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CHAPTER 8

PATIENT-TAILORED PLATE FOR BONE FIXATION AND ACCURATE 3D POSITIONING IN CORRECTIVE OSTEOTOMY

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ABSTRACT

A bone fracture may lead to malunion of bone segments, which gives discomfort to the patient and may lead to chronic pain, reduced function and finally to early osteoarthritis. A treatment option to realign the bone segments is a corrective osteotomy. In this procedure the surgeon tries to improve alignment by cutting the bone at, or near, the fracture location and fixes the bone segments in an improved position, using a plate and screws. Three-dimensional positioning is very complex and difficult to plan, perform and evaluate using standard 2D fluoroscopy imaging. This paper introduces a new technique that uses preoperative 3D imaging to plan positioning and to design a patient-tailored fixation plate that only fits in one way and realigns the bone segments as planned. The method is evaluated using artificial bones and renders realignment highly accurate and very reproducible ($d_{err} < 1.2 \pm 0.8$ mm and $\varphi_{err} < 1.8 \pm 2.1^\circ$). Application of a patient-tailored plate is expected to be of great value for future corrective osteotomy surgery.
INTRODUCTION

The annual occurrence of limb fractures is about 2.4% of the population [17]. These cases are mostly treated by non-operative alignment of the bone segments, followed by plaster application. In approximately 5% of the cases bone union is suboptimal [4]. If left untreated such malunions may lead to chronic pain, reduced function of the limb, and early osteoarthritis. An established surgical treatment option for dysfunction caused by malalignment is a corrective osteotomy [11],[12].

During corrective osteotomy, the bone is cut at, or near, the fracture site and the bone segments are repositioned to a more anatomical position, sometimes supported by a wedge-shaped bone graft. The positioning is followed by fixation, usually with a plate and screws, to keep the bone segments in place. During surgery, the surgeon therefore faces two major challenges: 1) repositioning of one bone segment with respect to another bone segment in three-dimensional (3D) space; a procedure which needs to be accomplished against large counter-acting tensile forces of surrounding tissue, and 2) fixation of the bone segments to maintain the achieved pose. Fixation itself should in no way alter the newly acquired position.

Standard corrective osteotomy of the distal radius is most-often planned using two orthogonal radiographs to find correction parameters for restoring the radial inclination, palmar tilt and ulnar variance, to normal [11]. However, 2D imaging techniques hide rotations about the bone axis [28] and may therefore cause a misinterpretation of the correction parameters [10]. With 3D imaging techniques, bone positioning can be planned and evaluated in all six degrees of freedom (DOF) (three displacements; three rotations), instead of using only three DOF as in standard surgery.

In conventional 2D based [11],[12] and recent 3D guided treatment [1],[4],10,19,21,22,29,34], several bone fixation techniques are used, such as external fixation using a Taylor Spatial Frame (TSF) [31] or internal plate fixation. TSFs are bulky, expensive, and adjustment of their six struts is error prone. Conventional L- or T-shaped internal fixation plates require bending of the plate in order to tightly fit on the bone surface. The final bone position is largely dependent on the skill of the surgeon in shaping and positioning the standard plate and also on his judgment of radiographic positioning parameters from intraoperative fluoroscopy images, when 2D evaluation is used. This renders bone positioning using a standard plate and screws, difficult and highly subjective [16].

Three-dimensional planning methods have been reported for preoperatively creating a patient-specific positioning wedge, using additive manufacturing techniques. During the procedure, these wedges are usually exchanged with a bone graft after plate fixation [1] although recently hydroxyapatite wedges were introduced [21] that serve as a positioning implant. However, Bilić et al. [2], have shown that an avascular wedge may (partially) be
resorbed during the healing phase, which may render the long-term outcome of such an osteotomy procedure unpredictable.

Angular-stable anatomical fixation plates have become very popular in the last decade [7]. The shape of these anatomical plates however, is designed to fit the average patient while a large degree of interindividual variation has been shown in bone shapes [27]. In addition, the anatomical plate permits attachment to the bone segments in different ways. The relative bone position therefore depends on the surgeon’s choice of positioning the plate for bone fixation.

This paper introduces 3D imaging techniques to design and create a patient-tailored plate preoperatively, which closely fits on the bone surface, and combines accurate positioning of bone segments in 3D space with bone fixation. The accuracy and reproducibility of the method is evaluated experimentally using artificial radii of a patient suffering from a malunion of the distal radius, eligible for a corrective osteotomy procedure.

**MATERIALS AND METHODS**

Utilization of a patient-tailored plate involves 1) planning the relative position of the bone segments in 3D space using the contralateral bone, 2) creating a predrilling and cutting guide, 3) creating a patient-tailored plate, and 4) the intraoperative procedure.

Custom-made planning software [4] for finding the repositioning parameters was extended for designing the drilling and cutting guide and the patient-tailored plate. This software enables interactive preoperative planning of the osteotomy cut, choosing a plate position, and choosing positions for the fixation screw holes. The thickness of the plate, the diameter of the predrilling holes and the screw diameter are set by the user and are taken into account when designing both the drilling and cutting guide, and the patient-tailored plate. The methods for guide and plate design are detailed in the following sections.

**Preoperative planning**

Preoperative 3D planning is based on a CT scan of the affected bone and the healthy contralateral bone. The contralateral bone is used as reference for planning the reconstruction of the affected bone. The affected bone is first segmented (see below) to create a 3D polygon, which is a virtual representation of the affected bone. A distal and a proximal segment are subsequently clipped, hereby excluding the malunited fracture region. Next, the clipped segments are aligned (by registration, see below) with the mirrored image of the healthy contralateral bone (Fig. 1). This yields two matrices: $M_d$, which aligns the distal segment with the reference bone, and $M_p$, which aligns the proximal segment with the reference bone. These matrices can be combined to find the correction matrix,
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\[ M_c = M_p^{-1}M_d \], which brings the distal segment of the affected bone from its affected position to the planned position.

In the above-described method, the affected bone is first segmented using threshold-connected region growing followed by a binary closing algorithm to fill residual holes inside the object and at the bone surface [4]. This intermediate segmentation result is used to initialize a Laplacian level-set segmentation growth algorithm [14], which slightly adjusts the edges towards the highest intensity gradient of the bone image. A polygon is finally extracted from the segmented image, which is used for visualization in 3D.

Intensity-based point-to-image registration is used for aligning the virtual representation of the bone segments of the affected side with the mirrored image containing the contralateral bone. To this end, points are selected by sampling the gray-level image 0.3-mm towards the inside (bright voxels) and outside (dark voxels) of the segmented bone. This results in a double-contour polygon, which includes the gray-level value at each vertex. Registration of these gray-level points with the gray-valued reference image renders bone alignment accurate [3,[4].

**Surgical drilling and cutting guide**

Patient-specific guides for cutting or drilling are increasingly used in surgery [5,10,11,19]. They enable accurate positioning of surgical instruments or implants with respect to the bone anatomy. In the present preoperative procedure, the surgeon is enabled to interactively set the position and orientation of the cutting plane (Fig. 2a).
Next, a virtual box is interactively positioned and a regular grid on the bounded plane formed by one side of this box (Fig. 2b) is projected onto the surface of the affected bone polygon. After tessellation between these points, the projected surface (red) is used to create a guide and plate by extrusion (towards plane normal, yellow arrow). c) Virtual drilling and cutting guide attached to the bone. d) A temporary plate is created by extrusion of the projected bone surface and subsequent drilling of holes at the same position as for the drilling and cutting guide. The temporary plate and affected bone are cut at the interactively indicated position and orientation (shown by line). e) After cutting, the distal bone and plate segments are repositioned using the correction matrix $M_c$. f) The missing piece is created by Bezier interpolation for smoothness, and concludes tailored plate design.

Fig. 2. Steps for creating a patient-tailored plate. a) Interactive selection of the cutting plane. b) Setting the virtual box for projecting a grid of points onto the bone surface. After tessellation between these points, the projected surface (red) is used to create a guide and plate by extrusion (towards plane normal, yellow arrow). c) Virtual drilling and cutting guide attached to the bone. d) A temporary plate is created by extrusion of the projected bone surface and subsequent drilling of holes at the same position as for the drilling and cutting guide. The temporary plate and affected bone are cut at the interactively indicated position and orientation (shown by line). e) After cutting, the distal bone and plate segments are repositioned using the correction matrix $M_c$. f) The missing piece is created by Bezier interpolation for smoothness, and concludes tailored plate design.

The patient-specific drilling and cutting guide (Fig. 2c) is created by first extruding the projected bone surface towards the plane normal (Fig. 2b, arrow), to create a 3D mold that
snugly fits on the bone surface. The extrusion length defines the thickness of the guide. A slit with a user-defined width (cutting blade thickness) is added at the position and orientation of the cutting plane defined above. Then, the surgeon is enabled to add holes to the guide for fixation of the cutting guide itself and for predrilling the bone for subsequent fixation of the patient-tailored plate with screws. Different hole positions, orientations and diameters can be chosen during this virtual planning step.

**Patient-tailored plate**

Creation of the tailored plate is done in a similar way as for the drilling and cutting guide although less user interaction is required. First the projected bone surface obtained above is again extruded in the same direction, to create a temporary plate that snugly fits on the bone surface (Fig. 2d). The extrusion length defines the plate thickness. Screw holes are added at the same predrill positions and orientations as defined for the drilling and cutting guide, although the hole diameters are adjusted to be in agreement with the required screw diameters. Next, the affected bone polygon and the temporary plate polygon are cut using the defined cutting plane and the distal bone and temporary plate segments are repositioned in 3D space as planned using the correction matrix $M_c$ (Fig. 2e). The missing piece between the two plate segments (Fig. 2e) is created by Bezier interpolation (Fig. 2f) (Appendix) between corresponding points. Bezier interpolation results in a smooth insert between the two plate pieces, altogether realizing the patient-tailored positioning and fixation plate.

**Surgical procedure**

In actual surgery, after resectioning, the surgical guide is positioned at the specific bone surface and is fixated with, e.g., Kirschner wires, using the planned fixation holes. The guide is subsequently used for predrilling screw holes, which are later used for plate fixation. The same guide is subsequently used to position and orient the oscillating surgical saw for cutting the bone (osteotomy) through the guiding slit. After osteotomy, the guide is removed and the patient-specific plate is first connected to the distal bone segment. It fits on the bone surface while the predrilled holes serve to further guide fixation of the plate to the distal bone segment using screws. The bone segments are subsequently extracted in order to align the proximal holes of the plate with the predrilled holes in the proximal bone segment. Positioning and fixation are then achieved by mounting the plate to the proximal bone segment using screws.
EXPERIMENTS

To evaluate the method, a CT scan is made of both arms of one single patient with a unilateral distal radius malunion, shown in Fig. 2. Preoperative planning, as described above, yields surface descriptions of the affected bone, the planned position of the proximal and distal segments, the drilling and cutting guide (thickness 10 mm, slit width 1 mm), the patient-tailored plate (thickness, 3.0 mm) and a reference bone (section 3.2). Five equal sets of the artificial bone of the affected radius, guides and plates are created of acrylonitrile butadiene styrene (ABS) by 3D printing (SST1200es 3D printer, Dimension Inc., Eden Prairie, MN). The resolution of this printer is 254μm.

In all experiments CT images are acquired with a Brilliance 64-channel CT scanner (Philips Healthcare, Best, The Netherlands) (isotropic voxel spacing of 0.45 mm) and included the whole radius.

Accuracy of position measurements
To test the accuracy and reproducibility of assessing positioning parameters, all five artificial affected radii were scanned before correction. One radius was segmented and a distal segment (10%) and a proximal segment (65%) were subsequently clipped for registration of the double-contour polygon with the remaining four radii images. The relative position of each distal segment with respect to its proximal segment was determined and the difference with the first bone (reference) yields the positioning error. This error value depends on manual initialization of the registration procedure, on the noise content of the images [4] and on possible shape differences due to 3D printing.

Positioning accuracy of a patient-tailored plate
To test the accuracy and reproducibility of positioning using a patient-tailored plate, the affected bone specimens are corrected using the described methodology. The plate is mounted to the bone segments using non-locking plastic screws instead of metal screws, to prevent image scattering in the evaluation scan. A reference bone is scanned together with these corrected bone specimens and serves to test the accuracy and reproducibility of positioning.

The end position is compared to the preoperatively planned position, in the same way as described in section 3.1, although the plate region was excluded for optimal registration (Fig. 3).

The reference bone (Fig. 4c) is created by virtually repositioning the distal segment of the affected bone using the correction matrix $M_c$ and by filling the gap between the bone segments using linear interpolation. Employing this reference bone to evaluate the accuracy of our method has the advantage of excluding positioning errors due to bilateral differences [15]. The reference bone is created using the same 3D printer.
Fig. 3. A reference radius (white) is constructed from the affected bone by adding a wedge-shaped insert to bring the distal end in alignment with the proximal segment, in agreement with the unaffected bone. Registration of the proximal (blue) and distal (green) segments with the reference bone (white) allow visualizing malalignment (green segment position). Six malalignment parameters (translations: $\Delta x$, $\Delta y$, $\Delta z$; rotations: $\Delta \phi_x$, $\Delta \phi_y$, $\Delta \phi_z$) are expressed in terms of an anatomical coordinate system.

Fig. 4. Artificial bone specimen showing: a) Drilling and cutting guide attached to the affected bone. The cutting blade is inserted into the slit for demonstration purposes. b) Bone after utilization of a patient-tailored plate showing a high degree of similarity with c) the reference bone.
Data analysis

Positioning parameters are represented by displacements \((x, y, z)\) along, and rotations \((\phi_x, \phi_y, \phi_z)\); rotation sequence: \(y, x, z\); the centroid of a bone segment serves as center of rotation) about three orthogonal axes of an anatomical coordinate system equally defined for each segmented radius (Fig. 3). The longitudinal gravitation axis is the \(z\)-axis. The \(x\)-axis is defined by the line perpendicular to the \(z\)-axis and passing through the tip of the radial styloid. The \(y\)-axis is perpendicular to the \(x\)- and \(z\)-axes according to the right-hand rule [15].

When evaluating residual displacement \((d_{\text{err}}(x), d_{\text{err}}(y), d_{\text{err}}(z))\), and orientation errors \((\phi_{\text{err}}(x), \phi_{\text{err}}(y), \phi_{\text{err}}(z))\) of the positioning parameters (Fig. 3) from a series of measurements, the average parameter value is used to represent the accuracy; the standard deviation (SD) represents the reproducibility. Positioning errors are also expressed in a parameter called the mean Target Registration Error (\(m\text{TRE}\)) [15]. In this parameter, corresponding polygon points are evaluated in the planned position (target) and in the achieved position. The average distance (\(m\text{TRE}\)) represents the goodness of alignment. The achieved position is found by registration of the distal and proximal segment, obtained during preoperative planning, with a CT scan of the corrected bone.

RESULTS

Accuracy of position measurements

Fig. 5 a-b show the accuracy and reproducibility of displacement and orientation parameters \((d_{\text{err}} < 0.06 \pm 0.23 \text{ mm} \text{ and } \phi_{\text{err}} < 0.27 \pm 0.28^\circ)\). The \(m\text{TRE}\) in this experiment was \(0.21 \pm 0.10 \text{ mm}\).

Positioning accuracy of a patient-tailored plate

After predrilling and cutting the plastic bone using the surgical guide (Fig. 4a), the plate snugly fits on the bone segments (Fig. 4b). Correction using the patient-tailored plates was followed by CT analysis and yielded residual displacement and orientation errors are shown in Fig. 5 c-d. It can be seen that both the relative displacements and orientations are achieved with high accuracy and reproducibility \((d_{\text{err}} < 1.2 \pm 0.8 \text{ mm} \text{ and } \phi_{\text{err}} < 1.8 \pm 2.1^\circ)\). The \(m\text{TRE}\) was \(1.6 \pm 0.6 \text{ mm}\) in this experiment.
DISCUSSION
This report described a patient-tailored plate for malunion treatment, which combines accurate positioning of bone segments in 3D space with customized fixation of these bone segments. A drilling and cutting guide is used for predrilling holes into the bone for subsequent screw fixation of the plate, and to position and orient the surgical saw for making the osteotomy intraoperatively. The method requires a single preoperative CT scan of the affected and contralateral bone, and planning software for guide and plate design. Additive manufacturing companies allow 3D printing of medical-grade guides and patient-specific plates, featuring fixation with locking or non-locking screws. This renders the method widely available, especially if planning, guide and plate design are provided by specialized third parties who can also take care of quality control, such as bone density versus screw positioning, screw diameter and
implant strength characteristics. This would avoid the high initial cost of alternative methods that are based on tool tracking [29,32] or systems that rely on robotic navigation [34]. The method therefore has a high potential for successful introduction in clinical practice.

When moving towards quantitative surgery [26][4] for accurate and reproducible positioning, we need accurate tools with known tolerances for planning, actual surgery and postoperative evaluation. In this study we showed (section 4.1) that the accuracy of preoperative position planning and postoperative evaluation is very high with residual translation errors of (mean ± SD) 0.06 ± 0.23 mm and rotation errors of 0.27° ± 0.28° [4]. However, when using the contralateral bone as reference for corrective surgery we cannot improve positioning better than bilateral asymmetry allows. Vroemen and coworkers [15] investigated the bilateral asymmetry for the radius and showed that the left and right distal radius segments are positioned slightly different with respect to the proximal segment by (Fig. 3): (Δx, Δy, Δz) -0.81±1.22mm, -0.01±0.64mm, and 2.63±2.03 mm; and (Δϕx, Δϕy, Δϕz): 0.13±1.00°, -0.60±1.35°, and 0.53±5.00°. They also showed that the relatively large displacement along the bone axis (Δz) can partly be compensated for by including the ulna in the plan. Except for remaining differences between rotations along the bone axis (Δϕz) the remaining parameters indeed show a high level of agreement. Since the contralateral side is normally considered the best reference, the 95% confidence interval (±1.96 SD) could describe acceptable tolerances in positioning, although the positioning thresholds for successful clinical outcome need yet to be determined.

Positioning using a patient-tailored plate has shown to be very accurate and reproducible (derr < 1.2 ± 0.8 mm and ϕerr < 1.8 ± 2.1°). The accuracy of our method is superior to that of related experimental studies that are based on 3D techniques, such as the method described by Westphal et al. [34] who used a robot-assisted technique for bone fracture reduction. They reported a similar translational accuracy but a rotational deviation, which is relatively large (derr < 1.57 mm and ϕerr < 4.50°). The accuracy of our method is comparable to that of Croitoru and coworkers [3] (derr < 1.0 ± 2.1 mm and ϕerr < 4.0 ± 6.5°) who used a navigation system for positioning bone segments. The reproducibility of our method, however, is much better. Oka and coworkers [21] recently validated their technique using cadaver specimens [22]. In this method a first guide is used to insert parallel pin pairs in the proximal and distal bone segment. After osteotomy a second guide is used to align the pins pairs, hence the bone segments. Their experimental evaluation method shows similar accuracy and reproducibility results (derr < 1.1 ± 0.6 mm and ϕerr < 1.1 ± 0.6 °) as our method. Dobbe et al. [4] used a similar method using parallel pin pairs but with a clamping tool for positioning the pin pairs in 3D space. This method achieved comparable accuracy and reproducibility results (derr < 1.2 ± 0.4 mm and ϕerr < 2.1 ± 1.6°). However, the latter two experimental evaluation methods did not include plate fixation, which may deteriorate postoperative positioning. Miyake et al., [20] proposed preoperative computer simulation for surgery using a standard volar locking plate. During actual surgery, they used a similar
drilling and cutting guide as we propose. Their radiographic evaluation showed average absolute differences with the unaffected side of 5° for the volar tilt (range [-2, 16]°) and 3° for the radial inclination (range [-3, 5]°). A disadvantage of their approach is the fact that a standard plate does not fit on the bone geometry when the bone is deformed due to a malunion. In addition, locking screws may introduce a positioning error if the screws lock into the plate with different distances between each of the bone segments and the plate, compared to what was planned. Reconsidering bilateral differences [15] described above, we can conclude that the patient-tailored plate shows to be more accurate in translational positioning than what is possible based on bilateral differences. The residual orientation deficit is slightly larger than average bilateral differences suggest. However, the spread in this biological parameter is quite large. For most individual cases, the patient-tailored plate will therefore position better than what is possible based on bilateral differences of the distal radius. Besides using the patient-tailored plate for corrective distal radius osteotomy, the new method is equally accurate as methods reported for, e.g., femoral or high tibial shaft malunion treatment [12,18] and for mandibular reconstruction after bone tumor resection [30,36]. To our knowledge however, clearly defined tolerances for these methods require more clinical and basic-science research to better define justifiable tolerances [26].

In our experimental evaluation, artificial bones, guides and plates were used based on a single bone morphology, which is a limitation of the study. Deformation of the 3D printed objects may have occurred, e.g., due to plastic deformation of the plate when removing it from the heated printer chamber. Small errors may also have been introduced in drill-hole positioning due to plastic deformation of either the bone or the drilling and cutting guide, since no coolant was used for drilling. This can be avoided either by using a coolant or by drilling through metal sleeve inserts. In addition, elastic deformation of the bone or plate, by compressive forces that are due to screw fixation, may also have contributed to the positioning error. Actual implants will be made of, e.g., titanium, with a much higher elastic modulus than the ABS plates used for our experiments, which will reduce deformation by compressive forces. Finally, the thickness of the drilling and cutting guide was 10 mm in our experiments. Increasing this thickness may have further enhanced orienting the drill. In fact, the total positioning variance is the result of the sum of variances of aforementioned causes. Better dealing with these limitations may further enhance positioning using a patient-tailored plate.

Actual surgery may introduce additional challenges, e.g., difficulties in placing the drilling and cutting guide due to the presence of ligaments, muscles and fascias, or unexpected bending of a plate due to large soft tissue tensile forces. Insertion of a bone graft or bone substitute can also be made more difficult if the plate is attached to the bone surrounded by soft tissue. On the other hand, it has been acknowledged that avascular wedges may (partially) be resorbed by the body [2]. Recently the use of a wedge is even considered not obligatory [35]. Actual surgery may benefit from using locking screws instead of compression fixation as used in our experiments. However, when using locking screws the surgeon needs to avoid a distance between
the snugly fitting plate and the bone, to prevent positioning errors as discussed above. Regarding to virtual planning of the osteotomy plane our method could benefit from using haptic control over mouse control for plane positioning as was proposed by Paul and coworkers [25]. Besides using the patient-tailored plate for corrective distal radius osteotomy, the method may be of interest for corrective osteotomy of other long bones, mandibular reconstruction and clavicular reconstruction as well. In all of these cases the contralateral side can equally be used as reference for reconstruction of the affected side. Even if a healthy reference is missing, the surgeon can plan the position of one (distal) bone segment with respect to another (proximal) bone segment in a manual fashion, e.g., guided by surrounding anatomy. A patient-tailored plate can thus generally be used to fixate bone segments in a planned position.

The two-step method of predrilling and cutting using a surgical guide, followed by the utilization of a patient-tailored plate for fixation and accurate 3D positioning at the same time, seems very easy to utilize during surgery since it does not require complex navigation or robotic equipment nor tracking tools. Custom treatment with a patient tailored plate may reduce the reoperation rate since repositioning is likely to be better than conventional malunion treatment using 2D imaging techniques and a standard anatomical plate. The patient-tailored plating technology is expected to have a great impact on future corrective osteotomy surgery.

**APPENDIX**

A patient-tailored plate is designed by virtually cutting the bone and temporary plate at a user-defined location and by repositioning the distal plate segment using the correction matrix $M_c$. (Fig. 2e). The cross-section of the plate (Fig. 6a, Plane 0) is positioned repetitively within the gap such that it smoothly runs from the proximal plate segment to the distal plate segment. This is achieved by extracting the angles of rotation $(\phi_x, \phi_y, \phi_z)$ from the rotation matrix that orients the cross-sectional points in “Plane 0” to “Plane N” (Fig. 6a), and by linear interpolation of the rotation angles for intermediate planes. Positioning of these N planes within the gap is done using cubic Bezier interpolation between the starting point ($P_0$), at the centroid of the cross-section points, and the end-point ($P_3 = M_c P_0$). The control points of this Bezier curve ($P_1$) and ($P_2$) are positioned at $P_1 = P_0 + c T_1$ and $P_2 = P_3 + c T_2$, with $T_1$ and $T_2$ the average tangent vector of the (transformed) cross-section points in the direction as shown by Fig. 6c. These control points define the curvature of the Bezier path. With this definition of the Bezier parameters ($P_0$, $P_1$, $P_2$, $P_3$), the centroids the cross-sectional planes $P(i)$ ($i = [0, N]$) follow a cubic Bezier curve:

$$
\bar{P}(i) = \left(1 - \frac{i}{N}\right)^3 P_0 + 3\left(1 - \frac{i}{N}\right)^2 \frac{i}{N} P_1 + 3\left(1 - \frac{i}{N}\right)\left(\frac{i}{N}\right)^2 P_2 + \left(\frac{i}{N}\right)^3 P_3, \quad i \in [0, N]
$$

(A.1)

A polygon mesh of the insert is created by tessellation between neighboring points (Fig. 6c).
Fig. 6. a) Creation of a plate insert by copying the cross section (Plane 0) to intermediate planes [0, N] showing smoothly varying orientations. b) The centroid of these planes follows a cubic Bezier curve defined by a starting point ($P_0$), an end point ($P_3$) and two control points ($P_1$, $P_2$). c) Tessellation between consecutive points yields a smooth polygon mesh of the insert.
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