead in adopting manuscript VIII's labeling throughout.

²⁷ Understood naively, the capital circuits may be taken for step-by-step descriptions of the stages capitalist production has to go through. From such a diachronous understanding of the capitalist production process, the phrase 'at any point in time' may seem incorrect. After all, if one step is taken at a time, the capitalist is either buying C, producing (P) or selling C'. But the crux of ongoing production is that no stage ever terminates: all commodities C that enter production P are replaced and when an end product C' is sold, it too is replaced by fresh produce. So from this point of view, the circuits must be understood synchronically, rather than diachronically and the aforementioned phrase is perfectly legitimate (the terminology of synchronic and diachronic was introduced by Reuten (2002b: 8-9)).

between the site of production and that of consumption, whereas consumption goods do, makes the distinction between the production department of *means of production* and that of *consumer goods* dialectically meaningful.

In his next assumption, Marx makes it clear that he intends to abstract from fixed capital for the time being:

- e. ‘From the point of view of social capital [...] – with a problem, where the question arises how the capital used up during the year can be replaced out of commodity production – only that part of the value of the employed fixed capital can initially be considered, which actually must be partially or completely replaced by new items of the same kind. Note therefore, that it is *assumed* in the following determination of the yearly value product (in means of consumption) that the *fixed part of the value* of the constant capital transferred to the product is only equal to that *part of the value of the fixed capital* that must be replaced in kind in order to start the reproduction anew at the same scale’ (1885M: 372).^{28 29}

Because we have already established that a minimum requirement for a unit of production’s continued existence is that a part of its *constant capital* is *fixed*, this assumption cannot be defended dialectically. According to Reuten, it is apparently introduced ‘so as to reduce the problem to its bare elements’ (1998: 195). Perhaps

²⁸ Where I use ‘determination’, Marx has ‘Darstellung’. *Darstellung* would literally translate something like ‘putting there’ or, equally, ‘positing there’. As such, it refers to analytical determination, the bringing into being of theoretical distinctions; systematic representation, the integration of previously distinguished parts, i.e. theoretical system building; as well as to their practical counterparts. It is exactly this double meaning that is aimed for by dialectical thinkers, for a chief contention in dialectics is that the full meaning of concepts can only be revealed when their analytical intrinsic meaning is mediated by the meanings of the words the concept is dialectically linked to. In this sense, a dialectical presentation continuously iterates between analysis and integration. Moreover, it is also claimed that processes going on in the real world can be identified that way. Thus, the German term ‘*Darstellung*’ aptly captures all the goals and claims of systematic dialectics. The fact that Engels has done away with this term in extensively rewriting the quoted sentences (cf. 1885E: 360) may therefore be a significant fact with respect to the reappraisal of Marx’s methodological stance.

²⁹ Reuten claims that this assumption was taken from manuscript VIII (1998: 195). But the table of origins (‘*Provenienzverzeichnis*’) in the apparatus (‘*Apparat*’) that accompanies Engels’ editorial manuscript shows it was really taken from manuscript II (Mega 2005: 925). Indeed, the block quote above was taken from the latter manuscript.

this is true. At first sight, it certainly seems to simplify the mathematics involved. On close inspection however, dismissal of this assumption does not significantly complicate the resulting model. So it is both undialectical and redundant. Hence the reconstruction of Marx's model in chapter 4 will proceed without it. But for now, it is Marx's use of dialectical reasoning in his model building that is under scrutiny and Marx clearly felt he had to adopt this assumption for a while and could only drop it later. Thus, he thought of it as a heuristic assumption.

Just before Engels moves on to the basic schematic for simple reproduction, he introduces another assumption in a rather offhand way:

- f. 'For our investigation of simple reproduction, we intend to use the following schema, in which c = constant capital, v = variable capital, s = surplus value, and the rate of valorization s/v is taken as 100 per cent' (Marx 1885E: 360; 1885F: 473), for both departments.

In the original manuscript VIII, Marx did not even bother to introduce this assumption in a proper sentence. He simply states: 'Percentage of valorization = 100%' (1885M: 728). This absence of comments and qualifications is surprising, because the statement is by no means self-evident.

Since, as we have seen, the appropriation of surplus value is predicated upon the existence of *variable capital*, the assumption that the ratio of s over v exhibits some degree of resilience can be granted on dialectical grounds provided that the average labor productivity is constant and given (which in turn can be argued for on the basis of assumption **b** and of **g** below). However, customers' motivation to purchase means of consumption is very different from their motivation to purchase means of production, so the sales of both departments have very different drivers. Therefore, it is very unlikely for the ratio s/v to be equal across departments, nor is there any dialectical reason for it to be a 100 per cent. In this respect Reuten rightly remarks: 'it seems a simplifying device without particular relevance to the problem at hand' (1998: 195).

Engels introduces the basic schematic for simple reproduction from manuscript VIII in almost the same sentence as assumption **f**. It is best summarized as follows:

$$\begin{array}{l}
 \text{‘} \quad \text{c} \quad \text{v} \quad \text{s} \quad \text{x} \\
 \text{I. } 4000 + 1000 + 1000 = 6000 \text{ (means of production)} \\
 \text{II. } \underline{2000 + 500 + 500 = 3000} \text{ (means of consumption)} \\
 \quad 6000 + 1500 + 1500 = 9000 \text{ (social gross product)}
 \end{array}$$

where:

I = department I, producing means of production (6000);

II = department II, producing means of consumption (3000);

c = constant capital, the value of the means of production applied;

v = variable capital, the value of the social labour power applied;

s = surplus value, the value that is added by labour minus the replacement of the variable capital advanced’ (Reuten 1998: 196; cf. Marx 1885M: 728-731; cf. 1885E: 360-361).³⁰

When one recognizes that in general – and quite apart from the particular values of the variables chosen – the value of the constant capital employed in any one year, must be equal to the value produced during that year (given the absence of accumulation), it follows that $c_I + c_{II} = x_I$. Using the rule of the excluded

³⁰ Marx’s original notation is a mess (but since it was all taken from draft scribbles, one cannot necessarily hold this against him). The figures are the same, but in a space comprising only three lines, we find three different notations:

1. $\text{c) } \text{v)}$
 $4000 + 1000; K = 5000$
2. $K = 5000; = 4000c + 1000v$
3. $[...] = 4000_c + 1000_v + 1000_m$ (1885M: 728)

Of these, only the first is partially compatible with mathematical conventions. Physicists and mathematical economists (among others) would be inclined to interpret the third as: ‘in the case of c the value of this variable is 4000’ and not as: ‘c = 4000’. The second seems to say: ‘4000 times c + 1000 times v’, which of course is not the interpretation intended. Engels considerably clears up Marx’s mess and renders the schemes much more insightful, but he also adopts this strange second notation (cf. 1885E: 360-361).

middle and eliminating terms found on both sides of the equal sign, the following proportionality condition is obtained (cf. Marx 1885M: 734; cf. 1885E: 365):

$$(v + s)_I = c_{II}.^{31}$$

Next, Marx assumes the value composition of capital ($c/(c + v)$) to be equal, constant and given across departments:

- g. ‘What is arbitrarily chosen here, for both departments I and II, is the ratio of *variable* to *constant capital*; arbitrary also is the *identity* of this ratio between the departments [...]. This *identity* is assumed here only for the sake of simplification, and the assumption of different ratios would not change anything at all in the conditions of the problem or its solution’ (1885M: 739; cf. 1885E: 370; 1885F: 483).^{32 33}

At first sight, it seems that the simplification Marx speaks of here can indeed not be defended dialectically. As we have seen, technological innovation geared towards accelerating accumulation is essential to capitalism and we have not identified any mechanism that would ensure that such innovations would leave the

³¹ Marx’s notation is the following: $(v + s)_I = c_{II}$. Engels rewrote this as: $I(v + s) = IIc$. In print, the v ’s, s ’s and c ’s were rendered subscripts. The printed version apparently led Reuten to comment: ‘Generalizing the schema, Marx uses the notation:

$$I_c + I_v + I_s = I$$

$$II_c + II_v + II_s = II$$

In what follows, we adopt the notation that has become conventional in modern Marxian economics:

$$c_1 + v_1 + s_1 = x_1 \text{ [...]you get the picture...]} \text{ (1998: 197).}$$

This is not true. Looking at the notations quoted above, it may be concluded that the notation that Reuten attributes to Marx is actually Engels’, while Marx leant towards the now conventional notation, albeit that he used Roman numerals instead of (the western notation of) Arab ones.

³² In this rare instance, Engels took over Marx’s sentence without any alterations, except for dispensing with Marx’s underlinings (rendered *italic* throughout the Mega, as well as in my quotations). Hence, I could rely on Fernbach’s translation for once.

³³ Yet, Reuten calculates $c/(c + v)$ that Marx implicitly uses for the two departments as $\gamma_I = 0,8$ and $\gamma_{II} = 0,67$. Since $0,8 \neq 0,67$, not even approximately, it may be concluded that the professed identity of the two departments in this respect is not consistently adopted by Marx throughout.

ratio between constant and variable capital unchanged. So ignoring technological development is no longer warranted at the level of abstraction the schemes of reproduction pertain to. Nevertheless, by the same token as assumption **d**, assumption **g** is dialectically defensible as a *heuristic* assumption. That is, if one first wants to investigate a stationary system, not only *accumulation*, but also technical change must be assumed away for now, provided they are both brought back in when modeling *expanded reproduction*. Although Marx fails to integrate technological change into his subsequent schemes of *expanded reproduction*, assuming a constant and given ratio v/c is at least dialectically justifiable as long as simple reproduction is under scrutiny. The assumption that this ratio is identical across departments is – as Marx stated – of course just a simplification without dialectical roots.³⁴

Immediately after the introduction of assumption **g**, Marx goes on to say that although the mentioned ratios may be arbitrarily chosen, the relationship described in the proportionality condition is a necessary one (1885M: 739), that, it might be added, clearly brings out the interdependence of the two departments.³⁵ With the proportionality condition we get a first glance at the fundamental relations between the two great departments ensuing from *capital's* necessary appropriation of means of production to be employed as *constant capital* and of labor power employed as *variable capital*. Since the whole analysis was geared towards this goal, this is an important intermediate result, the implications of

³⁴ Moreover, Reuten calculates the γ 's for the two departments that Marx implicitly uses in his schemes of *expanded reproduction* as $\gamma_I = 0,8$ and $\gamma_{II} = 0,67$. Since $0,8 \neq 0,67$, not even approximately, we may conclude that Marx implicitly dropped the most tricky part of his assumption when moving on to expanded reproduction. Thus, the assumption Marx *actually* worked with was even more justifiable than the one he *professed* to work with.

³⁵ At this point Engels apparently decided that a few lines of clarification were in order. He writes: 'If it $[(v + s)_I]$ were smaller than IIc [i.e. c_{II}], then department II could not completely replace its constant capital; if it were larger, then an unused surplus would be left over. In both cases, the assumption of simple reproduction would be injured' (1885E: 371) (Yes, 'injured', not 'destroyed' as Fernbach curiously translated the German word *verletzt*). Thus, Engels emphasizes the potential for crisis more than Marx does.

which Marx analyses at length in the next 30 odd pages (1885M: 740-768; cf. 1885E: 370-384, 401-430).³⁶

After his lengthy elaboration of what the proportionality condition entails for capitalist productive relationships, Marx reintroduces depreciation predicated on the existence of fixed capital (i.e. he drops assumption e above) and retains this for the remainder of his analysis of both simple and expanded reproduction. Concerning simple reproduction, the result of this is that capitalist crisis would ensue from either rising or falling depreciation rates, because department I would then respectively under- or overproduce. Hence, even in the assumed absence of accumulation, crises can easily occur (Marx 1885M: 768-769; cf. 1885E: 430-432). However, this can be seen directly when assumption e is never introduced in the first place (see chapter 4).

Only after all this analysis, Marx makes it explicit that he has abstracted from foreign trade all along:

- h.** ‘Capitalist production *never* exists *without foreign trade*. [...] Bringing foreign trade into an analysis of the *value of the product annually reproduced* can [...] only confuse things, without providing a new moment of the problem or its solution at any point. We therefore completely abstract from it’. (1885M: 772; cf. 1885E: 433-434)

This assumption follows directly from the locus of the schemes of reproduction within Marx’s overall systematic dialectics. That is, since the existence of many capitals, let alone foreign capitals, has not yet been determined, foreign trade is not even possible at this level of abstraction. Marx, however, argues for the adoption of this assumption on the grounds that the value relations between the two great departments would be unaffected by foreign exchange. So, in his mind, this too is best considered a negligibility assumption.

³⁶ The fact that Marx does not use algebraic means of generalization, but instead tries to infer general relationships from specified numerical examples, greatly contributes to the number of pages Marx requires for his analysis.

This concludes my discussion of Marx's model for simple reproduction and especially the way and order in which he conceives of and defends his choice of assumptions. As we have seen, only assumptions **a**, **c**, **d** and **h** can be defended dialectically. Of these, only **c**, **d** and **h** need to be called upon to justify the proportionality condition and the potential for crisis when actual wear and tear in department II does not match the demand for replacement of *constant capital* expected by department I. This is shown more clearly in the next chapter.

Either way, the schemes can only be considered successful if *accumulation* is brought back in first, for only then is our formal analysis back on a par with all preconditions for capitalism's reproduction dialectically determined so far. To this end, Marx next draws up and analyses a model for expanded reproduction, which will be evaluated in the next section.

4.1. Expanded Reproduction

For his analysis of expanded reproduction, Marx retains all the assumptions introduced above – except of course the one defining simple reproduction (i.e. **c**) and the one assuming fixed capital away (i.e. **e**) – and adds four more. The first of these is:

- i. there has '*already* [been] production on an expanded scale' (1885M: 791; cf. 1885E: 452)

According to Marx, this must be assumed because the means of production required by accumulating capital must somehow be on offer, before capital can decide to expand its scale of operations (Marx 1885M: 791; cf. 1885E: 452). Hence, someone must have somehow already brought them to market.

Contemplating this assumption Reuten remarks: 'Apparently Marx does not aim to set out the transition from simple to expanded reproduction' (1998: 204). Although this is probably true, it is a missed chance, for later on Marx *does* describe the transition from a steady 9% growth rate to a steady 10% rate of growth. If generalized, this description might just as easily apply to a steady

growth rate of 0% (as is the case for simple reproduction) accelerating to some positive number – the technique and algebra involved would be exactly the same (see chapter 4). Thus, it could have been shown how the dynamics of *expanded reproduction* are predicated upon static *simple reproduction* and the other way round, thus mutually validating each other. Hence, setting out the transition from simple to expanded reproduction would have brought out the model’s dialectical genesis and development more explicitly. At any rate, assumption **i** is redundant if Marx’s schemes are suitably generalized algebraically, as is done in the next chapter.

After the introduction of assumption **i**, Marx goes on to explain that yearly turnover must be hoarded in gradual lumps of depreciation allowances (‘one-sided sales’) and discrete lumps of investment (‘one-sided purchases’).³⁷ Although Marx does not explicitly say so, the existence of depreciation hoards implies that the value of c is now composed of two components: 1) means of production used up that year 2) depreciation allowances for means of production lasting longer than a year. On top of that he assumes the two cover each other (1885M: 795; 1885E: 456; cf. Reuten 1998: 202-203). Thus, the money required for maintenance is on average available from previous hoards, so that it may be assumed that whatever part of surplus value is invested that year is indeed used to expand a department’s productive capacity. In Marx’s words:

- j.** ‘The value of one-sided purchases (from c_{II} [i.e. c_{II}]) = one-sided sales (at c_{II} [i.e. c_{II}])’ (1885M: 795; 1885E: 456).

On average and in the aggregate (i.e. abstractly) the conclusion that surplus value is only used for accumulation and capitalist consumption must hold, for *many capitals*, let alone capitals performing different functions – such as financial capital – have not been introduced yet at the level of abstraction the reproduction schemes are supposed to analyze. So for now, the system must be analyzed

³⁷ The German terms are ‘bloßer Verkauf’ and ‘bloßer Kauf’ respectively. ‘Bloß’ would literally translate as ‘naked’, ‘bare’ or ‘only’, but certainly not as one-sided. But I admit the translation is actually clearer.

without recourse to credit. Consequentially, the only source of funds available to either department is yearly turnover. Now, on the current level of abstraction a capital department must be very big and hence employ a lot of very diverse means of production as *fixed capital*. All of these are most likely in different phases of their respective life cycles, so that the sum total of replacement expenditure equals the sum total of depreciation allowances for any given year (i.e. $\Sigma \text{ depreciation}_t = \Sigma \text{ replacement expenditure}_t$).³⁸ Thus, macroeconomically, the value of the *fixed capital* used up during a given year, equals that year's total replacement expenditure and hence the conclusion that accumulation is paid for out of surplus value is warranted on systematic dialectical grounds, albeit that assumption **j** may actually be unnecessary in this respect.

Next, 'Marx assumes a sufficient monetary accommodation for expanded reproduction' (Reuten 1998: 203). From the vantage point of the early 21st century, this is almost a no-brainer. After all, a company that decides to expand its production will in almost all cases borrow the necessary funds from the bank. In essence, banks conjure these funds into existence ex-nihilo: they just print the money requested. If the funds a company has thus acquired are invested successfully it is able to return the principal sum plus the required interest. Thus, one might say our money is essentially covered by production, so successful expansion of production goes hand-in-hand with a successful expansion of money. But in Marx's time things were not that straightforward, since money was still covered by gold to some degree. So Marx had to make his assumption explicit:

- k.** 'The only thing assumed here is that *the amount of money present in a country* is sufficient for both hoarding and accumulation.' (1885M: 800; cf. 1885E: 461)

³⁸ The famous Leontief input-output table interpretation of Marx's reproduction schemes stems from this observation. (By the by, the observation is still valid when e.g. a depreciation method based on historical cost is used, although part of a replacement in kind, would then be taken to reflect accumulation rather than replacement, so that accumulation of value would not coincide with accumulation in volume.)

From a dialectical point of view however, neither empirical reality (be it that of Marx's time or our own) needs to be fully accommodated yet (after all we have not introduced financial capital yet). Of course the concepts and language available to dialecticians cannot surpass the latest insights of the dialectician's time and are thus molded by empirics, but this influence is only an indirect one, until the most concrete stage of a dialectical presentation is reached. So far, it has been established that *capital* must *accumulate* to survive, is departmentalized by necessity and that neither *department* can survive without inputs from the other. If we aim to set out to determine how these abstract requirements can be upheld at the next stage of concretization (which is still far removed from empirical reality however), the mentioned assumption is fully warranted. For, if it is not satisfied in the abstract (i.e., in this case, in the aggregate and on average), accumulation would sooner or later grind to a halt.

Before Marx starts analyzing the effects of accumulation on the relations between capital's two great departments, he makes one final assumption:

1. It has been extensively explained in "*Capital*" (*Book I*) etc., that *labor power* [is] always available on the basis of capitalist production and how, if necessary, *more labor* can be made available without expansion of the employed amount of laborers or the mass of labor power. At this moment [it is] therefore not necessary to elaborate on this further, much more to assume that the part of the *newly created money capital* that can be transformed into *variable capital* will always find the labor power on hand to transform itself into. (1885M: 801-802; cf. 1885E: 463)

At first glance, the (possible) argument for adoption of this assumption seems to be similar to that regarding assumption **k**. That is, sufficient labor power, like sufficient monetary accommodation, must be on offer if *accumulation* is to be possible at all. Since assumption **g** (stating that the ratio of *constant* to *variable*

capital is constant) implies that variable capital must tag along with the accumulation rate for constant capital, this argument seems warranted.

However, in the quote above, Marx refers back to *Capital I* to justify his assumption. But in *Capital I* his chief argument is that labor expelling techniques result in a rising technical composition of capital - defined as the ratio 'between the mass of the means of production [*mp*] employed on the one hand, and the mass of labour necessary for their employment on the other [*l*]' (Reuten 2004: #) - , so the role variable capital plays in accumulation is an ever diminishing one (Reuten 2004: #). The problem with this argument is that it can only be adopted if assumption *g* is either dropped or modified.

In formal terms, Marx assumes that the growth rate for available labor power $\Delta n/n$ is always larger than the growth rate for variable capital_(i.e. employment) $\Delta v/v$ (i.e. $\Delta n/n > \Delta v/v$) Defining the value composition of capital as $\gamma = c / (c + v)$, one can write for the growth rate of variable capital_(i.e. employment) $\Delta v/v$:

$$\frac{\Delta v}{v} = \frac{1/\gamma' - 1}{1/\gamma - 1} g$$

In which *g* is the steady state growth rate for constant capital and γ' is the value composition of capital that goes with the latest technical composition of capital. If the latter is rising, so must the former. Hence the following condition must hold:

$$0 < \gamma' > \gamma < 1$$

Thus a rising γ will ensure that employment grows at a slower rate than constant capital, but given that some variable capital will always be employed, γ' will never reach unity and hence positive accumulation will always go hand in hand with rising employment (however marginally). Therefore, without assumption(s) on the development of $\Delta n/n$, a rising technical composition of capital is insufficient to justify the conclusion that labor power is always on hand.

A possible way out of this mess is suggested by Zarembka (2009). In his reappraisal of Rosa Luxemburg's contribution to Marxism he first identifies the dilemma sketched above and reaches the conclusion that Marx's assumptions can only hold when there is either a continual and unrealistically large population growth or potential growth of the proletariat. The latter implies that there is always someone somewhere that is not yet a wage worker and can be coerced to become one. Examples of these people in western societies include housemen (stay-at-home-husbands) and –women, children and retired people and in non-western societies also autarkic peasants and tribes. And indeed, these people still exist.

A defense of assumption I along those lines, however, runs counter to Marx's professed goal of investigating capitalism 'in its integrity, free from all disturbing subsidiary circumstances, [so that] we must treat the whole world as one nation, and assume that capitalist production is everywhere established and has possessed itself of every branch of industry.' (Marx 1867: 545, fn. 1 #cited by Zarembka 2009: 64 – original yet to be checked#). In short, he wanted to analyze an emerging societal system as if it were complete and chart out the structural interactions between the elements of this complete system. Although - as Reuten calls it – historical dialectics overarches systematic dialectics (2000: 140-152) and 'philosophy is its own time apprehended in thoughts' (Hegel 1821: 15; cf. Smith 1990: 4; cf. Smith 2003: 187), systematic and historical processes should be clearly distinguished analytically if one is to find out whether a system *qua* system is viable.³⁹ So, from a systematic dialectical perspective you cannot but agree with Marx's chosen standpoint and procedure and thus the problematic stands.

The bottom line is that without (a) specific assumption(s) concerning the development of labor power's availability (be it by means of population growth or by forceful expansion of the proletariat), the availability of labor power must be *assumed* as a condition of existence for *accumulation*. On closer scrutiny, the

³⁹ This is not to say that one of the conclusions following from a thorough dialectical analysis of a system could not be that it is only viable as long as some of its elements resist subsumption, but such a conclusion should not be embedded in the premise.

argument in *Capital I* Marx presumably refers to is just not sufficient to claim that the availability of labor power has been dialectically determined. All in all then, it must be concluded that the (possible) argument for adoption of assumption **I** must indeed be similar to that regarding assumption **k**.^{40 41}

With all these assumptions in place, Marx introduces the base scheme for expanded reproduction (the scheme for year 0, one might say):

$$\begin{array}{r}
 \text{'} \quad \text{c} \quad \text{v} \quad \text{s} \quad \text{x} \\
 \text{I.} \quad 4000 + 1000 + 1000 = 6000 \\
 \text{II.} \quad \underline{1500 + 750} + 750 = 3000 \\
 \quad \quad 5500 + 1750 + 1750 = 9000\text{' }
 \end{array}$$

(Reuten 1998: 207; cf. Marx 1885M: 810; cf. 1885E: 471).⁴²

Effectively these numbers imply that constant capital worth 500 (= $x_I - c$) is available for accumulation. Next, Marx assumes that department I accumulates half of its surplus value, i.e. 500 and spends 400 of this on constant capital and 100 on variable capital. Since accumulation in department I effectively means it has to buy more of its own produce, it is bound to succeed. Thus, the other department has to make due with whatever is left, i.e. an accumulation in constant capital worth 100. Due to assumptions **f** and **g** we end up with the following numbers after accumulation and expanded production has taken place:

$$\begin{array}{r}
 \text{'} \quad \text{c} \quad \text{v} \quad \text{s} \quad \text{x} \\
 \text{I.} \quad 4400 + 1100 + 1100 = 6600 \\
 \text{II.} \quad \underline{1600 + 800} + 800 = 3200
 \end{array}$$

⁴⁰ But I am open to suggestions on how to reinterpret Marx's reference to *Capital I* in such a way that his arguments may be upheld in both places.

⁴¹ Apart from Paul Zarembka, Jurriaan Bendien and Gerald Levy provided helpful comments on this problematic. But since Paul Zarembka actually wrote about it, his comments could be used most comprehensively. I am nevertheless very grateful to all three of these persons.

⁴² Marx's notation here is almost identical to Reuten's, albeit that he did not add up his numbers for c, v and m, but only mentions the total output as: 'Sum = 9000' (1885M: 810). Engels changed this back to the questionable notation 'I. 4000c + 1000v + 1000m = 6000' (1885E: 471; cf. footnote 28 of this chapter).

$$6000 + 1900 + 1900 = 9800'$$

(Reuten 1998: 211; cf. Marx 1885M: 812; cf. 1885E: 472).⁴³

If department I continues to accumulate at the same rate next period and department II continues to pick up the pieces, then, after the next round of accumulation and production the numbers become:

$$\begin{array}{r} \text{'} \\ \text{c} \quad \text{v} \quad \text{s} \quad \text{x} \\ \text{I.} \quad 4840 + 1210 + 1210 = 7260 \\ \text{II.} \quad \underline{1760 + 880} + 880 = 3520 \\ 6600 + 2090 + 2090 = 10780' \end{array}$$

(Reuten 1998: 212; cf. Marx 1885M: 813; cf. 1885E: 473).⁴⁴

Scrutinizing those numbers one can conclude that both departments now accumulate at the same rate again, but that this rate is higher than it could be on the basis of the first schematic given. But to achieve this, department II had to diminish its rate of accumulation in the intermediate period (the second schematic reveals this).

'Marx calculates the schema for three more periods' (Reuten 1998: 212) and then introduces a second example so as to show the effects of a diminishing rate of accumulation in department I. Only after that he verbally formulates the proportionality condition for expanded reproduction:

'It is self-evident that – on the assumption of accumulation, $v + m (I)$ [i.e. $v_I + m_I$] > than c_{II} [i.e. c_2] and not = c_{II} , as in simple reproduction, since 1) I incorporates part of its surplus product into its own productive capital, transforms it into constant capital, but cannot simultaneously replace it with means of consumption from II. 2) I has to supply the constant capital required

⁴³ Surprisingly, Marx starts mixing up notations again at this point (Engels' notation is still spurious, but at least consistently so).

⁴⁴ Marx miscalculated here, but Engels comprehensively corrected his numbers.

for accumulation within II out of its surplus product' (Marx 1885M: 817; cf. 1885E: 475).⁴⁵

Formalizing this, we would get:

$$(v + s)_I - \Delta c_I = c_{II} + \Delta c_{II} \text{ (amended from Reuten 1998: 209).}^{46}$$

Algebraic methods, of course would have given Marx this result a lot quicker, a lot clearer and in an immediately general form. Moreover, such methods show that conditions for transition to either a higher or lower rate of accumulation and for steady accumulation at the same rate can be captured in just a few formulas. On top of that, careful scrutiny of these formulas shows that a few of Marx's assumptions are redundant and others can easily be relaxed. The resultant clarity makes the model's contribution to and the way it is embedded in Marx's systematic dialectics much easier to evaluate. All these insights are conveyed in the next chapter.

5. Conclusions on the Role of Mathematics in Systematic Dialectical Investigation and Presentation

The first main aim for this chapter was to understand Marx's ontological and epistemological attitude towards mathematical concepts and means of investigation respectively regarding the study of capitalism as a system and to contrast this to Hegel's when required.

To get to grips with the ontology of the concepts pertaining to capitalism from a systematic dialectical perspective, section 1 has first positioned Marx's subject

⁴⁵ It is surprising that where Marx has 'transforms it' (1885M: 817, emphasis added), Engels has 'transforms five sixths' (1885E: 475). Thus, where Marx apparently draws a general conclusion from his overtly specific schemes, Engels (perhaps in pride of his superior calculations) sticks to a specific number that only holds for the values arbitrarily chosen by Marx.

⁴⁶ Reuten uses the now conventional notation (see footnote 29), but I wanted to stay in line with the notation Marx originally used for the proportionality condition for simple reproduction.

of investigation in the context of the grand architectonic of Hegel's *Encyclopädie*. This latter work is divided into parts I, II and III, which in turn are divided into subdivisions 1, 2 and 3 that are next subdivided into sections A, B and C and usually next into subsections a, b and c. Finally some of the subsections are subdivided into the now familiar α , β and γ . The parts, subdivisions, sections and subsections relate to each other in very much the same way as α , β and γ do. Thus, the logic (part I) considers thought in total abstraction of the world out there, whereas the philosophy of nature (part II) is concerned with the world out there only and the philosophy of mind (part III) discusses the impact of human thought (part I) on nature (part II) and the other way round. Representations of Marx's systematic dialectics mainly draw on the categories in Hegel's Logic (part I) and his philosophy of mind (part III).

Part I consists of 1) the doctrine of Being, 2) the doctrine of Essence and 3) the doctrine of the Notion. The conceptual progression in the first of these doctrines runs from A) Quality via B) Quantity to C) Measure and has been lengthily described in Chapter 2. The doctrine of Essence describes the type of concepts one needs to get to grips with more determinate qualities and it is concerned with A) Essence, B) Appearance and C) Actuality. The idea behind this is pretty simple: In Essence things are not as they Appear and Actual understanding therefore requires theory (Essence) as well as observations (Appearance). Finally the doctrine of the Notion describes how our C) Ideas are mediated and constrained by B) the Object under scrutiny and A) our potential for Subjective Understanding.

Since part I relates to thought only it can be conceived as a categorization of concepts that are applicable to Hegel's philosophical system as a whole as well as to its subfields considered in themselves. Thus the concepts applicable to the Logic are Being categories, those applicable to Nature are Essence categories and those found in the Philosophy of Mind are Notions. The latter in turn relate to 1) subjective Being (comprising Anthropology, Phenomenology of the Mind and Psychology), 2) Spirit objectified as Essence (society) and 3) absolute spirit (with philosophy as its ultimate notion).

Arthur holds that Marx's representation of capitalism roughly parallels Hegel's Logic, albeit that Marx is done with the doctrine of Being and Essence in less than one fifth of Capital I. Most of what Marx says therefore is said using Notions and more specifically Ideas in a Hegelian sense. But according to Arthur, Hegel thought that 'the Idea creates Nature'. And this statement in his eyes is widely of the mark with respect to nature, but spot on with respect to capitalism. After all, Capitalism is an unlikely system in which the abstract thought of *value* has gained material reality as *money*. In effect Arthur argues that Hegel's confusion led Marx to his correct presentation of capitalism as an inverted reality in which Ideas do indeed gain material reality. Thus all is well as long as Marx describes relations between value-expressions, but problems arise as soon as he tries to incorporate concrete people, for people may rebel against being treated as just another means of production.

Smith, by contrast, does not grant that Hegel was unconcerned with empirical reality. To him studiously and scholarly acquired concepts are embodiments of empirical facts, but when they can be dialectically interrelated they stand a greater chance of approximating the truth. Hence the Hegelian system has merits of its own, for instance regarding the proper position of Marx's capital vis-à-vis other scientific fields. Since Capital studies society it falls on level III-2. Within that level it is concerned with ethical life (III-2-C) and more specifically with civil society (III-2-C-b). So, in the last instance, capitalism is best described in terms of Essence categories.

Since such categories relate to the world out there only they abstract from volition and subjectivity, so that behaviors are law-like and subsuming a particular phenomenon under a law may be considered satisfactory as an explanation (just like in the natural sciences). According to Arthur of course systematic dialectics cannot deal with real people anyhow and must *therefore* distance itself from volition and subjectivity. So although he views capitalism mostly as a notion structure, Arthur's account implies a similar potential for the use of quantitative methods in the study of capitalism as Smith's. On top of that, value, actualized as money, is inherently quantitative and permeates all entities and concepts in the

economic domain. So those entities are not, like measurements in the natural sciences, a construction *imposed* on the world out there, but rather *constitute* the world of capitalism. Hence, a case can be made that capitalist entities are actually more quantitative in nature than those in the natural sciences. So, capitalist entities are ontologically quantitative and if capitalism is to function smoothly the great majority of the people should subject themselves to its value imperatives as though they were natural laws and since survival in capitalism depends on such subsumption most people (however grudgingly) will do exactly that.

As for the epistemological usefulness of mathematical means of investigation for the study of capitalism, we can be brief. Section 2 showed that 1) Marx by the end of his life had become quite conversant with the university textbooks on mathematics of his day, 2) endeavored to reform the basis for mathematics (especially the differential calculus) dialectically and 3) toyed with a lot of ideas for the application of mathematical and formal methods to his main studies in political economy. So, in sharp contrast with Hegel, he clearly thought that mathematics could be improved by dialectical methods and that dialectical presentations could be improved by augmenting them with mathematical techniques.

His most famous attempt at the latter concerns his schemes of reproduction. Despite their merits as models, Marx did not succeed in bringing these schemes on a par with his technical prowess in algebra, nor to fully integrate them into his dialectical presentation of capitalism, probably because he learned the wrong methods at the wrong time and mastery of mathematical techniques does not imply mastery of their applications to problems in other realms. Thus, there is ample room for improvement here and in the next chapter some promising avenues to this end will be suggested and worked out in some detail.

Now in order to evaluate the degree to which Marx succeeded in integrating his schemes of reproduction into the whole of his dialectical presentation in *Capital* (and pinpoint possible areas for improvement), we must first come to grips with this presentation itself and properly position these models therein. Since Marx never was very explicit concerning his method, such a presentation always

involves a substantial degree of interpretation. In this respect I especially draw on the interpretations of Smith (1990), Arthur (1993) and sometimes Reuten and Williams (1989), albeit that the mode of presentation is my own: the α - β - γ format introduced in previous chapters.

Marx's starting point is that any viable society must ensure procreation and socialization of useful products. In Reuten and Williams' terminology, a society that achieves this (i.e. *every* society by definition) is a α) *sociation*. But capitalist production is not inherently social, for it is undertaken in private units that are separated from the site of consumption, that is in β) *dissociation*. Dissociate produce in capitalism is resocialized through exchange: γ) the *association*. Generalized exchange implies production for exchange and thus goods become γ) *commodities* that are inherently α) *exchangeable* because they embody *use values* and can be owned and sold in discrete quantitatively delimited units, but when they are β) *bargained* over in the *exchange relation*, they present themselves as *exchange values*. As soon as a bargain is struck, incomparable use values get commensurated into the same dimension: γ) *value in exchange*.

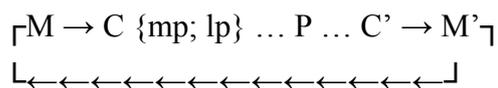
In one-off barter exchange this value appears only fleetingly during the exchange itself, i.e. it appears in its α) *simple commodity form*. But as more and more products are exchanged, the number of value relationships commodities can enter grows accordingly, so we have β) an *expanded commodity form of value*. Any commodity may next be singled out to serve as a general equivalent of value in its γ) *general commodity form*. If a significant part of society singles out the same commodity, it starts serving as a tangible embodiment of value: γ) *money*.

With the advent of *money* values become characteristics of *commodities* prior to the *bargain*. Hence, *money* first and foremost serves as α) *measure of value*. But since *money* sprang forth from the need to resocialize production through generalized *exchange*, it just as much serves as β) *means of circulation*. As such it appears to mediate between *commodities*, giving us the circuit of commodity (C) – money (M) – different commodity (C'). But since *money* can be immediately exchanged for any commodity, while commodities can do so only mediately via *money*, as of this point in the presentation it makes more sense to have a stock of

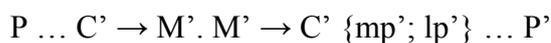
money than a stock of *commodities*. This means that $C - M - C'$, breaks down into $M - C - M$ and acquiring *money* instead of qualitatively different *commodities*, becomes the γ) *end of exchange*. However, $M - C - M$ only makes sense if money holdings grow in the process, giving us the circuit $M - C - M'$ (more *money*). With this, γ) *capital* can be introduced as ‘money which begets money’ or self-valorizing value. As such *capital* functions as a structural ground for ongoing capitalist production.

This begs the question how this self-valorization comes about. The answer lies in production: a qualitative transformation (of C into C') to be realized by means of *capital*. This is achieved by employing labor power as β) *variable capital* to transform means of production functioning as α) *constant capital*. Thus, all the requirements for an ongoing spiral of valorization have now been determined, laying the basis for γ) *accumulation*, the production of commodities on an ever-expanding scale.

If the required qualitative transformation and the forms *variable* and *constant capital* have to take (as labor power and means of production respectively) are articulated together we end up with a circuit. From the point of view of Capital’s overriding motive, this is α) *the money capital circuit*:



This gets expressed in the world as a continuous increase in the scale of production (P) and thus as β) *the production capital circuit*:



The money capital circuit commences with and culminates in exchange value, whereas the production capital circuit commences with and culminates in use value. If one takes the stock of finished products as beginning (C') and end (C'') of the capital circuit this tension is resolved since this stock represents only ideal

exchange-value to the potential seller, but use-value to the buyer. Thus we get γ) *the commodity capital circuit*:

$C' \rightarrow M'. M' \rightarrow C' \{mp'; lp'\} \dots P' \dots C''$

If means of production can enter into several capital circuits before their use-value is entirely exhausted and concomitantly their exchange-values transferred to the commodities they helped produce, they constitute α) *fixed capital*. If their use and exchange values are thus transferred in one go they constitute β) *circulating capital*.

Since *capital's* existence depends on the circuits it is engaged in, these circuits must at least achieve γ) *simple reproduction*. That is, valorized *commodity capital* must at least cover its cost of production by selling either α) *means of production* to other capitals or β) *consumption goods* to consumers. Since the latter disappear from the macroeconomic circuit, whereas the former reenter the circuit as inputs at the same time that they leave it as outputs, it makes sense to distinguish between departments producing means of production and consumption goods respectively. Modeling the relations within and between these two departments, shows that they are systematically interrelated and can therefore be treated as one organic whole: γ) *total social capital*. This whole must *accumulate* to stay in business and hence it must engage in γ) *expanded reproduction*.

The fact that *total social capital* is *expanding*, means that there is α) a *general rate of profit*. But since *capitals* must be specialized (as was foreshadowed in *dissociation*), there must be β) *many capitals* engaged in γ) *competition* leading to γ) *minimum prices of production*.

Marx introduces *simple* and *expanded reproduction* (and their models – or schemes) in the final chapters of the last part of *Capital II*. In section 4, I have evaluated whether the assumptions Marx makes in outlining these schemes follow from the systematic dialectical presentation in *Capital I* and *II* up to that point. In his chapter on simple reproduction, Marx assumes (in order of appearance):

- a.** Products are exchanged at their values
- b.** No revolution in values takes place in the component parts of the productive capital
- c.** All surplus value is consumed (so there is no accumulation)
- d.** Society's total yearly product breaks down into I) means of production and II) means of consumption
- e.** There is no fixed capital
- f.** The rate of valorization s/v (i.e. surplus value over variable capital) is 100% for both departments
- g.** The ratio of variable to constant capital is equal, constant and given across departments
- h.** There is no foreign trade.

Of these, assumptions **a**, **c**, **d**, **g** and **h** are dialectically defensible. **a** and **h** are warranted because at the current level of abstraction we are considering relations between the two great departments of capital, but have not introduced *many capitals* yet. Hence foreign countries cannot enter the equations yet (assumption **h**) and there are no competitors yet that could induce a divergence between prices and values (assumption **a**). **c** is acceptable on dialectical grounds because it ensures that reproduction of society's productive capital is first considered in α) conceptual isolation, before it is given room to move in the world through accumulation (a step reminiscent of β). Thus, in effect it is a dialectically defensible heuristic assumption. A similar argument holds for assumption **g** with respect to technological innovation, albeit that Marx failed to follow up on assumption **g** (by dropping or modifying it) when modeling expanded reproduction. But concerning *simple reproduction* **g** is perfectly defensible. Finally, **d** follows from the role *constant* and *variable capital* play in the *capital circuits* and the fact that *means of consumption* cross the institutional divide predicated on *dissociation*, whereas *means of production* do not. So, on the basis of the dialectical presentation so far this seems the best way to cut the cake at this juncture.

Whether and in what context **b** is dialectically defensible depends on how it is interpreted. If the emphasis is on ‘values’, it is dialectically defensible for simple reproduction only, because by virtue of assumptions **c** and **g** this stationary model has dispensed with both accumulation and technical change. Moreover, we have not introduced competition yet, so in the model all factors that could induce ‘a revolution in values’ are absent here and justifiably so. In the case of *expanded reproduction*, however, this argument cannot be upheld, for *accumulating capital* is bound to engage in technical innovation increasing the mass of products producible with a given combination of *constant* and *variable capital* and such innovations always correspond to revolutions in values. On the other hand, if ‘component parts’ is emphasized, the assumption is perfectly justified for both models. That is, the various forms *capital* takes (*variable, constant, fixed, circulating* etc.) and their roles in the *circuits* they must enter have been dialectically determined prior to drawing up these schemes. Thus, these component parts and the way they are interrelated have been dialectically determined before the modeling exercise starts, so it is safe to say that they will not undergo a revolution. This latter interpretation is supported by Marx’s explanatory text, stating: ‘[A]s far as revolutions in value are concerned they change nothing in the relations between the component parts’.

Assumption **e** is not dialectically defensible. After all, it has been abstractly determined that fixed capital is a necessary part of capital. Hence, it may not be assumed away at a more concrete level. This fact however, does not pose fundamental problems for the possible articulation of models within a dialectical presentation or the other way round, since **e** is actually a redundant assumption (see chapter 4).

Although the ratio of s/v that **f** speaks of is a dialectically defensible element, the assumption as put is way too strict. The mentioned ratio makes conceptual sense, since the appropriation of surplus value has been shown to be predicated on the existence of *variable capital*. And given assumption **b** and **g** it can also be granted that this ratio is more or less constant and given. However the stated 100 per cent and the assumed equality across departments are entirely arbitrary

choices. Assuming different ratios would complicate manipulations of the model, but as long as s/v is constant and given in each department, the results obtained will still be similar.

When drawing up his scheme for simple reproduction and its resultant proportionality condition, Marx only calls on assumptions **c**, **d** and **h**. The others are actually redundant until expanded reproduction is introduced, modeled and analyzed. Marx's scheme looks like this:

‘	c	v	s	x	
I.	4000	+ 1000	+ 1000	= 6000	(means of production)
II.	<u>2000</u>	+ <u>500</u>	+ <u>500</u>	= 3000	(means of consumption)
	6000	+ 1500	+ 1500	= 9000	(social gross product)

where:

I = department I, producing means of production (6000);

II = department II, producing means of consumption (3000);

c = constant capital, the value of the means of production applied;

v = variable capital, the value of the social labour power applied;

s = surplus value, the value that is added by labour minus the replacement of the variable capital advanced'

Abstracting from the numbers and focusing on the relations between the two departments, Marx comes up with the following proportionality condition:

$$(v + s)_I = c_{II}$$

What this says is that department II's constant capital must be paid out of department I's wages (i.e. the value of its *variable capital*) and surplus. Failing fulfillment of this condition, at least one of the two departments goes into crisis. Hence the two form one organic whole: *total social capital*.

Since assumption **c** is dialectically defensible as a heuristic assumption only, accumulation must be brought back in for the schemes to be considered successful. To this end, Marx drops **c** and **e** and moves on to *expanded reproduction*. To do so he introduces an extra four assumptions (listed in order of appearance):

- i.** There has already been production on an expanded scale
- j.** The sum total of replacement expenditure on *fixed capital* equals the sum total of depreciation allowances for *fixed capital* in each department
- k.** ‘[T]he amount of money present in a country is sufficient for both hoarding and accumulation.’
- l.** There is always enough labor power on hand

Apart from **i**, all these assumptions are dialectically justifiable. The upshot of **j** - and the argument for adopting it - is that only accumulation and capitalist consumption are paid for out of surplus value and all other costs are covered elsewhere. Since at this level of abstraction there is no financial capital, *capital departments* have no other sources of funds and this conclusion is fully warranted dialectically. Assumptions **k** and **l** basically describe conditions of existence of *accumulation* and must therefore feature in any model that aims to understand the ramifications of *accumulation* for *capital*'s interrelationships.

By means of assumption **i**, Marx prevents himself from having to set out the transition from simple to expanded reproduction. From a systematic dialectical perspective, this is a pity, for Marx does set out the transition from a steady state growth rate of 9% to one of 10% and setting out growth acceleration from 0% to some positive number would require exactly the same technique. Thus, he could easily have shown how the dynamics of *expanded reproduction* are predicated upon static *simple reproduction* and the other way round, thus mutually validating each other.

With all these assumptions in place, Marx introduces the base scheme for *expanded reproduction*. This is similar to that for *simple reproduction*, except that

the output of means of production is bigger than the constant capital used up that year, so that means of production worth $x_I - c$ are available for *accumulation*. Marx implicitly assumes that department I always fulfills its accumulation plans and department II picks up the pieces, so that all produced means of production are sold. As long as we have only determined the existence of two *capital departments* dialectically, department I will only need to appropriate more of its own produce to get its wish in this respect and is therefore indeed bound to succeed. In the same vein as assumptions **k** and **l**, the second part of this implicit assumption is dialectically defensible as a condition of existence of balanced accumulation. That is, we know capital must *accumulate* and if we want to get a handle on the ramifications thereof, we need to assume away all obstacles to it in order to be able to eventually find out how this trick is pulled off in the real world.

By calculating the numbers in his scheme for several production periods in which accumulation and expansion take place, Marx is able to show that the result of this assumption is that growth acceleration in department I leads to growth deceleration in II. But if department I sticks to the higher growth rate it has accelerated to for more than one period and II keeps picking up the pieces this effect lasts only one period after which both departments are growing at the higher rate. Only after that, Marx verbally formulates the proportionality condition for expanded reproduction, which in more formal terms boils down to:

$$(v + s)_I - \Delta c_I = c_{II} + \Delta c_{II}$$

On the basis of the above evaluation, it may be concluded that there are at least four forms of dialectically defensible assumptions. The first of these are *analytical*: they outline meaningful distinctions (such as that between the two capital departments), aggregates, concepts and relations or ratios (such as s/v or v/c) pertaining to the level of abstraction being modeled. Assumption **d** (and in their most lenient interpretations also **b** and **f**) is (are) of this type. Secondly, it may be that some influence (such as foreign trade) is *negligible* due to the level of abstraction that the model pertains to. **a**, **h** and **j** are of this type. Thirdly, an

assumption may formally mimic the *heuristics* of systematic dialectics, by α) conceptually isolating a - usually static - moment which is β) dynamized at the next stage of approximation. The tension between α) and β) may next be resolved in what may be called γ) a static dynamic model, e.g. describing a steady state, a predictable dynamism. Assumptions **c** and **g** are of this type (albeit that Marx unjustifiably retains **g** for expanded reproduction). Finally, assumptions may outline conditions of existence. That is, if one wants to see the dynamics of a moment by means of modeling it, obstacles to the dynamics concerned, must be assumed away (for the time being). **k** and **l** are of this type. The latter type of assumptions especially serve as guides to the further development of the dialectical presentation, for the conditions of existence assumed at one level must be materially grounded at a more concrete stage (or the consequences - such as crises - endured).

For Marx's schemes of reproduction this typology implies that assumptions **e** and **i** remain unjustifiable throughout and that **g** should be dropped or modified when the transition to expanded reproduction is made. For **e** and **i** this is no problem, because these assumptions were not required in the first place. But before it can be said that the model captures all essential characteristics of reproduction as it appears at the level of abstraction we are at, something must be done concerning **g**.⁴⁷ A possible way forward regarding this is suggested in chapter 4.⁴⁸

Alternatively, one may say that dropping **g** would not affect the proportionality conditions or the fact that growth acceleration in I leads to deceleration in II, but would only make the link between the development of *variable capital* and constant capital more stochastic. So, although it may not be the most elegant solution, one might also adopt a teleological stance regarding **g** and claim that the

⁴⁷ In effect **g** states that technological development is kept out of the model. A neoclassical economist would be elated if this were the only partially unjustifiable assumption in any of his or her models. So the approach developed here seems very promising if I may say so myself.

⁴⁸ It is quite possible to model the development of v/c (or one of its guises as c/v or $c/(c+v)$) independently and integrate it into the reproduction schemes. At the level of abstraction of these schemes v/c would then be modeled to fall with accumulation, as less labor intensive technology is adopted with each round of accumulation. That this dialectically determined trend is not borne out empirically (Kaldor # according to Blaug #) is immaterial at this juncture.

adoption of this assumption has not influenced the results the model set out to attain (i.e. to chart the relationships between departments and the conditions of existence for accumulation) and that therefore there has been no harm in its adoption. But such a defense is really an admission of weakness.

In the next chapter the typology just introduced is developed into a sort of ‘cookbook’ for model building. By way of illustration it opens with a recipe for reconstructing Marx’s reproduction schemes along dialectical lines using all four forms of dialectically justifiable assumptions in a predetermined order.

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Superscripts behind a publication year denote editions. The edition that was actually used is always cited first. Thus Hegel, Georg W. F. (1830³, 1817¹) means that I relied on the third edition of the *Encyclopädie* throughout the article and that the first edition of that work was published in 1817. If a work was published in several parts, the publication years of the different parts are cited without superscripts.

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