Optical noise and mesoscopic correlations in random media

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Weak localization of photon noise in random media

We present an experimental study of coherent backscattering (CBS) of photon noise from multiple scattering media. Using a pseudothermal light source we study the effect of weak localization on photon noise. In the noise spectrum we observe a continuous transition in the Fano factor enhancement from the shot noise regime to the wave fluctuations regime. These initial experiments on weakly scattering media demonstrate that sensitive noise measurements can be combined with the separation of path lengths present in coherent backscattering, opening up new opportunities for experiments on noise transport in the localization regime.

3.1 Introduction

In chapter 2 we investigated the total reflection of classical and quantum noise from TiO$_2$ random samples. However, those experiments did not allow to closely monitor the propagation of noise as function of the number of scattering events that light undergoes in the medium, nor did the experiments by Lodahl et al. [78], as they revolved around the total transmission of noise through random media. Moreover, in both experiments the classical noise source was given by the low frequency noise of the Ti:Sapphire laser, which did not allow for fine tuning of the noise intensity and spectral range of classical and shot noise. The goal of the experiments presented in this chapter is to extend the study of photon noise by devising a method to simultaneously follow the transport of noise as function of the order of scattering and to control the contribution of the classical noise.
A method to separate multiple scattering paths of different length is coherent backscattering (CBS). As explained in the chapter 1, CBS results from an interference effect between reciprocal paths in a random medium, and manifests itself as a twofold enhancement of the light intensity in the backscattering direction with respect to the diffuse background. Measurements of CBS have turned over the years from a striking evidence of weak localization of light to a tool for investigating the scattering properties of many different media \[86–88\]. The strong Anderson localization transition has received much attention since it was first proposed in 1958 \[89\], nevertheless it remains an elusive phenomenon \[90, 91\] and noise measurements could provide alternative means to investigate it. Furthermore, the impact of Anderson localization on non classical properties of light is yet an open question \[92\].

This chapter is structured as follows: in section 3.2 we describe the main principles that govern the statistics of light sources, in section 3.3 we calculate the fluctuations induced by a diffuser on a coherent beam and explain the analogy between a diffuser and a thermal source. Subsequently, in section 3.4 the intensity spectrum for a pseudothermal source is derived. In section 3.5 the experiments that lead to the weak localization of photon noise are described and finally, in section 3.6, we analyze and discuss the results.

### 3.2 Statistics of the light radiation

The statistical properties of the light can be assessed by using Mandel’s formula. Mandel’s formula is very useful to calculate the photocount distribution $P_M$ of the photons emitted by a source and is given by \[38, 93\]

$$P_M(n, t, T) = \int_0^\infty \frac{[\alpha W(t, T)]^n}{n!} \exp[-\alpha W(t, T)] P(W) dW,$$

with

$$W(t, T) = \int_t^{t+T} \dot{I}(t') dt',$$

where $n$ denotes the number of photons, $\dot{I}$ the instantaneous intensity, $\alpha$ the quantum efficiency of the detector. The measurement is carried out in the time interval $[t, t + T]$. With $P(W)$ is indicated the intensity distribution of the ensemble. One could interpret formula (3.1) in the following way: if we consider one single realization for the sequence of photons measured within time $T$ we would achieve a Poissonian photocount statistics. However, in general, different sequences would yield different results because $W(t, T)$ is not a deterministic variable but a random one. Therefore what is significant is the photocount distribution calculated over an ensemble of different realizations of the photon number sequences. Eq. (3.1) expresses then the Poissonian distribution weighted by the intensity distribution $P(W)$ of the ensemble.

In a true thermal source, like a discharge lamp or a light bulb, the radiating system is externally excited and subsequently generates light by means of sponta-
neous emission. Hence, all the elementary sources (atoms, molecules) emit light independently from one another.

A thermal source is characterized by a Rayleigh distribution for the intensity \( P_R(W) \) whereas, the photocount distribution \( P_M \) becomes of type Bose-Einstein, indicated with \( P_{BE}(n) \), and we have the expressions

\[
P_R(W) = \frac{1}{W} \exp \left( -\frac{W}{W} \right)
\]

and

\[
P_M = P_{BE}(n) = \frac{1}{\pi} \left( \frac{\pi}{\pi + 1} \right)^n,
\]

where with \( \pi \equiv \alpha W(t, T) \) we indicate the mean number of photocounts, also known as the occupation number.

By contrast, laser light is produced by stimulated emission, whereby all the elementary sources radiate in unison and the intensity distribution \( P_L(W) \) is given by

\[
P_L(W) = \delta(W - W),
\]

while the photocount distribution \( P_M \) is Poissonian, indicated with \( P_P(n) \), and results in

\[
P_M = P_P(n) = \frac{\pi^n \exp(-\pi)}{n!}.
\]

The variance of \( P_P(n) \) has the expression

\[
(\Delta n)_P^2 = \pi.
\]

Eqs. (3.4) and (3.6) can be obtained by inserting eqs. (3.3) and (3.5) in Mandel’s formula respectively, provided that the integration time \( T \) is much smaller than the coherence time \( \tau_c \) of the light. Eqs. (3.5) and (3.6) show that, in spite of the fact that all the different members of the intensity ensemble relative to \( W \) are exactly the same, we end up with a probability distribution for the photocounts. This situation highlights the particle nature of light: even the most stable source has intensity fluctuations. These fluctuations are proportional to the mean number of the emitted photons.

For a true thermal source, like a blackbody, Planck’s law predicts an average photon number given by [93]

\[
\frac{1}{\pi} = \exp \left( \frac{\hbar \omega}{k_T K} \right) - 1,
\]

with \( T_K \) being the temperature in Kelvin, \( h \) Planck’s constant, \( K \) Boltzmann’s constant and \( \omega \) the frequency of the light of concern. Eq. (3.8) shows that, at optical frequencies, a temperature above the one of the surface of the sun (6000 K) is needed to achieve an occupation number bigger than one. The variance of the Bose-Einstein distribution is

\[
(\Delta n)_{BE}^2 = \pi + \pi^2.
\]
An occupation number much smaller than one, like the one exhibited by true thermal sources in experimentally feasible configurations, depresses the classical fluctuations that are given by the $n^2$ term in eq. (3.9). An additional complication carried by thermal sources is that their signature, a Bose-Einstein probability distribution, shows up only when the measurements are performed within the coherence time $\tau_c$ of the source, as can be proved by using Mandel’s formula. Typical thermal sources have coherence times of the order of $10^{-10}\text{ s}$. Performing measurements within this time interval is extremely demanding from an experimental perspective. Measuring over many coherence times of the sources implies averaging out the interesting fluctuations that give rise to the quadratic term in eq. (3.9); in this case the measured photocount distribution of thermal light becomes indistinguishable from a Poissonian one.

If we want to investigate and use the classical properties of light it is then crucial to find a way to optimize the quadratic term of expression (3.9). Pseudothermal light is a solution to the complications displayed by true thermal sources as it mimics the same physics as the one responsible for thermal light but on an experimentally accessible scale. As it is explained in the next section pseudothermal light can be generated by illuminating a rotating ground glass diffuser with laser light. The analogy between the intensity statistics of a thermal source and the one generated by a rotating diffuser is at the base of the name pseudothermal source.

### 3.3 Diffuser as a pseudothermal source

In this section we calculate the intensity fluctuations and the intensity probability distribution of a thermal source, composed by an ensemble of radiating elements (atoms, molecules). Furthermore, we show the analogy with the statistical properties of the radiation produced by a ground glass diffuser illuminated by coherent light.

A ground glass diffuser, see fig. (3.1), is schematized as a collection of random glass defects that we call microareas. An incoming laser radiation that impinges on the diffuser is randomly scattered by the microareas. The diffuser rotates, therefore the incoming beam illuminates different microareas, that in turn act like independent sources. By independent we mean that there is no phase relationship among fields produced by different microareas. This is the key property that makes a ground glass diffuser a pseudothermal source: the microareas mimic the behaviour of the emitting atoms of molecules in a true thermal source.

To calculate the fluctuations induced by a large number of radiating scatterers we follow the approach presented in [22]. The scatterers in this model are the microareas of the diffuser.

Let us consider the field produced by a large number of scatterers $\nu$. The complex amplitude of the resulting field is given by

$$E(t) = E_0 \exp(-i\omega t) \sum_{j=1}^{\nu} \exp[i\phi_j], \quad (3.10)$$
3.3. Diffuser as a pseudothermal source

\[
I^2 = \left( \frac{\varepsilon_0 c}{2} \right)^2 E_0^4 \nu \sum_{ijkl} \exp \left( i (\phi_i - \phi_j + \phi_k - \phi_l) \right),
\]

(3.14)

To compute \( \overline{I^2} \) we use the same arguments as above and we end up with the situation presented in table (3.1). By adding all of the contributions in table (3.1) we have

\[
\overline{I^2} = \left( \frac{\varepsilon_0 c}{2} \right)^2 E_0^4 \left( \nu + 2 \nu (\nu - 1) \right).
\]

(3.15)

We now have everything we need to calculate \( \Delta I^2 \). By means of eqs. (3.13) and (3.15) we obtain

\[
\Delta I^2 = \overline{I^2} - \overline{I^2} = \left( \frac{\varepsilon_0 c}{2} \right)^2 E_0^4 \left[ (\nu + 2 \nu (\nu - 1)) - \nu^2 \right] = \left( \frac{\varepsilon_0 c}{2} \right)^2 E_0^4 \left[ \nu^2 - \nu \right].
\]

(3.16)

Table 3.1: Contributions to \( \overline{I^2} \)

<table>
<thead>
<tr>
<th>cases</th>
<th>( I^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i = j = k = l )</td>
<td>( \left( \frac{\varepsilon_0 c}{2} \right)^2 E_0^4 \nu )</td>
</tr>
<tr>
<td>( i = j ) and ( k = l ) with ( i \neq k )</td>
<td>( \left( \frac{\varepsilon_0 c}{2} \right)^2 E_0^4 \nu (\nu - 1) )</td>
</tr>
<tr>
<td>( i = l ) and ( j = k ) with ( i \neq j )</td>
<td>( \left( \frac{\varepsilon_0 c}{2} \right)^2 E_0^4 \nu (\nu - 1) )</td>
</tr>
<tr>
<td>remaining cases</td>
<td>0</td>
</tr>
</tbody>
</table>

with \( \phi_j \) being the phase of the \( j-th \) field and \( E_0 \) and \( \omega \) the amplitude and frequency of each radiating field, respectively. Our interest lies in calculating the intensity fluctuations, therefore we have to compute the quantity

\[
\Delta I^2 = \overline{I^2} - \overline{I^2},
\]

where the overbar denotes ensemble averaging. To calculate \( \Delta I^2 \) we proceed by separately calculating \( \overline{I^2} \) and \( \overline{I^2} \). The cycle averaged intensity \( I \) is given by

\[
I \equiv \frac{\varepsilon_0 c}{T} \int_0^T \text{Re} \{ E(t) \} \text{Re} \{ E(t)^* \} dt = \frac{\varepsilon_0 c}{2} E_0^2 \sum_{ij} \exp i (\phi_i - \phi_j),
\]

(3.12)

with \( T = \frac{2\pi}{\omega} \), \( c \) the speed of light, \( \varepsilon_0 \) the dielectric constant and where we have used eq. (3.10).

In order to calculate \( \overline{I^2} \) we recall that the phases \( \phi_i \) are random variables with uniform probability distribution that assumes values in \( [0, 2\pi] \). The only cases in which \( \overline{I^2} \neq 0 \) are then those for which \( \phi_i = \phi_j \). In all other cases we have \( \exp i (\phi_i - \phi_j) = 0 \). Therefore we get

\[
\overline{I} = \frac{\varepsilon_0 c}{2} E_0^2 \nu.
\]

(3.13)

Next we calculate \( \overline{I^2} \). By using eq. (3.12) we obtain

\[
\overline{I^2} = \left( \frac{\varepsilon_0 c}{2} \right)^2 E_0^4 \nu (\nu + 2 \nu (\nu - 1)).
\]

(3.15)
If the number of radiating scatterers is large \((\nu \gg 1)\) \(\Delta I^2\) becomes

\[
\Delta I^2 = \left(\frac{\varepsilon_0 c}{2}\right)^2 E_0^4 \nu^2 = T^2.
\] (3.17)

The intensity fluctuations \(\Delta I^2\) scale quadratically with the average intensity.

Having calculated \(\Delta I^2\) we can also recover the full probability distribution \(P(I)\). A way to calculate the probability distribution is to compute its higher order moments \(\overline{T^n}\). In general, computing the higher moments is a difficult task but, if the number of radiating scatterers \(\nu\) is large the following relationship holds

\[
\overline{T^n} = n! \overline{T^n}.
\] (3.18)

Eq. (3.18) can be deduced, following [42], by writing down the expression for \(\overline{T^n}\), given by

\[
\overline{T^n} = \sum_{\nu_1 \ldots \nu_n, \beta_1 \ldots \beta_n} E_{\nu_1} \ldots E_{\nu_n} E_{\beta_1} \ldots E_{\beta_n}.
\] (3.19)

If there is a random phase relationship between the fields, e.g. the fields are independent from one another in accordance with the reasoning that leads to table (3.1), the only contributions different from zero are of the type

\[
E_{\nu} E_{\beta} = \delta_{\nu\beta} I.
\] (3.20)

Since there are \(n!\) ways to combine the fields in eq. (3.19) so that eq. (3.20) holds true, eq. (3.18) is recovered for the moments.

From the expression for the moments we can extract the total probability distribution \(P_R(I)\), given by

\[
P_R(I) = \frac{1}{I} \exp \left( -\frac{I}{I} \right).
\] (3.21)

Eq. (3.21) represents the Rayleigh distribution, which is a milestone in statistical optics. The Rayleigh distribution occurs often in physics and is a consequence of the central limit theorem that applies because of the independency of the sources.

### 3.4 Intensity spectrum of a pseudothermal source

In this section we follow the theory presented in [94] to calculate the noise power spectrum \(S(\Omega)\) generated by a rotating diffuser. The schematics is presented in fig. (3.1).

The formula for the power spectrum is given by

\[
S(\Omega) = \int_{-\infty}^{+\infty} \left\langle \hat{j}(t) \hat{j}(t + \tau) \right\rangle \exp(-i\Omega \tau) \, d\tau,
\] (3.22)

where \(\Omega\) denotes the noise frequency, \(\hat{j}(t)\) the photocurrent operator and with \(\langle \rangle\) we indicate quantum averaging.
3.4. Intensity spectrum of a pseudothermal source

Figure 3.1: Setup consisting of a lens and a diffuser used to generate pseudothermal light. A laser beam of frequency $\omega$ and diameter $\sigma$ is focussed by a lens of focal lens $f$ and impinges on a ground glass diffuser rotating at speed $v$. With $A$ is indicated the cross section of the beam on the diffuser. The ground glass diffuser is modelled as a collection of random microareas. The rotation of the microareas gives rise to pseudothermal light.

Our goal is to evaluate the term $\langle \hat{j}(t) \hat{j}(t + \tau) \rangle$ and Fourier transform it according to eq. (3.22). In order to fully account for the wave and particle nature of light we resort to the quantum derivation outlined in [94]. We have

$$\langle \hat{I}(t) \hat{I}(t + \tau) \rangle = \langle : \hat{I}(t) \hat{I}(t + \tau) : \rangle - h 2 \epsilon_0 i \delta'(\tau) \langle \hat{E}(-)(t) \hat{E}(+)(t + \tau) \rangle$$

(3.23)

where $\hat{I}(t)$ denotes the intensity operator and the $::$ notation indicates normal ordering of the field operators $\hat{E}(-)(t)$ and $\hat{E}(+)$, that are given respectively by

$$\hat{E}(-)(t) = \left( \frac{1}{2\pi} \right)^2 \int d\omega \left( \frac{\hbar \omega}{2 \epsilon_0} \right)^{1/2} \hat{a}^\dagger(\omega) e^{i\omega t}$$

(3.24)

and

$$\hat{E}(+)(t) = \left( \frac{1}{2\pi} \right)^2 \int d\omega \left( \frac{\hbar \omega}{2 \epsilon_0} \right)^{1/2} \hat{a}(\omega) e^{-i\omega t}.$$ 

(3.25)

In eqs. (3.24) and (3.25) $\hbar = \frac{h}{2\pi}$ with $h$ being Planck’s constant and $\epsilon_0$ is the dielectric constant. By using the relationship

$$g^{(2)}(\tau) = \frac{\langle : \hat{I}(t) \hat{I}(t + \tau) : \rangle}{\langle \hat{I}(t) \rangle \langle \hat{I}(t + \tau) \rangle},$$

(3.26)

where $g^{(2)}(\tau)$ denotes the second order correlation function, we can rewrite eq. (3.23) as follows

$$\langle \hat{I}(t) \hat{I}(t + \tau) \rangle = g^{(2)}(\tau) \langle \hat{I}(t) \rangle \langle \hat{I}(t + \tau) \rangle - h 2 \epsilon_0 i \delta'(\tau) \langle \hat{E}(-)(t) \hat{E}(+)(t + \tau) \rangle.$$ 

(3.27)

The term $\langle \hat{E}(-)(t) \hat{E}(+)(t + \tau) \rangle$ is related to the first order correlation function $g^{(1)}(\tau)$ and we have

$$\langle \hat{E}(-)(t) \hat{E}(+)(t + \tau) \rangle = \frac{g^{(1)}(\tau) \langle \hat{I}(t) \rangle}{2 \epsilon_0} = \left( \frac{e}{\hbar \omega} \right) \left( \frac{1}{2 \epsilon_0} \right) \langle \hat{j}(t) \rangle g^{(1)}(\tau).$$

(3.28)
The relationship between the intensity $\hat{I}(t)$ and the photocurrent $\langle \hat{j}(t) \rangle$ is given by

$$
\langle \hat{j}(t) \rangle = \frac{e}{\hbar \omega} \langle \hat{I}(t) \rangle.
$$

(3.29)

Therefore, by exploiting eq. (3.29) we have

$$
\langle \hat{j}(t)\hat{j}(t+\tau) \rangle = g^{(2)}(\tau) \langle \hat{j}(t) \rangle \langle \hat{j}(t+\tau) \rangle - \left( \frac{e}{\hbar \omega} \right) h i \delta'(\tau) \langle \hat{j}(t) \rangle g^{(1)}(\tau).
$$

(3.30)

For chaotic light we can use the Siegert relationship, [95]

$$
g^{(2)}(\tau) = 1 + \left| g^{(1)}(\tau) \right|^2.
$$

(3.31)

What we need to calculate now is $g^{(1)}(\tau)$. Up to now we have not made use of the properties of the rotating diffuser that generates pseudo-thermal light. We use the result provided by [96] where $g^{(1)}(\tau)$ is calculated for a rotating ground glass diffuser and that yields

$$
g^{(1)}(\tau) = \frac{2\pi \sigma^2}{A} \nu s^2 \exp \left( i \omega \tau - \frac{v^2 \tau^2}{2} \left( \frac{k^2 \sigma^2}{f^2} + \frac{1}{4\sigma^2} \right) \right).
$$

(3.32)

In eq. (3.32), and with reference to fig. (3.1), $\sigma^2$ is the cross section of the incident beam, $\nu$ is the number of independent microareas we divide the diffuser into, $A$ is the illuminated area on the surface of the diffuser and $s$ is a term that takes into account diffraction from the microareas. By looking at eq. (3.32) we define the coherence time $\tau_c$ of the pseudo-thermal source as

$$
\frac{1}{\tau_c^2} = \frac{v^2}{2} \left( \frac{k^2 \sigma^2}{f^2} + \frac{1}{4\sigma^2} \right).
$$

(3.33)

It is instructive to consider eq. (3.33) for a series of realistic experimental conditions. As it is clear from fig. (3.2) to introduce excess fluctuations in the MHz regime we need to spin the diffuser with a linear velocity of a few ms$^{-1}$ and a focal length of $2 - 3$ cm.

We now have all the ingredients to calculate eq. (3.22). On inserting eq. (3.30) in eq. (3.22), and making use of eq. (3.32), we obtain

$$
S(\Omega) = \int_{-\infty}^{+\infty} \langle \hat{j}(t)\hat{j}(t+\tau) \rangle \exp(-i\Omega \tau) d\tau
$$

$$
= \langle \hat{j}(t) \rangle^2 \delta(\Omega)
$$

$$
+ \langle \hat{j}(t) \rangle^2 \frac{\tau_c}{2} \exp \left\{ -\frac{\Omega^2 \tau_c^2}{8} \right\}
$$

$$
+ \langle \hat{j}(t) \rangle \left( \frac{\omega - \Omega}{\omega} \right) \zeta,
$$

(3.34)
3.5. Weak localization of photon noise

Here, we present the first experimental investigation of weak localization of photon shot and pseudothermal noise from scattering media. The basic setup to induce and measure photon noise is depicted in fig. (3.3). It consists of a Ti:Sapphire laser (Spectra Physics-Tsunami) and a set of two lenses to focus light on the diffuser and collimate the exit beam respectively. The collimated exit beam is collected by an APD detector and the noise content of the signal is analyzed by the electronic

$$\zeta = e^{\frac{2\pi \sigma^2 \nu A^2}{\lambda}}.$$  \hspace{1cm} (3.35)

The first part of eq. (3.34) contains a delta function that represents the dc component of the spectrum. The second part of eq. (3.34) contains a frequency-dependent gaussian spectrum, whose amplitude scales quadratically with the intensity. This term is the classical noise contribution. Finally the third part of eq. (3.34) displays a flat, frequency independent term because, given the experimental conditions, the frequencies ratio yields one. This term is the shot noise contribution. The shot noise term is a consequence of the quantum nature of light, of its intrinsic granular nature.

The noise barrier in any optical system is given by the shot noise. A shot noise limited source, like some lasers, is thus the quietest possible source. It is feasible to beat this limit only by resorting to quantum optics tricks, like for example squeezing. Also in that case though, we are not eliminating noise, but only transferring noise from one quadrature to the other, so that one quadrature displays shot and excess noise while the other one exhibits a noise contribution which is below the shot noise level.

In the next section we will describe experiments that make use of the system composed by two lenses and a diffuser to generate pseudothermal light.
Weak localization of photon noise in random media

Figure 3.3: Basic setup to measure photon noise. A Ti:Sapphire laser is a photon shot noise source. The rotating ground glass diffuser produces pseudothermal light and the two lenses generate a collimated beam. The spectrum analyzer investigates the noise content of the signal.

Figure 3.4: Schematic overview of the experimental setup for measurement of noise and intensity coherent backscattering.

spectrum analyzer. By tuning the angular speed of the diffuser it is possible to push the transition frequency up to the MHz regime. We integrated these components into a coherent backscattering setup.

Coherent backscattering of both intensity and noise is measured in a beamsplitter configuration [97], as outlined in fig. (3.4). We used the 1.6 W output of a shot noise limited Ti:Sapphire oscillator operating in cw mode and at a wavelength of 780 nm. Light scattered from the sample was collected by a lens (f = 8 cm) and detected by an avalanche photodiode (APD) module (Hamamatsu C4777) that moves along the focal plane of the lens. An electronic spectrum analyzer (Agilent) was used to measure the noise power spectrum. The angular resolution amounts to 1.2 mrad and is determined by a 100 µm pinhole placed in front of the detector.
3.5. Weak localization of photon noise

Figure 3.5: Noise spectrum of light scattered from a gallium phosphide sample as function of backscattering angle and frequency. The CBS noise cones are clearly visible over the entire frequency range.

The sample was slightly tilted to prevent light which was specularly reflected from the sample to be detected. Speckle averaging was obtained by rotating the sample around its azimuthal axis using a spinning motor. We performed detection in the polarization conserving channel. Lock-in detection minimizes the influence of stray light. Pseudothermal light was generated by focusing the laser beam onto a rotating diffuser [98, 99]. By spinning the diffuser it was possible to introduce excess noise up to the MHz regime.

Our sample consists of a slab of porous GaP fabricated via an electrochemical etching technique, and showing negligible absorption at 780 nm [100]. The sample has thickness $L \simeq 70\, \mu m$ and $k_0 \simeq 17.7$, where with $k_0$ we indicate the incoming wavevector outside the medium.

Noise spectra were acquired with a resolution bandwidth (RBW) of 30 kHz and a video bandwidth (VBW) of 10 kHz over a frequency segment that spans 8.85 MHz, from 0.15 MHz to 10 MHz. Each point in the noise traces has been obtained by computing the average of 50 consecutive points in the spectrum. Intensity and noise cones were acquired in the polarization conserving channel and normalized to the intensity and noise power measured in the polarization nonconserving channel. The cones were further normalized to the value assumed on the left most angle. Noise spectra are shown in fig. (3.5) as function of angle and frequency. Cross sections at different frequencies are shown in fig. (3.6), along with the intensity CBS cone data and its square. The data highlight how the CBS shape changes from the shot noise regime at high frequencies to the pseudothermal noise regime at low frequencies. Furthermore, the comparison to the intensity data shows how the shot noise cone, obtained from the high frequency data, follows closely the intensity cone whereas the intensity CBS cone squared is an upper bound for the pseudothermal cones, obtained from the low frequency data. The enhancement produced by pseudothermal light exceeds the ordinary maximum value of 2 obtainable for the intensity cone.
Weak localization of photon noise in random media

Figure 3.6: Comparison between the measured CBS intensity and CBS noise signals. At high frequency the shot noise cones match the intensity cone. As the noise frequency is lowered the noise cones exceed the intensity cone and approach the maximum value given by the squared intensity.

3.6 Analysis

We start modelling our experiment by using eq. (3.34). The first term indicates the dc intensity and is filtered out in our experiments, therefore it will be discarded from the analysis. The second term in eq. (3.34) represents the wave fluctuations and quantifies the excess noise, while the third term accounts for the shot noise or particle fluctuations.

The measured photocurrent is related to the light intensity by \( \langle j \rangle = \gamma \langle I \rangle \) where \( \langle I \rangle \) is the intensity and \( \gamma \) a constant proportional to the responsivity of the APD detector. We can explicitly introduce the sample parameters by writing

\[
\langle I_{ab} \rangle = R_{ab} \langle I_a \rangle, \tag{3.36}
\]

\[
\langle I_{ab} \rangle^2 = R_{ab}^2 \langle I_a \rangle^2, \tag{3.37}
\]

where \( R_{ab} \) indicates the classical reflection coefficient from incoming channel \( a \) to outgoing channel \( b \), averaged over different realizations of the disorder (ensemble average). On inserting eqs. (3.36) and (3.37) in eq. (3.34) and performing ensemble average we obtain

\[
\langle S(\Omega)_{ab} \rangle = \phi R_{ab} \langle I_a \rangle + \gamma^2 R_{ab}^2 \langle I_a \rangle^2 \left( \frac{\tau_c}{2} \right) \exp \left( -\frac{\Omega^2 \tau_c^2}{8} \right), \tag{3.38}
\]

where we have highlighted the dependence of the photocurrent spectrum on the outgoing channel \( b \), and \( \phi = \gamma \zeta \). The frequency-dependent noise spectrum can be used to calculate a frequency-dependent Fano factor \( F \) defined as the ratio of the photocurrent fluctuation to the average photocurrent,

\[
F(\Omega)_{ab} = \frac{\langle S(\Omega)_{ab} \rangle}{\phi R_{ab} \langle I_a \rangle}, \tag{3.39}
\]
3.6. Analysis

Figure 3.7: Observed Fano factor as function of frequency obtained from the noise trace collected exactly at backscattering. A gallium phosphide sample with \( k_0 \ell \approx 17.7 \) has been used. The Fano factor has been extracted by dividing the full noise spectrum by the constant value obtained by fitting the high frequency noise data to a line. The solid line is a guide to the eye.

Fig. (3.7) shows the Fano factor as a function of noise frequency. In order to analyze the effect of coherent backscattering on the Fano factor, we analyze the Fano factor as a function of scattering angle and frequency. The instrumental response is divided out by normalizing both the average intensity and the noise spectra to those acquired in the polarization nonconserving channel, in which the CBS effect is absent. We define a Fano factor CBS enhancement \( \eta_{ab}(\Omega) \) as the ratio of the normalized noise spectra to the normalized intensity

\[
\eta_{ab}(\Omega) \equiv \frac{\langle S(\Omega)_{ab}^{pc} \rangle}{\langle S(\Omega)_{ab}^{pnc} \rangle} \left( \frac{I_{ab}^{pc}}{I_{ab}^{pnc}} \right)^{-1},
\]

where the subscript pc and pnc stand for polarization conserving and polarization nonconserving, respectively. By expanding eq. (3.40) we obtain

\[
\eta_{ab}(\Omega) = \frac{\phi + R_{ab}^{2 \cdot pc} \left( R_{ab}^{pc} \right)^{-1} \langle I_a \rangle \gamma^2 \left( \frac{a}{2} \right) \exp \left( -\frac{\Omega^2 r^2}{8} \right)}{\phi + R_{ab}^{2 \cdot pnc} \left( R_{ab}^{pnc} \right)^{-1} \langle I_a \rangle \gamma^2 \left( \frac{a}{2} \right) \exp \left( -\frac{\Omega^2 r^2}{8} \right)}. \tag{3.41}
\]

A 3D plot of \( \eta_{ab}(\Omega) \) is shown in fig. (3.8). Cross sections of the 3D plot are produced in fig. (3.10) for three representative noise frequencies as function of the scattering angle. The Fano factor enhancement \( \eta_{ab}(\Omega) \) is 1 away from the CBS cone, as here the polarization conserving and nonconserving channels yield the same noise and intensity.

It is interesting to consider the situation on backscattering, by calculating \( \eta_{aa}(\Omega) \). In order to do that we need to evaluate \( R_{aa}^{2 \cdot pc} \left( R_{aa}^{pc} \right)^{-1} \) and \( R_{aa}^{2 \cdot pnc} \left( R_{aa}^{pnc} \right)^{-1} \).
Figure 3.8: 3D plot for the measured Fano factor CBS enhancement $\eta_{ab}(\Omega)$ as function of frequency and angle. A gallium phosphide sample with $k_0 \ell \approx 17.7$ has been used. The plotted data have been obtained from the raw data by binning 50 consecutive data points along the frequency axis. $\eta_{ab}(\Omega)$ varies from 1 at high frequencies in the shot noise regime to a maximum value which is always lower than the intensity CBS enhancement. Values of $\eta_{ab}(\Omega)$ below 1 are due to spurious fluctuations.

The coherent backscattering effect due to interference is present in the polarization conserving channel \(^1\). Since in the polarization nonconserving case the coherent backscattering effect is absent we assume

\[
\frac{R_{aa}^{pnc}}{R_{aa}^{pnc}} = \frac{R_{ab}^2}{R_{ab}^2}.
\]

(3.42)

From now on we drop the subscripts $pc$ and $pnc$, as the term $R_{aa}$ will always be referred to the polarization conserving case. In order to assess the effect of mesoscopic interference in the coherent backscattering of noise, we consider the first order mesoscopic contribution in the reflectivity as obtained in [102]. By exploiting eq. (15) in [102] we have

\[
\overline{R_{ab}^2} = \overline{R_{ab}^2} \left( 2 - \frac{2}{N(1 - \frac{\ell}{L})} \right),
\]

(3.43)

and

\[
\overline{R_{aa}^2} = \overline{R_{aa}^2} \left( 2 - \frac{3}{N(1 - \frac{\ell}{L})} \right).
\]

(3.44)

As we have seen in chapter 2, the total conductance $g$, defined in transmission, is given by

\[
g = \frac{N \ell}{L}.
\]

(3.45)

\(^1\) While the polarization conserving channel contains the interference effect that gives rise to the coherent backscattering phenomenon, it does not reject single scattering events. Single scattering events do not have a reverse path and contribute to lowering the enhancement factor [101].
Figure 3.9: (left) Observed noise CBS enhancement factor on backscattering \( \eta_{aa} \) as function of frequency. The enhancement factor \( \eta_{aa}(\Omega) \) has been obtained by taking the maximum of \( \eta_{ab}(\Omega) \) at each frequency. The solid line is a fit of the theoretical line shape for \( \eta_{aa} \) to the data. (right) Calculations that illustrate the effect of the conductance \( g_r \) on the Fano factor CBS enhancement on backscattering \( \eta_{aa}(\Omega) \). Low \( g_r \) values induce stronger variations, especially at low frequencies. As \( g_r \) increases \( \eta_{aa} \) quickly converges towards the limit function in eq. (3.50), and deviations caused by high \( g_r \) values become difficult to be discerned. The inset highlights the different values reached by \( \eta_{aa} \) at \( \Omega = 0 \). The parameters \( \phi, \beta \) and \( \tau_c \) are fixed according to the values obtained in experiments.

In reflection, following the same procedure as the one adopted for \( g \) we define

\[
g_r \equiv \sum_{a,b} R_{ab}, \tag{3.46}
\]

and, using the fact that \( R_{ab} = (1 - \frac{\ell}{L}) \) we find

\[
g_r = N \left(1 - \frac{\ell}{L}\right). \tag{3.47}
\]

Formulas (3.43) and (3.44) express the fact that when \( g_r^{-1} \) contributions are negligible the reflected intensity follows a Rayleigh distribution. Deviations from Rayleigh statistics arise when interferences effect between multiple scattering paths start to play a role.

By using eqs. (3.41), (3.43) and (3.44) we obtain the following expression for the Fano factor enhancement on backscattering

\[
\eta_{aa}(\Omega) = \frac{\phi + R_{aa}\left(2 - \frac{2}{g_r}\right)\langle I_a \rangle \gamma^2 \left(\tau_c^2\right) \exp\left(-\frac{\Omega^2 \tau_c^2}{8}\right)}{\phi + R_{ab}\left(2 - \frac{2}{g_r}\right)\langle I_a \rangle \gamma^2 \left(\tau_c^2\right) \exp\left(-\frac{\Omega^2 \tau_c^2}{8}\right)}. \tag{3.48}
\]

According to random matrix theory, \( R_{aa} = 2R_{ab} \). To take into account a real experimental situation, where the enhancement on backscattering is usually lower
Figure 3.10: Noise CBS enhancement factor $\eta_{ab}$ as function of the scattering angle around the backscattering direction at three different frequencies.

than 2, we will assume $R_{aa} = (2 - \beta) R_{ab}$. After rearranging eq. (3.48) we find

$$\eta_{aa}(\Omega) = 1 + \frac{[2 (1 - \beta) \langle 1 - g^{-1}_r \rangle - (2 - \beta) g^{-1}_r ] \left( \frac{r_c}{2} \right) \exp \left( -\frac{\Omega^2 \tau^2}{8} \right)}{\psi + 2 (1 - g^{-1}_r) \left( \frac{r_c}{2} \right) \exp \left( -\frac{\Omega^2 \tau^2}{8} \right)},$$

(3.49)

with $\psi = \phi \left( \langle I_a \rangle R_{ab} \gamma^2 \right)^{-1}$. It is evident the sensitivity of $\eta_{aa}$ to a broad range of $g_r$ values. In our case, being the magnitude of $g^{-1}_r$ negligible, we can simplify eq. (3.49) and obtain

$$\eta_{aa}(\Omega)_{g^{-1}_r \to 0} = 1 + \frac{2 (1 - \beta) \left( \frac{r_c}{2} \right) \exp \left( -\frac{\Omega^2 \tau^2}{8} \right)}{\psi + 2 \left( \frac{r_c}{2} \right) \exp \left( -\frac{\Omega^2 \tau^2}{8} \right)}.$$  

(3.50)

In fig. (3.9, left panel), we show the measured Fano factor CBS enhancement on backscattering $\eta_{aa}$ in our experimental configuration as well as, in fig. (3.9, right panel), the calculated effect of a selection of values for the conductance $g_r$ on $\eta_{aa}$. The factor $\eta_{aa}(\Omega)$ is plotted as function of noise frequency and is fitted by using eq. (3.50). A good agreement is recovered.

The enhancement $\eta_{aa}$ depends on the balance of quantum and classical noise, respectively given by $\psi$ and the exponential term in the denominator of eq. (3.50). The behaviour of $\eta_{aa}$ at low and high frequency, in the classical and shot noise regimes, is given respectively by

$$\eta_{aa}(\Omega)_{\Omega \to 0} = 1 + \frac{2(1 - \beta) \frac{r_c}{2}}{2 \frac{r_c}{2} + \psi}$$

(3.51)

and

$$\eta_{aa}(\Omega)_{\Omega \to \infty} = 1.$$  

(3.52)

In the low-frequency limit, $\eta_{aa}(\Omega)$ approaches the coherent backscattering enhancement for the intensity, $2 - \beta$. The Fano factor at high frequencies, i.e. for the coherent state, is not modified by coherent backscattering.
In general, $\eta_{aa}(\Omega)$ contains both the shot noise and the pseudothermal contributions, as shown in eq. (3.50), with the frequency $\Omega$ gauging the two regimes as highlighted by eqs. (3.51) and (3.52).

In fig. (3.10) $\eta_{ab}(\Omega)$, the Fano factor CBS enhancement of the outgoing radiation, is plotted as function of the backscattering angle. The factor $\eta_{ab}(\Omega)$ displays no angular dependence in the full shot noise regime at high frequencies, whereas the interference effect responsible for the CBS cone, as the low frequency regime is approached, reflects itself on the development of an angular dependent Fano factor. A gradual transition as function of frequency between the full shot and pseudothermal noise case is also evident.

3.7 Conclusions

In this chapter we have studied for the first time photon noise measurements in the weak localization regime and shown that the transition from the full shot noise regime at high frequency to the full pseudothermal domain at low frequency can be investigated. We have experimentally demonstrated the enhancement of the Fano factor around the backscattering direction due to the CBS effect. Furthermore, we were able to extract the Fano factor CBS enhancement of the reflected radiation as function of both frequency and angle. It is remarkable that the enhancement of the Fano factor CBS depends on $g_r^{-1}$. The standard intensity coherent backscattering cone does not have such a contribution, whereas measuring photon noise gives access to $\overline{R^2}_{ab}$, from which the magnitude of $g_r^{-1}$ effects can be extracted.

We have shown that measurements that span the full noise spectrum, from the shot to the excess noise regime, have the advantage to provide simultaneously information about the first and second moment of the probability distribution of the intensity of the reflected radiation. More specifically, our theory indicates that the enhancement displayed by the noise cone reveals information about the mesoscopic correlations that give rise to deviations from Rayleigh statistics in the strongly scattering regime.

The use of the CBS technique has in addition the potential to study the first and second moment of the probability distribution as function of the path length.

Sensitive photon noise experiments can be combined with coherent backscattering experiments, opening up new avenues for studying quantum optical aspects of diffuse wave transport. Future experiments on photon noise in the strong scattering regime may be performed to explore mesoscopic quantum corrections and localization. Furthermore, noise measurements could prove themselves useful also in systems that display gain, random lasing and absorption [67, 72, 75].