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Team Production in
Competitive Labor Markets with Adverse Selection

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Abstract

Team production is a frequent feature of modern production processes. Combined with team incentives, team production creates externalities among workers as workers’ utility upon accepting a contract depends on their colleagues’ productivity. We study the effects of such externalities in a competitive labor market if workers have private information on their productivity. We find that in any competitive equilibrium there is Pareto-efficient separation of workers according to their productivity. We further find that externalities facilitate equilibrium existence, where arbitrarily small externalities can be sufficient to guarantee existence.

JEL classification: D82, D24, J30, L22.

Keywords: team production, competition, adverse selection, externality.

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1 Introduction

Many modern firms employ innovative human resource management practices that include team production, team incentives, and profit sharing (Ichniowski and Shaw 2003). Team production often comprises many tasks, all of which must be well executed for a team to be successful. A worker’s productivity then depends on the productivity of the other workers in his team. In particular, a worker will be less productive if matched with less productive co-workers.\(^1\) Combined with team incentives or profit sharing, team production then implies that the utility a worker gets upon accepting a job also depends on the characteristics of his colleagues. This stands in contrast to standard job market signaling and screening models that assume that the utility a worker gets upon accepting an employment contract depends exclusively on the terms of the contract and the worker’s own productivity.\(^2\)

Despite the practical and theoretical relevance of the subject, no paper has yet analyzed how team production – and the thereby arising externalities among workers – affect labor market competition and employment outcomes if workers have private information on their productivity. To the best of our knowledge, our paper is the first to fill this gap. We investigate a screening version of Spence’s (1973) job market signaling model while introducing a simple externality between workers. Firms compete for workers who have private information on their productivity. Employment contracts specify wages and some task level, with a worker’s costs of complying with the task level depending on his or her productivity. By appropriately combining wages and task levels firms can potentially screen workers according to their productivity. But contrary to the standard framework, a worker’s utility upon accepting a contract in our model does not only depend on his own type and the contract, but also on the average type of co-workers that are attracted by the respective firm.

We show that such externalities do not affect the well-known equilibrium characteristics: whenever there exists a competitive equilibrium, then firms make zero profit, workers with different productivity are separated, and the inefficient task levels required for separation are

\(^1\)Kremer (1993) illustrates this complementarity in production by an extreme but illuminating example: the explosion of the space shuttle Challenger in 1986, which happened because one single component, the O-ring, malfunctioned. Further examples of “O-ring production functions” are discussed in Dalmazzo (2002), Fabel (2004), and Jones (2008).

\(^2\)The discussed externalities among workers also arise naturally in partnerships that employ profit sharing. Partnerships with profit sharing are very common in many industries, including law, accounting, investment banking, management consulting, or medicine. See Hansmann (1996), Farrell and Scotchmer (1988), and Encinosa, Gaynor, and Rebitzer (2007). In these industries the quality of potential future partners might have an important impact on employment choices.
minimized. More intriguingly, however, we prove that externalities among workers facilitate equilibrium existence, where arbitrarily small externalities can be sufficient to guarantee existence. Since Rothschild and Stiglitz (1976) it is known that in the standard framework there does not exist a competitive equilibrium in pure strategies if the fraction of low-productivity workers is sufficiently small. The reason is that any competitive equilibrium must be a separating equilibrium in which high-productivity workers face the minimum task level needed to ensure separation from workers with low productivity. This minimum task level – and thus the inefficiency arising from private information – does not depend on the fraction of low-productivity workers. If the fraction of low-productivity workers is sufficiently small, the separating equilibrium can therefore be destroyed by a Pareto-dominating pooling contract that specifies zero task level and sets wages so as to make a small positive profit when accepted by all workers. Because pooling is ruled out in equilibrium, there exists no competitive equilibrium.

These arguments no longer hold in case of externalities among workers. Externalities entail that market entrants offering a pooling contract might not be able to attract high-productivity workers: given that none of the other high-productivity workers accepts the new contract, each high-productivity worker finds it optimal not to accept the contract, as he would then work with a low-productivity colleague for sure, which he dislikes. As the pooling contract makes losses when attracting only low-productivity workers, market entry is unprofitable. Hence, a competitive equilibrium exists. The externality essentially creates a coordination problem among workers in this case. Selecting the right equilibrium response to market entry ensures equilibrium existence.

The above result suggests that the externality has to be sufficiently large to ensure existence of equilibrium. We show that this is true only in situations in which all offered contracts are accepted in equilibrium. Firms, however, may also offer “preemptive contracts.” These contracts are not accepted by any workers in equilibrium, because they are offered by firms drawing only workers with low productivity. Following market entry of a new firm that attracts only low-productivity types, preemptive contracts may suddenly become appealing to high-productivity workers. We show that the maximum utility a high-productivity worker gets from a preemptive contract after market entry can be unbounded, even if the externality is arbitrarily small. Equilibrium existence is then guaranteed. In contrast to the first finding, coordination problems are not part of the argument in this case: for all equilibrium contract choices, the new firm always attracts all low-productivity workers, while all high-productivity workers accept a preemptive contract.
2 Related Literature

The present paper is closely related to Kosfeld and von Siemens (2009) and Kosfeld and von Siemens (2011). In these papers workers can be selfish – they maximize wages minus effort costs – or conditionally cooperative – they care for wages and effort costs, and in addition receive positive utility from mutual cooperation. Preferences are private information. These behavioral preferences create externalities similar to those in the present set-up because cooperative workers reap utility from mutual cooperation only if their colleagues are cooperative, as well. Selfish workers do not care about their colleagues’ preferences. The present analysis differs from the above two papers in a number of important aspects. First, the present model shows that externalities can arise in situations in which behavioral biases are not relevant. Second, it studies the use of latent (preemptive) contracts which are not considered in the previous studies. Thirdly, only in the present paper firms can use tasks to screen workers according to their productivity. In consequence, all firms make zero profit in equilibrium. In Kosfeld and von Siemens (2009, 2011) firms can separate conditionally cooperative from selfish workers only by reducing wages. Firms attracting conditionally cooperative workers thus might make strictly positive profits in equilibrium, since the screening constraint prevents them to pay out all proceeds as wages. Last but not least, in Kosfeld and von Siemens (2009, 2001) the behavioral externality has to be sufficiently large so as to make cooperation among workers feasible. Only the present paper investigates the impact of externalities on equilibrium existence as the former become arbitrarily small.

Further, our analysis connects to the theoretical literature on competition in markets with adverse selection. Extending Rothschild and Stiglitz’s original equilibrium concept Wilson (1977) and Riley (1979) allow principals to react to market entry. While this solves the equilibrium existence problem, equilibrium characteristics depend on whether principals can add or withdraw contracts after market entry. As argued by Hellwig (1987) explicitly modeling the principals’ strategic interaction generates interesting insights and solves the equilibrium existence problem; however, equilibrium predictions remain “very sensitive to the details of the game-theoretic specification” (p.320). Gale (1992) and Dubey and Geanakoplos (2002) depart from what the latter criticize as the “hybrid oligopolistic-competitive story.” In their Walrasian approach, market participants do not act strategically but consider the other parties’ behavior as unaffected by their own actions. Contracts are traded like consumption commodities, and the attractiveness – so to speak the price – of a contract is determined by

3See also Bester (1985) and Cho and Kreps (1987).
the forces of supply and demand rather than by the decision of principals. These assumptions guarantee equilibrium existence. Adverse selection arises in such a setting if the attractiveness of a contract depends on the types of market participants who accept this contract. In a sense, this externality is quite similar to the externality we study in our paper: in both cases the utility an agent gets upon accepting a contract depends on the acceptance decisions of the other agents. However, while in a Walrasian market the externality is inextricably connected to the existence of adverse selection, we can vary the strength of the externality while keeping the original adverse selection problem fixed. Our results thus add to the existing literature by showing that arbitrarily small externalities can be sufficient to ensure equilibrium existence.

3 Model

There is a continuum of workers with total mass normalized to one. Let \( \Theta = \{ \ell, h \} \subset \mathbb{R}^2 \) be the workers’ type space with \( h > \ell \). It is common knowledge that \( \mu_0 \in [0, 1] \) is the mass of type \( h \) workers, but individual types are private information. A countably infinite number of firms compete for workers. Firms can enter the market at zero costs to offer workers finite sets of contracts. Let \( n \in \mathbb{N} \) be the identity of a firm. A contract \( c = (t, w, n) \) describes a task level \( t \in \mathbb{R}_+ \) and wage benefits \( w \in \mathbb{R} \) that yields the worker some benefits upon acceptance. It includes the firm’s identity \( n \in \mathbb{N} \). The task level could represent a minimum number of working hours or some quality requirements. Firms can determine task levels and wages but are endowed with a fixed identity. Let \( C = \mathbb{R}_+ \times \mathbb{R} \times \mathbb{N} \) denote the contract space. Let \( C_n \) denote some set of contracts offered by firm \( n \) and let \( C = \bigcup_{n \in \mathbb{N}} C_n \) describe the total set of offered contracts.

In our model we consider a simple screening version of Spence’s (1973) job market signaling model. As commonly assumed in the literature the task level is not productive and thus serves as a pure and inefficient screening device. The strategic interaction between firms and workers is described by the following sequence of actions. First, firms simultaneously offer workers finite menus of contracts. Second, workers simultaneously chose among the set of offered contracts. Third, workers satisfy the task requirements and receive the wage benefits as specified by their contract choices. Fourth, payoffs and profits are realized. In the following we provide further details and specify our equilibrium concept.

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4 We essentially follow the exposition in Chapter 13 of Mas-Colell, Whinston, and Green (1995).
Our paper extends the existing literature by assuming that a worker’s utility upon accepting a contract can directly depend on the acceptance choices of the other workers. We assume that firms and workers do not have preferences over firms’ identities as such. Identities are only required since workers need not be indifferent between two contracts specifying the same task levels and wages if firms attract different average types of workers. Let the function $u: \Theta \times [\ell, h] \times \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$ describe agents’ preferences where $u(\theta, y; t, w)$ characterizes the utility of a worker of type $\theta$ who accepts contract $c = (t, w, n)$ if firm $n$ attracts workers of average type $y$. We assume that $u$ is at least twice partially differentiable in $y$, $t$, and $w$ with $u_t < 0$, $u_w > 0$, $u_{ew} = 0$, $u_{ee} \geq 0$, and $u_{ww} \leq 0$. Further, we assume that $u$ is unbounded above with respect to wage benefits $w$ and it is unbounded below with respect to task requirements $t$. Externalities among workers are formalized as follows.

**Assumption 1** (Externality: Workers). $u_y > 0$ for $y < \theta$ and $u_y = 0$ for $y \geq \theta$.

An worker’s utility upon accepting a contract offered by some firm is increasing in the average type of workers attracted by the same firm, as long as this average type is lower than the worker’s type.\(^5\) If a worker’s type is weakly lower than the average type, he no longer experiences a negative externality. A low type’s utility upon accepting a contract thus does not depend on the average type of worker attracted by the same firm.

Such negative externalities arise naturally in many labor market contexts. Low productivity workers can render the work environment less attractive for high productivity colleagues: less productive workers slow down production lines, they contribute less to team production, or they cause quality problems that impede efficient production. If high-productivity workers cannot – or do not want to – help those with low productivity, low productivity workers do not benefit from the presence of high productivity colleagues, while the former exert a negative externality on the latter. To fix ideas consider the following example we repeatedly will refer to throughout the paper.

**Example:** Suppose that after a worker has joined some firm he is matched into a team of two with another employee of the same firm. The two workers are promised a bonus if the team succeeds to meet a performance target. Like in Kremer (1993) production consists of many tasks, all of which must be well executed for the team to be successful. High-productivity

\(^5\)This implies that the average type of workers attracted by the same firm affects workers’ utility even if different types accept different contracts. We thus assume that firms either cannot fully prevent, or cannot credibly commit to fully prevent, that the externality spreads out among their workers. We further discuss this assumption at the beginning of Section 5.
workers can do the job, whereas the presence of at least one low-productivity worker causes
the team to fail. High-productivity workers then dislike being matched with a low-productivity
colleague. Low-productivity workers, on the other hand, do not care about their colleague’s
productivity as their own presence is already sufficient to ensure that they never get the team
bonus. High-productivity workers’ utility upon accepting the offered employment contract
thus depends on the fraction of low-productivity workers employed by the same firm. For
low-productivity workers this is not the case.\footnote{Although in the present example the externality originates from a particular production technology, it is mediated by the used employment contracts. Since we focus on the adverse selection problem, we do not make explicit the underlying moral hazard problem or derive optimal incentive contracts. Similar externalities can also arise for different reasons. For example, workers might derive intrinsic satisfaction from participating in a cooperative team production process, where teams work smoothly if and only if all team members have high productivity. Hamilton, Nickerson, and Owan (2003) provide empirical evidence for the existence of such non-monetary utility gains from working in teams.}

Firms offer contracts that specify a required task level $t$ and a base wage $w$. Part of the
compensation package is a team bonus $\gamma w$ with $\gamma > 0$. The bonus is exogenously tied to
the base salary; it is paid if and only if the team is successful, which happens if and only if
both team members are highly productive. Let $b(y)$ be the probability of being matched with
a high-productivity worker if $y$ is the average type of workers employed by firm $n$. We must
have $b(\ell) = 0$ and $b(h) = 1$. The firm might use some internal mechanisms to affect worker
matching, but our results hold as long as complete separation is impossible and $b$ is increasing
in $y$. Then a worker’s expected utility upon accepting contract $(t, w, n)$ is

$$
u(\theta, y; t, w) = \begin{cases} 
    w - t/\ell & \text{if } \theta = \ell \\
    w - t/h + b(y)\gamma w & \text{if } \theta = h
\end{cases}$$

(1)

where $t/\theta$ are the usual type-specific costs of completing tasks.

We often refer to workers’ preferences over different task levels and wages given that they are
pooled with some fixed average type of workers. Define a worker’s indifference curve $\bar{U}_\theta(y)$ in
the $(t, w)$-space as the set of all combinations of task levels and wages which – if offered by a
firm attracting workers of average type $y$ – yield workers of type $\theta$ constant utility. Formally,
this indifference curve is defined as

$$
\bar{U}_\theta(y) = \{(t, w) \in \mathbb{R}_+ \times \mathbb{R} : u(\theta, y; t, w) \equiv \bar{u}, \bar{u} \in \mathbb{R}\}.
$$

(2)

Indifference curves have slope $-u_t(\theta, y; t, w)/u_w(\theta, y; t, w)$. The latter does not depend on
the average attracted type for low types, but might depend on the average type for high
types. Following most of the literature on contracting we assume that the slope of workers’ indifference curves is decreasing in $\theta$ for all $y$. This is our version of the Spence-Mirrlees or single-crossing property. Requiring it to hold for all $y$ imposes a restriction: changing the average attracted type must not affect the slope of the indifference curves of high types so as to upset the single-crossing property. Moreover, we assume that for the two types the difference in the slope of the indifference curves is bounded away from zero, that is

$$-u_\ell(\ell, y; t, w)/u_w(\ell, y; t, w) + u_h(h, y; t, w)/u_w(h, y; t, w) > q$$

for some $q \in \mathbb{R}^+$. This ensures that any two indifference curves of the two types of workers intersect. Finally, we assume that the average attracted type $y$ has a monotone influence on the slope of high-type workers’ indifference curves: $\frac{\partial}{\partial y} \{-u_\ell(h, y; t, w)/u_w(h, y; t, w)\}$ is either always weakly positive or always strictly negative. We normalize a worker’s utility to zero in case he accepts no contract. We also assume that $u(\theta, \theta; 0, 0) = 0$ for all $\theta$. If a worker is pooled with workers of his type and accepts a contract that specifies zero task level and zero wage benefits, he thus gets a utility equal to his outside option.

Our results are entirely driven by the externality among workers, so that firms’ profit functions could be defined as usual. Yet, we show that including analogous externalities in the firms’ profit function causes no problems. Let the function $v : \Theta \times [\ell, h] \times \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$ describe firms’ profits, where $v(\theta, y; t, w)$ is firm $n$’s profit per worker of type $\theta$ accepting contract $c = (t, w, n)$ if the firm attracts workers of average type $y$. We assume that $v$ is at least once partially differentiable in $t$ and $w$ with $v_w < 0$ and $v_t = 0$. We focus on adverse selection and the associated costs of separation and thus take the task requirement to be non-productive. In this we follow Spence who assumes that education has no impact of worker productivity.\footnote{In case education or task requirements are productive, first-best levels might ensure separation so that there is no problem of adverse selection.}

The function $v$ is unbounded below with respect to wage benefits $w$. Firms prefer high types so that $v(h, y; t, w) > v(\ell, y; t, w)$ for all $(y, t, w)$.

**Assumption 2** (Externality: Firms). $v_y \geq 0$ for $y < \theta$ and $v_y = 0$ for $y \geq \theta$.

The presence of low types might reduce the firm’s profit per attracted worker of high type, but as the definition shows this externality can also be zero without affecting our results. Let $\eta(\theta; c, C)$ be the mass of workers of type $\theta$ accepting contract $c$ if set $C$ of contracts is offered. Firm $n$ who offers contracts $C_n \subseteq C$ then gets total profits

$$\sum_{\theta \in \Theta} \sum_{c \in C_n} \eta(\theta; c, C) v(\theta, y; t, w).$$
Firms get an outside option profit of zero if they do not attract any workers. We assume that \( v(\theta, \ell; 0, 0) > 0 \) for all \( \theta \). A firm thus gets more than its outside option if it can attract workers with a contract that specifies a zero wage. Together with the assumption on workers’ outside option, this ensures that mutually beneficial contracting between firms and workers is possible.

**Example (cont’d):** The profit of a firm that employs a worker of type \( \theta \) and promises base wage \( w \) is \( \theta - w \) if the worker’s team is not successful, and \( (1 + \gamma)\theta - (1 + \gamma) w \) if the team is successful. We can vary the importance of our externality by varying \( \gamma \), and we can do so without simultaneously affecting the problem of adverse selection in the labor market. If \( \gamma \) is zero we get a simple screening version of Spence’s job signaling model.

We focus on symmetric equilibria where all workers share the same type-dependent acceptance decisions and firms do not mix over their sets of offered contracts. Let \( \mathcal{P}(C) \) be the power set of contract space \( C \). Since we only consider symmetric equilibria we suppress indexation for a worker’s identity in the following. A worker’s completely specified strategy is an acceptance rule \( a : \Theta \times C \times \mathcal{P}(C) \rightarrow [0, 1] \) where \( a(\theta; c, C) \in [0, 1] \) is the probability with which he accepts contract \( c \in C \) if he is of type \( \theta \). An acceptance rule can only assign a positive acceptance probability to contracts that are offered. For all \( C \in \mathcal{C} \) and \( \theta \in \Theta \), this implies \( a(\theta; c, C) = 0 \) whenever \( c \not\in C \).

Workers’ acceptance rules define the mass \( \eta(\theta; c, C) \) of workers of type \( \theta \) accepting \( c \in C \). Let

\[
A(n; C) = \sum_{\theta \in \Theta} \sum_{c \in C_n} \eta(\theta; c, C)
\]  

be the total mass of workers attracted by firm \( n \) if it offers contracts \( C_n \subseteq C \). Unless \( A(n; C) \) equals zero the average type \( y(n; C) \) of workers attracted by firm \( n \) is directly determined as

\[
y(n; C) = \frac{1}{A(n; C)} \sum_{\theta \in \Theta} \sum_{c \in C_n} \eta(\theta; c, C) \theta.
\]  

If \( A(n; C) \) equals zero, \( y(n; C) \) is not pinned down by workers’ acceptance decisions. In our setting this does not cause any problems: if a worker accepts a contract offered by some firm that attracts no other workers, his own type determines the average type of workers attracted by the firm. As a single worker has mass zero he cannot influence the average type of worker attracted by a firm as soon as this firm draws a positive mass of workers.
In the presence of externalities, a worker’s utility might depend on the acceptance decisions of the other workers. We account for this as follows in our definition of equilibrium. Extending the notion of a competitive equilibrium from Rothschild and Stiglitz (1976), we define an equilibrium as an equilibrium set of offered contracts plus workers’ equilibrium acceptance rules. An equilibrium set $C^*$ of offered contracts is a finite set of contracts that satisfies two conditions. First, each firm gets at least zero profit in equilibrium. Otherwise, a firm could increase its profits by leaving the market.\footnote{We thus do not require that all contracts break even. However, the possibility for cross-subsidization does not drive any of our results.} Second, since the equilibrium set of contracts is finite, there always exists a firm $\tilde{n} \in \mathbb{N}$ who currently is not active on the market and gets zero profits. Then this firm $\tilde{n}$ must not be able to enter the market with a menu of contracts $C_{\tilde{n}}$ that can attract a strictly positive mass of workers while yielding positive overall profit. Perfect competition is thus formalized via a no-market-entry condition. Concerning the workers we require their equilibrium acceptance decisions $a^*$ to form a Bayesian equilibrium for all finite sets $C \subseteq C$ of offered contracts.\footnote{A Bayesian equilibrium need not exist if an infinite number of contracts is offered. We restrict the equilibrium set $C^*$ of offered contracts to be finite, and we only consider market entry by single firms who by assumption can only offer finite menus of contracts. It is thus sufficient to specify workers’ acceptance decisions for finite sets of offered contracts.} This requires that acceptance decisions maximize the worker’s utility given the distribution $\eta^*$ of workers across firms, and this distribution is consistent with the distribution of workers’ types and their corresponding acceptance decisions.

One of our main results is that there must be separation in any competitive equilibrium. Since firms’ identities as such are irrelevant, the model is silent on which firm offers which contract in equilibrium. However, workers’ utility functions and firms’ profit functions uniquely pin down the task levels and wages of contracts that are accepted by high and low types. These contracts include wages $(w_\ell, w_h)$ and task requirement $t_h$ that are implicitly defined by

\begin{align}
  v(\ell, y; 0, w_\ell) &= 0 \quad \quad (6) \\
  v(h, h; t_h, w_h) &= 0 \quad \quad (7) \\
  u(\ell, y; t_h, w_h) &= u(\ell, y; 0, w_\ell). \quad \quad (8)
\end{align}

As a low type’s utility does not depend on the average attracted type, the choice of $y$ in (6) and (8) is irrelevant. We show in the appendix that our assumptions on $v$ and $u$ guarantee the existence of a unique, finite, and strictly positive solution to (6) to (8) with $w_h > w_\ell$. We
define the sets of contracts

\[ C_\ell = \{ (t, w, n) \in C : t = 0 \text{ and } w = w_\ell \} \quad (9) \]

\[ C_h = \{ (t, w, n) \in C : t = t_h \text{ and } w = w_h \} \quad (10) \]

with generic elements \( c_\ell \in C_\ell \) and \( c_h \in C_h \).

**Definition 1** (Best Separating Equilibrium). In a best separating equilibrium firms offer contracts from both sets \( C_\ell \) and \( C_h \). Moreover, the following properties hold:

1. All workers accept some contracts. Workers of type \( \ell \) only accept contracts from \( C_\ell \) while workers of type \( h \) only accept contracts from \( C_h \).

2. Workers separate across firms according to their type. There are no contracts \( \{c, \tilde{c}\} \subseteq C^*_n \) offered by some firm \( n \) with \( a^*(h; c, C^*) > 0 \) and \( a^*(\ell; \tilde{c}, C^*) > 0 \).

Due to externalities there can be multiple equilibria at the acceptance stage. The following refinement rules out competitive equilibria that exist only because workers re-coordinate on better acceptance decisions once otherwise irrelevant new contracts are offered.

**Refinement** (No Switch). Consider an equilibrium set \( C^* \) of offered contracts. Suppose a new firm \( \tilde{n} \) enters the market with a set of contracts \( C_{\tilde{n}} \) that does not attract any workers, \( a^*(\theta; c, C^* \cup C_{\tilde{n}}) = 0 \) for all \( \theta \in \Theta, c \in C_{\tilde{n}} \). Then workers keep their old acceptance decisions, \( a^*(\theta; c, C^* \cup C_{\tilde{n}}) = a^*(\theta; c, C^*) \) for all \( c \in C^* \).

### 4 Results

Before we come to our main result concerning equilibrium existence (Proposition 1) we derive conditions for the existence of a best separating equilibrium if firms do not offer contracts that are not accepted in equilibrium. This allows us to characterize the importance of latent contracts for equilibrium existence. The following lemma shows that if all offered contracts are accepted in equilibrium, then the utility loss of high types from being pooled with only low types must exceed the minimum screening costs high types have to incur in order to ensure separation in equilibrium. The lemma prepares the ground for our main result on arbitrarily small externalities. All proofs can be found in the appendix.

**Lemma 1** (Best Separating Equilibrium - No Preemptive Contracts). There exists a best separating equilibrium for all \( \mu_0 \in (0, 1) \) in which firms offer only contracts in \( C_\ell \cup C_h \) if and only if \( u(h, h; t_h, w_h) \geq u(h, \ell; 0, w_h) \).
Contrary to the situation without externalities, market entry with a pooling contract need not be profitable even if the fraction of low-type workers is arbitrarily small. Indeed, in one equilibrium at the acceptance stage all workers optimally accept the new pooling contract. But in another equilibrium the high types do not accept the new contract because the new firm only attracts low types. Since the new firm would then make losses, it does not enter the market. The externality thus creates multiple equilibria at the acceptance stage. By specifying the right equilibrium response to market entry – high-productivity workers essentially fail to coordinate their contract choices – equilibrium existence is guaranteed. Note that our result is not driven by a particular specification of out-of-equilibrium beliefs. If a new firm does enter the market, it attracts all low types whose acceptance decisions thus pin down beliefs. Instead, the multiplicity of Bayesian equilibria at the acceptance stage solves the equilibrium existence problem.

Example (cont’d): In our example the best separating contracts specify \( w_\ell = \ell \) for low-productivity workers, and \( w_h = h \) and \( t_h = \ell(h - \ell) \) for high-productivity workers. These contracts can form a competitive equilibrium for all \( \mu_0 \) if and only if

\[
\gamma h \geq \frac{\ell(h - \ell)}{h}. \tag{11}
\]

Choosing their contract high-productivity workers can secure themselves the bonus \( \gamma h \), but they incur screening costs \( t_h/h = \ell(h - \ell)/h \). By accepting the pooling contract they save the screening costs, but as they are matched with low-productivity workers they loose the bonus. If the relation \( \gamma \) of bonus to base salary – which in our example corresponds to a measure of the externality among workers – is sufficiently large, there always exists a best separating equilibrium. As shown by Rothschild and Stiglitz (1976) this condition cannot be fulfilled if the externality \( \gamma \) disappears.

Lemma 1 describes conditions for the existence of a best separating equilibrium in which firms do not offer any contracts that are not in \( C_\ell \) or \( C_h \). We now demonstrate that an arbitrarily small externality can be sufficient to guarantee equilibrium existence if firms offer contracts that are never accepted in equilibrium. The sole purpose of these “preemptive contracts” is to prevent market entry.

The argument runs as follows. Consider a firm that attracts only low types in equilibrium. Suppose this firm also offers some contract that would be very attractive for high types if it attracted only high types, but in equilibrium this contract is not accepted as the firm only
attracts low types. Now if another firm enters the market and draws all low types, the old
firm offering the preemptive contract attracts no workers any more. It thus suddenly becomes
very attractive for high types. If the new firm cannot draw any high types, it gets negative
profits and there is no market entry.\footnote{The argument resembles the line of reasoning in Riley (1979) who allows firms to offer new contracts and thereby to skim off the good types after market entry. In our model such contracts are already offered in equilibrium.}

To analyze preemptive contracts in our setup, let $\Gamma$ be the set of all combinations of wages
and task levels that do not attract any high types if offered by a firm who currently attracts
only low types. $\Gamma$ contains all $(t, w)$ that satisfy
\begin{align}
    u(\ell, y; t_{\ell}, w_{\ell}) & \geq u(\ell, y; t, w) \quad (12) \\
    u(h, h; t_{h}, w_{h}) & \geq u(h, \ell; t, w) \quad (13)
\end{align}

As a low type’s utility does not depend on the average attracted type, the choice of $y$ in (12)
is irrelevant. Because preemptive contracts do not attract any workers in equilibrium, they
cannot cause any losses. No constraint concerning the profits of the offering firm is needed.
Define
\[ U_p = \sup_{(t, w) \in \Gamma} \{ u(h, h; t, w) \} \quad (14) \]
as the supremum of the utility which high types can get if they accept a preemptive contract
which is offered by a firm that – after market entry – attracts no workers. We obtain the
following result.

**Lemma 2** (Supremum Utility Preemptive Contracts). Consider the supremum utility $U_p$
that high-type workers can get by accepting a preemptive contract after market entry.

1. If $\frac{\partial}{\partial y} \{ -u_i(h, y; t, w) / u_w(h, y; t, w) \} \geq 0$ then $U_p = u(h, h; t_p, w_p)$, where $(t_p, w_p)$ is im-
plicitly defined by $u(\ell, y; 0, w_{\ell}) = u(\ell, y; t_p, w_p)$ and $u(h, h; t_{h}, w_{h}) = u(h, \ell; t_p, w_p)$.

2. If $\frac{\partial}{\partial y} \{ -u_i(h, y; t, w) / u_w(h, y; t, w) \} < 0$ then $U_p = +\infty$.

Figure 1 illustrates Lemma 2. Set $\Gamma$ is the area under the two indifference curves $\bar{U}_\ell(y)$ and
$\bar{U}_h(\ell)$ for some $y$ (since the average attracted type is irrelevant for low types). If increasing
the average attracted type does not flatten the indifference curves of high types, the supremum
utility $U_p$ is finite and is attainable by accepting a preemptive contract with task requirement
and wage $(t_p, w_p)$. But if increasing $y$ flattens the indifference curves of high types, moving
up along $\bar{U}_h(\ell)$ increases the high types’ utility without violating any constraint. In this case
$U_p$ equals plus infinity. It is now possible to characterize sufficient and necessary conditions for the existence of a best separating equilibrium.

**Proposition 1** (Best Separating Equilibrium). There exists a best separating equilibrium for all $\mu_0 \in (0,1)$ if and only if $U_p \geq u(h,\ell;0,w_h)$. By Lemma 2 this implies that $U_p$ can be arbitrarily large even for arbitrarily small externalities. Equilibrium existence is then guaranteed.

Note that our main result does not arise because the externality creates multiple Bayesian equilibria at the acceptance stage and workers fail to coordinate on the right contract choices: there are no multiple equilibria at the acceptance stage if $U_p$ is arbitrarily large. In this case low-type workers strictly prefer the new pooling contract no matter what high-type workers do. Once all low-type workers are attracted by the market entrant, each high-type worker can get an arbitrarily large utility by accepting the preemptive contract. They thus prefer the latter to the new pooling contract even if all other high-type workers join the new firm. In any equilibrium at the acceptance stage, the market entrant thus makes losses. Profitable market entry is not possible. Lemma 2 and Proposition 1 imply that arbitrarily small externalities can ensure equilibrium existence. In the following we show with the help of our example that the required flattening of the high types’ indifference curves might well be plausible.

**Example (cont’d):** In our example a high-productivity worker is indifferent to – and thus optimally rejects – any preemptive contract that specifies for task level $\tilde{t}$ a base salary $\tilde{w} =$
\( w + \frac{\ell}{h} \) where
\[
w = h(1 + \gamma) - \frac{1}{h}(h - \ell).
\] (15)

Among these contracts we look for a preemptive contract that satisfies two conditions. First, the preemptive contract is not accepted by low-productivity workers. Using these workers’ equilibrium utility yields the following condition
\[
\tilde{\ell} \geq \frac{\ell h}{h - \ell} \left( h(1 + \gamma) - \ell - \frac{1}{h}(h - \ell) \right).
\] (16)

Second, the preemptive contract must attract all high-productivity workers in case there is market entry. If the pooling contract only attracts low-productivity workers, a high-productivity worker who joins the new firm never gets the bonus and thus does not earn more than utility \( h \). He prefers the preemptive contract in case
\[
\tilde{\ell} \geq \frac{h}{\gamma} \left( h(1 + \gamma) - \frac{1}{h}(h - \ell) \right).
\] (17)

Market entry can thus be prevented by a preemptive contract that specifies a task level sufficiently large so as to satisfy both (16) and (17). Such a contract can be easily found for any level of externality \( \gamma \), thus an arbitrarily small externality is sufficient to guarantee equilibrium existence.

In our setup latent contracts, i.e., contracts that are not accepted in equilibrium, may prevent market entry. This connects our analysis to the literature on competitive equilibria in markets with moral hazard and nonexclusive contracting. With nonexclusive contracting principals are limited in their ability to prevent workers from contracting with other principals. Analyzing the insurance market, Arnott and Stiglitz (1991) show that there might then exist competitive equilibria in which agents buy insurance from only one firm that makes strictly positive profits. Market entry is prevented by latent insurance policies. The argument is the following: if a new firm offers a more attractive insurance policy, agents accept the new contract. But they subsequently find it attractive to use previously latent contracts to buy additional insurance. This destroys the agents’ incentives to avoid accidents so that the new insurance policy makes expected losses. Anticipating this, no firm enters the market.\(^{11}\) Our results show that if there are externalities among workers, similar arguments hold in markets with adverse selection and exclusive contracting.

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\(^{11}\)In a related article, Hellwig (1983) gives firms the option to communicate to other firms any contractual relationship with an agent. Firms might condition the terms of their contracts on this information. Results remain similar. For a more recent contribution to this literature, see Bisin and Guaitoli (2004).
So far we have focused on the sufficient and necessary conditions under which externalities ensure the existence of a best separating equilibrium. We now proceed to show that whenever there exists a competitive equilibrium in pure strategies, it is a best separating equilibrium. We therefore have segregation on the labor market despite the existence of externalities between workers. The main step is to show that there cannot be pooling in equilibrium.

**Lemma 3 (No Pooling).** In any competitive equilibrium there are no contracts \( \{c, \hat{c}\} \subseteq C^*_n \) offered by some firm \( n \) with \( a^*(\ell; c, C^*) > 0 \) and \( a^*(h; \hat{c}, C^*) > 0 \).

Adapting the arguments by Rothschild and Stiglitz (1976) it is now easy to show that any competitive equilibrium must be a best separating equilibrium. Since only low types exert an externality on high types, low types behave as if there were no externalities at all. In any competitive equilibrium they must thus get their contracts from set \( C_\ell \). If high types do not get their contracts from \( C_h \), then a new firm can enter the market and offer a contract that brings these workers closer to the best separating equilibrium. This contract is designed so as to never attract low types. High types can then be certain that they will be either alone or among themselves whenever they accept the new contract. A strictly positive mass of high types is thus attracted while low types stick to their old contract choices. The new contract then yields the firm strictly positive profits, and there is market entry.

**Proposition 2 (Description and Existence of Competitive Equilibrium).** Any competitive equilibrium is a best separating equilibrium as characterized in Definition 1. By Proposition 1 a competitive equilibrium thus exists if and only if \( U_p \geq u(h, \ell; 0, w_h) \).

As in Rothschild and Stiglitz (1976) any competitive equilibrium is characterized by Pareto-dominant separation. Although externalities facilitate equilibrium existence, they do not affect equilibrium predictions. Moreover, sufficient and necessary conditions for the existence of a best separating equilibrium are sufficient and necessary conditions for the existence of any competitive equilibrium.

## 5 Discussion

In this paper we show that negative externalities among workers can mitigate the equilibrium existence problem in competitive markets with adverse selection. In a sense, a particular form of externality exists in standard adverse selection models, as well: the presence of low-productivity types prevents firms from offering high wages to workers with high-productivity without requiring them to acquire some minimum task level that ensures separation. The externalities we have in mind are more direct as they arise once workers contract with the
same firm. It is crucial that these externalities lie beyond the control of firms. To see this
suppose a firm can credibly guarantee to protect high types from low types – for example
by structural means that separate types within the organization, or by committing to pre-
cisely specified contractual terms. Since this makes pooling contracts attractive again, there
is market entry and the equilibrium existence problem continues to exist. However, it is
not clear whether firms can indeed credibly commit to eliminate externalities among their
workers. For example, having separate plants for workers of different productivity might be
prohibitively expensive or impossible given the production technology. Moreover, the ex-
ternalities might arise because firm adopt certain production technologies or organizational
practices like team production. Externalities then cannot be avoided without fundamentally
changing the production process. Finally, our main result holds even if the effective extern-
ality – the externality that remains after firms do their best to separate workers according to
their types – is arbitrarily small. Unless firms can fully eliminate all externalities completely,
equilibrium existence is thus guaranteed.

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Appendix (Proofs)

Definition of a Best Separating Equilibrium

\[ v(\ell, y; 0, 0) > 0 \] and \( v \) is unbounded below concerning wage benefits \( w \). Then \( v_w < 0 \), con-
 tinuity of \( v \), and the intermediate value theorem imply that there exists a unique, finite, and
strictly positive \( w_\ell \) satisfying (6). By the same argument there exists a unique, finite, and
strictly positive \( w_h \) satisfying (7) where \( v_t = 0 \) implies that \( w_h \) does not depend on \( t_h \).

\[ v(h, y; t, w) > v(\ell, y; t, w) \] for all \((y, t, w)\). Then \( v(h, h; t_h, w_\ell) > v(\ell, h; t_h, w_\ell) = 0 \) where the
last equality follows from \( v_t = 0 \) and \( v_y(\ell, y; t, w) = 0 \) for all \((y, t, w)\). This yields \( w_h > w_\ell \) so
that \( u(\ell, y; 0, w_h) > u(\ell, y; 0, w_\ell) \) since \( u_w > 0 \). Function \( u \) is unbounded below concerning
task level \( t \). Then \( u_\ell < 0 \), continuity of \( u \), and the intermediate value theorem imply the
existence of a unique, finite, and strictly positive \( t_h \) satisfying (8).
Proof of Lemma 1

We first show that workers get more than their outside option in a best separating equilibrium. This holds directly for low types since $w_{\ell}>0$, $u_w>0$, and $u(\ell,y;0,0) = 0$ for all $y$. By definition $u(\ell,h;\hat{t}_h,\hat{w}_h) = u(\ell,h;0,\ell)$. This yields $u(h,h;\hat{t}_h,\hat{w}_h) > u(h,h;0,\ell)$ because $\hat{t}_h > 0$ and the single crossing property holds for all $y$. However, $u(h,h;0,\ell) > 0$ follows from $w_{\ell}>0$, $u_w>0$, and $u(h,h;0,0) = 0$. In equilibrium high types thus get more than their outside option. We next show the main result from the lemma.

Part 1: Sufficiency

Consider a best separating equilibrium in which firms only offer contracts in $C_{\ell} \cup C_h$. By definition this equilibrium cannot be upset by a firm that enters the market and then attracts only one type of workers.

Suppose firm $\tilde{n}$ enters the market with contracts $\tilde{c}_{\ell} = (\ell, \tilde{w}_{\ell}, \tilde{n})$ and $\tilde{c}_h = (\hat{t}_h, \hat{w}_h, \tilde{n})$ for type $\ell$ and $h$. As otherwise type $\ell$ optimally rejects, $u_y(\ell, y; t, w) = 0$ and $u_t < 0$ imply $\tilde{w}_{\ell} > w_{\ell}$. Because they do not care for the average attracted type, type $\ell$'s acceptance decisions do not depend on the behavior of type $h$. If $a^*(\ell; \tilde{c}_{\ell}, C^* \cup C_{\tilde{n}})$ is strictly positive in one equilibrium at the acceptance stage, $\tilde{n}$ must attract all type $\ell$ in any equilibrium at the acceptance stage.

$v_y(\ell, y; t, w) = 0$, $v_t = 0$, and $\tilde{w}_{\ell} > w_{\ell}$ imply $v(\ell, y; \tilde{t}_{\ell}, \tilde{w}_{\ell}) < 0$. Firm $\tilde{n}$ makes losses unless it can attract $h$. The proof proceeds to show that for every equilibrium at the acceptance stage with $a^*(h; \tilde{c}_h, C^* \cup C_{\tilde{n}}) > 0$ in which the firm breaks even, there exists another equilibrium with $a^*(h; \tilde{c}_h, C^* \cup C_{\tilde{n}}) = 0$. Given these acceptance decisions firm $\tilde{n}$ makes losses and thus does not enter the market.

Suppose $a^*(h; \tilde{c}_h, C^* \cup C_{\tilde{n}}) > 0$ and firm $\tilde{n}$ makes no losses. If $y_{\tilde{n}}$ is its average attracted type, $v(h, y_{\tilde{n}}; \hat{t}_h, \hat{w}_h) > 0$ holds. With $w_{\ell} < 0$, $u_{w} > 0$, $u_t < 0$, and $v_t = 0$, this implies that type $h$ who accept $\tilde{c}_h$ cannot get more than $u(h, y_{\tilde{n}}; 0, \hat{w}_h)$ where $\hat{w}_h$ solves $v(h, y_{\tilde{n}}; 0, \hat{w}_h) = 0$. Take $w_h$ from Definition 1. The task is not productive so that $v_y(h, y; t, w) \leq 0$ and $y_{\tilde{n}} \leq h$ imply $w_h \geq \hat{w}_h$. Then $a^*(\ell; \tilde{c}_{\ell}, C^* \cup C_{\tilde{n}}) = 1$ and $a^*(h; \tilde{c}_h, C^* \cup C_{\tilde{n}}) = 0$ with $y_{\tilde{n}} = \ell$ form an equilibrium at the acceptance stage by the following arguments. Type $\ell$ act optimally by the above arguments. Further, type $h$ who accept $\tilde{c}_h$ get $u(h, \ell; 0, \hat{w}_h)$ and thus less than $u(h, \ell; 0, \hat{w}_h)$. They get $u(h, h; t_h, w_h)$ by accepting $c \in C_h$. As $u(h, h; t_h, w_h) \geq u(h, \ell; 0, w_h)$ they choose $c_h$. 

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Part 2: Necessity

Take a best separating equilibrium and consider a sequence \( \{\mu_k\}_{k \in \mathbb{N}} \) of prior probabilities with \( \mu_k < 1 \) for all \( k \in \mathbb{N} \) but \( \lim_{k \to \infty} \mu_k = 1 \). Define \( y_k = \ell + \mu_k(h - \ell) \) and \( \epsilon_k = 1/k \).

Given \( \mu_k \) suppose a firm \( n \) enters the market with a pooling contract \( c_k = (0, w_k - \epsilon_k, n) \) where \( w_k \) is implicitly defined by \( \mu_k v(h, y_k; 0, w_k) + (1 - \mu_k)v(\ell, y_k; 0, w_k) = 0 \). It thus gets strictly positive profits if it can attract all workers. Then \( v_i = 0 \) and continuity of \( v \) imply \( \lim_{k \to \infty} w_k = w_h \). If \( u(h, h; t_h, w_h) < u(h, \ell; 0, w_h) \) then continuity of \( u \) implies that there exists a \( K \in \mathbb{N} \) so that for all \( k \geq K \) we have \( u(h, \ell; 0, w_k) > u(h, h; t_h, w_h) \).

In the following take some \( \mu_k \) with \( k \geq K \).

Given prior probability \( \mu_k \) and market entry by firm \( n \), type \( \ell \) accept \( c_k \) since \( w_k > w_\ell \). As \( u(h, \ell; 0, w_h) > u(h, h; t_h, w_h) \) type \( h \) prefer \( c_k \) to any \( c_h \in C_h \) even if \( y(n; C^* \cup c_k) = \ell \).

The single-crossing property and \( t_h > 0 \) imply that high types prefer contract \( c_h \in C_h \) to any contract \( c_\ell \in C_\ell \) even if the firm that offers \( c_\ell \) does not attracting any low types. Only contracts in \( C_\ell \cup C_h \) are offered. Transitivity implies \( a^*(h; c_k, C^* \cup c_k) = 1 \) and \( a^*(\ell; c_k, C^* \cup c_k) = 1 \) in any equilibrium at the acceptance stage. As it thereby gets strictly positive profits, firm \( n \) enters the market.

Proof of Lemma 2

We look for the supremum \( U_p \) as defined in (14). First, suppose that both (12) and (13) are slack. Then one can increase the wage benefits \( w \) without violating any constraint so that \( U_p > u(h, h; t, w) \) for such \( (t, w) \). Second, suppose that (12) is binding while (13) is slack. Take marginal changes \( dw, dt > 0 \) with \( dw/dt = -u_t(\ell, y; t, w)/u_w(\ell, y; t, w) \). This keeps (12) satisfied. By the single-crossing property (13) holds while \( u(h, h; t + dt, w + dw) > u(h, h; t, w) \).

Thus, \( U_p > u(h, h; t, w) \) for such \( (t, w) \). Third, consider \( (t_p, w_p) \) so that by definition both (12) and (13) are binding. By the boundedness condition (3) such a point of intersection exists. Consider marginal changes \( dw, dt > 0 \) with \( dw/dt = -u_t(\ell, \ell + dt, w + dw)/u_w(h, \ell; t_p, w_p) \).

This keeps (13) satisfied while slackening (12) by the single-crossing property. There are then two cases.

First, suppose \( \partial / \partial y \{-u_t/u_w\} \geq 0 \). Then \( u(h, h; t_p, w_p) \geq u(h, h; t_p + dt, w_p + dw) \) so that contracts with \( (t_p, w_p) \) provide type \( h \) with the maximum utility \( U_p = u(h, h; t_p, w_p) \). Second, suppose \( \partial / \partial y \{-u_t/u_w\} < 0 \). Then \( u(h, h; t_p, w_p) < u(h, h; t_p + dt, w_p + dw) \) where by (3) this increase in \( u(h, h; t_h, w_k) \) is bounded away from zero. Because one can find similar contract
adjustments for all \((t, w)\) such that (12) is slack while (13) is binding, there exists an infinite sequence \(\{(t_k, w_k)\}_{k \in \mathbb{N}}\) with \((t_k, w_k) \in \Gamma\) for all \(k \in \mathbb{N}\) so that \(\lim_{k \to \infty} u(h; h; t_k, w_k) = +\infty\). We then have \(U_p = +\infty\). \(Q.E.D.\)

Proof of Proposition 1

Consider a best separating equilibrium in which firm \(n\) offers a contract \(c_\ell \in C_\ell\) that attracts type \(\ell\). Suppose \(n\) also offers a contract \(c_n \notin C_\ell \cup C_h\) while \(c_n \in \Gamma\). This contract \(c_n = (t_n, w_n, n)\) is not accepted in equilibrium, but it changes the conditions under which a newly offered contract \(\tilde{c}_h\) can attract type \(h\). Contract \(\tilde{c}_h\) need not draw type \(h\) if and only if they have a better option in \(C^*\). Type \(h\) who accept \(c_n\) can now get \(u(h; h; t_n, w_n)\) once \(n\) no longer attracts any type \(\ell\). There exists \(c_n \in \Gamma\) so that they reject \(\tilde{c}_h\) for all \(\mu_0 \in (0, 1)\) if and only if \(U_p \geq u(h; \ell; w_h, 0, \tilde{n})\). \(Q.E.D.\)

Proof of Lemma 3

Consider a competitive equilibrium in which firm \(n\) offers one or several contracts and attracts all types of workers. Let \(y^*(n; C^*) \in (\ell, h)\) be the average type of worker it attracts in equilibrium. The proof proceeds in two steps.

Part 1: Strictly Positive Profits with Low Types

Let \(c = (t, w, n)\) be a contract offered by \(n\) that attracts \(\ell\) while \(v(\ell, y; t, w) > 0\). Suppose a new firm \(\tilde{n}\) enters the market with contract \(\tilde{c} = (t, w + \epsilon, \tilde{n})\) with \(\epsilon > 0\). Then \(u_w > 0\) and \(u_y = 0\) for type \(\ell\) imply \(u(\ell, y; t, w + \epsilon) > u(\ell, y; t, w)\) for any \(y\) and \(\tilde{y}\). Consequently, \(a^*(\ell; \tilde{c}, C^* \cup \tilde{c}) = 1\) and contract \(\tilde{c}\) can attract at least all type \(\ell\) in any equilibrium at the acceptance stage. Further, \(v_y(\ell, y; t, w) = 0\) and \(v(\ell, y; t, w) > 0\) imply \(v(\ell, \ell; t, w + \epsilon) > 0\) for small \(\epsilon\). Finally, \(v(h, y; t, w + \epsilon) > v(\ell, y; t, w + \epsilon)\) for all \(y\) so that \(\tilde{c}\) always yields firm \(\tilde{n}\) strictly positive profits no matter what workers are attracted. As \(\tilde{c}\) attracts at least workers of type \(\ell\) the original situation cannot form an equilibrium.

Part 2: Weakly Negative Profits with Low Types

Now suppose there is no \(c \in C_n^*\) that attracts \(\ell\) and \(v(\ell, y^*(n; C^*); t, w) > 0\). As firm \(n\) otherwise gets strictly negative profits in equilibrium, there must exist \(\tilde{c} = (\tilde{t}, \tilde{w}, n) \in C_n^*\) that attracts \(h\) and \(v(h, y^*(n; C^*); \tilde{t}, \tilde{w}) \geq 0\). Suppose a new firm \(\tilde{n}\) enters the market and offers contract \(\tilde{c} = (\tilde{t}, \tilde{w} - \epsilon, \tilde{n})\) with \(\epsilon > 0\).
Together with the optimality of the original contract choice, $u_w > 0$ and $u_y(\ell, y; t, w) = 0$ imply $u(\ell, y; t, w) \geq u(\ell, y; \hat{t}, \hat{w} - \epsilon)$ for any $y$ and $\hat{y}$. Then $a^*(\ell; \hat{c}, C^* \cup \hat{c}) = 0$ so that $\hat{c}$ never attracts type $\ell$ in any equilibrium at the acceptance stage. Because we restrict attention to symmetric acceptance decisions, there are two cases.

First, suppose $\hat{c}$ attracts nobody. Then Refinement (No Switch) requires $a^*(\theta; c, C^* \cup \hat{c}) = a^*(\theta; c, C^*)$ for all $\theta \in \Theta$, $c \in C^*$. Type $h$ get utility $u(h, y^*(n; C^*); \hat{t}, \hat{w})$ in equilibrium. Then $y^*(n; C^*) < h$ and $u_y(h, y; t, w) < 0$ imply $u(h, h; \hat{t}, \hat{w} - \epsilon) > u(h, y^*(n; C^*); \hat{t}, \hat{w})$ for small $\epsilon$. A worker who alone accepts $\hat{c}$ determines the average type attracted by $\tilde{n}$. Rejection of $\tilde{n}$ thus cannot be optimal for type $h$. Second, suppose $\hat{c}$ attracts type $h$. In both cases firm $\tilde{n}$ then gets strictly positive profits from entering the market because $v_w < 0$, $v_y(h, y; \hat{t}, \hat{w}) \geq 0$, and $y^*(n, C^*) < h$ imply $v(h, h; \hat{t}, \hat{w} - \epsilon) > v(h, y^*(n, C^*); \hat{t}, \hat{w}) \geq 0$.

Q.E.D.

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