Three essays on the insurance of income risk and monetary policy

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The primary focus of this thesis is on the explicit incorporation of individual heterogeneity to address two important questions. First, do the households have more information about their income risk as compared to what is assessed by an econometrician? I find that there is a systematic income risk gap: household's perceived income risk is at least 12 percent lower than what is estimated by an econometrician. Second, is the provision of public insurance necessarily welfare improving in the standard incomplete markets model? I show quantitatively that this may not be the case if one group of individuals receive significantly more public insurance as compared to other groups over time.

Swapnil Singh holds a Bachelor degree in Economics from the Indian Institute of Technology, Kanpur and MPhil in Economics from the Tinbergen Institute. In September 2013, as a PhD student he joined the Macroeconomics and International Economics group at the University of Amsterdam. Here he worked under the excellent supervision of Prof. Dr. Roel Beetsma and Dr. Christian Stoltenberg. His primary interest lies in quantitative and empirical aspects of incomplete market models.

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Universiteit van Amsterdam

Three Essays on the Insurance of Income Risk and Monetary Policy

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Three Essays on the Insurance of Income Risk and Monetary Policy

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Abstract

The present dissertation contains three chapters: the first two chapters are associated with incomplete market models and the last chapter is concerned with optimal monetary policy in the Dynamic Stochastic General Equilibrium (DSGE) framework.

The first chapter is concerned with structurally estimating the difference between idiosyncratic income risk estimated by an econometrician and what is actually perceived by households. Using the US micro-data to inform the consumption risk sharing model, I estimate that the perceived income risk is at least 12 percent lower than the risk estimated by an econometrician. Accounting for this gap, the model jointly explains three distinct risk sharing measures that are not captured in the standard model without a gap: (i) the cross-sectional variance of consumption, (ii) the covariance of consumption with income growth, and (iii) the income conditional-mean of household consumption.

In the second chapter, using the March Current Population Survey, I show that over the last two decades, married households in the United States received increasingly more public insurance against labor income risk, whereas the opposite was true for single households. To evaluate the welfare consequences of this trend, I perform a quantitative analysis. As a novel contribution, I expand the standard incomplete markets model à la Aiyagari (1994) to include two groups of households: married and single. The model allows for changes in the marital status of households and accounts for transition dynamics between steady states. I show that the divergent trends in public insurance have a significant detrimental effect on the welfare of both married and single households. Higher public insurance crowds out the private savings of married households, thus decreasing their mean wealth. In the long run, lower wealth decreases mean consumption for married households, driving the decline in their welfare. For singles, transition dynamics play a major role. Although in response to lower public insurance they save more and can afford higher mean consumption in the new steady state, the welfare loss from lower initial consumption after the policy change offsets this welfare gain.

In the third chapter, I study the role of factor demand linkages in the design of optimal monetary policy. A two-sector New-Keynesian model is developed in which sectors are connected through factor demand linkages and differ in price stickiness. I find two important results: first, the presence of factor demand linkage induces amplification effects in resource mis-allocation and, hence, the concern for price stability becomes more important. Second, the optimal price index is not the same as the aggregate price index, although it does not depend upon factor demand linkages. Furthermore, based on the micro-founded loss function we derive a policy rule that implements the optimal allocation.

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I wanted to write an honest acknowledgment. This has proved to be a difficult task for me. I had to sit down and think about many past moments of my life where numerous people gave me their unconditional support. Thinking about all those moments I realize that I wouldn't have been able to finish my PhD without their help.

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From my promoter Roel Beetsma I learned patience and time management in doing research. There were numerous times when we will discuss about some ideas. I will present my arguments and he will present his. After the discussion he will let me implement my arguments. However, later on I will realize that his arguments were right. He was teaching me to recognize the correct solution from the group of many solutions. I can only imagine how much patient he was to not to tell me that his solution is right and I should implement it. He is the one who told me that I make grammatical mistakes, but never chided me for it. I am grateful to him for patiently letting me discover my own solutions.

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Co-author List

- Chapter 1, entitled "*How much do households really know about their future income??*" is a joint work with supervisor Dr. Christian Stoltenberg, University of Amsterdam. Christian involved me in the project in April 2014 and then we developed the paper further together. It is a joint project in all parts but as the idea further developed, I focused more on the computational challenges of the project and Christian focused on the theoretical results and writing.

- Chapter 3, entitled "*Optimal monetary policy under sector interconnections*" is a joint work with my supervisor Prof. Dr. Roel Beetsma. This paper was a part of my MPhil thesis project in which I came up with the idea. I was producing the results in the paper. However, Roel has not only provided constant guidance but also using his experience, has made the paper much more accessible.

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Chapter 1

How Much Do Households Really Know About Their Future Income?

1.1 Introduction

What is households' income uncertainty when they decide about their savings to insure against undesirable fluctuations of their consumption? The answer to this question is of central importance to understand consumption risk sharing; only what households don't know yet constitutes uncertainty they seek to hedge. Typically, households' income uncertainty measures stem from aggregating earnings across household members and income types in the population. As pointed out by Browning, Hansen, and Heckman (1999) and again more recently by Cunha and Heckman (2016), this procedure may however create a disconnect between the uncertainty as assessed by an *econometrician* and income uncertainty as perceived by households. Quantifying households' perceived income uncertainty is however a prerequisite for evaluating the welfare effects of hotly-debated reforms such as changes in the progressivity of the tax system. As our main contribution, we develop a risk-sharing model suitable for policy analysis that can distinguish between households' perceived income uncertainty and income uncertainty as measured by an econometrician in a systematic and consistent way. We find that accounting for households' perceived income uncertainty is key to understand risk sharing of households in the United States.[1]

We consider an environment in which risk averse households seek insurance against idiosyncratic fluctuations of their disposable income. As the new element here, we explicitly extend households' information set by signals that inform households on their income in the next period with certain precision. While the stochastic income process constitutes the income uncertainty as assessed by an econometrician, the joint process of signals and income represents households' income uncertainty. The difference between the two income uncertainties depends on the precision of the signals as households' advance information; the more precise are the signals, the lower is households' forecast error for income growth and the lower is households' perceived income uncertainty. The extension of households' information set is motivated by the observation that households typically have more information than their current income to predict their future earnings. Thus, the signals capture a wide spectrum of information relevant for future changes in disposable income that are already known to households before the actual change occurs. Examples for this type of fore-knowledge are information on future performance bonuses, promotions, demotions or wage cuts, wage rises, changes in income taxes and transfer but also changes in marital status; a divorce can be interpreted as a signal for a negative income shock while a marriage signals a positive income shock.

In reality, households can smooth income shocks in a variety of ways, involving progressive taxation, family transfers, informal networks or default. To capture these various insurance possibilities, we employ a general-equilibrium model with endogenous solvency constraints stemming from limited contract enforcement as proposed by Alvarez and Jermann (2000). In this model, households have access to a full set of formal and informal insurance contracts with the drawback that these contract are not enforceable under all circumstances.

As our main novel finding, we discover that households have advance information on their future income such that their income uncertainty is lower than what is typically considered in

[1] Throughout the paper, we use the terms uncertainty and risk interchangeably, and consider known probabilities of random events which differs from the concept of Knightian uncertainty where the probabilities are unknown.

consumption risk sharing models. Employing U.S. micro data to inform the theoretical model, we find that advance information reduces households' mean-squared forecast error for income by 12 percentage points. This implies a systematic gap between the income uncertainty as perceived by households and the income uncertainty as estimated by an econometrician. Accounting for this gap, the model jointly explains three distinct consumption risk sharing measures that are not captured without advance information: (i) the unconditional variance of households consumption in the cross section, (ii) the covariance of current consumption and income growth and (iii) the income-conditional mean of household consumption.

As our main theoretical result, we show that with limited contract enforcement advance information reduces consumption risk sharing. The rationale for this surprising result is that more precise signals decrease the value of insurance for high-income households which limits opportunities for risk sharing between households. Models with limited contract enforcement but without advance information tend to overstate the degree of household risk sharing. Thus, the decrease in risk sharing resulting from households with advance information improves the fit of the model to the data.

In the quantitative exercise, we characterize cross-sectional long-run distributions of consumption, income and wealth across households with advance information. To do that, we develop a stochastic model with an explicit specification of the joint distribution of income and signals as a methodological contribution that is applicable to a large set of macroeconomic models with a recursive structure. Keeping track of the joint distribution of signals and income also imposes a challenge to the computation of optimal allocations with limited contract enforcement.

Given an uncertainty gap of 12 percent, we analyze the quantitative implications for the income-conditional distribution of consumption. We find that advance information also here substantially improves the fit to the data. In particular, the model (almost) perfectly tracks the income-conditional mean of consumption both low, medium and high income earners. Further, advance information helps to attenuate a non-linearity present in limited contract enforcement models without information but absent in the data. In the absence of information, the limited commitment model implies a variance of consumption conditional on a high income that is equal to zero. With informative signals, the conditional variance is positive, bringing the model closer to the data.

Related literature We are not the first to find that households know more than econometricians about their future earnings.[2] The main differences to existing papers are first that we find a quantitative important role for advance information in general equilibrium. Further, we employ a limited commitment model to explicitly capture both formal and informal insurance arrangements while previous papers focused on a standard incomplete markets model with self insurance.

Most closely related to our paper are Blundell, Pistaferri, and Preston (2008), Kaplan and Violante (2010) and Heathcote, Storesletten, and Violante (2014a) who studied the role of advance information in standard incomplete markets environments. Blundell et al. (2008) pointed out that advance information may result in counterfactual non-zero correlations of current consumption growth with future income growth. This is problematic because these correlations serve as a key indicator for consumption smoothing. Further, Kaplan and Violante (2010) showed that the resulting increase in risk sharing with advance information is quantitatively not important enough to account for the cross-sectional dispersion of consumption in the data. With our paper, we clarify that the effect of advance information on risk sharing depends both qualitatively and quantitatively on the particular consumption-savings model employed. While we can confirm the earlier findings on advance information in the standard incomplete markets model (Blundell et al., 2008, Kaplan and Violante, 2010), a model with endogenous solvency constraints due to limited enforcement leads to very different conclusions. Without inducing counterfactual correlations of current consumption with future income growth, a limited enforcement model with advance information bridges the gap to several consumption insurance measures observed in the data.

Employing a structural-analytical framework, Heathcote et al. (2014a) considered two different type of shocks, “uninsurable shocks” and “insurable shocks”. The former shocks can only be

[2] Exemplary papers in that literature are Cunha and Heckman (2016), Cunha, Heckman, and Navarro (2005), Guvenen (2007), Guvenen and Smith (2014), Huggett, Ventura, and Yaron (2006), Primiceri and van Rens (2009).

smoothed via labor supply decisions, self insurance in a non-state contingent bond or via government interventions. The latter type of shocks can (by assumption) be perfectly insured and can be interpreted as perfectly forecastable shocks through the lens of a standard incomplete markets model. Our contribution is to highlight that when households use a large variety of insurance possibilities perfectly forecastable shocks does not necessarily enhance but may actually restrict the degree of risk sharing.

Guvenen and Smith (2014) employ a life-cycle standard incomplete markets model and Bayesian indirect inference to jointly estimate how much households initially know about their future income (when learning about the nature of their income process during their working life) and the degree of partial insurance households can achieve. As main substantive finding and similar to us, the authors argue that households' perceived income uncertainty is smaller than what is typically considered in macroeconomic models. Our work is complementary to Guvenen and Smith (2014). First, we model advance information in general equilibrium with an infinite time horizon while they consider a partial equilibrium setting with a finite time horizon. Second, we employ an explicit insurance model in which advance information can result in less insurance while in their environment advance information leads to more insurance.

Methodologically, our paper draws heavily on Alvarez and Jermann (2000), Kehoe and Levine (1993) and Krueger and Perri (2006, 2011) who studied the theoretical and quantitative properties of constrained efficient allocations with limited contract enforcement. Aiyagari (1994) pioneered in characterizing invariant distributions of consumption and assets in the standard incomplete markets model in general equilibrium. Building on these papers, Broer (2013) compared the cross-sectional implications of both models with the data. We extend the limited contract enforcement model and the standard incomplete markets model with a role for information to study how households' perceived income uncertainty – instead of the uncertainty assessed by an econometrician – affects consumption risk sharing of U.S. households.

Hirshleifer (1971) was among the first to point out that better information makes risk-averse agents ex-ante worse off if such information leads to evaporation of risks that otherwise could have been shared in a competitive equilibrium with full insurance and perfect contract enforcement. Schlee (2001) provided conditions under which better public information about idiosyncratic risk is undesirable. Similar to these authors, we also find that better public information can result in less risk sharing. The difference is that the negative effect relies on the importance of the limited enforceability of contracts and arises only when consumption insurance is not full but partial. If enforcement frictions were absent and insurance full, information would not affect consumption allocations in the limited commitment model.

The remainder of the paper is organized as follows. In the next section, we present our economic environment. In Section 1.3, we provide analytical results on the effects of information on conditional and unconditional consumption moments. Section 1.4 is devoted to the quantitative implications of advance information for risk sharing of U.S. households. The last section concludes.

1.2 Environment

Preferences and endowments Consider an economy with a continuum of households indexed by i. The time is discrete and indexed by t from zero onward. Households have preferences over consumption streams and evaluate it conditional on the information available at $t = 0$

$$U\left(\{c_t^i\}_{t=0}^{\infty}\right) = (1-\beta)\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i), \tag{1.1}$$

where the instantaneous utility function $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing, strictly concave and satisfies the Inada conditions.

Household i's disposable income in period t is given by a stochastic process $\{y_t^i\}_{t=0}^{\infty}$, where the set of possible income realizations in each period is time-invariant and finite $y_t^i \in Y \equiv \{y_1, ..., y_N\} \subseteq \mathbb{R}_{++}$, ordered. The history y^t is $(y_0, ...y_t)$. The income is independent across households and evolves across time according to a first-order Markov chain with time-invariant transition matrix $\pi_{jk} > 0$ for all j, k whose elements are the conditional probabilities of next period's income y_k

given current period income y_j. The Markov chain induces a unique invariant distribution of income $\pi(y)$ such that average income (or aggregate labor endowment) is $\bar{y} = \sum_y y\pi(y)$.

Information and utility allocation Each period $t \geq 0$, household i receives a public signal $k_t^i \in Y$ that informs about income realizations in the next period. The signal has as many realizations as income states and its precision κ is captured by the probability that signal and future income coincide, $\kappa = \pi(y_{t+1} = y_j | k_t = y_j)$, $\kappa \in [1/N, 1]$. Uninformative signals are characterized by precision $\kappa = 1/N$, perfectly informative signals by $\kappa = 1$. The realizations of the public signal follow an exogenous first-order Markov process with the same conditional probabilities as income, i.e., $\pi(k' = y_j | k = y_i) = \pi(y' = y_j | y = y_i)$. Hence, at each point in time the agents can find themselves in one of the states $s_t = (y_t, k_t)$, $s_t \in S$, where S is the Cartesian product $Y \times Y$ and $s^t = (y^t, k^t) = (s_0, \ldots s_t)$ is the history of the state.

Using the assumptions on income and signals, the probabilities for the distribution of future income conditional on today's state s is given by[3]

$$\pi(y'|s) = \pi\left(y' = y_j | k = y_m, y = y_i\right) = \frac{\pi_{ij}\kappa^{\mathbf{1}_{m=j}}\left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=j}}}{\sum_{z=1}^{N}\pi_{iz}\kappa^{\mathbf{1}_{m=z}}\left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=z}}} \quad , \tag{1.2}$$

where $\mathbf{1}_{m=j}$ is an indicator function and equals one if the signal and the actual realization of income coincide. The logic of the formula is a signal extraction with two independent signals on future income realizations, current income and the public signal. Both signals enter the signal extraction weighted with their precision, income with transition probability π_{ij} and signals with precision κ.

For example, with uninformative signals ($\kappa = 1/N$) the conditional expectation of income y_j tomorrow given signal k_j and income y_i can be computed as

$$\pi\left(y' = y_j | k = y_j, y = y_i\right) = \frac{\pi_{ij}\frac{1}{N}}{\frac{1}{N}\sum_{z=1}^{N}\pi_{iz}} = \pi_{ij}.$$

[3] Appendix A.5 provides details on the derivation of the formulas for the joint distribution of income and signals. Further, we provide arguments for the assumptions on the signal process.

With signals following an exogenous process, the conditional distribution of signals and income can be combined to a time-invariant Markov transition matrix P_s with conditional probabilities $\pi(s'|s)$ as elements

$$\pi(s'|s) = \pi\left(y' = y_j, k' = y_l | k = y_m, y = y_i\right) = \pi(k' = y_l | k = y_m)\pi\left(y' = y_j | k = y_m, y = y_i\right). \tag{1.3}$$

Households differ with respect to their initial utility entitlements w_0 and the initial state s_0 which constitute their information set in period $t = 0$. The utility allocation is $h = \{h_t(w_0, s^t)\}_{t=0}^{\infty}$ and the consumption allocation c can be obtained as $c = \{C\left(h_t(w_0, s^t)\right)\}_{t=0}^{\infty}$, where $C : \mathbb{R} \to \mathbb{R}_+$ is the inverse of the period utility function u. Thus, the allocation depends on initial utility promises and on the history of income and signals s^t.

Insurance arrangements To protect their consumption from undesirable fluctuations, households can trade insurance contracts that cover the complete state space but have limited enforceability (or limited commitment) as in Kehoe and Levine (1993) and Krueger and Perri (2011). As illustrated in Figure 1.1, enforcement of the insurance contracts is limited because each period after receiving the current income and the public signal, households decide to participate in risk-sharing contracts implementing the allocation c or to deviate into autarky forever, consuming only their endowment.[4] The limited enforceability of contracts gives rise to participation constraints for each history s^t for all periods t:

$$(1-\beta)u\left(C(h_t)\right) + \beta \sum_{s_{t+1}} \pi(s_{t+1}|s^t)\, \mathrm{U}_{t+1}\left(\{C(h_\tau)\}_{\tau=t+1}^{\infty}\right) \geq U^{Aut}(s_t), \tag{1.4}$$

with $h_t = h_t(w_0, s^t)$, $h_\tau = h_\tau(w_0, s^\tau)$ and

$$U^{Aut}\left(s_t\right) \equiv (1-\beta)u(y_t) + \beta \sum_{y_{t+1}} \pi(y_{t+1}|s_t)\, \mathrm{U}_{t+1}\left(\{y_\tau\}_{\tau=t+1}^{\infty}\right) \tag{1.5}$$

[4] In quantitative analysis, we allow also for self insurance in the outside option.

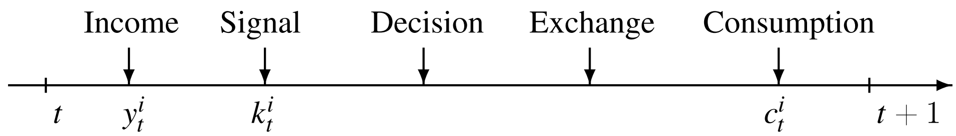


Figure 1.1: Timing of events with limited contract enforcement and public information

as the value of the outside option (autarky) and $U_\tau = (1-\beta)\,\mathrm{E}_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} u\,(c_t)$.

Efficient allocations Let $\Phi_0 = \Phi_0(w, s)$ be an initial distribution over w_0 and the initial shocks $s_0 = (y_0, k_0)$. In the following definitions, we capture constrained feasible and efficient allocations.

Definition 1 *An allocation $\{h_t(w_0, s^t)\}_{t=0}^{\infty}$ is constrained feasible if*

(i) the allocation delivers the promised utility w_0

$$w_0 = \mathrm{U}\left(\{C(h_t)\}_{t=0}^{\infty}\right);$$

(ii) the allocation satisfies for each state s^t participation constraints (1.4)

(iii) and the allocation is resource feasible

$$\sum_{s^t} \int \left[C(h_t) - y_t\right] \pi(s^t|s_0) d\Phi_0 \leq 0 \quad \forall t. \tag{1.6}$$

Atkeson and Lucas (1992) show by applying a duality argument that efficient allocations can be computed by minimizing resource costs to deliver the promises made in Φ_0. The notion of efficiency is summarized in the following definition.

Definition 2 *An allocation $\{h_t(w_0, s^t)\}_{t=0}^{\infty}$ is efficient if it is constrained feasible and there does not exist another constrained feasible allocation $\{\hat{h}_t(w_0, s^t)\}_{t=0}^{\infty}$ with respect to Φ_0 that requires fewer resources in one period t*

$$\exists t: \quad \sum_{s^t} \int \left[C(\hat{h}_t) - C(h_t)\right] \pi(s^t|s_0)\,\mathrm{d}\,\Phi_0 < 0.$$

Summing up, an efficient allocation respects promises and participation incentives, is resource feasible and there is no other allocation that uses fewer resources that is also constrained feasible.

Stationary efficient allocations As in Krueger and Perri (2011), we restrict ourselves to computing stationary efficient allocations – allocations in which the distribution of current utility and utility promises is constant across time. As shown by Atkeson and Lucas (1992), a stationary allocation is efficient if it is a solution to a dynamic programming problem (described in the next paragraph) and if it satisfies resource feasibility. In the next section, we will show that these allocations can be also interpreted as allocations stemming from a competitive equilibrium.

Given utility promise w, state $s = (y, k)$ and an inter-temporal price $R \in (1, 1/\beta]$, a planner (or financial intermediary) chooses a portfolio of current utility h and future promises $w'(s')$ for each future income realization y' and signal k'. The portfolio $(h, \{w'(s')\})$ is required to minimize the discounted resources costs, to deliver the promised value $w(s)$ and to satisfy participation constraints. Formally, the minimization problem reads as follows

$$V(w,s) = \min_{h,\{w'(s')\}} \left[\left(1 - \frac{1}{R}\right) C(h) + \frac{1}{R} \sum_{s'} \pi(s'|s) V\left(w'(s'), s'\right) \right] \tag{1.7}$$

s.t.

$$w = (1-\beta)h + \beta \sum_{s'} \pi(s'|s) w'(s') \tag{1.8}$$

$$w'\left(s'\right) \geq U^{Aut}\left(s'\right), \forall s'. \tag{1.9}$$

The solution is characterized by the first order conditions:

$$V_w\left(w'(s'), s'\right) \frac{1-\beta}{(R-1)\beta} \geq C'\left(h\right) \tag{1.10}$$

$$V_w\left(w'(s'), s'\right) \frac{1-\beta}{(R-1)\beta} = C'\left(h\right), \quad \textit{if}\; w'(s') > U^{Aut}(s'), \tag{1.11}$$

where we have applied the envelope condition

$$\lambda = \frac{\partial V(w,s)}{\partial w} = \frac{R-1}{(1-\beta)R} C'(h), \tag{1.12}$$

where λ is the Lagrange multiplier associated with the promise keeping constraint (1.8). The result of the minimization problem are policy functions $h(w,s), \{w'(w,s;s')\}$. Following similar arguments as in Krueger and Perri (2011), $w' \in W = [\underline{w}, \overline{w}]$, $\underline{w} = \min_{s'} U^{Aut}(s')$ and $\overline{w} = \max_{s'} U^{Aut}(s')$. The state space therefore comprises $Z = W \times S$ with Borel σ algebra $\mathcal{B}(Z)$ and typical subset $(\mathcal{W}, \mathcal{S})$. The Markov process for income and signals P_s together with the policy functions w' generate a law of motion for the probability measure $\Phi_{w,s}$. Let $Q_R\left[(w,s), (\mathcal{W}, \mathcal{S})\right]$ be the probability that an agent with utility promise w and income-signal state s transits to the set $\mathcal{W} \times \mathcal{S}$:

$$Q_R\left[(w,s), (\mathcal{W}, \mathcal{S})\right] = \sum_{s' \in \mathcal{S}} \begin{cases} \pi(s'|s) & \text{if } w'(w,s) \in \mathcal{W} \\ 0 & \text{else} \end{cases}$$

It follows that the law of motion for the probability measure $\Phi_{w,s}$ is given by

$$\Phi'_{w,s} = H(\Phi_{w,s}) = \int Q_R\left[(w,s), (\mathcal{W}, \mathcal{S})\right] \Phi_{w,s}(\mathrm{d}\, w \times \mathrm{d}\, s).$$

An allocation is stationary if $\Phi_{w,s} = H(\Phi_{w,s}) = \Phi_0$.[5]

Definition 3 *An efficient allocation in the limited commitment economy is a stationary utility allocation $\{h(s,w)\}$ and an invariant probability measure $\Phi_{w,s}$ induced by the problem (1.7)-(1.9) satisfying resource feasibility* (1.6).

1.3 Analysis

In this section, we provide analytical results on the effect of public information on consumption risk sharing. First, we describe how efficient allocations with public information can be decentralized in

[5] The probability measure can be shown to exist and to be unique. The proof relies on an argument that requires the state space of promises to be compact which follows from analogous arguments as in Krueger and Perri (2011).

a competitive equilibrium. As the key result in this section, we show that better public information reduces risk-sharing possibilities in the limited commitment model and thus results in a riskier consumption distribution when risk sharing is partial. Further, we analyze how the precision of public signals affect the conditions for full perfect risk sharing and autarky as efficient allocations.

1.3.1 Competitive equilibrium and decentralization

In a decentralized version of the economy, households are heterogenous in initial asset holdings, income and signals. We start with defining a competitive equilibrium as in Krueger and Perri (2011) that follows Kehoe and Levine (1993) and derive prices to decentralize the efficient allocations. Denote by $p_t(s^t)$ the period-zero price of a unit of period-t consumption faced by a household following history s^t. A household with initial wealth a_0, initial endowment y_0, and signal k_0 chooses an allocation $\{c_t(a_0, s^t)\}_{t=0}^{\infty}$ that provides the highest utility subject to their intertemporal budget constraint

$$c_0(a_0, s_0) + \sum_{t=1}^{\infty} \sum_{s^t|s_0} p_t(s^t) c_t(a_0, s^t) \leq y_0 + \sum_{t=1}^{\infty} \sum_{s^t|s_0} p_t(s^t) y_t + a_0 \tag{1.13}$$

and their participation incentives for each history s^t in each period t

$$(1-\beta)u\left(c_t\right) + \beta \sum_{s_{t+1}} \pi(s_{t+1}|s^t)\, \mathrm{U}\left(\{c_\tau\}_{\tau=t+1}^{\infty}\right) \geq$$
$$(1-\beta)u(y_t) + \beta \sum_{y_{t+1}} \pi(y_{t+1}|s^t)\, \mathrm{U}_{t+1}\left(\{y_\tau\}_{\tau=t+1}^{\infty}\right), \tag{1.14}$$

where $c_t = c_t(a_0, s^t)$ and $c_\tau = c_\tau(a_0, s^\tau)$.

Definition 4 *A competitive equilibrium with limited commitment is a price system $\{p_t(s^t)\}_{t=0}^{\infty}$ and an allocation $\{c_t(a_0, s^t)\}_{t=0}^{\infty}$ such that*

(i) given prices, the allocation of each household (a_0, s_0) solves the household's problem;

(ii) all markets clear.

An efficient allocation with public information can be decentralized as a competitive equilibrium which is captured in the following proposition.

Proposition 1 *A stationary efficient allocation* $\{C(h_t(w_0, s^t))\}_{t=0}^{\infty}$ *can be decentralized as a competitive equilibrium allocation* $\{c_t(a_0, s^t)\}_{t=0}^{\infty}$ *with prices and initial asset holdings given by*

$$p_t(s^t) = \frac{\pi(s^t|s_0)}{R^t} \quad \textit{and} \quad a_0 = c(w_0, s_0) - y_0 + \sum_{t=1}^{\infty} \sum_{s^t|s_0} \frac{\pi(s^t|s_0)}{R^t} \left[c(w_0, s^t) - y_t\right].$$

The proof is provided in Appendix A.1. This proposition is important for two reasons. First, we can interpret the stationary allocations stemming from the recursive social planner problem (1.7)-(1.9) more realistically as allocations generated by a competitive equilibrium. Second, the distribution of assets across households instead of utility promises offers the possibility to analyze the effects of public signals on wealth inequality.

1.3.2 Information, perfect risk sharing and autarky

To analytically characterize the effects of public information, we abstract in this section from a number of features. We assume that the set of possible income realizations consists of two states only $y_h = \bar{y} + \delta_y$ and $y_l = \bar{y} - \delta_y$, where $\delta_y > 0$ is the standard deviation of the income process. Further, the states are equally likely and the income realizations are independent across time and agents. Correspondingly, public signals are i.i.d. as well and can indicate either a high income ("good" or "high" signals) or a low income ("bad" or "low" signals) in the future. Efficient allocations may feature either full insurance, partial insurance or no insurance against income risk (autarky). The first case is analyzed in the following proposition.

Proposition 2 (Perfect Risk Sharing) *Consider efficient allocations.*

1. *There exists a unique cutoff point,* $0 < \bar{\beta}(\kappa) < 1$*, such that for any discount factor* $1 < \beta \geq \bar{\beta}(\kappa)$ *the efficient allocation for any signal precision is perfect risk sharing.*

2. *The cutoff point* $\bar{\beta}(\kappa)$ *is increasing in the precision of the public information.*

The proof is provided in Appendix A.2. The cutoff point for perfect risk sharing is determined by the tightest participation constraint which is the one of the high income agent receiving a high signal on future income. The long-term gains from risk sharing can only outweigh the desire to leave the arrangement if agents are sufficiently patient. Furthermore, the value of the outside option at the tightest participation constraint is increasing in the precision of the signal.

On the other extreme, autarky may be the only constrained feasible allocation and therefore efficient. In the next proposition, we provide conditions for this case.

Proposition 3 (Autarky) *Autarky is efficient if and only if*

$$\beta < \frac{u'(y_h)}{u'(y_l)}.$$

The proof is provided in Appendix A.3. Thus, with public information the condition for autarky to be efficient is identical to the condition in absence of public information analyzed in Krueger and Perri (2011). The empirically relevant case is the one when risk sharing is neither perfect nor absent but partial which we analyze below.

1.3.3 Information and partial risk sharing

In this sub-section, we analytically characterize how better public information affects the unconditional and conditional distribution of consumption with partial risk sharing. As the key result here, we find that better public information has a negative effect on risk sharing.

We continue to employ on an i.i.d. income with two equally likely states. Further, we consider allocations that do not depend on the history of the state s^t but only on the current state s_t. We refer to such allocations as memoryless allocations which are defined as follows.

Definition 5 *An allocation $\{h_t(w_0, s^t)\}_{t=0}^{\infty}$ is a memoryless allocation (denoted h_{ML}) if:*

$$\forall s^t \quad \{h_t(w_0, s^t)\}_{t=0}^{\infty} = \{h_t(s_t)\}_{t=0}^{\infty} \equiv h_{ML}.$$

While these allocation are in general not efficient, they allow us to prove the effects of public information on the consumption distribution analytically. A utility allocation in this simplified setting is $h_{ML} = \left\{u\left(c_i^j\right)\right\}$ where for notational convenience we have $c_i^j = C(h(y = y_j, k = y_i))$. Social welfare is

$$V_{rs} = (1-\beta)\frac{1}{4}\sum_{t=0}^{\infty}\sum_{j\in\{l,h\}}\sum_{i\in\{l,h\}}\beta^t u\left(c_{i,t}^j\right). \tag{1.15}$$

Resource feasibility requires in each period t

$$\frac{1}{4}\sum_{j\in\{l,h\}}\sum_{i\in\{l,h\}} c_{i,t}^j = \frac{1}{2}\sum_{j\in\{l,h\}} y_{j,t}. \tag{1.16}$$

Participation is constrained by rational incentives that take into account the public signal. For illustrative purposes, we focus here only on high-income agents. The participation constraints are thus for a good public signal

$$\begin{aligned}(1-\beta)u(c_h^h) + \beta(1-\beta)\left[\kappa V_{rs}^h + (1-\kappa)V_{rs}^l\right] + \beta^2 V_{rs} &\geq V_{h,out}^h \\ &= (1-\beta)u(y_h) + \beta(1-\beta)\left[\kappa u(y_h) + (1-\kappa)u(y_l)\right] + \beta^2 V_{out}, \end{aligned} \tag{1.17}$$

and for a bad public signal

$$\begin{aligned}(1-\beta)u(c_l^h) + \beta(1-\beta)\left[(1-\kappa)V_{rs}^h + \kappa V_{rs}^l\right] + \beta^2 V_{rs} &\geq V_{l,out}^h \\ &= (1-\beta)u(y_h) + \beta(1-\beta)\left[(1-\kappa)u(y_h) + \kappa u(y_l)\right] + \beta^2 V_{out}, \end{aligned} \tag{1.18}$$

with

$$V_{rs}^h = \frac{1}{2}\left[u(c_h^h) + u(c_l^h)\right], \quad V_{rs}^l = \frac{1}{2}\left[u(c_h^l) + u(c_l^l]\right)$$

and

$$V_{rs} = \frac{1}{2}\left[V_{rs}^h + V_{rs}^l\right], \quad V_{out} = \frac{1}{2}\left(u(y_h) + u(y_l)\right).$$

As a next step, we provide the definition of an optimal allocation in this environment.

Definition 6 *An optimal memoryless allocation is a consumption allocation* $\{c_i^j\}$ *that maximizes households' utility (1.15) subject to resource feasibility (1.16) and enforcement constraints.*

In this environment, we can analytically show how unconditional and conditional moments of consumption are affected by information precision. As summarized in the following proposition, better public signals lead to less risk sharing and higher consumption dispersion.

Proposition 4 (Information and risk sharing) *Consider an optimal memoryless allocation with partial risk sharing such that enforcement constraints (1.17)-(1.18) are binding. An increase in information precision has the following effects on the consumption allocation:*

1. *The conditional mean of consumption of high-income agents increases and the conditional mean of low-income agents decreases.*

2. *The conditional standard deviation of consumption of high-income agents increases.*

3. *The unconditional standard deviation of consumption increases.*

The proof is provided in Appendix A.4.

Better public information results in lower average consumption of low-income agents and in higher average consumption but as well in higher consumption dispersion of high-income agents. This makes the consumption distribution riskier from an ex-ante perspective.

To get intuition, consider an increase in the precision of public signals. By (1.17) and (1.18), this results in an increase in the value of the outside option for high-income agents with a good public signal and a decrease for agents with a bad public signal. As captured by the changes in the outside option values, agents with a bad signal are more willing while the agents with a good signal are less willing to share their current high income. Thus, consumption of high-income agents spreads out and the conditional standard deviation of high-income increases. Thereby, the changes in the value of the outside option of high-income agents with a good signal ($V_{h,out}^h$) and with a bad signal ($V_{l,out}^h$) are symmetric:

$$\frac{\partial V_{h,out}^h}{\partial \kappa} = -\frac{\partial V_{l,out}^h}{\partial \kappa}.$$

For informative signals, the high-income agents with a good public signal have a lower marginal utility of consumption and thus require more additional resources than the high-income agents with a bad public signal are willing to give up. In sum, conditional mean consumption of high-income agents increases which by resource feasibility reduces the risk-sharing possibilities for low-income agents. As a consequence, the allocation becomes riskier ex ante and the unconditional standard deviation of consumption increases as well.

In this section, we have shown that better public information results in a riskier allocation ex ante such that the standard deviation of consumption increases. Further, better public information results in higher consumption of high-income and lower consumption of low-income agents. Thus, better public information has the potential to improve the predictions of the limited commitment model for the unconditional and conditional distribution of consumption. In the next section, we quantitatively explore whether a limited commitment model extended with a role of advance information can indeed reproduce the consumption moments observed in the data and can be used to **quantify** households unobserved perceived income risk.

1.4 Quantitative Results

In this section, we provide quantitative results on the effect of advance information on risk sharing of households in the United States. We start by explaining the data employed in the quantitative exercise and the calibration. Then, we first illustrate the quantitative implications of advance information for constrained-efficient consumption allocation in the endowment economy as described in Section 1.2. To distinguish households' perceived income uncertainty from the income uncertainty as measured by an econometrician and to compute our main quantitative results, we employ a production economy with capital and endogenous solvency constraints. As a robustness exercise, we also study the role of advance information in a standard incomplete markets model – both in an endowment and a production economy with capital.

1.4.1 Data and calibration

To facilitate comparison with related studies in particular to Krueger and Perri (2006) and Broer (2013), we employ the Consumer Expenditure Interview Survey (CEX), and follow these authors in their methodology. In particular, we decompose consumption and income inequality in between and within group inequality. Between-group inequality are differences in household income and consumption attributable to observable characteristics for example education, region of residence, etc., and assume that households cannot insure against these observable characteristics. Income inequality devoid of between group inequality component is called within group inequality. This residual measure of inequality is the focus of this paper as it is caused by the idiosyncratic income shocks and hence, depending on the insurance available against these shocks, consumption inequality will not exactly mirror income inequality.

As measure of household consumption, we employ non-durable consumption (ND+) which also includes an estimate for service flows from housing and cars. For households' disposable income, we use after-tax labor earnings plus transfers (LEA+). Consistent with voluntary participation, we thus take the mandatory public insurance as given and focus on private insurance. LEA+ comprises the sum of wages and salaries of all household members, plus a fixed fraction of self-employment farm and non-farm income, minus reported federal, state, and local taxes (net of refunds) and social security contributions plus government transfers.

We drop the households who report zero or only food consumption, whose head is older than 64 years or younger than 21 years, with negative or zero earnings or have negative working hours, which have positive labour income but no working hours, which live in the rural area or their weekly wage is below the minimum wage and which are not present in all interviews. To facilitate a comparison between households of different size, the consumption and income measures are divided by adult equivalence scales as in Dalaker and Naifeh (1997).

To compute within group inequality, we follow Krueger and Perri (2006) and Blundell et al. (2008), and regress the logs of household consumption and income on the set of observed characteristics dummies. The dummies include region, marital status, race, education, experience, occupation, sex and age. The residuals of the regression are treated as consumption and income shock.

Model parameters Our annual calibration is designed to highlight the differences between a standard limited commitment model without information as entertained in Broer (2013) and a model with information. Therefore, we set a number of corresponding parameters to the same values. In particular, we consider a period utility function that exhibits constant relative risk aversion with parameter $\sigma = 1$. The discount factor β is chosen to yield an annual gross interest rate of $R = 1.025$ in general equilibrium. In the production economy with capital in Section 1.4.3, we further use a Cobb-Douglas production function $AF(K, L)$ with a capital-production elasticity of 0.30. Given R, we choose the depreciation of the capital stock δ and the technology parameter A to yield a real wage rate of unity and an aggregate wealth-to-income ratio of 2.5 as for example estimated by Kaplan and Violante (2010) based on the Survey of Consumer Finances (SCF).

Following the practice in the literature, the income specification comprises persistent and transitory income components. Log income of household i is modeled as

$$\ln(y_{it}) = z_{it} + \epsilon_{it}, \qquad z_{it} = \rho z_{it-1} + \eta_{it},$$

where ϵ_{it} and η_{it} are independent, serially uncorrelated and normally distributed with variances σ_ϵ^2 and σ_η^2, respectively. The persistence parameter ρ is set to 0.9989 which is the value originally found by Storesletten et al. (2004). Given the persistence parameter, we identify the variances $\sigma_\epsilon^2, \sigma_\eta^2$ from the cross-sectional within-group income variance and auto-covariance in the CEX data as the averages of the years 1999–2003. The method proposed by Tauchen and Hussey (1991) is used to approximate the persistent part of income by a Markov process with three states and time-invariant transition probabilities, and the transitory part is modeled with two exogenous states of equal probability. We normalize the value of all income states such that mean income (or aggregate labor endowment) is equal to 1. For each of the 6 income states, there are therefore 6 public signals such that the joint income-signals state S is approximated by 36 states which is higher than the 14 states typically considered in related studies (Broer, 2013, Krueger and Perri, 2006). The increase in the number of states leads to a numerical challenge for computing consumption allocations in general equilibrium.[6]

Insurance measures To measure the extent of consumption smoothing from the data, we focus on two measures: (1) the covariance of consumption and income growth, and (2) relative variance of log-consumption with respect to log-income. The first measure captures the sensitivity of consumption growth to income growth. Following Mace (1991), the sensitivity is captured by the coefficient $\beta_{\Delta y}$ in the following regression equation

$$\Delta c_{it} = \psi + \beta_{\Delta y}\Delta y_{it} + \nu_t + \nu_{it} \tag{1.19}$$

where ψ is a constant, ν_t a vector of time dummies and ν_{it} a residual; Δc_{it} and Δy_{it} are the growth rates of consumption and income of individual i in period t. When the coefficient $\beta_{\Delta y}$ is zero, then consumption growth is perfectly insured against changes in income growth. The higher is the coefficient, the less insurance is achieved.

The second measure is defined as one minus the ratio of the cross-sectional unconditional variance of logged consumption over logged income:

$$RS = 1 - \frac{\mathrm{var}_c}{\mathrm{var}_y} \tag{1.20}$$

On one extreme, if $\mathrm{var}_c = \mathrm{var}_y$, then $RS = 0$, and there is no private insurance against fluctuations in disposable. On the other hand, if $\mathrm{var}_c = 0$ then $RS = 1$ implying full insurance against income shocks. In Table 1.1, we summarize the calibrated parameters in the upper part and unconditional moments of consumption and income from the CEX data in the lower part. The value of $\beta_{\Delta y}$ is

[6] In Appendix A.6, we describe our algorithm for computing allocations in the *LC* model in more detail. With 500 points on the promises grid, we solve in each iteration step for 666,000 variables. In a standard model without information and 14 income states as in Broer (2013), the corresponding number of variables is 105,000.

Table 1.1: Baseline parameters and CEX moments

	Parameter	Value
σ	Risk aversion	1
α	Elasticity of capital in production function	0.3000
R	Gross interest rate	1.0250
ρ	Auto-correlation	0.9989
var_z	Variance persistent	0.2505
var_ϵ	Variance transitory	0.1149
S	Income-signal states	36
var_y	Variance log income	0.3654
var_c	Variance log consumption	0.1462
$\beta_{\Delta y}$	Regression coefficient	0.1078

equal to 11 percent with a standard error of 0.0035; the insurance ratio is $1 - \frac{\mathrm{var}_c}{\mathrm{var}_y} = 0.60$ which implies 40 percent of income shocks transfer to consumption.

Information and reduction in perceived income uncertainty To interpret the effects of an increase in information precision κ, we compute the percentage reduction of households' perceived income uncertainty $\tilde{\kappa}$ as measured by the reduction in the mean-squared forecast error resulting from conditioning expectations on signals

$$\tilde{\kappa} = \frac{\mathrm{MSFE}_y - \mathrm{MSFE}_s}{\mathrm{MSFE}_y}, \tag{1.21}$$

with

$$\mathrm{MSFE}_y = \sum_y \pi(y) \sum_{y'} \pi(y'|y) \left[y' - \mathrm{E}(y'|y)\right]^2$$

$$\mathrm{MSFE}_s = \sum_s \pi(s) \sum_{y'} \pi(y'|s) \left[y' - \mathrm{E}(y'|s)\right]^2,$$

and $\pi(s)$ as the joint invariant distribution of income and signals. Thus, $\tilde{\kappa}$ captures the difference in income uncertainty as measured by an econometrician in the aggregate and the income uncertainty as perceived by households. For this reason, we refer to $\tilde{\kappa}$ as the uncertainty gap. If signals are uninformative, $\tilde{\kappa}$ is equal to zero and if signals are perfectly informative, $\tilde{\kappa}$ equals one. For a given increase in κ, the reduction in perceived income risk is the smaller the more persistent income is.

Outside option For the quantitative results, we allow for self insurance in the outside option. In case of defaulting to the outside option and consistent with U.S. bankruptcy law, households loose all their consumption claims. Further, access to financial markets is restricted. While agents can save unlimited amounts in a non-state contingent bond with gross return $R^{Aut} > 0$, they cannot borrow. Thus, the value of the outside option is a solution to an optimal savings problem that can be written in recursive form as follows

$$v(s,a) = \max_{0 \leq a' \leq y + aR^{Aut}} \left[(1-\beta)u(aR^{Aut} + y - a') + \beta \sum_{s'} \pi(s'|s) v'(s',a') \right].$$

We set $R^{Aut} = R$, implying that in the outside option households can save at the intertemporal price $1/R$ also used in the recursive problem (1.7)-(1.9). Thus, the value of the outside option will be conditional on R and is given by

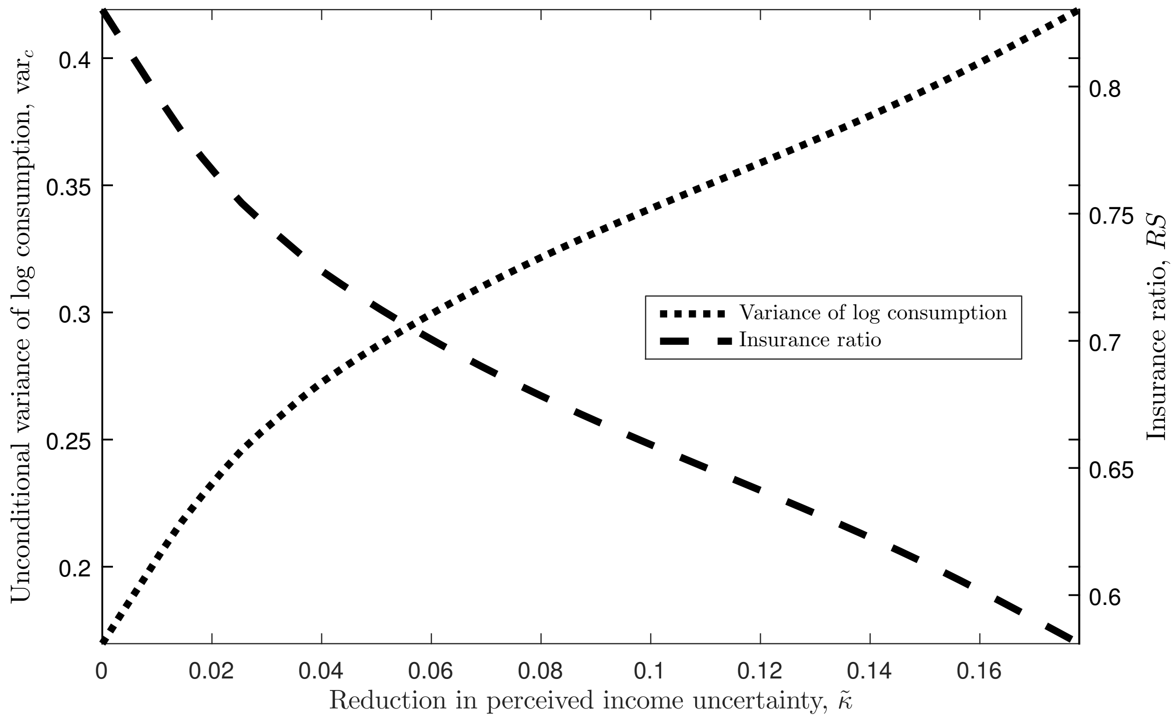
$$U_R^{Aut}(s) = v(s,0).$$

1.4.2 Advance information and constrained-efficient allocations

In the simplified environment of the analytical section, we showed that the unconditional variance of consumption is increasing in information precision. Thus, insurance of income shocks is decreasing in information precision because the consumption allocation becomes riskier *ex-ante*. We find that this negative effect of information on insurance also applies in the fully-fledged environment with efficient history-dependent insurance contracts and a persistent income process. Further, the negative effect of information on risk sharing is quantitatively important for the unconditional and income-conditional moments of consumption.

In Figure 1.2, we plot on the left axis the unconditional variance of logged consumption and on the right axis the resulting insurance ratio in the stationary equilibrium. As shown in the third part

Figure 1.2: The unconditional variance of logged consumption and insurance ratio in the limited commitment endowment economy.

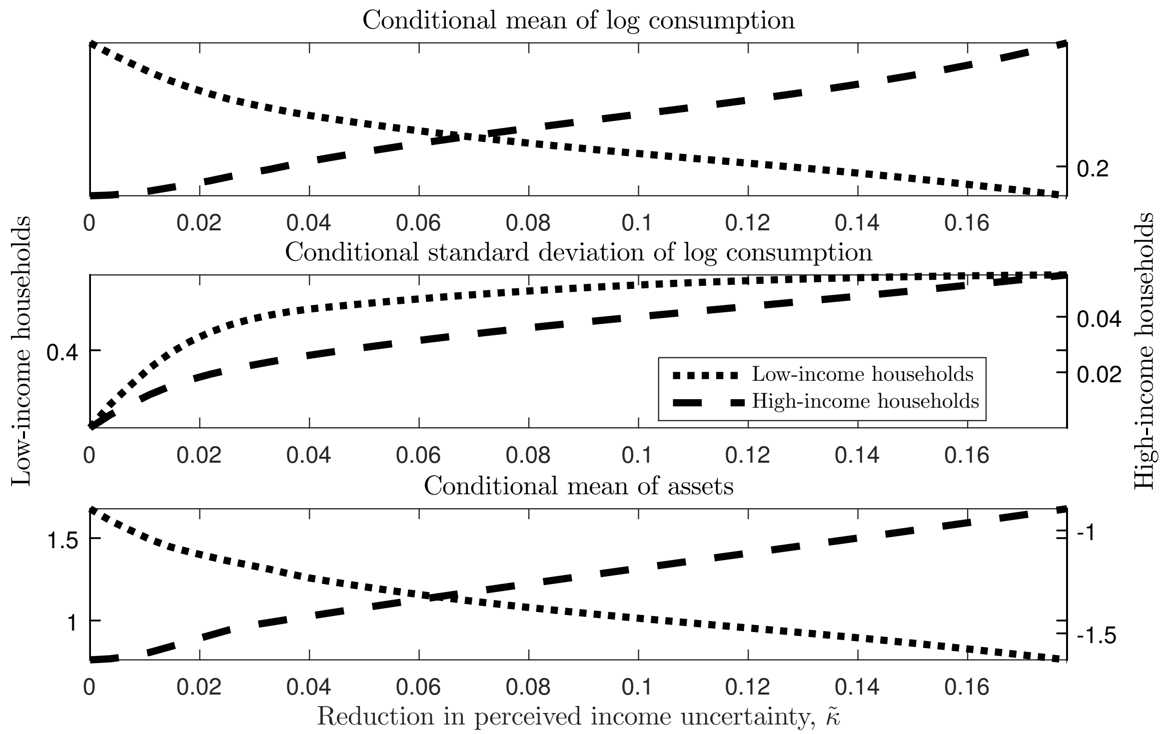


Notes: The present figure plots the unconditional variance of logged consumption and insurance ratio in the limited commitment endowment economy. The $x-$axis picks the reduction in perceived income uncertainty, $\tilde{\kappa}$; the left $y-$axis represents the unconditional variance of consumption; the right $y-$axis the insurance ratio.

of Proposition 4, the unconditional variance of consumption is increasing in the precision of information, or equivalently, is increasing in the difference between the income uncertainty as measured by an econometrician and the income uncertainty as perceived by households. Correspondingly, the insurance ratio on the right axis decreases when income risk is reduced. In the standard model without advance information ($\tilde{\kappa} = 0$), the insurance ratio is equal to 83 percent which is 23 percent higher than the 60 percent observed in the CEX data. This finding is typical for the limited commitment model that is known for predicting too much consumption insurance for standard calibrations. The insurance ratio is sensitive with respect to advance information. Reducing the income risk by 18 percent, results in an insurance ratio of 0.58.

Figure 1.3 illustrates the effect of advance information on the conditional mean and standard deviation of consumption for low-and high-income households. The results confirm to a large

Figure 1.3: The effect of advance information on conditional moments of consumption in the limited commitment endowment economy.



Notes: The present figure plots the conditional mean and standard deviation of the logged consumption with respect to increase in advance information. The $x-$axis picks the reduction in perceived income uncertainty, $\tilde{\kappa}$; the left $y-$axis represents low-income households, the right $y-$axis high-income households. The upper panel is the conditional mean of log consumption, the middle panel is the conditional standard deviation of log consumption and the lower panel pictures the conditional mean of assets holdings.

extent the analytical results of the first and second part of Proposition 4 derived in the simplified environment of the previous section. However, there is one important exception. With memoryless allocations, the conditional standard deviation of low-income households is equal to zero, in efficient allocations the standard deviation is positive.

While the conditional mean consumption of low-income households decreases, consumption of high-income households increases when income risk is resolved by advance information. When we decentralize efficient allocations as described in Section 1.3.1, the changes in consumption are reflected in corresponding changes in assets holdings across income groups; mean asset holdings of low-income households decrease while high-income households hold more assets on average when income risk is resolved (see the lower panel of Figure 1.3). Further, the conditional standard deviation of consumption increases for low-and high-income households. Quantitatively, the standard deviation of low-income households is affected stronger than the dispersion of consumption among high-income households: it increases by 25 log points while for high-income agents, the increases equals 6 log points when households' income risk reduces by 18 percent.

To facilitate comparison with existing studies without advance information, we now continue with a production economy with capital and limited commitment.

1.4.3 A stationary production economy with endogenous solvency constraints

The production economy with endogenous solvency constraints is in detail explained in Krueger and Perri (2006). In the following, we refer to this economy as the *limited commitment production economy*. Here, we directly focus on stationary allocations. Households trade a complete set of one-period zero coupon assets $a(s')$ priced at $q(s,s')$ with financial intermediaries that live for one period and invest into capital. As in Alvarez and Jermann (2000), households face state-contingent endogenous credit limits $A(s')$ that are not "too tight". Given asset holdings a, state $s = (y,k)$, and prices $w, \{q(s,s')\}$, households' problem can be written recursively as

$$V(a,s) \quad = \quad \max_{c,\{a'(s')\}} \left[(1-\beta)u(c) + \beta \sum_{s'} \pi(s'|s) V(a'(s'),s') \right]$$

subject to a budget and solvency constraints

$$c + \sum_{s'} q(s,s')a'(s') \quad \leq \quad wy + a \tag{1.22}$$

$$a'(s') \quad \geq \quad A(s'), \quad \forall s', \tag{1.23}$$

where wy is labor income. The endogenous credit limits are pinned down by outside option values $U^{aut}(s')$

$$A(s') = \min_{a(s')} \left\{ a(s') : V'(a(s'), s') \geq U^{Aut}(s') \right\}, \quad \forall s'. \tag{1.24}$$

The result of the maximization problem are policy functions $c(a, s), \{a'(a, s; s')\}$.

Here, households differ with respect to initial asset asset holdings and initial shocks where the heterogeneity is captured by the invariant probability measure $\Phi_{a,s}$.

A representative firm hires labor L and capital K at rental rates w and r to maximize profits where the production of consumption goods Y takes place via a linear homogenous production function

$$Y = AF(L, K),$$

where A is a constant productivity parameter, L is the aggregate labor endowment (in efficiency units) and K the aggregate capital stock. Capital depreciates at rate δ.

The stationary recursive competitive equilibrium is summarized in the following definition.

Definition 7 *A stationary recursive competitive equilibrium in the limited commitment production economy comprises a value function $V(a, s)$, a price system $R, w, q(s, s')$, an allocation $K, c(a, s), \{a'(a, s; s')\}$, a joint probability measure of assets and state $\Phi_{a,s}$, and endogenous credit limits $A(s')$ such that*

(i) $V(a, s)$ is attained by the decision rules $c(a, s)$, $\{a'(a, s; s')\}$ given $R, w, q(s, s')$

(ii) Endogenous credit limits are determined by outside option values according to (1.24)

(iii) The joint distribution of assets and state $\Phi_{a,s}$ induced by $\{a'(a, s; s)\}$ and P_s is stationary

(iv) No arbitrage applies

$$q(s, s') = \frac{\pi(s'|s)}{R}$$

(v) Factor prices satisfy

$$R - 1 = AF_K(1, K) - \delta$$
$$w = AF_L(1, K)$$

(vi) The asset market clears

$$R'K' = \int \sum_{s'} a'(a, s; s')\pi(s'|s) \, \mathrm{d}\, \Phi_{a,s}.$$

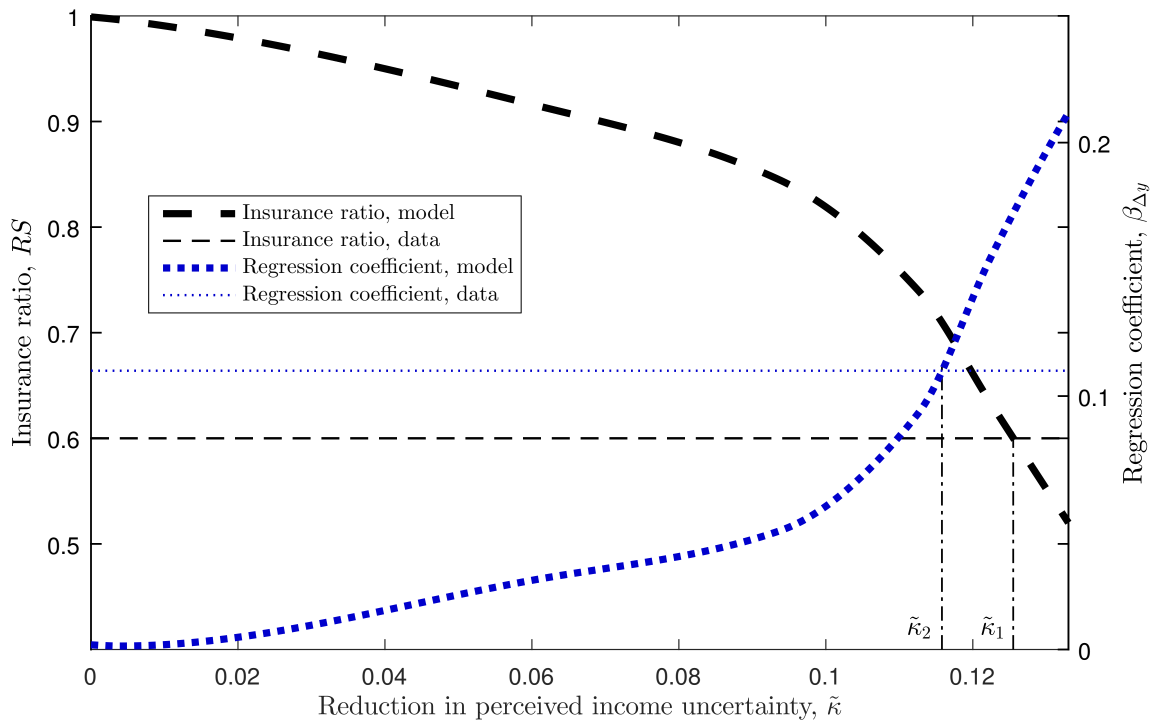
Quantifying advance information To discipline the free parameter κ, we choose the parameter such that the risk sharing predicted by the model matches two distinct insurance measures observed in the data. The insurance ratio as the first measure characterizes the cross-sectional dispersion of consumption. As the second measure, we employ the regression coefficient of current consumption growth with respect to income growth as a measure to determine the sensitivity of consumption with respect to changes in income. In general, we therefore expect to pin down two values for the reduction in households' perceived income uncertainty $\tilde{\kappa}_1, \tilde{\kappa}_2$ that yield insurance measures in the model that are consistent with the measure observed in the CEX.

For the first insurance measure, we use the cross-sectional variance of consumption in the invariant distribution. For the second insurance measure, we employ stationarity and simulate the model for 300,000 time periods and discard the first 100,000 periods to ensure convergence. Then we estimate correlations of consumption and income growth using the simulated data.

As displayed in Figure 1.4, households' advance information has a strong quantitative effect on consumption insurance that allows us to capture both insurance measures with advance information. Without information, risk sharing is nearly perfect such that the insurance coefficients equals one (left axis) and the regression coefficients β_{Δ_y} is close to zero (right axis).[7] The insurance ratio is matched for a reduction in income uncertainty of $\tilde{\kappa}_1 = 0.1236$, the regression coefficient

[7] Krueger and Perri (2006) provide similar results for the standard model without information. With capital, risk sharing improves compared to an endowment economy because in autarky households loose their claims on the capital stock which decreases the value of their outside option. To receive partial insurance with capital, Broer (2013) intro-

Figure 1.4: Effect of advance information on consumption insurance in the limited commitment production economy.



Notes: The figure reports the effect of advance information on consumption insurance in the limited commitment economy. The $x-$axis picks the reduction in perceived income uncertainty $\tilde{\kappa}$; the left $y-$axis represents the insurance ratio; the right $y-$axis the regression coefficient of current consumption growth on current income growth; the black dashed line is the insurance ratio of 0.60, the black doted line is the regression coefficient of 0.11 from the CEX, 1998–2003.

is matched for $\tilde{\kappa}_2 = 0.1158$. Thus, both insurance measures are jointly matched for a reduction in income risk of 12 percent. This is a remarkable result because in general the two insurance measures have to coincide only in the extreme cases when risk sharing is either perfect or absent.

Blundell et al. (2008) argue that including advance information in the SIM model may lead to correlations of current consumption with future income growth that are not consistent with the data. To test for a potential role of advance knowledge of future income shocks, Blundell et al. (2008) used household panel data from the Panel Study of Income Dynamics (PSID) to estimate correlations of current consumption growth $\Delta c_{i,t} = \log(c^i_t) - \log(c^i_{t-1})$ with future income growth $\Delta y_{i,t+j} = \log(y^i_{t+j}) - \log(y^i_{t+j-1})$ for $j \geq 1$. Through the lens of a standard incomplete markets

duces the possibility that households can with a probability of 12 percent return to insurance after defaulting. Using this return probability, we compute $RS = 0.89$ and $\beta_{\Delta y} = 0.06$.

model, if there was advance knowledge of income shocks, the correlation in the data should be significantly different from zero because consumption should adjust before the shock has occurred. However, Blundell et al. (2008) estimate correlations that are not significantly different from zero with p-values larger than 0.25.

The limited commitment model with information is consistent with that evidence. As reported in the first column of Table 1.2, the correlation of current consumption growth with future income growth is not significantly different from zero for the standard model with $\tilde{\kappa} = 0$. This pattern does not change for informative signals. As displayed in the second and third column, for $\tilde{\kappa}_1 = 0.1236$ (yields the insurance ratio from the data) and for $\tilde{\kappa}_2 = 0.1158$ (yields the regression coefficient from the data), only the correlation of current income growth and current consumption growth is significantly different from zero. Consistent with Blundell et al. (2008), the correlations of current consumption growth with future income growth are not significantly different from zero with p-values larger than 89 percent. Unlike in a standard incomplete markets model, advance information in the *LC* model does not induce counterfactual correlations of current consumption growth with future income growth.[8]

The logic for this result can be intuitively rationalized in the limited commitment endowment economy. In the limited commitment model, the size of the income uncertainty is not directly relevant because in principle there is a complete set of securities available for insurance. Consumption insurance is imperfect because the enforcement of insurance contracts is limited by the outside option to live in autarky. In the optimal insurance contract with partial insurance, the planner encourages high-income agents with binding enforcement constraints to transfer resources today in exchange for insurance of income shocks in the future. Insurance involves both promising to decouple future income and future consumption (insurance across states), and to smooth consumption across periods. This logic is strengthened further by more precise signals. When signals become more precise, the outside option becomes more attractive for agents with a high income. To accommodate this change and to encourage these agents to transfer resources today, the planner

[8] The CEX is a revolving panel in which households drop out after one year. For this reason, we cannot estimate correlations of current consumption with future income growth.

Table 1.2: Income and consumption growth regression: limited commitment production economy

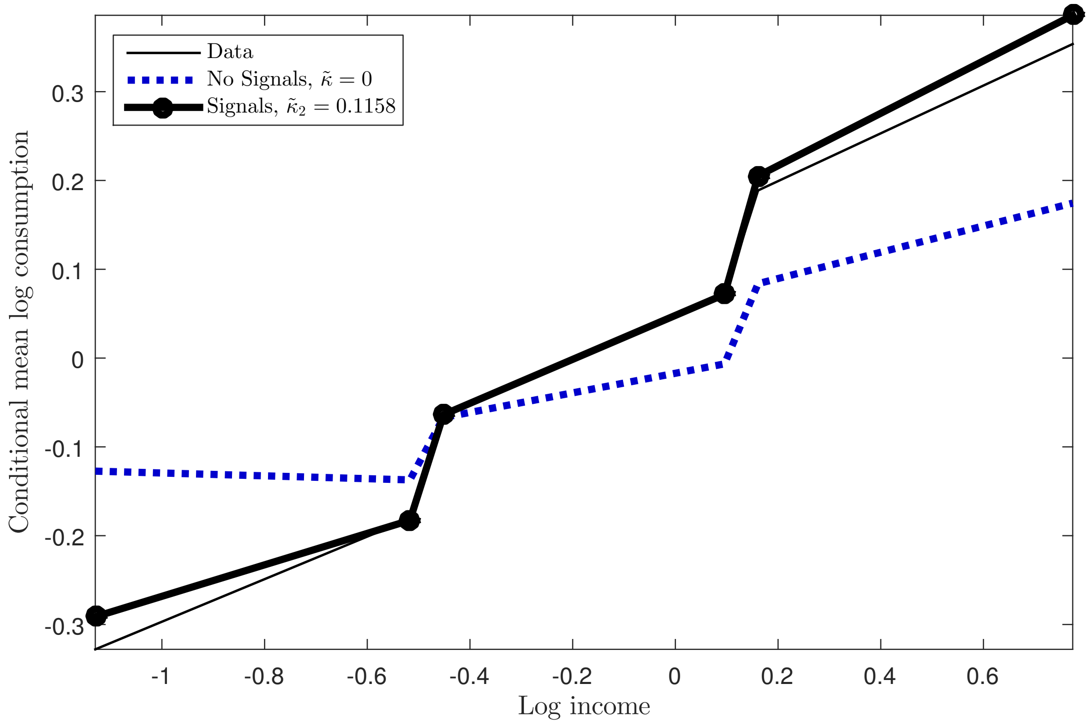
	No signals, $\tilde{\kappa} = 0.00$	$\tilde{\kappa}_2 = 0.1158$	$\tilde{\kappa}_1 = 0.1236$	**CEX Data**
$\beta_{\Delta y_t}$	0.00	0.11	0.16	**0.11**
β_2	−0.00	0.01	0.01	–
Test $\text{cov}(\Delta c_t, \Delta y_t), p$-values	0.00	0.05	0.05	**0.00**
Test $\text{cov}(\Delta c_t, \Delta y_{t+1}), p$-values	0.00	0.89	0.92	–

Notes: In the table, we provide regression coefficients and their p values for the regression $\Delta c_t^i = \beta_0 + \beta' \Delta y^i + \epsilon_t^i$, with $\beta = [\beta_{\Delta y_t}, \beta_2]'$ and $\Delta y^i = [\Delta y_t^i, \Delta y_{t+1}^i]'$ for different precisions of signals.

promises even more consumption smoothing across time and states. Nevertheless, according to Proposition 4, transfers from high-to-low income agents are reduced such that the dependency of current consumption and current income strengthens while more consumption smoothing across periods prevents a higher correlation of current consumption with future income growth.

Aguiar and Bils (2015) and Attanasio et al. (2012) argued that the consumption expenditures reported in the CEX Interview Survey are exposed to measurement error that result in biased estimates of cross-sectional consumption inequality measures. In particular, Aguiar and Bils (2015) claimed that the insurance ratio might be smaller than the 60 percent as directly measured using the CEX data that we employ as one of the insurance measures. Nevertheless, the uncertainty gap we identify is not very sensitive with respect to a potentially downward-biased estimate for consumption inequality for two reasons. First, using the regression coefficient as an alternative insurance measure yields very similar numbers for advanced information. This measure is less prone to measurement error because the regression coefficient employs growth rates as a ratio and therefore corrects for time-invariant multiplicative measurement error. Additionally, we find that the regression coefficient in the CEX of 0.11 is very close to the corresponding coefficient that we estimate using PSID data of 0.12. Second, even if the correct insurance ratio was higher than the number computed directly from the CEX, the identified uncertainty gap would only be mildly affected. As illustrated in Figure 1.4, consumption insurance reacts sensitively to advance information. For example, if the insurance ratio was with 50 percent lower than the 60 percent as directly observed in the CEX, the identified reduction in perceived income uncertainty changes only mildly

Figure 1.5: Conditional mean of logged consumption: data and the limited commitment production economy for different precisions of signals.



Notes: The figure reports the conditional mean of logged consumption from the data and the limited commitment production economy for different precisions of the signals. The $x-$axis represents the log income and $y-$axis represents the conditional mean of log consumption. Income steps represent percentiles: $[17th, 33th, 50th, 67th, 83th]$. Solid line captures the conditional means for the years 1999–2003 in the CEX.

from 12 to 13 percent. In that sense, the uncertainty gap of 12 percent can also be interpreted as a lower bound.

Kaplan and Violante (2010) conclude that advance information cannot reconcile insurance ratios or regression coefficients in a life-cycle standard incomplete markets model and in the data. In Section 1.4.5, we also allow for signals in a standard infinite horizon incomplete markets (SIM) model and confirm the earlier findings of Kaplan and Violante (2010).

We find that the picture changes when we alternatively employ the limited commitment model with explicit insurance contracts. Here, advance information on future income shocks can bridge the gap to the data because risk sharing in the limited commitment model is more sensitive to information than in the SIM model. Further, advance information does not induce counterfactual correlations of current consumption growth with future income growth. Correspondingly, we can quantify households' advance information by matching the insurance ratio or, alternatively, by capturing the regression coefficient of consumption on income growth. Our main quantitative finding is that both insurance measure can be jointly explained when households' perceived income uncertainty is reduced by 12 percent. In the following, we fix information precision at this value, and analyze the implications for the joint distribution of income and consumption as "over-identifying restrictions".

1.4.4 Information and conditional moments of consumption

To compare conditional moments from the data and models, our procedure is the following. We start with the stationary distribution of income implied by the Tauchen and Hussey (1991)'s procedure and compute the conditional mean and variance corresponding to this stationary distribution in each model. For the data, we employ the percentiles from the stationary income distribution and compute the moments for the percentiles, accordingly. For our calibration, this corresponds to the following percentiles: $[17th, 33th, 50th, 67th, 83th]$. For example, households with a high income represent the top 17 percent of income earners.

Throughout this section, we use the conditional consumption moments as over-identifying restrictions and compare the standard model without signals to the case of informative signals. For informative signals, we consider precisions $\tilde{\kappa}_1 = 0.1236$ and $\tilde{\kappa}_2 = 0.1158$ that capture the insurance ratio and the regression coefficient β_{Δ_y}. Insurance is however perfect in the standard limited commitment production model without signals. To facilitate a fair comparison, we employ the results derived in the endowment economy for the standard model.[9]

In Figure 1.5, we plot the conditional mean of log consumption for the data, standard model and for informative signals of precision $\tilde{\kappa}_2 = 0.1158$. In the absence of signals, the average consumption of low-income households is too high compared to the data while the consumption of

[9] Alternatively employing the standard model with the possibility to return from autarky to insurance as in Broer (2013) yields similar conditional consumption moments.

Table 1.3: Conditional moments of consumption: model versus data

	No signals, $\tilde{\kappa} = 0$	$\tilde{\kappa}_2 = 0.1158$	$\tilde{\kappa}_1 = 0.1236$	**CEX Data**
MSE, $\mathrm{E}\left[\log(c)\|y\right]$, normalized	34.15	1	4.67	–
$\mathrm{E}\left[\log(c)\|y_h\right] - \mathrm{E}\left[\log(c)\|y_l\right]$	0.30	0.68	0.80	**0.68**
MSE, $\mathrm{STD}\left[\log(c)\|y\right]$, normalized	3.48	1	0.91	–
$\frac{\mathrm{STD}[\log(c)\|y_h]}{\mathrm{STD}[\log(c)\|y_l]}$	0	0.38	0.36	**0.95**

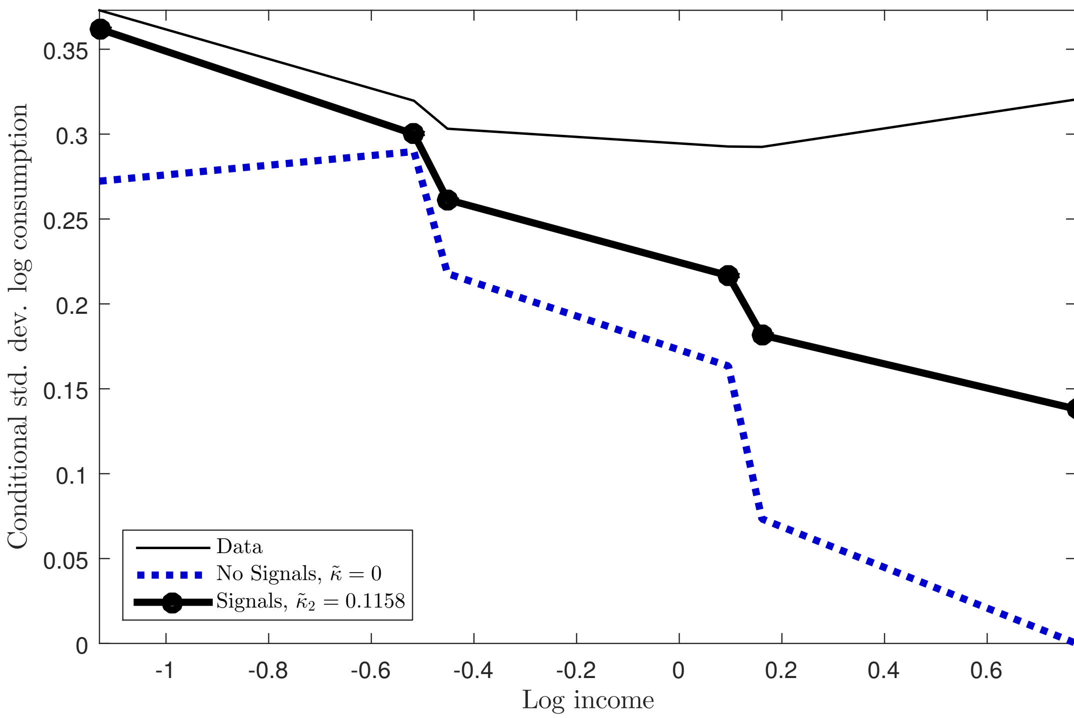
Notes: The table provides the mean squared deviations of model and data for the conditional means and standard deviations of consumption expressed relative to signals with $\tilde{\kappa}_2 = 0.1158$; the table also provides spreads between average consumption and the standard deviation of low-and high income households.

high-income agents is too low. Further, indicating also too much insurance for low-income states, average consumption is constant for the two low-income groups in the absence of information; in the CEX data, average consumption is increasing for all income states. With informative signals, household consumption becomes more dispersed. Consistent with the first part of Proposition 4, we find that average consumption of low-income households decreases while consumption of high-income households increases, leading to a more dispersed consumption distribution and a better fit to the data. Further and as in the data, the conditional mean of consumption is increasing in income over all incomes states. Overall, the conditional mean of consumption is tracked in an almost perfect way for informative signals over all six income groups.

As displayed in Figure 1.6, advance information results in a higher conditional standard deviation for all income groups. In particular, information leads to an increase in consumption dispersion conditional on a high income; in the absence of information, the standard deviation is zero while with informative signals it is positive and increasing in precision. While the conditional standard deviation is tracked reasonably well for low-and middle-income earners, the distance to the data increases of higher income groups.

Quantitatively, the fit of the conditional consumption distribution to the data is substantially improved by advance information. As displayed in Table 1.3, the mean-squared deviations of the conditional mean of consumption between model and data are approximately 34 times as large in the standard model than for $\tilde{\kappa}_2 = 0.1158$; for $\tilde{\kappa}_1 = 0.1236$, the mean deviations are 4.5 times higher than for $\tilde{\kappa}_2 = 0.1158$ but still over 7 times lower than in the standard model. Further, the

Figure 1.6: Conditional standard deviation of logged consumption: data and the limited commitment production economy for different precisions of the signals.



Notes: The figure reports the conditional standard deviation of logged consumption from the data and the limited commitment production economy for different precisions of the signals. The $x-$axis represents the log income and the $y-$axis the conditional standard deviation of logged consumption. Income steps represent percentiles: $[17th, 33th, 50th, 67th, 83th]$. Solid line captures the conditional standard deviations for the years 1999–2003 in the CEX.

spread between average consumption of high-and low income households in the CEX data of 0.68 is perfectly captured by signals with $\tilde{\kappa}_2 = 0.1158$.

There is also some improvement in fit for the conditional standard deviation of consumption but the improvement is not as striking as for the conditional mean. Relative to the standard model, the mean-square error is 3.5 times smaller for $\tilde{\kappa}_2 = 0.1158$, and approximately 4 times smaller for $\tilde{\kappa}_1 = 0.1236$. Further, the ratio of the conditional standard deviations for high-and low-income households increases from 0 in the standard model to 0.4 with advance information. This increase is however too small to capture the ratio of almost 1 observed in the CEX.

1.4.5 Advance information in a standard incomplete markets model

As a robustness exercise, we integrate public signals in a standard incomplete markets (SIM) model. Confirming earlier results by Kaplan and Violante (2010), we find little support for a prominent role of advance information in the SIM-model.

Environment While preferences and endowments are as described in Section 1.2, households in the standard incomplete markets economy can only trade in a single non-state contingent bond with gross return R and face an exogenous borrowing limit $\bar{a}$. There are no enforcement frictions and we directly focus on stationary allocations. The model we consider is similar to Huggett (1993) and relies on a market structure with a continuum of households as in Aiyagari (1994). Given asset holdings a, state $s = (y, k)$, and an interest rate R, households' problem can be written recursively as

$$V(a,s) \quad = \quad \max_{c,a'} \left[(1-\beta)u(c) + \beta \sum_{s'} \pi(s'|s) V(a', s') \right]$$

subject to a budget and a borrowing constraint

$$\begin{aligned} c + a' &\leq y + Ra \\ a' &\geq -\bar{a}. \end{aligned}$$

Here, households differ with respect to initial asset asset holdings and initial shocks where the heterogeneity is captured by the probability measure $\Psi_{a,s}$. The state space is given by $M = A \times S$, where $A = [-\bar{a}, \infty)$.

The stationary recursive competitive equilibrium is summarized in the following definition.

Definition 8 *A stationary recursive competitive equilibrium in the standard incomplete markets economy comprises a value function $V(a,s)$, an inter-temporal price R, an allocation*

$c(a,s), a'(a,s)$ a joint probability measure of assets and the state $\Psi_{a,s}$, and an exogenous borrowing limit $\bar{a}$ such that

(i) $V(a,s)$ is attained by the decision rules $c(a,s)$, $a'(a,s)$ given R

(ii) The joint distribution of assets and state $\Psi_{a,s}$ induced by $a'(a,s)$ and P_s is stationary.

(iii) The bond market clears

$$\int a'(a,s)\,\mathrm{d}\,\Psi_{a,s} = 0.$$

Quantitative results As emphasized by Blundell et al. (2008) and Kaplan and Violante (2010), in a SIM model better information on future income realizations allows households to improve on their consumption-savings decisions, and risk sharing improves. Thus, better information here has a positive effect by improving individual decision which is referred to as a Blackwell (1953) effect of information. For generating the quantitative results, we employ for for the common parameters the same parameter values as in the corresponding limited commitment economy. Wolff (2011a) finds that 19 percent of all U.S. households are borrowing constrained. For this reason, we choose an exogenous borrowing limit $\bar{a}$ to yield in equilibrium 19 percent borrowing-constrained households in the standard model without information.

In line with earlier findings by Kaplan and Violante (2010), we find that insurance ratios improve monotonically in information precision but the improvement is too small to capture the insurance ratio of 60 percent observed in the data even for very informative signals. In the absence of signals, the model implies that households insure about 25 percent of all fluctuations in their after-tax income. As an extreme case, if information precision amounts to $\kappa = 0.99$ – corresponding to a reduction of income uncertainty $\tilde{\kappa}$ of 97 percent, the insurance ratio reaches 0.33. Thus, the increase in insurance by better information is quantitatively too small to capture the insurance observed in CEX data.

Further as displayed in Table 1.4, as signals become informative the SIM model predicts that current consumption growth is counter-factually correlated with future income growth. For uninformative signals and informative signals with precisions up to $\tilde{\kappa} = 0.75$, current consumption

Table 1.4: Income and consumption growth regression: SIM model

	No signals, $\tilde{\kappa} = 0.00$	$\tilde{\kappa} = 0.16$	$\tilde{\kappa} = 0.76$	**CEX Data**
$\beta_{\Delta y_t}$	0.28	0.25	0.17	**0.11**
β_2	-0.02	0.00	0.12	–
Test $\text{cov}(\Delta c_t, \Delta y_t), p$-values	0.00	0.00	0.02	**0.00**
Test $\text{cov}(\Delta c_t, \Delta y_{t+1}), p$-values	0.77	0.99	0.10	–

Notes: In the table, we provide regression coefficients and their p values for the regression $\Delta c_t^i = \beta_0 + \beta' \Delta y^i + \epsilon_t^i$, with $\beta = [\beta_{\Delta y_t}, \beta_2]'$ and $\Delta y^i = [\Delta y_t^i, \Delta y_{t+1}^i]'$.

growth is uncorrelated with income growth one period ahead on a 10-percent significance level (see the first three columns). However, the regression coefficient of current consumption with current income growth of 0.17 is still too high compared to the 0.11 estimated in the CEX data. From $\tilde{\kappa} = 0.76$ onwards, the correlation of current consumption growth with income growth one period in the future is statistically significantly different from zero and with a coefficient of $\beta_2 = 0.12$ also economically significant (see the forth column). The non-zero correlation is inconsistent with the evidence provided in Blundell et al. (2008) who find correlations of current consumption growth with future income growth not significantly different from zero. Further even for $\tilde{\kappa} = 0.97$, the regression coefficients is with 0.15 too high compared to the data.

The logic behind the non-zero correlation of current consumption with future income growth in the SIM model can be rationalized as follows. In the SIM model, better information reduces directly the income fluctuations households want to insure. Knowing future income allows for better insurance of income risk given the limited option to use a non-state contingent bond. Thus, before the shock realizes households' consumption today reacts to the part of the future income shock that is known, and consumption today is correlated with future income when signals become precise enough.

For $\tilde{\kappa} = 0.75$ as the highest value for $\tilde{\kappa}$ that yields no counterfactual correlation of current consumption with future income growth, the insurance ratio of 32 percent however falls short compared to the 60 percent as observed in the CEX data.

A production economy with capital Alternatively, we consider capital as a physical asset instead of the financial asset a in the endowment economy. Instead of (iii) in Definition 8, asset market clearing now requires

$$\int a'(a,s)\,\mathrm{d}\,\Psi_{a,s} = K'.$$

Further, the rental prices of the production factors equal their marginal products. For all values of $\tilde{\kappa}$, the capital-to-income ratio is kept constant at 2.5 and the wage is normalized to unity. As before, the exogenous borrowing limit is chosen to yield 19 percent borrowing-constrained households with uninformative signals.

We compute quantitative similar results on the effect of advance information as in the endowment economy. The insurance ratio RS increases from 37 percent without information to 44 percent with signals that reveal 97 percent of all future income uncertainty. Thus, the increase in insurance amounts to 7 percent (compared to 8 percent in the endowment economy). The regression coefficient $\beta_{\Delta y}$ decreases from 0.26 without advance information to 0.15 for signals with $\tilde{\kappa} = 0.97$. As in the endowment economy however, from $\tilde{\kappa} = 0.76$ onwards, current consumption growth is correlated with future income growth with an economically and statistically significant coefficient of 0.11.

1.5 Conclusions

In this paper, we have developed a framework to address the issue of a potential disconnect between households' income uncertainty and the income uncertainty as measured by an econometrician raised by Browning, Hansen, and Heckman (1999) and Cunha and Heckman (2016). To that end, we have developed a risk sharing model that can distinguish between the two types of uncertainties in a systematic and consistent way. To quantify the difference in the perception of uncertainty, we have employed a general equilibrium model with endogenous borrowing constraints. Using U.S. micro data, we have found that there is a systematic uncertainty gap: households' perceived income uncertainty is 12 percent lower than the uncertainty estimated by an econometrician that is typically used in consumption risk sharing models. For this uncertainty gap, the model jointly explains three distinct consumption insurance measures that are not captured in the absence of advance information: (i) the cross-sectional variance of consumption, (ii) the covariance of consumption with income growth, and (iii) the income-conditional mean of household consumption.

With their recent paper, Heathcote, Storesletten, and Violante (2016) contribute to a lively debate on the optimal progressivity of taxes in the United States. One of the main arguments in favor for a progressive tax system is that it helps to insure idiosyncratic earnings uncertainty when private insurance is limited. Thereby, a higher tax progressivity reduces the earnings risk after taxes. Computing the optimal tax progressivity requires a precise estimate for households' earnings uncertainty. In particular, if there is a systematic uncertainty gap as suggested in this paper and income uncertainty is actually lower than what is typically considered, less tax progressivity might be desirable than conventional wisdom suggests.

One of the limitations of this paper is that the precision of households' advance information is not a choice variable. One possible avenue for future research is to consider the precision of signals a choice variable; higher precision of signals could for example result in higher resource, utility, or time costs. Then the precision of signals can differ across households in the population reflecting the financial situation of a household and the importance of better information on future earnings for making accurate consumption-savings decisions.

Chapter 2

Public Insurance of Married versus Single Households in the US: Trends and Welfare Consequences

2.1 Introduction

In the US, income tax and transfer laws are conditional on the marital status of individuals. This conditionality is due to several reasons. It reflects the preference of the policy makers who want to maintain a balance in the tax treatment of families with different marital status, CBO (1997). Also, some policy makers use tax laws to promote the institution of marriage. For example, one out of four goals of the Temporary Aid to Needy Families (1996) program was to "encourage the formation and maintenance of two-parent families", Kominos (2006)[1] . Furthermore, these tax and transfer policies change over time due to the introduction of new laws. As an example, Table 2.1 reports the number of income tax bills, in which marital status plays an important role, referred and enacted in the US Senate in the last four (111^{th}-114^{th}) Congress sessions. It shows that in just four sessions only, Congress enacted 18 income tax bills which are conditional on marital status.

[1] Temporary Aid to Needy Families was a block grant system under the aegis of the Personal Responsibility and Work Reconciliation Act, 1996.

As tax and transfer policies provide public insurance against labor income risk, their conditionality on marital status and change over time can induce different levels and trends of public insurance received by married and single households. This paper asks two questions: first, is the evolution of public insurance against idiosyncratic labor income risk significantly different for married and single households in the US? If the answer is yes, then what are the quantitative implications for the insurance[2] and welfare of married and single households?

The contribution of this paper is twofold: empirical and quantitative. As an empirical contribution, using the March Current Population Survey dataset, I report how public insurance against labor income risk has changed over time for married and single households. Public insurance is defined as one minus the ratio of after-tax and transfer labor income risk over before-tax and transfer labor income risk. More reduction in after-tax and transfer labor income risk increases public insurance. I show that smoothed trends in public insurance are monotonic over time and married households benefited relative to single households. Over the sample period 1992-2015, married households experienced a 19 percent increase in public insurance whereas single households experienced a 13 percent decrease.

Given this empirical fact, I explore whether this *relative* change in public insurance has significant implications for risk sharing and welfare in the economy. To answer this question, as a quantitative contribution, I expand the standard incomplete markets (SIM) model á la Aiyagari (1994) to include two groups of households: married and single. The transition between married and single is allowed, and is determined by an exogenous probability. The market is assumed to be exogenously incomplete. Due to market incompleteness, households have a precautionary motive to save, and they do so by using a risk-free non-state-contingent bond. This precautionary saving determines private insurance. Another type of insurance in the model economy is public insurance provided by the government through progressive taxation of labor income. As public insurance affects after-tax and transfer income risk, it also effects the precautionary motive to save, and by

[2] Insurance in this paper refers to consumption insurance against idiosyncratic labor income risk. Furthermore, insurance and risk sharing are synonymous and will be used interchangeably.

Table 2.1: Number of income tax bills, conditional on marital status, referred and enacted by US Senate during the last four (111^{th}-114^{th}) Congress sessions.

Purpose of the bill	**Referred**	**Enacted**
Income tax credits	225	6
Income tax deduction	111	2
Tax treatment of families	39	2
Income tax rates	81	6
State and local taxation	18	2

Notes: The source of this data is https://www.govtrack.us/.

extension the amount of savings in the economy. The response of the precautionary motive to save to changes in public insurance will determine the welfare effects, as explained below.

I calibrate the model to the US data. For this I follow the method proposed by Castañeda et al. (2003) which allows me to match the wealth distribution in the US almost perfectly. After calibrating the model I implement the changes in public insurance as found in the data: an increase for married households and a decrease for singles. As there is a one-to-one mapping between public insurance and tax progressivity in the model, an increase (decrease) in public insurance implies an increase (decrease) in tax progressivity for married (single) households. The central result of the paper is that higher public insurance for married households leads to better total insurance but lower welfare for them, whether transition between steady states is taken into account or not. Hence, the model suggests that tax and transfer policies that are supposed to benefit married households can actually deteriorate their welfare. For single households, their total insurance and welfare decrease.

The key to understanding why the increase in total insurance for married households does not translate into higher welfare, lies in the heterogeneous saving response of married and single households. For a moment, consider the changes in total insurance only. When public insurance increases for married households, their precautionary motive to save decreases, leading to a reduction in their savings. This translates into lower private insurance for married households. In other words, public insurance crowds out private insurance. However, as shown by Krueger and Perri (2011) in the framework of the standard incomplete markets model, public insurance compensates more than a one for one decrease in private insurance. In this model also, I observe that total insurance increases for married households. The opposite happens with singles: in response to lower public insurance they save more and experience higher private insurance but lower total insurance.

The heterogeneous saving response also explains the welfare results. The measure of welfare adopted in this paper is the utilitarian welfare. It allows to evaluate the welfare effects of the relative change in public insurance by comparing two economies: one without any change in public insurance and another where married households experience a relative increase in public insurance. This welfare measure gives the percentage change in consumption in the economy without any changes in public insurance which would allow to achieve the same expected utility as observed in the economy where public insurance changes. I find that the change in welfare of married households is -0.4 percent if the transition is not taken into account, and -0.08 percent if the transition between steady states is taken into account. Following Koehne and Kuhn (2015), the welfare metric can be decomposed into (i) an inequality effect [0.2], (ii) an uncertainty effect [-0.16], (iii) a level effect [-0.43] and (iv) a transition effect [0.30]. These effects come into the picture because the decrease in savings and the relative increase in public insurance affects inequality, uncertainty and mean consumption for married households. As public insurance increases, the ex-ante inequality decreases due to better redistribution. Hence, the inequality effect for married households is positive. However, higher public insurance does not reduce consumption uncertainty as much as it would if private savings remained constant. As private savings of married households decrease, consumption uncertainty does not decrease significantly, which yields the negative uncertainty effect. Furthermore, due to the decrease in savings, long-term mean consumption of married households decreases, and this is captured by the negative level effect. The transition effect is strongly positive because, as married households start to dis-save in response to the relative increase in public insurance, they substitute it by more consumption initially. This increase in consumption slowly decreases towards the low-level of consumption in the long run. However, taking the whole transition path into account implies that the initial increase in consumption matters more (due to the discounting of future consumption), making the transition effect positive. In total, the level effect dominates, leading to a decline in the welfare of married households. The opposite logic applies to the change in welfare for single households. When the transition is not taken into account, their welfare effect is 0.46 percent. However, taking transition into account reverses the result: welfare declines by -0.21 percent which again can be decomposed into the inequality effect (-0.28), the uncertainty effect (0.02), the level effect (0.73) and the transition effect (-0.67). One important point to notice here is that the importance of the transition effect differs for the two groups. For married households, the transition effect counteracts the level effect by less as compared to the case of single households.

Having two groups of agents is the assumption that allows for the counter-intuitive welfare results for married households. Providing more public insurance cannot lead to a decrease in welfare in the standard incomplete markets model (Aiyagari (1994)). In that model, a tax and transfer policy change in the standard incomplete markets model applies to all individuals. To understand why these two models offer different predictions for welfare in response to the change in public insurance, consider an economy composed only of married households. When these households experience an increase in public insurance, they decrease their savings. The interest rate increases to clear the market. Hence, the elasticity of the interest rate with respect to the public insurance against the labor income risk is high. As capital income of an individual is the product of the interest rate and savings, the decrease in savings is, to some extent, compensated by the significant increase in the interest rate. Furthermore, the decrease in the interest rate implies a higher wage rate and hence higher labor income. Lower capital income and higher labor income almost offset each other leading to minor changes in mean consumption. Thus the level effect in the standard incomplete markets model is weaker and is dominated by the positive transition effect. In the model presented in this paper, when married households decrease their savings in response to the increase in public insurance, the interest rate does not respond as much as in the standard incomplete markets model. This is because the decrease in savings by married households is counteracted by the increase in savings by singles, leading to the low elasticity of the interest rate. As a consequence, the capital income of married households declines significantly. Also, as the interest rate does not respond too much, this implies an almost stagnant wage rate. This means that married households experience a significant decline in their capital income but almost no change in their labor income. The opposite happens with singles. In short, in the standard incomplete markets model, aggregate variables are more responsive to the change in public insurance. However, in the model presented in this paper, aggregate variables are almost mute with respect to the relative change in public insurance. This difference explains the difference in the welfare results between the standard incomplete markets model and this model.

The last section of the paper shows that the results are qualitatively robust with respect to the calibration methods. Although the calibration method of Castañeda et al. (2003) allows the model to match the wealth distribution, it fails in matching consumption insurance as is found in the data. To make sure that the welfare results are not very sensitive to the calibration method, in Section 2.6 I implement the calibration method proposed by Krueger and Perri (2006), Broer (2013) in which the model matches the consumption insurance moments very well. Qualitatively, welfare results remain robust to calibration exercises although their magnitudes change.

Related Literature: The idea that taxes can provide insurance has been studied extensively in the macroeconomics literature. Diamond and Mirrlees (1978) derive the optimal policy for insurance against the earnings-ability risk. Varian (1980) assumes that observed income inequality in a society is solely due to the income shocks an individual receives. He derives the optimal re-distributive tax policy based on the trade off between the benefit from providing public insurance and the cost from reduced incentives. Eaton and Rosen (1980) show how re-distributive taxes can provide insurance to individuals who have imperfect information at the time when they choose their labor supply. However these papers left one question on the table: whether public insurance through progressive taxation improves the total insurance or not? Recently Krueger and Perri (2011) provided the results which show that the answer to this question depends upon the underlying frictions due to which markets are incomplete. If market is incomplete for exogenous reasons as in Aiyagari (1994), public insurance crowds out private insurance. However, this crowding out is more than one for one, hence total insurance in the economy increases. Furthermore, this is also welfare improving. Hence improved welfare and total insurance go hand in hand. The most important contribution of this paper is that it shows how relative changes in tax progressivity, where different groups in an economy face different tax progressivity over time, can result in improved total insurance at the cost of reduced welfare.

The method adopted in this paper, evaluating tax and transfers policies by employing dynamic macroeconomic model with heterogeneity, is based on the work by Aiyagari (1995), Ventura (1999), Nishiyama and Smetters (2005), Conesa et al. (2009), Mitman (2016), and among others. The paper which comes closest in the spirit and method adopted in this paper is Guner et al. (2012): it focuses on the effect of the US tax reforms on the labor supply of married and single households. It shows that marital status plays an important role in the determination of extensive margin of labor supply in response to tax reforms.

Note that, although this paper distinguishes individuals and their tax progressivity by marital status, the insight provided by this paper is very general. For example, progressivity of taxes can also be conditional on age of individuals, Heathcote et al. (2014b). Hence if different age individuals experience different public insurance/tax progressivity over time this also can lead to reduction in welfare due to heterogeneous response in savings.

The outline of the paper is as follows: Section 2.2 explains and reports the empirical results related to labor income tax progressivity. Section 3.2 expands the Aiyagari (1994) model by including two groups of households: married and single. Section 2.4 describes different calibration methods. Section 2.5 reports the results when model is calibrated according to Castañeda et al. (2003) method, whereas section 2.6 reports the results when model is calibrated according to Tauchen (1986) method. Section 3.5 concludes the paper.

2.2 Data

I use March Current Population Survey[3] (CPS henceforth) to report the changes in public insurance results. The CPS is the monthly survey of the US households conducted together by the US Census Bureau and Bureau of Labor Statistics. It is the primary source of information regarding the labor force, employment, unemployment and demographic characteristics of the US population. I focus on Annual Social and Economic (ASEC) Supplement which refers to the sample surveyed in March and is known as March CPS. This supplement extends the usual question of other month surveys to include questions on income, taxes, cash and non-cash benefits, migration and work experience[4] .

Sample Selection: The sample is restricted to a 24 year period, 1992-2015. I intend to focus on the labor income before and after-tax and transfer at the family level. However, in CPS the basic unit of observation is a housing unit and not the family unit[5] . To overcome this problem, I drop those households in which more than one family is living, making the household and the family unit same. As the focus is on the labor income, the households in which the head's labor income is not the main source of income are dropped. Furthermore, the households in which head is below (above) 25 (65) years of age, head is not working full time or worked less than 30 hours per week in last year are dropped. Also to construct the homogeneous dataset, households which are below the poverty line, or have the negative before-tax labor income are also dropped. This procedure gives the sample of 686,423 households in which 242,296 households are single households, and 444,127 households are married with a spouse present. Next, I define the measure of labor income, taxes and transfers taken in this paper.

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[3] I use the harmonized micro dataset provided by Integrated Public Use Microdata Series for CPS (IPUMS-CPS, Flood et al. (2015).

[4] Due to the timing of the survey design, the data on employment and income refer to the preceding year whereas the demographic data refer to the year of survey.

[5] In the technical language of CPS, a household is different from the family. A household is a group of persons, related or unrelated living together in a dwelling unit. The family is the group of persons who are related by blood, marriage or adoption and are living together.

Measure of labor income: The labor income of the household is the sum of wages and salaries of the head (and spouse if present and working) and the fixed fraction of self-employment farm and non-farm income. The value of fixed fraction is taken from Diáz-Giménez et al. (1997) and set equal to 0.864.

Measure of taxes: There are three types of taxes taken into consideration: federal taxes, state taxes and social security payroll taxes collected under the Federal Insurance Contributions Act (FICA). Note that these numbers are not collected from the survey respondents, as in the harmonized data provided by IPUMS, the amount of taxes were not determined by the direct questioning of respondents. On the contrary, these numbers are generated by the Census Bureau's tax model.

Measure of transfers: The measure of transfers is the sum of child tax credit, additional child tax credit, the dollar value of food stamps and income from (supplemental) social security, welfare, unemployment, retirement and worker compensations. Furthermore, depending upon the status of individual, income from veteran or survivor or disability benefits is added.

All nominal amounts are corrected by adult equivalence scale (Dalaker and Naifeh (1998)) and consumer price index corresponding to the year 1999. It is important to mention here that I follow the path of Krueger and Perri (2006) and use adult equivalence scales and not the method proposed by Aguiar and Hurst (2013). This is because of two reasons. First, Aguiar and Hurst (2013) are concerned with the life-cycle profile of consumption, implying that the age and family size are correlated. As the model presented here is an infinite horizon model, the concern of Aguiar and Hurst (2013) is not of much importance. Second, choice of family size control has more implications for the consumption as compared to the labor income.

Table B.1 in appendix reports the sample size, mean of age, labor income before tax, total taxes and transfers over the years for married and single households.

2.2.1 Public Insurance

Public insurance against idiosyncratic labor income risk reduces the after-tax and transfer income risk. Hence, one way to measure it is by asking how much income risk is reduced by taxes and transfers. For this, I need a measure of before and after-tax and transfer labor income risk. To measure this, I follow the methodology employed by Katz and Autor (1999): decompose the labor income inequality in between and within group inequality. The between-group inequality is attributable to observed characteristics of the individual, for example, education, sex, race, the region of residence, etc. As these characteristics are already known to the individual, the differences arising in labor income due to these characteristics do not come under the purview of the labor income risk. The within-group inequality is the labor income inequality minus the between-group inequality and is the measure of labor income risk[6] . Formally, denoting Y_{it} as the after-tax and transfers[7] log labor income of individual i in period t, X_{it} as the vector of observed individual characteristics, I can write

$$Y_{it} = X_{it}\mathcal{B} + y_{it} \tag{2.1}$$

where $\mathcal{B}$ is the vector of OLS estimated returns to observable characteristics and y_{it} is the residual. Due to the assumption of orthogonality between the estimated value and the residuals I can write

$$\mathrm{var}(Y_{it}) = \mathrm{var}(X_{it}\mathcal{B}) + \mathrm{var}(y_{it}) \tag{2.2}$$

In equation (2.2), $\mathrm{var}(Y_{it})$ is the total labor income inequality whereas $\mathrm{var}(X_{it}\mathcal{B})$ and $\mathrm{var}(y_{it})$ denote the between and within group inequality respectively. As mentioned above, $\mathrm{var}(y_{it})$ is also the measure of labor income risk.

[6] Note that this is not the ideal definition of income risk, where I would correct for unobserved heterogeneity across individuals by taking individual fixed effects into account. As CPS is not a panel dataset, I cannot take fixed effects into account. However, as the focus is on the trend, I will assume that the role of unobserved heterogeneity in determining the size of within-group inequality did not change over time.

[7] The same argument holds for before-tax and transfer labor income

Using these definitions of before and after-tax and transfer income risk, the public insurance can be defined as

$$\mathcal{GI} = 1 - \frac{\mathrm{var}(y_{it})}{\mathrm{var}(\tilde{y}_{it})} \tag{2.3}$$

where $\mathrm{var}(\tilde{y}_{it})[\mathrm{var}(y_{it})]$ refers to before [after] tax income risk. If $\mathrm{var}(y_{it}) = \mathrm{var}(\tilde{y}_{it})$, then $\mathcal{GI} = 0$, implying that taxes and transfers do not provide any insurance. However if $\mathrm{var}(y_{it}) = 0$, then taxes and transfers provide full insurance against labor income shocks.

2.2.2 Empirical Results

In this section, I report the public insurance results based on the method explained above. For this, I proceed in two steps. First, the levels of tax progressivity are reported. In the second step, I report how the levels of tax progressivity changed over time for married and single households.

Level of public insurance: Table 2.2 report the levels of public insurance for the sample period 1992-2015. Some important points to notice. First, before and after tax labor income risk of single individuals is significantly higher as compared to married households. In the case of before-tax and transfer income risk, singles face 53 percent more income risk whereas in the case of after-tax and transfer, singles have 37 percent more income risk. Second, taxes and transfers indeed reduce the after-tax and transfer labor income risk. For the whole sample, the reduction is about 27 percent. However, the reduction in income risk is more for singles, 32 percent, as compared to married, 24 percent. Hence single households face more income risk, but also receive more public insurance through taxes and transfers.

Trend in public insurance: The discussion till now reveals that the married households receive lower public insurance as compared to the single household. In this section, I show that these levels of public insurance do not remain same over time and follow a systematic trend.

Table 2.2: Estimated public insurance.

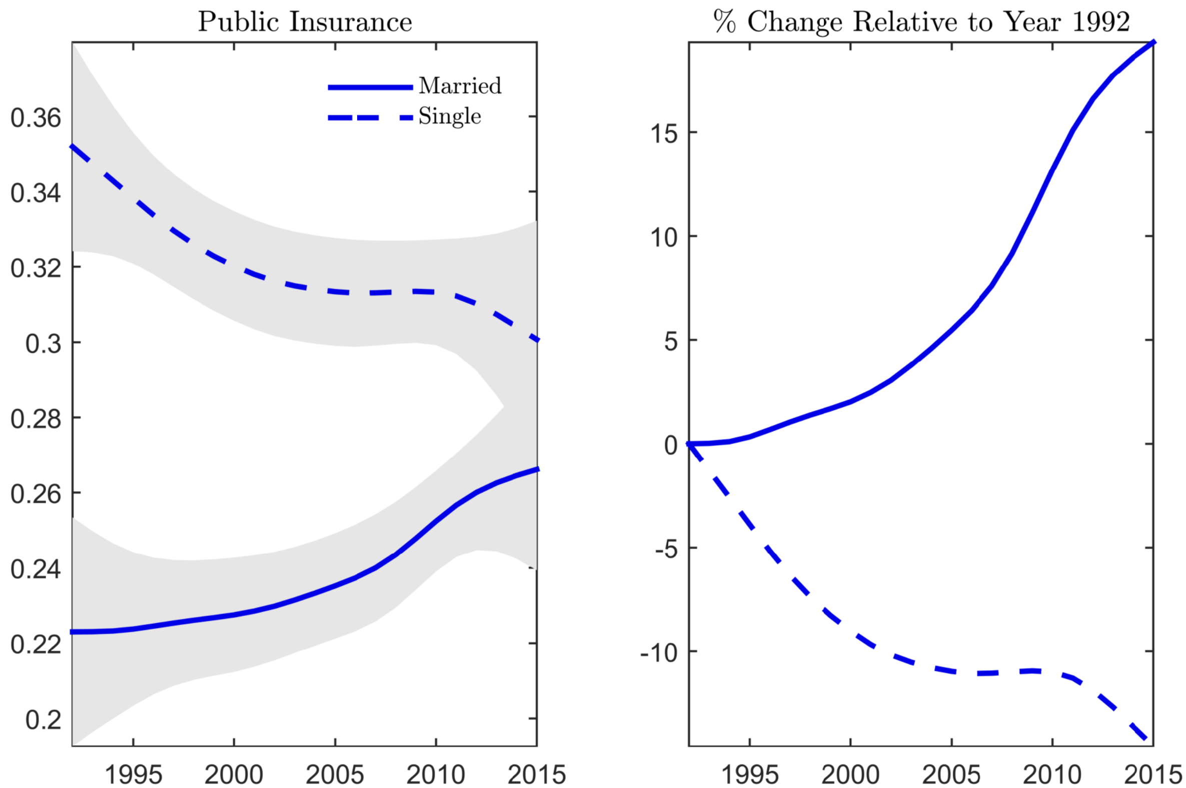
Sample	**Sample Size**	$\text{var}(\tilde{y}_{it})$	$\text{var}(y_{it})$	$\mathcal{GI} = 1 - \frac{\text{var}(y_{it})}{\text{var}(\tilde{y}_{it})}$
Whole	686423	0.370	0.268	0.275
		(0.001)	(0.001)	
Married	444127	0.307	0.234	0.238
		(0.001)	(0.001)	
Single	242296	0.471	0.322	0.316
		(0.002)	(0.002)	

Note: This table reports the variance of residual income for the whole sample as well as married and single households. The last column reports the measure of public insurance. Standard errors are reported in open brackets below the concerned statistic. Bootstrap procedure with 1000 repetition is used to compute the standard errors.

More importantly, the trends of public insurance are significantly different for married and single households. Figure 2.1 plots the public insurance trend for married and single household using the above-mentioned method. The left panel reports the actual trends and the right panel report the percentage change with respect to the base year 1992. The estimated public insurance was smoothed using the Hodrick-Prescott filter (Hodrick and Prescott (1997)) with the smoothing factor 100 (Backus and Kehoe (1992)). The most important conclusion from Figure 2.1 is that the public insurance received by married and single households follow a different trend. Over the sample period 1992-2015, it increased for married households by 19 percent and decreased for singles by about 13 percent.

Summarizing, the level of public insurance against idiosyncratic labor income risk is not the same for all households and depends on their marital status. Single households have received higher public insurance as compared to the married households. However, over time this difference has decreased, as insurance received by single (married) households decreased (increased). As the presence of public insurance affects the precautionary motive to save and hence affects the private and total insurance, change in public insurance implies a change in total insurance and which can also affect the welfare of the individuals. Furthermore, as different marital status households face different trend in progressivity, this also has a distributional implication within the economy and

Figure 2.1: Public insurance trend and change, 1992-2015.



Notes: The left panel reports the trends of public insurance against the labor income risk for the married and single households. The shaded region is the 95% confidence interval based on the standard error computed using 1000 bootstrap repetitions. The right panel reports the percentage change in public insurance, with 1992 as the base year.

hence can affect the welfare more. In next section, I develop a two group standard incomplete markets model to quantitatively assess the total insurance and welfare implication of change in public insurance.

2.3 Model

In this section I extend the standard SIM model, à la Aiyagari (1994) to include two groups of individuals[8] , namely married (denoted by m) and singles (denoted by s). The groups are indexed

[8] As the empirical evidence was based on the labor income adjusted by the adult equivalence scale, in the model, individual and household are synonyms.

by $j = \{m, s\}$. Individuals in the economy live for infinite time periods. Time is discrete and is denoted by t, starting from $t = 0$. Each group at time t contains a continuum of individuals with measure g_t^j. The measure of all individuals in this economy is equal to one i.e. $g_t^m + g_t^s = 1$. The individuals can transit between the married and single groups with exogenous probabilities[9] . The transition matrix between married and single is given as

$$\mathcal{P} = \begin{bmatrix} \pi_{mm} & \pi_{ms} \\ \pi_{sm} & \pi_{ss} \end{bmatrix} \tag{2.4}$$

where π_{ij} denotes the probability of transition from state i to j such that $\pi_{mm} + \pi_{ms} = 1$ and $\pi_{sm} + \pi_{ss} = 1$.

The objective of the individual in group j is to maximize the expected lifetime utility, given as,

$$U^j = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^j) \tag{2.5}$$

where β is the subjective discount factor, $\mathbb{E}_0$ is the expectation based on the information set[10] at time $t = 0$ and $u(\cdot)$ is the period utility function which is strictly increasing and strictly concave in its argument and satisfies the Inada conditions. I assume that the subjective discount factor and the period utility function of the individuals do not depend upon the group to which individual belongs. As the utility function is independent of labor supply, I implicitly assume that labor is supplied inelastically.

Market Structure: Individuals in group j can only save in the form of real capital, denoted by a^j. The presence of this non-state-contingent bond allows individuals to smooth consumption across time as well as state. The gross return of this bond is denoted by R. Individuals are also

[9] This is a strong assumption, as individuals explicitly decide to marry and divorce. In future work I intend to include endogenous marriage decision.

[10] The information set contains the transition probability between married and single and vice versa, and also the information about the labor income process, described later.

allowed to borrow and the exogenous borrowing limit is denoted by $\underline{a}$.[11]

Labor Endowment Process: In period t, a household in group $j = \{m, s\}$ receives a random labor endowment denoted by $\tilde{l}_t^j$. The set of random labor endowment is time-invariant and finite and is given by an ordered set $\tilde{\mathcal{L}}^j = \{\tilde{l}^{j,1}, \cdots, \tilde{l}^{j,N}\}$. The labor endowment is independent and identically distributed across households and evolves across time according to a first order Markov chain process with transition matrix $\mathcal{P}^j$. The before tax labor income is given by $\tilde{y}^j = w\tilde{l}^j$ where w is the economy wide wage rate. I assume the law of large numbers for each group, implying that the fraction of agents with labor income $\tilde{y}^{j'}$ next period given the labor income $\tilde{y}^j$ in present period is $\pi^j(\tilde{y}^{j'}|\tilde{y}^j)$. Note that the transition probabilities are different for each group, which can be the case if the labor endowment process is different for each group. I assume that the transition probabilities have a unique measure Π^j. The average income in each group is normalized to one i.e. $\sum_{\tilde{y}^j} \Pi^j(\tilde{y}^j)\tilde{y}^j = 1$.

Labor Income Tax: Government provides public insurance through the tax policy denoted by $\tau^j(w\tilde{l}_t^j)$. As the aim is to establish how the ex-ante welfare and the risk allocation gets affected due to the progressive taxes, I assume that tax system is revenue neutral within each group i.e. net revenues generated by the tax system within each group are equal to zero and hence there is no redistribution between the groups[12] . Furthermore I assume a one-parameter family of tax system as in Krueger and Perri (2011) so that in policy experiments the progressivity of tax system can be varied in a transparent way. The tax system for the group j is given by a marginal tax rate τ^j and a transfer ϕ^j. Since the mean of before tax labor income is normalized to one, this implies the

[11] The borrowing constraint is not indexed by the group. This is because I assume that both married and single households face the same borrowing constraint. If $\underline{a} = 0$ then it implies that no borrowing is allowed and individuals can only save. Furthermore, the lower bound on borrowing can come into existence because of a requirement that individual is always able to pay back her debt. It may also be the case that lower bound is exogenously set.

[12] It is important to note that no redistribution between the groups through tax and transfers does not imply that there is no redistribution between the groups at all. As both groups are interacting the same market and face one same prices, the change in these prices will induce redistribution between the groups.

constant transfer $\phi^j = w\tau^j$. Hence the after tax labor income (y^j) is given as

$$y^j = (1-\tau^j)\tilde{y}^j + w\tau^j \tag{2.6}$$

Higher τ^j implies more transfer from individuals with high labor income to individuals with low labor income. Hence the higher value of τ^j implies higher extent of public insurance.

Firms: There is a continuum of competitive firms who have access to the constant returns to scale (CRS) production technology, denoted as $Y = Af(K,L)$ where K and L are the total capital and labor inputs respectively, A is the technology parameter and $f(\cdot,\cdot)$ is the production function. The reason for assuming the CRS production function is that then the size of the firms doesn't matter and hence I can assume the existence of a representative firm. Furthermore, production function is strictly increasing and strictly concave in both of its argument i.e $f_K, f_L > 0$ and $f_{KK}, f_{LL} < 0$. Every period capital depreciates at the rate δ. Inputs are rented by the firm in the competitive factor markets.

Aggregate State Variables: In this economy individuals are characterized by the group to which they belong and the pair (a^j, y^j) which I call individual states. The aggregate state of the economy is the distribution of the agents across these states. There are two distributions corresponding to each group, denoted by $\lambda^m(a^m, y^m)$ and $\lambda^s(a^s, y^s)$. These two distributions are the probability measures over the Borel sets of the compact set $S^j = A^j \times Y^j$, and determine the aggregate amount of capital and labor in the economy, which are given as

$$K = \sum_j g^j \int_{S^j} a^j \, d\lambda^j, \quad L = \sum_j g^j \int_{S^j} \tilde{l}^j \, d\lambda^j$$

The gross rental rate of capital and wage rate are $R = 1 + f_K(K,L) - \delta$, $w = f_L(K,L)$ respectively. This implies that the factor prices are the functions of probability measures λ^j i.e. $R = R(\lambda^m, \lambda^s)$ and $w = w(\lambda^m, \lambda^s)$.

Household Problem: The household's problem is to maximize (2.5) given the sequence of inter-temporal budget constraint and the borrowing constraint. The problem for the married individual can be cast in a recursive problem given as,

$$V^m(a^m, y^m; \lambda^m, \lambda^s) = \max_{c^m, a'} \left\{ u(c^m) + \beta \sum_{j=m,s} \pi_{mj} \sum_{y^{j'}} \pi^j(y^{j'}|y^j) V^{j'}(a', y^{j'}; \lambda^{m'}, \lambda^{s'}) \right\} \tag{2.7}$$

$s.t.$

$$c^m + a' = w(\lambda^m, \lambda^s)[(1-\tau^m)\tilde{l}^m + \tau^m] + R(\lambda^m, \lambda^s)a^m \tag{2.8}$$

$$a' \geqslant -\underline{a} \tag{2.9}$$

The superscript prime denote the next period variables. The problem for the single household can be written in the same way. The equation (2.7) is the value function of the married individual with initial wealth a^m and labor endowment y^m. Given a^m, y^m and λ^m, λ^s the married individual chooses the present consumption c^m and savings a'. Note that I don't index the savings by the group. This is because next period, the married individual can transit to being single. Two assumptions are made to write the problem in this way. First, if individual transit from married to single and vice versa, the wealth level of the individual does not change. This implies that transition between married and single state do not affect the wealth level. Second, I assume that if the individual transfers from married to single, his associated income process also changes immediately. Another way of saying this is that the transition between the marital status and income process are perfectly correlated. So, if a married individual switches to being single, his income process will be the associated single income process, immediately.

To solve the model, I focus on the stationary recursive competitive equilibrium, defined below.

Definition 9 ***Stationary Recursive Competitive Equilibrium***

A stationary recursive competitive equilibrium consists of the type distribution of individuals $\lambda^m(a^m, y^m)$, $\lambda^s(a^s, y^s)$, the gross rental rate of capital $R(\lambda^m, \lambda^s)$, rental rate of labor $w(\lambda^m, \lambda^s)$, and for each group the value function $V^m(a^m, y^m; \lambda^m, \lambda^s), V^s(a^s, y^s; \lambda^m, \lambda^s)$, optimal policy functions $a'(a^m, y^m; \lambda^m, \lambda^s)$, $a'(a^s, y^s; \lambda^m, \lambda^s)$ $c^j(a^j, y^j; \lambda^m, \lambda^s)$, and the aggregate capital $(K(\lambda^m, \lambda^s))$ and labor $(L(\lambda^m, \lambda^s))$ such that

- *Given prices $R(\lambda^m, \lambda^s)$ and $w(\lambda^m, \lambda^s)$, the policy functions $a'(a^j, y^j; \lambda^m, \lambda^s), c^j(a^j, y^j; \lambda^m, \lambda^s)$ solves the individual's optimization problem* (2.7) *and $V^j(a^j, y^j; \lambda^m, \lambda^s)$ are the associated value functions.*
- *Firm maximize the profit*
- *For all $(\mathcal{A}^j, \mathcal{Y}^j)$ the probability measure λ^j are invariant*

$$\lambda^j(\mathcal{A}^j \times \mathcal{Y}^j) = \int_{A^j \times Y^j} Q^j((a^j, y^j), \mathcal{A}^j \times \mathcal{Y}^j) \, \mathrm{d}\, \lambda^j(a^j, y^j) \tag{2.10}$$

  where Q^j is the transition matrix for group j.
- *Capital and labor market clears*

The change in public insurance implies substantial redistribution, both within and between the groups, in the short run. Hence, focusing only on the steady-states can be very misleading. Therefore, this paper explicitly takes transition dynamics into account to compute the welfare and insurance effect of the relative change in public insurance. The definition of recursive competitive equilibrium with transition is given below.

Definition 10 ***Recursive Competitive Equilibrium***

A recursive competitive equilibrium consists of the sequence of the type distribution of individuals $\{\lambda_t^m, \lambda_t^s\}_{t=0}^{\infty}$, the sequence of the gross rental rate of capital $\{R(\lambda_t^m, \lambda_t^s)\}_{t=0}^{\infty}$, the sequence of the rental rate of labor $\{w(\lambda_t^m, \lambda_t^s)\}_{t=0}^{\infty}$, and for each group the sequence of value functions

$\{V_t^m(a_t^m, y_t^m; \lambda_t^m, \lambda_t^s)\}_{t=0}^{\infty}$, $\{V_t^s(a_t^s, y_t^s; \lambda_t^m, \lambda_t^s)\}_{t=0}^{\infty}$, the sequence of policy functions $\{c_t^m(a_t^m, y_t^m; \lambda_t^m, \lambda_t^s)\}_{t=0}^{\infty}$, $\{c_t^s(a_t^s, y_t^s; \lambda_t^m, \lambda_t^s)\}_{t=0}^{\infty}$, $\{a_{t+1}(a_t^m, y_t^m; \lambda_t^m, \lambda_t^s)\}_{t=0}^{\infty}$, $\{a_{t+1}(a_t^s, y_t^s; \lambda_t^m, \lambda_t^s)\}_{t=0}^{\infty}$ and the sequence of aggregate capital and labor, $\{K_{t+1}(\lambda_t^m, \lambda_t^s)\}_{t=0}^{\infty}$, $\{L_t(\lambda_t^m, \lambda_t^s)\}_{t=0}^{\infty}$ such that

- *Given prices $R(\lambda_t^m, \lambda_t^s)$ and $w(\lambda_t^m, \lambda_t^s)$, the policy functions $a_{t+1}(a_t^j, y_t^j; \lambda_t^m, \lambda_t^s), c_t^j(a_t^j, y_t^j; \lambda_t^m, \lambda_t^s)$ solves the individual's optimization problem* (2.7) *and $V_t^j(a_t^j, y_t^j; \lambda_t^m, \lambda_t^s)$ are the associated value functions.*

- *Firm maximize the profit every period*

- *For all $(\mathcal{A}^j, \mathcal{Y}^j)$ the probability measure λ_t^j and λ_{t+1}^j satisfy*

$$\lambda_{t+1}^j(\mathcal{A}^j \times \mathcal{Y}^j) = \int_{A^j \times Y^j} Q_t^j((a_t^j, y_t^j), \mathcal{A}^j \times \mathcal{Y}^j) \, \mathrm{d}\, \lambda_t^j(a_t^j, y_t^j) \tag{2.11}$$

  where Q_t^j is the transition matrix for group j at time period t.

- *Capital and labor market clears*

It is important to note that this definition of recursive equilibrium with transition is written under the assumption that the dynamics introduced by the change in tax progressivity are deterministic in nature. This is also the basis of the solution algorithm.

2.4 Functional Forms and Calibration

To assess the welfare and risk sharing implications of differential change in tax progressivity, the underlying model should quantitatively account for not only the observed earnings and wealth inequality, but also the consumption risk sharing observed in the data. Due to the structure of the standard incomplete markets model, the predictions about the wealth inequality and consumption risk sharing rely on the underlying earnings process. The literature uses two methods predominantly to generate the earnings process. First method is based on Tauchen (1986), Tauchen and Hussey (1991) procedure which generates the symmetric transition matrix for the earnings process. The advantage of using this method is that it can match the consumption risk sharing as observed in the data fairly well, but fails to generate the observed wealth inequality. The second method, proposed by Castañeda et al. (2003), generates the earnings process which can match the observed wealth inequality. However this method fails in matching the observed consumption risk sharing. To solve this dilemma, I report the results for both types of calibration. The major part of the paper will focus on Castañeda et al. (2003) method, which is described below in this section. However the last section will report the results when calibration is based on Tauchen (1986) method of discretization. The conclusion of implementation of both types of calibration is that, qualitatively, results remain the same although the magnitudes change.

Labor income process: The group specific (idiosyncratic) labor productivity levels and the transition matrices $\mathcal{P}^j, j = \{m, s\}$ are chosen to match the US wealth distribution. The labor income process is discretized in three states which implies that there are in total 24 free parameters (18 from income transition matrix and six from income states). However, the weighted labor productivity is normalized to 1 and the rows of the transition matrices add up to one. This reduces the free parameters to 18.

To match the US wealth distribution, I focus on the percentage of wealth owned by the bottom 60 percent and top 40 percent and the Gini index. Based on the Survey of Consumer Finances 1992, the Gini index value is set equal to 0.71 instead of 0.83 as is found in SCF 1992 by Wolff (2011b). The lower value of Gini index is obtained after dropping the individuals in top 1 percent of the wealth distribution. The reason behind dropping these individuals is two-fold. First, the primary focus of this paper is on labor income. Alvaredo et al. (2013) and references cited there in emphasize the importance of capital income and not labor income for the households in top 1 percent of wealth distribution. Hence I drop these households. Second reason is based on survey design. SCF employs two methods for random sampling. One method using the area-probability sample identify the households with characteristics that are broadly distributed in the population. This sample is called area-probability sample. Second method is focused on disproportionately including the wealthy families and this sample is called list sample. The response in area probability sample is about 70 percent whereas in list sample response rate is just one third. Furthermore, the response rate of the wealthiest families was only half of that level. This implies, even though SCF focuses on wealthy families to create a complete picture of wealth inequality, very wealthy families response rate distorts this picture. Hence I drop those households.

The (log) labor income is assumed to follow the AR(1) process. The variance of the log labor income process i.e. labor income risk is equal to the group specific after-tax and transfer income risk in the year 1992. This is equal to 0.221 for married group and 0.301 for the single group. I take after-tax and transfer risk because then in the policy experiment I can only focus on the trend of the tax progressivity. There is uncertainty associated with the autocorrelation coefficient of the AR(1) process. Domeij and Heathcote (2004) report that autocorrelation coefficient lies between 0.88 and 0.96. For this paper, I set this coefficient equal to 0.9. The estimated labor income productivity level, transition matrix and stationary distribution for the married and single households are,

$$\tilde{\mathcal{L}}^m = \begin{pmatrix} 0.6581 \\ 0.6719 \\ 1.9068 \end{pmatrix} \qquad \mathcal{P}^m = \begin{pmatrix} 0.9562 & 0.0353 & 0.0086 \\ 0.0731 & 0.8499 & 0.0770 \\ 0.0200 & 0.0602 & 0.9198 \end{pmatrix} \qquad \Pi^m = \begin{pmatrix} 0.5020 \\ 0.2267 \\ 0.2713 \end{pmatrix}$$

$$\tilde{\mathcal{L}}^s = \begin{pmatrix} 0.5512 \\ 0.6325 \\ 1.8443 \end{pmatrix} \qquad \mathcal{P}^s = \begin{pmatrix} 0.9563 & 0.0276 & 0.0160 \\ 0.0539 & 0.8622 & 0.0839 \\ 0.0201 & 0.0591 & 0.9208 \end{pmatrix} \qquad \Pi^s = \begin{pmatrix} 0.4371 \\ 0.2303 \\ 0.3326 \end{pmatrix}$$

Some comments are in order. First, as the number of free parameters are greater than the targeted moments, the calibration process is under-identified. Second, the estimated income processes are different from what is estimated by Castañeda et al. (2003). This is because they calibrate their model without dropping the households in the top 1 percent of the wealth distribution. Third, the estimated income processes for the married and single households are different. This is because of the difference in labor income risk faced by these households.

Functional Forms: The agent's instantaneous utility function is assumed to be of constant relative risk aversion type (CRRA) given as

$$u(c) = \begin{cases} \frac{c^{(1-\sigma)}}{1-\sigma}, & \sigma \neq 1 \\ \log(c), & \sigma = 1 \end{cases}$$

The production function is assumed to be of Cobb Douglas type, given as $F(K,L) = K^{\alpha}L^{1-\alpha}$ where α is the output elasticity of capital.

Model parameters: As the main emphasis of the paper is to show how the more progressive labor income tax affects risk sharing and welfare, it is important that the benchmark economy captures the wealth stock properly. This is because accumulated wealth affects the level of risk sharing. Hence following Kaplan and Violante (2010), I target aggregate wealth to income ratio equal to 2.5. This number is computed from the Survey of Consumer Finances, 1989 and 1992 waves. To calculate this number wealth is defined as the total net worth of an individual and income is pre-tax labor income and capital income. To target this statistic, I set gross interest rate (R) equal to 1.025[13] , the depreciation rate of capital (δ) equal to 0.1357 and find the subjective discount factor β which gives aggregate wealth to income ratio equal to 2.5. On average, the single individual's population share in the economy is about 33 percent. So I set the share of married individuals in the economy (g^m) equal to 0.67 and hence $g^s = 0.33$. The borrowing constraint is set exogenously to $a^j = -1, j = \{m, s\}$ i.e. both married and single individuals can borrow up to the average income.

[13] This is taken from Broer (2013)and is the average ex-ante real interest rate on six-month US Treasury bills between 1998 and 2003.

Married single transition probabilities: To calibrate the probability of transition from married to single and vice versa, I use the National Survey of Family Growth 2006-2010 wave, Copen et al. (2012). From this survey, focusing only on white men and women, the probability of remaining married after five years of marriage is 0.8 for women and 0.82 for men. The weighted average is equal to 0.809[14] . Assuming that the survival probability of marriage follows the exponential distribution with parameter λ_{mm}, I can write

$$\exp(-5\lambda_{mm}) = 0.809$$

which implies $\lambda_{mm} = 0.0423$. Hence the probability of remaining married after one year is $\pi_{mm} = \exp(-\lambda_{mm}) = 0.958$ and transition to single is $\pi_{ms} = 1 - \exp(-\lambda_{mm})$. To calibrate the transition probabilities for the single individuals I follow different procedure because it is not straight forward to estimate these probabilities. I ask, given the transition probabilities of the married individuals, which transition probabilities I need for single individuals so that the share of single and married individuals in the economy is equal to g^m and g^s respectively. This procedure gives $\pi_{ss} = 0.929$ and $\pi_{sm} = 0.071$.

Table 2.3 summarizes the model parameters and the target statistics for the benchmark economy[15] .

2.5 Effects of Change in Tax Progressivity

The objective of this section is to quantify the effect of change in public insurance of married and single households, as documented in Section 2.2, on consumption risk sharing (or synonymously insurance) and welfare in the economy. As reported in Figure 2.1, over the sample period

[14] Sample size of white women is 21,703 and of white men is 17,813

[15] I do not target the consumption risk sharing measure in this calibration exercise. There are two reasons behind it. First, at this moment, model is calibrated on CPS dataset (variance of income is taken from this dataset) which does not have consumption data. Hence there is no consumption risk sharing statistic which I can target. Second, I cannot use information from the Consumption and Expenditure Survey (CEX) data because of the difference between the CPS and CEX sample survey design.

Table 2.3: Model parameter and target statistics for the benchmark model.

Calibration statistic	Parameter	Value
Preferences and Technology		
Subjective discount factor	β	0.964
Capital income share	α	0.300
Capital depreciation rate	δ	0.136
Technology parameter	A	1.060
Gross interest rate	R	1.025
CRRA utility parameter	σ	1
Marriage characteristics parameters		
Probability, married to single	π_{ms}	0.042
Probability, single to married	π_{sm}	0.071
Share, married	g^m	0.630
Share, single	g^s	0.370
Exogenous borrowing constraint	$\underline{a}$	1

Target Statistic	Data	Model
Wealth to income ratio	2.50	2.50
% Wealth, bottom 60%	4.10	5.26
% Wealth, top 40%	95.8	94.7
Wealth Gini	0.71	0.68
Variance labor income, married	0.221	0.221
Variance labor income, single	0.301	0.301
Autocorrelation, labor income	0.900	0.900

Notes: The top table reports the calibrated value of the model parameters. The bottom table reports the targeted statistics, based on the calibration method proposed by Castañeda et al. (2003)

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1992-2015, the public insurance against labor income risk increased for married households and decreased for single households. Since there is one-to-one mapping between public insurance and tax progressivity in the present model, this implies tax progressivity increased for married households and decreased for singles. The primary objective of this section is to show that this relative change in tax progressivity has non-trivial consequences for the welfare. The conclusion of the exercise is this: the married households, despite increase in their tax progressivity which reduces their after-tax and transfer labor income risk, suffer a welfare loss, whether transition is taken into account or not. This is despite the increase in insurance achieved by married households. On the other hand, single households experience welfare gain when transition is not taken into account. However, when transition is taken into account, they loose in terms of welfare. For the comparison, if the economy was populated by only married households, then the increase in tax progressivity will result in unambiguous welfare gain.

Section 2.5.1 describes the insurance and utilitarian welfare measures. Furthermore, utilitarian welfare measure is decomposed into uncertainty effect, inequality effect, level effect and the transition effect.

2.5.1 Insurance and Welfare Measures

Insurance measure: Following Krueger and Perri (2011) I define the total insurance of group j as one minus the ratio of standard deviation of consumption with respect to pre-tax income:

$$\mathcal{TI}^j = 1 - \frac{\mathrm{var}(c^j)}{\mathrm{var}(\tilde{y}^j)} \tag{2.12}$$

Intuitively, (2.12) tells us how much the variability of before-tax and transfer income transfers to consumption. If $\mathrm{var}(c^j) = 0$, this implies $\mathcal{TI}^j = 1$ implying that income shocks are perfectly insured. But if $\mathrm{var}(c^j) = \mathrm{var}(\tilde{y}^j)$ then $\mathcal{TI}^j = 0$ implying that income shocks completely transfer to consumption. I can further decompose the total insurance in public or government insurance ($\mathcal{GI}^j$) and private insurance ($\mathcal{PI}^j$), defined as

$$\mathcal{GI}^j = 1 - \frac{\mathrm{var}(y^j)}{\mathrm{var}(\tilde{y}^j)}, \quad \mathcal{PI}^j = 1 - \frac{\mathrm{var}(c^j)}{\mathrm{var}(y^j)} \tag{2.13}$$

Given the assumed functional form of tax policy, (2.6), there is one to one monotonous positive relationship between τ^s and $\mathcal{GI}^j$. Higher τ^j implies higher public insurance.

Welfare measure: I denote the benchmark economy with superscript A and the economy after the policy reform by superscript B. The utilitarian welfare of group $j = \{m, s\}$ in economy A

(similarly for economy B) is defined as

$$U^{j,A} = \int_{A^j \times Y^j} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^j | a_0^j, y_0^j) \,\mathrm{d}\, \lambda^j (a_0^j, y_0^j) \tag{2.14}$$

For the logarithmic utility function, the utilitarian welfare change for group $j = \{m, s\}$ due to the policy reform, denoted by Δ^j is given as

$$\Delta^j = \exp[(1-\beta)(U^{j,B} - U^{j,A})] - 1$$

where U_B^j is calculated by taking the value function based on the consideration of the transition path, but the stationary distribution of benchmark economy A. The variable Δ^j can be interpreted as the percentage change in consumption in economy A, such that the expected utility in economy A is same as in economy B.

In order to understand the different implications of the policy reform, following Flodén (2001),Benabou (2002) and Koehne and Kuhn (2015), I can decompose Δ^j into welfare components arising from changes in inequality, levels, uncertainty and transition. For an individual in group $j = \{m, s\}$ with initial asset a_0^j and initial labor income y_0^j a certainty equivalent of consumption in economy A can be defined as

$$\frac{u(\bar{c}^{j,A}(a_0^j, y_0^j))}{(1-\beta)} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{j,A}(a_0^j, y_0^j))$$

The mean certainty equivalent of consumption of group j in economy A is denoted by $\bar{C}^{j,A}$ and is defined as

$$\bar{C}^{j,A} = \int \bar{c}^{j,A}(a_0^j, y_0^j) \,\mathrm{d}\, \lambda^{j,A}(a_0^j, y_0^j)$$

whereas the mean consumption of group j in economy A is defined as

$$C^{j,A} = \int c^j(a_0^j, y_0^j)\, \mathrm{d}\, \lambda^{j,A}(a_0^j, y_0^j)$$

As I am interested in determining the welfare effect due to change in inequality and uncertainty, I have to first determine how much the individuals in the economy dislike inequality and uncertainty. For group j in economy A denote the price of inequality by $p_{ineq}^{j,A}$ and the price of uncertainty by $p_{unc}^{j,A}$. The $p_{ineq}^{j,A}$ can be determined by comparing the consumption of single individual with the average consumption. Formally, it is given as

$$u((1 - p_{ineq}^{j,A})\bar{C}^{j,A}) = \int u(\bar{c}^{j,A}(a_0^j, y_0^j))\, \mathrm{d}\, \lambda^{j,A}(a_0^j, y_0^j)$$

Note that the certainty equivalent of consumption and its mean determine the cost of inequality. Using the actual consumption will does not allow to separate the cost of uncertainty from the cost of inequality. The cost of uncertainty for group j in economy A is defined as

$$u((1 - p_{unc}^{j,A})C^{j,A}) = u(\bar{C}^{j,A})$$

In words, the cost of uncertainty quantifies how much individuals are willing to let go of the mean consumption to consume mean certainty equivalent of consumption. Using these definitions I can define the different components of welfare change. These are given as

$$\begin{aligned}
\Delta_{ineq}^j &= \frac{1 - p_{ineq}^{j,B}}{1 - p_{ineq}^{j,A}} - 1 \\
\Delta_{unc}^j &= \frac{1 - p_{unc}^{j,B}}{1 - p_{unc}^{j,A}} - 1 \\
\Delta_{lev}^j &= \frac{C^{j,B}}{C^{j,A}} - 1 \\
\Delta_{trans}^j &= \exp[(1 - \beta)(U^{j,B} - U_{ss}^{j,B})] - 1
\end{aligned}$$

Table 2.4: Different welfare measures and their symbols.

Welfare Measures	Symbol
Utilitarian welfare, steady state	Δ^j_{ss}
Utilitarian welfare	Δ^j
Inequality effect	Δ^j_{ineq}
Uncertainty effect	Δ^j_{unc}
Level effect	Δ^j_{lev}
Transition effect	Δ^j_{trans}

Note: The table reports the different measures of welfare and their symbols. The superscript j refers to the married (m) and singles (s) group.

where $U^{j,B}_{ss}$ is the utilitarian welfare of group j under economy B when transition is not taken into account.

Proposition 5 $\Delta^j = (1 + \Delta^j_{ineq})(1 + \Delta^j_{unc})(1 + \Delta^j_{lev})(1 + \Delta^j_{trans}) - 1$

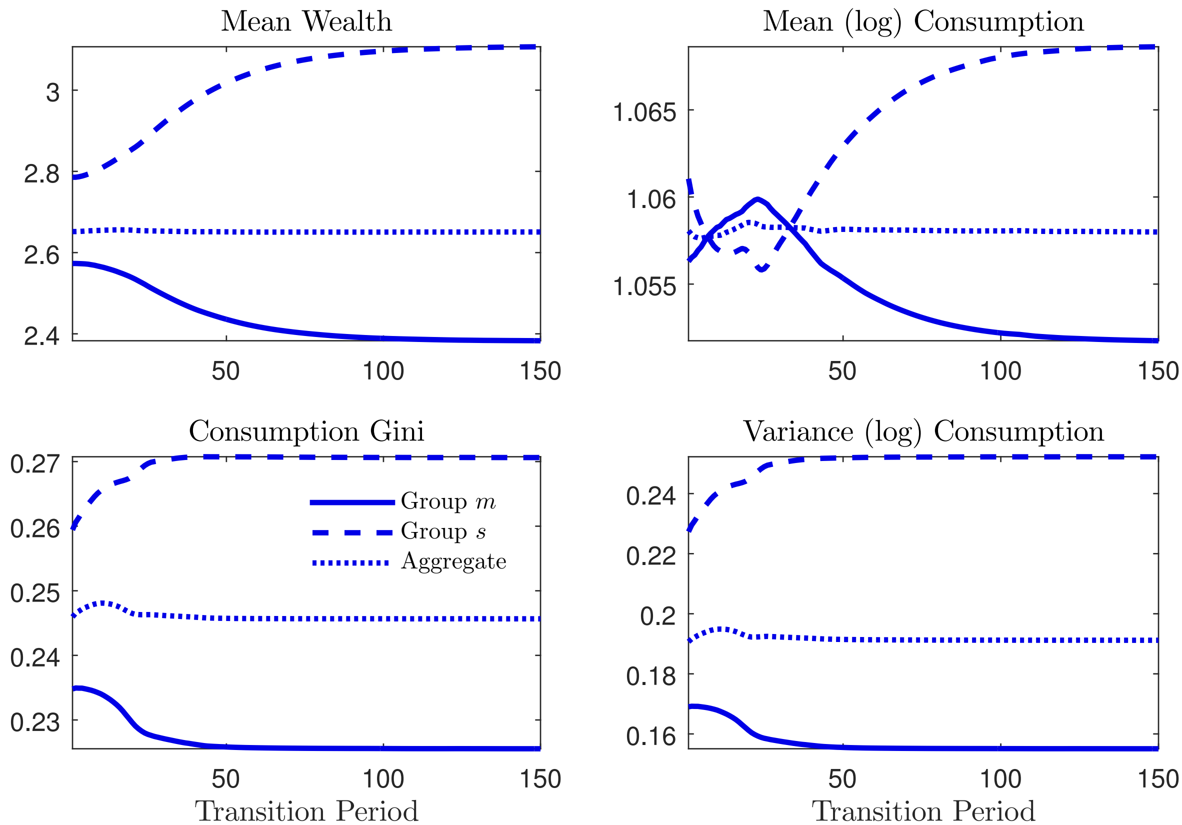
The proof follows from the method proposed by Flodén (2001), Benabou (2002), Domeij and Heathcote (2004) and Koehne and Kuhn (2015). Note that under this particular welfare decomposition, all the changes which were not attributed to inequality, uncertainty or level effect is relegated to the transition part.

Table 2.4 summarizes the different welfare measures.

2.5.2 Results

Figure 2.2 report the evolution of mean savings, mean log(consumption), Gini index of consumption and variance of log(consumption) when individuals in group m and s experience the change in tax progressivity. The reason for reporting these variables is that it allows me to explain the utilitarian welfare change and its components. The top left panel reports the change in group specific mean savings and the aggregate savings in the economy. It can be seen that there is no visible change in the aggregate savings as compared to group specific changes, which are significant. Savings of group m decreased and group s increased.

Figure 2.2: Evolution of endogenous variables in response to change in tax progressivity.



Notes: The figure reports the evolution of endogenous variables in response to the change in tax progressivity which mimics the change in public insurance as reported in Figure 2.1.

This is because, as progressivity of labor income taxes increases for group m individuals, it lead to the decrease in precautionary motive to save, leading to the decrease in savings. Opposite happens with group s[16] . The aggregate savings are insignificantly affected due to the opposite behavior of two groups. Due to this change in saving behavior, as group m (s) individuals start to de-cumulate (accumulate) their savings, their initial consumption increases (decreases). However, in long term, as the market clearing interest rate is not much affected, the mean wealth of group m

[16] At this point it is important to realize that, qualitatively, this behavior of group specific and aggregate savings will still hold if only married group experienced increase in tax progressivity, but single household's tax progressivity remain the same. This is because due to the decrease in precautionary savings by married households, aggregate interest rate will rise, implying that single households will start to save more.

Table 2.5: Percentage welfare change and decomposition.

Welfare measure	**Two Group Model**		**One Group Model**
	Group m	Group s	
Utilitarian, steady state (Δ^j_{ss})	- 0.38	0.46	0.37
Utilitarian (Δ^j)	-0.08	-0.21	0.19
Inequality (Δ^j_{ineq})	0.20	-0.28	0.12
Uncertainty (Δ^j_{unc})	-0.16	0.02	0.30
Level (Δ^j_{lev})	-0.43	0.73	-0.05
Transition (Δ^j_{trans})	0.30	-0.67	-0.18

Notes: The table reports the welfare measures for the married and single households in response to the change in tax progressivity which mimics the change in public insurance as reported in Figure 2.1. The results for one group model is generated by keeping all the parameters in the model same, except for setting the share of married individuals in economy, g^m, equal to one.

individuals is low as compared to the single individuals. Hence the long term mean consumption of group m (s) individuals is low (high).

Now focusing on the bottom panel of Figure 2.2, we see that the Gini index and variance of log consumption increased for group s and decreased for group m. The variance of consumption increases for group s, because the decrease in tax progressivity leads to the reduction in total insurance. Opposite happens with group m.

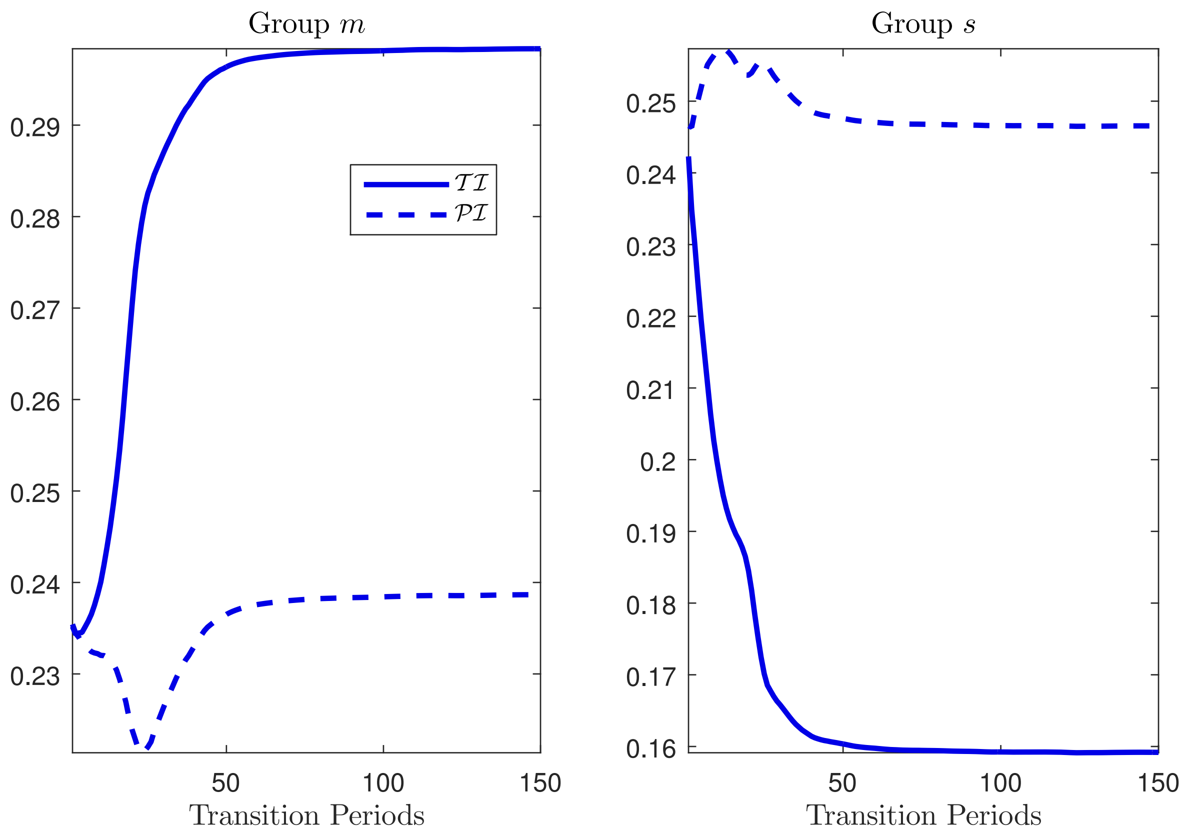
Table 2.5 reports the welfare results of the change in tax progressivity. The second and third column report the result for the group m and s, whereas the last column refers to the one group model. There are some important points to notice. First, in two group model, by comparing the steady state, group m suffered welfare loss whereas group s experience welfare gain. However if the group m experiences the same tax progressivity change in one group model, then there is welfare gain. Second, when transition is taken into account, the utilitarian welfare change for both group is negative. However, the result of one group model is robust to the transition effect and utilitarian welfare change remains positive. These two observations imply that reducing the after-tax and transfer labor income risk of individuals does not necessarily imply that those individuals will be better off. Third, decomposition of welfare change shows that the origin of negative welfare change is different for two groups. For group m, uncertainty and level effect play an important role, whereas for group s it is inequality and transition effect. Below I discuss why these effects behave differently for group m and s.

Inequality effect: The consumption Gini index has increased for group s but decreased for group m. This is because, due to the decrease in tax progressivity, group s individuals started to increase their savings. However, the individuals with history of positive shocks will accumulate disproportionately more wealth as compared to individuals with history of negative shocks, and this translates into higher dispersion in wealth as well as consumption. For the opposite reason, group m experienced decrease in consumption Gini index. As group s becomes more unequal, individuals are ready to pay more price to reduce the inequality, and hence inequality effect for group s is negative.

Uncertainty effect: Variance of log consumption for married households decreased and increased for single households. So it comes as a surprise that uncertainty effect is negative for married households and positive for single households. This welfare effect implies that price of uncertainty has increased for married households. To visualize this, Figure 2.3 plots the total and private insurance of group m and s. More progressive taxes crowd out the private insurance, more than one by one. Hence total insurance increases but private insurance decreases for married households. Opposite is true for the single households. As individuals are concerned with after-tax and transfer income and the extent to which it transfers to the consumption (in other words private insurance), married households are ready to pay higher price to remove the uncertainty. Hence the uncertainty effect is negative for married households. Similar but opposite argument applies to single households.

Level effect: This effect depends upon the mean of consumption in steady states. From Fig-

Figure 2.3: Total and private insurance measures.



Notes: The figure reports the evolution of total and private insurance in response to the change in tax progressivity which mimics the change in public insurance as reported in Figure 2.1. The solid (dashed) line refers to total (private) insurance.

ure 2.2 we can see that mean consumption of group m is lower in final steady state, while for group s it is higher. Hence level effect is negative for group m but positive for group s.

Transition effect: After level effect, transition effect plays the most important role in the determination of utilitarian welfare. It is positive for group m and negative for group s. The intuition behind it is bit ambiguous. As mentioned in Section 2.5.1 all the effects which cannot be attributed to the inequality, uncertainty or level affect are summed up in the transition effect. More importantly, except for transition effect, all other welfare effects rely on the comparison of two steady states. However, looking at the change in mean of log consumption over the transition period, it decreased for group s initially but increased for group m. This is because, after the tax progressivity change, group s starts to save more whereas group m started to dis-save. In later periods, group s (m) experienced an increase (decrease) in consumption. Due to the presence of the subjective discount factor, the initial period consumption contributes to the lifetime utility more as compared to later period consumption. Hence transition effect is positive for group m, as their initial mean consumption increased. Opposite is true for group s.

2.6 Tauchen Calibration

As discussed in Section 2.4, calibration method of Castañeda et al. (2003) allows me to match the wealth distribution. However it fails in matching the insurance coefficient as it is observed in data. The objective of this section is to check whether the welfare results are robust to the specification in which calibration allows me to match the insurance coefficient.

As CPS does not provide the consumption data, I focus on the Consumption and Expenditure Survey (CEX). For this I use the same sample as used by Broer (2013) and Krueger and Perri (2006). In this sample, the income is supposed to capture the sources of household revenues which are independent of consumption and saving decisions. Hence income is after-tax and transfer labor earnings. The labor earnings is measured as the sum of wages and salaries of households members and the fixed fraction farm and non-farm income. Taxes refer to federal, state and local taxes and contributions to social security whereas transfers refer to welfare, food stamps and unemployment insurance. The measure of consumption refers to the expenditure on non-durable consumption and the flow of services from durable goods.

The measure of insurance, as defined above, is one minus the variance of residual consumption over residual income. I find that the insurance achieved by the married and single households, based on after-tax and transfer labor income, is 0.57 and 0.58 respectively. This value is close to the value found by Broer (2013) which is equal to 0.61, when marital status is not taken into consideration.

Table 2.6: Labour income calibration according to Tauchen (1986) method and target statistics.

Labour Income Statistic	**Symbol**	**Value**
Income persistence, AR(1)	ρ	0.9989
Variance labor income, married	$\mathrm{var}(y_t^m)$	0.34
Co-variance labor income, married	$\mathrm{Cov}(y_t^m, y_{t-1}^m)$	0.22
Variance labor income, single	$\mathrm{var}(y_t^s)$	0.38
Co-variance labor income, single	$\mathrm{Cov}(y_t^s, y_{t-1}^s)$	0.24
Target Statistic	**Data**	**Model**
Wealth to income ratio	2.50	2.50
Insurance, married	0.57	0.45
Insurance, single	0.58	0.45
Wealth Gini	0.71	0.22
% Wealth, bottom 60%	4.10	21.97
% Wealth, top 40%	95.80	78.03

Notes: The table reports the calibration and target statistics for the Tauchen (1986) calibration. The top table reports the calibration of the labor income process based on the Consumption and Expenditure Survey data. The bottom table reports the target statistics.

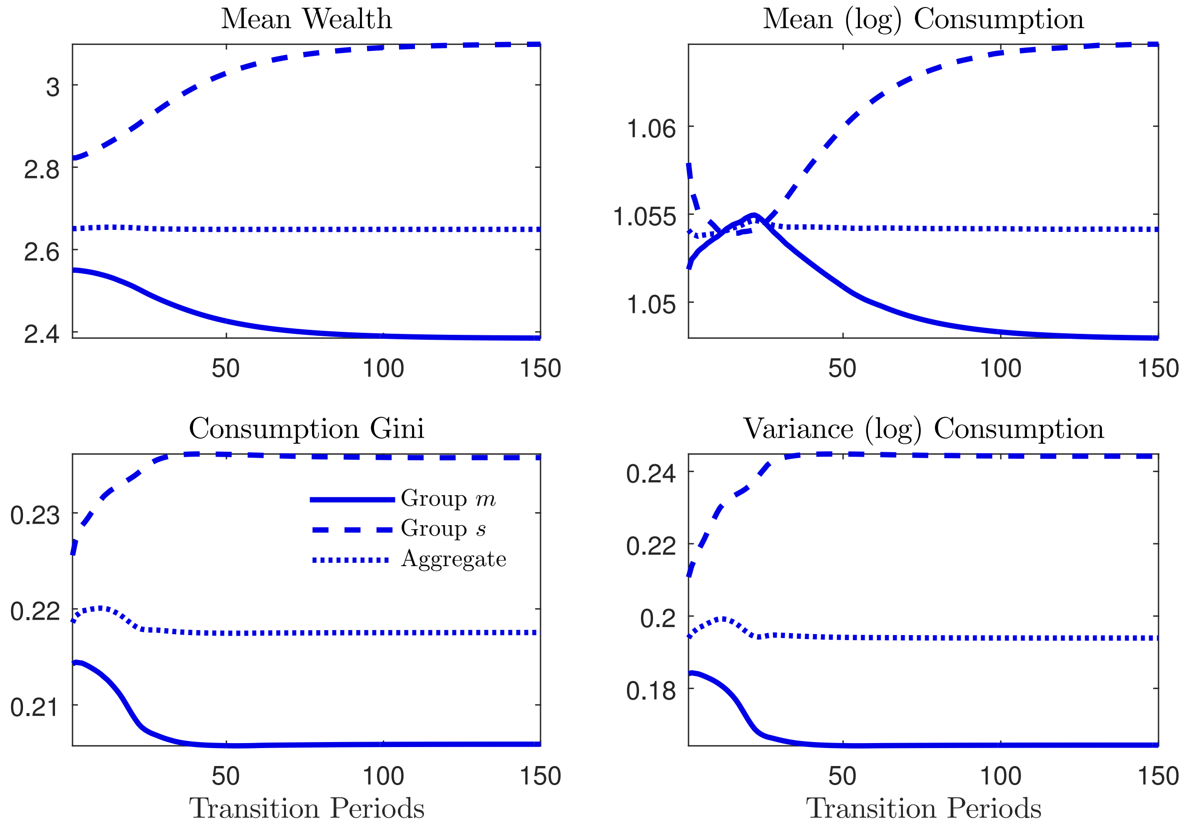
For the discretization of labor income, it is modeled as (I remove the superscript j because the same procedure is followed for married and single group)

$$\log(y_t) = z_t + \epsilon_t, \quad z_t = \rho z_{t-1} + \eta_t \tag{2.15}$$

where ϵ_t, η_t are independent, serially uncorrelated and normally distributed with mean zero and variances σ_ϵ^2 and σ_η^2 respectively. Following Storesletten et al. (2004) I set $\rho = 0.9989$. Using the CEX sample for the period 1991-2003 I estimate the values of σ_ϵ^2 and σ_η^2 for married and single group. Then using the Tauchen and Hussey (1991) method, the income process is approximated by five persistent and two transitory income states.

Table 2.6 reports the calibrated labor income and the target statistics. All other calibration parameters are the same as in previous calibration. As predicted, this calibration matches the insurance coefficient and aggregate wealth to income ratio reasonably well, but fails in matching

Figure 2.4: Evolution of endogenous variable in response to the change in tax progressivity under Tauchen (1986) calibration.



Notes: The figure reports the evolution of endogenous variables in response to the change in tax progressivity which mimics the change in public insurance as reported in Figure 2.1. For this figure, the model is calibrated using the Tauchen (1986) method.

the wealth inequality. From SCF 1992 survey, the bottom 60 percent of the population had 4.1 percent of the total wealth where as the model predicts 21.97 percent. Same mis-prediction holds for the top 40 percent of the population.

Figure 2.4 reports the evolution of the variables in response to the same tax progressivity changes as was used for Figure 2.2. We can see that the change in calibration procedure has no impact on the qualitative evolution of the variables over the transition path. Although the aggregate mean wealth and mean consumption do not respond to the change in tax progressivity, group specific variables exhibit significant heterogeneous behavior. But the magnitude of the changes is different. For the Castañeda et al. (2003) type calibration, mean wealth of group s increased by

Table 2.7: Percentage welfare change and decomposition, Tauchen (1986) calibration.

Welfare measure	Group m	Group s
Utilitarian, steady state (Δ^j_{ss})	- 0.23	0.53
Utilitarian (Δ^j)	0.02	-0.11
Inequality (Δ^j_{ineq})	0.14	-0.20
Uncertainty (Δ^j_{unc})	-0.00	0.08
Level (Δ^j_{lev})	-0.37	0.65
Transition (Δ^j_{trans})	0.24	-0.63

Notes: The table reports the welfare measures for the married and single households in response to the change in tax progressivity which mimics the change in public insurance as reported in Figure 2.1. For this table, the model is calibrated using the Tauchen (1986) method.

11.52 percent, but in present calibration it only increased by 8.18 percent. Does these different magnitudes have implications for the welfare effect?

Table 2.7 reports the utilitarian welfare change and its decomposition. The most important point to notice is that the welfare effects are qualitatively robust to the change in the calibration and the intuition from the previous calibration follows. However their combination leads to the welfare gain for group m (consumption increases by 0.02 percent) whereas in previous calibration it decreased by 0.08 percent if transition is taken into account. Group s still experiences the decrease in consumption by 0.1 percent as compared to in previous calibration, where its consumption decreased by 0.2 percent. The overall conclusion of this exercise is that under Tauchen calibration, the magnitude of welfare effects are muted, but qualitative effects remain the same.

2.7 Conclusion

The objective of this paper was two-fold: empirical and quantitative. As an empirical contribution, using the March Current Population Survey dataset I show that the tax and transfer policies have benefited married households more in terms of the public insurance against the labor income risk. Over the sample period 1992-2015, the public insurance received by the married households increased by 19 percent, but the single households experienced a 13 percent decrease. As public insurance effects the saving behavior of individuals, the relative change has an implication for the total insurance and welfare in an economy. Hence, for the quantitative contribution, I expanded the standard incomplete markets model to include two groups of households: married and single. The main contribution was in showing that relatively more public insurance to the married households can increase their total insurance, but decrease their welfare. Hence better insurance and better welfare do not go hand in hand if there is a relative change in public insurance. This counter-intuitive result comes into the picture because, in response to the relative change in public insurance, the married households decrease their savings and the single households increase their savings.

Future directions: To make the exposition simple, this paper made two strong assumptions: first, the labor supply is assumed to be inelastic and second, the decision to remain single or married is determined by the exogenous probability. In future, this paper intends to relax these two assumptions. The second important direction which this paper can take is to understand how the change in tax progressivity affects the within household decision making about the labor supply.

Chapter 3

Optimal Monetary Policy Under Sector Interconnections

3.1 Introduction

The recent monetary policy literature has been concerned with optimal monetary policy in a multi-sector economy (Aoki (2001), Woodford (2010) and Petrella and Santoro (2011)). The focus of this literature is on two-sector models in which sectors differ in terms of price stickiness. The monetary authority then faces the problem of stabilizing an appropriately-defined inflation measure and an appropriately-defined output gap. This problem differs in two ways from that of optimal monetary policy in a one-sector New-Keynesian model: the presence of price indices for individual sectors introduces a relative price component in the economy, while aggregate inflation is a weighted average of the sectoral inflation rates. These complications make the problem of optimal monetary policy non-trivial. For example, in the context of a two-sector economy with a sector with fully flexible prices and a sector with sticky prices, Aoki (2001) shows that the optimal monetary policy involves targeting the inflation rate in the sticky-price sector. The intuition for this result is that while price fluctuations in the flexible sector lead to only transitory and non-distorting relative price fluctuations, relative price fluctuations originating in the sticky-price sector cause distortions in resource allocation and, hence, reduce welfare.

Although the result of Aoki (2001) provides an important insight, it is highly theoretical in nature. For example, the empirical results by Bils and Klenow (2004) suggests that there does not exist a sector with fully flexible prices, although they do find that price stickiness differs across sectors.[1] In particular Bils and Klenow (2004) report that the sector with the lowest degree of price stickiness is transportation sector where 40 percent of the prices change every month, while the most sticky sector is medical care where only 10 percent of the prices are revised every month. Moreover, Kim and Kim (2006) and Holly and Petrella (2012) argue that inter-sectoral factor demand linkages are important for the transmission of both sectoral and aggregate shocks.

Keeping these two empirical facts in mind, this paper answers the following question: "what is the optimal monetary policy in a multi-sector economy, in which the sectors differ in the degree of price stickiness and are interconnected through inter-sectoral factor demand linkages?" In order to answer this question we develop a two-sector New-Keynesian model with two specific features. First, the sectors differ in terms of price stickiness, while none of the sectors are fully flexible. Second, the sectors are interconnected through inter-sectoral factor demand linkages.[2] To introduce factor demand linkages we assume that intermediate firms use final output as the input in producing goods. This assumption is justified by the fact that most sectors produce both for final product use and for use of their products by other sectors.

The objective of the present paper is to explore how factor demand linkages and the difference in price stickiness affect optimal monetary policy. To analytically characterize the latter we follow Woodford (2003) and derive the loss function and the implied optimal policy rule. We show that both factor demand linkages and the difference in price stickiness play an important role determining the optimal policy rule. As the factor demand linkage becomes stronger, implying that a larger fraction of the final produce is used as input in production and less labour is employed, the coefficients associated with the endogenous variables in the optimal interest rate rule tend towards zero.

[1] See Table 2 in Bils and Klenow (2004).

[2] For brevity, from now on we will refer to "intersectoral factor demand linkages" as "factor demand linkages".

This is because consumption responds less to productivity shocks as the labor share in production becomes relatively less important due to stronger factor demand linkages. This implies that in an economy with extremely high factor demand linkages, the monetary authority will be less concerned with stabilizing inflation as a given inflation rate leads to smaller consumption distortions. As for the difference in price stickiness, with one sector becoming relatively more sticky compared to the other sector, the monetary authority will react relatively more aggressively to inflation in the more sticky price sector.

Factor demand linkages play an important role, because they amplify economic fluctuations. When a sector experiences a positive productivity shock, this raises individual consumption through an increase in the real wage. The resulting increase in the demand for final output leads to an increase in the demand from both intermediate sectors, which in turn raises demand for final output further, because the latter serves as an input into intermediate goods production. Hence, a multiplier process arises through which economic fluctuations are magnified.

Our final analytical result concerns the characterization of the optimal price index. We show that the optimal price index targeted by the monetary authority is the weighted sum of the prices in the two sectors, with the weights being a non-linear function of the degrees of price stickiness of the two sectors. The weight of a sector increases with the price stickiness of the sector. Hence, in the optimal price index prices of more sticky sector get a higher weight. This corroborates the finding of Aoki (2001), but in a more general setting. Importantly, the factor demand linkage does not affect the optimal price index, although it affects the trade-off between stabilization of the output gap and stabilization of the optimal price index.

Section 3.4 contains our most interesting and novel result. We show that for relatively low values of the factor demand linkage and holding constant the price stickiness in the flexible sector, at relatively low levels of sticky-sector price stickiness the welfare cost associated with sectoral productivity shocks is increasing in the degree of sticky-sector price stickiness and decreasing at relatively high levels of price stickiness. This result is novel, because the standard finding is that an increase in price stickiness produces higher welfare losses when monetary policy is chosen optimally. The intuition for our result is as follows. In the standard model the monetary authority faces a trade-off between output and aggregate inflation stabilization. However, in the present framework, due to the presence of two sectors, the objective of the monetary policy becomes to stabilize output and two sectoral inflation rates. With the interest rate as the *only* instrument, the monetary authority can only trade off stabilization of two endogenous variables at a same time. The fluctuation of the remaining variable, in our case the more flexible sector inflation, is dependent on the extent of the factor demand linkage in the economy.

We also explore the costs of sub-optimal policies. The first arises from the monetary authority not knowing the true value of the factor demand linkages in the economy. We find that setting monetary policy optimally, but based on an incorrect assumption about the strength of the factor demand linkages, only has limited a limited welfare cost. However, basing monetary policy on a welfare loss function with sectoral inflation weights based solely on relative sector sizes, rather than on an appropriate account of the differences in sectoral stickiness leads to substantially larger welfare losses.

Our paper is related to a number of contributions in the literature. First, while the current paper focuses on a closed economy, the modeling set-up borrows from a large monetary policy literature in an open-economy framework. Benigno (2004) finds that if the regions in a two-region currency area feature the same nominal rigidity, then optimal monetary policy focuses on weighted-average inflation with relative weights based on the relative economic sizes of the regions. Lombardo (2006) shows that this result breaks down in the presence of other types of asymmetries. Beetsma and Jensen (2005) adds to this literature by focusing on fiscal stabilization under commitment and discretion.As Horvath (1998, 2000) make clear, factor demand linkages are an important feature of any economy and determine to a large extent how the sectoral and the aggregate shocks are propagated through the economy. In a recent paper Bouakez et al. (2009) shows how the co-movement between spending on durables and non-durables due to monetary policy shock can be explained by the presence of factor demand linkages. Our paper complements this work by exploring the role of optimal monetary policy in presence of factor demand linkages. However, closest in spirit to our paper is Petrella and Santoro (2011). They also explore optimal monetary policy under factor demand linkages. However, apart from a number of modelling differences, our paper also differs from that paper in terms of the analysis. We explicitly derive the optimal policy rule that allow us to characterize analytically how the weights associated with the different endogenous variables vary with respect to the sectors' relative weights in the economy, the different degrees of price stickiness of the sectors and strength of the factor demand linkages. This is important, as we show that the monetary authority responds in a non-trivial way to variations in the factor demand linkages. Furthermore, we also explore welfare losses for situations in which monetary policy is sub-optimal.

The remainder of this paper is organized as follows. Section 3.2 presents the model, Section 3.3 derives the sticky price equilibrium and the optimal monetary policy rule. Section 3.4 presents the results. Section 3.5 concludes.

3.2 Model

There are three types of agents in the economy: a representative household, firms and the government. The economy is divided into two sectors that differ in their degrees of price stickiness. The more sticky sector is referred to as sector s, while the less sticky sector is referred to as sector f. To model the factor demand linkage in an intuitive way, we assume that intermediate firms buy output from final output producers and use this as an input into the production of their own product. As the final output producers combine outputs from all the intermediate producers in order to produce final output, this assumption connects all intermediate firms with each other and introduces the factor demand linkage. The government runs a balanced budget in each period, which allows us to eliminate one constraint in solving the model. Price stickiness introduces a nominal friction whereas monopolistic competition introduces a real rigidity. Each component of the economy is described in detail below.

3.2.1 Households

The representative household's lifetime utility is

$$\max_{\{C_t,N_t,B_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right], \tag{3.1}$$

where β is the subjective discount factor, C_t is the consumption index of the representative household and N_t is the labor supplied. The consumption index C_t is the combination of the two indices of the consumption goods produced in the sectors s and f, to be discussed later. The period t inter-temporal budget constraint of the household is

$$P_t C_t + \mathbb{E}_t Q_{t,t+1} B_{t+1} \leq W_t N_t + B_t + \Upsilon_t - T_t, \tag{3.2}$$

where P_t is the price index associated with the consumption index, B_{t+1} is the portfolio of one-period state-contingent Arrow-Debreu securities, $Q_{t,t+1}$ is the price of B_{t+1}, W_t is the nominal wage, Υ_t are firm profits and T_t stands for net tax payments. The objective of the household is to maximize lifetime utility (3.1) given the period-t budget constraint (3.2). The first-order conditions of the household's problem are given as

$$\frac{N_t^{\phi}}{C_t^{-\sigma}} = \frac{W_t}{P_t}, \qquad 1 = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t}{\Pi_{t+1}} \right], \tag{3.3}$$

where R_t is the risk-free short-term nominal interest rate. The first equation regulates the intratemporal trade-off between consumption and leisure, while the second equation is the Euler equation for consumption. Following Woodford (2003), we have assumed that there are no monetary frictions making it worthwhile for households to hold liabilities of the monetary authority that earn a sub-standard rate of return. We further assume that the monetary authority can control the short-term nominal interest rate. Ruling out arbitrage we can write $R_t = \frac{1}{\mathbb{E}_t Q_{t,t+1}}$. The gross inflation rate is given as $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$. For future use, we define the nominal stochastic discount factor as

$$Q_{t,\tau} = \beta^{\tau-t} \frac{U_{C,\tau}}{U_{C,t}} \frac{P_t}{P_\tau},$$

where U denotes the period utility function and $U_{C,t}$ denotes the marginal utility of consumption in period t. The expression is the factor by which financial markets discount random nominal income in any future period τ to assess the claim to that income in period t.

3.2.2 Firms

The economy is divided into the more-sticky price sector s and the less-sticky price sector f. There is a unit mass of monopolistically competitive intermediate goods producing firms located on the interval $[0, 1]$, with the sector s firms located on the sub-interval $[0, n)$ and sector f firms located on the sub-interval $[n, 1]$. Firms are indexed by the subscript i. Firms in both sectors can change prices under the Calvo (1983) price-setting mechanism. Hence, every period a fixed fraction of the firms in each sector gets the chance to reset their prices. The fraction of price-resetting firms in the more-sticky sector is lower than in the less-sticky sector. For production each firm hires labor in a competitive market, buys input from the final goods producers and produces output using the given production technology.

Besides the intermediate goods producers, there are the final goods producers. A key extension of the existing literature is that the intermediate goods producers use final output as input into their own production. Because the final output producers combine inputs from all the intermediate goods producers into the final product, this introduces factor demand linkages.

The final goods producers are perfectly competitive and operate in two stages.[3] First, using a Dixit and Stiglitz (1977) CES aggregation technology they combine the various intermediate

[3] By "perfectly competitive" we mean that the final goods producers are price takers. Hence, they take the price of final output and the prices charged by the intermediate producers as given.

outputs produced in a given sector into a sectoral product. Second, again using a CES aggregation technology they combine the two sectoral products into the final product of the economy. Aggregate final output is

$$Y_t = \frac{Y_{s,t}^{\eta} Y_{f,t}^{1-\eta}}{\eta^{\eta}(1-\eta)^{1-\eta}},$$

where $Y_{s,t}$ and $Y_{f,t}$ are the sectoral aggregate outputs, where the parameter η captures the weight of sector s in the final product. The aggregate final-good price index is a composite of price indices $P_{s,t}$ and $P_{f,t}$ of the sectors s, respectively f:

$$P_t = P_{s,t}^{\eta} P_{f,t}^{1-\eta}. \tag{3.4}$$

The final goods producers' problem is to minimize the cost of production. The resulting demands for input from the two sectors are

$$Y_{s,t} = \eta T_t^{\eta-1} Y_t, \qquad Y_{f,t} = (1-\eta) T_t^{\eta} Y_t, \tag{3.5}$$

where $T_t = \frac{P_{s,t}}{P_{f,t}}$ is the relative price index of the two sectors.

The sector-level aggregation technologies are

$$Y_{s,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n Y_{s,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \qquad Y_{f,t} = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\epsilon}} \int_n^1 Y_{f,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}},$$

where ϵ is the elasticity of substitution between each pair of intermediate goods, while $Y_{a,t}(i)$ is the output of intermediate producer i in sector $a = \{s, f\}$. Hence, the demand for each intermediate product is

$$Y_{s,t}(i) = \frac{1}{n}\left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{-\epsilon} Y_{s,t}, \qquad Y_{f,t}(i) = \frac{1}{1-n}\left(\frac{P_{f,t}(i)}{P_{f,t}}\right)^{-\epsilon} Y_{f,t}, \tag{3.6}$$

where $Y_{a,t}(i)$ is the demand faced by intermediate producer i in sector a, $P_{a,t}(i)$ is the price charged by producer i in sector a, and $P_{a,t}$ is the sector a price index. The sectoral price indices are defined as

$$P_{s,t} = \left[\frac{1}{n}\int_0^n P_{s,t}(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}, \qquad P_{f,t} = \left[\frac{1}{1-n}\int_n^1 P_{f,t}(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}. \tag{3.7}$$

The production technology of intermediate producer i in sector $a = \{s, f\}$ is given by

$$Y_{a,t}(i) = Z_{a,t} X_{a,t}(i)^{\chi} N_{a,t}(i)^{1-\chi}$$

where $Z_{a,t}, a = \{s, f\}$ is a sector-specific productivity shock, $X_{a,t}(i)$ is the amount of the final output used as input by producer i in sector a and $N_{a,t}(i)$ is the labor input. The parameter χ is the share of final output as input in the production of $Y_{a,t}(i)$. Hence, a higher value of χ implies a stronger factor demand linkage in the economy.

The intermediate good producer faces two problems. First, given the demand for its output and taking the prices of the production factors as given, she minimizes the cost of production. This yields her inputs demands:

$$X_{a,t}(i) = \Gamma^{1-\chi} \left(\frac{W_t}{P_t}\right)^{1-\chi} \frac{Y_{a,t}(i)}{Z_{a,t}}, \qquad N_{a,t}(i) = \Gamma^{-\chi} \left(\frac{W_t}{P_t}\right)^{-\chi} \frac{Y_{a,t}(i)}{Z_{a,t}}, \quad a = \{s, f\}, \tag{3.8}$$

where $\Gamma = \frac{\chi}{1-\chi}$ is a constant. As the production technology of each intermediate producer has constant returns to scale, the real marginal cost *within* a sector will be the same for all producers and is given as

$$MC_{a,t} = \frac{\Gamma^{1-\chi}}{\chi} \left(\frac{W_t}{P_t}\right)^{1-\chi} \frac{1}{Z_{a,t}}, \qquad a = \{s, f\} \tag{3.9}$$

Second, assuming a Calvo (1983) price setting mechanism, whenever the intermediate good producer gets the chance to reset her price, she does this with the aim of maximizing nominal profits, while taking into account the uncertainty of again being able to revise this price in the future. Hence, she resets her price to:

$$P^*_{a,t} = \frac{\epsilon}{(\epsilon - 1)(1 + \tau_a)} \frac{\mathbb{E}_t \sum_{\tau=t}^{\infty} (\theta_a \beta)^{\tau-t} MC_{a,\tau} C_\tau^{-\sigma} P_{a,\tau}^{\epsilon-1} P_\tau Y_\tau}{\mathbb{E}_t \sum_{\tau=t}^{\infty} (\theta_a \beta)^{\tau-t} C_\tau^{-\sigma} P_{a,\tau}^{\epsilon-1} Y_\tau} \tag{3.10}$$

where $1 - \theta_a$ is the fraction of firms in sector a that get the chance to reset their price and τ_a is the subsidy provided by the government to intermediate producers in sector a. Hence, parameter θ_a measures the degree of price stickiness in sector a. Notice that the new optimal price is the same for all the producers within the same sector.

The remainder of this subsection focuses on sector s, while the derivations for sector f yield corresponding expressions. For solution purposes, following Woodford (2010), we can write (3.10) as

$$\mu_s \frac{H_{s,t}}{K_{s,t}} = T_t^{1-\eta} \left[\frac{1}{1-\theta_s} - \left(\frac{\theta_s}{1-\theta_s} \right) \Pi_{s,t}^{\epsilon-1} \right]^{\frac{1}{1-\epsilon}} \tag{3.11}$$

where $\Pi_{s,t}$ is sector level inflation in sector s and $\mu_s = \frac{\epsilon}{(\epsilon-1)(1+\tau_s)}$. It is convenient to write $H_{s,t}$ and $K_{s,t}$ in recursive form as

$$H_{s,t} = \frac{\Gamma^{1-\chi}}{\chi} \frac{1}{Z_{s,t}} C_t^{-\sigma\chi} N_t^{\phi(1-\chi)} T_t^{(1-\eta)(\epsilon-1)} Y_t + \theta_s \beta \mathbb{E}_t \left(\Pi_{s,t+1}^{\eta\epsilon} \Pi_{f,t+1}^{(1-\eta)\epsilon} H_{s,t+1} \right) \tag{3.12}$$

$$K_{s,t} = C_t^{-\sigma} T_t^{(1-\eta)(\epsilon-1)} Y_t + \theta_s \beta \mathbb{E}_t \left(\Pi_{s,t+1}^{\eta(\epsilon-1)} \Pi_{f,t+1}^{(1-\eta)(\epsilon-1)} K_{s,t+1} \right) \tag{3.13}$$

Aggregating the outputs produced by sector s, one can write

$$Y_{s,t} D_{s,t} = Z_{s,t} X_{s,t}^{\chi} N_{s,t}^{1-\chi} \tag{3.14}$$

where $D_{s,t} = \frac{1}{n} \int_0^n \left(\frac{P_{s,t}(i)}{P_{s,t}} \right)^{-\epsilon} di$ is the measure of price dispersion in sector s arising from the price stickiness. Using Jensen's inequality it can be proved that $D_{s,t} \geq 1$. Hence, compared to the situation with flexible prices, price stickiness depresses the aggregate sectoral production in the sector, because the price dispersion produces a suboptimal composition of the individual intermediates that are aggregated into the composite sectoral product. The price dispersion $D_{s,t}$ can be written in a recursive form as

$$D_{s,t} = (1-\theta_s) \left[\frac{1}{1-\theta_s} - \left(\frac{\theta_s}{1-\theta_s} \right) \Pi_{s,t}^{\epsilon-1} \right]^{\frac{\epsilon}{\epsilon-1}} + \theta_s \Pi_{s,t}^{\epsilon} D_{s,t-1} \tag{3.15}$$

3.2.3 The Government

The government runs a balanced budget. Hence, aggregate government purchases G_t equal aggregate net tax revenues T_t:

$$G_t = T_t \equiv \tau_s P_{s,t} Y_{s,t} + \tau_f P_{f,t} Y_{f,t},$$

where $\tau_a,\ \ a = \{s, f\}$ is the rate at which the revenues of sector a are taxed.

3.2.4 The Exogenous Processes

The exogenous process $Z_{a,t}$ of sector a is given by

$$Z_{a,t} = Z_{a,t-1}^{\rho_{z,a}} \exp(\epsilon_{a,t}), \tag{3.16}$$

Hence, the logarithm of $Z_{a,t}$ follows a first-order autoregressive processes with AR(1) persistence parameter $\rho_{z,a}$ and shock $\epsilon_{a,t}$, which is assumed to be independent, serially uncorrelated and normally distributed with mean zero and variance σ_a^2.

3.2.5 Competitive Equilibrium

A stationary competitive equilibrium is defined as the set of stationary processes W_t/P_t, N_t, C_t, R_t, Y_t,$Y_{s,t}$, $Y_{f,t}$, $X_{s,t}$, $X_{f,t}$, T_t, $N_{s,t}$, $N_{f,t}$, $H_{s,t}$, $H_{f,t}$, $K_{s,t}$, $K_{f,t}$, Π_t, $\Pi_{s,t}$, $\Pi_{f,t}$, $D_{s,t}$, $D_{f,t}$, $Z_{s,t}$, $Z_{f,t}$ such that the first-order conditions for an optimum hold and the goods and labor markets clear. Two additional conditions are given by relative price identity, $T_t = \frac{\Pi_{s,t}}{\Pi_{f,t}} T_{t-1}$, and the aggregate inflation identity, $\Pi_t = \Pi_{s,t}^{\eta} \Pi_{f,t}^{1-\eta}$. The initial values $Z_{s,-1}, Z_{f,-1}, T_{-1}, D_{s,-1}$ and $D_{f,-1}$, given. We still lack one equation to solve the model, which is the monetary policy rule implemented by the central bank. We will derive it in the next section.

3.3 Sticky Price Equilibrium and Loss Function

Until further notice, we assume that the output subsidy τ_a for sector $a = \{s, f\}$ is set such that it removes the monopolistic competition distortion. Hence, $\tau_a = \frac{1}{\epsilon - 1}$. We then proceed as follows. First, given that the steady state is efficient, we log-linearize the model around this efficient steady state. Second, following the lead of Woodford (2003) and using the log-linearized equations, we derive the loss function. Based on the loss function, we finally derive the optimal monetary policy rule.

In the sequel, for a generic variable X, we define $\widetilde{x} = \widehat{x} - x^*$, where $\widehat{x}$ is the log-linearized value of x under the sticky-price equilibrium and x^* is the log-linearized value of x under the flexible-price equilibrium. Flexible prices refer to the situation in which all intermediate good producers can change the price of their product each period.[4]

3.3.1 The Sticky-Price Equilibrium

Here, we provide a first-order log-linearization of the structural equations. Under a first-order approximation, our price dispersion measure does not play any role, as it is of second order. The dynamics under sticky prices, derived in Appendix C, are given by

$$\widetilde{y}_t = \mathbb{E}_t \widetilde{y}_{t+1} - \Xi(r_t - \mathbb{E}_t(\eta \pi_{s,t+1} + (1-\eta)\pi_{f,t+1}) - r_t^{n*}) \tag{3.17}$$

$$\pi_{s,t} = \kappa_s \{\Theta \widetilde{y}_t - (1-\eta)\widetilde{T}_t\} + \beta \mathbb{E}_t \pi_{s,t+1} \tag{3.18}$$

$$\pi_{f,t} = \kappa_f \{\Theta \widetilde{y}_t + \eta \widetilde{T}_t\} + \beta \mathbb{E}_t \pi_{f,t+1} \tag{3.19}$$

$$\Delta \widetilde{T}_t = \pi_{s,t} - \pi_{f,t} - \Delta T_t^* \tag{3.20}$$

[4] Note that we can define the output gap here as the difference between the logarithm of output under the sticky-price and the flexible-price equilibrium. This is so, because with the appropriate subsidy in place, allocations under flexible prices correspond to those under the efficient equilibrium.

where $\Delta T_t^* \equiv \Delta z_{f,t} - \Delta z_{s,t}$, $\Xi \equiv \sigma^{-1}(1 + \chi(\phi + \sigma))$, $\Theta \equiv \frac{(1-\chi)(\phi+\sigma)}{1+\chi(\phi+\sigma)}$ and Δ is the first difference operator. Finally, T_t^* is the log-linearised value of the relative price under the flexible-price equilibrium in period t. Hence, we have four equations and five endogenous variables, i.e. $\widetilde{y}_t, \pi_{s,t}, \pi_{f,t}, \widetilde{T}_t$ and r_t. We still need a monetary policy equation to close the model and solve for the complete set of endogenous variables.

We avoid choosing an arbitrary monetary policy equation, but rather derive an optimal rule based on the micro-foundations provided by our model. To this end, we derive the relevant loss function by taking the second-order approximation to the utility function of the representative household. Note that, with the appropriate subsidy in place, the flexible-price equilibrium corresponds to the efficient equilibrium and we only need a first-order approximation of the structural equations in order to derive the second-order approximation of the utility function.[5]

The loss function is

$$L = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\lambda_y \widetilde{y}_t^2 + \lambda_{\pi,s} \pi_{s,t}^2 + \lambda_{\pi,f} \pi_{f,t}^2 \right), \tag{3.21}$$

where $\kappa_1 \equiv \left(\frac{1}{1+\chi(\phi+\sigma)}\right)$, $\kappa_s \equiv \frac{(1-\theta_s\beta)(1-\theta_s)}{\theta_s}$, $\kappa_f \equiv \frac{(1-\theta_f\beta)(1-\theta_f)}{\theta_f}$, $\lambda_y \equiv \left(\frac{\sigma+\phi}{2}\right)\kappa_1^2$, $\lambda_{\pi,s} \equiv \frac{\eta\epsilon}{2\kappa_s}$ and $\lambda_{\pi,f} \equiv \frac{(1-\eta)\epsilon}{2\kappa_f}$. It is easy to see that κ_a for sector a is decreasing in θ_a, hence $\lambda_{\pi,a}$ is increasing in θ_a. Further, if θ_a converges zero, then κ_a goes to infinity and $\lambda_{\pi,a}$ goes to zero. In maximizing the representative individual's utility, the monetary authority trades off the output gap against the sector level inflation rates, given the optimal weights just stated. In the extreme case in which the prices in sector f is fully flexible, the price index of this sector does not enter the loss function. Hence, fluctuations in these prices leave the representative individual's welfare unaffected. This result is in line with Aoki's (2001) finding that in an economy with a sticky- and a fully flexible price sector, the monetary authority should ignore the price fluctuations in the latter sector. Hence, we show here that this result also holds in the presence of factor demand linkages.

[5] In order to ensure that the real wage is the same across individuals, even though there are two sectors, we can either assume perfect income risk sharing among the individuals or the existence of a perfectly competitive labour market.

The key contribution of this paper is to determine the role of factor demand linkages for the economy. Proposition 6 shows how they affect individual welfare:

Proposition 6 *The weights associated with sector level inflation relative to output in the individual's loss function are positively related to the strength χ of the factor demand linkage.*

Proof. Note that the weight associated with inflation in sector a, $\lambda_{\pi,a}$, does not depend on factor demand linkage parameter χ. However, the weight associated with the output gap, λ_y, does depend on χ. It is easy to show that $\frac{\partial \lambda_y}{\partial \chi} < 0, \chi \in (0, 1)$. Hence, the weights of sectoral inflation relative to output, $\frac{\lambda_{\pi,s}}{\lambda_y}$ and $\frac{\lambda_{\pi,f}}{\lambda_y}$, are increasing in χ. ■

The intuition for this proposition is as follows: a stronger factor demand linkage amplifies the response of the economy to technology shocks. Therefore, the misallocation of resources resulting from sectoral inflation rates deviating from their steady states starts to weigh relatively more heavily in the individual's loss function.

3.3.2 The Monetary Policy Rule

The aim of the central bank in to minimize the loss function (3.21), subject to the underlying economic dynamics given by (C.5), (3.19), and (C.9). We assume that the central bank is not constrained in setting the short-term nominal interest rate r_t and, hence, there is no need to include the Euler equation (C.4) as a constraint[6]. Hence, we can write the Lagrangian as

$$\begin{aligned}\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ &\frac{1}{2} \left(\lambda_y \widetilde{y}_t^2 + \lambda_{\pi,s} \pi_{s,t}^2 + \lambda_{\pi,f} \pi_{f,t}^2 \right) \\ &+ \Lambda_{s,t} (\pi_{s,t} - \kappa_s (\Theta \widetilde{y}_t - (1-\eta) \widetilde{T}_t) - \beta \mathbb{E}_t \pi_{s,t+1}) + \Lambda_{f,t} (\pi_{f,t} - \kappa_f (\Theta \widetilde{y}_t + \eta \widetilde{T}_t) - \beta \mathbb{E}_t \pi_{f,t+1}) \\ &+ \Lambda_{T,t} (\widetilde{T}_t - \widetilde{T}_{t-1} - \pi_{s,t} + \pi_{f,t} + T_t^* - T_{t-1}^*) \Big\},\end{aligned}$$

[6] Through simulations we verify that the short-term nominal interest rate never touches the zero lower bound and, hence, that the monetary authority is never constrained in setting r_t. As long as this is the case, the Lagrange multiplier associated with the Euler equation will be zero.

where $\Lambda_{s,t}, \Lambda_{f,t}$ and $\Lambda_{T,t}$ are the period-t Lagrange multipliers associated with (C.5), (3.19), respectively (C.9). The first-order conditions with respect to $\pi_{s,t}, \pi_{f,t}, \widetilde{y}_t$ and $\widetilde{T}_t$ are, respectively,

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \pi_{s,t}} &= 0 \Rightarrow \lambda_{\pi,s}\pi_{s,t} + \Lambda_{s,t} - \Lambda_{s,t-1} - \Lambda_{T,t} = 0, \\
\frac{\partial \mathcal{L}}{\partial \pi_{f,t}} &= 0 \Rightarrow \lambda_{\pi,f}\pi_{f,t} + \Lambda_{f,t} - \Lambda_{f,t-1} + \Lambda_{T,t} = 0, \\
\frac{\partial \mathcal{L}}{\partial \widetilde{y}_t} &= 0 \Rightarrow \lambda_y \widetilde{y}_t - \kappa_s \Theta \Lambda_{s,t} - \kappa_f \Theta \Lambda_{f,t} = 0, \\
\frac{\partial \mathcal{L}}{\partial \widetilde{t}_t} &= 0 \Rightarrow \kappa_s (1-\eta)\Lambda_{s,t} - \kappa_f \eta \Lambda_{f,t} + \Lambda_{T,t} - \beta \mathbb{E}_t \Lambda_{T,t+1} = 0.
\end{aligned}
$$

In the appendix, we show that these first-order conditions together with (C.5), (3.19), and (C.9)) satisfy the Blanchard-Kahn conditions. Hence the solution of this system of difference equations is unique and bounded.[7] As the technology shock processes are assumed to be stationary, the long run values of the endogenous variables will correspond to their steady-state values. It can be observed that the steady-state values of the Lagrange multipliers associated with (C.5), (3.19) and (C.9) are zero. Hence, optimal monetary policy in the long run does not depend on the choice of the aggregate price index, because the sectoral inflation rates are both at their steady state values. However, in the short run, the monetary authority responds in an optimal way to technology shocks. To see how, sum the first two first-order conditions to yield,[8]

$$
\lambda_{\pi,s} p_{s,t} + \lambda_{\pi,f} p_{f,t} + \Lambda_{s,t} + \Lambda_{f,t} = \lambda_{\pi,s} p_{s,t-1} + \lambda_{\pi,f} p_{f,t-1} + \Lambda_{s,t-1} + \Lambda_{f,t-1}. \tag{3.22}
$$

From (3.22) we see that the combination $\lambda_{\pi,s} p_{s,t} + \lambda_{\pi,f} p_{f,t} + \Lambda_{s,t} + \Lambda_{f,t}$ is constant over time. Define $\widetilde{p_t} = \lambda_{\pi,s} p_{s,t} + \lambda_{\pi,f} p_{f,t}$. Hence, in response to a technology shock, the monetary authority

[7] We prove this claim in Appendix F.

[8] We use that $\pi_{a,t} = p_{a,t} - p_{a,t-1}$, where $p_{a,t} = \log P_{a,t}$.

optimally targets in the short run the price index $\widetilde{p_t}$ and not the index $p_t = \eta p_{s,t} + (1-\eta)p_{f,t}$. This leads us to Proposition 7:

Proposition 7 *The optimal price index for the monetary authority to target is* $\widetilde{p_t} = \lambda_{\pi,s} p_{s,t} + \lambda_{\pi,f} p_{f,t}$.

The weights associated with the optimal price index are independent of the factor demand linkages, but depend on the weight of each sector in the entire economy in combination with the degree of price stickiness in that sector. More price stickiness in a sector implies a higher weight of that sector in the optimal price index, while under full flexibility the sectoral price index drops out of the optimal price index, in line with Aoki (2001).

By eliminating the Lagrange multipliers, we can obtain the optimal monetary policy rule. It amounts to the following optimal interest rate rule (OIR):

$$\Delta r_t = \mathcal{A}_{\pi_s}\pi_{s,t} + \mathcal{A}_{\pi_f}\pi_{f,t} + \mathcal{A}_{\Delta\pi_s}\Delta\pi_{s,t} + \mathcal{A}_{\Delta\pi_f}\Delta\pi_{f,t} + \mathcal{A}_{\Delta\widetilde{y}_0}\Delta\widetilde{y}_t + \mathcal{A}_{\Delta\widetilde{y}_1}\Delta\widetilde{y}_{t-1} + \mathcal{A}_{\Delta T}\Delta\widetilde{T}_t + \Delta r_t^{n*} \tag{3.23}$$

where

$$\mathcal{A}_{\pi_s} \equiv \frac{\epsilon\sigma(1-\chi)\eta\kappa_f}{\beta}, \qquad \mathcal{A}_{\pi_f} \equiv \frac{\epsilon\sigma(1-\chi)(1-\eta)\kappa_s}{\beta}, \quad \mathcal{A}_{\Delta\pi_s} \equiv \frac{\eta}{\beta}, \qquad \mathcal{A}_{\Delta\pi_f} \equiv \frac{1-\eta}{\beta}$$
$$\mathcal{A}_{\Delta\widetilde{y}_0} \equiv \frac{\Theta}{\lambda_y\beta\Xi}\left[\frac{\lambda_y}{\Theta} + \frac{\Theta\eta\epsilon\kappa_s}{2} + \frac{\Theta(1-\eta)\epsilon\kappa_f}{2} - (\eta\kappa_s + (1-\eta)\kappa_f)\Xi\lambda_y\right] + \frac{1}{\beta\Xi}(\eta\kappa_f + (1-\eta)\kappa_s),$$
$$\mathcal{A}_{\Delta\widetilde{y}_1} \equiv -\frac{1}{\beta\Xi}, \quad \mathcal{A}_{\Delta T} \equiv -\frac{(\kappa_s-\kappa_f)\eta(1-\eta)}{\beta}\left[2\epsilon\sigma(1-\chi)-1\right].$$

A number of observations regarding (3.23) are warranted. First, the OIR describes a unit-root process, hence the short-term interest rate is a difference stationary process. Second, the OIR does not prescribe a purely contemporaneous relation between the short term nominal interest rate, inflation and the output gap. It contains lags of the output gap, the sectoral inflation rates, the relative price index and the natural rate. The presence of these lagged variables confirms the finding of Woodford (2010) that optimal monetary policy will be history dependent and any rule which takes into account only the future paths of inflation, the output gap and the relative price index will not be able to implement the optimal allocation. In other words, the OIR requires commitment, because under discretion the monetary authority would focus only on the future path of endogenous variables. Third, the OIR shown here is not unique. There exist other rules that can induce the same optimal equilibrium pattern of responses to exogenous disturbances. For example, one can substitute (C.5) into (3.23), and the resulting rule will give the same equilibrium responses as (3.23). However, the new rule will not be a direct rule (see Giannoni and Woodford (2003)), as it involves feedback from past, present and expected future real disturbances. Fourth, the OIR is robust in the sense that its derivation does not exploit any of the statistical properties of the technology shocks. Fifth, the formulation of the OIR is symmetric in the sector s and f variables. Sixth, note that $\Delta\widetilde{T}_t$ is given as $\pi_{s,t} - \pi_{f,t} - \Delta T^*$. Hence, (3.23) could be simplified by expressing it in terms of the responses to $\pi_{s,t}$, $\pi_{f,t}$ and ΔT^* (and the other variables). However, the current formulation makes clear that the optimal monetary policy rule responds to both the individual sector inflation developments and relative price movements between the sectors, in line with the fact that production losses arise from dispersion of prices within each sector and dispersion between the sectoral price levels. Finally, the rule is optimal from a timeless perspective (see Woodford (2003)).

Given that the values of χ and η lie in the interval $(0,1)$, the coefficients of $\pi_{s,t}, \pi_{f,t}, \Delta\pi_{s,t}$ and $\Delta\pi_{f,t}$ in (3.23) are positive. The monetary authority not only responds to higher inflation in each of the sectors by raising the nominal interest rate, but also tightens monetary policy in response to a positive change in inflation in each of the sectors. Also, the difference between the coefficients of $\pi_{s,t}$ and $\pi_{f,t}$ is positive, and, hence, the response to inflation in the more sticky sector is larger if $\frac{\kappa_f}{\kappa_s} > \frac{1-\eta}{\eta}$, which always holds under our calibration. Moreover, an increase in the price stickiness of sector s or a reduction in the price stickiness in sector f raises the difference in the coefficients of $\pi_{s,t}$ and $\pi_{f,t}$ and, hence, induces the monetary authority to raise the weight to inflation in the stickier sector compared to that in the more flexible sector.

3.3.3 Numerical Investigation of the OIR

To further enhance our intuition about the responses of the monetary authority to inflation, the output gap and relative price developments, we numerically investigate the weights attached to these variables in (3.23). Hence, we need to calibrate the model first. We focus on the US economy and calibrate our two sectors to the goods and services producing sectors. The services producing sector is the more sticky one and, hence, it will serve as the sector s in our model, while the goods producing sector serves as the sector f. We follow standard practice and calibrate the model at the quarterly frequency. Most of the parameters we can fix at standard values used in the literature. We set the discount factor β at 0.99, implying a steady-state quarterly nominal interest rate of roughly one percent. Because steady-state inflation is zero, also the quarterly real interest rate is approximately one percent. Using the evidence from Smets and Wouters (2007), the inter-temporal elasticity of substitution is set at 0.5 implying that $\sigma = 2$. The elasticity of substitution between differentiated goods is set at 11. The Frisch inverse elasticity of the labor supply is fixed at 2, [9] while the AR(1) parameters of the technology shocks are fixed at 0.95 and variances of the sectoral shocks are fixed at 0.009. The price stickiness parameters are based on Bils and Klenow (2004) and Nakamura and Steinsson (2008). The median price duration of the services producing sector is 13 months, whereas that of the goods producing sector is 3.3 months. This implies that $\theta_s = 0.77$ and $\theta_f = 0.09$. Following Horvath (2000) we set the expenditure share of services, hence the weight η of the sector s at 0.62, implying that the weight of the sector f is 0.38. To calibrate the factor demand linkage, we follow Petrella and Santoro (2011) and use the Bureau of Economic Analysis input-output table named "The Use of Commodities by Industries" for the year 1992[10]

[9] There is a debate about the value of the Frisch inverse elasticity of the labor supply. Micro estimates put the value below 1, while most of the macro calibrations assume a value above 2. Peterman (2016) points out that the micro and macro estimates differ, because of the use of different definitions of Frisch inverse elasticity of the labor supply.

Table 3.1: Calibration of Parameters.

Parameters	Symbol	Value	Source
Subjective discount factor	β	0.99	
Inter-temporal elasticity of substitution	$1/\sigma$	0.5	Smets and Wouters (2007)
Intra-temporal elasticity of substitution	ϵ	11	Petrella and Santoro (2011)
Frisch inverse elasticity of labor supply	ϕ	2	Peterman (2016)
Factor demand linkage	χ	0.307	Bureau of Economic Analysis
Weight of sector *s*	η	0.62	Horvath (2000)
Price stickiness sector *s*	θ_s	0.77	Nakamura and Steinsson (2008)
Price stickiness sector *f*	θ_f	0.09	Nakamura and Steinsson (2008)
Auto-correlation tech. shock sector *s*	ρ_s	0.95	Petrella and Santoro (2011)
Auto-correlation tech. shock sector *f*	ρ_f	0.95	Petrella and Santoro (2011)
Variance shock sector *s*, *f*	σ_s^2, σ_f^2	0.009	Smets and Wouters (2007)

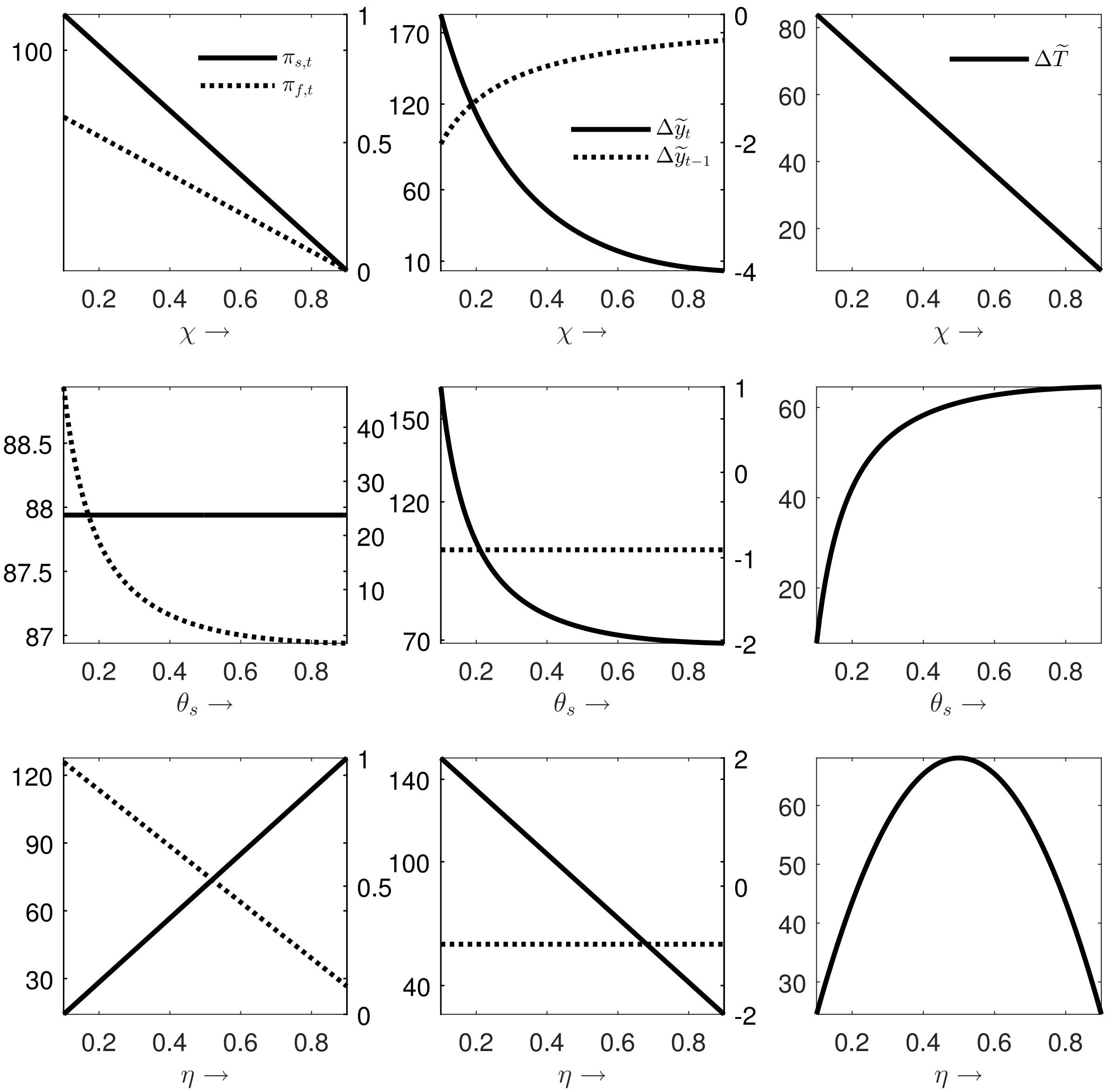
Notes: The table reports the baseline calibration for the deep parameters of the model.

. According to the table, the factor demand linkage from services to manufacturing is equal to $\chi = 0.41$, whereas from manufacturing to services it is equal to 0.14. We use the sector weights to compute the weighted average of factor demand linkage, which is equal to $\chi = 0.307$. This value represents the amount of input flow from the services sector to the goods producing sector. Table 3.1 summarizes the calibration.

Figure 3.1 plots the coefficients of $\pi_{s,t}, \pi_{f,t}, \Delta\widetilde{y}_t$, $\Delta\widetilde{y}_{t-1}$ and $\Delta\widetilde{T}_t$ in the optimal interest rate rule as a function of the factor demand linkage (top row), sector *s* price stickiness (middle row), and the sector *s* weight (bottom row) while keeping all the other parameters at the values listed in Table 3.1. The left panels reports the coefficients associated with $\pi_{s,t}$ and $\pi_{f,t}$, the middle panel report the coefficients associated with $\widetilde{y}_t, \widetilde{y}_{t-1}$ and $\widetilde{y}_{t-2}$ and the right panel report the coefficient associated with $\Delta\widetilde{T}_t$. Notice that the coefficients of $\Delta\pi_{s,t}$ and $\Delta\pi_{f,t}$ are proportional to the weights of the respective sectors and independent of the factor demand linkage and the degree of price stickiness. Hence, we do not plot them.

[10] As reported by Petrella and Santoro (2011), the input-output table is remarkably stable and hence, focusing on the year 1992 does not affect our analysis. We choose this year to make our results comparable to the previous studies which use this value of factor demand linkage. We also provide robustness results with respect to the different values of the factor demand linkage

Figure 3.1: Coefficients in the OIR (3.23) as functions of χ, θ_s and η.



Notes: The figure reports the responses of the coefficients attached to sectoral inflation, the current and lagged output gaps and the relative price in the optimal interest rate (3.23) when the three key parameters χ, θ_s and η are varied in the range $(0.1, 0.9)$ while all the other parameters are fixed at their calibrated values reported in the Table 3.1. The top row reports the result when χ is varied and other parameters are fixed. Similarly, middle (bottom) row reports the effects of varying θ_s (η). The left panel reports the coefficients associated with sectoral inflation, the middle panel reports the coefficients on the current and lagged output gaps and the right panel reports the coefficient on the relative price coefficient. For the left panels, the left axis refers to the coefficient on sector s inflation and the right panel refers to the coefficient on sector f. For the middle panels the left axis refers to $\Delta\widetilde{y}_t$ and the right axis refers to $\Delta\widetilde{y}_{t-1}$.

Now, we discuss the intuition behind the behaviour of the coefficients. First focus on the behavior of coefficients with respect to the change in the factor demand linkage. The coefficient of inflation in sector s indeed exceeds that of inflation in sector f, implying that monetary authority puts more emphasis on the inflation in sector s. Both coefficients fall with an increase in the factor demand linkage and they converge to zero when the factor demand linkage converges to unity. The same happens with the coefficients associated with $\Delta\widetilde{y}_t, \Delta\widetilde{y}_{t-1}$ and $\Delta\widetilde{T}_t$. The intuition is as follows: as χ represents the fraction of final output used as input by the intermediate firms, an increase in χ lowers the fraction of final output used for consumption. In the limit, when χ approaches one, there is no consumption and optimal monetary policy does not need to concern itself with stabilizing inflation, as it will not cause any welfare losses.

We now turn to the effect of sector s price stickiness (θ_s) on the coefficients in the OIR. An increase in θ_s implies that the monetary authority will put more emphasis on the sector s inflation and by extension aggregate inflation. This will also imply that inflation stabilization will become more important as compared to output stabilization. Hence, in absolute terms, the relative weight on sector s inflation will go up with respect to sector f inflation and output gap. The middle panels support this intuition. The weights associated with sector f inflation and output gap decrease relative to sector s inflation. Note that the weight associated with the output gap do not converge to zero. This is because factor demand linkage is still positive and the monetary authority still wants to stabilize output. The middle-right panel shows that the monetary authority puts more weight on the relative price gap as θ_s increases. To see the intuition, recall that $\Delta\widetilde{T}_t = \pi_{s,t} - \pi_{f,t} - \Delta T_t^*$. More price stickiness in the sticky sector also produces larger price dispersion between the sectors, hence leads to larger production losses, ceteris paribus. Hence, it is optimal for the monetary authority to respond more vigorously to a larger deviation of the difference between sectoral inflation rates from the movement in the efficient inflation differential.

Finally, the bottom row reports the behavior of the coefficients when the weight of sector s i.e. η increases. As the sector s share increases, stabilizing inflation in sector s becomes more important and stabilizing inflation in sector f becomes less important. When the share of the relatively sticky sector increases, the relative importance of the output stabilization decreases as compared to inflation in sector s, hence the weights on the output gap variables fall in absolute magnitude. The coefficient of $\Delta\widetilde{T}_t$ (lower-right panel) first increases with η and then decreases. As is immediate from its expression, it reaches a maximum at $\eta = 0.5$. As discussed, movements in relative prices produce losses arising from misallocation between the sectors. Note that the coefficient on the relative price movement between the sectors is largest when the sectors are equally large. In this case the loss from price dispersion between the sectors is largest. Hence, an increase in the share η of the sticky sector initially induces the monetary authority to work harder to curb the difference in inflation movements between the sectors, while for values of η exceeding one-half, a further increase in the relative size of the sticky sector leads the monetary authority to pay less attention to relative inflation developments.

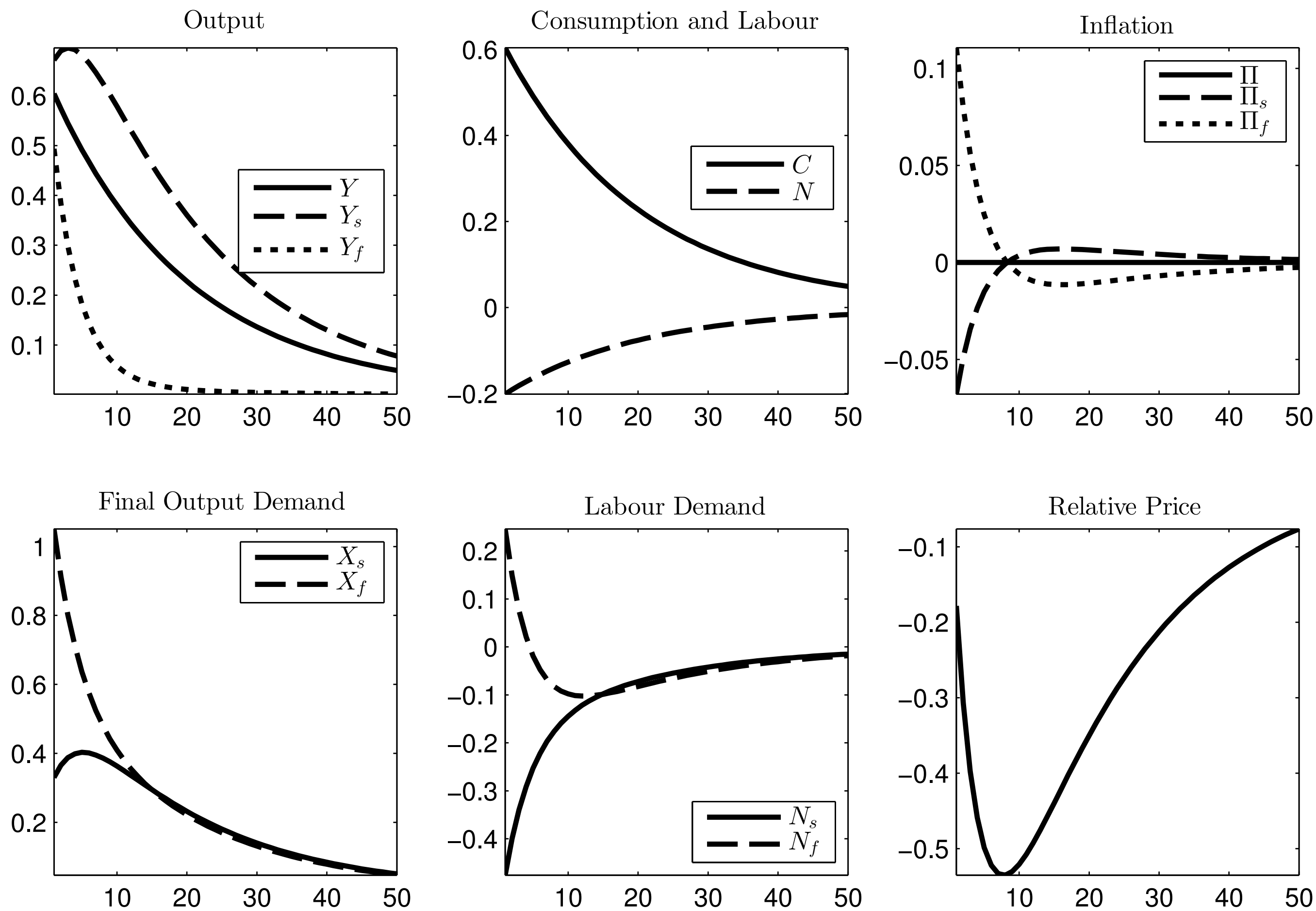
3.4 Results

In this section we first explore the impulse responses of the key endogenous variable with respect to the sectoral productivity shocks. Second, using the loss function (3.21) we analyze the welfare cost under the OIR for different degrees of price stickiness in sector s. Here, we also analyse how the welfare effect of price stickiness interacts with the factor demand linkage. Third, we explore the robustness of the OIR (3.23).

3.4.1 The Impulse Response Functions

Figure 3.2 plots the responses of the endogenous variables when sector s receives a positive technology shock. We do not separately report the responses to the shock in sector f, because they are qualitatively the same, though of different magnitude. In response to the positive productivity shock in sector s, aggregate (final) output, consumption and aggregate inflation increase, while the labour supply falls. The productivity shock raises the real wage, which leads households to raise consumption and lower the labour supply, as the income effect dominates the substitution effect.

Figure 3.2: Impulse response functions following a positive technology shock in sector s.



Notes: The figure reports the impulse response functions of the endogenous variables to a one standard-deviation positive productivity shock in sector s. The $y-$axis reports the percent deviation of the variable from the steady state.

Higher consumption boosts aggregate demand and, for the market to clear, aggregate output has to increase. Due to the presence of the factor demand linkage, output in both sectors goes up, although the effect is strongest for sector s, as this is where the shock originated. Further, the increase in the real wage raises the marginal cost of production in sector f, thereby boosting inflation in sector f. However, for sector s, the effect of the increase in productivity dominates that of the increase in the real wage, thereby leading to lower inflation in sector s. As the price stickiness of sector s is $\frac{\theta_s}{\theta_f} = 8.5$ times higher than the price stickiness of sector f, the increase in inflation in sector f is substantially higher than the decrease in inflation in sector s. Hence, aggregate inflation of the intermediates sectors increases with a positive productivity shock in sector s.

Due to the factor demand linkage, the increased demand for final output also leads to increased demand for intermediates. Interestingly, we observe (lower left panel) that on impact the demand for intermediate output by the flexible-sector exceeds that by the sticky sector. This is due to the positive productivity shock in sector s, as it allows the firms in sector s to economize on their inputs. However, this effect vanishes as the productivity shock slowly starts to disappear. Initially, the productivity shock also allows sector s producers to economize more on labor input, which on impact experience a larger fall in sector s than in sector f. The initial decrease in relative price can be explained by the differential behavior of sector level inflation described above.

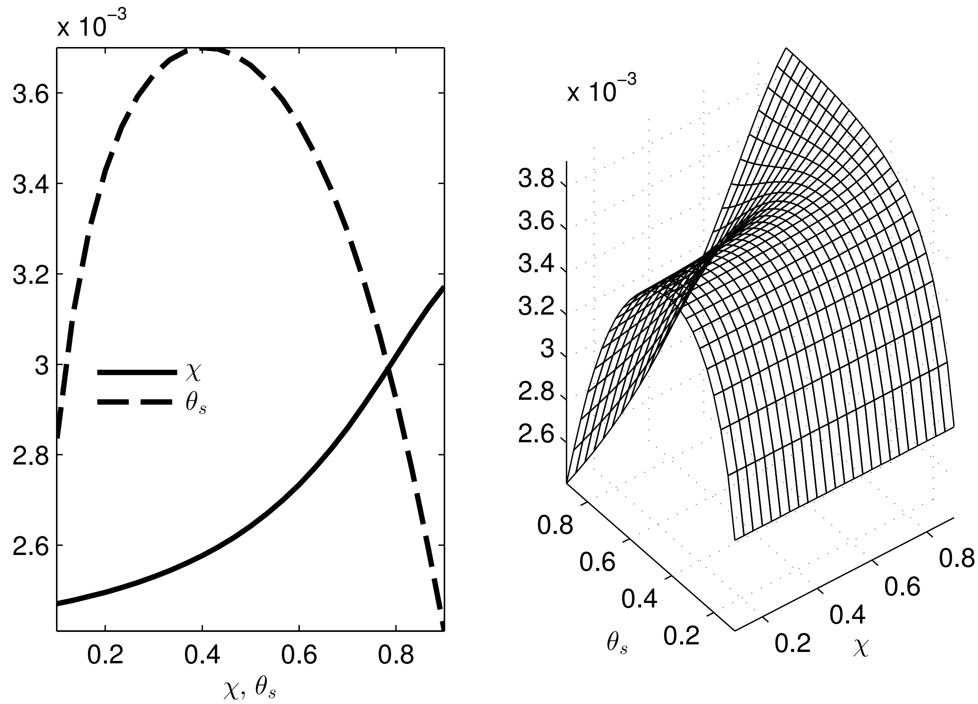
3.4.2 Relative Welfare Cost

The objective of this subsection is to explore how the heterogeneity in sectoral price stickiness and the magnitude of factor demand linkage s affect the welfare cost associated with sectoral productivity shocks. Following Lucas (1987), we express welfare effects in terms of the permanent and constant increase in consumption c needed to bring utility in the actual setting to the same level as when consumption is always and with certainty at its steady state level C. In Appendix C.9 we show that the welfare loss in terms of a permanent and constant consumption fall relative to non-stochastic steady state consumption is

$$\frac{c}{C} = 1 - \left[(1-\sigma)\left((1-\beta)(\mathbb{A} - L) + \frac{1}{1+\phi}\right)\right]^{\frac{1}{1-\sigma}} \tag{3.24}$$

where L is the loss (equation (3.21)) under the optimal monetary policy given the value of deep parameters and $\mathbb{A} = \frac{\sigma+\phi}{(1-\beta)(1-\sigma)(1+\phi)}$. To compute (3.24) we first solve the model and then simulate it for 1000 periods. Then, using (3.21) and the simulated series for the output gap and sectoral inflation, we can compute the loss L. The left panel of Figure 3.3 reports the welfare costs, when

Figure 3.3: Welfare costs of sectoral productivity shocks as functions of χ and θ_s.



Notes: The left panel shows the welfare cost in terms of certainty equivalent consumption of sector level productivity shocks when χ or θ_s are varied, while all the other parameters are kept at their baseline values in Table 3.1. The right panel allows χ and θ_s to be simultaneously varied, keeping all the other parameters at their baseline values. The welfare cost computations are based on the simulation of the model for 1000 periods.

parameter χ or θ_s are varied, while all the other parameters are kept at their baseline values in Table 3.1. The right panel varies χ and θ_s simultaneously, again keeping the other parameters at their baseline values.

Holding all the other parameters at their baseline values, the welfare cost rises with an increase in the factor demand linkage (left-hand panel of Figure 3.3). Intuitively, a stronger factor demand linkage has a stronger amplifying effect on the response of the economy to shocks. This amplification effect arises from the spill-over of the original shock to the other intermediate sector via the demand by the final output sector, a second round effect from the other intermediate sector to the initial intermediate sector, and so on.

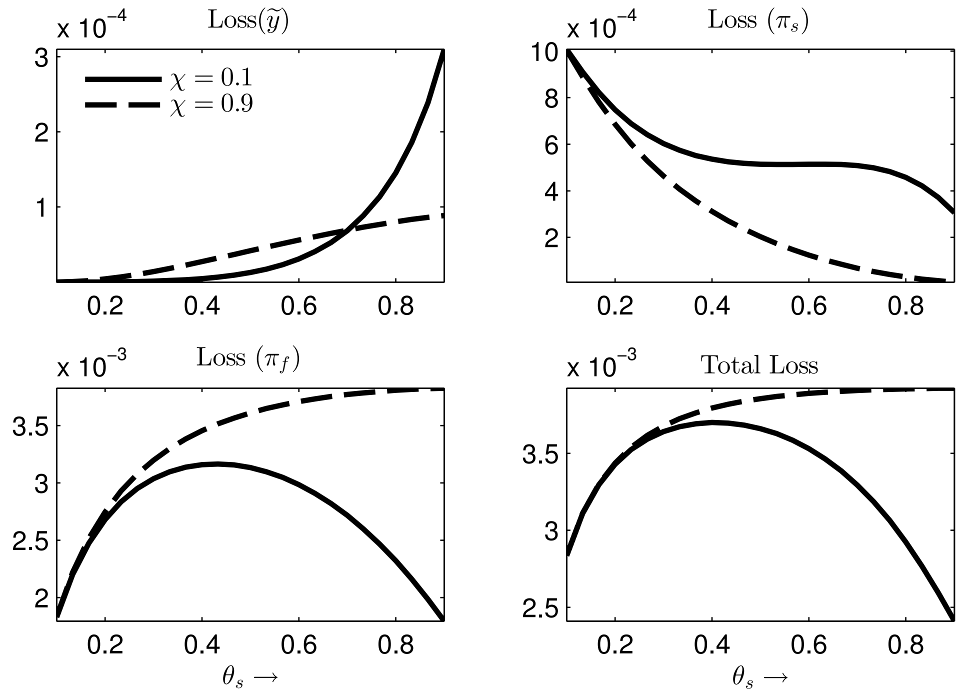
The effect of varying θ_s is of particular interest. Figure 3.3 suggests that the profile of the welfare loss as a function of θ_s depends on the magnitude of the factor demand linkages. For weak factor demand linkages, the welfare loss reaches a maximum in θ_s on the zero-one interval for θ_s, while for relatively strong factor demand linkages, the welfare loss is monotonically increasing in θ_s. The non-monotonicity of the welfare loss in sticky-sector price stickiness at low levels of χ goes against the accepted wisdom in the literature that higher price stickiness entails higher welfare costs.

To see the intuition, it is important to realize first that the present problem of output and inflation stabilization is different from the standard monetary policy problem in Woodford (2003), where the monetary authority faces a trade-off between two endogenous variables, whereas in the present case, there are three endogenous variables that need to be stabilized: the output gap and two sectoral inflation rates. However, the monetary authority only has one instrument, the interest rate, for stabilization.

There is a one-to-one mapping between the welfare loss as expressed in (3.24) and the loss computed via (3.21). Figure 3.4 plots the three components of the latter and the total for low and high factor demand linkages, i.e. $\chi = \{0.1, 0.9\}$, and varying sticky-sector price stickiness θ_s over the range $(0.1, 0.9)$. The figure is based on solving the model and then simulating it over one thousand periods. We plot the average quadratic terms in the output gap and the sectoral inflation rates multiplied by their respective coefficients in the welfare loss function.

We observe from the bottom-right panel that the "total loss" reaches a maximum in θ_s for low values of the factor demand linkage and is monotonically increasing θ_s when factor demand linkages are strong. The total loss is the sum of the components in the preceding three panels of the figure. When θ_s is low, output gap stabilization is relatively important. For example, if $\chi = \theta_s = 0.1$, $(\lambda_y, \lambda_{\pi_s}, \lambda_{\pi_f}) = (1.02, 0.42, 0.25)$. Hence, for low θ_s, sector s inflation fluctuations are relatively large. However, as θ_s increases, the monetary authority clamps down more and more on sticky-sector inflation leading to an increasing loss associated with output gap fluctuations and a falling loss associated with sticky-sector inflation fluctuations, despite the increasing weight on sticky-sector inflation fluctuations in the loss function. Most interesting is the pattern of the loss associated with flexible sector inflation fluctuations. As the weight on sticky-sector inflation

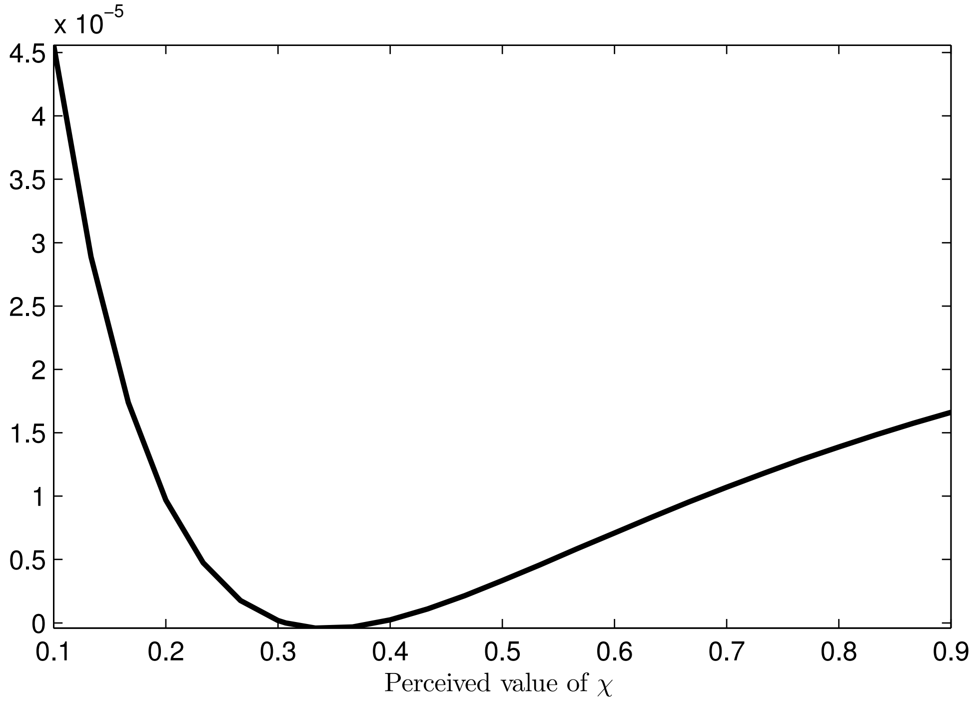
Figure 3.4: Loss and its decomposition associated with sectoral productivity shock for different values of χ and θ_s



Notes: The figure plots the total loss associated with sectoral productivity shock and its decomposition in three parts: loss due to fluctuations in output gap and sectoral inflation. The value is computed using the loss function equation (3.21). The solid line refers to $\chi = 0.1$ and dashed line to $\chi = 0.9$ whereas θ_s is in the range $(0.1, 0.9)$. The bottom-right panel reports the total loss and the bottom-left panel reports the loss associated with sector f inflation. The top left(right) panel report the loss associated with fluctuations in the output gap (sector s inflation). The computations are based on the simulation of the model for 1000 periods.

fluctuations increases starting from a low value of θ_s, also flexible sector inflation fluctuations receive less attention and the loss associated with these fluctuations increases. However, when θ_s increases further, the loss patterns for low and high factor demand linkages start to diverge, with the loss associated with flexible sector inflation fluctuations starting to fall if χ is low, while continuing to rise if χ is high. Apparently, when χ is low the monetary authority starts to clamp down relatively more of flexible sector inflation fluctuations at the cost of relatively higher sticky-sector inflation fluctuations: we see that the loss associated with sticky-sector inflation fluctuations is falling at a slower pace in θ_s when sector demand linkages are high.

Figure 3.5: Additional welfare cost of a misperceived factor demand linkage



Notes: This figure plots as a function of the monetary authority's perception of the factor demand linkage the additional welfare cost as a fraction of non-stochastic steady-state consumption when the authority follows the OIR (3.23) based on its own perception. The true value of χ is its baseline value of 0.307. The additional welfare cost is based on the simulation of the model for 1000 periods.

3.4.3 Robustness of Optimal Monetary Policy

In this subsection we explore the welfare loss when our OIR is based on a value of the factor demand linkage that differs from its true value or when the monetary authority conducts monetary policy purely on the basis of the relative sizes of the two sectors in the economy, while ignoring differences in price stickiness. Both cases are relevant for policymakers, as it is hard in practice to pinpoint the actual values of the factor demand linkage or price stickiness. Hence, a rule that produces only limited additional welfare loss when it is conducted under incorrect parameter value assumptions should be attractive for policymakers.

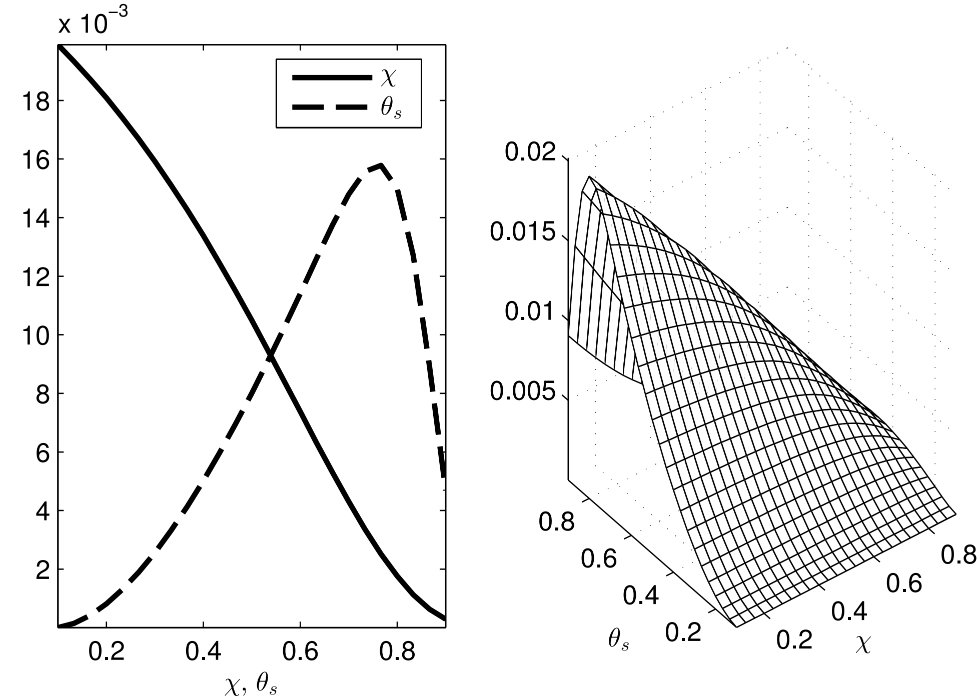
Uncertainty about the Factor Demand Linkage

In this section we explore the additional welfare cost when the monetary authority's perception of the factor demand linkage deviates from the true factor demand linkage. Denote by c^{o} the certainty-equivalent consumption loss in the "optimal" case when the authority has the correct perception about the factor demand linkage and by c^{no} the certainty-equivalent consumption loss in the "non-optimal" case when the monetary authority bases its policy on an incorrect perception of the factor demand linkage. Using the (3.24), we calculate the additional welfare cost associated with the incorrect perception as $\left(\frac{c^{\mathrm{no}}}{C} - \frac{c^{\mathrm{o}}}{C}\right) \times 100$, where C is again non-stochastic steady-state consumption. Figure 3.5 plots the additional welfare cost against the perceived value of the factor demand linkage. Obviously, the additional welfare cost is zero when the perceived factor demand linkage equals the true factor demand linkage. The additional welfare loss is increasing in the distance between the perceived and the true factor demand linkage. However, overall at the current calibration this additional welfare cost is very small and always below one-hundredth of a percent of non-stochastic steady-state consumption.

Disregarding Differences in Sector Level Price Stickiness

Figure 3.6 depicts the additional expected welfare cost when the monetary authority conducts policy on the basis of the sector weights in the economy, while disregarding the differences in sector level price stickiness. Concretely, the monetary authority follows the optimal rule based on the loss function (3.21) with $\lambda_{\pi_s} = \eta$ and $\lambda_{\pi_f} = 1 - \eta$, while all parameters are given by their baseline values. The welfare cost is computed in the same manner as in previous section. Denoting by c^{o} the certainty-equivalent consumption loss in the optimal case and by c^{no} the certainty-equivalent consumption loss in the non-optimal case when the monetary authority disregards the sector level price stickiness, Figure 3.6 depicts for different values of χ and θ_s the additional certainty-equivalent consumption loss relative to non-stochastic steady consumption, $\left(\frac{c^{\mathrm{no}}}{C} - \frac{c^{\mathrm{o}}}{C}\right) \times 100$. The left-hand panel of the figure is based on varying either χ or θ_s, while the right panel is based on varying both parameters simultaneously.

Figure 3.6: Additional welfare cost when inflation weights are based on relative sector sizes ignoring differences in price stickiness



Notes: The figure plots the additional welfare cost when the monetary authority bases its policy on a loss function with weights based on relative sector sizes, hence ignores differences in price stickiness across sectors. The benchmark economy is the one in which monetary policy is optimal given the utility-based welfare loss function. The left panel depicts the additional welfare loss when all parameters other than those on the horizontal axis are fixed at their baseline values (Table 3.1), while the right panel makes this assumption for all parameters other than χ and θ_s, which are varied simultaneously. The welfare cost computation is based on the simulation of the model over 1000 periods.

The figure leads to several observations. First, in the left-hand panel the additional welfare cost of attaching the wrong weights to inflation is minimal when $\theta_s = \theta_f$. The reason is that when price stickiness is identical in both sectors, the relative weights on the inflation terms in the welfare-based loss function are indeed proportional to the sector weights. Second, the welfare cost of selecting monetary policy based on the wrong inflation weights in the loss function can be non-negligible for the current calibration and reach a maximum certainty-equivalent consumption loss of around 2 percent. Third, for given sticky-sector price stickiness, the welfare cost is monotonically decreasing with the factor demand linkage. There are two reasons for this. Disregarding price stickiness and focusing on relative sector weights only, leads the monetary authority to attach a too low relative weight to sticky-sector inflation in its loss function. An increase in χ partially corrects this, because it reduces the weight λ_y on the output gap as $\frac{\partial \lambda_y}{\partial \chi} < 0$, while leaving λ_{π_s} and λ_{π_f} unchanged. While the importance of sticky-sector inflation relative to flexible-sector inflation thus remains unchanged, the importance of sticky-sector inflation relative to the output gap gains in importance, thereby shifting policy somewhat into the direction of the optimal policy setting. The other reason is that, because of the stronger inflation spill-overs across the sectors when the factor demand linkage increases, the distinction between clamping down on sticky sector inflation versus clamping down on flexible sector inflation becomes less important. Hence, the relative overemphasis on clamping down flexible sector inflation when attaching weights in accordance to the sector sizes becomes less harmful for welfare. Not surprisingly, from the right-hand panel we observe that for low values of θ_s, the effect of an increase in the factor demand linkage on the additional welfare loss is very small. The reason is that at low values of θ_s the additional welfare loss associated with the wrong inflation weights is only small. Fourth, in line with the intuition given earlier, for a given factor demand linkage, the welfare cost first rises and then decreases with the increase in sticky-sector price stickiness.

3.5 Conclusion

This paper has explored the consequences for optimal monetary policy of factor demand linkages between sectors, while allowing for different degrees of price stickiness in the sectors of the economy, thereby calibrating our model set-up to the U.S. situation. We find that factor demand linkages amplify the fluctuations of the economy, because price fluctuations in one sector feed into the other sector. This has consequences for optimal monetary policy, which should become relatively more concerned with stabilizing inflation as opposed to stabilizing the output gap when factor demand linkages become stronger. We derive the micro-founded loss function and the optimal interest rate rule in the presence of a subsidy which eliminates the distortion from monopolistic competition, and we characterize how optimal monetary policy is affected by the deep model parameters. Interestingly, the factor demand linkage does not directly affect the relative weights on the sectoral inflation rates, which are driven by the degrees of price stickiness in the sectors. However, if factor demand linkages increase the loss function weight on the output gap falls relative to the weights on the sectoral inflation rates. The cross-sector amplification effects of inflation become stronger, inducing the monetary authority to more actively clamp down on inflation. We also show that, when factor demand linkages are weak and holding stickiness in the other sector constant, the welfare cost of productivity shocks may be non-monotonic in the degree of price stickiness of the sticky sector, with the welfare cost initially rising and then falling in price stickiness. This result arises from the fact that the monetary authority has only one instrument to achieve an optimal trade-off among three variables.

Many extensions of the analysis in this paper are possible. For example, it focuses on a closed economy and it considers only one type of asymmetry between the sectors, viz. differences in price stickiness. Future work could introduce additional elements of realism by relaxing these assumptions, for example by introducing an open-economy setting in which also foreign inputs are allowed in the production of intermediates and by allowing for differences in wage stickiness between sectors. Also, it will be interesting to see the implications of factor demand linkages in the presence of state-dependent pricing (for example, see Dotsey et al. (1999)).

Appendix A

A.1 Proof of Proposition 1

The first order condition for competitive equilibrium consumption $c_t = c_t(a_0, s^t)$ requires

$$\beta^t \pi(s^t|s_0)u'(c_t) \leq \lambda p_t(s^t),$$

where λ is the Lagrange multiplier on the intertemporal budget constraint (1.13). Asset prices are determined by households with the highest willingness to pay for the asset. These are unconstrained households. For two consecutive periods, the first order conditions for households with slack participation constraints are

$$\beta^t \pi(s^t|s_0)u'(c_t) = \lambda p_t(s^t),$$
$$\beta^{t+1} \pi(s^{t+1}|s_0)u'(c_{t+1}) = \lambda p_{t+1}(s^{t+1}).$$

Dividing those we obtain:

$$\beta \frac{u'(c_{t+1})}{u'(c_t)} = \frac{\pi(s^t|s_0)p_{t+1}(s^{t+1})}{p_t(s^t)\pi(s^{t+1}|s_0)} \tag{A.1}$$

Consider a consumption allocation from the planner problem $\{C(h_t(w_0, s^t))\}_{t=0}^{\infty}$. The allocation is an equilibrium allocation if it satisfies the optimality condition of a household with non-binding participation constraints in periods t and $t+1$ over history s^{t+1}:

$$\beta \frac{u'(c_{t+1})}{u'(c_t)} = \frac{1}{R}. \tag{A.2}$$

Combining the optimality conditions of planner and households results in

$$\frac{1}{R} = \frac{\pi(s^t|s_0)p_{t+1}(s^{t+1})}{p_t(s^t)\pi(s^{t+1}|s_0)},$$

which implies that

$$p_t(s^t) = \frac{\pi(s^t|s_0)}{R^t}.$$

Finally, the initial wealth that makes the allocation (w_0, s_0) affordable is given by

$$a_0 = c(w_0, s_0) - y_0 + \sum_{t=1}^{\infty} \sum_{s^t|s_0} \frac{\pi(s^t|s_0)}{R^t} \left[c(w_0, s^t) - y_t \right].$$

A.2 Proof of Proposition 2

Let $\bar{V}_{rs} = u(\bar{y})$ be the period-social welfare under perfect risk sharing. First, perfect risk sharing provides the highest *ex-ante* utility among the consumption-feasible allocations. The existence of $\bar{\beta}(\kappa)$ follows from monotonicity of participation constraints in β and $\bar{V}_{rs} > V_{out}$. A higher β increases the future value of perfect risk sharing relative to the allocation in the equilibrium without transfers, leaving the current incentives to deviate unaffected. Therefore, if the participation constraints are not binding for $\bar{\beta}(\kappa)$, they are not binding for any $\beta \geq \bar{\beta}(\kappa)$. The cutoff point is characterized by the tightest participation constraint, i.e., by the participation constraint with the highest value of the outside option. Solving this constraint yields a unique solution for $\bar{\beta}(\kappa)$ in $(0, 1)$ because $u(\bar{y}) < u(y_h)$. Second, the tightest constraint at the first best allocation is

$$u(\bar{y}) \geq (1-\beta)u(y_h) + \beta(1-\beta)\kappa u(y_h) + \beta(1-\beta)(1-\kappa)u(y_l) + \beta^2 V_{out}.$$

Differentiating the constraint fulfilled with equality with respect to κ using the implicit function theorem gives

$$\frac{\partial \bar{\beta}(\kappa)}{\partial \kappa} = \frac{\beta(1-\beta)[u(y_h) - u(y_l)]}{(1-\beta)[u(y_h) - \kappa u(y_h) - (1-\kappa)u(y_l)] + \beta[\kappa u(y_h) + (1-\kappa)u(y_l) - u(y_l)]} \geq 0$$

and results in a positive sign for $y_h > y_l$.

A.3 Proof of Proposition 3

First, if $\beta \leq u'(y_h)/u'(y_l)$, autarky is a solution to the recursive problem because (1.10) is satisfied for autarky. From the other end, we have to show that it is not possible to construct a resource-feasible distribution that dominates the stationary distribution $\Phi^{Aut}(\{U^{Aut}(y), y\}) = q(y)$ when the condition in the proposition is satisfied. Consider the following distribution $\hat{\Phi}$ over utility promises, income and public signals with a one-period history

$$\hat{\Phi}(\{U^{Aut}(y_h, k_h), y_h\}) = \pi_h/2, \quad \hat{\Phi}(\{U^{Aut}(y_l, k_h), y_l\}) = (1-\pi_h)(1-\pi_h)/2,$$
$$\hat{\Phi}(\{U^{Aut}(y_l, k_l), y_l\}) = (1-\pi_h)(1-\pi_h)/2 \quad \hat{\Phi}(\{U^{Aut}(y_h, k_l), y_h\}) = \pi_h/2$$
$$\hat{\Phi}(\{\tilde{\omega}_{k_h}^{k_h}, y_l\}) = (1-\pi_h)\pi_h/4 \quad \hat{\Phi}(\{\tilde{\omega}_{k_l}^{k_h}, y_l\}) = (1-\pi_h)\pi_h/4$$
$$\hat{\Phi}(\{\tilde{\omega}_{k_h}^{k_l}, y_l\}) = (1-\pi_h)\pi_h/4 \quad \hat{\Phi}(\{\tilde{\omega}_{k_l}^{k_l}, y_l\}) = (1-\pi_h)\pi_h/4$$

where $\tilde{\omega}_{k_i}^{k_j} = U^{Aut}(y_l, k_i, n_h) + \varepsilon_{k_i}^{k_j}$ for small $\varepsilon_{k_i}^{k_j}$ and and the upper (lower) index indicates the previous (current) period signal.

Let $\{\delta_{ij}^m\}$, $i, j, m \in \{l, h\}$ denote transfers in terms of utility where the first index is current income, the second index denotes the current public signal and the upper index is previous period public signal. The transfers are defined by

$$
\begin{aligned}
\tilde{\omega}_{k_h}^{k_h} &= (1-\beta)(u(y_l) + \delta_{lh}^{h}) + \beta\left[(1-\kappa)U^{Aut}(y_l) + \kappa U^{Aut}(y_h)\right] \\
\tilde{\omega}_{k_h}^{k_l} &= (1-\beta)(u(y_l) + \delta_{lh}^{l}) + \beta\left[(1-\kappa)U^{Aut}(y_l) + \kappa U^{Aut}(y_h)\right] \\
\tilde{\omega}_{k_l}^{k_h} &= (1-\beta)(u(y_l) + \delta_{ll}^{h}) + \beta\left[\kappa U^{Aut}(y_l) + (1-\kappa)U^{Aut}(y_h)\right] \\
\tilde{\omega}_{k_l}^{k_l} &= (1-\beta)(u(y_l) + \delta_{ll}^{l}) + \beta\left[\kappa U^{Aut}(y_l) + (1-\kappa)U^{Aut}(y_h)\right]
\end{aligned}
$$

for the low-income agents and by

$$
\begin{aligned}
U^{Aut}(y_h, k_h) &= (1-\beta)(u(y_h) - \delta_{hh}^{h}) + \beta\left[(1-\kappa)\frac{\tilde{\omega}_{k_l}^{k_h} + \tilde{\omega}_{k_h}^{k_h}}{2} + \kappa U^{Aut}(y_h)\right] \\
U^{Aut}(y_h, k_h) &= (1-\beta)(u(y_h) - \delta_{hh}^{l}) + \beta\left[(1-\kappa)\frac{\tilde{\omega}_{k_l}^{k_h} + \tilde{\omega}_{k_h}^{k_h}}{2} + \kappa U^{Aut}(y_h)\right] \\
U^{Aut}(y_h, k_l) &= (1-\beta)(u(y_h) - \delta_{hl}^{h}) + \beta\left[\kappa\frac{\tilde{\omega}_{k_l}^{k_l} + \tilde{\omega}_{k_h}^{k_l}}{2} + (1-\kappa)U^{Aut}(y_h)\right] \\
U^{Aut}(y_h, k_l) &= (1-\beta)(u(y_h) - \delta_{hl}^{l}) + \beta\left[\kappa\frac{\tilde{\omega}_{k_l}^{k_l} + \tilde{\omega}_{k_h}^{k_l}}{2} + (1-\kappa)U^{Aut}(y_h)\right],
\end{aligned}
$$

for the high-income agents, where $U^{Aut}(y_h) = 0.5(U^{Aut}(y_h, k_h, n_h) + U^{Aut}(y_h, k_l, n_h))$, $U^{Aut}(y_h), \tilde{w}$ are defined correspondingly.

Outside options do not depend on previous period signals, thus, $\delta_{hh}^{h} = \delta_{hh}^{l} = \delta_{hh}$ and $\delta_{hl}^{h} = \delta_{hl}^{l} = \delta_{hl}$. The marginal utility of low-income agents before the the transfer and the Pareto weight are identical for each combination of past and current public signal. Thus, low-income agents should optimally receive the same transfer, $\delta_{li}^{j} = \epsilon/[4(1-\beta)]$ for all i, j, where the sum of transfers to low income agents is normalized to $\epsilon/(1-\beta)$. The utility of high-income agents is equal to their outside option and therefore independent from the particular transfer scheme. The transfers of high-income agents can be directly derived from their outside option value, reflecting their expected future gain from transferring in the current period. It follows that the transfer scheme can be summarized by

$$\delta_l \equiv \sum_{i,j} \delta_{li}^j = \frac{\epsilon}{1-\beta} \quad \delta_{hh} = \beta\epsilon\frac{(1-\kappa)}{4(1-\beta)} \quad \delta_{hl} = \beta\epsilon\frac{\kappa}{4(1-\beta)}.$$

The distribution $\hat{\Phi}$ requires the following increase in resources

$$\begin{aligned}\Delta &= \pi_h(1-\pi_h)c'[u(y_l)]\delta_l/4 - \frac{\pi_h}{2}c'[u(y_h)](\delta_{hh}+\delta_{hl}) \\ &= \frac{\pi_h(1-\pi_h)\epsilon}{4(1-\beta)}\left[\frac{1}{u'(y_l)} - \frac{\beta}{2(1-\pi_h)u'(y_h)}\right].\end{aligned} \tag{A.3}$$

If

$$\beta < \left[\frac{(1-\pi_h)u'(y_h)}{u'(y_l)}2\right],$$

the constructed allocation $\hat{\Phi}$ violates resource feasibility. Using $\pi_h = 1/2$ results in the expression stated in the proposition.

A.4 Proof of Proposition 4

Proof. With partial risk sharing, the two participation constraints of high-income agents are binding but the correspond constraint for low-income agents are slack.[1] They are given by the following expressions

$$\begin{aligned}F(c_h^h, c_l^h) \equiv{}& (1-\beta)u(c_h^h) + \beta(1-\beta)\kappa V_{rs}^h + \beta(1-\beta)(1-\kappa)V_{rs}^l + \beta^2 V_{rs} \\ & - (1-\beta)u(y_h) - \beta(1-\beta)\left(\kappa u(y_h) + (1-\kappa)u(y_l)\right) - \beta^2 V_{out} = 0,\end{aligned}$$

[1] A proof for this result can be found for example in Lepetyuk and Stoltenberg (2013). In history-dependent arrangements also participation constraints of low-income agents are occasionally binding.

$$G(c_h^h, c_l^h) \equiv (1-\beta)u(c_l^h) + \beta(1-\beta)(1-\kappa)V_{rs}^h + \beta(1-\beta)\kappa V_{rs}^l + \beta^2 V_{rs}$$
$$- u(y_h) - \beta(1-\beta)\left((1-\kappa)\, u(y_h) + \kappa u(y_l)\right) - \beta^2 V_{out} = 0,$$

1. The derivative of the conditional mean of consumption of high-income agents is given by

$$\frac{1}{2}\left(\frac{\partial c_h^h}{\partial \kappa} + \frac{\partial c_l^h}{\partial \kappa}\right) = \frac{x}{2}\left(\frac{F_{c_l^h} + G_{c_l^h}}{F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h}} - \frac{F_{c_h^h} + G_{c_h^h}}{F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h}}\right),$$

with

$$x \equiv \beta(1-\beta)(u(y_h) - V_{rs}^h - u(y_l) + V_{rs}^l) \geq 0.$$

At the optimal memoryless allocation, we have that

$$F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h} > 0$$

which follows from autarky not being the optimal memoryless allocation (see Lemma 1 in Section A.4.1). Further, the partial derivatives are

$$\begin{aligned}
F_{c_h^h} &= (1-\beta)\left[u'(c_h^h) + \beta\frac{\kappa}{2}u'(c_h^h) - \beta\frac{1-\kappa}{2}u'(c^l)\right] + \frac{\beta^2}{4}\left[u'(c_h^h) - u'(c^l)\right]\\
F_{c_l^h} &= (1-\beta)\left[\beta\frac{\kappa}{2}u'(c_l^h) - \beta\frac{1-\kappa}{2}u'(c^l)\right] + \frac{\beta^2}{4}\left[u'(c_l^h) - u'(c^l)\right]\\
G_{c_l^h} &= (1-\beta)\left[u'(c_l^h) + \beta\frac{1-\kappa}{2}u'(c_l^h) - \beta\frac{\kappa}{2}u'(c^l)\right] + \frac{\beta^2}{4}\left[u'(c_l^h) - u'(c^l)\right]\\
G_{c_h^h} &= (1-\beta)\left[\beta\frac{1-\kappa}{2}u'(c_h^h) - \beta\frac{\kappa}{2}u'(c^l)\right] + \frac{\beta^2}{4}\left[u'(c_h^h) - u'(c^l)\right].
\end{aligned}$$

After some steps of tedious but straightforward algebra, one gets

$$\frac{1}{2}\left(\frac{\partial c_h^h}{\partial \kappa} + \frac{\partial c_l^h}{\partial \kappa}\right) = \frac{x}{2}(2-\beta)\frac{u'(c_l^h) - u'(c_h^h)}{F_{c_h^h}G_{c_l^h} - F_{c_l^h}G_{c_h^h}} \geq 0$$

because $c_h^h \geq c_l^h$ follows from a higher outside option value of agents with a good public signal

$$\begin{aligned} u(y_h) + \beta(1-\beta)\left(\kappa u(y_h) + (1-\kappa)\,u(y_l)\right) + \beta^2 V_{out} \geq \\ u(y_h) + \beta(1-\beta)\left((1-\kappa)\,u(y_h) + \kappa u(y_l)\right) + \beta^2 V_{out}. \quad \text{(A.4)} \end{aligned}$$

Non-binding participation constraints for low-income agents result in equal consumption of agents with a good and bad public signal. Resource feasibility then implies that conditional consumption of low-income agents decreases in signal precision.

2. The conditional standard deviation of low-income agents is not affected because consumption of these agents is equalized in optimal memoryless allocations. The variance of consumption of high-income agents is

$$\mathrm{var}(c|y^h) = \frac{1}{2}\left[(c_h^h - \bar{c}^h)^2 + (c_l^h - \bar{c}^h)^2\right], \quad \text{(A.5)}$$

with

$$\bar{c}^h = \frac{c_h^h + c_l^h}{2}.$$

From the first order conditions at the optimal memoryless allocation, we get

$$F_{c_l^h} + G_{c_l^h} > 0 \quad F_{c_h^h} + G_{c_h^h} > 0.$$

Thus, the derivatives of high-income agents' consumption with respect to signal precision satisfy

$$\frac{\partial c_h^h}{\partial \kappa} \geq 0 \quad \frac{\partial c_l^h}{\partial \kappa} \leq 0.$$

From Part 1, we know that the conditional mean of high-income agents is increasing in signal precision but less than c_h^h because c_l^h decreases. Thus, both terms in (A.5) increase such that the conditional standard deviation of high-income agents increases.

3. The unconditional variance of consumption is

$$\mathrm{var}(c) = \frac{1}{4}\left[(c_h^h - \bar{y})^2 + (c_l^h - \bar{y})^2 + 2(c^l - \bar{y})^2\right].$$

From Part 1, we get that $c^l < \bar{y}$ is decreasing in signal precision, where the inequality follows from perfect risk sharing not being feasible. From Part 2, $c_h^h > \bar{y}$ is increasing while $c_l^h > \bar{y}$ is decreasing in signal precision but from Part 1, we get that the conditional mean of consumption of high-income agents increases in signal precision. This implies that $\mathrm{d}\, c_h^h \geq -\mathrm{d}\, c_l^h$. Thus, all terms on the right-hand side increase in signal precision and the unconditional standard deviation of consumption increases.

■

A.4.1 Lemma on autarky with memoryless allocations

In the proof of Proposition 4, we make use of the following property.

Lemma 1 *Assume that participation constraints of high-income agents are binding. If*

$$(2-\beta)u'(y_h) - \beta u'(y_l) < 0$$

then autarky is not the optimal memoryless allocation.

Proof. The optimal memoryless arrangement can be analyzed as a fixed-point problem expressed in terms of the period value of the arrangement.

The fixed-point problem is constructed as follows. Let W be the unconditional expected value of an arrangement before any signal has realized. We restrict attention to $W \in [V_{out}, \bar{V}_{rs})$ because per assumption participation constraints for high-income households are binding. The binding participation constraints are given by the following

$$
\begin{aligned}
&u(c_h^h) + \frac{\beta\left\{\kappa\nu[u(c_h^h)+u(c_l^h)]/2 + (1-\kappa)(1-\nu)u(2\bar{y}-(c_h^h+c_l^h)/2)\right\}}{\kappa\nu+(1-\kappa)(1-\nu)} \\
&= u(y_h) + \frac{\beta\left[\kappa\nu u(y_h) + (1-\kappa)(1-\nu)u(y_l)\right]}{\kappa\nu+(1-\kappa)(1-\nu)} + \frac{\beta^2}{1-\beta}(V_{out}-W),
\end{aligned}
\tag{A.6}
$$

$$
\begin{aligned}
&u(c_l^h) + \frac{\beta\left\{(1-\kappa)\nu[u(c_h^h)+u(c_l^h)]/2 + \kappa(1-\nu)u(2\bar{y}-(c_h^h+c_l^h)/2)\right\}}{(1-\kappa)\nu+\kappa(1-\nu)} \\
&= u(y_h) + \frac{\beta\left[(1-\kappa)\nu u(y_h) + \kappa(1-\nu)u(y_l)\right]}{(1-\kappa)\nu+\kappa(1-\nu)} + \frac{\beta^2}{1-\beta}(V_{out}-W),
\end{aligned}
\tag{A.7}
$$

and resource feasibility is used. The objective function of the problem to compute the optimal memoryless arrangement is given by the following expression

$$
V_{rs}(W) \equiv \frac{1}{4}\left[u(c_h^h(W)) + u(c_l^h(W)) + 2u(2\bar{y}-(c_h^h(W)+c_l^h(W))/2)\right].
$$

The optimal memoryless arrangement should necessary solve the fixed-point problem $W = V_{rs}(W)$. We will show that $V_{rs}(W)$ is strictly increasing. $V(W)$ is also strictly concave, therefore there exist at most two solutions to the fixed-point problem.

From the participation constraints (A.6) and (A.7), the derivative of $V(W)$ is given by

$$
V_{rs}'(W) = \frac{1}{4}\left[(u'(c_h^h)-u'(c_l))\frac{\partial c_h^h}{\partial W} + (u'(c_l^h)-u'(c_l))\frac{\partial c_l^h}{\partial W}\right]
$$

which is strictly increasing in W because perfect risk sharing is not constrained feasible which implies that $\partial c_h^h/\partial W$ and $\partial c_l^h/\partial W$ are negative and $c_h^h, c_l^h \neq \bar{y}$.

By construction, one solution to the fixed-point problem is V_{out}. The concavity of $V_{rs}(W)$ implies that the derivative of $V_{rs}(W)$ at V_{out} is higher than at any partial risk-sharing allocation and there are at most two solutions. Therefore, when autarky is not the optimal memoryless arrangement the derivative of $V'_{rs}(w)$ at V_{out} must larger than 1

$$V'_{rs}(W) = \frac{1}{4}\left[(u'(y_h) - u'(y_l)\right]\left(\frac{\partial c_h^h}{\partial W} + \frac{\partial c_l^h}{\partial W}\right)\Big|_{\{c_i^j\}=\{y_j\}} > 1$$

The two derivatives are

$$\frac{\partial c_h^h}{\partial W} = -\frac{\begin{vmatrix} \beta^2 & P_{c_l^h} \\ \beta^2 & Q_{c_l^h} \end{vmatrix}}{\begin{vmatrix} P_{c_h^h} & P_{c_l^h} \\ Q_{c_h^h} & Q_{c_l^h} \end{vmatrix}}, \quad \frac{\partial c_l^h}{\partial W} = -\frac{\begin{vmatrix} P_{c_h^h} & \beta^2 \\ Q_{c_h^h} & \beta^2 \end{vmatrix}}{\begin{vmatrix} P_{c_h^h} & P_{c_l^h} \\ Q_{c_h^h} & Q_{c_l^h} \end{vmatrix}},$$

with the auxiliary derivatives P, Q as the partial derivatives of the binding participation constraints (A.6) and (A.7) evaluated at the autarky allocation given by

$$\begin{aligned}
P_{c_h^h} &= (1-\beta)[u'(y_h) + \frac{\beta}{2}\kappa u'(y_h) - \frac{\beta}{2}(1-\kappa)u'(y_l)] \\
P_{c_l^h} &= P_{c_h^h} - (1-\beta)u'(y_h) \\
Q_{c_l^h} &= (1-\beta)[u'(y_h) + \frac{\beta}{2}(1-\kappa)u'(y_h) - \frac{\beta}{2}\kappa u'(y_l)] \\
Q_{c_h^h} &= Q_{c_l^h} - (1-\beta)u'(y_h).
\end{aligned}$$

Using these expression, the sum of the partial derivatives with respect to W evaluated at $\{c_i^j\} = \left\{y_j\right\}$ is given by

$$\begin{aligned}
\left(\frac{\partial c_h^h}{\partial W} + \frac{\partial c_l^h}{\partial W}\right) &= -\frac{2\beta^2(1-\beta)u'(y_h)}{(1-\beta)u'(y_h)[P_{c_h^h} + Q_{c_l^h} - (1-\beta)u'(y_h)]} \\
&= \frac{-4\beta}{(1-\beta)\left[u'(y_h)\left(\frac{2}{\beta}+1\right) - 2u'(y_l)\right]}.
\end{aligned}$$

Using this expression in $V'_{rs}(W)$ and collecting terms eventually results in

$$u'(y_h)(2-\beta) > \beta u'(y_l).$$

Under this condition, the optimal memoryless arrangement is not the outside option and thus characterized by risk sharing. It implies that at the optimal allocation $V'(W) < 1$ which leads to

$$F_{c_h^h} G_{c_l^h} - F_{c_l^h} G_{c_h^h} > 0$$

which is used in the proof of Proposition 4. ■

A.5 Details on the joint distribution of income and signals

In this subsection, we explain how to derive the formulas (1.2) and (1.3) stated in the main text. Further, we explain the logic behind the assumption that the stochastic process for signals shares the transition probabilities with the process for income.

A.5.1 Derivation of the formulas on the joint distribution of income and signals

We start with the derivation of the conditional probability of income. Using the general formula for calculating conditional probabilities, we receive

$$\pi\left(y' = y_j | k = y_m, y = y_i\right) = \frac{\pi\left(y' = y_j, k = y_m, y = y_i\right)}{\pi\left(k = y_m, y = y_i\right)}.$$

The conditional probability of income can be simplified using the identity

$$\sum_{z=1}^{N} \pi\left(y' = y_z | k = y_m, y = y_i\right) = 1$$

to replace the denominator with the following expression

$$\pi\left(k=y_m, y=y_i\right)=\sum_{z=1}^{N} \pi\left(y'=y_z, k=y_m, y=y_i\right).$$

The joint probability in the numerator is

$$\pi\left(y'=y_j, k=y_m, y=y_i\right)=\pi_{ij} \kappa^{\mathbf{1}_{m=j}}\left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=j}},$$

where π_{ij} is the Markov transition probability for moving from income i to income z. For all income states that are not indicated by the signal, $j \neq m$, we assume here that their probability of occurrence conditional on the signal is identical and therefore equals $(1-\kappa)/(N-1)$. For the conditional probability of income, the general formula can then be written as

$$\pi\left(y'=y_j | k=y_m, y=y_i\right)=\frac{\pi_{ij} \kappa^{\mathbf{1}_{m=j}}\left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=j}}}{\sum_{z=1}^{N} \pi_{iz} \kappa^{\mathbf{1}_{m=z}}\left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=z}}} \tag{A.8}$$

which resembles (1.2) in the main text. For example, with two equally likely persistent income states, the conditional probability of receiving a low income y_l in the future conditional on a high signal $k=y_h$ and a low income today is given according to (A.8) by

$$\pi\left(y'=y_l | k=y_h, y=y_l\right)=\frac{(1-\kappa) \pi_{11}}{(1-\kappa) \pi_{11}+\left(1-\pi_{11}\right) \kappa}.$$

The joint transition probability $\pi(s'|s)=\pi\left(y', k'|k, y\right)$ can be computed by combining the conditional probability of income with an assumption on the signal process. With signals following an exogenous first-order Markov process, the conditional probability $\pi\left(y', k'|k, y\right)$ is given by

$$\pi\left(y'=y_j, k'=y_l | k=y_m, y=y_i\right)=\pi_{ml} \frac{\pi_{ij} \kappa^{\mathbf{1}_{m=j}}\left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=j}}}{\sum_{z=1}^{N} \pi_{iz} \kappa^{\mathbf{1}_{m=z}}\left(\frac{1-\kappa}{N-1}\right)^{1-\mathbf{1}_{m=z}}} \quad \forall k', \tag{A.9}$$

where compared to (1.3), we used $\pi(k' = y_l | k = y_m) = \pi_{ml}$ because the signal process is characterized by the same transition probabilities as income. In the following, we argue why we choose signals that share the transition probabilities with income.

Consistency requirements with exogenous signals

We assumed that the signal realizations share the transition probabilities with the stochastic income process. In the following, we argue why we make this assumption by comparing implications of this assumption to alternative stochastic processes for signals. In our environment, income is the fundamental uncertainty and signals provide additional information on future realizations of income without affecting the realizations of income themselves. More formally, the later implies that the joint distribution of income and signals should satisfy the following two consistency requirements.

Consistency Requirement I: The marginal distribution of the joint invariant distribution $\pi(s) = \pi(y,k)$ equals the invariant distribution of income $\pi(y)$ i.e.

$$\hat{\pi}(y) = \sum_{k \in Y} \pi(y,k) \doteq \pi(y)$$

Consistency Requirement II: The conditional distribution of income $\pi(y'|y)$ follows from integrating over the signals

$$\hat{\pi}(y'|y) = \sum_{k \in Y} \pi(y'|y,k)\pi(k|y) \doteq \pi(y'|y).$$

In the following proposition, we show that when signals follow the same stochastic process as income, the two requirements are satisfied. If signals were to follow a different process then at least one of the requirements is violated. For the analytical results, we consider an income process with two values y_l and y_h and a symmetric transition between income states. The transition matrix for these two income states is given as P=

$$\begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix}$$

where rows represent the present income state and columns represent the future income states. For $p = 0.5$, income states is i.i.d.

Proposition 8 *Consider a Markov income process with transition matrix P.*

(i) If signals follow the same stochastic process as income then both consistency requirements are satisfied.

(ii) Consider a Markov process for signals with transition matrix $\tilde{P}$

$$\tilde{P} = \begin{bmatrix} \tilde{p} & 1-\tilde{p} \\ 1-\tilde{p} & \tilde{p} \end{bmatrix}$$

and $0 < \tilde{p} < 1$, $\tilde{p} \neq p$. Then Consistency Requirement II is violated.

Proof.

(i) When signals follow the same transition probabilities as income, the transition probabilities of s can be computed using (1.3) and are then summarized in the transition matrix P_s. For example, the probability of a low income and a low signals conditional on a low income and signal is

$$\pi\left(y' = y_l, k' = y_l | k = y_l, y = y_l\right) = p\frac{\kappa p}{(1-\kappa)(1-p) + p\kappa}.$$

The unique stationary distribution corresponding to the transition matrix P_s is given by

$$\pi(y,k) = \begin{bmatrix} \pi(y_l,k_l) \\ \pi(y_l,k_h) \\ \pi(y_h,k_l) \\ \pi(y_h,k_h) \end{bmatrix} = \begin{bmatrix} \kappa p - \frac{p}{2} - \frac{\kappa}{2} + \frac{1}{2} \\ \frac{\kappa}{2} + \frac{p}{2} - \kappa p \\ \frac{\kappa}{2} + \frac{p}{2} - \kappa p \\ \kappa p - \frac{p}{2} - \frac{\kappa}{2} + \frac{1}{2} \end{bmatrix}$$

Adding the first two and last two rows show that Consistency Requirement I is satisfied. Further, the probabilities of signals conditional on income can be computed from the invariant distribution. For example, the probability of a low signal conditional on a low income can be computed as

$$\pi(k=y_l|y=y_l) = \frac{\kappa p - \frac{p}{2} - \frac{\kappa}{2} + \frac{1}{2}}{\kappa p - \frac{p}{2} - \frac{\kappa}{2} + \frac{1}{2} + \frac{\kappa}{2} + \frac{p}{2} - \kappa p} = 2\kappa p - \kappa - p + 1.$$

To check for the Consistency Requirement II, we consider present income $y = y_l$ and future income $y' = y_l$ (the other transitions can be computed in the same way and are omitted here)

$$\begin{aligned}
&\hat{\pi}(y'=y_l|y=y_l) = \sum_{k\in Y} \pi(y'=y_l|y=y_l,k)\pi(k|y=y_l) \\
&= \pi(y'=y_l|y=y_l,k=y_l)\pi(k=y_l|y=y_l) + \pi(y'=y_l|y=y_l,k=y_h)\pi(k=y_h|y=y_l) \\
&= \frac{\kappa p}{\kappa p + (1-\kappa)(1-p)}(2\kappa p - \kappa - p + 1) + \frac{p(1-\kappa)}{\kappa(1-p)+p(1-\kappa)}(\kappa + p - 2\kappa p) \\
&= p
\end{aligned}$$

which is also satisfied. From the other side, for the transition from low income today to low income in the future, Requirement II calls for

$$p \doteq \pi(y'=y_l|y=y_l,k=y_l)\hat{\pi}(k=y_l|y=y_l) + \pi(y'=y_l|y=y_l,k=y_h)[1-\hat{\pi}(k=y_l|y=y_l)],$$

	Consistency requirements	
	I, $\max(\lvert\widehat{\pi}(y) - \pi(y)\rvert)$	II, $\max(\lvert\widehat{\pi}(y'\lvert y) - \pi(y'\lvert y)\rvert)$
i.i.d. signals	0.0115	0.1620
persistent signals	$3.70e-16$	$2.11e-16$

Table A.1: Consistency requirement results with persistent income

which has as unique solution $\hat{\pi}(k = y_l | y = y_l) = 2\kappa p - \kappa - p + 1$ which completes the proof of part (i).

(ii) The general symmetric transition matrix for signals $\tilde{P}$ results in a joint transition matrix for signals and income $\tilde{P}_s$ and in a unique invariant distribution for income and signals $\tilde{\pi}(y, k)$ with a unique conditional probability $\tilde{\pi}(k = y_l | y = y_l)$. If an only if $\tilde{p} = p$, it is $\tilde{\pi}(k = y_l | y = y_l) = \hat{\pi}(k = y_l | y = y_l) = 2\kappa p - \kappa - p + 1$. Thus, Requirement II is violated for $\tilde{p} \neq p$. Requirement I is satisfied because $\sum_k \tilde{\pi}(y_l, k) = 1/2 = \sum_k \tilde{\pi}(y_h, k)$ for any $0 < \tilde{p} < 1$.

■

As an immediate implication of the proposition, i.i.d. signals violate Requirement II when income is persistent. When income has more than two states, i.i.d. signals violate not only the second requirement. In Table A.1, we also compare both signal processes using the income process employed for computing the quantitative results in Section 1.4 for $\kappa = 0.99$ as an extreme case. As displayed in the first row of the table, i.i.d. signals fail both consistency requirements. The inconsistency following from i.i.d. signals is not negligible. On average, i.i.d. signals imply a perceived income transition that differs from the true transition by 16 percent. Persistent signals with the same persistence as income continue to satisfy both requirements (see the second row).[2]

[2] We use the method proposed by Tauchen and Hussey (1991) to approximate the income process. This method implies a symmetric transition matrix.

A.6 Numerical algorithm: limited commitment endowment economy

To solve the limited commitment model with signals, we follow the policy function algorithm proposed by Coleman (1990) and extended for these models by Krueger and Perri (2011). Given a discount factor β and state (w, s), we search for the optimal piecewise linear functions $\{w'_{s'}(w, s)\}_{s' \in S}$ and $h(w, s)$. We apply the Ridder algorithm and check for convergence on the discount factor. The algorithm can be summarized in the following steps

Step 1: Solve for the autarky values and get the maximum and minimum values of the outside option $(\underline{w}, \overline{w})$.

Step 2: Create the grid for the promised values $w_{grid} = \{\underline{w}, \cdots, \overline{w}\}$.

Step 3: Guess an initial value for $V_w^0(w, s)$, where the subscript refers to differentiation with respect to w.

Step 4: Use the first order conditions (1.10) and (1.11) and the promise keeping constraint (1.8) to solve for $h(w, s)$ and $\{w'_{s'}(w, s)\}$ for each $(w, s) \in w_{grid} \times S$[3]. The solution at each grid point involves two steps.

- First check which of the participation constraints are binding. The ones which are binding, replace them with the corresponding autarky value. Solve for the remaining values. For N states, then we have to solve for each grid point $N + 1$ equations. We can solve this as there are N first order conditions and one promise keeping constraint. If $M \leq N$ participation constraints are binding then we drop those variables and the corresponding first order conditions.
- After solving, again check whether any $w'_{s'}(w, s)$ violates the participation constraint in state s'. If yes, then replace the corresponding promise with the autarky value.

[3] Note that $h(w, s)$ and $\{w'_{s'}(w, s)\}$ are not restricted to be on the w grid.

Step 5: Update $V_w^0(w,s)$ to $V_w^1(w,s)$ using the envelope condition (1.12).

Step 6: Check whether convergence in $h(w,s), \{w'_{s'}(w,s)\}$ and $V_w(w,s)$ is achieved. Otherwise go back to Step 4 with the updated value of $V_w^1(w,s)$

In the next step, use policy functions $h(w,s)$ and $w'_{s'}(w,s)$ (that are piece-wise linear in w) to compute the invariant distribution in the following manner. For each grid point (w,s) find $\underline{w}_{s'}(w,s)$ and $\overline{w}_{s'}(w,s)$ such that

$$\underline{w}_{s'}(w,s) = \max\{w \in w_{grid} | w \leq w'_{s'}(w,s)\}$$
$$\overline{w}_{s'}(w,s) = \min\{w \in w_{grid} | w > w'_{s'}(w,s)\}$$

Then find $\alpha_{s'}(w,s)$ which solves the equation

$$\alpha_{s'}(w,s)\underline{w}_{s'}(w,s) + (1 - \alpha_{s'}(w,s))\overline{w}_{s'}(w,s) = w'_{s'}(w,s)$$

Using $\alpha_{s'}(w,s)$ define the Markov transition matrix $Q : (W \times S) \times (W \times S) \to [0,1]$ as

$$Q[(w,s),(w',s')] = \begin{cases} \pi(s'|s)\alpha_{s'}(w,s) & \text{if} \quad w' = \underline{w}_{s'}(w,s) \\ \pi(s'|s)(1 - \alpha_{s'}(w,s)) & \text{if} \quad w' = \overline{w}_{s'}(w,s) \\ 0 & \text{otherwise} \end{cases}$$

The invariant distribution $\Phi_{w,s}$ is then computed as the normalized eigenvector of Q corresponding to the unit eigenvalue. Using the invariant distribution compute the excess demand

$$d(\beta) = \int C(w,s)\,\mathrm{d}\,\Phi_{w,s} - \int y\,\mathrm{d}\,\Pi(y)$$

and check whether it is satisfied. If not, decrease β if $d(\beta)$ is in surplus and increase β if it is in deficit, and go back to Step 4. We use a Ridder algorithm until convergence on the discount factor is achieved and excess demand equals zero.

A.7 Numerical algorithm: limited commitment production economy

Given initial wealth a, state $s = (y,k)$, and an interest rate R, households' problem can be written recursively as

$$V(a,s) \quad = \quad \max_{c,\{a'\}} \left[(1-\beta)u(c(a,s)) + \beta \sum_{s'} \pi(s'|s) V'(a'(a,s;s'),s') \right]$$

subject to a budget and a borrowing constraint

$$c + \sum_{s'} \frac{\pi(s'|s)a'(a,s;s')}{R} \quad \leq \quad y + a \tag{A.10}$$

$$a'(a,s;s') \quad \geq \quad A(s'), \quad \forall s'. \tag{A.11}$$

The borrowing limits satisfy the following equations

$$U^{Aut}(s') = V'[A(s'),s'], \quad \forall s'. \tag{A.12}$$

The first order conditions are

$$u'[c(a,s)](1-\beta) = \lambda = V_a(a,y) \tag{A.13}$$

$$\beta V_a'[a'(a,s;s'),s'] \leq \frac{u'[c(a,s)](1-\beta)}{R}, \quad \forall s', \tag{A.14}$$

where $V_a'[a'(a,s;s'),s']$ denotes the derivative of the value function with respect to $a'(a,s;s')$. Consider N income states such that $s \in S = (s_1, s_2, ... s_{N^2})$. Consider a grid for a. Start with a guess of the value function V_0 and for the derivative $V_{a,0}$. From the guess of the value function, back out the state-dependent borrowing limits $A_0(s')$ from (A.12).

1. For each pair a,s, solve for the policy functions $c_0(a,s), \{a_0'(a,s;s')\}$ using the N^2+1 first order conditions (A.14) and (A.10). Start with the strict equality for all s' and solve. Check

borrowing constraints. If not satisfied in some state s', set $a_0'(a,s;s') = A_0(s')$ and solve again for $c_0(a,s)$ and the remaining $a_0'(a,s;s')$ until no borrowing constraint is violated.

2. Update the derivative of the value function with respect to a using the envelope condition and the policy function for consumption

$$V_{a,1}(a,s) = u'[c_0(a,s)](1-\beta)$$

3. Update the value function according to the Bellman equation to receive V_1

$$V_1(a,s) = (1-\beta)u[c_0(a,s)] + \beta \sum_{s'} \pi(s'|s) V_0[a_0'(a,s;s'),s']$$

4. Continue until convergence in the policy functions, the derivative of the value function and in the value function $V_n(a,s) = V_{n+1}(a,s) = V(a,s)$ is achieved.

5. Then update the borrowing limits solving the following equation for A_1

$$V[A_1(s'),s)] = U^{aut}(s').$$

6. Continue until convergence in the policy functions, in the value function (and its derivative) and in the borrowing limits is achieved.

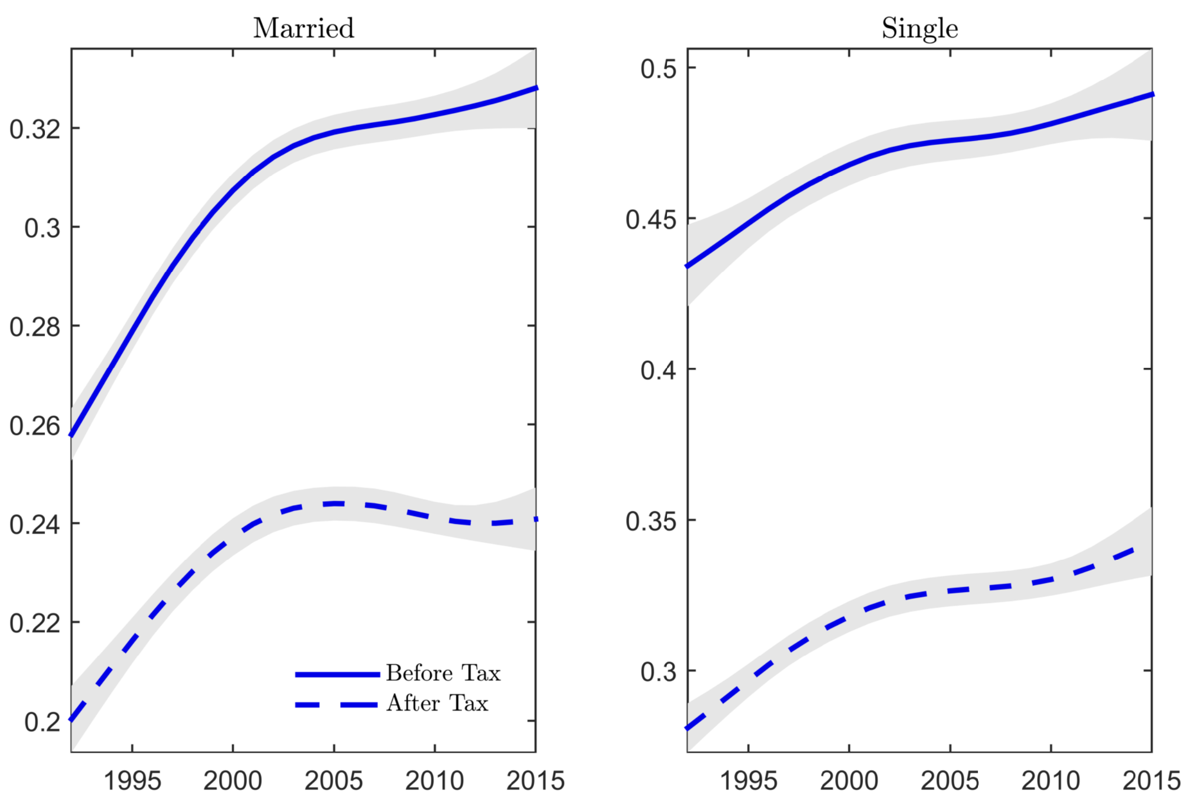
The computation of the invariant distribution $\Phi_{a,s}$ follows the same steps as in the endowment economy. The excess demand on the goods market now reads

$$d_K(\beta) = \int c(a,s)\,\mathrm{d}\,\Phi_{a,s} + K' - K(1-\delta) - AF(L,K).$$

Appendix B

B.1 Figures and Tables

Figure B.1: Before and after-tax and transfer income risk of married and single households.



Note: *The shaded region represent the 95 percent confidence interval computed using 1000 bootstrap samples.*

Figure B.1 reports the before and after tax income risk of married and single households over the period 1992-2015. Some important points to notice. Before and after tax income risk of married and single households has increased. However after 2004, the reduction in income risk due to taxes and transfer is very strong for married households. So even though they experienced increase in before-tax and transfer income risk, their after-tax and transfer income risk did not change much.

Table B.1: Group specific mean of age, income, taxes and transfers in CPS 1992-2015 sample.

	Single					Married				
Year	N	Age	Income	Taxes	Transfers	N	Age	Income	Taxes	Transfers
1992	8392	41	28318	7010	932	17828	42	25864	6475	537
1993	8258	42	28140	7021	997	17762	42	26135	6519	615
1994	7839	42	28324	7156	989	16650	42	28797	7152	867
1995	8007	42	29193	7544	945	16501	42	27254	7386	607
1996	7300	42	29934	7398	1016	14384	42	29143	7351	572
1997	7535	42	30966	7877	1013	14506	42	29209	7492	591
1998	7540	42	30721	7876	935	14066	43	30264	7942	554
1999	7812	42	30963	7984	974	14122	43	31299	8290	580
2000	7917	42	31757	8848	912	14020	43	31218	8757	563
2001	11892	42	30472	7782	1084	23229	42	32063	8553	558
2002	12296	43	31596	8120	1085	23005	43	32317	8531	598
2003	12036	43	31357	7398	1107	22430	43	32557	7952	647
2004	11573	43	30921	7161	1090	21890	43	32713	7536	668
2005	11626	43	31057	7465	1190	21132	43	32891	8062	915
2006	11655	44	31402	7721	1143	20798	43	32606	7624	880
2007	11744	44	31679	8079	1092	20807	44	33491	7956	850
2008	11955	44	31918	8268	1066	20462	44	33398	7989	795
2009	11689	44	30994	7595	1036	20472	44	33334	7899	835
2010	11249	44	31686	7813	1297	19536	44	33451	7785	1037
2011	11049	44	31722	8055	1273	18651	44	33002	8092	1007
2012	11050	45	31589	7299	1177	18195	45	33676	7661	948
2013	10890	45	32199	7452	1073	18427	45	33407	7459	872
2014	10776	45	31492	7990	1030	18159	45	34011	8578	827
2015	10710	45	31745	8309	972	17937	45	34444	9353	831
All years	242790	43	30964	7726	1072	444969	43	31701	7864	747

Notes: The present table reports the mean of age, labor income, taxes and transfers for married and single households in the CPS 1992-2015 sample. The age refers to the head's age. Taxes is the sum of federal, state and social security payroll taxes. The measure of transfers is the sum of child tax credit, additional child tax credit, the dollar value of food stamps and income from (supplemental) social security, welfare, unemployment, retirement and worker compensations. Furthermore, depending upon the status of individual, income from veteran or survivor or disability benefits is added.1

Table B.2: Estimated after and before-taxes and transfer labor income risk and public insurance for married households.

Year	$\mathrm{var}(y_{it})$	$\mathrm{var}(\tilde{y}_{it})$	$\tau^{IR} = 1 - \frac{\mathrm{var}(y_{it})}{\mathrm{var}(\tilde{y}_{it})}$
1992	0.198	0.262	0.244
1993	0.203	0.270	0.248
1994	0.208	0.262	0.208
1995	0.204	0.260	0.217
1996	0.230	0.304	0.244
1997	0.237	0.314	0.247
1998	0.232	0.311	0.255
1999	0.228	0.301	0.241
2000	0.234	0.297	0.214
2001	0.250	0.332	0.248
2002	0.256	0.325	0.212
2003	0.242	0.325	0.255
2004	0.242	0.321	0.245
2005	0.234	0.319	0.266
2006	0.241	0.319	0.244
2007	0.258	0.325	0.205
2008	0.251	0.308	0.185
2009	0.234	0.310	0.246
2010	0.246	0.338	0.271
2011	0.225	0.327	0.312
2012	0.233	0.326	0.285
2013	0.232	0.318	0.273
2014	0.234	0.321	0.270
2015	0.260	0.344	0.243

Notes: The table reports the after and before-taxes and transfers labor income risk (second and third column, respectively) and the public insurance for married households. The labor income is calculated using Katz and Autor (1999) methodology and refers to the variance of the residual logged income.

Table B.3: Estimated after and before-taxes and transfer labor income risk and public insurance for single households.

Year	$\mathrm{var}(y_{it})$	$\mathrm{var}(\tilde{y}_{it})$	$\tau^{IR} = 1 - \frac{\mathrm{var}(y_{it})}{\mathrm{var}(\tilde{y}_{it})}$
1992	0.286	0.452	0.367
1993	0.283	0.432	0.344
1994	0.275	0.436	0.368
1995	0.280	0.444	0.368
1996	0.316	0.469	0.327
1997	0.326	0.481	0.322
1998	0.320	0.475	0.327
1999	0.305	0.452	0.324
2000	0.299	0.456	0.345
2001	0.333	0.489	0.319
2002	0.350	0.491	0.288
2003	0.319	0.477	0.332
2004	0.328	0.485	0.324
2005	0.315	0.473	0.334
2006	0.348	0.480	0.276
2007	0.317	0.474	0.331
2008	0.325	0.463	0.298
2009	0.328	0.472	0.305
2010	0.324	0.493	0.343
2011	0.312	0.490	0.364
2012	0.327	0.487	0.328
2013	0.349	0.504	0.308
2014	0.345	0.475	0.274
2015	0.348	0.499	0.301

Notes: The table reports the after and before-taxes and transfers labor income risk (second and third column, respectively) and the public insurance for single households. The labor income is calculated using Katz and Autor (1999) methodology and refers to the variance of the residual logged income.

Appendix C

C.1 Household Problem

The objective of the household is to maximize (3.1) subject to (3.2). As the household budget constraint will hold with equality in every period, we can form the Lagrangian as

$$\mathcal{L} = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right] + \beta^t \lambda_t^c \left(W_t N_t + B_t + \Upsilon_t - T_t - P_t C_t - \mathbb{E}_t Q_{t,t+1} B_{t+1} \right) \right\}$$

where λ_t^c is the Lagrange multiplier associated with household budget constraint in period t. Intuitively it is the gain in utility by one extra unit of income. The first order conditions (FOC) with respect to C_t, N_t, B_{t+1} are given as

$$C_t^{-\sigma} = \lambda_t^c P_t, \quad N_t^{\phi} = \lambda_t^c W_t, \quad \mathbb{E}_t Q_{t,t+1} = \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \right)$$

Substituting, $\mathbb{E}_t Q_{t,t+1} = \frac{1}{R_t}$, $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$, we can write the FOC's as

$$\frac{N_t^{\phi}}{C_t^{-\sigma}} = \frac{W_t}{P_t}, \qquad 1 = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t}{\Pi_{t+1}} \right]$$

The stochastic discount factor is defined as

$$Q_{t,\tau} = \beta^{\tau-t} \frac{\lambda_\tau^c}{\lambda_t^c} = \beta^{\tau-t} \frac{C_\tau^{-\sigma}}{C_t^{-\sigma}} \frac{P_\tau}{P_t}$$

C.2 Firm Problem

C.2.1 Second Stage Final Good Producer Problem

The problem of the second stage final good producer is to minimize the cost of production given the demand for final output and sector level price indices. Formally, we can state the problem as

$$\min_{Y_{s,t},Y_{f,t}} P_{s,t}Y_{s,t} + P_{f,t}Y_{f,t} \qquad s.t. \qquad Y_t = \frac{Y_{s,t}^{\eta}Y_{f,t}^{1-\eta}}{\eta^{\eta}(1-\eta)^{1-\eta}}$$

Forming the Lagrangian,

$$\mathcal{L} = P_{s,t}Y_{s,t} + P_{f,t}Y_{f,t} + \lambda_t^{P2}\left[Y_t - \frac{Y_{s,t}^{\eta}Y_{f,t}^{1-\eta}}{\eta^{\eta}(1-\eta)^{1-\eta}}\right]$$

where λ_t^{P2} is the marginal cost of production in period t. The first order conditions with respect to $Y_{s,t}$ and $Y_{f,t}$ are given as

$$\lambda_t^{P2}\ \eta\ \frac{Y_{s,t}^{\eta-1}Y_{f,t}^{1-\eta}}{\eta^{\eta}(1-\eta)^{1-\eta}} = P_{s,t}, \qquad \lambda_t^{P2}\ (1-\eta)\ \frac{Y_{s,t}^{\eta}Y_{f,t}^{-\eta}}{\eta^{\eta}(1-\eta)^{1-\eta}} = P_{f,t}$$

Taking the ratio of these two first order conditions we get

$$\frac{P_{f,t}}{P_{s,t}} = \left(\frac{1-\eta}{\eta}\right)\frac{Y_{s,t}}{Y_{f,t}}$$

Substituting the ratio $\frac{Y_{s,t}}{Y_{f,t}}$ in production function will give the demand of output from sector s and f. We can also write

$$\frac{\lambda_t^{P2}}{\eta^{\eta}(1-\eta)^{1-\eta}}\left(\frac{Y_{s,t}}{Y_{f,t}}\right)^{\eta-1}$$

Substituting for $\frac{Y_{s,t}}{Y_{f,t}}$ we will get,

$$\lambda_t^{P2} = P_{s,t}^{\eta} P_{f,t}^{1-\eta}$$

Define $\lambda_t^{P2} = P_t$, where P_t is the aggregate price index.

C.2.2 First Stage Final Goods Producer Problem

Here we will solve for the final goods producers problem belonging to sector s, as for sector f it is exactly similar. The first stage final goods producers problem is to minimize the cost of aggregation, given the demand for final output from sector s, the prices charged by intermediate producer and the production technology. So formally we can write

$$\min_{Y_{s,t}(i), i\in[0,n)} \int_0^n P_{s,t}(i) Y_{s,t}(i) di, \qquad s.t. \qquad Y_{s,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n Y_{s,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$$

Forming the Lagrangian we get

$$\mathcal{L} = \int_0^n P_{s,t}(i) Y_{s,t}(i) di + \lambda_t^{P1} \left(Y_{s,t} - \left[\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n Y_{s,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \right)$$

where λ_t^{P1} is the sector marginal cost of production in period t. The first order condition with respect to $Y_{s,t}(i)$ and $Y_{s,t}(j)$, $i, j \in [0, n)$ are given as

$$\lambda_t^{P1} \left(\frac{Y_{s,t}}{n}\right)^{1/\epsilon} Y_{s,t}(i)^{-1/\epsilon} = P_{s,t}(i), \quad \lambda_t^{P1} \left(\frac{Y_{s,t}}{n}\right)^{1/\epsilon} Y_{s,t}(j)^{-1/\epsilon} = P_{s,t}(j)$$

Taking the ratio of these two first order conditions we get

$$\frac{Y_{s,t}(i)}{Y_{s,t}(j)} = \left(\frac{P_{s,t}(i)}{P_{s,t}(j)}\right)^{-\epsilon}$$

From the first FOC we get,

$$Y_{s,t}(i) = \left(\frac{P_{s,t}(i)}{\lambda_t^{P1}}\right)^{-\epsilon} \frac{Y_{s,t}}{n}$$

Substituting this in the production technology, and solving for λ_t^{P1} we will get

$$\lambda_t^{P1} = \left[\frac{1}{n}\int_0^n P_{s,t}(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$$

where $\lambda_t^{P1} = P_{s,t}$ is the sector s price index. Now substituting

$$Y_{s,t}(i) = \left(\frac{P_{s,t}(i)}{P_{s,t}(j)}\right)^{-\epsilon} Y_{s,t}(j)$$

in the production function and solving for $Y_{s,t}(j)$ we will get

$$Y_{s,t}(j) = \frac{1}{n}\left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{-\epsilon} Y_{s,t}$$

C.2.3 Intermediate Producer Problem

The intermediate producer faces two problem. First, minimize the cost of production given the demand, the factor prices and the technology. Second, under Calvo (1983) price setting mechanism, when it gets a chance to change prices, it does so in order to maximize the future nominal profits. Formally writing the first problem as

$$\min_{X_{s,t}(i), N_{s,t}(i)} P_t X_{s,t}(i) + W_t N_{s,t}(i) \qquad s.t. \qquad Y_{s,t}(i) = Z_{s,t} X_{s,t}(i)^{\chi} N_{s,t}(i)^{1-\chi}$$

Forming the Lagrangian

$$\mathcal{L} = P_t X_{s,t}(i) + W_t N_{s,t}(i) + \lambda_{s,t}^m [Y_{s,t}(i) - Z_{s,t} X_{s,t}(i)^{\chi} N_{s,t}(i)^{1-\chi}]$$

where $\lambda^m_{s,t}$ is the marginal cost of production for producer i in sector s. The point to notice here is that marginal cost is not varying from producer to producer. This is because of the assumption of constant returns to scale production technology. The first order condition with respect to $X_{s,t}(i)$ and $N_{s,t}(i)$ are given as

$$\lambda^m_{s,t} Z_{s,t} \chi X_{s,t}(i)^{\chi-1} N_{s,t}(i)^{1-\chi} = P_t, \qquad \lambda^m_{s,t} Z_{s,t}(1-\chi) X_{s,t}(i)^{\chi} N_{s,t}(i)^{-\chi} = W_t$$

The ratio of the two first order conditions are given as

$$\frac{X_{s,t}(i)}{N_{s,t}(i)} = \left(\frac{\chi}{1-\chi}\right)\frac{W_t}{P_t}$$

From the first FOC we get

$$P_t = \lambda^m_{s,t} Z_{s,t} \chi \left(\frac{N_{s,t}(i)}{X_{s,t}(i)}\right)^{1-\chi} \Rightarrow \frac{\lambda^m_{s,t}}{P_t} = \frac{\Gamma^{1-\chi}}{\chi}\left(\frac{W_t}{P_t}\right)^{1-\chi}\frac{1}{Z_{s,t}}$$

where $\Gamma = \frac{\chi}{1-\chi}$ and $\frac{\lambda^m_{s,t}}{P_t}$ is the real marginal cost of production. We define $MC_{s,t} = \frac{\lambda^m_{s,t}}{P_t}$. The production technology for the intermediate producer i in sector s can be written as

$$Y_{s,t}(i) = Z_{s,t}\left(\frac{X_{s,t}(i)}{N_{s,t}(i)}\right)^{\chi} N_{s,t}(i)$$

Substituting for $\frac{X_{s,t}(i)}{N_{s,t}(i)}$ we get

$$N_{s,t}(i) = \Gamma^{-\chi}\left(\frac{W_t}{P_t}\right)^{-\chi}\frac{Y_{s,t}(i)}{Z_{s,t}}$$

and hence $X_{s,t}(i) = \Gamma^{1-\chi}\left(\frac{W_t}{P_t}\right)^{1-\chi}\frac{Y_{s,t}(i)}{Z_{s,t}}$. The second problem of the intermediate producer can be formalized as

$$\max_{P_{s,t}(i)} \mathbb{E}_t \sum_{\tau=t}^{\infty} \theta_s^{\tau-t} Q_{t,\tau}\left[P_{s,t}(i)(1+\tau_s) - MC_{s,\tau}P_\tau\right] Y_{s,\tau}(i)$$

where $Y_{s,\tau}(i) = \frac{1}{n}\left(\frac{P_{s,t}(i)}{P_{s,\tau}}\right)^{-\epsilon} Y_{s,\tau}$, τ_s is the subsidy given by the government in order to remove the monopolistic power which accrued to all the intermediate firms because of monopolistic competition, $Q_{t,\tau}$ is the stochastic discount factor. Substituting for $Y_{s,\tau}(i)$, and $Q_{t,\tau}$ and taking the first order derivative we will get

$$P_{s,t}(i)\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}C_\tau^{-\sigma}P_{s,\tau}^{\epsilon-1}Y_\tau = \frac{\epsilon}{(\epsilon-1)(1+\tau_s)}\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}C_\tau^{-\sigma}MC_{s,\tau}P_{s,\tau}^{\epsilon-1}P_\tau Y_\tau$$

Set $\tau_s = \frac{1}{\epsilon-1}$, government removes the price markup and hence there is no distortion in prices due to monopolistic competition. Hence, if a producer i in sector s gets the chance to change the prices then it chooses the price given as

$$P_{s,t}^* = \frac{\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}C_\tau^{-\sigma}MC_{s,\tau}P_{s,\tau}^{\epsilon-1}P_\tau Y_\tau}{\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}C_\tau^{-\sigma}P_{s,\tau}^{\epsilon-1}Y_\tau}$$

Next, log-linearize the optimal price around the steady state with constant prices. The optimal price can be written as

$$P_{s,t}^*\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}C_\tau^{-\sigma}P_{s,\tau}^{\epsilon-1}Y_\tau = \mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}C_\tau^{-\sigma}MC_{s,\tau}P_{s,\tau}^{\epsilon-1}P_\tau Y_\tau$$

First consider the left hand side. Define $\widehat{x_t} = \frac{X_t - X}{X}$ where X is the variable X_t's steady state[1] . Using this notation we can write

$$P\sum_{\tau=t}^{\infty}(\theta_s\beta)C^{-\sigma}P^{\epsilon-1}Y + P\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)C^{-\sigma}P^{\epsilon-1}Y\widehat{p_{s,t}^*} + P\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)C^{-\sigma}P^{\epsilon-1}Y(-\sigma\widehat{c_\tau})$$
$$P\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)C^{-\sigma}P^{\epsilon-1}Y((\epsilon-1)\widehat{p_{s,\tau}}) + P\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)C^{-\sigma}P^{\epsilon-1}Y\widehat{y_\tau}$$

[1] For the relative price in period t we define the notation $\widehat{T_t} = \frac{T_t - T}{T}$. This is done just to make the notation clear.

which reduces to

$$PC^{-\sigma}P^{\epsilon-1}Y\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}(1+\widehat{p}^*_{s,t}-\sigma\widehat{c_\tau}+(\epsilon-1)\widehat{p}_{s,\tau}+\widehat{y_\tau})$$

Now log-linearizing the right hand side of the equation,

$$\sum_{\tau=t}^{\infty}(\theta_s\beta)C^{-\sigma}MC_sP^{\epsilon-1}PY+\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)C^{-\sigma}MC_sP^{\epsilon-1}PY(-\sigma\widehat{c_\tau})+$$
$$\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)C^{-\sigma}MC_sP^{\epsilon-1}PY\widehat{mc}_{s,\tau}+\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)C^{-\sigma}MC_sP^{\epsilon-1}PY((\epsilon-1)\widehat{p}_{s,\tau})+$$
$$\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)C^{-\sigma}MC_sP^{\epsilon-1}PY\widehat{p_\tau}+\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)C^{-\sigma}MC_sP^{\epsilon-1}PY\widehat{y_\tau}$$

Equating the left hand and right hand side of log-linearized equation and making use of the fact that $MC_s=1$ we get

$$\widehat{p}^*_{s,t}=(1-\theta_s\beta)\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}\{\widehat{mc}_{s,\tau}+\widehat{p_\tau}\}$$

From the definition of relative price index, $T_t=\frac{P_{s,t}}{P_{f,t}}$ and using the aggregate price index (3.4) we can write

$$\widehat{p_t}=\widehat{p}_{s,t}-(1-\eta)\widehat{t_t}=\widehat{p}_{f,t}+\eta\widehat{T_t}$$

Hence we can write

$$\widehat{p}^*_{s,t}=(1-\theta_s\beta)\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}\{\widehat{mc}_{s\tau}+\widehat{p_\tau}\}=(1-\theta_s\beta)\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}\{\widehat{mc}_{s\tau}+\widehat{p}_{s,\tau}-(1-\eta)\widehat{T_\tau}\}$$

Similarly we can derive for sector f, which is given as

$$\widehat{p}^*_{f,t}=(1-\theta_f\beta)\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_f\beta)^{\tau-t}\{\widehat{mc}_{f\tau}+\widehat{p_\tau}\}=(1-\theta_f\beta)\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_f\beta)^{\tau-t}\{\widehat{mc}_{f\tau}+\widehat{p}_{f,\tau}+\eta\widehat{T_\tau}\}$$

Now we will derive the New Keynesian Phillips Curve, (NKPC), for the sector s. For sector f similar derivation follows. Consider the equation

$$\widehat{p}^{*}_{s,t} - \widehat{p}_{s,t-1} = (1-\theta_s\beta)\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}\{\widehat{mc}_{s\tau} + \widehat{p}_{s,\tau} - \widehat{p}_{s,t-1} - (1-\eta)\widehat{T}_{\tau}\}$$

which can be written further as

$$\widehat{p}^{*}_{s,t} - \widehat{p}_{s,t-1} = (1-\theta_s\beta)\mathbb{E}_t\sum_{\tau=t}^{\infty}\widehat{mc}_{s,\tau} - (1-\eta)\widehat{T}_{\tau} + (1-\theta_s\beta)\mathbb{E}_t[(\theta_s\beta)^0(\widehat{p}_{s,t} - \widehat{p}_{s,t-1}) +$$
$$(\theta_s\beta)^1(\widehat{p}_{s,t+1} - \widehat{p}_{s,t} + \widehat{p}_{s,t} - \widehat{p}_{s,t-1}) + (\theta_s\beta)^2(\widehat{p}_{s,t+2} - \widehat{p}_{s,t+1} + \widehat{p}_{s,t+1} - \widehat{p}_{s,t} + \widehat{p}_{s,t} - \widehat{p}_{s,t-1}) + \cdots]$$

which can be further written as

$$\widehat{p}^{*}_{s,t} - \widehat{p}_{s,t-1} = (1-\theta_s\beta)\mathbb{E}_t\sum_{\tau=t}^{\infty}\widehat{mc}_{s,\tau} - (1-\eta)\widehat{T}_{\tau} + (1-\theta_s\beta)\mathbb{E}_t[(\theta_s\beta)^0\pi_{s,t} +$$
$$(\theta_s\beta)^1(\pi_{s,t+1} + \pi_{s,t}) + (\theta_s\beta)^2(\pi_{s,t+2} + \pi_{s,t+1} + \pi_{s,t})\cdots]$$

Now in another step of manipulation we get

$$\widehat{p}^{*}_{s,t} - \widehat{p}_{s,t-1} = (1-\theta_s\beta)\mathbb{E}_t\sum_{\tau=t}^{\infty}\{\widehat{mc}_{s,\tau} - (1-\eta)\widehat{T}_{\tau}\} + \mathbb{E}_t[(\theta_s\beta)^0\pi_{s,t} + (\theta_s\beta)^1(\pi_{s,t+1} + \pi_{s,t}) +$$
$$(\theta_s\beta)^2(\pi_{s,t+2} + \pi_{s,t+1} + \pi_{s,t})\cdots]$$
$$-[(\theta_s\beta)^1\pi_{s,t} + (\theta_s\beta)^2(\pi_{s,t+1} + \pi_{s,t}) + (\theta_s\beta)^3(\pi_{s,t+2} + \pi_{s,t+1} + \pi_{s,t})\cdots]$$

which results in to

$$
\begin{aligned}
\widehat{p}^*_{s,t} - \widehat{p}_{s,t-1} &= (1-\theta_s\beta)\mathbb{E}_t \sum_{\tau=t}^{\infty}\{\widehat{mc}_{s,\tau} - (1-\eta)\widehat{T}_\tau\} + \\
&\quad \mathbb{E}_t\big[(\theta_s\beta)^0\pi_{s,t} + (\theta_s\beta)^1(\pi_{s,t+1} + \pi_{s,t}) + (\theta_s\beta)^2(\pi_{s,t+2} + \pi_{s,t+1} + \pi_{s,t})\cdots\big] \\
&= (1-\theta_s\beta)\mathbb{E}_t \sum_{\tau=t}^{\infty}\{\widehat{mc}_{s,\tau} - (1-\eta)\widehat{T}_\tau\} + \mathbb{E}_t \sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}\pi_{s,\tau} \\
&= (1-\theta_s\beta)\{\widehat{mc}_{s,t} - (1-\eta)\widehat{T}_t\} + \pi_{s,t} + \\
&\quad (1-\theta_s\beta)\mathbb{E}_t \sum_{\tau=t+1}^{\infty}(\theta_s\beta)^{\tau-t}\{\widehat{mc}_{s,\tau} - (1-\eta)\widehat{t}_\tau\} + \mathbb{E}_t \sum_{\tau=t+1}^{\infty}(\theta_s\beta)^{\tau-t}\pi_{s,\tau} \\
&= (1-\theta_s\beta)\{\widehat{mc}_{s,t} - (1-\eta)\widehat{T}_t\} + \pi_{s,t} + \theta_s\beta\mathbb{E}_t\{\widehat{p}^*_{s,t+1} - \widehat{p}_{s,t}\}
\end{aligned}
$$

Under the Calvo price setting mechanism we know the price index in sector s evolves as

$$P^{1-\epsilon}_{s,t} = \theta_s P^{1-\epsilon}_{s,t-1} + (1-\theta_s)P^{*(1-\epsilon)}_{s,t}$$

Hence log-linearizing this equation gives us

$$\pi_{s,t} = (1-\theta_s)(\widehat{p}^*_{s,t} - \widehat{p}_{s,t-1})$$

Hence we can write $\widehat{p}^*_{s,t} - \widehat{p}_{s,t-1}$ as

$$
\begin{aligned}
\widehat{p}^*_{s,t} - \widehat{p}_{s,t-1} &= (1-\theta_s\beta)\{\widehat{mc}_{s,t} - (1-\eta)\widehat{T}_t\} + (1-\theta_s)(\widehat{p}^*_{s,t} - \widehat{p}_{s,t-1}) + \theta_s\beta\mathbb{E}_t\{\widehat{p}^*_{s,t+1} - \widehat{p}_{s,t}\} \\
\widehat{p}^*_{s,t} - \widehat{p}_{s,t-1} &= \frac{(1-\theta_s\beta)}{\theta_s}\{\widehat{mc}_{s,t} - (1-\eta)\widehat{T}_t\} + \beta\mathbb{E}_t\{\widehat{p}^*_{s,t+1} - \widehat{p}_{s,t}\}
\end{aligned}
$$

Multiplying both sides by $(1-\theta_s)$ and using the fact that $\pi_{s,t} = (1-\theta_s)(\widehat{p}^*_{s,t} - \widehat{p}_{s,t-1})$, we get the NKPC for sector s which is given as

$$\pi_{s,t} = \kappa_s\{\widehat{mc}_{s,t} - (1-\eta)\widehat{T}_t\} + \beta\mathbb{E}_t\pi_{s,t+1}$$

where $\kappa_s = \frac{(1-\theta_s\beta)(1-\theta_s)}{\theta_s}$. Similarly for sector f we get the NKPC which is given as

$$\pi_{f,t} = \kappa_f\{\widehat{mc}_{f,t} + \eta\widehat{T}_t\} + \beta\mathbb{E}_t\pi_{f,t+1}$$

where $\kappa_f = \frac{(1-\theta_f\beta)(1-\theta_f)}{\theta_f}$

C.3 Aggregation and the Steady State

Demand faced by intermediate producer i in sector s is given by (3.6). Using the production technology of intermediate producer we can write

$$Z_{s,t}X_{s,t}(i)^{\chi}(i)N_{s,t}(i)^{1-\chi} = \frac{1}{n}\left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{-\epsilon}Y_{s,t}$$

Given the constant returns to scale of production we know

$$\frac{X_{s,t}(i)}{N_{s,t}(i)} = \frac{X_{s,t}}{N_{s,t}}$$

where $X_{s,t}, N_{s,t}$ are the aggregate output and labor demand from sector s respectively. Hence we can further write

$$Z_{s,t}\left(\frac{X_{s,t}}{N_{s,t}}\right)^{\chi} N_{s,t}(i) = \frac{1}{n}\left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{-\epsilon} Y_{s,t}$$
$$Z_{s,t}\left(\frac{X_{s,t}}{N_{s,t}}\right)^{\chi} \int_0^n N_{s,t}(i)di = \int_0^n \frac{1}{n}\left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{-\epsilon} Y_{s,t}$$
$$Z_{s,t}X_{s,t}^{\chi}N_{s,t}^{1-\chi} = Y_{s,t}D_{s,t}$$

where $D_{s,t} = \frac{1}{n}\int_0^n \left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{-\epsilon}$ is the measure of price dispersion in sector s. Similarly we get

$$Z_{f,t}X_{f,t}^{\chi}N_{f,t}^{1-\chi} = Y_{f,t}D_{f,t}$$

It can be proved using Jensen's inequality that $D_{s,t}, D_{f,t} \geq 1$. Hence presence of measure of relative price dispersion, will always push the equilibrium output below its flexible-price equilibrium level. Now the labour demand from firm i in sector s is given by (3.8). The aggregate labour demand by sector s is given by

$$\int_0^n N_{s,t}(i) = \Gamma^{-\chi}\left(\frac{W_t}{P_t}\right)^{-\chi} \frac{1}{Z_{s,t}} \int_0^n Y_{s,t}(i)$$
$$= \Gamma^{-\chi}\left(\frac{W_t}{P_t}\right)^{-\chi} \frac{Y_{s,t}}{Z_{s,t}} \frac{1}{n} \int_0^n \left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{-\epsilon}$$

which yields

$$N_{s,t} = \Gamma^{-\chi}\left(\frac{W_t}{P_t}\right)^{-\chi} \frac{Y_{s,t}}{Z_{s,t}} D_{s,t}, \qquad N_{f,t} = \Gamma^{-\chi}\left(\frac{W_t}{P_t}\right)^{-\chi} \frac{Y_{f,t}}{Z_{f,t}} D_{f,t} \tag{C.1}$$

Similarly,

$$X_{s,t} = \Gamma^{1-\chi}\left(\frac{W_t}{P_t}\right)^{1-\chi}\frac{Y_{s,t}}{Z_{s,t}}D_{s,t} \qquad X_{f,t} = \Gamma^{1-\chi}\left(\frac{W_t}{P_t}\right)^{1-\chi}\frac{Y_{f,t}}{Z_{f,t}}D_{f,t} \tag{C.2}$$

Note that we can write $D_{a,t}, a \in \{s, f\}$ in recursive form as (see Appendix for proof)

$$D_{a,t} = (1-\theta_a)\left[\frac{1}{1-\theta_a} - \left(\frac{\theta_a}{1-\theta_a}\right)\Pi_{a,t}^{\epsilon-1}\right]^{\frac{\epsilon}{\epsilon-1}} + \theta_a \Pi_{a,t}^{\epsilon} D_{a,t-1} \tag{C.3}$$

C.3.1 The Steady-State for Endogenous Variables

We derive the steady state of endogenous variables, and use the above mentioned code to compute the steady state of Lagrange multipliers associated with the constraints of the Ramsey problem. The steady state of generic variable X_t is denoted by $\bar{X}$. In the steady state, all endogenous variables are constant, and technology shocks $Z_{s,t}$ and $Z_{f,t}$ have the value $\bar{Z}_s = \bar{Z}_f = 1$. Conjecture the solution for which steady state value of aggregate gross inflation rate is 1, i.e. $\bar{\Pi} = 1$. Using the relative price identity, we know that $\bar{\Pi}_s = \bar{\Pi}_f$, and using the aggregate inflation equation we get, $\bar{\Pi} = \bar{\Pi}_s = \bar{\Pi}_f = 1$. Then using (C.3) we get $\bar{D}_s = \bar{D}_f = 1$. From (3.12), (3.13), we know

$$\frac{\bar{H}_s/\bar{K}_s}{\bar{H}_f/\bar{K}_f} = 1$$

Using this we can write

$$\bar{T} = \frac{\mu_s}{\mu_f}\frac{\bar{H}_s/\bar{K}_s}{\bar{H}_f/\bar{K}_f}$$

Hence $\bar{T} = \frac{\mu_s}{\mu_f}$. From (3.11) we know

$$\mu_s \frac{\bar{H}_s}{\bar{K}_s} = \bar{T}^{1-\eta}, \qquad \mu_f \frac{\bar{H}_f}{\bar{K}_f} = \bar{T}^{-\eta}$$

Defining $\mu = \mu_s^{\eta} \mu_f^{1-\eta}$ we can write $\frac{\bar{H}_s}{\bar{K}_s} = \frac{\bar{H}_f}{\bar{K}_f} = \frac{1}{\mu}$. From (3.12) and (3.13) we can write

$$\frac{\bar{H}_s}{\bar{K}_s} = \frac{\Gamma^{1-\chi}}{\chi} (\bar{C}^{\sigma} \bar{N}^{\phi})^{1-\chi}$$

which gives us

$$\bar{C}^{\sigma} \bar{N}^{\phi} = \left(\frac{\chi}{\mu}\right)^{\frac{1}{1-\chi}} \frac{1}{\Gamma}$$

From (C.2) we can write

$$\begin{aligned}
\bar{X}_s + \bar{X}_f &= \Gamma^{1-\chi} (\bar{C}^{\sigma} \bar{N}^{\phi})^{1-\chi} (\eta \bar{T}^{\eta-1} + (1-\eta)\bar{T}^{\eta}) \bar{Y} \\
&= \left(\frac{\chi}{\mu}\right) \left[\eta \bar{T}^{\eta-1} + (1-\eta)\bar{T}^{\eta}\right] \bar{Y}
\end{aligned}$$

where we also used (3.5). Now using goods market clearing condition we know $\bar{C} + \bar{X}_s + \bar{X}_f = \bar{Y}$, and labor market clearing condition $\bar{N} = \bar{N}_s + \bar{N}_f$, we can further write $\bar{C} = k_{CY}\bar{Y}, \bar{N} = k_{NY}\bar{Y}$ where k_{CY}, k_{NY} are given as

$$k_{CY} = 1 - \left(\frac{\chi}{\mu}\right) \left[\eta \bar{T}^{\eta-1} + (1-\eta)\bar{T}^{\eta}\right], \qquad k_{NY} = \left(\frac{\chi}{\mu}\right)^{-\frac{\chi}{1-\chi}} (\eta \bar{T}^{\eta-1} + (1-\eta)\bar{T}^{\eta})$$

We know $\frac{\bar{H}_s}{\bar{K}_s} = \frac{1}{\mu} = \frac{\Gamma^{1-\chi}}{\chi} (\bar{C}^{\sigma} \bar{N}^{\phi})^{1-\chi}$. Substituting for $\bar{C}$ and $\bar{N}$ we get $\bar{Y}$ which is given as

$$\bar{Y} = \left[\left(\frac{\chi}{\mu}\right)^{\frac{1}{1-\chi}} \frac{1}{\Gamma} \frac{1}{k_{CY}^{\sigma} k_{NY}^{\phi}}\right]^{\frac{1}{\sigma+\phi}}$$

Since all the remaining steady state values of endogenous variables depend on $\bar{Y}$, all those values can be determined. Using (3.3) the steady state value of short term nominal interest rate is given by $\bar{R} = \beta^{-1}$.

C.4 The Flexible-Price Equilibrium

The flexible-price equilibrium refers to the equilibrium when prices in both sectors are fully flexible. This means, the degree of price stickiness in both sectors, is zero. Under flexible-price equilibrium, (3.10) gives us

$$P_{f,t} = P_t MC_{f,t}, \qquad P_{s,t} = P_t MC_{s,t}$$

Using (3.4) we can substitute for $P_{s,t}$ and $P_{f,t}$,

$$MC_{s,t}^{\eta} MC_{f,t}^{1-\eta} = 1$$

Substituting for $MC_{s,t}$ and $MC_{f,t}$ using (3.9) we get

$$\left(\frac{W_t}{P_t}\right)^{1-\chi} = \frac{\chi}{\Gamma^{1-\chi}} Z_{s,t}^{\eta} Z_{f,t}^{1-\eta}$$

Loglinearizing around steady state, and representing the log linearized variable by $*$ we get

$$w_t^* - p_t^* = \frac{1}{1-\chi}(\eta z_{s,t} + (1-\eta) z_{f,t})$$

Log linearizing marginal costs, using (3.9) we get

$$
\begin{aligned}
mc^*_{s,t} &= (1-\chi)(w^*_t - p^*_t) - z_{s,t} \\
&= (1-\eta)(z_{f,t} - z_{s,t}) \\
mc^*_{f,t} &= (1-\chi)(w^*_t - p^*_t) - z_{f,t} \\
&= \eta(z_{s,t} - z_{f,t})
\end{aligned}
$$

Using the definition of relative price measure, i.e. $T_t = \frac{P_{s,t}}{P_{f,t}}$ we get the log linearized version as

$$
\begin{aligned}
T^*_t &= p^*_{s,t} - p^*_{f,t} \\
&= mc^*_{s,t} - mc^*_{f,t} \\
&= z_{f,t} - z_{s,t}
\end{aligned}
$$

Using (3.5) we get

$$
\begin{aligned}
y^*_{s,t} &= (\eta - 1)T^*_t + y^*_t \\
y^*_{f,t} &= \eta T^*_t + y^*_t
\end{aligned}
$$

The aggregate demand for final good as an input by sector s is given by ($D_{s,t} = 1$ under flexible prices)

$$
X_{s,t} = \Gamma^{1-\chi} \left(\frac{W_t}{P_t} \right)^{1-\chi} \frac{Y_{s,t}}{Z_{s,t}}
$$

Log-linearized version of this equation gives us

$$x^*_{s,t} = (1-\chi)(w^*_t - p^*_t) + y^*_{s,t} - z_{s,t}$$

Substituting for $(w^*_t - p^*_t)$ and $y^*_{s,t}$ gives $x^*_{s,t} = y^*_t$. Similarly, $x^*_{f,t} = y^*_t$. Hence the final goods market clearing condition implies $Yy^*_t = Cc^*_t + X_s x^*_{s,t} + X_f x^*_{f,t}$. Using the steady state values we get $y^*_t = c^*_t$.Aggregate labor demand from sector s is given as

$$N_{s,t} = \Gamma^{-\chi}\left(\frac{W_t}{P_t}\right)^{-\chi}\frac{Y_{s,t}}{Z_{s,t}}$$

The log-linearized version of this equation is given as

$$n^*_{s,t} = -\chi(w^*_t - p^*_t) + y^*_{s,t} - z_{s,t}$$

Substituting for $(w^*_t - p^*_t)$ and $y^*_{s,t}$ and after some manipulations we get

$$n^*_{s,t} = -(w^*_t - p^*_t) + y^*_t$$

Similarly we get $n^*_{f,t} = -(w^*_t - p^*_t) + y^*_t$. Using labor market clearing condition i.e. $N_t = N_{s,t} + N_{f,t}$ we get

$$n^*_t = -(w^*_t - p^*_t) + y^*_t$$

where we have used the fact that $\frac{N_s}{N} = \eta$ and $\frac{N_s}{N} = 1-\eta$. Using (3.3), i.e. labor supply equation derived from the household's optimization problem we get

$$\frac{N_t^{\phi}}{C_t^{-\sigma}} = \frac{W_t}{P_t}$$

which on log-linearization gives us

$$\phi n_t^* + \sigma c_t^* = w_t^* - p_t^*$$

Substituting $n_t^* = -(w_t^* - p_t^*) + y_t^*$, $(w_t^* - p_t^*)$ and $c_t^* = y_t^*$ and after some manipulations we get

$$y_t^* = c_t^* = x_{s,t}^* = x_{f,t}^* = \frac{(1+\phi)}{(\phi+\sigma)(1-\chi)}(\eta z_{s,t} + (1-\eta) z_{f,t})$$

Hence we get n_t^*, which is given as

$$n_t = n_{s,t}^* = n_{f,t}^* = \left(\frac{1-\sigma}{(\phi+\sigma)(1-\chi)}\right)(\eta z_{s,t} + (1-\eta) z_{f,t})$$

We also get sector level output, $y_{s,t}^*$ and $y_{f,t}^*$

$$\begin{aligned} y_{s,t}^* &= \left(\frac{(1-\sigma) + (\phi+\sigma)\chi}{(\phi+\sigma)(1-\chi)}\right)(\eta z_{s,t} + (1-\eta) z_{f,t}) + z_{s,t} \\ y_{f,t}^* &= \left(\frac{(1-\sigma) + (\phi+\sigma)\chi}{(\phi+\sigma)(1-\chi)}\right)(\eta z_{s,t} + (1-\eta) z_{f,t}) + z_{f,t} \end{aligned}$$

From (3.3), log linearizing Euler equation gives us

$$c_t^* = \mathbb{E}_t c_{t+1}^* - \sigma^{-1}(r_t^* - \mathbb{E}_t \pi_{t+1}^*)$$

Define $r_t^{n*} = r_t^* - \mathbb{E}_t \pi_{t+1}^*$ the ex-ante natural real interest rate. Then we get

$$\begin{aligned}
r_t^{n*} &= \sigma(\mathbb{E}_t(c_{t+1}^* - c_t^*)) \\
&= \frac{\sigma(1+\phi)}{(\phi+\sigma)(1-\chi)} \mathbb{E}_t(\eta \Delta z_{s,t+1} + (1-\eta)\Delta z_{f,t+1}) \\
&= \frac{\sigma(1+\phi)}{(\phi+\sigma)(1-\chi)} \left[\eta(\rho_{z,s} - 1) z_{s,t} + (1-\eta)(\rho_{z,f} - 1) z_{f,t}\right]
\end{aligned}$$

C.5 Sticky Price Equilibrium

In the derivation below, for endogenous variable X , we define the following form: $\widetilde{x} = \widehat{x} - x^*$, where $\widehat{x}$ is the log linearized value of x in sticky price equilibrium, x^* is the log-linearized value of x in flexible price equilibrium[2] . The log linearized version of marginal cost in sector s is given by

$$\begin{aligned}
\widehat{mc}_{s,t} &= (1-\chi)(\widehat{w}_t - \widehat{p}_t) - z_{s,t} \\
&= (1-\chi)(\widetilde{w}_t - \widetilde{p}_t) + (1-\chi)(w_t^* - p_t^*) - z_{s,t} \\
&= (1-\chi)(\widetilde{w}_t - \widetilde{p}_t) + (1-\eta) T_t^*
\end{aligned}$$

[2] Except for the relative price T_t which is defined as $\widetilde{T} = \widehat{T} - T^*$

where the last equality follows from the definition of flexible price equilibrium. Substituting this in the NKPC of the sector s we get

$$
\begin{aligned}
\pi_{s,t} &= \kappa_s\{\widehat{mc}_{s,t} - (1-\eta)\widehat{T}_t\} + \beta\mathbb{E}_t\pi_{s,t+1} \\
\pi_{s,t} &= \kappa_s\{(1-\chi)(\widetilde{w}_t - \widetilde{p}_t) + (1-\eta)T_t^* - (1-\eta)\widehat{T}_t\} + \beta\mathbb{E}_t\pi_{s,t+1} \\
\pi_{s,t} &= \kappa_s\{(1-\chi)(\widetilde{w}_t - \widetilde{p}_t) - (1-\eta)\widetilde{T}_t\} + \beta\mathbb{E}_t\pi_{s,t+1}
\end{aligned}
$$

The log linearized version of marginal cost in sector f is given by

$$
\begin{aligned}
\widehat{mc}_{f,t} &= (1-\chi)(\widehat{w}_t - \widehat{p}_t) - z_{f,t} \\
&= (1-\chi)(\widetilde{w}_t - \widetilde{p}_t) - \eta T_t^*
\end{aligned}
$$

Substituting this in the NKPC of the sector f we get

$$
\pi_{f,t} = \kappa_f\{(1-\chi)(\widetilde{w}_t - \widetilde{p}_t) + \eta\widetilde{T}_t\} + \beta\mathbb{E}_t\pi_{f,t+1}
$$

Now our aim is to substitute out $(\widetilde{w}_t - \widetilde{p}_t)$ and express NKPC's in terms of relative output gap i.e. $(\widehat{y}_t - y_t^*)$ and relative price gap $(\widehat{T}_t - T_t^*)$. The log linearized version of household labor supply equation, (3.3) is given as

$$
\begin{aligned}
\phi\widehat{n}_t + \sigma\widehat{c}_t &= \widehat{w}_t - \widehat{p}_t \\
\phi\widetilde{n}_t + \sigma\widetilde{c}_t &= \widetilde{w}_t - \widetilde{p}_t
\end{aligned}
$$

So if we can express $\widetilde{n}_t$ and $\widetilde{c}_t$ in terms of $\widetilde{y}_t$ then we can express NKPC's in terms of output gap. From the labor market clearing condition in sector s and f we get

$$\widehat{n}_{s,t} = -\chi(\widehat{w}_t - \widehat{p}_t) + \widehat{y}_{s,t} - z_{s,t}$$

$$\widetilde{n}_{s,t} + n^*_{s,t} = -\chi(\widetilde{w}_t - \widetilde{p}_t) - \chi(w^*_t - p^*_t) + \widetilde{y}_{s,t} + y^*_{s,t} - z_{s,t}$$

From the flexible price equilibrium we know,

$$n^*_{s,t} = -\chi(w^*_t - p^*_t) + y^*_{s,t} - z_{s,t}$$

Hence we get,

$$\widetilde{n}_{s,t} = -\chi(\widetilde{w}_t - \widetilde{p}_t) + \widetilde{y}_{s,t}$$

Similarly from sector f labor market clearing condition we get

$$\widetilde{n}_{f,t} = -\chi(\widetilde{w}_t - \widetilde{p}_t) + \widetilde{y}_{f,t}$$

The sector level goods market clearing condition is given by (3.5) which in log linearized form is given by

$$\widehat{y}_{s,t} = (\eta - 1)\widehat{T}_t + \widehat{y}_t, \qquad \widehat{y}_{f,t} = \eta\widehat{T}_t + \widehat{y}_t$$

$$\Rightarrow \widetilde{y}_{s,t} = (\eta - 1)\widetilde{T}_t + \widetilde{y}_t, \qquad \widetilde{y}_{f,t} = \eta\widetilde{T}_t + \widetilde{y}_t$$

Substituting $\widetilde{y}_{s,t}, \widetilde{y}_{f,t}$ in the sector level labor market clearing condition gives us

$$\widetilde{n}_{s,t} = -\chi(\widetilde{w}_t - \widetilde{p}_t) + (\eta - 1)\widetilde{T}_t + \widetilde{y}_t, \qquad \widetilde{n}_{f,t} = -\chi(\widetilde{w}_t - \widetilde{p}_t) + \eta\widetilde{T}_t + \widetilde{y}_t$$

The labor market clearing condition implies

$$\widehat{n_t} = \frac{N_s}{N}\widehat{n}_{s,t} + \frac{N_f}{N}\widehat{n}_{f,t}$$

where $\frac{N_s}{N} = \eta$ and $\frac{N_f}{N} = 1 - \eta$. Substituting this, we get

$$\widehat{n_t} = \eta\widehat{n}_{s,t} + (1-\eta)\widehat{n}_{f,t}$$

$$\widetilde{n_t} + n_t^* = \eta\widetilde{n}_{s,t} + (1-\eta)\widetilde{n}_{f,t} + \eta n_{s,t}^* + (1-\eta)n_{f,t}^*$$

From the flexible price equilibrium we know that $n_t^* = n_{s,t}^* = n_{f,t}^*$. Hence we can write

$$\widetilde{n_t} = \eta\widetilde{n}_{s,t} + (1-\eta)n_{f,t}$$

Substituting for $\widetilde{n}_{s,t}$ and $\widetilde{n}_{f,t}$ we get

$$\widetilde{n_t} = -\chi(\widetilde{w_t} - \widetilde{p_t}) + \widetilde{y_t}$$

Substituting for $(\widetilde{w_t} - \widetilde{p_t}) = \phi\widetilde{n_t} + \sigma\widetilde{c_t}$ we get

$$\widetilde{n_t} = -\frac{\chi\sigma}{1+\chi\phi}\widetilde{c_t} + \frac{1}{1+\chi\phi}\widetilde{y_t}$$

The final goods market clearing condition implies

$$Y\widehat{y_t} = C\widehat{c_t} + X_s\widehat{x}_{s,t} + X_f\widehat{x}_{f,t} \Rightarrow Y\widetilde{y_t} = C\widetilde{c_t} + X_s\widetilde{x}_{s,t} + X_f\widetilde{x}_{f,t}$$

The steady state value of $X_{s,t}, X_{f,t}, C_t$ are $X_s = \eta\chi Y, X_f = (1-\eta)\chi Y, C = (1-\chi)Y$. Our aim at this moment is to express $\widetilde{x}_{s,t}$ and $\widetilde{x}_{f,t}$ in terms of $\widetilde{y}_t$. We know from the aggregation,

$$X_{f,t} = \Gamma^{1-\chi}\left(\frac{W_t}{P_t}\right)^{1-\chi}\frac{Y_{f,t}}{Z_{f,t}}D_{f,t}, \qquad X_{s,t} = \Gamma^{1-\chi}\left(\frac{W_t}{P_t}\right)^{1-\chi}\frac{Y_{s,t}}{Z_{s,t}}D_{s,t}$$

Log linearized version of these equations gives us

$$\begin{aligned}
\widehat{x}_{s,t} &= (1-\chi)(\widehat{w}_t - \widehat{p}_t) + \widehat{y}_{s,t}, \qquad \widehat{x}_{f,t} = (1-\chi)(\widehat{w}_t - \widehat{p}_t) + \widehat{y}_{f,t} \\
\Rightarrow \widetilde{x}_{s,t} &= (1-\chi)(\widetilde{w}_t - \widetilde{p}_t) + \widetilde{y}_{s,t}, \qquad \widetilde{x}_{f,t} = (1-\chi)(\widetilde{w}_t - \widetilde{p}_t) + \widetilde{y}_{f,t}
\end{aligned}$$

Substituting for $\widetilde{y}_{s,t}$ and $\widetilde{y}_{f,t}$ and the steady state values we get

$$X_s\widetilde{x}_{s,t} + X_f\widetilde{x}_{f,t} = \chi(1-\chi)Y(\widetilde{w}_t - \widetilde{p}_t) + \chi Y\widetilde{y}_t$$

Substituting $(\widetilde{w}_t - \widetilde{p}_t) = \phi\widetilde{n}_t + \sigma\widetilde{c}_t$, we get

$$X_s\widetilde{x}_{s,t} + X_f\widetilde{x}_{f,t} = \chi(1-\chi)Y(\phi\widetilde{n}_t + \sigma\widetilde{c}_t) + \chi Y\widetilde{y}_t$$

Substituting this in the final goods market clearing condition implies

$$\begin{aligned}
Y\widetilde{y}_t &= C\widetilde{c}_t + X_s\widetilde{x}_{s,t} + X_f\widetilde{x}_{f,t} \\
Y\widetilde{y}_t &= C\widetilde{c}_t + \chi(1-\chi)Y(\phi\widetilde{n}_t + \sigma\widetilde{c}_t) + \chi Y\widetilde{y}_t \\
Y(1-\chi)\widetilde{y}_t &= C\widetilde{c}_t + Y(1-\chi)\chi(\phi\widetilde{n}_t + \sigma\widetilde{c}_t) \\
\widetilde{y}_t &= (1+\chi\sigma)\widetilde{c}_t + \chi\phi\widetilde{n}_t
\end{aligned}$$

where in the last equality we used the fact that $C = (1-\chi)Y$. Substituting for $\widetilde{n}_t$ and solving further we get

$$\widetilde{y}_t = (1+\chi(\phi+\sigma))\widetilde{c}_t$$

Hence $\widetilde{n}_t$ can be expressed as

$$\begin{aligned}
\widetilde{n}_t &= -\frac{\chi\sigma}{1+\chi\phi}\widetilde{c}_t + \frac{1}{1+\chi\phi}\widetilde{y}_t \\
&- \frac{\chi\sigma}{1+\chi\phi}\widetilde{c}_t + \frac{(1+\chi(\phi+\sigma))}{1+\chi\phi}\widetilde{c}_t \\
&= \widetilde{c}_t
\end{aligned}$$

So $(\widetilde{w}_t - \widetilde{p}_t)$ can be expressed as

$$\begin{aligned}
\widetilde{w}_t - \widetilde{p}_t &= (\phi+\sigma)\widetilde{c}_t \\
&= \frac{\phi+\sigma}{1+\chi(\phi+\sigma)}\widetilde{y}_t
\end{aligned}$$

Define $\frac{(1-\chi)(\phi+\sigma)}{(1+\chi(\phi+\sigma))} = \Theta$. Then we can write the NKPC's as

$$\begin{aligned}
\pi_{s,t} &= \kappa_s\{\Theta\widetilde{y}_t - (1-\eta)\widetilde{T}_t\} + \beta\mathbb{E}_t\pi_{s,t+1} \\
\pi_{f,t} &= \kappa_f\{\Theta\widetilde{y}_t + \eta\widetilde{T}_t\} + \beta\mathbb{E}_t\pi_{f,t+1}
\end{aligned}$$

The log linearized household Euler equation is given by

$$\widehat{c}_t = \mathbb{E}_t\widehat{c}_{t+1} - \sigma^{-1}(r_t - \mathbb{E}_t\pi_{t+1})$$

Writing in terms of relative gap we get

$$\widetilde{c}_t = \mathbb{E}_t\widetilde{c}_{t+1} - \sigma^{-1}(r_t - \mathbb{E}_t\pi_{t+1} - r_t^{n*})$$

Using the aggregate price index (3.4) we can write

$$\pi_{t+1} = \eta\pi_{s,t+1} + (1-\eta)\pi_{f,t+1}$$

Hence the Euler equation becomes

$$\widetilde{c}_t = \mathbb{E}_t\widetilde{c}_{t+1} - \sigma^{-1}(r_t - \mathbb{E}_t(\eta\pi_{s,t+1} + (1-\eta)\pi_{f,t+1}) - r_t^{n*})$$

Substituting $\widetilde{c}_t$ in terms of $\widetilde{y}_t$ and defining $\sigma^{-1}(1+\chi(\sigma+\phi)) = \Xi$ we get

$$\widetilde{y}_t = \mathbb{E}_t\widetilde{y}_{t+1} - \Xi(r_t - \mathbb{E}_t(\eta\pi_{s,t+1} + (1-\eta)\pi_{f,t+1}) - r_t^{n*})$$

We know that relative price is defined as

$$T_t = \frac{P_{s,t}}{P_{f,t}}$$

which can be further written as

$$\begin{aligned} T_t &= \frac{P_{s,t}}{P_{s,t-1}}\frac{P_{s,t-1}}{P_{f,t-1}}\frac{P_{f,t-1}}{P_{f,t}} \\ &= \frac{\Pi_{s,t}T_{t-1}}{\Pi_{f,t}} \end{aligned}$$

The log linearized version is given as

$$
\begin{aligned}
\widehat{T}_t &= \pi_{s,t} - \pi_{f,t} + \widehat{T}_{t-1} \\
\widehat{T}_t - T_t^* &= \pi_{s,t} - \pi_{f,t} + \widehat{T}_{t-1} - T_{t-1}^* + T_{t-1}^* - T_t^* \\
\widetilde{T}_t &= \pi_{s,t} - \pi_{f,t} + \widetilde{T}_{t-1} - (T_t^* - T_{t-1}^*) \\
\Delta\widetilde{T}_t &= \pi_{s,t} - \pi_{f,t} - \Delta T_t^*
\end{aligned}
$$

where $\Delta x_t = x_t - x_{t-1}$. As $T_t^* = z_{f,t} - z_{s,t}$, $\Delta T_t^* = \Delta z_{f,t} - \Delta z_{s,t}$. So the sticky price equilibrium is given by

$$
\widetilde{y}_t = \mathbb{E}_t\widetilde{y}_{t+1} - \Xi(r_t - \mathbb{E}_t(\eta\pi_{s,t+1} + (1-\eta)\pi_{f,t+1}) - r_t^{n*}) \tag{C.4}
$$

where $\Xi = \sigma^{-1}(1 + \chi(\phi + \sigma))$.

$$
\pi_{s,t} = \kappa_s\{\Theta\widetilde{y}_t - (1-\eta)\widetilde{T}_t\} + \beta\mathbb{E}_t\pi_{s,t+1} \tag{C.5}
$$

$$
\tag{C.6}
$$

$$
\pi_{f,t} = \kappa_f\{\Theta\widetilde{y}_t + \eta\widetilde{T}_t\} + \beta\mathbb{E}_t\pi_{f,t+1} \tag{C.7}
$$

$$
\tag{C.8}
$$

where $\Theta = \frac{(1-\chi)(\phi+\sigma)}{(1+\chi(\phi+\sigma))}$

$$
\Delta\widetilde{T}_t = \pi_{s,t} - \pi_{f,t} - \Delta T_t^* \tag{C.9}
$$

C.6 Deriving Optimal Interest Rate Rule

The four first-order conditions corresponding to the minimization of loss function are given as

$$\lambda_{\pi,s}\pi_{s,t} + \Lambda_{s,t} - \Lambda_{s,t-1} - \Lambda_{T,t} = 0$$

$$\lambda_{\pi,f}\pi_{f,t} + \Lambda_{f,t} - \Lambda_{f,t-1} + \Lambda_{T,t} = 0$$

$$\lambda_y \widetilde{y}_t - \kappa_s \Theta \Lambda_{s,t} - \kappa_f \Theta \Lambda_{f,t} = 0$$

$$\kappa_s(1-\eta)\Lambda_{s,t} - \kappa_f \eta \Lambda_{f,t} + \Lambda_{T,t} - \beta \mathbb{E}_t \Lambda_{T,t+1} = 0$$

Using third first-order condition we can write

$$\kappa_s \Lambda_{s,t} + \kappa_f \Lambda_{f,t} = \frac{\lambda_y}{\Theta}\widetilde{y}_t$$

Multiplying the first first-order condition by κ_s, second by κ_f, adding the resulting two equations and solving for $\Lambda_{T,t}$ gives,

$$\Lambda_{T,t} = \frac{\kappa_s \lambda_{\pi,s}}{\kappa_s - \kappa_f}\pi_{s,t} + \frac{\kappa_f \lambda_{\pi,f}}{\kappa_s - \kappa_f}\pi_{f,t} + \frac{\lambda_y}{\Theta(\kappa_s - \kappa_f)}\Delta\widetilde{y}_{t-1}$$

Hence $\beta \mathbb{E}_t \Lambda_{T,t+1}$ is given as

$$\beta \mathbb{E}_t \Lambda_{T,t+1} = \frac{\kappa_s \lambda_{\pi,s}}{\kappa_s - \kappa_f}\beta \mathbb{E}_t \pi_{s,t+1} + \frac{\kappa_f \lambda_{\pi,f}}{\kappa_s - \kappa_f}\beta \mathbb{E}_t \pi_{f,t+1} + \frac{\lambda_y}{\Theta(\kappa_s - \kappa_f)}\beta \mathbb{E}_t \Delta\widetilde{y}_{t+1}$$

Using the NKPC of sector s and f, (C.5) and (3.19) respectively, we get

$$\beta \mathbb{E}_t \pi_{s,t+1} = \pi_{s,t} - \kappa_s \Theta \widetilde{y}_t + \kappa_s(1-\eta)\widetilde{T}_t$$

$$\beta \mathbb{E}_t \pi_{f,t+1} = \pi_{f,t} - \kappa_f \Theta \widetilde{y}_t - \kappa_f \eta \widetilde{T}_t$$

Similarly using the Euler equation, (C.4), we can write,

$$
\begin{aligned}
\beta\mathbb{E}_t\Delta\widetilde{y}_{t+1} &= \beta\Xi(r_t - \eta\mathbb{E}_t\pi_{s,t+1} - (1-\eta)\mathbb{E}_t\pi_{f,t+1} - r_t^{n*}) \\
&= \beta\Xi(r_t - r_t^{n*}) - \eta\Xi\pi_{s,t} - (1-\eta)\Xi\pi_{f,t} + \Xi\Theta(\eta\kappa_s + (1-\eta)\kappa_f)\widetilde{y}_t - \eta(1-\eta)\Xi(\kappa_s - \kappa_f)\widetilde{T}_t
\end{aligned}
$$

Hence we can write $\beta\mathbb{E}_t\Lambda_{T,t+1}$ as

$$
\begin{aligned}
\beta\mathbb{E}_t\Lambda_{T,t+1} = &\left[\frac{\kappa_s\lambda_{\pi,s}}{\kappa_s-\kappa_f} - \frac{\lambda_y\eta\Xi}{\Theta(\kappa_s-\kappa_f)}\right]\pi_{s,t} + \left[\frac{\kappa_f\lambda_{\pi,f}}{\kappa_s-\kappa_f} - \frac{\lambda_y(1-\eta)\Xi}{\Theta(\kappa_s-\kappa_f)}\right]\pi_{f,t} + \\
&\frac{\lambda_y\beta\Xi}{\Theta(\kappa_s-\kappa_f)}(r_t - r_t^{n*}) + \\
&\left[-\frac{\kappa_s^2\Theta\lambda_{\pi,s}}{\kappa_s-\kappa_f} - \frac{\kappa_f^2\lambda_{\pi,f}\Theta}{\kappa_s-\kappa_f} + \frac{\Xi\lambda_y(\eta\kappa_s + (1-\eta)\kappa_f)}{\kappa_s-\kappa_f}\right]\widetilde{y}_t + \\
&\left[\frac{\kappa_s^2\lambda_{\pi,s}(1-\eta)}{\kappa_s-\kappa_f} - \frac{\kappa_f^2\lambda_{\pi,f}\eta}{\kappa_s-\kappa_f} - \frac{\eta(1-\eta)\Xi\lambda_y}{\Theta}\right]\widetilde{T}_t
\end{aligned}
$$

Hence $\Lambda_{T,t} - \beta\mathbb{E}_t\Lambda_{T,t+1}$ is given as

$$
\begin{aligned}
\Lambda_{T,t} - \beta\mathbb{E}_t\Lambda_{T,t+1} = &\frac{\lambda_y\eta\Xi}{\Theta(\kappa_s-\kappa_f)}\pi_{s,t} + \frac{\lambda_y(1-\eta)\Xi}{\Theta(\kappa_s-\kappa_f)}\pi_{f,t} + \\
&\left[\frac{\lambda_y}{\Theta(\kappa_s-\kappa_f)} + \frac{\kappa_s^2\Theta\lambda_{\pi,s}}{\kappa_s-\kappa_f} + \frac{\kappa_f^2\lambda_{\pi,f}\Theta}{\kappa_s-\kappa_f} - \frac{(\eta\kappa_s + (1-\eta)\kappa_f)\Xi\lambda_y}{\kappa_s-\kappa_f}\right]\widetilde{y}_t \\
&- \frac{\lambda_y}{\Theta(\kappa_s-\kappa_f)}\widetilde{y}_{t-1} - \left[\frac{\kappa_s^2\lambda_{\pi,s}(1-\eta)}{\kappa_s-\kappa_f} - \frac{\kappa_f^2\lambda_{\pi,f}\eta}{\kappa_s-\kappa_f} - \frac{\eta(1-\eta)\Xi\lambda_y}{\Theta}\right]\widetilde{T}_t - \\
&\frac{\lambda_y\beta\Xi}{\Theta(\kappa_s-\kappa_f)}(r_t - r_t^{n*})
\end{aligned}
$$

which can be further written as

$$
\Lambda_{T,t} - \beta\mathbb{E}_t\Lambda_{T,t+1} = \rho_s\pi_{s,t} + \rho_f\pi_{f,t} + \rho_y\widetilde{y}_t - \frac{\lambda_y}{\Theta(\kappa_s-\kappa_f)}\widetilde{y}_{t-1} - \rho_T\widetilde{T}_t - \rho_r(r_t - r_t^{n*})
$$

where

$$\rho_s = \frac{\lambda_y \eta \Xi}{\Theta(\kappa_s - \kappa_f)}, \qquad \rho_f = \frac{\lambda_y (1-\eta)\Xi}{\Theta(\kappa_s - \kappa_f)}$$

$$\rho_y = \frac{\lambda_y}{\Theta(\kappa_s - \kappa_f)} + \frac{\kappa_s^2 \Theta \lambda_{\pi,s}}{\kappa_s - \kappa_f} + \frac{\kappa_f^2 \lambda_{\pi,f}\Theta}{\kappa_s - \kappa_f} - \frac{(\eta\kappa_s + (1-\eta)\kappa_f)\Xi\lambda_y}{\kappa_s - \kappa_f}$$

$$\rho_T = \frac{\kappa_s^2 \lambda_{\pi,s}(1-\eta)}{\kappa_s - \kappa_f} - \frac{\kappa_f^2 \lambda_{\pi,f}\eta}{\kappa_s - \kappa_f} - \frac{\eta(1-\eta)\Xi\lambda_y}{\Theta}, \qquad \rho_r = \frac{\lambda_y \beta \Xi}{\Theta(\kappa_s - \kappa_f)}$$

Defining $F_t = \Lambda_{T,t} - \beta \mathbb{E}_t \Lambda_{T,t+1}$ we can write last first-order condition as

$$-\kappa_s(1-\eta)\Lambda_{s,t} + \kappa_f \eta \Lambda_{f,t} = F_t$$

Additionally using the equation, $\kappa_s \Lambda_{s,t} + \kappa_f \Lambda_{f,t} = \frac{\lambda_y}{\Theta}\widetilde{y}_t$ we can solve for $\Lambda_{f,t}$, which is given as,

$$\Lambda_{f,t} = \frac{F_t}{\kappa_f} + \frac{\lambda_y(1-\eta)}{\Theta\kappa_f}\widetilde{y}_t$$

Substituting $\Lambda_{f,t}$ in the second first-order condition and solving for r_t we get

$$\begin{aligned} r_t = r_{t-1} &+ \frac{\kappa_s\kappa_f}{\kappa_s - \kappa_f}\frac{\lambda_{\pi,f}}{\rho_r}\pi_{f,t} + \frac{\kappa_s\kappa_f}{\kappa_s - \kappa_f}\frac{\lambda_{\pi,s}}{\rho_r}\pi_{s,t} + \frac{\rho_s}{\rho_r}\Delta\pi_{s,t} + \frac{\rho_f}{\rho_r}\Delta\pi_{f,t} + \\ &\frac{\kappa_f}{\rho_r}\left[\frac{\rho_y}{\kappa_f} + \frac{\lambda_y(1-\eta)}{\Theta\kappa_f} + \frac{\lambda_y}{\Theta(\kappa_s - \kappa_f)}\right]\Delta\widetilde{y}_t \frac{\lambda_y}{\Theta\rho_r(\kappa_s - \kappa_f)}\Delta\widetilde{y}_{t-1} - \frac{\rho_T}{\rho_r}\Delta\widetilde{T}_t + \Delta r_t^{n*} \end{aligned}$$

C.7 Proof of Claim in Footnote 17

The four first order conditions from the minimization of loss function with (C.5), (3.19) and (C.9) as the constraint are given as

$$\lambda_{\pi,s}\pi_{s,t} + \Lambda_{s,t} - \Lambda_{s,t-1} - \Lambda_{T,t} = 0 \qquad (\text{OMP}, \pi_s)$$

$$\lambda_{\pi,f}\pi_{f,t} + \Lambda_{f,t} - \Lambda_{f,t-1} + \Lambda_{T,t} = 0 \tag{OMP, π_f}$$

$$\lambda_y \widetilde{y_t} - \kappa_s\Theta\Lambda_{s,t} - \kappa_f\Theta\Lambda_{f,t} = 0 \tag{OMP,y}$$

$$\kappa_s(1-\eta)\Lambda_{s,t} - \kappa_f\eta\Lambda_{f,t} + \Lambda_{T,t} - \beta\mathbb{E}_t\Lambda_{T,t+1} = 0 \tag{OMP,t}$$

From (OMP,y) we can write $\widetilde{y_t}$ as

$$\widetilde{y_t} = \left(\frac{\Theta}{\lambda_y}\right)\left(\kappa_s\Lambda_{s,t} + \kappa_f\Lambda_{f,t}\right)$$

Substituting this (C.5), (3.19), (C.9), (OMP, π_s), (OMP, π_f) and (OMP,t), and defining $X_t = \left(\Lambda_{s,t-1}, \Lambda_{f,t-1}, \widetilde{t}_{t-1}, \pi_{s,t}, \pi_{f,t}, \Lambda_{T,t}\right)'$, $X_{t+1} = \left(\Lambda_{s,t}, \Lambda_{f,t}, \widetilde{t}_t, \mathbb{E}_t\pi_{s,t+1}, \mathbb{E}_t\pi_{f,t+1}, \mathbb{E}_t\Lambda_{T,t+1}\right)'$ we can write the above system of equations as

$$A\mathbb{E}_tX_{t+1} = BX_t + C\Delta t_t^*$$

where A, B are 6×6 matrices of coefficients, C is a 6×1 matrix of coefficient. A, B and C are given as

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \kappa_s(1-\eta) & -\kappa_f\eta & 0 & 0 & 0 & -\beta \\ \frac{\kappa_s^2\Theta^2}{\lambda_y} & \frac{\kappa_s\kappa_f\Theta^2}{\lambda_y} & -\kappa_s(1-\eta) & \beta & 0 & 0 \\ \frac{\kappa_s\kappa_f\Theta^2}{\lambda_y} & \frac{\kappa_f^2\Theta^2}{\lambda_y} & \kappa_f\eta & 0 & \beta & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 & 0 & -\lambda_{\pi,s} & 0 & 1 \\ 0 & 1 & 0 & 0 & -\lambda_{\pi,f} & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}'$$

Determinant of A is β^3 and hence under the accepted parametrization, matrix A is always invertible. Hence we can write $\mathbb{E}_t X_{t+1}$ as

$$\mathbb{E}_t X_{t+1} = A^{-1}BX_t + A^{-1}C\Delta t_t^*$$

There are three predetermined variables and three non predetermined variables. In order to have unique bounded solution, there should be exactly three eigen values of $A^{-1}B$ with modulus greater than 1. It can be shown computationally, the for all the combination of $\theta_s, \theta_f, \eta, \chi$ and ϵ, within the permissible range of values, there are always exactly three eigen values with modulus greater than one. Hence, using Blanchard Kahn (1980) conditions, we can say that the solution for $\mathbb{E}_t X_{t+1}$ is unique and bounded. Now, analytically, $A^{-1}B$ and $A^{-1}C$ are given as

$$A^{-1}B = \begin{bmatrix} 1 & 0 & 0 & -\lambda_{\pi,s} & 0 & 1 \\ 0 & 1 & 0 & 0 & -\lambda_{\pi,f} & -1 \\ 0 & 0 & 1 & 1 & -1 & 0 \\ -\frac{\Theta^2\kappa_s^2}{\beta\lambda_y} & -\frac{\Theta^2\kappa_s\kappa_f}{\beta\lambda_y} & \frac{\kappa_s(1-\eta)}{\beta} & \frac{\lambda_y+\kappa_s\lambda_y+\Theta^2\kappa_s^2\lambda_{\pi,s}-\eta\kappa_s\lambda_y}{\beta\lambda_y} & \left(\frac{\kappa_s}{\beta\lambda_y}\right)(\kappa_s\lambda_{\pi,f}\Theta^2-\lambda_y(1-\eta)) & \frac{\kappa_s\Theta^2(\kappa_f-\kappa_s)}{\beta\lambda_y} \\ -\frac{\Theta^2\kappa_s\kappa_f}{\beta\lambda_y} & -\frac{\Theta^2\kappa_f^2}{\beta\lambda_y} & -\frac{\eta\kappa_f}{\beta} & \left(\frac{\kappa_f}{\beta\lambda_y}\right)(\kappa_s\lambda_{\pi,s}\Theta^2-\eta\lambda_y) & \frac{\lambda_{\pi,f}\Theta^2\kappa_f^2+\eta\lambda_y\kappa_f+\lambda_y}{\beta\lambda_y} & \frac{\Theta^2\kappa_f^2-\kappa_s\kappa_f\Theta^2}{\beta\lambda_y} \\ \frac{\kappa_s(1-\eta)}{\beta} & -\frac{\kappa_f\eta}{\beta} & 0 & -\frac{\kappa_s(1-\eta)\lambda_{\pi,s}}{\beta} & \frac{\eta\kappa_f\lambda_{\pi,f}}{\beta} & \frac{\kappa_s(1-\eta)+\kappa_f\eta+1}{\beta} \end{bmatrix}$$

$$A^{-1}C = \begin{bmatrix} 0 & 0 & 1 & \frac{(1-\eta)\kappa_s}{\beta} & -\frac{\eta\kappa_f}{\beta} & 0 \end{bmatrix}'$$

C.8 Derivation of Loss Function

The linearization is done around the steady state with flexible prices, and no shocks. Government subsidy is in place, hence markup is equal to one.

The period t utility function in the model is given as

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \tag{L1}$$

We know we can write C_t as $\exp(\log C_t)$ and approximating this expression by second order Taylor expression we get

$$\begin{aligned}
C_t &= \exp(\log C_t) = \exp(\log C) + \exp(\log C)(\log C_t - \log C) + \frac{1}{2}\exp(\log C)(\log C_t - \log C)^2 \\
C_t &= C + C\widehat{c_t} + \frac{C}{2}\widehat{c_t^2} \\
&= C\left[1 + \widehat{c_t} + \frac{1}{2}\widehat{c_t^2}\right]
\end{aligned}$$

and hence we get

$$\frac{C_t - C}{C} = \widehat{c_t} + \frac{1}{2}\widehat{c_t^2}$$

Similarly for N_t we get

$$\frac{N_t - N}{N} = \widehat{n_t} + \frac{1}{2}\widehat{n_t^2}$$

For the notational simplicity, we denote $U(C_t, N_t) = U_t$, $U_c(C_t, N_t) = U_{ct}$, $U_n(C_t, N_t) = U_{nt}$ and so on. $U, U_c, U_n, \cdots$ have the same interpretation, evaluated at the steady state value of C_t, N_t. Taking the second order approximation of period t utility we get

$$
\begin{aligned}
U_t &= U + U_c(C_t - C) + \frac{1}{2}U_{cc}(C_t - C)^2 + U_n(N_t - N) + \frac{1}{2}U_{nn}(N_t - N)^2 + O(||z^3||) \\
&= U + U_c C\left(\frac{C_t - C}{C}\right) + \frac{1}{2}U_{cc}C^2\left(\frac{C_t - C}{C}\right)^2 + U_n N\left(\frac{N_t - N}{N}\right) + \frac{1}{2}U_{nn}N^2\left(\frac{N_t - N}{N}\right)^2 + O(||z^3||)
\end{aligned}
$$

where $O(||z^3||)$ represents the term of third or higher order. Substituting for $\frac{C_t - C}{C}$ and $\frac{N_t - N}{N}$, we get

$$
U_t = U + U_c C\left(\widehat{c_t} + \frac{1}{2}\widehat{c_t^2}\right) + \frac{1}{2}U_{cc}C^2\left(\widehat{c_t} + \frac{1}{2}\widehat{c_t^2}\right)^2 + U_n N\left(\widehat{n_t} + \frac{1}{2}\widehat{n_t^2}\right) + \frac{1}{2}U_{nn}N^2\left(\widehat{n_t} + \frac{1}{2}\widehat{n_t^2}\right)^2 + O(||z^3||)
$$

From (L1) we know that $U_c C = C^{1-\sigma}$, $U_{cc}C^2 = -\sigma C^{1-\sigma}$, $U_n N = -N^{1+\phi}$, $U_{nn}N^2 = -\phi N^{1+\phi}$. Substituting these in the expression above and neglecting the third or higher order terms we get

$$
U_t - U = C^{1-\sigma}\left(\widehat{c_t} + \frac{1-\sigma}{2}\widehat{c_t^2}\right) - N^{1+\phi}\left(\widehat{n_t} + \frac{1+\phi}{2}\widehat{n_t^2}\right) + O(||z^3||) \qquad \text{(L2)}
$$

Our objective is to express this expression in terms of output gap $\tilde{y}_t$ and aggregate inflation π_t. For this first consider eliminating $\widehat{n_t}$. The labour market clearing condition gives us

$$
N_t = N_{s,t} + N_{f,t}
$$

The sector level labor market clearing condition gives us

$$N_{s,t} = \Gamma^{-\chi}\left(\frac{W_t}{P_t}\right)^{-\chi}\frac{Y_{s,t}}{Z_{s,t}}D_{s,t}, \qquad N_{f,t} = \Gamma^{-\chi}\left(\frac{W_t}{P_t}\right)^{-\chi}\frac{Y_{f,t}}{Z_{f,t}}D_{f,t}$$

Consider for sector s labour market clearing condition. Taking the logarithm on both sides we get, (if X_t is a variable then we denote $\log X_t$ as x_t)

$$n_{s,t} = -\chi(w_t - p_t) + y_{s,t} - z_{s,t} + d_{s,t} + \log(\Gamma^{-\chi})$$

where $d_{s,t} = \log(D_{s,t})$. Subtracting the steady state counter part

$$n_{s,t} - n_s = -\chi(w_t - p_t) + \chi(w - p) + y_{s,t} - y_s - z_{s,t} + d_{s,t}$$

Note that the logarithm of the steady state value of $D_{s,t}$ is zero, as all the prices in sector s are constant and equal to the sector level price index. Hence $D_s = 1$, therefore $\log(D_s) = 0$. We can further write

$$\widehat{n}_{s,t} = -\chi(\widehat{w}_t - \widehat{p}_t) + \widehat{y}_{s,t} - z_{s,t} + d_{s,t} \tag{L3}$$

Although during log-linearization we neglected $d_{s,t}$ but in second approximation to utility we cannot do that as $d_{s,t}$ is of second order. Forgetting about (L3) for some time and focusing on $d_{s,t}$, which is given as

$$d_{s,t} = \log\left[\frac{1}{n}\int_0^n \left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{-\epsilon} di\right]$$

From the definition of sector s price index we know

$$
\begin{aligned}
P_{s,t}^{1-\epsilon} &= \left[\frac{1}{n}\int_0^n P_{s,t}(i)^{1-\epsilon} di\right] \\
1 &= \left[\frac{1}{n}\int_0^n \left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{1-\epsilon} di\right]
\end{aligned}
$$

Hence $\mathbb{E}_i\left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{1-\epsilon} = 1$. Now consider $\left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{1-\epsilon}$, which can be written as

$$
\begin{aligned}
\left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{1-\epsilon} &= \exp\left((1-\epsilon)\log\left(\frac{P_{s,t}(i)}{P_{s,t}}\right)\right) \\
&= \exp\left((1-\epsilon)(p_{s,t}(i) - p_{s,t})\right)
\end{aligned}
$$

Define $\widehat{p_{s,t}}(i) = (p_{s,t}(i) - p_{s,t})$. Note that $\widehat{p_{s,t}}(i)$ is not the deviation of $\log(P_{s,t}(i))$ from its steady state counter part. Hence we can write

$$
\begin{aligned}
\left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{1-\epsilon} &= \exp\left((1-\epsilon)\widehat{p_{s,t}}(i)\right) \\
&= \exp(0) + \exp(0)(1-\epsilon)\widehat{p_{s,t}}(i) + \exp(0)\frac{(1-\epsilon)^2}{2}\widehat{p_{s,t}}(i)^2 \\
&= 1 + (1-\epsilon)\widehat{p_{s,t}}(i) + \frac{(1-\epsilon)^2}{2}\widehat{p_{s,t}}(i)^2
\end{aligned}
$$

Hence we can write $1 = \frac{1}{n}\int_0^n \left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{-\epsilon} di = \mathbb{E}_i\left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{1-\epsilon}$ as

$$
\begin{aligned}
\mathbb{E}_i\left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{1-\epsilon} &= \mathbb{E}_i(1 + (1-\epsilon)\widehat{p_{s,t}}(i) + \frac{(1-\epsilon)^2}{2}\widehat{p_{s,t}}(i)^2) \\
1 &= 1 + (1-\epsilon)\mathbb{E}_i\widehat{p_{s,t}}(i) + \frac{(1-\epsilon)^2}{2}\mathbb{E}_i(\widehat{p_{s,t}}(i)^2)
\end{aligned}
$$

Therefore we can write $\mathbb{E}_i \widehat{p_{s,t}}(i)$ as

$$\mathbb{E}_i \widehat{p_{s,t}}(i) = -\frac{(1-\epsilon)}{2} \mathbb{E}_i (\widehat{p_{s,t}}(i)^2)$$

Now focusing on the measure of price dispersion in sector s i.e. $D_{s,t} = \frac{1}{n}\int_0^n \left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{-\epsilon} di$ as

$$\begin{aligned}
D_{s,t} &= \frac{1}{n}\int_0^n \left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{-\epsilon} di \\
&= \frac{1}{n}\int_0^n \exp(-\epsilon(\widehat{p_{s,t}}(i))) \\
&= \frac{1}{n}\int_0^n 1 - \epsilon \widehat{p_{s,t}}(i) + \frac{\epsilon^2}{2}\widehat{p_{s,t}}(i)^2 \\
&= 1 - \epsilon\ \mathbb{E}_i \widehat{p_{s,t}}(i) + \frac{\epsilon^2}{2}\ \mathbb{E}_i (\widehat{p_{s,t}}(i)^2)
\end{aligned}$$

Substituting $\mathbb{E}_i \widehat{p_{s,t}}(i) = -\frac{(1-\epsilon)}{2} \mathbb{E}_i (\widehat{p_{s,t}}(i)^2)$ we get

$$\begin{aligned}
D_{s,t} &= 1 + \frac{\epsilon(1-\epsilon)}{2}\mathbb{E}_i (\widehat{p_{s,t}}(i)^2) + \frac{\epsilon^2}{2}\mathbb{E}_i (\widehat{p_{s,t}}(i)^2) \\
&= 1 + \frac{\epsilon}{2}\mathbb{E}_i (\widehat{p_{s,t}}(i)^2)
\end{aligned}$$

Since $d_{s,t} = \log(D_{s,t})$ we can write

$$\begin{aligned}
d_{s,t} &= \log(D_{s,t}) \\
&= \log(1 + \frac{\epsilon}{2}\mathbb{E}_i (\widehat{p_{s,t}}(i)^2)) \\
&\approx \frac{\epsilon}{2}\mathbb{E}_i (\widehat{p_{s,t}}(i)^2)
\end{aligned}$$

Hence we can write (L3) as

$$
\begin{aligned}
\widehat{n}_{s,t} &= -\chi(\widehat{w}_t - \widehat{p}_t) + \widehat{y}_{s,t} - z_{s,t} + d_{s,t} \\
&= -\chi(\widehat{w}_t - \widehat{p}_t) + \widehat{y}_{s,t} - z_{s,t} + \frac{\epsilon}{2}\mathbb{E}_i(\widehat{p}_{s,t}(i)^2)
\end{aligned}
$$

Similarly we can write for sector f, which is given as

$$
\widehat{n}_{f,t} = -\chi(\widehat{w}_t - \widehat{p}_t) + \widehat{y}_{f,t} - z_{f,t} + \frac{\epsilon}{2}\mathbb{E}_i(\widehat{p}_{f,t}(i)^2)
$$

From the labor market clearing condition we know

$$
\widehat{n}_t = \frac{N_s}{N}\widehat{n}_{s,t} + \frac{N_f}{N}\widehat{n}_{f,t}
$$

Using $\frac{N_s}{N} = \eta$, $\frac{N_f}{N} = 1 - \eta$ we can write

$$
\begin{aligned}
\widehat{n}_t &= \eta\left(-\chi(\widehat{w}_t - \widehat{p}_t) + \widehat{y}_{s,t} - z_{s,t} + \frac{\epsilon}{2}\mathbb{E}_i(\widehat{p}_{s,t}(i)^2)\right) + (1-\eta)\left(-\chi(\widehat{w}_t - \widehat{p}_t) + \widehat{y}_{f,t} - z_{f,t} + \frac{\epsilon}{2}\mathbb{E}_i(\widehat{p}_{f,t}(i)^2)\right) \\
&= -\chi(\widehat{w}_t - \widehat{p}_t) + \eta\widehat{y}_{s,t} + (1-\eta)\widehat{y}_{f,t} - \left(\eta z_{s,t} + (1-\eta)z_{f,t}\right) + \frac{\eta\epsilon}{2}\mathbb{E}_i(\widehat{p}_{s,t}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\widehat{p}_{f,t}(i)^2)
\end{aligned}
$$

We know that $\eta z_{s,t} + (1-\eta)z_{f,t} = (1-\chi)(w_t^* - p_t^*)$. Substituting this in the equation above and using the fact that $\widetilde{w}_t = \widehat{w}_t - w_t^*$ we can write

$$
\widehat{n}_t = -\chi(\widetilde{w}_t - \widetilde{p}_t) + \eta\widehat{y}_{s,t} + (1-\eta)\widehat{y}_{f,t} - (w_t^* - p_t^*) + \frac{\eta\epsilon}{2}\mathbb{E}_i(\widehat{p}_{s,t}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\widehat{p}_{f,t}(i)^2)
$$

Furthermore we know, $\widehat{y}_{s,t} = (\eta - 1)\widehat{t}_t + \widehat{y}_t$, $\widehat{y}_{f,t} = \eta\widehat{t}_t + \widehat{y}_t$. Substituting this in the above equation we get

$$\widehat{n}_t = -\chi(\widetilde{w}_t - \widetilde{p}_t) + \widehat{y}_t - (w_t^* - p_t^*) + \frac{\eta\epsilon}{2}\mathbb{E}_i(\widehat{p}_{s,t}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\widehat{p}_{f,t}(i)^2)$$

We also know that $y_t^* = \frac{1+\phi}{(\phi+\sigma)(1-\chi)}(\eta z_{s,t} + (1-\eta)z_{f,t}) = \frac{(1+\phi)(1-\chi)}{(\phi+\sigma)(1-\chi)}(w_t^* - p_t^*)$. So we can further write the above equation as

$$\widehat{n}_t = -\chi(\widetilde{w}_t - \widetilde{p}_t) + \widehat{y}_t - (w_t^* - p_t^*) + \frac{\eta\epsilon}{2}\mathbb{E}_i(\widehat{p}_{s,t}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\widehat{p}_{f,t}(i)^2)$$
$$\widehat{n}_t = -\chi(\widetilde{w}_t - \widetilde{p}_t) + \widehat{y}_t - \frac{\phi+\sigma}{1+\phi}y_t^* + \frac{\epsilon}{2}\mathbb{E}_i(\widehat{p}_{s,t}(i)^2) + \frac{\epsilon}{2}\mathbb{E}_i(\widehat{p}_{f,t}(i)^2)$$

Writing $\widehat{y}_t = \widetilde{y}_t + y_t^*$, the equation further reduces to

$$\widehat{n}_t = -\chi(\widetilde{w}_t - \widetilde{p}_t) + \widehat{y}_t - \frac{\phi+\sigma}{1+\phi}y_t^* + \frac{\eta\epsilon}{2}\mathbb{E}_i(\widehat{p}_{s,t}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\widehat{p}_{f,t}(i)^2)$$
$$\widehat{n}_t = -\chi(\widetilde{w}_t - \widetilde{p}_t) + \widetilde{y}_t + \left(1 - \frac{\phi+\sigma}{1+\phi}\right)y_t^* + \frac{\eta\epsilon}{2}\mathbb{E}_i(\widehat{p}_{s,t}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\widehat{p}_{f,t}(i)^2)$$
$$\widehat{n}_t = -\chi(\widetilde{w}_t - \widetilde{p}_t) + \widetilde{y}_t + \left(\frac{1-\sigma}{1+\phi}\right)y_t^* + \frac{\eta\epsilon}{2}\mathbb{E}_i(\widehat{p}_{s,t}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\widehat{p}_{f,t}(i)^2)$$

Furthermore we know that $\widetilde{w}_t - \widetilde{p}_t = \left(\frac{\phi+\sigma}{1+\chi(\phi+\sigma)}\right)\widetilde{y}_t$. Substituting this in the above equation we get

$$\widehat{n_t} = \left(\frac{1}{1+\chi(\phi+\sigma)}\right)\widetilde{y_t} + \left(\frac{1-\sigma}{1+\phi}\right)y_t^* + \frac{\eta\epsilon}{2}\mathbb{E}_i(\widehat{p_{s,t}}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\widehat{p_{f,t}}(i)^2)$$

Now coming onto the main objective of deriving loss function. First notice that $C^{1-\sigma} = N^{1+\phi}$, as

$$C^{1-\sigma} = \left(\chi^{\frac{\chi(1+\phi)}{(1-\chi)(\sigma+\phi)(1-\chi)^{\frac{1+\phi}{\sigma+\phi}}}}\right)^{1-\sigma} = N^{1+\phi}$$

Secondly, we can write $\widehat{c_t} + \frac{1-\sigma}{2}\widehat{c_t^2}$, using $\widetilde{c_t} = \widehat{c_t} - c_t^*$ as

$$\widetilde{c_t} + \frac{1-\sigma}{2}\widetilde{c_t^2} + (1-\sigma)\widetilde{c_t}c_t^* + t.i.p + O(||z^3||)$$

where $t.i.p$ refers to terms independent of policy. Then using the relation $\widetilde{c_t} = \left(\frac{1}{1+\chi(\phi+\sigma)}\right)\widetilde{y_t}$ and using $y_t^* = c_t^*$ we can write

$$\left(\frac{1}{1+\chi(\phi+\sigma)}\right)\widetilde{y_t} + \frac{1-\sigma}{2}\left(\frac{1}{1+\chi(\phi+\sigma)}\right)^2\widetilde{y_t^2} + (1-\sigma)\left(\frac{1}{1+\chi(\phi+\sigma)}\right)\widetilde{y_t}y_t^* + t.i.p + O(||z^3||)$$

For the present moment, in order to simplify the notation, define $\kappa_1 = \left(\frac{1}{1+\chi(\phi+\sigma)}\right)$. Hence we can write

$$\widetilde{c_t} + \frac{1-\sigma}{2}\widetilde{c_t^2} + (1-\sigma)\widetilde{c_t}c_t^* = \kappa_1\widetilde{y_t} + \frac{1-\sigma}{2}\kappa_1^2\widetilde{y_t^2} + (1-\sigma)\kappa_1\widetilde{y_t}y_t^* + t.i.p + O(||z^3||)$$

Now coming onto simplifying the notation of $\widehat{n_t} + \frac{1+\phi}{2}\widehat{n_t^2}$. As we had derived above, $\widehat{n_t}$ is given as

$$\widehat{n_t} = \left(\frac{1}{1+\chi(\phi+\sigma)}\right)\widetilde{y_t} + \left(\frac{1-\sigma}{1+\phi}\right)y_t^* + \frac{\eta\epsilon}{2}\mathbb{E}_i(\widehat{p_{s,t}}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\widehat{p_{f,t}}(i)^2)$$

Defining $\kappa_2 = \left(\frac{1-\sigma}{1+\phi}\right)$ we can simplify the notations and write

$$\widehat{n_t} = \kappa_1 \widetilde{y_t} + \kappa_2 y_t^* + \frac{\eta\epsilon}{2}\mathbb{E}_i(\widehat{p_{s,t}}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\widehat{p_{f,t}}(i)^2)$$

So $\widehat{n_t} + \frac{1+\phi}{2}\widehat{n_t^2}$ becomes

$$\begin{aligned}\widehat{n_t} + \frac{1+\phi}{2}\widehat{n_t^2} = \kappa_1 \widetilde{y_t} + \left(\frac{1+\phi}{2}\right)\kappa_1^2 \widetilde{y_t^2} + (1+\phi)\kappa_1\kappa_2\widetilde{y_t} y_t^* + \\ \frac{\eta\epsilon}{2}\mathbb{E}_i(\widehat{p_{s,t}}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\widehat{p_{f,t}}(i)^2) + t.i.p + O(||z^3||)\end{aligned}$$

Now using $C^{1-\sigma} = N^{1+\phi}$ and substituting $\widehat{n_t} + \frac{1+\phi}{2}\widehat{n_t^2}$, $\widetilde{c_t} + \frac{1-\sigma}{2}\widetilde{c_t^2} + (1-\sigma)\widetilde{c_t}c_t^*$ in (L2) we get

$$-C^{1-\sigma}\left[\left(\frac{\sigma+\phi}{2}\right)\kappa_1^2\widetilde{y_t^2} + \frac{\eta\epsilon}{2}\mathbb{E}_i(\widehat{p_{s,t}}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\widehat{p_{f,t}}(i)^2)\right]$$

Now our aim is to express the price dispersion $\frac{\epsilon}{2}\mathbb{E}_i(\widehat{p_{s,t}}(i)^2)$, $\frac{\epsilon}{2}\mathbb{E}_i(\widehat{p_{f,t}}(i)^2)$ to relate to the variability of inflation rate. Focus on sector s, (same derivation applied to sector f), we can write $\mathbb{E}_i(\widehat{p_{s,t}}(i)^2)$ as

$$\begin{aligned}\mathbb{E}_i\left(\widehat{p_{s,t}}(i)^2\right) &= \mathbb{E}_i\left(p_{s,t}(i) - p_{s,t}\right)^2 \\ &\approx \mathbb{E}_i\left(p_{s,t}(i) - \mathbb{E}_i p_{s,t}(i)\right)^2 \\ &= var\ (p_{s,t}(i))\end{aligned}$$

We denote $var\ (p_{s,t}(i))$ by $\mathfrak{D}_{s,t}$. We first prove that $\mathfrak{D}_{s,t}$ can be written in recursive form as

$$\mathfrak{D}_{s,t} = \theta_s \mathfrak{D}_{s,t-1} + \left(\frac{\theta_s}{1-\theta_s}\right)\pi_{s,t}^2$$

To prove this, first define $\bar{P}_{s,t} = \mathbb{E}_i p_{s,t}$. Hence we can write

$$
\begin{aligned}
\bar{P}_{s,t} &= \mathbb{E}_i p_{s,t}(i) \\
\bar{P}_{s,t} - \bar{P}_{s,t-1} &= \mathbb{E}_i (p_{s,t}(i) - \bar{P}_{s,t-1}) \\
&= \theta_s \mathbb{E}_i \left(p_{s,t-1}(i) - \bar{P}_{s,t-1} \right) + (1-\alpha)(p^*_{s,t} - \bar{P}_{s,t-1}) \\
&= (1-\theta_s)(p^*_{s,t} - \bar{P}_{s,t-1})
\end{aligned}
$$

Furthermore, we can write

$$
\begin{aligned}
var_i \; p_{s,t}(i) &= var_i (p_{s,t}(i) - \bar{P}_{s,t-1}) \\
&= \mathbb{E}_i \left[(p_{s,t}(i) - \bar{P}_{s,t-1})^2 \right] - \left(\mathbb{E}_i (p_{s,t}(i) - \bar{P}_{s,t-1}) \right)^2 \\
&= \theta_s \mathbb{E}_i \left[(p_{s,t}(i) - \bar{P}_{s,t-1})^2 \right] + (1-\theta_s)(p^*_{s,t} - \bar{P}_{s,t-1})^2 - (\bar{P}_{s,t} - \bar{P}_{s,t-1})^2 \\
&= \theta_s var_i \; (p_{s,t-1}(i)) + \left(\frac{\theta_s}{1-\theta_s} \right) \left(\bar{P}_{s,t} - \bar{P}_{s,t-1} \right)^2
\end{aligned}
$$

Following Woodford (2003) we can write $\bar{P}_{s,t} \approx \log P_{s,t}$. Hence we get the final expression

$$
\mathfrak{D}_{s,t} = \theta_s \mathfrak{D}_{s,t-1} + \left(\frac{\theta_s}{1-\theta_s} \right) \pi^2_{s,t}
$$

Iterating forward we get

$$
\mathfrak{D}_{s,t} = \theta_s^{t+1} \mathfrak{D}_{-1} + \sum_{k=0}^{t} \theta_s^{t-k} \left(\frac{\theta_s}{1-\theta_s} \right) \pi^2_k
$$

So we can write

$$
\sum_{t=0}^{\infty} \beta^t \mathfrak{D}_{s,t} = \frac{\theta_s}{(1-\theta_s)(1-\theta_s \beta)} \sum_{t=0}^{\infty} \beta^t \pi^2_{s,t}
$$

Now summing up all the results. Second order approximation of utility at time t is given as

$$\frac{1}{C^{1-\sigma}}\left[\left(\frac{C_t^{1-\sigma}}{1-\sigma}-\frac{N_t^{1+\phi}}{1+\phi}\right)-\left(\frac{C^{1-\sigma}}{1-\sigma}-\frac{N^{1+\phi}}{1+\phi}\right)\right]\approx-\left(\left(\frac{\sigma+\phi}{2}\right)\kappa_1^2\widetilde{y}_t^2+\frac{\epsilon}{2}(\eta\mathfrak{D}_{s,t}+(1-\eta)\mathfrak{D}_{f,t})\right)$$

Hence we get,

$$\sum_{t=0}^{\infty}\beta^t\frac{1}{C^{1-\sigma}}\left[\left(\frac{C_t^{1-\sigma}}{1-\sigma}-\frac{N_t^{1+\phi}}{1+\phi}\right)-\left(\frac{C^{1-\sigma}}{1-\sigma}-\frac{N^{1+\phi}}{1+\phi}\right)\right]\approx-\sum_{t=0}^{\infty}\beta^t L_t+t.i.p+O(||z^3||) \quad \text{(C.10)}$$

where L_t is given as

$$\left(\frac{\sigma+\phi}{2}\right)\kappa_1^2\widetilde{y}_t^2+\frac{\epsilon\eta}{2}\frac{\theta_s}{(1-\theta_s)(1-\theta_s\beta)}\pi_{s,t}^2+\frac{\epsilon(1-\eta)}{2}\frac{\theta_f}{(1-\theta_f)(1-\theta_f\beta)}\pi_{f,t}^2$$

C.9 Derivation of (3.24)

Define period t utility as $U_t=\frac{C_t^{1-\sigma}}{1-\sigma}-\frac{N_t^{1+\phi}}{1+\phi}$ and its steady state counter part as $U=\frac{C^{1-\sigma}}{1-\sigma}-\frac{N^{1+\phi}}{1+\phi}$. Then from the equation (C.10) we know,

$$\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t U_t-\frac{1}{1-\beta}U=-C^{1-\sigma}\ \ \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t L_t$$

where the first term on the left-hand side is the lifetime expected utility and the second term is the steady-state lifetime utility. Denoting the right hand side as $-C^{1-\sigma}L$ and using that at the steady state $C^{1-\sigma}=N^{1+\phi}$, we can write

$$\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t U_t-\mathbb{A}C^{1-\sigma}=-C^{1-\sigma}L \quad \text{(C.11)}$$

where $\mathbb{A} = \frac{\sigma+\phi}{(1-\beta)(1-\sigma)(1+\phi)}$. Now, define c as the permanent decrease in the steady state consumption level, such that following equality holds

$$\begin{aligned}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t = \sum_{t=0}^{\infty} \beta^t \left[\frac{(C-c)^{1-\sigma}}{1-\sigma} - \frac{N^{1+\phi}}{1+\phi}\right] &= \frac{1}{1-\beta}\left[\frac{(C-c)^{1-\sigma}}{1-\sigma} - \frac{N^{1+\phi}}{1+\phi}\right] \\ &= \frac{1}{1-\beta}\left[\frac{C^{1-\sigma}\left(1-\frac{c}{C}\right)^{1-\sigma}}{1-\sigma} - \frac{N^{1+\phi}}{1+\phi}\right]\end{aligned}$$

This equation reduces to

$$\left(1-\frac{c}{C}\right)^{1-\sigma} = (1-\sigma)\left(\frac{(1-\beta)\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t}{C^{1-\sigma}} + \frac{1}{1+\phi}\right)$$

Now using (C.11) we can write

$$\left(1-\frac{c}{C}\right)^{1-\sigma} = (1-\sigma)\left(\frac{(1-\beta)(\mathbb{A}C^{1-\sigma} - C^{1-\sigma}L)}{C^{1-\sigma}} + \frac{1}{1+\phi}\right)$$

which reduces to equation (3.24).

$$\frac{c}{C} = 1 - \left[(1-\sigma)\left((1-\beta)(\mathbb{A}-L) + \frac{1}{1+\phi}\right)\right]^{\frac{1}{1-\sigma}}$$

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Dutch Summary

Deze dissertatie bevat drie hoofdstukken. De eersten twee hoofdstukken sluiten aan bij de literatuur over incomplete marktmodellen en het laatste hoofdstuk houdt zich bezig met monetair beleid in een Dynamisch Stochastisch Algemeen Evenwichtsmodel (DSAE).

Het eerste hoofdstuk schat in een structureel model het verschil tussen idiosyncratisch inkomensrisico geschat door een econometrist en wat daadwerkelijk wordt waargenomen door huishoudens zelf. Ik schat dat het waargenomen inkomensrisico ten minste 12 procent lager is dan het risico geschat door een econometrist, waarbij ik idiosyncratische informatie aan het consumptie risicodelingsmodel verstrek met behulp van US micro-data. Wanneer ik rekening houd met dit verschil, verklaart het model drie aparte risicodelingsmaatstaven die niet worden verklaard in het standaard model dat geen rekening houdt met dit verschil: (i) de cross-sectionele variantie van consumptie (ii) de covariantie van consumptie met economische groei, en (iii) de voorwaardelijke verwachte huishoudconsumptie gegeven het inkomen.

In het tweede hoofdstuk laat ik zien, gebruik makend van de March Current Population Survey, dat over de laatste twee decennia getrouwde huishoudens in de Verenigde Staten steeds meer sociale verzekering krijgen tegen risico in het arbeidsinkomen, terwijl het tegenovergestelde gebeurt voor alleenstaande huishoudens. Om de consequenties van deze trend op welvaart te analyseren, doe ik een kwantitatieve analyse. Mijn bijdragen aan de literatuur is dat ik het standaard incomplete marktmodel (Aiyagari (1994)) uitbereid zodat het twee soorten huishoudens bevat: getrouwd en alleenstaand. Het model staat toe dat huishoudens hun burgerlijke status kunnen veranderen en houdt rekening met de transitie dynamiek tussen steady states. Ik laat zien dat de uiteenlopende trends in sociale verzekering een significant schadelijk effect hebben op de welvaart van zowel getrouwde als alleenstaande huishoudens. Meer sociale verzekering verdringt private besparingen van getrouwde huishoudens, dit leidt tot een afnamen in hun gemiddeld vermogen. In de lange termijn leidt een lager vermogen tot een lagere gemiddelde consumptie, dit drijft hun welvaartsverlies. Voor alleenstaande speelt de transitie dynamiek een belangrijke rol. Hoewel zij meer sparen in reactie op een lagere sociale verzekering en zich een hogere gemiddelde consumptie kunnen veroorloven in de nieuwe steady state, wordt deze winst in welvaart teniet gedaan door het verlies in welvaart als gevolg van een lagere initiÃńle consumptie na de verandering in het beleid.

In het derde hoofdstuk, bestudeer ik de rol van sectorale factor vraag verbindingen op het optimaal monetair beleid. Ik ontwikkel een twee-sector New-Keynesian model, waar sectoren zijn verbonden door sectorale factor vraag en verschillen in de starheid van de prijzen. Ik vind twee belangrijke resultaten. Ten eerste, de aanwezigheid van sectorale factor vraag verbindingen leiden tot versterkte misallocatie van productiefactoren en, daarom, wordt de zorg over prijs instabiliteit belangrijker. Ten tweede, is de optimale prijs index niet hetzelfde als de geaggregeerde prijs index, ondanks dat dit niet afhangt van sectorale factor vraag verbindingen. Daarnaast leid ik een beleidsregel af die de optimale allocatie implementeert gebaseerd op de verlies functie, welke gefundeerd is op het levenslange nut van de consumenten.

The Tinbergen Institute is the Institute for Economic Research, which was founded in 1987 by the Faculties of Economics and Econometrics of the Erasmus University Rotterdam, University of Amsterdam and VU University Amsterdam. The Institute is named after the late Professor Jan Tinbergen, Dutch Nobel Prize laureate in economics in 1969. The Tinbergen Institute is located in Amsterdam and Rotterdam. The following books recently appeared in the Tinbergen Institute Research Series:

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