Corrections to "Construction of the exact Fisher information matrix of Gaussian time series models by means of matrix differential rules"

Klein, A.; Mélard, G.; Niemczyk, J.

Link to publication

Citation for published version (APA):

General rights
It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations
If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: https://uba.uva.nl/en/contact, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.
Corrections to
"Construction of the exact Fisher information matrix of a Gaussian time series model by means of matrix differential rules"

André Klein, Guy Mélard and Jerzy Niemczyk

www.fee.uva.nl/ke/UvA-Econometrics

Department of Quantitative Economics
Faculty of Economics and Econometrics
Universiteit van Amsterdam
Roetersstraat 11
1018 WB AMSTERDAM
The Netherlands
Corrections to "Construction of the exact Fisher information matrix of Gaussian time series models by means of matrix differential rules"

ANDRÉ KLEIN, GUY MÉLARD, AND JERZY NIEMCZYK *

Abstract

— This note contains some corrections and amplifications that were deduced from the implementation and testing of the method.

Keywords: Matrix differentiation; Vector autoregressive moving average model; Fisher information matrix.

1. Introduction

During implementation of the method described in Klein et al. [1], some minor mistakes became apparent and also a severe failure concerning the initialization stage in Section 5, Remark 4.

* A. Klein is at the Department of Quantitative Economics, University of Amsterdam, Roetersstraat 11, 1018 WB Amsterdam, The Netherlands (phone +31-20-5254245, fax: +31-20-5254349, e-mail: A.A.B.Klein@uva.nl).

G. Mélard is with Université Libre de Bruxelles, ECARES CP114, Avenue Franklin Roosevelt 50, B-1050 Bruxelles, Belgium (phone: +32-2-6504604, fax: +32-2-6504012, e-mail: gmelard@ulb.ac.be). This author has benefited from an IAP-network in Statistics, contract P5/24, financially supported by the Belgian Federal Office for Scientific, Technical and Cultural Affairs (OSTC). J. Niemczyk was with ECARES when this paper was written. He is presently with Department of Quantitative Economics, University of Amsterdam (phone: +31-20-5257363, fax: +31-20-5254349, e-mail: J.Niemczyk@uva.nl).
2. Some minor corrections

(1) In (48), last term, replace $I_n$ by $I_m$

(2) In (54), line 3, replace $F^T$ by $F^T_t$

(3) Also in (54), last term, replace $M_{mn,n}$ by $M_{mn,m}$

(4) In (58), first line, replace $M_{n,n^2}$ by $M_{n^2,n}$

(5) On page 229, ten occurrences of $\text{vec}F^\theta$ should be replaced by $F^\theta$ but most of these relations are useless, as seen in the next Section.

3. Modified initialization stage

Indeed, the initialization stage described in Section 5, Remark 4, is failed. It worked perfectly for the simple example (submitted in the first version of the paper but omitted at referee’s request), a vector moving average (VMA) of order 1, i.e. the case where $p = 0$ and $s = 1$. But it proved to be wrong when comparing the results for more general univariate ARMA processes with the program (Pmeklex, due to Zahaf) built on the basis of [3]. The whole page 229, except the last sentence, should be replaced by the following:

"Note that $\mathbb{E}\{w_1 \otimes w_1\} = \text{vec}Q$. Initial covariances involving only the process $\tilde{x}_t$, like $\mathbb{E}\{\tilde{x}_1 \otimes \tilde{x}_1\}$ or $\mathbb{E}\{(\tilde{x}_\theta)^T \otimes \tilde{x}_1\}$ or $\mathbb{E}\{\tilde{x}_1^\theta \otimes \tilde{x}_\theta\}$, make use of the initial values discussed in Remarks 1 and 2. For example $\mathbb{E}\{\tilde{x}_1 \otimes \tilde{x}_1\} = P_{1|0}$. Note that the process $x_t$ is stationary and doesn’t depend on the observations made at times $t = 1, ..., N$.

In order to have a stable solution for all the equations involving the process $x_t$, we can"
run the recurrence equations of Section 4 for $t = 1, \ldots, N_0$, where $N_0$ is large enough, watching for convergence. This needs initial values which can be any non-zero matrices of appropriate sizes. When we have the stable solutions, we use them as the initial values of the recurrence equations of Section 4 but for $t = 1, \ldots, N$, where $N$ is the number of observations."

Indeed, it can be seen that the original initialization $x_1 = Fw_0$ is not compatible with $\mathbb{E}\{\hat{z}_1\hat{z}_1^T\} = B_1$ and is therefore wrong. For example, for a univariate AR(1) process (i.e. the case where $m = 1, p = 1, s = 0$) described by the equation $z_t = \theta_1 z_{t-1} + w_t$ with $Q = 1$, the state vector is $x_t = \theta_1 z_{t-1}$ and its variance is $\theta_1^2/(1 - \theta_1^2)$. When we replace $\Phi$ and $F$ by $\theta_1$ in (65) we obtain effectively $\theta_1^2/(1 - \theta_1^2)$ as a stable solution of that equation. However, following the wrong original approach with the stated initial value $\mathbb{E}[x_1 \otimes x_1] = (F \otimes F) \text{vec}Q = \theta_1^2$, we have of course a time-dependent solution which is not appropriate. This illustrates the mistake. Another solution is possible but it would involve $P_{1|0}$ which is not fully computed here and similar matrices for expectations involving derivatives $\tilde{x}_1^\theta$.

4. Conclusion

A related paper [2] contains a discussion of the implementation in the MATLAB environment as well as numerical aspects related to it. Note that computing the derivatives of the autocovariances is the subject of an other paper [4] where some algebraic aspects related to the problem are mentioned.

Acknowledgements

The authors wish to thank Toufik Zahaf for helpful and valuable comments and sugges-
References


