Consumer demand with social interactions: a simulation study

Soetevent, A.R.

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Adriaan R. Soetevent*

University of Amsterdam

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Abstract

How do social interactions affect consumer demand? In this paper, I investigate the effects that different social structures have on social welfare and on the budget shares of various categories of goods. A society is modeled in which households' demand for goods is described by a Linear Expenditure System with Social Interactions (LES-SI).

In the context of the LES-SI, I find that social interactions can lead to a considerable reallocation of resources over goods. The differences in effect are small for the different social structures. Optimal taxes and subsidies set by a welfare maximizing social planner are explicitly calculated. Interestingly, the budget share and optimal tax rate of the most conspicuous good do not necessarily increase when the degree of interactions increases. The presence of social interactions does not make the tax and subsidy instrument more effective in terms of the increase in social welfare that can be obtained.

Keywords: social interactions, Linear Expenditure System, simulation, social structure

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*Department of Economics, University of Amsterdam, FEE/AE, Roetersstraat 11, 1018 WB Amsterdam, The Netherlands, Ph: +31 - (0) 20 - 525 73 51; E-mail: a.r.soetevent@uva.nl. This paper resulted from my PhD research at the Univeristy of Groningen. Helpful comments by Peter Kooreman, Bert Schoonbeek, Marco Haan, Tom Wansbeek and participants at several seminars are gratefully acknowledged.

1 Introduction

In the last decade, a rising surge could be observed in economic studies on social interactions, that is, interactions between agents which are not regulated by a price mechanism. Surveys by Glaeser and Scheinkman (2000) mark this development. In a number of empirical studies with microeconomic data, researchers have estimated economic models that allow for social interactions between agents or households. Examples include studies concerning labor supply (Aronsson, Blomquist and Sacklén, 1999), saving decisions (Kapteyn, 2000) and consumer demand (Alessie and Kapteyn, 1991; Kapteyn *et al.*, 1997). For reasons of identification, all these studies make simplifying assumptions with regard to the reference weights that specify the importance an agent attaches to the consumption of another agent.[1]

In another, primarily theoretical, branch of literature it is just the interaction topology that is the object of study. Ioannides (2003) assesses the importance of how agents are connected by comparing the effects of different stylized topologies of social interactions. An important distinction made in this literature is between *global* and *local* interaction topologies. In models with global interactions, agents assign a non-zero reference weight to *all* other agents in the population. An example of such a model is the mean field model studied by Brock and Durlauf (2001a). An interaction is called local if agents only interact directly with a limited number of other agents in the population. Examples include the nearest neighbor setup where agents only assign positive weights to the agents who are next to them according to some measure (see e.g. Ioannides, 1997 and chapter 6 in Young, 1998). In still another approach that is not considered in this paper, links between agents are assumed to be stochastic instead of deterministic. The links between agents are in this case modeled using random graph theory. In this way, probabilistic results are obtained for general interaction structures. See e.g. Kirman (1983), Kirman, Oddou and Weber (1986) and Horst and Scheinkman (2003)

[1] Kapteyn *et al.* (1997, p. 669) note for example that "the main problem in estimating the model is, of course, created by the large number of reference weights..." and Ioannides (2003, p. 1) remarks that "Even in economics, we often make some very specific assumptions about interaction patterns in order to obtain analytically tractable models for particular problems." Early work by Kapteyn (1977) and Kapteyn, Van Praag and Van Herwaarden (1978) tried to estimate reference patterns directly but this led to complicated models which seem to have got out of fashion in more recent empirical work.

In this paper, I try to establish a connection between this theoretical work on interaction topologies and the empirical studies on consumer demand. To this end, the effects of three different interaction topologies on consumer demand are compared in the context of a Linear Expenditure System that is extended to allow for preference interdependencies, the LES-SI. In the first specification, each individual attaches equal weight to all other agents in society. This is a global interactions model and I refer to this model as the 'complete model', since the topology can be visualized as a complete graph in which the agents are the nodes and the equal and positive reference weights the edges. The second model specifies a local interaction structure by dividing society in a number of non-overlapping peer groups, based on socio-economic characteristics. I refer to it as the '(non-overlapping) cliques model'. In the third model, agents are arranged on a closed one-dimensional lattice and each agent gives positive reference weights to $2R$ of his neighbors and these weights decrease with the distance to their residence. Just as the second specification, it is a model with a local interaction topology. The difference is that peer groups are now overlapping. Since the model can be represented as a $2R$-regular graph, I call it the 'cyclical model with degree of regularity $2R$', or in short: the cyclical model.

In empirical work on neighborhood effects, in which individual outcomes are regressed on the average neighborhood outcome of the same variable, like e.g. Case and Katz (1991), the underlying topological structure is complete: Individuals in a certain neighborhood are assumed to give equal weight to the other individuals in the same neighborhood, without regard to differences in personal characteristics.

The clique type of social structure is commonly assumed in empirical studies on interdependencies between households based on individual level data. In most of these studies, cliques or *social groups* are defined using certain household characteristics that are provided by the data, like e.g. the age of the head of the household and his or her educational attainment.[2] In the simplest version, equal weights are assumed between agents in the same social groups and zero weights between agents in different groups. An example is Aronsson *et al.* (1999). The demand systems proposed by Alessie and Kapteyn (1991) and Kapteyn *et al.*

[2]The exception is Woittiez and Kapteyn (1998) who use direct survey questions and factor analysis to infer the average values of variables of interest in the peer group of an individual.

(1997) include a parameter that allows agents to give positive weight to agents outside their own social group. In their analysis of high school teen behavior, Kooreman and Soetevent (2004) define two social groups within school classes based on gender. They allow for differences between own-gender and cross-gender interactions. The circular interaction topology is for example used in Ellison (1993) and in the empirical study on interactions in crime of Glaeser, Sacerdote and Scheinkman (1996).

The current paper analyzes the different impact these three interaction topologies have on aggregate consumption behavior. I do this by means of simulation. I construct a fictitious city called SIcity whose inhabitants consume seven categories of goods. The way households in SIcity spend their income on the different goods is prescribed by the LES-SI demand system. Households are heterogeneous with respect to income and family size. Moreover, I try to keep close to reality by letting the income and family size distribution mimic the corresponding distributions in a sample of Dutch households. The parameters of the LES-SI system are chosen in such a way that the resulting budget shares and elasticities of the goods are comparable to those found in empirical studies. The three interaction topologies are compared by looking at both the average budget shares of the goods and the maximum value of a (utilitarian) social welfare function that is obtainable for a social planner.[3]

The three questions addressed in the simulations are: (a) In which way is the change in social welfare due to social interactions dependent on the underlying social structure? (b) How does for each of the social structures social welfare change when one or more of the households experience an income shock, e.g. by winning a lottery prize? (c) How can government enhance social welfare by means of taxes and subsidies?

The third question is also considered in Kooreman and Schoonbeek (2004), but a difference with their approach is that I impose societal budget neutrality instead of budget neutrality at the level of individual households. Societal budget neutrality reflects the government's wish to enhance welfare without a change in net aggregate outlays. Due to the absence of individual budget neutrality, taxes and subsidies can lead to an increase in welfare for two reasons:

[3] The welfare properties of the LES-SI are studied extensively in Kooreman and Schoonbeek (2004).

by the internalization of the social cost of consumption, or by a reallocation effect of means over households. A novelty in this paper is that for the first time, optimal taxes and subsidies are calculated numerically for the case with heterogeneous consumers.

The main results of the paper are that the loss in social welfare due to social interactions seems modest for all social structures, but that considerable reallocations of resources across goods may occur. Social interactions have least effect in the cliques model. Imposing taxes on some goods and giving subsidies to others increases social welfare, but the presence of social interactions does not lead to an appreciable extra gain over the one obtainable in a society without preference interdependencies. Interestingly, increases in the degree of interactions does not always lead to increases in the budget share of the most conspicuous good. Consequently, more interdependency does not always lead to higher taxation of the most conspicuous good.

The paper proceeds as follows. In the next section, the LES-SI system is introduced and discussed as well as the three interaction topologies used in the simulations and the social welfare function that is employed. In section 3, I will introduce SIcity and its inhabitants. A justification is given for the choice of the household characteristics in SIcity and for the parameters of the underlying LES-SI system. Section 4 discusses the results of the simulations. Section 5 concludes.

2 The model

2.1 The Linear Expenditure System with Social Interactions

Demand analysis studies the question how consumers spend their budget over the set of the available goods, given total outlays and the prices of the goods. The relationship between demand, total expenditures and prices can be expressed as

$$q_{gn} = v_{gn}(y_n, p), \qquad g = 1, \ldots, G, \quad n = 1, \ldots, N. \tag{1}$$

In these Marshallian demand functions, q_{gn} denotes the quantity demanded of good g by consumer n, y_n is his or her total outlay and p is the price vector

$p = (p_1, \ldots, p_G)'$. Particular specifications of these demand functions are estimated since the 1950s, beginning with Stone's (1954) estimation of a Linear Expenditure System (LES), using British macroeconomic data. The LES is a particular member of the class of models described by (1).

Besides the analysis of macroeconomic time-series data, demand analysis is also concerned with the explanation of behavioral differences between households. In older empirical studies, like Working (1943) and Leser (1963), attention was restricted to the estimation of Engel curves of the form $q_{gn} = v_g^*(y_n)$, due to the absence of price variability across the households observed. In the 1950s, microeconomic repeated cross-section data became available due to the start-up of yearly expenditure surveys in several countries.[4] This made the empirical estimation of demand systems of the form (1) possible. An example is Blundell and Robin (1999) who studied the Quadratic Almost Ideal Demand System.

In this simulation study, I will focus on changes in the behavior of households when interactions between the households are taken into account using the LES as the underlying demand system. The utility function for consumer n that underlies the LES without preference interdependence is specified as

$$U_n = \sum_{g=1}^{G} \gamma_g \ln(x_{gn} - b_{gn}), \tag{2}$$

with $\gamma_g > 0$, $\forall g$, $\sum_{g=1}^{G} \gamma_g = 1$, and $x_{gn} > b_{gn}$, $\forall g$, $\forall n$. Maximization of U_n with respect to x_{gn}, $\forall g$, subject to the budget constraint $\sum_{g=1}^{G} p_g x_{gn} = y_n$, yields the demand equations

$$x_{gn} = b_{gn} + \frac{\gamma_g}{p_g}(y_n - \sum_{h=1}^{G} p_h b_{hn}). \tag{3}$$

The quantities b_{gn} are often interpreted as 'subsistence', 'necessary', or 'committed' quantities, being the quantities that a household at least has to buy in order to function. Households whose demand system is a LES subsequently divide the remaining or 'supernumerary' income $y_n - \sum_{h=1}^{G} p_h b_{hn}$ among the goods in fixed proportions $\gamma_1, \ldots, \gamma_G$.

[4] For example, the British Family Expenditure Survey has been carried out annually since 1957, the Japanese Family Income and Expenditure Survey since 1963, the Dutch Budget Survey since 1978 and the US Consumer Expenditure Survey since 1980.

The LES is attractive from an expositional point of view since its parameters have straightforward behavioral interpretations and since explicit solutions can be derived for many variables of interest. One disadvantage of the LES is that the marginal budget shares are independent of prices as well as expenditures and are equal to the γ's in (2). Another problem is that for household n, the LES utility function U_n is defined only in the region of the commodity space for which the quantity x_{gn} bought of commodity $g, \forall g$, is larger than b_{gn}. See equation (2). This is called the limited-domain problem.

In order to make the model more flexible, I add a demographic translation which allows for differences in household composition. The specific form of this transformation is the same as the one applied by Kapteyn *et al.* (1997). I denote the size of household n — defined as the number of household members — by f_n, $\forall n$, and I assume that the household's committed expenditures on good g increase with $\delta_g f_n$, where $\delta_1, \ldots, \delta_G$ are good-specific. Let $\tilde{x}_{gn}$ be defined as

$$\tilde{x}_{gn} \equiv x_{gn} - \delta_g f_n,$$

which can be interpreted as the household-size adjusted quantity of good g that is bought by household n. Following Pollak (1976), Kapteyn *et al.* (1997) and Kooreman and Schoonbeek (2004), social interactions are introduced by making the subsistence expenditure dependent on the consumption by others in the following way:

$$b_{gn} = b_{g0} + \delta_g f_n + \beta_g \sum_{k=1}^{N} w_{nk} \tilde{x}_{gk}. \tag{4}$$

The non-negative reference weight w_{nk}, with $w_{nn} = 0$ and $\sum_{k=1}^{N} w_{nk} = 1$, $\forall n$, denotes the relative importance that household n attaches to the consumption by household k. These weights are assumed constant across goods. As a result of the limited-domain problem, the region in the commodity space for which the LES utility function is defined, is reduced. The part $b_{g0} + \delta_g f_n$ can be interpreted as the subsistence expenditure on good g when preference interdependencies do not play a role. The coefficient β_g is good-specific and is a measure of the degree of conspicuousness of good g. When β_g is positive, the quantity of good g that household n believes to be necessary is increased through the interaction with other households. This can be interpreted as a positive social cost.

Define the last part of the right hand side of equation (4) as the social cost function:

$$s_{gn}(x_{g,-n}) \equiv \beta_g \sum_{k=1}^{N} w_{nk}(x_{gk} - \delta_g f_k), \tag{5}$$

where $x_{g,-n} \equiv (x_{g1}, x_{g2}, \ldots, x_{g,n-1}, x_{g,n+1}, \ldots, x_{gN})$.

With positive social costs ($\beta_g > 0$), consumption decisions of other households – corrected for the size of these households – have a negative net effect on the households utility of consumption with the effect that the household has to buy a larger quantity of the good to obtain the same level of utility that would have been obtained when no contact existed with other households. This effect may occur because the actions of others affect your preferences (you are jealous when your neighbor drives a new car, and as a result derive less utility of driving your own) or because the actions of others affect your constraints (your neighbor buys a new car and parks it on the parking lot in front of your house). Negative values of β_g can be interpreted as *benefits* of social interaction. For example, when your neighbor buys flowers for his garden which you also like, your neighbor's action has a positive externality that increases your well-being.

Substitution of (4) and (5) in (3) and using the expression for $\tilde{x}_{gn}$, gives consumer n's *reaction functions*, that is, his optimal demands as a function of the consumption of others:

$$\begin{aligned} x_{gn} &= b_{g0} + \delta_g f_n + s_{gn}(x_{g,-n}) \\ &+ \frac{\gamma_g}{p_g}\left[y_n - \sum_{h=1}^{G} p_h b_{h0} - \tilde{\delta} f_n - \sum_{h=1}^{G} p_h s_{hn}(x_{h,-n})\right], \ \forall g, \ \forall n, \end{aligned} \tag{6}$$

where $\tilde{\delta}$ is defined as $\tilde{\delta} \equiv \sum_{g=1}^{G} \delta_g p_g$. I refer to this system as the LES-SI.

Let $x_n \equiv (x_{1n}, \ldots, x_{gn})'$, $x \equiv (x'_1, \ldots, x'_N)'$ and $y \equiv (y_1, \ldots, y_N)'$. An allocation $\hat{x}$ is called a Nash equilibrium if for all $n = 1, \ldots, N$

$$\begin{aligned} U_n(\hat{x}) = \max_{x_n} U_n(\hat{x}_1, \ldots, \hat{x}_{n-1}, x_n, \hat{x}_{n+1}, \ldots, \hat{x}_N) \\ \text{s.t. } \textstyle\sum_{g=1}^{G} p_g x_{gn} = y_n. \end{aligned} \tag{7}$$

For the LES-SI case, the Nash solution can be obtained analytically due to the linearity of the model. Kooreman and Schoonbeek (2004) give this solution and in appendix A, I provide an extension for the case with a demographic translation. Moreover, I allow for the possibility of different households facing different prices.

In order to get a little bit of feel for the LES-SI system, consider the following situation: In a particular society, all households all equal (i.e. $y_n = \bar{y}, f_n = f$, and $x_{gn} = x_g \, \forall n$); they all give the same reference weights to all other households (i.e. $w_{nk} = \frac{1}{N-1}, \forall n, k \neq n$) and all goods are equally conspicuous (i.e. $\beta_g = \beta, \ \forall g$). Under which conditions does the introduction of social interactions not change the allocation of resources of the households? From (6), one observes that social interactions leave allocations unaffected whenever

$$s_{gn}(x_{g,-n}) - \frac{\gamma_g}{p_g} \sum_{h=1}^{G} p_h s_{hn}(x_{h,-n}) = 0, \ \forall g, \ \forall n.$$

In the specific case of homogenous households, these conditions reduce to

$$\beta p_g(x_g - \delta_g f) = \beta[\gamma_g \sum_{h=1}^{G} p_h(x_h - \delta_h f)].$$

The solution $\beta = 0$ corresponds to the situation without social interactions. With social interactions x_g has to satisfy

$$p_g x_g = p_g \delta_g f + \gamma_g(\bar{y} - f \sum_{h=1}^{G} p_h \delta_h), \ \forall g.$$

From the characteristics of the LES it follows that this condition is satisfied if and only if the part of the subsistence expenditures that is independent of family size, b_{g0}, is 0 for all goods g. Intuitively, this can be seen as follows: Whereas the weighted sum in the social cost function (5) sums over family-size adjusted quantities, this function does not discount the quantities b_{g0} that households deem necessary irrespective of family-size and social interactions. For this reason, the introduction of social interactions has the effect of making households aware of the other households' independent subsistence expenditures b_{g0}, thereby leading to a higher demand for goods for which b_{g0} is large. Notice in particular that no restrictions are imposed on (relative) prices.

The social cost function $s_{gn}(x_{g,-n})$ may take on many forms, depending on how and to which extent households are influenced by the consumption decisions of other households. In the next subsection I discuss a number of specifications that differ with respect to the specific form of the reference weights w_{nk}. In subsequent simulations, I compare the effect of these different specifications on the allocational decisions made, using the concept of social welfare that will be introduced in subsection 2.3.

2.2 Social topologies

One can make different assumptions about the reference group of an individual, that is, the set of people by whom an individual is influenced. In fact, two parameters have to be determined: a) who interacts with whom (e.g., households living in the same neighborhood or households with similar incomes)? and b) how strong are these interactions? On basis of these two questions, I introduce three different social topologies: the complete model, the cyclical model and the cliques model.[5]

The complete model

Consider a society consisting of N units (individuals or households). In the complete model, it is assumed that all households give an equal weight to the allocation decisions of all other households in society such that:

$$w_{nk} = 1/(N-1), \qquad n = 1, \ldots, N; \quad k \neq n, k = 1, \ldots, N.$$

The cyclical model

In the cyclical model, a household is influenced by physically neighboring households. The farther households live apart, the less they influence each other. In the application, I assume a circular city, where people live at equidistance from their neighbors. So, the direct neighbors of the household at position 1 are the household at position 2 and the household at position N. The particular weighing scheme I employ, is

$$w_{nk} = w_{kn} = \frac{R - c(n,k,R,N)}{R(R+1)} \quad \text{with}$$

[5] A possible fourth topology could be based on the social reference space model as put forward by e.g. Kapteyn, Van Praag and Van Herwaarden (1978). In the social reference space model, people's reference group is determined on basis of a few clearly described characteristics, like age of the head of household, family size and education level. The weight that people give to other people depends on the proximity of these people as measured by these socioeconomic variables. In Gärtner (1974), the weights w_{nk} are dependent on the differences in consumption between households n and k.

$$c(n,k,R,N) = \left\{\left|N \cdot I(|n-k| > N/2) - |n-k|\right| - 1\right\} \cdot I(|n-k| \leq R),$$

where $I(\cdot)$ denotes an indicator function. In this weighing scheme, R denotes the range of neighbors affecting the preferences of household n and $c(\cdot)$ denotes the number of households between households n and k. To give an example, suppose that $N = 10$ and that household $n = 5$'s preferences are affected by the consumption decisions of the three neighboring households living on the left and right of it. Then, $R = 3$ and the influence of the nearest two neighbors is $\frac{R-c(5,6,3,10)}{R(R+1)} = \frac{R-c(5,4,3,10)}{R(R+1)} = \left[3 - \left(\left| - |5-4|\right| - 1\right)\right] / (3 \cdot 4) = 3/12$, of the one but nearest neighbors $2/12$ and of the other two neighbors $1/12$. Note that $\sum_{k=1}^{N} w_{nk} = 1$ and that for all n and k the w_{nk}'s approach $1/N$ when $R \to N/2$ and $N \to \infty$. Thus, given $R = N/2$, the cyclical and complete model are identical if $N \to \infty$.

There are a lot of different weighing schemes one can think of to express neighborhood effects. For our purposes, the straightforward linear weighing scheme above suffices.

The cliques model

In this model, society is segregated in T non-overlapping subsets of households, $A_1, A_2, \ldots, A_T$, with $T < N$, and $N_t = |A_t|$ denotes the number of households in group A_t.[6] Household's preferences are influenced equally by all other households within the same subset, but are not at all affected by households in other subsets. In other words, within each subset a weighing scheme as in the complete model applies. In our imaginary city, one can think of leisure activities like sporting or playing an instrument in an orchestra having this effect.

The weight matrix is block-diagonal and symmetric with individual weights defined as

$$w_{nk} = \begin{cases} 1/(N_t - 1) & \text{if } (k \in A_t) \wedge (n \in A_t \backslash k) \text{ for some } t \in \{1, 2, \ldots, T\}; \\ 0 & \text{otherwise.} \end{cases}$$

In the simulations with the cliques model, four subsets are defined, based on income and household size. Again, $\sum_{k=1}^{N} w_{nk} = 1, \ \forall n$.

[6] Note that when the number of subsets is inflated to N, with each subset consisting of one household only, the cliques model reduces to the ordinary LES without social interactions.

2.3 Social welfare

A yardstick is needed that enables us to make a sensible comparison between the different social topologies. This yardstick is provided by the concept of the utilitarian social welfare function. This function gives a value to each allocation x by weighing the utility an individual household n derives from this allocation by a factor a_n, $\forall n$, and add them up:

$$\sum_{n=1}^{N} a_n U_n(x_1, \ldots, x_N). \tag{8}$$

Note that social welfare as defined here is a cardinal concept and for this reason sensitive to monotonically increasing transformations. A social planner may derive a Pareto efficient allocation by maximizing (8) with respect to x_{gn}, $\forall g$, $\forall n$, subject to the individual budget constraints

$$\sum_{g=1}^{G} p_g x_{gn} = y_n, \quad \forall n, \tag{9}$$

for any choice of weights a_n satisfying $a_n > 0$, $\forall n$. Following Kooreman and Schoonbeek (2004), I will further take $a_n = 1$, $\forall n$, saying that every household is considered equally important. This seems a reasonable objective, though one may imagine other yardsticks for social welfare, for example one based on the household in society that is worst off in terms of welfare. Assuming that there is a unique solution to the social welfare problem, I denote this general solution by $x^{Pareto}(p, y) = (x_1^{Pareto}(p, y)', \ldots, x_N^{Pareto}(p, y)')'$. $W^*(p, y)$ denotes the optimal value of the social welfare function, i.e.

$$W^*(p, y) = \sum_{n=1}^{N} U_n(x^{Pareto}(p, y)).$$

Inspection of the first order conditions of the problem reveals that no closed form solution of $x^{Pareto}(p, y)$ can be obtained for the case with general reference weight w_{nk}.

In principle, two comparisons can be made with this yardstick. On the one hand, one can compare social welfare obtained in a society of non-cooperative agents where social interactions are absent with the level of social welfare obtained in a society where they do play a role. This comparison measures the welfare loss from interdependent preferences and is the subject of subsection 4.1. On the other hand, one can look at a society with a certain amount of interdependence and compare the social welfare corresponding to the Nash allocation reached by non-cooperative agents with the level that corresponds to the Pareto efficient solution. This point of view is taken by Kooreman and Schoonbeek. If this difference is positive, there is a case for an intervening government levying taxes and giving subsidies on goods in order to internalize the externalities from interdependent preferences and leading the non-cooperative households to the Pareto efficient allocation. This is the kind of comparison I make in subsection 4.3.

3 Design of SIcity

In this section, the design of the fictitious city called SIcity is explained. Successively, the population characteristics, the choice of the number of goods and the choice of parameter values of the LES-SI system is discussed. The choice of the parameter values is based on estimates found in the empirical studies by Alessie and Kapteyn (1991) and Kapteyn *et al.* (1997).

3.1 Population

Incomes in SIcity are assumed to be lognormally distributed with $\mu = 10$ and $\sigma^2 = 0.2$. The mean income is then $\exp(\mu + \sigma^2/2)$ =€24,343 and the standard deviation €11,454. For comparison: the expenditures of the 10,076 household entries in the budget surveys that were conducted by Statistics Netherlands between 1992-96 averaged €21,315 and had a standard deviation of €10,164.

The size of the households in SIcity follows a bimodal distribution with the probabilities as given in table 1. The values are in accordance with numbers from the budget surveys over the period 1992-96. I make the simplifying assumption of no correlation between household income and family size. A sample of 200 households is drawn from the population.[7] The income and size of each household is determined by a random draw from the income and household size distributions as specified above. In the cliques model, four subsets of households

[7]This modest size is chosen for mere computational reasons. Simulated cities with a more realistic number of inhabitants can be obtained by inflating the population in a straightforward manner. The results are similar to the ones reported.

are specified, depending on whether the household has more than two members and whether the households earns more than €23,000. All simulations reported in this paper work with the same sample of 200 households.

Table 1: Household size distribution

Size household	1	2	3	4	5	6	7	8
Probability	.17	.31	.15	.23	.10	.02	.01	.01

3.2 Goods

The goods that are consumed by the households of SIcity are categorized into seven different classes, which are labeled, 'food', 'housing', 'clothing', 'medical care', 'education and entertainment', 'transportation' and the remainder category 'other expenditures'. The reason for choosing these particular classes is that these are also the categories specified in the aforementioned papers of Alessie and Kapteyn (1991) and Kapteyn *et al.* (1997). This has the advantage that the input parameters in the current simulation study can be based on their empirical estimates. I assume that the prices of all goods equal one. Since the analysis is static, this does not involve a loss of generality because all goods can be redefined in units with price one.

3.3 Parameter choices for the LES-SI system

For the realism of the simulation results, it is important to choose reasonable input parameters. My particular choice of δ_g and γ_g ($g = 1, \ldots, G$) coincide with the estimates reported in Kapteyn *et al.* of the model without interdependence. One may object that Kapteyn *et al.* reject this model in favor of the model with interdependence. For our results however, that does not seem to make much of a difference. The estimates of the γ's are similar for both cases and the δ's only lead to a translation of the constant terms b_{g0}, whose values cannot be identified by Kapteyn *et al.*[8] The parameter values chosen are listed in table 2.

[8] The problem with using the estimates that Kapteyn *et al.* report for the model with interdependence, is that it is – by the different social topologies that are employed – unclear which of the models considered in this paper is comparable with the model with interdependencies that is estimated in Kapteyn *et al.* For this reason, I chose to make the model without social interactions comparable to theirs.

Table 2: Parameter values.

	b_0	δ_g	γ_g	β_g
food	2900	0.729	0.126	-0.060
housing	1500	-0.317	0.327	0.030
clothing	450	0.177	0.080	0.180
medical care	1150	0.148	0.099	0.120
education + entertainment	250	-0.194	0.171	0.020
transportation	250	-0.583	0.177	0.080
other expenditures	225	0.040	0.020	0.000

The subsistence expenditures b_{0g}, $\forall g$, are chosen in such a way that for each good, the average budget share in the model without social interactions is comparable with the average budget share for the same category of goods reported by Alessie and Kapteyn. See table 3. (Alessie and Kapteyn do not list transportation expenditures as a separate category.) Moreover, when social interactions are absent, the own-price (ϵ_{ii}) and income (ϵ_g) elasticities, evaluated at the population mean of income and household size, are realistic: food and medical care for example are necessary goods.[9] This is in accordance with the finding by Alessie and Kapteyn. They find that housing is also a necessary good. In my case, housing has an income elasticity somewhat larger than one. With regard to the parameters of conspicuousness β_g, I note that clothing and medical care are chosen to be most, and food to be least conspicuous. In general, the order of the conspicuousness of goods as imposed by picking values β_g is comparable with the order found by Alessie and Kapteyn. The conspicuousness of medical care, which Alessie and Kapteyn consider partly an artefact, is relatively less in the current model and the same holds for the conspicuousness of education and entertainment.

[9] Formulas for the calculation of these elasticities can be found in the appendix. The income and price elasticities are in accordance with the conditions $\sum_g \bar{z}_g \epsilon_g = 1$ and $\sum_g \bar{z}_g \epsilon_{gi} + \bar{z}_g = 1$, $\forall i$, respectively, with $\bar{z}_g = p_g(\overline{x_{gn}/y_n})$, $\forall g$. (See for example Deaton and Muellbauer (1980) for a statement of these conditions.)

Table 3: Budget shares and income and own-price elasticities, evaluated at the mean point

	budget shares				elasticities	
				AK (1991):		
	min	max	mean	mean	ϵ_g	ϵ_{ii}
food	0.156	0.438	0.234	*0.231*	0.592	-0.497
				(0.070)		
housing	0.221	0.317	0.290	*0.325*	1.099	-0.857
				(0.100)		
clothing	0.067	0.079	0.075	*0.083*	1.048	-0.771
				(0.045)		
medical care	0.106	0.173	0.124	*0.140*	0.829	-0.634
				(0.046)		
education + entertainment	0.034	0.158	0.124	*0.128*	1.286	-0.934
				(0.071)		
transportation	0.034	0.163	0.128	–	1.290	-0.937
other expenditures	0.021	0.034	0.025	–	0.839	-0.609

Parameter values as given in table 2.
Fifth column: budget shares as reported by Alessie and Kapteyn (1991); Standard errors in parentheses.

4 Simulations

4.1 The effect of social structure on social welfare

In this section I will assess the effect an increase in the degree of social interactions has on social welfare for the complete model, the cliques model and the cyclical model (the latter with $R = 10$). This increase is generated by multiplying the parameters β_g in equation (5) with a factor ν, $\forall g$. The value $\nu = 0$ corresponds to the case without social interactions. I restrict $\nu\beta_g$ to the interval $(-1, +1)$, $\forall g$. This assumption is needed in the derivation of the Nash-equilibrium (see equation (10)). In practice, I take more restricted values of ν, since — because of the limited domain problem — quantities x_{gn} become negative for some combinations of g and n when ν is too large (values larger than ≈ 1.7 for the complete and cyclical model and larger than ≈ 2.3 for the cliques model).

The value of the social welfare function is plotted against the magnitude ν of the social interactions in figure 1. This figure shows that the loss of welfare is, for all considered levels of interdependence, somewhat less for the cliques model (0.96% when $\nu = 1.7$) than for for the complete and the cyclical model (about 1.1%). The reason for this is that in the cliques model, the peer groups are non-overlapping, such that each household's utility is only influenced by a subset of the other households in society. Indeed, if there are as many peer groups as households, there is no welfare loss irrespective the value of ν, since in that case, the model is effectively equal to the model without interdependency. Further note that the loss in welfare for different levels of social interactions is about equal for the cyclical model and the complete model. This comes as no surprise since the models are similar for large values of R and N.[10]

An interesting question in this respect is to what extent this result is due to the specific choice of $R = 10$. In figure 2, the value of the social welfare function in the cyclical model is plotted against R, with the magnitude of social interactions held constant at $\nu = 1$. For increasing values of R, the level of social

[10] Ioannides (2002) shows that for the case with identical consumers and $R = 1$ for the cyclical model, aggregate consumption is the same for the complete and cyclical model when the number of households is large. It is easy to show that this result also holds for the case with heterogenous consumers and $R > 1$, with weights for the cyclical model as specified in subsection 2.2.

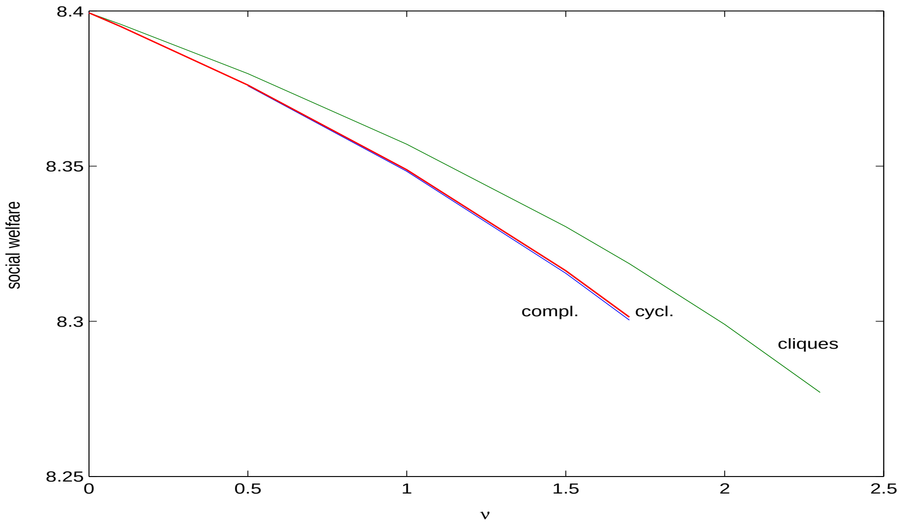

Figure 1: Loss in social welfare for the different social topologies; $R = 10$ for the cyclical model.

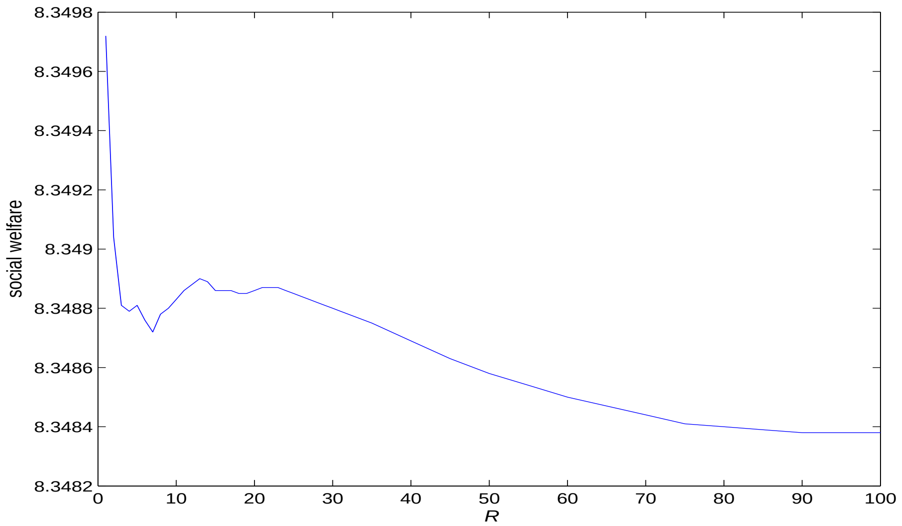

Figure 2: The effect of knowing your neighbors when preferences are interdependent: the cyclical model with $\nu = 1$.

welfare drops and approaches the level obtained under the complete model. The most interesting observation when looking at the figure is that the level of welfare is relatively most sensitive to adding the first four neighboring households (on both sides of the household) to the peer group but that further extension of the peer group of the household leads to a small recovery in social welfare. Presumably, the first links are most important to make the cyclical model similar to the complete model.

In figure 3, the development of the average budget shares under increasing social interactions is plotted for the complete model (plots for the cliques and cyclical model show similar patterns and are not reported here). As social interactions increase, strictly increasing budget shares are observed for clothing (38.5% on average for ν increasing from zero to 1.7) and medical care (20.1%) and decreasing budget shares for food (-14.7%), education and entertainment (-8.9%) and housing (-4.5%). A summary is given in table 4.

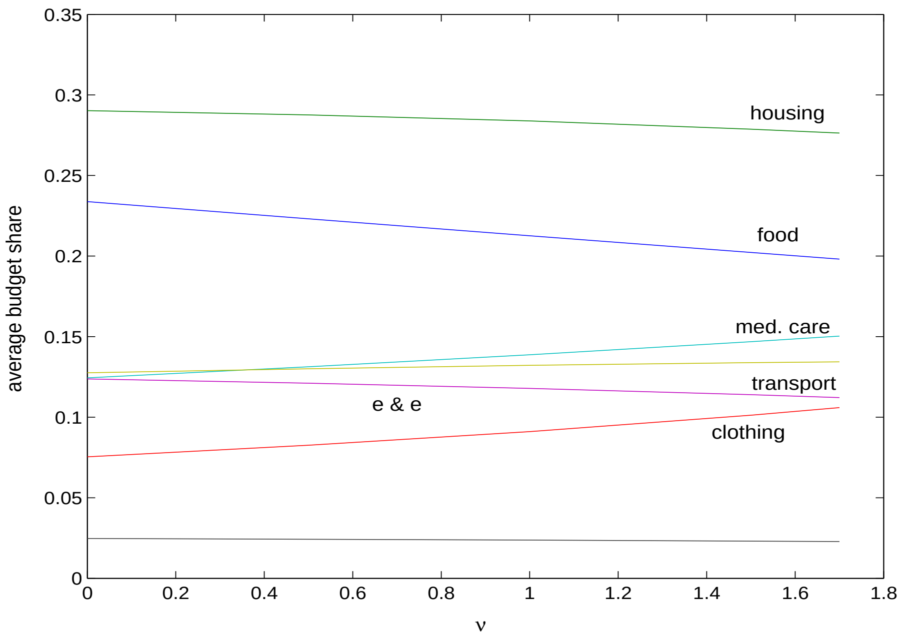

Figure 3: Change in average budget shares: complete model.

Whereas figure 1 shows only a small decrease in social welfare due to social interactions, table 4 points to the fact that social interactions lead to a considerable reallocation of resources. As expected, there is a clear correspon-

Table 4: Change in budget share caused by social interactions (ν=1.7, $N = 200$).

	% change in budget share		
	complete	cliques	cyclical
food	-15.3	-13.7	-15.2
housing	-4.8	-4.0	-4.7
clothing	40.5	35.1	40.0
medical care	20.8	19.0	20.7
education + entertainment	-9.3	-8.0	-9.2
transportation	5.3	4.1	5.3
other expenditures	-7.9	-6.7	-7.8

Note: R=10 for the cyclical model.

dence between the relative change in budget share and the conspicuousness of a good, as represented by the parameter β_g. When looking at the change in budget shares, a difference is observed between the complete and cyclical model on the one hand and the cliques model on the other. For the former two, the increase in the consumption of conspicuous goods like clothing, medical care and transportation is the largest.

From figure 3 one might conclude that there is a linear relationship between the budget shares of the goods and the value of ν. This however, is not the case. Intuitively, this can be seen by looking at the Nash equilibrium solution of (7) for the complete model with identical consumers and no demographic translation (that is, $y_n = \bar{y}$, $\forall n$ and $\delta_g = 0$, $\forall g$, respectively) and assuming unit prices for all goods:[11]

$$x_g^{Nash} = \frac{b_{g0}}{1-\nu\beta_g} + \frac{\gamma_g \left(\bar{y} - \sum_{h=1}^{G} \frac{b_{h0}}{1-\nu\beta_h}\right)}{(1-\nu\beta_g)\sum_{h=1}^{G} \gamma_h/(1-\nu\beta_h)}. \tag{10}$$

Firstly, one observes that the denominator in the second term on the right hand side is smallest for the most conspicuous good, that is, the good with the largest value of β_g. Moreover, the difference with the denominator of the other goods increases with ν, such that for ν large enough, the largest share of the supernumerary income $\left(\bar{y} - \sum_{h=1}^{G} \frac{b_{h0}}{1-\nu\beta_h}\right)$ is spent on the most conspicuous good. Secondly, notice that $\sum_{h=1}^{G} b_{h0}/(1-\nu\beta_h)$ increases with ν. In other words, supernumerary income itself becomes smaller with the degree of interdependency as households deem more and more expenditures necessary to function. As a

[11] This equation is a special case of equation (A.4) in the appendix.

Table 5: Average budget share and social welfare when one household receives an income shock of €100,000 ($N = 200$); **w** = winners **nw** = non-winners.

	$\nu = 0$		$\nu = 1$					
			complete		cliques		cyclical	
	w	**nw**	**w**	**nw**	**w**	**nw**	**w**	**nw**
food	14.3	23.4	13.9	19.5	14.0	19.5	13.9	19.5
housing	32.1	29.0	32.0	29.2	32.0	29.2	32.0	29.2
clothing	7.9	7.5	8.2	8.9	8.2	8.9	8.2	8.9
med. care	10.3	12.4	10.5	13.1	10.5	13.1	10.5	13.1
ed. + ent.	16.4	12.4	16.3	12.8	16.3	12.8	16.3	12.8
transp.	16.9	12.8	17.0	14.1	17.0	14.1	17.0	14.1
other exp.	2.1	2.5	2.1	2.3	2.1	2.3	2.1	2.3
				social welfare				
inc. shock	**10.025**	**8.399**	**10.017**	**8.347**	**10.018**	**8.356**	**10.017**	**8.347**
no inc. sh.	—	**8.399**	—	**8.348**	—	**8.357**	—	**8.349**

consequence, a "most conspicuous good takes all" pattern is observed for increasing values of ν only when supernumerary income is large enough. In this case, the second term in (10) dominates over the first term. However, when supernumerary is small due to large values of b_{g0}, the first term dominates and the good with the largest value of b_{g0} will still have the highest budget share for high values of ν.

4.2 The social interaction effect of income shocks experienced by some of the households

What are the implications for the other households when one of the households in society receives an income shock of €100,000? To answer this question, I run 200 simulations in each of which, one of the households receives an income shock of €100,000 (starting with household 1 and ending with household 200). I refer to a household that experiences an income shock as a "winner" and to the household that do not experience an income shock as "non-winners". Average changes in both budget shares and social welfare are reported for winners and non-winners and for all topologies in table 5, with the magnitude of interaction fixed at $\nu = 1$.

First look at the complete model. For non-winners, the budget shares of conspicuous goods are larger when social interactions are present (for example, the budget share of clothing increases from 7.5 to 8.9 per cent). For winners, the increase in the budget share of conspicuous goods occurs is less pronounced – e.g. 7.9 to 8.2 per cent for clothing. As compared to non-winners, the smaller change in the budget share of food, clothing and medical care for winners can be explained by the relatively low income elasticity of these goods. For the three models, the changes in budget shares are similar. In the cliques model with its non-overlapping peer groups, the decrease in welfare of non-winners is somewhat smaller.

4.3 Imposing taxes and giving subsidies

In subsection 2.3, it was shown that $W^*(p, y)$ is the maximum level of social welfare that can be obtained by maximizing (8) subject to (9), given prices p and incomes y. In this section, I follow (and borrow from) Kooreman and Schoonbeek in asking the question whether the loss in welfare resulting from non-cooperative Nash behavior in the presence of interdependent preferences can be completely eliminated by means of taxes and subsidies.

I denote the price vector consumers face when taxes and subsidies are imposed by $q = (q_1, \ldots, q_G)'$. So, good g is taxed when $q_g > p_g$ and subsidized when $q_g < p_g$. The general problem of optimal taxation is formulated by Kooreman and Schoonbeek (2000) as:

$$\max_q V(q, p, y) = \max_q \sum_{n=1}^{N} a_n U_n(\hat{x}_1(q, y), \ldots, \hat{x}_N(q, y)) \tag{11}$$

s.t.

$$\sum_{g=1}^{G} (q_g - p_g)\hat{x}_{gn}(q, y) = 0, \quad \forall n. \tag{12}$$

In this equation $\hat{x}_n(q, y)$ denotes the Nash equilibrium consumption bundle of household n, given the price vector q and total outlay y. Let $V^*(p, y)$ denote the optimal value of the objective function (11).

Under some mild assumptions on the functional form of the utility functions and under the assumption that all consumers receive the same consumption bundle $x^{Pareto}(p, y)$ in the Pareto optimal allocation and that $a_n = 1, \forall n$, the authors prove that there exists a unique solution $q^* = (q_1^*, \ldots, q_G^*)' > 0$ such that $\hat{x}(q^*, y) = x^{Pareto}(p, y)$. As a result $V^*(p, y) = V(q^*, p, y) = W^*(p, y)$.

The approach here differs in two respects. First, we consider a population of *heterogeneous* consumers and second, we do not require taxes and subsidies to satisfy individual budget neutrality, but (the less restrictive) collective budget neutrality. This amounts to replacing (12) with

$$\sum_{n=1}^{N}\sum_{g=1}^{G}(q_g - p_g)\hat{x}_{gn}(q, y) = 0. \tag{13}$$

Collective budget neutrality is a realistic constraint when a government, upon imposing new taxes and subsidies, is more concerned with repercussions on its own spendings than on the reallocation of income that is caused by these measures. Note however, that due to consumer heterogeneity, one cannot apply the result in Kooreman and Schoonbeek (2004) to assert that the loss of social welfare can be completely eliminated.[12]

With heterogeneous consumers and (13), the difference between the maximum value $V^*(p, y)$ of (11) and (13) and $W^*(p, y)$ of (8) is the sum of a reallocation effect of income across households and a remainder term. This difference reflects the extent to which the loss of social welfare caused by non-cooperative behavior can be eliminated by imposition of taxes and subsidies respectively. In other words, it reflects the usefulness of the tax and subsidy instrument for a government. However, the effect of reallocation may also be obtained when preferences are independent ($\nu = 0$). For this reason, I will compare the difference between $V^*(p, y)$ and $W^*(p, y)$ with the increase in social welfare that can be obtained in a society *without* social interactions to assess the extra increase that can be obtained by the presence of social interactions.

For computational reasons, I subdivide the selection of 200 households into ten small societies of 20 households each.[13] A corresponding reduction for the value of R is made in the cyclical model to $R = 3$. In figure 4, the average change in social welfare for different values of ν is shown for the complete and cliques model.[14] The lines without taxes depict the Pareto optimal solution

[12] In the corresponding working paper Kooreman and Schoonbeek (2000), it is shown that it is possible to completely eliminate the loss of welfare if one is able to impose consumer specific taxes and subsidies.

[13] Otherwise the matrix of reference weights becomes large, which makes the optimization procedure for finding the optimal prices q very slow, since equation (A.4) has to be evaluated many times.

[14] The pattern for the cyclical model is similar to that of the complete model and for this reason not shown.

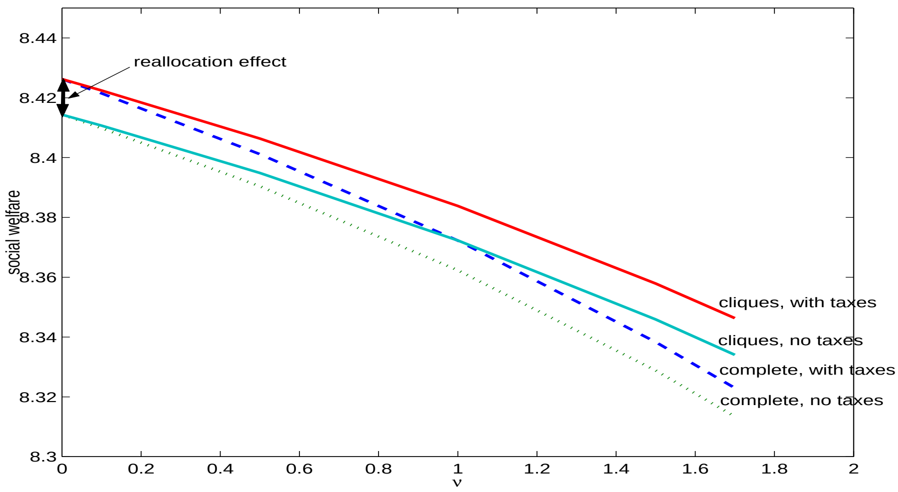

Figure 4: Elimination of welfare loss by imposing taxes and subsidies for the complete model and the cliques model. (Average over ten cities with $N = 20$.)

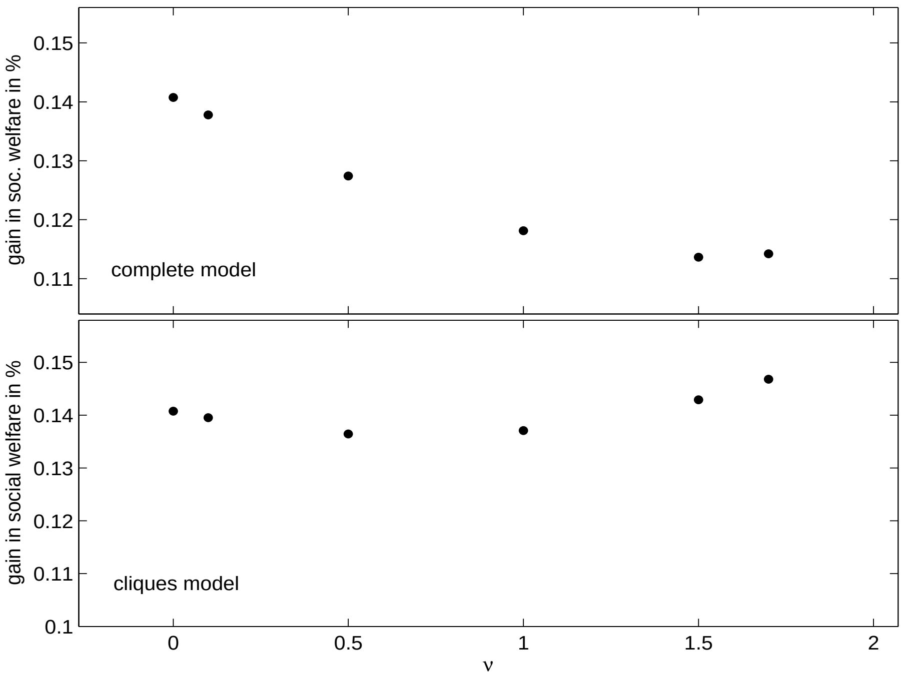

Figure 5: Percentage gain in social welfare obtainable by imposition of taxes and giving subsidies for different degrees of interaction ν. (Average over ten cities with $N = 20$.)

under unit prices and the lines 'with taxes' show the solution to (11) and (13). The corresponding optimal prices q^* are shown in table 6. Figure 5 shows that the gain in social welfare is somewhat larger for the cliques model than for the complete model (about 0.14% against 0.12% on average).

However, figure 4 also shows that a gain in welfare can also be obtained when social interactions are absent. In other words, the elimination of the loss in welfare is due to an income reallocation effect, and is not caused by the presence of interdependent preferences. For the complete model, the gain in social welfare even seems to decrease as the degree of interaction becomes larger (see figure 5). With regard to the optimal prices q^*, notice that as social interactions increase, the most conspicuous goods are taxed most heavily and the less conspicuousness goods receive a subsidy, conform expectations, see table 6. For medical care, one sees that the optimal prices first increase with ν and then decrease. This is due to the fact that for the parameter values given in table 2, the "most conspicuous good takes all" pattern holds for large values of ν: initially, the consumption of the conspicuous good ($\beta_4 = 0.12$) has to be discouraged by increasing its price, but for ν large enough, the consumption of medical care decreases since households want to free money to buy more clothing, the most conspicuous good ($\beta_3 = 0.18$). For the same reason, the taxes on clothing keep rising with ν.

5 Summary and Conclusions

The effects of preference interdependencies on the allocation of resources and social welfare were analyzed in the context of the Linear Expenditure System with Social Interactions (LES-SI). The main findings are that, due to social interactions, a considerable reallocation of resources over goods may occur. In the example (with $\nu = 1.7$), the budget share of a conspicuous good like clothing increases on average with 38%. For high levels of interactions, all resources are spent on the most conspicuous good when the initial budget share of the other goods is low enough.

Comparing three different social structures, I find that the complete and cyclical model lead to similar changes in social welfare and budget shares. Social interactions have the smallest effects in the cliques model, where the reference

Table 6: Changes in social welfare maximizing prices when the level of social interaction increases.

ν	0.000	0.100	0.500	1.000	1.500	1.700
			complete model			
food	1.177	1.166	1.117	1.051	0.980	0.950
housing	1.020	1.020	1.018	1.013	1.006	1.002
clothing	1.088	1.105	1.176	1.262	1.341	1.367
medical care	1.258	1.260	1.262	1.255	1.232	1.218
education + entertainment	0.775	0.775	0.776	0.782	0.796	0.806
transportation	0.770	0.776	0.799	0.834	0.879	0.900
other expenditures	1.257	1.248	1.212	1.161	1.104	1.080
social welfare	**8.426**	**8.421**	**8.401**	**8.372**	**8.338**	**8.323**
			cliques model			
food	1.177	1.169	1.136	1.089	1.037	1.015
housing	1.020	1.019	1.013	1.004	0.993	0.988
clothing	1.088	1.105	1.173	1.264	1.355	1.391
medical care	1.258	1.262	1.275	1.285	1.286	1.283
education + entertainment	0.775	0.774	0.768	0.762	0.759	0.759
transportation	0.770	0.774	0.790	0.812	0.838	0.850
other expenditures	1.257	1.250	1.224	1.186	1.143	1.125
social welfare	**8.426**	**8.422**	**8.406**	**8.384**	**8.358**	**8.346**
			cyclical model ($R = 3$)			
food	1.177	1.166	1.117	1.050	0.978	0.948
housing	1.020	1.020	1.018	1.013	1.006	1.002
clothing	1.088	1.105	1.176	1.261	1.339	1.365
medical care	1.258	1.260	1.262	1.254	1.230	1.216
education + entertainment	0.775	0.775	0.777	0.783	0.798	0.808
transportation	0.770	0.776	0.799	0.835	0.880	0.903
other expenditures	1.257	1.248	1.211	1.160	1.103	1.078
social welfare	**8.426**	**8.421**	**8.401**	**8.372**	**8.338**	**8.323**

Note: the average is taken over ten societies of 20 households each.

groups are non-overlapping. With regard to the cyclical model, I find that adding the first neighboring households to a household's reference group leads to a relatively large change in social welfare but further increases do not have much effect. The effect on social welfare when one of the households receives an income shock of €100,000 is small and similar for all three models.

The last part of the study deals with the question how government can enhance social welfare by imposing taxes on some goods and giving subsidies to others. Contrary to Kooreman and Schoonbeek (2004), I consider a society with heterogenous consumers and collective budget neutrality. A gain in social welfare can be obtained by setting optimal prices. However, since a similar gain is obtainable when social interactions are absent, I conclude that the tax and subsidy instrument does not become more effective when social interactions play a role in the allocation of resources. Finally, I argue that – in the context of the LES-SI – the budget share of the most conspicuous good not always increases with the degree of interactions. As a consequence, the optimal tax for the most conspicuous does not always increase with the degree of social interactions.

A possible future extension of this study is to include more refined tax-schedules, for example by making a distinction between single member and multiple member households. The equations in the appendix allow for such household specific taxes. Other issues are the implementation of a sensitivity analysis for the choice of parameter values; investigation of the effect of other kinds of income shocks and the consideration of other measures of social welfare.

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6 Appendix: The LES with Social Interactions – Derivations

For convenience, I repeat here (6) which gives the system of equations that must be satisfied by the Nash equilibrium:

$$\begin{aligned} x_{gn} &= b_{g0} + \delta_g f_n + s_{gn}(x_{g,-n}) \\ &+ \frac{\gamma_g}{p_{gn}} \left[y_n - \sum_{h=1}^{G} p_{hn} b_{h0} - \tilde{\delta}_n f_n - \sum_{h=1}^{G} p_{hn} s_{hn}(x_{h,-n}) \right], \ \forall g,\ \forall n, \end{aligned} \quad \text{(A.1)}$$

where $\tilde{\delta}_n$ is defined as $\tilde{\delta}_n \equiv \sum_{g=1}^{G} \delta_g p_{gn}$. Note that I added an index 'n' to the prices to allow for household specific prices.

In order to derive a more concise expression for the quantities in the Nash equilibrium, I introduce some additional notation:

$$\begin{array}{ll} x^g \equiv (x_{g1}, \ldots, x_{gN})', \ \forall g; & X \equiv (x^1, \ldots, x^G)' \\ b_0 \equiv (b_{10}, \ldots, b_{G0})'; & f \equiv (f_1, \ldots, f_N)' \\ y \equiv (y_1, \ldots, y_N)'; & \delta \equiv (\delta_1, \ldots, \delta_G)' \\ D \equiv diag(\delta_1, \ldots, \delta_G); & P_g \equiv diag(p_{g1}, \ldots, p_{gN}), \ \forall g \\ B \equiv diag(\beta_1, \ldots, \beta_G); & \Gamma \equiv diag(\gamma_1, \ldots, \gamma_G) \\ M \equiv diag(\delta' p_1, \delta' p_2, \ldots, \delta' p_N), & \text{with } p_j = (p_{1j}, p_{2j}, \ldots, p_{Gj})'. \end{array}$$

$$W \equiv \begin{pmatrix} w_{11} & \ldots & w_{1N} \\ \vdots & & \vdots \\ w_{N1} & \ldots & w_{NN} \end{pmatrix}; \quad P^* \equiv \begin{pmatrix} p_{11} & \ldots & p_{G1} \\ \vdots & & \vdots \\ p_{1N} & \ldots & p_{GN} \end{pmatrix}$$

$$P^{\Delta^+} \equiv \begin{pmatrix} P_1 & P_2 & \ldots & P_G \end{pmatrix}'; \quad P^{\Delta^-} \equiv \begin{pmatrix} P_1^{-1} \\ P_2^{-1} \\ \vdots \\ P_G^{-1} \end{pmatrix}$$

Further, let ι_N denote an $N \times 1$ vector of ones, and I_G, I_N, and I_{GN} denote identity matrices of dimensions $G \times G$, $N \times N$ and $GN \times GN$, respectively. vec$X$ denotes the $GN \times 1$ vector obtained by stacking the columns of matrix X one underneath the other.

Using this notation, one can rewrite (A.1) as

$$[I_N - \beta_g W + \gamma_g \beta_g W] x^g + \gamma_g P_g^{-1} \sum_{\substack{h=1 \\ h \neq g}}^{G} \beta_h P_h W x^h = \delta_g (I_N - \beta_g W) f + b_{g0} \iota_N$$

$$+ \gamma_g P_g^{-1} y - \gamma_g P_g^{-1} P^* b_0 + \gamma_g P_g^{-1} \left[\sum_{h=1}^{G} \beta_h \delta_h P_h W f - M f \right], \ \forall g. \quad \text{(A.2)}$$

or, using the Kronecker product, as

$$[I_{GN} - B \otimes W + (\Gamma \otimes I_N)P^{\Delta^-}\{(B \otimes I_N)P^{\Delta^+}\}'(I_G \otimes W)]\text{vec}X = \delta \otimes f$$

$$-(DB \otimes W)(\iota_G \otimes f) + b_0 \otimes \iota_N + (\Gamma \otimes I_N)P^{\Delta^-}y - (\Gamma \otimes I_N)P^{\Delta^-}P^* b_0$$

$$+\{(\Gamma \otimes I_N)P^{\Delta -}\}\left[\{(BD \otimes I_N)P^{\Delta +}\}'(\iota_G \otimes Wf) - Mf\right]. \tag{A.3}$$

Kapteyn *et al.* (1997, Lemma 2) prove that in case all prices are equal to unity, the $GN \times GN$ matrix between brackets on the left hand side of (A.3) is non-singular. One can verify that their proof easily extends to the more general case. Consequently, the explicit expression of the Nash equilibrium is given by

$$\begin{aligned}
\text{vec}X &= [I_{GN} - B \otimes W + (\Gamma \otimes I_N)P^{\Delta^-}\{(B \otimes I_N)P^{\Delta^+}\}'(I_G \otimes W)]^{-1} \\
&\quad \Big[\delta \otimes f - (DB \otimes W)(\iota_G \otimes f) \\
&+ \quad b_0 \otimes \iota_N + (\Gamma \otimes I_N)P^{\Delta^-}y - (\Gamma \otimes I_N)P^{\Delta^-}P^* b_0 \\
&+ \quad \{(\Gamma \otimes I_N)P^{\Delta -}\}\Big[\{(BD \otimes I_N)P^{\Delta +}\}'(\iota_G \otimes Wf) - Mf\Big]\Big].
\end{aligned} \tag{A.4}$$

The vector $\hat{x} = \hat{x}(p, y)$ which is used to denote the Nash equilibrium follows directly from (A.4).[15]

[15] The expression equivalent to (A.4) for the case when all households face the same prices is

$$\begin{aligned}
\text{vec}X &= [I_{GN} - B \otimes W + P^{-1}\gamma\beta' P \otimes W]^{-1} \\
&\quad \delta \otimes f - B\delta \otimes Wf + b_0 \otimes \iota_N + P^{-1}\gamma \otimes y \\
&- \quad (b_0' p)P^{-1}\gamma \otimes \iota_N + (\beta' P\delta)P^{-1}\gamma \otimes Wf - (\delta' p)P^{-1} \otimes f \quad ,
\end{aligned}$$

with $\beta \equiv (\beta_1, \ldots, \beta_G)'$; $\gamma \equiv (\gamma_1, \ldots, \gamma_G)'$, and $P \equiv diag(p_1, \ldots, p_G)$.