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Costly innovators versus cheap imitators:  
a discrete choice model*

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Abstract

Two alternative ways to an innovative product or process are R&D investment  
or imitation of others’ innovation. In this article we propose a discrete choice model  
with costly innovators and free imitators and study the endogenous dynamics of  
price and demand in a market with many firms producing a homogeneous good.  
The basic idea is that imitation works better the more innovators are around, with  
a trade off between the advantages of the two strategies. First we look at innovation  
as costs reduction in a perfectly competitive market. Here we also study the sta-  
bilizing or destabilizing effect of memory and asynchronous updating of strategies.  
Then we introduce endogenous technological progress and analyze the determinants  
of the speed of price reduction as well as the occurrence of an initial oscillatory phase  
that precedes convergence. An extension of the model introduces product differentia-

tion addressing the effects of innovation on demand. While the basic version of  
the model have stable equilibrium or cyclical behaviour, there are conditions for  
chaotic behaviour of price and agents’ choices. This is the case of long memory and  
asynchronous updating of strategies, as well as with innovations affecting demand.  
These results indicate how the dynamical interplay of innovators and imitators can  
contribute to markets variability.

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1 Introduction

Innovation is an important strategy of firm competition. Two alternative ways to innovate are R&D investment or imitation of others’ innovation (Nelson and Winter, 1982). Imitation can be perceived not just as free riding but also as exploitation of external sources. This can involve public knowledge such as published research but also spillovers and leakages from private knowledge (Spence, 1984). Nevertheless innovation in strict sense is the production or implementation of a novelty resulting from R&D activity. Considering the taxonomy of Malerba (1992), innovation and imitation refer to learning by searching and learning from spillovers, where in the latter we may include all different kinds of information flows, from knowledge leakages to pure product copying activity. Here we will consider imitation in a similar way to Conlisk (1980) model of costly optimizers versus cheap imitators, being interested in the interplay between innovation and imitation as strategies and its effect on innovation dynamics. Technology is a non-rival partially excludable good (Romer, 1990), which makes direct imitation possible, although with different levels of difficulty from case to case. In the end, it will always be possible to copy a design of a new product once this is in production. Benoit (1985) addresses non-patentable innovations and study the interplay of innovators and imitators in the strategic setting of a duopoly. We consider a population of firms instead, where innovation and imitation are two alternative choices that identify ex post an agent.

Heterogeneity seems to be central to the process of innovation, in scientific as well as applied R&D. It gives rise to a differentiated cost structure or to differentiated products. Our aim is to examine the impact of heterogeneity of agents on innovation dynamics, understanding what internal forces (within an industry) determine the prevalence of innovation or imitation. One may ask then what is the interplay between technological conditions and market forces, the mutual interaction between opportunities given by science and appropriability of related technologies on the one side and market-induced effects as changes in prices and demand on the other.

The empirical evidence presents a substantial unexplained inter-firm and intra-sectoral variance of innovation proxies (R&D expenditure, innovative output, patenting activity, etc.) after allowing for the firms’ size effect (Dosi, 1988). This suggests other factors of influence on the innovative process, such as heterogeneity of firms’ strategies and firms interactions. For instance imitation would be missed by data on patents while it is an important constituent of innovation dynamics. Moreover innovation and imitation are not just static approaches of firms to market competition. It is likely that a considerable portion of innovation proxies’ variance derives from the interplay between innovation and imitation and from its evolving character, with the possibility that firms switch strategy.
Consequently, it is important that a theoretical model of innovation dynamics is able to capture the evolving nature of innovation through the adaptive strategy of firms.

If one is interested in micro-motives underlying innovation dynamics, the paradigm of a representative rational profit maximizing firm appears not appropriate. A more suitable approach is to consider different sensible strategies or rules open to a firm when posing to itself the question of innovation and its possible forms in order to gain competitive advantage. This is in line with Schumpeter’s hypothesis of routinization of innovation (Schumpeter, 1942), and more generally with Simon (1957) idea that information gathering and processing costs are an obstacle to the fully rational behaviour of firms. In order to answer to such behavioural diversity we address innovation and imitation adopting the evolutionary adaptive model based on discrete choice of Brock and Hommes (1997). As Conlisk (1980), we model a mechanism where some kind of negative feedback makes it profitable to change rule by agents when their type become too diffuse. Differently from Conlisk, in our model the economy evolves not due to an exogenous stochastic process but under the endogenous action of innovators and imitators. An endogenous model of the interplay between sophisticated and naive agents is Sethi and Franke (1995). However this model as well as Conlisk’s model is globally stable: if it was not for exogenous random shocks the economy in these two models would converge to a homogeneous equilibrium where all agents use the cheap strategy. In our model different long run dynamics are possible beyond stable equilibrium, without any exogenous stochastic process.

We consider an industry where one technology underlies the production of one good. In a competitive market firms need to catch up either via innovation or imitation in order to recover in terms of profitability and market competitiveness (Dosi, 1988). Our model mainly deals with innovation as cost reduction: with an evolutionary dynamics of innovators and imitators aiming at cost reduction, we bring together the Iwai (1984) and Conlisk (1980) models. A first version of the model considers innovator firms opposed to non-innovator ones. In a second version we introduce the imitation externality and study the endogenous dynamics of innovation and imitation.

There are two main assumptions in our model: the first is a deterministic positive outcome of innovation investment, the second is ignoring the mutual effects of technological progress and agents decisions. The dynamics of price and agents’ fractions evolves on the underlying technological progress. With this restriction we focus on the behavioural relative effects which may explain the excess variance of innovation proxies.

The model in extended in various directions: first we introduce technological progress and study its effects on the industrial dynamics. Separately we add product differentiation to cost reduction and study innovation effects on the demand beside the effects on supply. Moreover we study the effect of memory and asynchronous updating in agents’ decisions,
trying to capture persistence of behaviours.

Resuming, we address questions such as: what role does heterogeneity of firms play in the dynamics of innovation? What are the behavioural determinants in an industry leading to more or less innovation? What conditions maximize social welfare, which takes into account not only innovation per se but also its costs? Are there long run equilibria where everybody innovates or imitates? What characteristics would be responsible for an industry being stable or unstable, evolving fast or slowly? Ultimately, quoting Hahn (1970), is it true that the pursuit of private interests, as firms’ profits in this case, produce not chaos but coherence? Depending on the relative intensity of forces internal to the industry, we see that firms’ choices and market price may become unstable, accounting for a process as innovation where an equilibrium may be never attained and an evolving dynamics is the most evident feature (Freeman and Soete, 1997).

Beside the market equilibrium of demand and supply in a market with heterogeneous firms, our model more generally addresses interacting agents that make a choice about whether or not to invest in order to be more productive. Beyond the industrial dynamics issues just presented we may think to other possible interpretations in this sense, that concern other economic and social contexts. An example is the decision about investment in higher education that in principle leads to higher wages. Within such interpretation highly educated people take the place of innovators in the model, while imitators are those who save the cost of education at the expense of a lower income.

The paper is organized as follows. Section 2 introduces the discrete choice evolutionary model of innovators and imitators, and the main model on innovation as costs reduction is developed. Section 3 proposes the extension where also effects on demand are considered, allowing for product differentiation beside costs reduction. Section 4 concludes.

2 A discrete choice model of innovation

The theoretical framework for our analysis is Brock and Hommes (1997) adaptive model of expectation rules ($BH$ henceforth). In our case costly innovators and free imitators take the place of costly rational and free naive expectations. The agents’ payoff depends on benefits and costs of innovation, while agents’ population (firms) is heterogeneous not for their price expectations, as in $BH$, but for their supply function. In short, we model the endogenous evolutionary selection of heterogeneous production technologies.

Consider an industry with $N$ firms producing the same good in a perfectly competitive market. Innovation here is production cost reduction. Imitation is simply relying on the old worse technology. Firms divide in innovators, with fraction $n_t^{INN}$ and imitators, with fraction $n_t^{IM} = 1 - n_t^{INN}$. The quantity $S^h(p_t)$ supplied in period $t$ by type $h$ is a
function of price which depends on type \( h \) cost structure, with \( h = INN, IM \). Choosing strategy (innovation or imitation) amounts to choose the production cost structure. The performance of strategy \( h \) is measured by a weighted average of past profits:

\[
U^h_t = wU^h_{t-1} + \pi^h_t
\]

Parameter \( w \) controls the memory of the system. Agents are boundedly rational and every period they choose the best performing strategy with some probability. Adopting the discrete choice model of BH such probabilities have a logit (or ‘Gibbs’) distribution:

\[
n^I_{NN} = \alpha n^I_{NN} + (1 - \alpha) \frac{e^{\beta U^I_{t-1}}}{e^{\beta U^I_{t-1}} + e^{\beta U^M_{t-1}}}, \quad n^I_{IM} = 1 - n^I_{NN}
\]

The intensity of choice \( \beta \) measures how sensitive firms are to select the best strategy. For \( \beta = 0 \) agents split equally among the different types. On the other hand, \( \beta = \infty \) represents the neoclassical limit where all agents choose the optimal strategy. The fitness of a strategy \( h \) is not known to agents with infinite precision (bounded rationality). One defines a random fitness \( U^h_t = U^h_t + \epsilon^h_t \) where the noise \( \epsilon^h_t \) (iid across agents) represents imperfect knowledge of strategy \( h \) fitness. The multinomial logit probabilities come as a limit after considering an infinite number of agents. Parameter \( \beta \) is inversely related to the variance of that noise (Hommes, 2006). Another interpretation of this noise term is the uncertainty related to the innovation process.

The parameter \( \alpha \in [0, 1] \) is the fraction of firms that stick to their previous strategy. Such asynchronous updating accounts for the persistence of firms behaviour. In a first stance we set ourselves in the simplifying setting of synchronous updating (\( \alpha = 0 \)) without memory (\( w = 0 \)). In this case the probability, or fraction, of innovators at time \( t \) is

\[
n^I_{NN} = \frac{e^{\beta \pi^I_{t-1}}}{e^{\beta \pi^I_{t-1}} + e^{\beta \pi^M_{t-1}}} = \frac{1}{1 + e^{-\beta \Delta \pi_{t-1}}}
\]

with \( \Delta \pi_t \equiv \pi^I_{t-1} - \pi^M_{t-1} \). This setup recalls the quantal response game of McKelvey and Palfrey (1995). The difference is that choices here are based on past experience.

As anticipated in the introduction, we ignore the mutual effects of technological advance and agents decisions, focusing on the relative effects of innovators and imitators interplay. In order to model this relative dynamics we assume that innovation is like buying a shortcut which results in lower production costs in one period. Time is discrete and in each period the market clears in a Walrasian equilibrium:

\[
D(p_t) = n^I_{NN} S^I_{NN}(p_t) + n^I_{IM} S^I_{IM}(p_t)
\]

This equation results from aggregation of demand over consumers and of supply over
firms, after dividing by the total number $N$ of firms which is assumed to be constant.\footnote{Aggregation of supply gives $S_t = \sum_{i=1}^{N_{t}^{\text{INN}}} S_{t}^{\text{INN}} + \sum_{j=1}^{N_{t}^{\text{IM}}} S_{t}^{\text{IM}}$, but innovators and imitators are indistinguishable among themselves ($S_{t}^{\text{INN}} = S_{t}^{\text{IM}}$ and $S_{t}^{\text{IM}} = S_{t}^{\text{IM}}$ for all $i,j$), then $S_t = N_{t-1}^{\text{INN}} S_{t}^{\text{INN}} + N_{t-1}^{\text{IM}} S_{t}^{\text{IM}}$. Dividing by the number of firms $N$ one obtains the right-hand side of (4).}

The total supply is a convex combination of innovators’ and imitators’ contributions. Each agent’s supply is determined by profit maximization in one period. Profits of a firm of type $h$ in period $t$ are $\pi_t^h = p_t q_t^h - c^h(q_t^h)$, with $q_t^h = S_t^h(p_t)$. We choose a quadratic cost function such as $c^h(q) = \frac{q^2}{2s} + C^h$, where $C^h$ represents the fixed costs of strategy $h$. Innovators obtain lower marginal costs, expressed by $s_{t}^{\text{INN}} > s_{t}^{\text{IM}}$. In each period innovation comes at a positive cost, $C^{\text{INN}} = C > 0$ (the cost of buying the shortcut for that period), while imitation is free, $C^{\text{IM}} = 0$.\footnote{In principle imitators have the advantage of not replicating an unsuccessful innovation. Here we assume that an innovation is always successful.} Notice how the cost reduction from innovation is larger for larger values of output: $\Delta c \simeq -\frac{q}{2s}\Delta s$. This means that larger firms profit more from innovation.

A quadratic cost function gives linear supply for each type: $S_{t}^{\text{INN}}(p_t) = s_{t}^{\text{INN}} p_t$ and $S_{t}^{\text{IM}}(p_t) = s_{t}^{\text{IM}} p_t$. The total supply lies in between the lines with slopes $s_{t}^{\text{INN}}$ and $s_{t}^{\text{IM}}$. Within this assumption profits are as follows:

$$
\pi_t^{\text{INN}} = \frac{1}{2}s_{t}^{\text{INN}} p_t^2 - C, \quad \pi_t^{\text{IM}} = \frac{1}{2}s_{t}^{\text{IM}} p_t^2
$$

(5)

The difference of profits between the two strategies $\Delta \pi = \frac{1}{2} \Delta s l p_t^2 - C$ is central in this model. In particular we have $\Delta \pi = 0$ for $p = \bar{p} \equiv \sqrt{2C/\Delta s}$. The total costs per unit of output are: $\gamma^{\text{INN}} = p/2 + C/s^{\text{INN}} p$ and $\gamma^{\text{IM}} = p/2$. We see that $\gamma^{\text{INN}} > \gamma^{\text{IM}}$ and they are equal in the limit of an infinite price.\footnote{If everybody was innovator, the competitive price that makes marginal costs $q/s$ equal to average costs $C/q+q/(2s)$ would be $\bar{p} = \sqrt{2C/s}$. In our model this is a limit case.} This is a first indication that innovators like the price to be high, although their aggregate effects is exactly in the opposite direction, as we will see. Consider a linearly decreasing demand $D(p_t) = a - dp_t$ ($d > 0$): the market equilibrium equation (4) becomes

$$
a - dp_t = \pi_t^{\text{INN}} s_{t}^{\text{INN}} p_t + \pi_t^{\text{IM}} s_{t}^{\text{IM}} p_t
$$

(6)

2.1 A simple model: costly innovative vs. cheap sluggish firms

The next step is about modelling the competitive advantage of innovators, which boils down to specifying $s_{t}^{\text{INN}}$ and $s_{t}^{\text{IM}}$. Assume that $s_{t}^{\text{INN}}$ and $s_{t}^{\text{IM}}$ are constant and consider the market equilibrium (6). Solving for $p_t$}

$$
p_t = \frac{a}{d + s_{t}^{\text{INN}} \pi_t^{\text{INN}} + s_{t}^{\text{IM}} \pi_t^{\text{IM}}}
$$

(7)
where fractions depend on last period price:

\[
n_t^{\text{INN}} = \frac{1}{1 + e^{-\beta \left[ \frac{1}{2} p_{t-1} (s^{\text{INN}} - s^{\text{IM}}) - C \right]}}
\]

(8)

When everybody innovates the price is \( p_t^{\text{INN}} = a/(d + s^{\text{INN}}) \), while \( p_t^{\text{IM}} = a/(d + s^{\text{IM}}) \) is the price with only imitators. Assume that \( s^{\text{INN}} \) and \( s^{\text{IM}} \) are constant and \( s^{\text{INN}} > s^{\text{IM}} \); we have \( p_t^{\text{IM}} > p_t^{\text{INN}} \). In figure 1 we show an example. The more innovators are around, the less steep is the average supply curve and the lower is the price. Equation (8) tells that when \( p_t \to 0 \) imitators prevail: when the price is too low, it is difficult to profit from the innovation advantage. The opposite is true when the price grows larger, with prevalence of innovators. The value of price is bounded, as one sees from equation (7): since \( p = p(n^{\text{INN}}) \) is monotonic decreasing we have \( p_t^{\text{INN}} \leq p \leq p_t^{\text{IM}} \).

The steady state price, where demand and supply intersect, is between \( p_t^{\text{IM}} \) and \( p_t^{\text{INN}} \). To find it one substitutes the fractions expression (8) in the expression of price (7) and looks for fixed points. The model describes a one-dimensional system which can be described by either the price or the fraction of innovators (or imitators). If we substitute (8) into (7) we obtain a map for the price:

\[
f(p) = \frac{a}{d + s^{\text{IM}} + \frac{\Delta s}{1 + e^{-\beta \left[ \frac{1}{2} p (\Delta s - C) \right]}}}
\]

such that \( p_{t+1} = f(p_t) \). Here \( \Delta s \equiv s^{\text{INN}} - s^{\text{IM}} > 0 \). The equilibria are fixed points of \( f \).

**Proposition 1** The map \( f \) is decreasing, then \( \exists! \) steady state \( p^* \), with three cases:

- **very low** \( \beta \) (myopic agents): \( f'(p^*) \simeq 0 \), then \( p^* \) is stable
- **with large** \( \beta \) (rational agents), two cases are possible:

4 We can think of this limit as a situation with only one innovator: If \( N >> 1 \) we have \( n^{\text{INN}} \simeq 0 \).
– high fixed costs, $\Delta \pi^* = \frac{1}{2} s (p^*)^2 - C < 0$: as $\beta$ gets larger it reaches a value $\beta_1$ s.t. $f^*_{\beta_1}(p^*) = -1$. The steady state loses stability and a 2-cycle occurs.

– low fixed costs, $\Delta \pi^* > 0$: as $\beta$ gets larger $f'(p^*) \to 0$, then $p^*$ remains stable.

The proof is given in appendix A. The steady state $p^*$ is stable whenever $f'(p^*) > -1$ and unstable otherwise. When the steady state is unstable the system ends oscillating indefinitely in a cycle of period 2. Are innovation and imitation forces stabilizing or destabilizing? In the steady state $p = p^*$ the fraction of innovators is equal to

$$n^*_{INN} = \frac{1}{1 + e^{-\beta \Delta \pi^*}} \quad (10)$$

Since $p^*_{INN} \leq p \leq p^*_{IM}$ and $n^*_{INN}(p_t)$ is monotonic (eq. 8), two cases are possible:

- high $C \Rightarrow \Delta \pi^* < 0$, $n^*_{INN} < 1/2$ and imitators are more frequent as $\beta$ grows, with $n^*_{IM} \to 1$ for $\beta \to \infty$. $\exists \beta = \beta_1$ where the system loses stability (proposition 1).

- low $C \Rightarrow \Delta \pi^* > 0$, $n^*_{INN} > 1/2$ with $n^*_{INN} \to 1$ for $\beta \to \infty$. Moreover $f(p^*) \to 0$, which means the steady state remains stable.

This means that imitation is a destabilizing force, while innovation is stabilizing. Nevertheless, the two strategies always coexist. If $\Delta \pi$ is finite (the price is finite), imitators or innovators can disappear only in the limit of rational agents ($\beta \to \infty$).

One way to model the competitive advantage obtained by marginal costs reduction is to relate the supply function to the fixed cost of $R&D$, for instance with $s^*_{INN} = se^{bC}$ and $s^*_{IM} = s$ ($b, s > 0$). The exponential effect of $R&D$ expenditure is common in innovation models (Dopfer, 2005). If we take the exponential of a random variable, this can account for the uncertain outcome of innovation and make the model stochastic.\(^5\) Innovation reduces the marginal costs of producing a given quantity: $c'(q) = \frac{2}{s}$ for imitators and $c'(q) = \frac{se}{se^{bC}}$ for innovators. This description of the innovation effect is similar to Iwai (1984). But equivalently it can describe the increase in productivity of capital. This setting recalls Spence (1984) model of competition driven by $R&D$, provided that time is discrete and firms are homogeneous but for their choice about innovation. Net realized profits (11) and agents fractions (8) now become

$$\pi^*_{INN} = \frac{1}{2} se^{bC} p^2 - C, \quad \pi^*_{IM} = \frac{1}{2} sp^2 \quad (11)$$

$$n^*_{INN} = \frac{1}{1 + e^{\beta \left[\frac{1}{2} p^2 - s (1-e^{bC}) + C\right]}}, \quad n^*_{IM} = 1 - n^*_{INN} \quad (12)$$

\(^5\)This source of randomness comes on top of the noise term in the random fitness function mentioned before and does not average away in the limit of infinitely many agents.
For $C = 0$ we have $n_{t}^{\text{INN}} = n_{t}^{\text{IM}} = 1/2 \forall t$, because the two rules coincide.\(^6\) Another case where we see equally distributed types is $\Delta \pi_t = 0$, which realizes when the price is $\overline{p} = \sqrt{2C/\left[s(e^{bC} - 1)\right]}$. This price does not coincide with the equilibrium price (when this exists). In other words, in the equilibrium the difference of profits $\Delta \pi$ between the two rules is different from zero, in general. The reason is bounded rationality of agents, which is expressed by a finite value of the intensity of choice $\beta$. The price $\overline{p}$ does not depend on $\beta$, because $\beta$ does not enter the definition of profits. The fixed points of (9) can be computed numerically. In the following table we report the values of $\overline{p}$ and $p^*$ for a few different combinations of $b$ and $C$ ($\beta = 1$) for which a stable steady state exists:

From the table we see the following: first, both $p^*$ and $\overline{p}$ are decreasing in $b$ and in $C$.

<table>
<thead>
<tr>
<th></th>
<th>$b = 0.5$</th>
<th>$b = 1$</th>
<th>$b = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C = 0.25$</td>
<td>$\overline{p} = 1.37$, $p^* = 1.28$</td>
<td>$\overline{p} = 0.94$, $p^* = 1.21$</td>
<td>$\overline{p} = 0.47$, $p^* = 0.90$</td>
</tr>
<tr>
<td>$C = 0.5$</td>
<td>$\overline{p} = 1.33$, $p^* = 1.22$</td>
<td>$\overline{p} = 0.88$, $p^* = 1.07$</td>
<td>$\overline{p} = 0.38$, $p^* = 0.54$</td>
</tr>
<tr>
<td>$C = 1$</td>
<td>$\overline{p} = 1.24$, $p^* = 1.12$</td>
<td>$\overline{p} = 0.76$, $p^* = 0.82$</td>
<td>$\overline{p} = 0.23$, $p^* = 0.20$</td>
</tr>
<tr>
<td>$C = 1.5$</td>
<td>$\overline{p} = 1.16$, $p^* = 1.02$</td>
<td>$\overline{p} = 0.66$, $p^* = 0.63$</td>
<td>$\overline{p} = 0.13$, $p^* = 0.08$</td>
</tr>
</tbody>
</table>

Table 1: Values of $\overline{p} = \sqrt{2C/\left[s(e^{bC} - 1)\right]}$ and the equilibrium price $p^*$ for different combinations of the benefits of innovation $b$ and the innovation costs $C$, with $\beta = 1$.

Second, it may be either $\overline{p} > p^*$ or $\overline{p} < p^*$. The latter case realizes with low $C$ and high $b$. We know from (9) that $f(\overline{p}) = a/(d + s^{\text{INN}}) = p^*_{\text{INN}}$: to have equal profits in equilibrium everybody must be innovator. This is not always true, as we will see later. Nevertheless, for all these values of parameters we register a comovement of prices $\overline{p}$ and $p^*$. For $b = 1.5$ and $C = 1.5$ we have $\overline{p} \simeq p^*$. In this case the equilibrium value $p^*$ is attained very slowly.

We have seen analytically, that only convergence to a stable steady state or convergence to a period 2 cycle can realize. We fix the demand curve with $a = 4$ and $d = 1$. Figure 2 reports an example where the system converges to a stable steady state $n_{t}^{\text{INN}} \simeq 0.55$. Innovators slightly prevail in the long run, and accordingly the equilibrium value of $\Delta \pi$ is positive. In general Innovators and imitators coexist in this model, due to the finite value of $\beta$. Whether one or the other type prevail depends on the relative value of innovation benefits and costs, as we will see. Nevertheless, innovation benefits are always enough to compensate some innovators for investing in $R&\text{D}$. This case is similar to Grossman and Stiglitz (1976) heterogeneous equilibrium of informed and uninformed agents. In our model the role of information is replaced by the degree of rationality of agents, as measured by the intensity of choice $\beta$: if the intensity of choice is relatively

\(^{6}\)Our specification of the innovation outcome is calibrated to give equal fractions with no innovation.
large, the steady state becomes unstable and the system ends up oscillating (Figure 3). A similar result is obtained with relatively large innovation costs $C$. This cyclical behaviour reflects a minority game character of this model: when a strategy becomes too frequent agents start moving to the other strategy. The economic reason is the following: innovation drives down the price. At some point the profits from innovation become too low (even negative, due to the fixed costs $C$, see equation 5), so that imitation becomes preferable. Agents start switching to imitative behaviour then, and starts to go up. But then innovation profits increase more than imitation profits (due to heterogeneous productivity, which is larger for innovators by the factor $e^{bC}$). At some point innovation profits will return to be largest, agents will switch to innovation again, and the story repeats. This periodical dynamics is the basis to reproduce cycles of innovation once persistence of behaviours is added to the model through asynchronous updating.

The long run behaviour of the system for different values of a parameter is well represented by a bifurcation diagram\textsuperscript{7,8} In the case of $\beta$ we obtain the diagram of innovators fraction presented in Figure 4. When $\beta \simeq 2.2$ the steady state loses stability and agents fractions attain a 2-cycle. As $\beta$ gets larger the 2-cycle converges to $(0,1)$, meaning that when agents are rational, this industry experiences an almost complete periodical switching with everybody alternately innovating and imitating.

Different levels of innovation benefits $b$ may also lead to cycles. Moreover a tradeoff exists, so that not necessarily higher innovation benefits are good for innovators (Figure \textsuperscript{7}This is made of the realizations of a variable after a given number of periods (1000 here). \textsuperscript{8}The numerical analysis has been done with the software package $E$&$F$ Chaos (Diks et al., 2008).
5): here we have a period doubling near \( b = 1.1 \) and a period halving near \( b = 5.5 \). In other words there is a region of \( b \) where the market is unstable, while outside this region agents converge to an equilibrium proportion of innovators and imitators. In appendix A we show that as \( \beta \) and/or \( s \) increase, a period doubling value \( b_1 \) and a period halving value \( b_2 \) appear, with the width \( b_2 - b_1 \) becoming wider as \( \beta \) and/or \( s \) increase further. The effect of \( b \) on the frequency of innovation is positive for relatively small values and negative for large values. This is due to a double effect of innovation on innovators’ profits. A positive (direct) effect comes from the factor \( e^{bC} \): the larger \( b \) the larger \( \pi_{INN} \) (equation 11). The negative (indirect) effect realizes through the price: since innovation drives down the price, this effect is stronger the larger \( b \) is. The price effect hampers innovators’ profits more than imitators’, as we have seen. If the price effect is prevailing, innovators become less frequent as \( b \) gets larger, as it clearly happens in equilibrium for large values of \( b \) in the example of figure 5.

To sum up, while for \( \beta \) there is a threshold value where the steady state loses stability and a 2 cycle appears, for \( b \) there are two values at which the system undergoes a qualitative change of its dynamics. This is clear from the parameter basin of attraction in figure 6. For small values of \( C \) the system is stable, no matter the value of \( b \). For larger \( C \) the system attains a 2 cycle but only if \( b \) is not too low or too high. The example of figure 5 falls into this case. The same is true for \( C \) and \( b \) exchanged: there are values of \( b \) where a 2-cycle appears for mid-range values of \( C \).
2.2 Asynchronous updating of strategies

It is realistic to think that only a fraction of firms update their strategy in a given period. Such asynchronous updating is described by equation (2), that we rewrite here for the case without memory:

\[ n_t^{\text{INN}} = \alpha n_{t-1}^{\text{INN}} + (1 - \alpha) \frac{e^{\beta n_{t-1}^{\text{IM}}}}{e^{\beta n_{t-1}^{\text{IM}}} + e^{\beta n_{t-1}^{\text{INN}}}}, \quad n_t^{\text{IM}} = 1 - n_t^{\text{INN}} \quad (13) \]

A fraction \( \alpha \in [0, 1] \) of agents stick to their strategy in a given period \( t \), while the rest update according to the discrete choice model described before. One possible effect of asynchronous updating is a stabilizing effect. For instance in the case of the example of figure 4, if we assume that only half of the agents update strategy each period setting \( \alpha = 0.5 \), we obtain a much higher bifurcation value for \( \beta \) (figure 7), meaning that equilibrium is attained for a larger range of values of the intensity of choice \( \beta \). The value of the intensity of choice is critical in dictating the effect from asynchronous updating: if \( \beta \) is low, asynchronous updating has a stabilizing effect. But if \( \beta \) is relatively large, asynchronous updating can lead to very irregular behaviour. This is shown in figure 8, where we compare the bifurcation diagram of innovation benefits \( b \) with low \( \beta \) (left) and large \( \beta \) (right), when half agents update every period. When \( \beta \) is large periods 3 and 4 appear,
as well as chaotic dynamics for large intervals of $b$. The same effect we observe with a bifurcation diagram of the updating fraction $\alpha$ itself (figure 9), where we see that also innovation costs play a role: when $β$ and $C$ are relatively low (left), the system converges either to a 2-cycle or to a stable equilibrium. A higher cost of innovation destabilizes the dynamics, giving place to cycles of period 4 (centre). By increasing $β$ while keeping $C$ large, we obtain irregular dynamics for middle values of the updating fraction $\alpha$ (right). The examples of figure 9 indicate that in general, when most of agents stick to their strategy (large $\alpha$), the industry converges to a stable equilibrium. When only a small fraction of agents update instead (low $\alpha$), the dynamics is periodical with period 2. Middle values of the updating fraction $\alpha$ may lead to irregular dynamics, if intensity of choice and innovation cost are large enough.
Another aspect of the irregular behaviour introduced by asynchronous updating is a mild persistence of strategies. The time series of figure 10 is an example where for many periods oscillations stop almost completely, and start again in a very irregular fashion.

2.3 Long memory decision

Up to here we have assumed that firms base their decision on last period profits. Now we study the effect of long memory decisions, assuming that firms also look at previous periods and that each period counts less the more far away it is. The introduction of memory adds to the realism of the model, because very likely a change of strategy is characterized by some friction due to organizational reasons or simply additional costs. Moreover, memory of past profits is a way to introduce indirectly cumulativeness of knowledge, although only in agents choices and not in their effects (production costs).

For simplicity we do not consider asynchronous updating here: every period the probability of a firm innovating has the following distribution:

\[ n_t^{\text{INN}} = \frac{e^{\beta U_{t-1}^{\text{INN}}}}{e^{\beta U_{t-1}^{\text{INN}}} + e^{\beta U_{t-1}^{\text{IM}}}}, \]

\[ U_t^{\text{INN}} = wU_{t-1}^{\text{INN}} + \pi_t^{\text{INN}} \]  

(14)

Profits are given by (11). If \( w = 0 \) we are back in the situation considered before. If \( w \neq 0 \) the utility of period \( t \) from choosing strategy \( h \) is

\[ U_t^h = \sum_{i=0}^{t} w^i \pi_{t-i}^h = \pi_t^h + w \pi_{t-1}^h + w^2 \pi_{t-2}^h + \cdots + w^{t-1} \pi_1^h + w^t \pi_0^h \]  

(15)

The memory parameter \( w \in [0, 1] \) resembles a discount factor and firms discount the past with geometrically increasing power every period. The larger \( w \) is, the more past periods are taken into account in the strategy decision.

The effect of memory results to be either stabilizing or destabilizing, depending on other factors. In particular the innovation benefits \( b \) happen to be critical in this sense. If \( b \) is relatively low, memory is stabilizing, reducing the parameter space where the system has cyclical behaviour (figure 11).

![Figure 11: Bifurcation diagram of the intensity of choice \( \beta \) for \( n_t^{\text{INN}} \) with low innovation benefits \( (b = 0.5) \). Right has no memory. Left has \( w = 0.5 \). \( (C = 1, s = 2, a = 4, d = 1) \).](image-url)
Slightly higher values of innovation benefits make the effect of memory destabilizing: in figure 12 we see an example where for $w = 0.5$ the dynamics of decisions become very irregular for several values of the intensity of choice. In general, the combined effect of innovation cost and benefits, intensity of choice and memory, is destabilizing. This we see for instance by looking at the basin of attraction of the system (figure 13). Although the effect of memory can be destabilizing in a dynamical sense, increasing the periodicity of oscillations or even leading to chaotic behaviours, it may well reduce the variance of such oscillations. If one is mainly interested in the variability of an industry state, as the price, this effect may be way more important. Figure 14 reports examples of time series with increasing memory. A moderate amount of memory ($w = 0.5$) makes the dynamics very irregular, but it reduces the price variability $\Delta p$ by more than a half. With very strong memory $w = 0.9$ the variance drops by more than 75%. Concluding, memory of
past profits in firms’ decision may cause a loss of coherence in terms of the dynamics but leads to a lower price variability.

2.4 Technological progress

In this section we study the effect of technological progress on the discrete choice dynamics, by relaxing the hypothesis that innovation does not cumulate: in each period the achievements of innovation build on a technological frontier, pushing down production costs. The technological frontier is made of all past periods contributions, each weighted by the fraction of innovators in that period. We let imitators enjoy the technological frontier of production technology. Innovators build on the frontier, obtaining a better production technology in reason of their actual innovation investment. Formalizing these assumptions we define the period $t$ productivity of innovators and imitators as follows:

$$s_{t}^{INN} = se^{f(\sum_{i=1}^{t-1} n_{i}^{NN})+bC}, \quad s_{t}^{IM} = se^{f(\sum_{i=1}^{t-1} n_{i}^{NN})}$$

where $f$ represents the speed of technological progress. Equation (16) tells that innovators competitive advantage over imitators amounts to the factor $e^{bC}$, as before. The speed of technological progress $f$ measures how much the technological outcome of each period cumulate and how strong are the technological spillovers that allow any firm to access the the “state of the art” technology. There are two main assumptions in this definition of technological progress: the first is that in each period innovation cumulates at the same rate, which gives a constant rate of technological progress $f$. The second assumption is that $f$ is independent on $b$ and $C$. A more precise definition would require the overall rate of technological progress to be equal to $fbC$, but this happens not to change the results of the analysis. By substituting the factors $s_{t}^{INN}$ and $s_{t}^{IM}$ as just defined in the equation for price at time $t$ (7) we obtain

$$p_{t} = \frac{a}{d + se^{f(\sum_{i=1}^{t-1} n_{i}^{NN})}(n_{t}^{INN}e^{bC} + n_{t}^{IM})}$$

As expected, the introduction of technological progress causes the industry to converge always to an equilibrium. Unexpectedly, this equilibrium may be one with a non-zero price. As we will see this is due to the presence of imitators that can not be driven out of the market in some cases. Beside this, two main questions that one would ask to a model of technological progress are the following: first how fast technological progress is and what are the main factors that affect its speed, and secondly, whether this convergence is achieved smoothly or through a transitory oscillating phase, instead.

Figure 15 reports an example where the industry presents oscillations before converging to an equilibrium. Such equilibrium is characterized by a non zero price and a low but positive fraction of innovators.
Figure 15: Time series of price (left) and innovators fraction (right) with $\beta = 5$, $b = 2$, $C = 1$, $f = 0.1$ ($a = 4$, $d = 1$, $s = 2$).

A relatively large intensity of choice $\beta$ is causes oscillations in the initial phase of the market. This effect is inherited from the basic model, where we have a change from stable equilibrium to period 2 cycles for $\beta$ above some threshold. The same is true for innovation benefits $b$ and cost $C$. On the contrary, a larger cumulativeness of innovation $f$ dampens oscillations and makes the convergence to equilibrium faster (figure 16).

Figure 16: Time series of price with $f = 0.1$ (left) and $f = 0.5$ (right), with $\beta = 5$, $b = 1$, $C = 1$, $a = 4$, $d = 1$, $s = 2$.

An unexpected result of this analysis is that price may not converge to 0, despite innovation cumulates. Equation (17) gives a necessary and sufficient condition for this:

**Proposition 2** The price converges to 0 if and only if $\sum_{t=0}^{\infty} n_t^{\text{NN}}$ diverges.

A standard result tells that a necessary condition for the convergence of a series is that its argument has a limit larger than zero, which in our case boils down to $\lim_{t\to\infty} n_t^{\text{NN}} > 0$. In other words, whenever some innovator is left in the market, the price can go down down, and it will eventually converge to 0 if there is always at least one innovator. The tricky part of the story is when $\lim_{t\to\infty} n_t^{\text{NN}} = 0$. This is only a necessary condition for convergence of the series, but not sufficient: $\sum_{t=0}^{\infty} n_t^{\text{NN}}$ may converge and in this case the price converges to a limit value $\hat{p} > 0$. The intuition is that when innovators are driven out of the market there is nobody that can push the price down further, and the industry stagnates profiting from the fixed rent of a constant price.

The intensity of choice is critical in determining the convergence of the market to a non-zero price: a low $\beta$ causes a smooth disappearance of innovators, with consequent
convergence of price to zero (figure 17). When $\beta$ is large instead, innovators disappear quickly after the oscillatory phase and the price goes down very slowly, converging seemingly to a positive value (figure 18). Figures 17 and 18 show also the time series of the sum $\sum_t n_t^{NN}$, which is a measure of technological progress: this is relatively smooth and slow with low $\beta$, while a large $\beta$ lead to an initial fast progress followed by a sudden interruption after which the industry enters a stagnation with almost not progress at all.

A numerical analysis can not rule out convergence of price to zero, when this is too slow to be perceived in the finite time horizon of the computation. Nevertheless, for many practical purposes, knowing that such convergence is very slow is enough. A scenario where $p_t \to 0$ after a time very long compared to say the oscillatory phase, does not differ from a scenario where actually $p_t \to \hat{p} > 0$. In other words, we may not know in the example of figure 18 if the price converges to zero or not after an infinite time, but for the knowledge of the industrial dynamics within a suitable time frame this information is useless. For a practical study of market values in the long run we can still use a bifurcation diagram forgetting about convergence and just looking at the value of say the price after some long time. This allows to evaluate the effect of factors as the intensity of choice and the cumulativeness of technology on the price at a time long after the possible oscillatory phase. Figure 19 shows the value of price after 1000 periods. When the intensity of choice is low, the price is almost zero and it is insensitive to $\beta$. After some level this sort of equilibrium price becomes larger for larger $\beta$. The
effect saturates when $\beta$ is very large. A higher rate of technological cumulation $f$ has the following effect: it extends the region where the market converges to zero price and makes the response to $\beta$ more similar to a step function, with a lower level of the price for large $\beta$. A similar analysis of the rate of technological cumulation $f$ reveals a weird pattern of the long run value of the price (figure 20): when the intensity of choice is large, this price level does not change monotonically with $f$, revealing a complex mechanism of non-linear interactions between the choice mechanism and the market dynamic equilibrium. To conclude, cumulation of technological knowledge dramatically changes the dynamics of the model, with convergence to a stable equilibrium as the only possible outcome. Possibly the market presents an initial oscillatory phase, with decaying amplitude before stabilizing on a convergence path. Higher cumulation of technological knowledge dampens such oscillations and fastens convergence. Moreover cumulation of knowledge drives the price down to zero unless innovators are quickly driven out of the market. in this case technological progress stops abruptly and the price stabilizes on a non-zero level.

3 Adding imitation

Suppose that all firms have access to innovation, either by innovating or by imitating the innovators. Moreover assume that imitation efficiency is proportional to the frequency of innovators. In other words, the more innovators there are, the easier it is to imitate
them. In the following we extend the previous model with such imitation externality and study the endogenous dynamics of innovators and imitators. Again we focus on the endogenous selection dynamics of the two agents’ types, discarding the mutual effects of such dynamics and technological progress: innovation is a production shortcut and every period imitators try to free ride on it, without being able to replicate it perfectly.

The idea of imitation as a cheap heuristic opposed to a costly one is already in Conlisk (1980)’s model of costly optimizers versus cheap imitators, and in a general sense it goes back to Grossman and Stiglitz (1976)’s model of informed and uninformed agents in a competitive asset market, where uninformed agents try to exploit the free information carried by the price and such information is more revealing the more agents pay the cost of being informed. The main idea behind these models as well as ours is that it may be more efficient for some agents to exploit other agents than do the work themselves.

Technology is a non-rival, partially excludable good (Romer, 1990). Innovators may try but cannot completely prevent others from copying an innovative design. The innovator have a competitive advantage from being the first and supposedly the best in an initial stage of the market (scale advantages and falling costs by learning earlier). Innovators bear the cost of R&D for that advantage though. Imitators profit from the innovation activity performed by innovators and do not pay any fixed cost, assuming that the activity of copying a design costs almost nothing compared to innovation costs. However, being followers, imitators are never as good as innovators. As before, we focus on short term advantages from innovation ignoring mutual effects with technological progress: innovation investments give an informational advantage that does not cumulate.

The replicability of an innovation is dictated by internal and external factors: the first one is essentially the complexity of technological knowledge. The external factors are patents and intellectual property rights. However, even when the latter protect private knowledge, there are always spillovers and leakages. The relative importance and interaction of such factors is highly sector and technology specific (Dosi et al., 2006). What we claim here is that the evolving interaction of innovators and imitators is a common and general ground able to explain a good deal of firms’ behaviours and industrial dynamics. The interplay between innovation and imitation results in two opposing forces that take an industry through qualitatively different dynamic scenarios. Depending on the relative value of parameters as innovation fixed costs, appropriability of knowledge, benefits from innovation, the market may miss a stable equilibrium and enter an unstable environment where agents’ concentration and market price fluctuate indefinitely. Moreover, if also effects on demand from product differentiation are considered, there may be even more dramatic changes, with irregular paths of industry and market dynamics.

In the following we study the effect of different factors as replicability of innovation
as well as innovation benefits and costs. Beside analyzing the stability of the system, we are interested in understanding how the imitation externality affect the innovation rate.

Assume that imitators’ supply is proportional to the density of innovators: $s^{IM} = s^{IM}(n) = \mu n^{INN}_{t-1} e^{BC}$, where $\mu \in [0, 1]$ is the effectiveness of imitation. This parameter accounts for the static level of replicability of technological knowledge and expertise. The limit case $\mu = 1$ refers to an innovation which is perfectly replicable. The factor $\mu n^{INN}_{t-1}$ expresses a dynamic replicability of innovation and is close to Spence (1984) measure of spillovers externalities. The innovators’ supply is again $s^{INN} = e^{BC}$. There are two reasons why in this model imitators “enjoy” the benefits of innovation measured by $e^{BC}$: the first is that once innovation is replicated it gives the same benefits to imitators, up to the replicability factor $\mu n^{INN}_{t-1}$. Secondly, the larger innovation benefits are, the higher the incentive to imitate (Benoit, 1985).

With the above specification of imitators’ and innovators’ supply functions the market equilibrium equation (6) reads

$$a - dp_t = n^{INN}_{t} e^{BC} p_t + n^{IM}_{t} \mu n^{INN}_{t-1} e^{BC} p_t$$

Equation (18) says that supply comes in the form of a quantity $se^{BC}p_t$ for both types, because also imitators enjoy the outcome of the R&D investment performed by innovators in that period. On the other hand the contribution of imitators is weighted by $\mu n^{INN}_{t-1}$: even a perfectly replicable innovation ($\mu = 1$) never gives to imitators the same productivity of innovators, since $n^{INN}_{t-1} < 1$.

**Proposition 3** For a given price $p$ the supply is maximum when everybody is innovator. Supply is 0 when everybody imitates.

The proof is in appendix B. If we rewrite the market equation as

$$a - dp_t = n^{INN}_{t} e^{BC} p_t + (1 - n^{INN}_{t}) n^{INN}_{t-1} \mu se^{BC} p_t$$

the contribution of imitators recalls very well Iwai (1984), where the rate of change of firms with unit cost up to a level is equal to the product of their proportion and the proportion of firms with higher unit cost. We solve (19) for the equilibrium price:

$$p_t = \frac{a}{d + \mu n^{INN}_{t-1} e^{BC} [1 + \mu (1 - n^{INN}_{t})]}$$

In the limit case where everybody innovate ($n^{INN}_{t-1} = 1$) the price is $p^{INN}_{t} = a/(d + se^{BC})$. If everybody imitates the price goes up to $p^{IM}_{t} = a/d$. An example is given in Figure 21. The difference of realized profits reads

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9We assume there is always at least one innovator in the industry.
Substituting this in expression (3) we obtain

\[ n_{t+1}^{INN} = \frac{1}{1 + e^{-\beta \Delta \pi}} \left( \frac{1}{1 + e^{\beta \left[ \frac{1}{2} \sigma^{2} e^{bC} (1 - \mu n_{t}^{INN}) - C \right]}} \right) \]  

(22)

The price \( \bar{p} \) for equal profits (that gives equal proportions) is \( \bar{p} = \sqrt{2C/|se^{bC}(1 - \mu/2)|} \); the easier imitation, the higher such price. Because of innovators’ externality the system becomes a two-dimensional one: its dynamics is described by equations \( n_{t}^{INN} = g(n_{t-1}^{INN}, p_{t-1}) \) and \( p_{t} = h(n_{t-1}^{INN}, p_{t-1}) \), where \( (n_{t}^{INN}, p_{t}) \) is the state variable and \( g \) and \( h \) are two functions. The analytical study of steady states for such a system is quite complicated. Nevertheless a partial knowledge of its dynamical properties can be obtained with some straightforward computation. By substituting the price (20) into (22) one obtains a map \( n_{t+1}^{INN} = f(n_{t}^{INN}) \) (although doing so we lose information about the price):

\[ f(x) = \frac{1}{1 + e^{\beta \left[ \frac{1}{2} \sigma^{2} e^{bC} (1 - \mu x) + C \right]}} \]  

(23)

A steady state \( n_{*}^{INN} \) is such that \( f(n_{*}) = n_{*} \). The long run dynamics depends crucially on the intensity of choice \( \beta \). Limit cases are summarized in the following proposition.

**Proposition 4** Limit conditions for equation (23):

- **very low \( \beta \) (myopic agents):** \( n_{*}^{INN} \approx 1/2 \): innovators and imitators split equally.
- **very large \( \beta \) (rational agents), two cases are possible:**
  - high fixed costs, \( \Delta \pi^* = \frac{1}{2} \Delta s(p^*)^2 - C < 0 \Rightarrow \lim_{\beta \to \infty} n_{*}^{INN} = 0 \) (all imitators).
  - low fixed costs, \( \Delta \pi^* = \frac{1}{2} \Delta s(p^*)^2 - C < 0 \Rightarrow \lim_{\beta \to \infty} n_{*}^{INN} = 1 \) (all innovators).

These limits exist if replicability \( \mu \) and innovation costs \( C \) are low enough.
 Computations are reported in appendix C. This proposition in particular shows the ambiguous effect of rationality on innovation. Figure 22 presents two bifurcation diagrams with respect to the intensity of choice $\beta$. Increasing the rationality parameter $\beta$ the market becomes unstable. In the diagram with one bifurcation the number of innovators increases with $\beta$ when the market has a stable steady state. For larger $\beta$ the dynamics is periodical, and the average number of imitators $\hat{n}_{IM} = (n_{IM}^{up} + n_{IM}^{down})/2$ goes down after the bifurcation. A lower replicability $\mu$ may lead to a bounded instability region for $\beta$, with a period halving bifurcation. We have the following proposition:

**Proposition 5** If replicability $\mu$ and costs $C$ are low enough, the steady state is stable for low values and large values of the intensity of choice $\beta$. This means that $\exists \beta_1, \beta_2$ with $\beta_1 < \beta_2$ such that $\beta_1$ is a period doubling and $\beta_2$ a period halving bifurcation.

The proof is in appendix C. This proposition tells again the destabilizing effect of imitation: when imitating is easier (larger $\mu$) the instability range of $\beta$ is wider.

Although the model with innovation externality is two-dimensional, a numerical analysis reveals properties similar to the preliminary one-dimensional model. Consider a perfectly replicable innovation ($\mu = 1$), with small benefits ($b << 1$): $e^{bc} \simeq 1$, and the map (23) becomes

$$f(x) = \frac{1}{1 + e^{\beta \left[ \frac{1}{2} a^2 x \left( x - \frac{1}{2} \right)^2 + C \right]}}$$

In the domain $[0, 1]$ the map $f_{\mu=1}(x)$ is decreasing. This means that we have a stable steady state and a limit 2-cycle separated by a period-doubling bifurcation, as in the

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**Figure 22**: Bifurcation diagrams of $\beta$ for $n^{NN}$, with $\mu = 0.7$ (period doubling bifurcation near $\beta = 1.15$) and $\mu = 0.27$ (period doubling and period halving bifurcations). Here $s = 2$, $b = 0.2$ and $C = 1$.

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\[10\] This is because $f'_{\mu=1}(x) < 0$ if and only if $I(x) = 0$, where $I(x) = d + sx(2 - x) + 4s(1 - x)^2$. Since $x \equiv n^{NN} \in [0, 1]$ the above inequality is always satisfied. Moreover $f_{\mu=1}(x) = 0$ if $d + sx(2 - x) = 0$, which means $x = x_1 \equiv 1 - \sqrt{1 + d/s}$ or $x = x_2 \equiv 1 - \sqrt{1 - d/s}$; but $x_1 < 0$ and $x_2 > 1$. 

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preliminary model. Figure 23 reports two examples with $\beta = 1$, $s = 2$, $a = 4$, $\mu = 1$, $d = 1$. In the example with $C = 0.4$ the fixed point is stable, while the case with $C = 4$ presents an unstable fixed point, with limit cycle of period 2. More generally, the innovation costs determine the stability of the system, similarly to the intensity of choice. In general a positive innovation cost can make the system unstable:

**Proposition 6** When $C = 0$ the steady state is such that $n^*_I = 1$ and it is globally stable. When $C > 0$ and $\mu$ is not too low the limit cycle converges to $(0,1)$ as $\beta$ grows.

Figure 24 exemplifies this result with two bifurcation diagrams of $\beta$, one with zero costs of innovation and the other with positive costs.

Differently from the preliminary model here the two types of agents do not split equally when innovation costs are zero, because of the externality from innovators to imitators. On the other hand, imitators never disappear completely due to bounded rationality ($\beta$ finite), like in the preliminary model: as we read from equations (21) and (22) even with
$C = 0$ and $\mu < 1$ the market converge to an equilibrium with very few imitators but still in a positive amount (no cycles are possible with $C = 0$).

A larger replicability of innovation leads to less innovators (figure 25), a consequence of lower incentives to do R&D (Spence, 1984). Moreover, above a threshold value of replicability $\hat{\mu}$ the system becomes unstable. Nevertheless, the average fraction of innovators over time keeps decreasing as $\mu$ becomes larger. The effect of replicability on innovation is recognized to be not univocal, in the sense that a higher replicability does not necessarily hampers innovation (Dosi et al., 2006). This is due to the cumulativeness of knowledge and technological advance. In the present model we cannot capture that, because we focus on the second order dynamics of the interplay between innovation and imitators, which is superimposed on technological advance and deals with information and not with knowledge.

The introduction of memory in agents decisions strongly affects the dynamics of the model. We have seen already without imitation that memory complicates the dynamics but reduces the variability of market indicators, as the price. Now we check if this is still true for the model with imitation. Figure 26 reports the same two examples of figure 11 with replicability $\mu = 0.4$. There is no a stabilizing effect on the dynamics this time, because with $w = 0.5$ we obtain periodic as well as chaotic orbits, while without imitation.

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![Figure 25: Bifurcation diagram of $\mu$ for innovators' fraction ($\beta = 2$, $s = 2$, $C = 1$, $b = 0.2$).](image1)

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![Figure 26: Bifurcation diagram of the intensity of choice $\beta$ for $n^{IN}$ with $\mu = 0.4$ and $b = 0.5$. Right has no memory. Left has $w = 0.5$. ($C = 1$, $s = 2$, $a = 4$, $d = 1$).](image2)
we have only stable equilibrium in the same conditions. Different levels of replicability do not affect much this result. Regarding the effect of memory on price variability, this is even more stabilizing than without imitation, as we see from figure 27, which proposes the same example of figure 14 with replicability $\mu = 0.4$. With memory $w = 0.5$, only the last three periods count for more than 5% compared to present time profits, nevertheless the price variance drops by nearly two thirds.

![Figure 27: Time series of price with different memory: $w = 0$ (left), $w = 0.5$ (centre), $w = 0.9$ (right).](image)

4 Innovation that affects demand

The model so far has addressed innovation that results in production costs reduction. As we argued, not just process but also product innovation may well lead to cost reduction, like for instance with information data storage devices, video screens, photovoltaic cells, etc. But we would lose much of technological and economic innovation if we do not consider all changes resulting in products that are innovative for their design, functionalities, options, etc. Examples are mobile telephones, cars, televisions, sport tools and so on. In general all products for which design and non-core functionalities matter in consumers’ choice. In a word, we now want to extend the model beyond innovation as costs reduction and consider also innovation that increases products differentiation. This type of innovation fits particularly well our assumption of negligible technological advance: now a new product is a somewhat different version but it is not necessarily “better” than old products. Nevertheless, on top of differentiation, $R&D$ investment still gives a cost reduction shortcut as in the previous model, which can be intended as a positive byproduct on generic innovative activity. Caulkins et al. (2007) propose a model of fashion cycles with innovators and imitators in continuous time. They focus on dynamic optimization and do not consider market dynamics, which is the focus of our model instead.

We consider the market for a slightly differentiated product, where firms act as innovators or imitators. Imitation can possibly rebalance non-equilibrium scenarios, reducing the competitive advantage of innovating firms (Winter, 1995). We do not consider the occurrence of radical innovation, which would increase the variety of the market and
change the technological landscape. Here innovation means the introduction of some new feature that do not change the core technology of the product or the main structure of the productive process. Firms may come up with new designs or new functionalities, or just extend the range of applications once a major discovery has been achieved. An example is the laser industry in the 80s and the 90s: innovation in those years lead to a proliferation of different products without a dominant design. In particular innovation involved to achieve new regions of the light spectrum (Klepper and Sleeper, 2005).

We present a model where innovation affects both supply and demand through costs reduction (as before) and product differentiation, respectively. After innovating or imitating, firms engage in monopolistic competition (Chamberlin, 1962; Spence, 1976; Romer, 1990). If many firms innovate or imitate at the same time, we can appeal to the law of large numbers: the price adjustment of each firm is averaged away and we can use the approximation of a uniform price. In their model of monopolistic competition and product diversity Dixit and Stiglitz (1977) show how the relative effect of a single firm’s price change on the overall quantity index is of the order of $1/N$ where $N$ is the total number of firms. If the number of firms is large, the price is driven down to competitive level. In many cases a firm offers more than one version of a product on the same market. Such a firm may well be innovating and imitating at the same time: ten different products appear as ten different firms to our model. This consideration increases the number $N$ considerably and makes our assumption of uniform price more plausible.

In a market with monopolistic competition and perfectly informed consumers there cannot be price dispersion since everybody would buy from the lowest price firm (Salop, 1977). Spence (1976) uses the same simplification dealing with products that are symmetric with respect to demand and costs: it does not implies they also are perfect substitutes. We assume that products are homogeneous also in terms of substitutability, in the sense that any two of them are substitutable to the same level.\footnote{12} A change in product differentiation is likely to reduce substitutability. Since this is determined by consumer preferences (Spence, 1976), an innovation (or imitation) of this kind must affect the market demand. It seems straightforward to assume that innovation lowers the degree of substitutability of firms’ products. A decline in product substitutability shifts the demand curve outwards (Lin and Saggi, 2002). Our idea is that such a differentiation force is proportional to the density of innovators. If we call $\xi \in [0,1]$ the substitutability of any two products when everybody is imitator, we express

\footnote{11}This is a typical approach in micro-founded macroeconomics, for instance when aggregating firms’ price decisions to study the dynamics of inflation. See for instance Calvo (1983).

\footnote{12}Here the substitution matrix is the identity matrix multiplied by a factor smaller than one.
the substitutability at time $t$ as

$$\xi_t = \xi (1 - n_t^{INN})$$

(25)

A dynamic substitutability translates innovation into product differentiation. How does product differentiation translates into demand change? Within the assumption of a linear demand $D(p) = a - dp$, under the simplified condition of symmetric products, one can show that demand coefficients $a$ and $d$ depend negatively on substitutability $\xi_t$ and on the total number $N$ of products (Lambertini and Mantovani, 2009). In particular the whole demand is proportional to the factor $1 + 1/[1 + \xi (N - 1)]$. We introduce in our model the parameter $\sigma = \xi N$, and call this “generalized substitutability”. Innovation affects this generalized substitutability according to (25) and demand becomes:

$$D_t(p_t) = (a - dp_t) \left[1 + \frac{1}{1 + \sigma (1 - n_t^{INN})}\right]$$

(26)

In the limit of perfect competition ($\xi \to 1$ and $N \to \infty$) we are back to the model studied so far. Figure (28) contains an intuitive picture of supply and demand: beside the supply effect of innovation there is a demand effect. More innovators expand the market and the demand line rotates clock-wise. This effects raises the price. Which effect will prevail, it depends on the choice of parameters. The market equilibrium in period $t$ becomes

$$(a - dp_t) \left[1 + \frac{1}{1 + \sigma (1 - n_t^{INN})}\right] = n_t^{INN} se^{bc} p_t + n_t^{IM} \mu n_t^{INN} se^{bc} p_t$$

(27)
Solving for price in equilibrium we find

\[ p_t = \frac{a[2 + \sigma(1 - n_t^{NN})]}{d[2 + \sigma(1 - n_t^{NN})] + sn_t^{NN}\beta C[1 + \mu(1 - n_t^{NN})][1 + \sigma(1 - n_t^{NN})]} \]  

(28)

This should be compared to equation (20) of the model without effects on demand. The increased complexity due to a dynamic substitutability produces a chaotic market dynamics: price and agents’ concentration may not converge to stable equilibria or to regular periodic patterns but end up oscillating in an irregular and unpredictable way. Figure 29 shows the time series of innovators in one simulation: the fluctuations lack any regularity. On top of this, there is a persistence of imitative behaviour with jumps to innovation. Now the qualitative behaviour of agents’ choices changes dramatically when parameters are changed. The new parameter \( \sigma = \xi N \) is actually a measure of the competition degree of the market (Aghion et al., 2001). An increased level of competition lowers the frequency of innovators, as one sees from the bifurcation diagram of Figure 30. This effect happens to be stronger for larger values of the imitation efficiency \( \mu \). In the conditions of this example the dynamics become very irregular when \( \sigma \) is between 70 and 100. Moreover, new periodic orbits appear, as 3-cycles for instance. The effect
of replicability $\mu$ is mathematically identical to that of $\sigma$ (although it must be $\mu \leq 1$). Figures 31 and 32 presents bifurcations diagram of $\mu$ for innovators’ fraction and price, respectively. This example shows how the demand effect of innovation may prevail: here a larger replicability $\mu$ reduces innovation and lowers the time average of price.

![Figure 31: Bifurcation diagram of $\mu$ for $n^{INN}$ ($\beta = 5, a = 4, d = 1, s = 2, b = 0.5, C = 1, \sigma = 80$).](image)

![Figure 32: Bifurcation diagram of $\mu$ for price ($\beta = 5, a = 4, d = 1, s = 2, b = 0.5, C = 1, \sigma = 80$).](image)

Concluding, the increased complexity of innovation that hits both supply and demand of an industry may present a very irregular behaviour. When this happens, the paths of market and industry variables miss any coherence, not converging to a stable equilibrium neither to a regular cyclical oscillation. The trajectories of agents’ fractions and product price may be unpredictable for a large set of parameters values. Moreover, the combination of demand and supply effects of innovation captures the persistence of agents’ behaviours to some extent, even without introducing memory. Irregularity and persistence of dynamics are two characters that one finds in several industries where demand plays an important role and where switching of strategy by firms is more frequent, as the service sector, for instance.
5 Conclusions

We have addressed the dynamics of innovation and imitation in two cases: first, in a perfectly competitive market, with innovation as costs reduction. Secondly introducing also innovation as product differentiation. We have studied the evolutionary interaction of innovators and imitators in a large population of firms, using a discrete choice mechanism. The basic idea of our model is that the evolutionary dynamics of agents affect endogenously the supply function of imitators, and in particular that imitation works better the more innovators there are. The extended version of the model is based on the idea that substitutability of products decreases when innovation affects demand through product differentiation and that such effect is stronger the larger the fraction of innovators.

Our work proposes an endogenous model of innovation and imitation which is not necessarily globally stable. This has resulted from the application of a decision mechanism based on discrete choice to production costs reduction and product differentiation. By modelling the interactions between innovators and imitators and their effects on supply and demand, we describe the opposing forces that lead a firm to innovate or to imitate, which may be relevant for industrial dynamics. In particular, we can see how and why an industry presents a prevalence of one or the other attitude, and to which extent one type can outweigh the other and what are the conditions for more or less innovation.

The main insights of the model are the following: there are two opposite effects of innovation on profits, a direct effect and an indirect effect through the price. Such tension may produce a cyclical behaviour of price and agents’ fractions. In particular, an increased value of the innovation benefits may happen to lower the number of innovators.

Asynchronous updating of strategies and long memory in agents’ utility may be either stabilizing or destabilizing, depending on the importance of factors as rationality, innovation benefits and costs. The destabilizing effects realizes with a loss of coherence in the dynamics, leading to higher periodicity cycles or even to chaotic dynamics. Despite such dynamic destabilization, memory is usually stabilizing in terms of persistence of agents’ behaviour and variance of market indicators.

The model extended with endogenous technological progress presents a different scenario, with convergence to a stable equilibrium after an initial oscillatory phase. Higher cumulation of technology dampens oscillations and fastens convergence. The long run equilibrium may be one with zero price in cases where innovators never disappear from the market, or an equilibrium with a positive level of price and no innovators, if they are driven out too quickly.

The imitation externality is destabilizing: when an innovation is easier to replicate, the regions of parameters’ values for which the system is cyclical are larger. The effect of
rationality of agents on innovation is ambiguous. Depending on the conditions a larger intensity of choice may lead either to more or less innovators.

When we consider only supply effects, the dynamics of the model is always coherent, with either convergence to a stable steady state or to a period 2 cycle. Product differentiation beside costs reduction produces irregular dynamics even without memory or asynchronous updating. This extended version of the model presents a supply effect and a demand effect of innovation which affect the price in opposite ways. The total effect on the price is not univocal and it depends on the value of parameters. There are conditions where more innovation leads to higher prices and conditions where the opposite is true. Regarding the effect of substitutability and competition, a higher degree lowers innovation in general.

**Appendix A  Existence and stability of steady states in the preliminary model**

The first derivative of price map (9) is as follows:

\[ f'(p) = \frac{-a \Delta s^2 \beta p e^{-\beta (\frac{A_s p^2}{2} - C)}}{\left[1 + e^{-\beta (\frac{A_s p^2}{2} - C)} \right]^2 \left[d + sM + \frac{\Delta s}{1 + e^{-\beta (\frac{A_s p^2}{2} - C)}} \right]^2} \]  

(29)

It is always \( f'(p) < 0 \) \( \forall p > 0 \), which means \( \exists 1! p = p^* \) such that \( f(p^*) = p^* \). In the steady state \( p^* \) the first derivative is given by

\[ f'(p^*) = -\frac{(p^*)^3 \beta \Delta s^2}{a \left[1 + e^{\beta (\frac{A_s p^2}{2} - C)} \right]^2} \]

(30)

where we used \( f(p^*) = p^* \). We have the following cases:

1. \( \lim_{\beta \to 0} f'(p^*) = 0 \)
2. if \( \Delta \pi^* = \frac{1}{2} \Delta s(p^*)^2 - C < 0 \) then \( \lim_{\beta \to \infty} f'(p^*) = -\infty \)
3. if \( \Delta \pi^* = \frac{1}{2} \Delta s(p^*)^2 - C > 0 \) then \( \lim_{\beta \to \infty} f'(p^*) = 0 \)

In the second case \( \exists \beta = \beta_1 \) such that \( f'_{\beta_1}(p^*) = -1 \) (bifurcation value). There is only one bifurcation value for \( \beta \): substituting \( f'_{\beta_1}(p^*) = -1 \) into (30) one gets

\[ \frac{(p^*)^3}{a} \Delta s \beta = \left[1 + e^{\beta (\frac{A_s (p^*)^2}{2} - C)} \right]^2 \]

(31)

The left hand side is a line through the origin, while the right hand side is decreasing for \( \Delta \pi^* < 0 \) (condition for the existence of a bifurcation). Then \( \exists 1! \beta = \beta_1 \) that satisfies the equation above.
Other parameters may present more than one bifurcation value. This is the case of the innovation benefits rate $b$, defined as $s^{\text{INN}} = s^{\text{IM}} e^{bC}$. If we rewrite equation (31) with this specification we have

$$\frac{(p^*)^3}{a} s(e^{bC} - 1)\beta = \left\{1 + e^{\beta \left[\frac{1}{2} (e^{bC} - 1)(p^*)^2 - C]\right]} \right\}^2$$

(32)

Since the right hand side is positive, convex and increasing in $b$, there may be two values $b_1$ and $b_2$ with $b_1 < b_2$ that satisfy (32). The first bifurcation value $b_1$ is a period doubling bifurcation, while $b_2$ is a period halving bifurcation. Let’s define $B = e^{bC}$ (monotonic transformation) and the function $g(B)$ for the right hand side of (32). The first and second derivatives of $g$ are

$$g'(B) = \left\{1 + e^{\beta \left[\frac{1}{2} (e^{bC} - 1)(p^*)^2 - C]\right]} \right\} \beta s(p^*)^2 e^{\beta \left[\frac{1}{2} (e^{bC} - 1)(p^*)^2 - C]\right]}$$

(33)

$$g''(B) = \frac{1}{2} \beta^2 s^2 (p^*)^4 e^{\beta \left[\frac{1}{2} (e^{bC} - 1)(p^*)^2 - C]\right]} \left\{1 + 2e^{\beta \left[\frac{1}{2} (e^{bC} - 1)(p^*)^2 - C]\right]} \right\}$$

(34)

The larger the intensity of choice $\beta$ or the larger the inverse of marginal production costs $s$, the more convex is $g$. At the same time, the larger $\beta$ or $s$ and the steeper the left hand side of (32). Consequently, as parameters $\beta$ and $s$ become larger an instability region $[b_1, b_2]$ appears in the benefits rate domain, which is wider for larger values of $\beta$ and $s$.

**Appendix B  Proof of proposition 3**

Given a value of the price $p$, the supply side of equation (18) can be written as a function of $n ≡ n^{\text{INN}}$:

$$S(n) = se^{bC} f(n)$$

(35)

where $f(n) = n + \mu n(1 - n)$. Its derivative is $f'(n) = 1 + \mu - 2\mu n$. It holds $f'(n) \geq 0$ for $n \in [0, 1]$ and $\mu \in [0, 1]$, with $f'(n) = 0$ when $n = 1$ and $\mu = 1$. This means that $f(n)$ is increasing and that supply is maximum when everybody chooses to be an innovator. On the other hand, $f(n = 0) = 0$.

**Appendix C  The steady state with imitation**

The first derivative of the map (23) can be written as follows:

$$f'(x) = f(x)[f(x) - 1] \frac{\mu sa^2 e^{bC}}{2} \left\{d + se^{bC}x[1 + \mu(1 - x)]\right\} - 2(\mu x - 1)se^{bC}(1 + \mu - 2\mu x)$$

$$\left\{d + se^{bC}x[1 + \mu(1 - x)]\right\}^3$$

(36)

Stability of the unique steady state changes as $\beta$ changes:

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• for $\beta$ low (myopic agents) $\lim_{\beta \to 0} f'(x) = 0$, stable steady state.

• for $\beta$ large (rational limit) with $\mu$ and $C$ low enough, $\lim_{\beta \to 0} f'(x) = 0$, again.

This is because of proposition 4 which says that $\lim_{\beta \to 0} f(x) = 0$ if $\Delta \pi^* < 0$ and $\lim_{\beta \to 0} f(x) = 1$ if $\Delta \pi^* > 0$. In both cases $f'(x)$ converges to 0, but the limits above exist only when replicability $\mu$ and innovation costs $C$ are low enough.

References


