The maturity of sovereign debt issuance in the euro area

Roel Beetsma a,1, Massimo Giuliodori b,1, Jesper Hanson c, Frank de Jong d

a University of Amsterdam, European Fiscal Board, CEPR, CESifo, Netspar, Tinbergen Institute, the Netherlands
b University of Amsterdam, Tinbergen Institute, the Netherlands
c International Monetary Fund and Department of Economics and Econometrics, University of Amsterdam, P.O. Box 15867, 1001 NJ Amsterdam, the Netherlands
d Department of Finance, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, the Netherlands

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ABSTRACT

We use information on new sovereign debt issues in the euro area to explore the drivers behind the debt maturity decisions of governments. We set up a theoretical model for the maturity structure that trades off the preference for liquidity services provided by short-term debt, roll-over risk and price risk. The average debt maturity is negatively related to both the level and the slope of the yield curve. A panel VAR analysis shows that positive shocks to risk aversion, the probability of non-repayment and the demand for the liquidity services of short-term debt all have a positive effect on the yield curve level and slope, and a negative effect on the average maturity of new debt issues. These results are partially in line with our theoretical framework. A forecast error variance decomposition suggests that changes in the probability of non-repayment as captured by the expected default frequency extracted from credit default spreads are the most important source of shocks.

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1. Introduction

One of the most important choices sovereign debt managers face is the maturity structure of the outstanding stock of debt. With Treasury yield curves that are upward sloping most of the time, they can reduce average annual funding costs by tilting the issuance of new debt towards shorter maturities. However, this also raises roll-over risks, as outstanding debt has to be refinanced more frequently. The Eurozone sovereign debt crisis demonstrated that even developed economies can be subject to debt roll-over risks. Such risks may re-emerge when rising interest rates raise budgetary pressures on high-debt countries or when the ability of governments to service the rising debt resulting from current corona crisis gets called into question. Hence, from the perspective of analyzing roll-over risks, exploring the determinants of the maturity structure of the public debt is relevant for the euro area.

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1 Address: Department of Economics and Econometrics, University of Amsterdam, P.O. Box 15867, 1001 NJ Amsterdam, the Netherlands.
E-mail addresses: r.m.w.j.beetsma@uva.nl (R. Beetsma), m.giuliodori@uva.nl (M. Giuliodori), j.hanson@uva.nl (J. Hanson), f.dejong@tilburguniversity.edu (F. de Jong)

2 Sandbu (2020) argues that Eurozone member states should lengthen the maturity of their public debt in order to reduce roll-over risks, facilitated by the ECB raising the upper limit on the maturities eligible for its sovereign debt purchases.

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This paper addresses the determinants of the public debt maturity structure using a newly-constructed comprehensive database of sovereign bond issues in the euro area over the period 1999–2017. Our empirical analysis is motivated with a theoretical framework that trades off the benefits of the liquidity services and lower costs of issuing short-term debt against the likelihood of a debt roll-over crisis resulting from unexpected increases in the probability of non-repayment. Analyzing such trade-offs in deciding the maturity structure of the public debt is the subject of an expanding literature. Broner et al. (2013) develop a two-period model, in which at the start the government decides about the amount of short (one-period) versus long (two-period) debt. Uncertain repayment risk, which materializes after one period, potentially causing a roll-over crisis, leads the government to prefer issuing long-debt, while risk-aversion to bond price fluctuations leads international investors to prefer short debt. Hence, issuing more long debt raises debt-servicing costs, but lowers roll-over risks. Using a dataset constructed for emerging markets, it is confirmed that risk-premiums on long-term debt are higher than on short-term debt and that the difference increases during crises, causing a shift towards more short debt issuance. Similar findings for emerging economies are obtained by Arellano and Ramanarayanan (2012), Bai et al. (2015) and Perez (2017). Greenwood et al. (2015b) also develop a two-period model, again with the government at the start deciding the amount of short versus long debt. Investors derive utility not only from consumption, but also from the monetary services associated with holding safe short-term debt. Further, they feature uncertain risk aversion, which materializes only after one period. These elements produce a trade-off between short debt, the issuance costs of which are suppressed by its attractiveness in terms of providing liquidity services, and long debt which avoids roll-over risk associated with the uncertain risk aversion. Our model combines the Broner et al. (2013) setup with a debt maturity choice in the presence of fiscal risk with the liquidity services of safe short-term debt found in Greenwood et al. (2015b).

Our empirical analysis focuses on the maturity structure of new debt issues rather than that of the complete stock of outstanding debt, because the maturity structure of the latter is only a slow-moving variable and, hence, it would be more difficult to unearth the driving factors behind the debt managers’ choice of the maturity structure. The analysis is conducted in a number of steps, first linking the average maturity of newly-issued euro-area public debt to the level and slope of the yield curve, followed by a panel VAR analysis in which we link the weighted average maturity of new debt and the level and slope of the yield curve to the fundamental shocks in our theoretical framework, i.e. shocks to risk aversion, repayment risk and the demand for the liquidity services of short-term debt. Deploying a panel VAR analysis allows us to trace the effects of a shock to an endogenous variable on the other variables in the system, not only within a specific time period, but also in terms of the dynamics through time. We find that a positive shock to risk aversion, the risk of debt non-repayment and the demand for the liquidity services of short debt all raise the level and the slope of the yield curve, while they reduce the average maturity of newly-issued debt.

Our paper is related to the literature on fiscal insurance, which suggests that debt management can provide insurance against fiscal shocks, thereby contributing to a smoother tax profile (Missale, 2012). Lucas and Stokey (1983) show that governments can optimize their tax profile through the issuance of contingent securities. Angeletos (2002) and Buera and Nicolini (2004) demonstrate that the same can be achieved by issuing non-contingent debt at different maturities. Debortoli et al. (2017) introduce imperfect commitment, whereas Niepelt (2014) models imperfect commitment in combination with the social costs of default. Nosbusch (2008) focuses on the case in which governments can only issue two maturities, while Lustig et al. (2008) endogenize inflation. Faraglia et al. (2008) find limited empirical evidence for OECD countries over the period 1970–2000 that debt management has helped to insulate the public finances against fiscal shocks. Finally, in the context of a debt sustainability analysis, Athanasopoulou et al. (2018) optimize the maturity structure of public debt, while trading off refinancing risk and borrowing costs.


The relationship between the average maturity of the government debt and its level is explored by Blanchard and Missale (1994) and De Haan et al. (1995), who find that it was negative prior to the introduction of the euro. A potential explanation is that a shorter average maturity resulted from the need (forced upon by the capital markets) to reduce the temptation to inflate away high debt burdens. Greenwood et al. (2015a) instead find that the maturity of US Treasury issuance is positively

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3 The liquidity services we model here refer to liquidity in the “Keynesian” sense and not to the tradability of financial market instruments.

4 Greenwood et al. (2015b) further analyze the effect of short-term debt issuance on financial stability through its crowding-out effect on the maturity transformation in the financial sector. In particular, banks also issue safe short-term debt, e.g. deposits, to finance long assets. The role of a safe short-term debt instrument in terms of its liquidity services, and therefore the premium on it, is increasing in its supply. Hence, more issuance of safe short-term public debt crowds out the private issuance of such debt (see also Kacperczyk et al., 2018). However, these considerations are beyond the scope of the current analysis. We also abstract from strategic default and possible debt repayment incentives of short term debt, as in Arellano and Ramanarayanan (2012), as these considerations seem less relevant for the countries and period that we study.

5 Kacperczyk et al. (2018) find that Treasury bills generally carry a safety premium. In line with this finding, Jiang et al. (2018) provide strong evidence of liquidity services of short-term debt by showing a “convenience yield effect” of U.S. Treasuries on the determination of the dollar exchange rate.
related to the debt-to-GDP ratio, which is the result of a trade-off between roll-over risks and the demand for liquid T-bills in their framework. Controlling for the short rate, Greenwood and Vayanos (2014) find a positive relationship between the maturity-weighted-debt-to-GDP ratio to bond yields and future returns. Focusing on the reverse relationship, in an analysis for the euro area Equiza-Goñi (2016) suggests that extending debt maturities may result in lower debt in the long run.

Our analysis differs in various ways from previous work by (i) constructing a theoretical framework that allows us to simultaneously analyze the trade-offs among the price risk of long-term debt, the provision of liquidity services by safe short-term debt and the roll-over risks associated with short-term debt, and (ii) exploring the consistency of the framework’s predictions with the empirical relationship between the maturity of newly-issued Eurozone debt and the yield curve, as well as with the empirical role of the factors driving both debt maturity and the yield curve.

The remainder of this paper is organized as follows. Section 2 constructs our theoretical model. Section 3 defines our measure of the weighted average maturity (WAM) of new debt issuances and describes our data. Section 4 presents the empirical results, first linking the WAM to the yield curve and then investigating the fundamental driving factors of the yield curve and the WAM. Finally, Section 5, which concludes the main text of the paper, summarizes the empirical results and draws some policy conclusions.

2. The theoretical model

In this section we develop a theoretical model that distinguishes different fundamental shocks affecting the yield curve and the choice of the public debt maturity structure. The model builds upon Broner et al. (2013) by adding the liquidity services provided by short-term debt, and yields empirically testable implications about the responses of the maturity structure and the yield curve to shocks to risk aversion, the expected debt repayment probability and the demand for the liquidity services of short debt.

Broner et al. (2013) model decisions on the maturity structure of the government debt in a small open economy that borrows from international investors. In this three-period model, model, investors face fiscal risk that follows from an uncertain revenue stream in the third period. All else equal, risk averse investors prefer short-term debt to limit their exposure to the price risk associated with holding long-term debt. However, issuing more short-term debt also enhances the risk of a roll-over crisis which requires costly fiscal adjustment and makes issuing long-term debt more attractive.

The model by Broner et al. (2013) focuses on emerging markets and does not consider the potential liquidity services that investors may derive from holding safe short-term government debt. These services are a key element in the model by Greenwood et al. (2015b), which focuses on how the preference for safe short-term debt affects the optimal maturity structure of the government debt. However, fiscal risk in their model is limited to a random discount factor.

Our theoretical model combines both approaches. Hence, it combines in one model fiscal risk, price risk and a potential safety premium on sovereign debt. This setup is particularly suitable in the context of euro-area sovereign debt (see e.g. Coeuré, 2016):

- Including default risk is not unreasonable in a model of sovereign debt issuance of euro-area countries: privately-held Greek government debt was subject to a haircut in 2012, the European Stability Mechanism (ESM) Treaty mentions the possibility of debt restructuring ("private sector involvement") and euro-area sovereign bonds have dual-limb collective action clauses since 2013. Further, in December 2018 the Eurogroup expressed the intention to introduce single-limb collective action clauses and to enable the ESM to facilitate a dialogue between Member States and private investors in the case of a debt restructuring (Eurogroup, 2018).6

- At the same time, euro-area sovereign debt is used as a safe asset in financial transactions and in investors’ portfolios. For some euro area countries, the safe status of their debt was called into question during the recent sovereign debt crisis, which resulted into higher sovereign bond yields and flight-to-safety episodes.

2.1. Government and international investors

The government:

There are three periods, labeled 0, 1 and 2. The government maximizes a two-period expected utility function with government consumption $G_t$ as its argument:

$$U = E_0 \left[ u(G_1) + \sum_{s=1}^{S} Pr(s) u(G_{2s}) \right]$$

where $u(\cdot)$ is twice differentiable, increasing and strictly concave, $u(0) = 0$, $u'(0) = -\infty$, $s$ is the state of the economy in period 2, $S$ the number of possible states and $Pr(s)$ the stochastic probability that state $s$ occurs in period 2. The government has an initial outstanding stock of short-term (maturing in period 1) and long-term (maturing in period 2) debt. In period 0, the government can adjust its maturity structure by replacing the existing stock of debt with principal values $B_{01}$ and $B_{02}$ for

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6 The introduction of a Sovereign Debt Restructuring Framework is a recurring element in discussions about the future of the EMU (see e.g. Regling, 2018, and Bénassy-Quéré et al., 2018).
short-, respectively long-term, debt with new debt with principal values \((B_{01}, B_{02})\), subject to the following budget constraint:

\[
P_{01}B_{01} + P_{02}B_{02} = P_{01}B_{01}^* + P_{02}B_{02}^*
\]

where \(P_{01}\) and \(P_{02}\) are the prices of short and long debt in period 0.

Newly-issued short-term debt with principal value \(B_{12}\) in period 1 is needed to finance the repayment of the maturing stock of short-term debt and government consumption in period 1. Hence, the government budget constraint in period 1 is:

\[
B_{01} + G_1 = P_{12}B_{12},
\]

where \(P_{12}\) is the price of new one-period debt issued in period 1. We assume that default never takes place in period 1. This requires \(P_{02}\) not to be too high (see below), so that new short-term debt can still be issued in period 1.

In period 2 the government receives an exogenous flow of fiscal revenues \(y\), which is stochastic and can take on two values:

- \(y = \mathcal{Y}\) with probability \(\pi > 0\) repayment.
- \(y = 0\) with probability \(1 - \pi\) default.

Hence, period 2 features two possible states, a “good” one, in which \(y = \mathcal{Y}\) and all outstanding debt is repaid, and a “bad” one, in which \(y = 0\) and none of the outstanding debt is repaid. Because the world “ends” in period 2, no new debt is issued in period 2. When viewed from the perspective of period 0, the chance \(\pi\) of the good state occurring in period 2 is uncertain.

International Investors:

International investors derive utility from consumption in periods 0, 1 and 2, as well as from the liquidity services associated with holding short-term sovereign debt issued in period 0. Similar to Greenwood et al. (2015b), we assume that these liquidity services cannot be provided by the short-term debt issued in period 1, because short-term debt issued in period 1 is subject to default risk. Also, long-term debt issued in period 0 is unable to provide liquidity services as it is subject to price risk in period 1 and, therefore, not safe. The utility function of the representative international investor is thus equal to

\[
U = C_0 + E_0[m_1C_1 + m_1m_2C_2] + v(B_{01})
\]

where \(v(B_{01})\) represents the liquidity services enjoyed by investors from holding safe short-term government debt issued in period 0. We assume that \(v' > 0\) and \(v'' < 0\). Further, \(m_1\) and \(m_2\) are stochastic discount factors that materialize in periods 1 and 2. These are assumed to be unaffected by the maturity structure chosen by the government. We assume that the risk-free short-term interest rate is zero in both periods, so \(E_0[m_1] = E_1[m_2] = 1\). We make this simplification in order to focus on the trade-off between the liquidity-providing benefits and lower costs of holding short-term debt against the higher likelihood of a debt roll-over crisis as a determinant of the debt maturity structure. Hence, in our theoretical model we abstract from uncertainty about the short risk-free interest rate.

In period 0 short-term debt is riskless, so:

\[
P_{01} = E_0[m_1] + v'(B_{01}) = 1 + v'(B_{01})
\]

Long-term debt issued in period 0 and short-term debt issued in period 1 carry credit risk. The price of period-1 short-term bonds is equal to \(P_{12} = E_1[\chi m_2]\), where \(\chi\) is an indicator denoting repayment in period 2, hence \(\pi = E_1[\chi]\). For convenience, and without loss in terms of results, we assume that the correlation between \(\chi\) and \(m_2\) is zero, hence international investors are risk-neutral with respect to period-1 short-term bonds, so:

\[
P_{12} = \pi
\]

The price of period-0 long-term bonds is equal to \(P_{02} = E_0[P_{12}m_1] = E_0[\pi m_1]\). We assume that the international investors are risk-averse with respect to period-0 long-term bonds and demand a premium to carry the risk of changing repayment probabilities. This implies that \(\pi\) and \(m_1\) are negatively correlated, and the price of the two-period bond is

\[
P_{02} = \sigma \pi_0
\]

where \(\pi_0 = E_0[\pi]\) and \(\sigma < 1\) is a constant parameter, which captures the risk premium required by the international investors in period 0.

2.2. Derivation of the optimal maturity

To summarize, the timing of events is as follows. In period 0, the government chooses the optimal maturity structure \((B_{01}, B_{02})\) of the public debt, given the inherited maturity structure \((B_{01}, B_{02})\), while investors choose their bond holdings, resulting in the prices for short- and long-term debt.

In period 1, the probability \(\pi\) of a good state in period 2 materializes and, given this probability, the government decides about the amount of public consumption in period 1, which, together with the amount of maturing short-term debt, determines the amount of new short-term debt to be issued in that period.
The government repays its debt in period 2 to the maximum possible extent given its available resources, which excludes the possibility of strategic default, and it allocates the remainder of its revenues in that period to government consumption. Hence, the feasible maximum amount of short-term debt the government can enter period 1 with is \( P_{12} \). With this level of short-term debt when entering period 1, the amount of long-term debt issued in period 0 must be zero and all the government’s income in the good state in period 2 will be used to pay off its short-term debt. Hence, if \( B_{01} = P_{12} \), government consumption in periods 1 and 2 is zero in all states of the world.\(^7\) For \( B_{01} < P_{12} \) there will be strictly positive solutions for government consumption in period 1 and in period 2 in the good state (in the bad state in period 2, government consumption is zero).

We solve the government’s optimization problem backwards.

**Period 1:**

Period-2 government consumption in the good state is:

\[
G_{2g} = y - B_{02} - B_{12}
\]

With the period 1 government budget constraint in (3) and the bond price \( P_{12} \) in (5), we obtain, for given initial maturity structure, the relationship between public consumption in period 1 and in the good state in period 2:

\[
G_{2g} = y - B_{02} - \frac{G_1 + B_{01}}{\pi}
\]

Substituting into the government’s objective function (1) and differentiating with respect to \( G_1 \), the first-order condition for period 1 is\(^8\)

\[
u'(G_1) = u'(G_{2g})
\]

**Period 0:**

We now turn to the government’s choice of the optimal maturity structure in period 0. Using the expressions for the bond prices in period 0, \( P_{01} = 1 + \frac{\nu'(B_{01})}{\pi} \) and \( P_{02} = \sigma \pi_0 \), we can write the period-0 government budget constraint as:

\[
B_{02} = B_{02} - \frac{(1 + \nu'(B_{01}))(B_{01} - B_{01})}{\sigma \pi_0}
\]

Hence, government consumption in the good state in period 2 is:

\[
G_{2g} = y - B_{02} + \frac{(1 + \nu'(B_{01}))(B_{01} - B_{01})}{\sigma \pi_0} - \frac{G_1 + B_{01}}{\pi}
\]

Substitution into the government’s utility function yields:

\[
U^* = E_0 \left[ u(G_1) + \pi u(G_{2g}) \right] = E_0 \left[ u(G_1) + \pi u \left( y - B_{02} + \frac{(1 + \nu'(B_{01}))(B_{01} - B_{01})}{\sigma \pi_0} - \frac{G_1 + B_{01}}{\pi} \right) \right]
\]

where the superscript * denotes the optimum, as evaluated in period 1. Differentiating \( U^* \) with respect to \( B_{01} \) yields the first-order condition:

\[
E_0 \left[ \left( u'(G_1) + \pi u'(G_{2g}) \frac{\partial G_{2g}}{\partial B_{01}} \frac{dG_1}{dB_{01}} + \pi u'(G_{2g}) \frac{\partial G_{2g}}{\partial B_{01}} \right) \right] = 0.
\]

Substituting \( \frac{\partial G_{2g}}{\partial B_{01}} = -\frac{1}{\pi} \) from (7) and exploiting the first-order condition of period 1, the period-0 first-order condition reduces to \( E_0 \left[ \pi u'(G_{2g}) \frac{\partial G_{2g}}{\partial B_{01}} \right] = 0 \). Using (7) again, this can be written out as:

\[
E_0 \left[ \pi u'(G_{2g}) \left( 1 + \nu'(B_{01}) + \left( B_{01} - B_{01} \right) \nu'(B_{01}) - \frac{1}{\pi} \right) \right] = 0
\]

This first-order condition can be rewritten further as:

\[
\left[ 1 + \nu'(B_{01}) + \left( B_{01} - B_{01} \right) \nu'(B_{01}) \left( 1 + \sigma \nu_0 \frac{u'(G_{2g})}{E_0 u'(G_{2g})} \right) \right] = \sigma
\]

\(^7\) To rule out any chance of not repaying the maturing short-run debt in period 1 and allowing for positive consumption in period 1 and in period 2 in the good state, we impose the restriction that \( B_{01} < \pi y \), where \( \pi > 0 \) is the lowest possible probability of a good state in period 2. Hence, \( P_{12} = \pi \) is the lowest possible price of short-term debt in period 1.

\(^8\) Broner et al. (2013) assume a government utility function which is concave in period 1 consumption and linear in period 2 consumption. In period 1, the government chooses the amount of fiscal adjustment. For fiscal adjustment at the internal (unconstrained) optimum, period 2 government consumption cannot be guaranteed to be positive if one excludes the possibility of default in the good state. The result is that fiscal adjustment may have to be set at a level higher than its internal optimum. The current setup abstracts from these complications.
Because \( \sigma < 1 \), for a positive solution to \( \nu'(B_{01}) \), we need that \( \text{Cov}_0 \left( \frac{\nu'(G_{02})}{\sigma_0 \nu(G_{02})} \right) < \sigma - 1 < 0 \). Using a first-order Taylor approximation of \( \nu'(G_{02}) \) around the point \( \pi = \pi_0 \) and assuming CARA utility, i.e. \( u(x) = -\exp(-\alpha x) \), the Appendix A shows that we can write the first-order condition (9) as:

\[
[1 + \nu'(B_{01}) + (B_{01} - B_{00}) \nu'(B_{01})] \left[ 1 - \alpha \text{Var}_0(\pi) \left( \frac{G_{02}(\pi_0)}{\pi_0} \right) \right] = \sigma
\]

The Appendix A also shows that

\[
\frac{G_{02}(\tau_0)}{\pi_0} = \frac{1}{\pi_0} \left( \frac{1}{1 + \pi_0} \right)^2 \left[ \frac{\eta + B_{01} - B_{02}}{\nu(B_{01} - B_{02})} + \frac{1 + \nu'(B_{01})}{\sigma \pi_0} \right] > 0
\]

Hence, for (10) to have a solution, we will from now on assume that

\[
\alpha \text{Var}_0(\pi) \left( \frac{\eta + B_{01} - B_{02}}{\pi_0(1 + \pi_0)^2} \right) < 1
\]

In other words, the variance of the repayment probability and the CARA coefficient are assumed to be not too high.

Finally, the Appendix A also shows that the second-order condition is fulfilled under weak assumptions. In particular, we make the simplifying assumption that the initial amount of short-term debt is optimal (again, indicated by superscript \(^*\)), i.e. \( B_{00} = B_{01} \). This assumption eliminates the income effects associated with changes in bond prices. In the sequel, we maintain this assumption.\(^9\)

2.3. Testable propositions

We are now ready to explore a number of implications of our theoretical setup. In this subsection we show the comparative statics for three different shocks:

- An increase in investor risk aversion via a reduction in \( \sigma \).
- A reduction in expected fiscal revenue through a fall in the expected likelihood \( \pi_0 \) that the state in period 2 is good.
- An exogenous increase in the preference for liquidity services \( \nu'(B_{01}) \), i.e. an episode of increased flight-to-safety or a flight-to-liquidity.

In the empirical analysis below, we characterize the yield curve by its level and its slope. The level is defined as the average between the short-term and the long-term yield, i.e. as \( \left( \frac{1}{\pi_0} + \sqrt{\frac{1}{\pi_0}} \right)/2 \), while the spread or slope is defined as the long-term minus the short-term yield, i.e. as:

\[
\sqrt{\frac{1}{\pi_0}} - \frac{1}{\sqrt{\pi_0}} = \sqrt{\frac{1}{\sigma \pi_0}} - \frac{1}{1 + \nu(B_{01})} = \frac{1}{\sigma} \left[ \sqrt{\frac{\sigma}{\pi_0}} + \alpha \text{Var}_0(\pi) \left( \frac{G_{02}(\pi_0)}{\pi_0} \right) - 1 \right]
\]

where the second equality is obtained using the first-order condition (10) evaluated at \( B_{01} = B_{02} \). A sufficient, but by no means necessary, condition for the spread to be positive is that \( \pi_0 < \sigma \). A higher variance in the repayment probability and a higher coefficient of absolute risk aversion on the side of the government both raise the spread.

Our first proposition deals with an increase in investor risk aversion:

**Proposition 1.** An increase in the risk aversion of international investors, i.e. a reduction in \( \sigma \), leads in period 0 to:

(i) an upward shift in the level of the yield curve,
(ii) an ambiguous effect on the slope of the yield curve, and
(iii) a shortening of the maturity structure, i.e. a higher \( B_{01} \) and a lower \( B_{02} \).

Regarding Part (i), the upward shift in the yield curve level follows directly from falling prices \( P_{01} \) and \( P_{02} \) of both short- and long-term bonds. In turn, the effect on \( P_{01} \) follows immediately from (4) and the effect on \( P_{02} \) follows immediately from (6). Regarding Part (ii), we are not able to establish an unambiguous effect of \( \sigma \) on the slope of the yield curve. While an increase in risk aversion has a direct positive effect on the slope, there is an opposite negative effect resulting from a shortening of the maturity structure. The Appendix A demonstrates Part (iii) by differentiating (10) and evaluating at \( B_{01} = B_{02} \). The optimal maturity structure, determined by the trade-off between the risk-premium on the long-term bond and the liquidity premium of the short-term bond, is altered such that the first-order condition for \( B_{01} \) continues to hold.

\(^9\) Hence, when doing the comparative statics, we will always differentiate with respect to \( B_{01} \) first, after which we impose \( B_{00} = B_{01} \).
Concretely, when risk aversion increases, hence the risk premium on the long-term bond rises, the liquidity services provided by short-term debt have to increase to restore the equilibrium. This is accomplished by shortening the maturity structure.

Next, we have the effect of a reduction in expected fiscal revenue. This is modelled by a reduction in the expected probability \( \pi_0 \) of a good state, i.e. of debt repayment, in period 2:

**Proposition 2.** A reduction in the expected probability of repayment \( \pi_0 \) leads in period 0 to:

(i) an ambiguous effect on the level of the yield curve,

(ii) an increase in the slope of the yield curve, and

(iii) a lengthening of the maturity structure, i.e. a lower \( B_{01} \) and a higher \( B_{02} \).

Part (i) of Proposition 2 follows immediately from the reduction in the short-term bond yield, because \( P_{01} \) rises, and the increase in the long-term bond yield, because \( P_{02} \) falls. The effects on \( P_{01} \) and \( P_{02} \) follow immediately from (4) and (6). Finally, since the effects on the short-term and the long-term yields go into opposite directions, we are unable to establish an unambiguous effect on the yield curve level (Part (ii)). The Appendix A demonstrates Part (iii) by differentiating (10) and evaluating at \( B_{01} = B_{01}^0 \). Issuing long-term debt is relatively expensive compared to short-term debt. However, the government refrains from only issuing short-term debt, because of the roll-over risk in period 1. A reduction of \( \pi_0 \) makes the government less wealthy, which, with constant absolute risk aversion, increases its relative risk aversion. With a given variance of the actual repayment probability around \( \pi_0 \) this leads to higher (expected) marginal utilities of the government in periods 1 and 2 if the actual probability of repayment in period 2 falls below the expected repayment probability, which induces the government to issue more long-term debt in order to limit these fluctuations in marginal government utility.

Finally, there is the effect of an increase in the demand for the liquidity services of short debt:

**Proposition 3.** Assume that \( v(B_{01}) = \gamma f(B_{01}), \) where \( \gamma \) is a positive constant. An increase in \( \gamma \) leads to:

(i) a downward shift in the level of the yield curve,

(ii) an increase in the slope of the yield curve, and

(iii) a shortening of the maturity structure, i.e. a higher \( B_{01} \) and a lower \( B_{02} \).

To prove Parts (i) and (ii), start by differentiating (4), which yields \( \frac{dP_{01}}{dC_{01}} = f' (B_{01}) + \gamma f'' (B_{01}) \frac{dC_{01}}{dC_{01}} \). The Appendix A shows that \( \gamma f''' (B_{01}) \frac{dC_{01}}{dC_{01}} > -\frac{1}{\gamma} f' (B_{01}), \) hence \( P_{01} \) rises and the short-term bond yield falls. By (6), \( P_{02} \) remains unchanged and, hence, the long-term yield remains unaltered. Parts (i) and (ii) now follow immediately. The Appendix A demonstrates Part (iii) by differentiating (10) and evaluating at \( B_{01} = B_{01}^0 \).

We summarize the effects of Propositions 1 – 3 in Table 1.

### 3. Data sources and description

We compile a database of all public debt auctions by Germany, The Netherlands, France, Belgium, Italy and Spain over the period 1999–2017. The countries in our sample are the six largest issuers of public debt in the euro area. In total, these countries account for more than 90% of the outstanding stock of debt of the euro area. Preferably, we would also include some of the smaller peripheral countries, in particular Greece, Ireland and Portugal, in our analysis. However, during part of our sample period these countries had no access to the capital market. This implies a reduction in the number of observations for the smaller peripheral countries, in particular Greece, Ireland and Portugal, in our analysis. However, during part of our sample period these countries were effectively unable to optimally choose their maturity structure based on the trade-offs captured by our theoretical framework. For these reasons we do not include these countries in the analysis.

The data on the public debt auctions are taken from Bloomberg, which reports for each auction its date, the maturity of the new issue and the total amount allotted. We cross-check the Bloomberg data with data from the countries’ debt management offices. For a more detailed discussion, see also Beetsma et al. (2018a,b).

For each country we calculate the weighted average time to maturity (WAM) of newly-issued debt as:

\[
WAM_t = \frac{\sum_{m=2}^{50} m \cdot AUC_m \cdot S_{m,t}}{\sum_{m=2}^{50} AUC_m \cdot S_{m,t}}
\]

where \( \sum_{m=2}^{50} AUC_m \cdot S_{m,t} \) denotes the volume of maturity-\( m \) debt auctioned in period \( t \), which we set to be quarterly. Constructing monthly measures for the WAM is possible, but due to the fact that public debt issuance occurs relatively infrequently we construct the WAM only at the quarterly frequency. The range for \( m \) is limited by the fact that we exclude bill issuance with a
Note: this table summarizes the theoretically expected effect of the described changes in the risk factors on the level and slope of the yield curve and the maturity structure, with a plus indicating a shift from short to long debt and a minus indicating a shift from long to short debt. A "?" indicates that the direction cannot be unambiguously determined.

Table 1
Effects of risk factors on yield curve and maturity structure.

<table>
<thead>
<tr>
<th>Risk factor</th>
<th>Level</th>
<th>Slope</th>
<th>Maturity Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increase in the investors' risk aversion</td>
<td>+</td>
<td>?</td>
<td>-</td>
</tr>
<tr>
<td>Reduction in expected probability of repayment</td>
<td>?</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Increase in preference for liquidity services</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

10 For Spain the shortest maturity we include in calculating the WAM is 18 months.
11 For example, Fig. 2B in Greenwood et al. (2015b) exhibits a cyclical pattern over the year in bill issuance for the U.S. that seems related to deadlines for tax payments. De Haan (2009) shows that of the total intervention of 80.5 billion euros in the Dutch financial sector in the fall of 2008, the Dutch Treasury funded 65.5 billion through bills. Indeed, including bills would cause erratic and rather volatile patterns in the WAM.
12 Data collected from the ECB (2019) show that in 2017, as a share of total debt issuance, issuance of foreign-currency debt by Germany was 5%, by Belgium and France 3%, and by the other countries in our sample essentially zero. In 2014, the latest year for which we have the relevant figures, indexed debt issuance as a share of total debt issuance was 4% for Germany, 3% for France and Italy, and zero for the other sample countries.
13 The VSTOXX is the Euro Stoxx 50 Volatility Index. The Euro Stoxx 50 is the most closely followed equity index of the Eurozone – see https://www.macroption.com/vstoxx/.
14 Bekaert et al. (2013) decompose the Chicago Board of Exchange VIX, which is essentially the U.S. counterpart of the VSTOXX, into a risk aversion component and an uncertainty component (based on stock market volatility).
15 For convenience, we define the PVS as the negative of the measure in Pflueger et al. (2018). This way an increase in the degree of risk aversion causes a movement of the VSTOXX and the PVS into the same direction, which allows for easier comparison.
17 See, for example, Afonso et al. (2011) on the determinants of credit ratings.
between the Kreditanstalt für Wiederaufbau (KfW) loan rate and the rate on German public debt of the same maturity.\textsuperscript{18,19} Because KfW loans are guaranteed by the German government, their default risk is identical to that on regular public debt. Hence, the difference between the two rates should be attributable to differences in liquidity. Conceptually, the safety premium on

\textsuperscript{18} The series for the KfW-Bund spread was kindly made available by Roberto de Santis of the ECB. For more information, see De Santis (2014).

\textsuperscript{19} The KfW is a German state-owned development bank, founded after World War II as part of the Marshall Plan.
short-term public debt arises from its money-like properties, such as its extreme safety and its use as collateral in financial transactions and by banks as liquid assets to back short-term liabilities. Finally, following Krishnamurthy and Vissing-Jorgensen (2012), our secondary proxy for the demand for the liquidity services of short debt is the spread between the BBB and AAA Merrill Lynch euro-area corporate bond indices with a maturity from 1 to 3 years, which we download from Datastream.

Fig. 3 plots the aforementioned variables. The VSTOXX peaks during periods commonly seen as turbulent, in particular the second half of 2008 and the end of 2011, while the PVS reaches its highest point towards the end of 2008. For Italy and Spain the 5-year EDF tends to peak around the beginning of 2011 and the end of 2012. Credit ratings are relatively close until mid-2010, when the euro-area sovereign debt crisis starts to erupt and they start to diverge more widely. The credit rating downgrades tend to be less concentrated than the increases in the EDFs. The KfW-Bund spread reaches particularly high values towards the end of 2008 and the first half of 2009 and in the second half of 2011 and in 2012. These periods also roughly correspond to those in which the BBB - AAA euro-area corporate bond spread is at its largest. Table A.1 in the Additional Appendix (not for publication) reports the correlations of the variables in Fig. 3. All the correlations are positive and in many instances quite high.

4. Empirical results

To relate our paper to the literature that explores the relationship between the maturity structure and the yield curve, and see if its main findings are consistent with the data obtained from the debt auctions, we start our empirical analysis with the estimation of the relationship between the WAM and the yield curve for the euro area countries in our sample. Such a reduced-form regression cannot serve as a formal test of the above propositions, because the theory treats the WAM and the yield curve as endogenous and, unlike in the regression, the WAM and the yield curve level and slope will all change simultaneously in response to the underlying shocks. However, the regression could serve as an initial step to gauge the potential relevance of the theory by exploring whether the signs of the coefficient estimates correspond to those predicted by the propositions. In particular, Proposition 1 predicts a negative relationship between the WAM and level of the yield curve, while Proposition 3 predicts a positive relationship. Further, Proposition 2 predicts a positive relationship between the WAM and the yield curve slope, while Proposition 3 predicts a negative relationship.

Next, using a panel VAR analysis we explore how the underlying shock sources affect the properties of the yield curve and the average maturity of new debt issues, thereby providing direct evidence on the hypotheses derived above. The VAR structure is motivated by the fact that the shocks we consider may need time to propagate into the yield curve and the maturity structure, while there may also be feedback effects among the endogenous variables. We close the empirical analysis with a variance decomposition in order to assess the relative importance of the different shocks in explaining the fluctuations in the yield curve and the weighted average maturity of new debt issues.

---

20 In Krishnamurthy and Vissing-Jorgensen (2012) the US short-term safety premium as captured by the BBB - AAA corporate spread is shown to decrease if the supply of US Treasuries increases. They conclude that this is due to a safety preference that gets satisfied when the supply increases. In their paper, the safety premium is assumed to reflect the utility derived by investors from holding safe short-term debt. Greenwood et al. (2015b) explicitly refer to Krishnamurthy and Vissing-Jorgensen (2012) when they introduce into their model a utility term associated with safe short-term debt holdings.
Fig. 3. Variables proxying for the shocks. Notes: The VSTOXX measures the implied volatility of near-term EuroStoxx 50 options and is taken from Thomson Reuters Datastream. PVS ("price volatile stocks") measures the macroeconomic risk appetite as the stock market value of low-volatility stocks minus that of high-volatility stocks. It was kindly provided by Pflueger et al. (2018). EDF is the 5-year expected default frequency, obtained from Moody’s. Rating is a credit rating index constructed out of sovereign credit ratings from the three major credit rating agencies Fitch, Moody’s and Standard & Poor’s. Data are from Oxford Economics obtained via Datastream. The original index, which ranges from 0 to 20, is inverted such that a value of 0 corresponds to the highest possible rating level and is assigned to a country that has an AAA rating from all three credit rating agencies. KfW is the spread between the Kreditanstalt für Wiederaufbau (KfW) loan rate and the rate on German sovereign debt, kindly provided by Roberto de Santis of the ECB. Corporate spread is the spread between the BBB and AAA Merrill Lynch euro-area corporate bond indices with a maturity from 1 to 3 years, which we download from Datastream. The KfW and the corporate spread are expressed in percentage points on an annual basis. The other variables have no dimension. Country labels: DE = Germany, NL = Netherlands, FR = France, BE = Belgium, IT = Italy, ES = Spain.
4.1. The relationship between the WAM and the yield curve

In our theoretical model the risk-free rate for all maturities is zero. In reality, the short-term risk-free rate is determined by monetary policy, while the long-term risk-free rate is determined by the expectations of future short-term risk-free rates plus inflation risk premia. Moreover, the risk-free rate is subject to stochastic shocks. As these elements are not present in our theoretical model, we amend our empirical set-up to incorporate a non-zero risk-free rate. For the risk-free rate we take the overnight index swap (OIS). We define our theoretical model, we amend our empirical set-up to incorporate a non-zero risk-free rate. For the risk-free rate we take plus inflation risk premia. Moreover, the risk-free rate is subject to stochastic shocks. As these elements are not present in our theoretical model, we amend our empirical set-up to incorporate a non-zero risk-free rate.

- The “level” of the risk-free rate:
  \[ \text{OIS\_LEVEL}_i = (\text{OIS}_{1\text{t}} + \text{OIS}_{10\text{t}})/2 \]

- The “slope” of the risk-free rate:
  \[ \text{OIS\_SLOPE}_i = \text{OIS}_{10\text{t}} - \text{OIS}_{1\text{t}} \]

- The “level” of the yield curve of country \( i \) in our sample:
  \[ \text{Y\_LEVEL}_i = \frac{(\text{Y}_{1\text{it}} - \text{OIS}_{1\text{t}}) + (\text{Y}_{10\text{it}} - \text{OIS}_{10\text{t}})}{2} = \frac{\text{Y}_{1\text{it}} + \text{Y}_{10\text{it}}}{2} - \text{OIS\_LEVEL}_i \]

- The “slope” of the yield curve of a country in our sample:
  \[ \text{Y\_SLOPE}_i = \text{Y}_{10\text{it}} - \text{Y}_{1\text{it}} - \text{OIS\_SLOPE}_i \]

The baseline regression equation for the relationship between the WAM and the yield curve, controlling for the level of the risk-free rate, reads:

\[ \text{WAM}_{it} = c_i + \delta_t + \mu \sum_{j=1}^{5} D_{jt} + \beta_1 \text{Y\_LEVEL}_{it} + \beta_2 \text{Y\_SLOPE}_{it} + \gamma \text{OIS\_LEVEL}_i + \epsilon_{it} \]  \( (13) \)

where \( c_i \) is a constant, \( \delta_t \) a time trend, \( D_{jt} \) a dummy for season \( j \), and \( \epsilon_{it} \) a disturbance term. Notice that we do not include time fixed effects. The reason is that we have common variables, in particular \( \text{OIS\_LEVEL}_i \), which for reasons of multicollinearity cannot be included together with time fixed effects. Including the \( \text{OIS\_LEVEL}_i \) is important for the analysis, because we need to control for the general level of the risk-free rate in order uncover the role of the yield curve for the weighted average maturity.

We estimate this equation at the quarterly frequency using pooled OLS with robust standard errors. To avoid feedback effects from the dependent variable to the explanatory variables, we always use beginning-of-quarter values for yields, so \( \text{Y\_LEVEL}_{it}, \text{Y\_SLOPE}_{it}, \) and \( \text{OIS\_LEVEL}_i \) refer to values are the beginning of quarter \( t \), while the \( \text{WAM}_{it} \) always measures the weighted average maturity of new issues during quarter \( t \). This way we avoid feedback effects from the dependent variable to the explanatory variables. Hence, there is no need to estimate the model with instrumental variables. We include quarter dummies to account for possible seasonal issuance patterns. For instance, countries typically issue less debt during the summer months and in December, and we cannot a priori exclude that this lower issuance activity is systematically related to the maturity of the issues. We estimate equation \((13)\) at the country level and as a panel with country-fixed effects and country-specific time trends. The latter allow to account for potential country-specific trends in the weighted average maturity of new debt issuances.

Baseline regression \((13)\) thus links the WAM to the level and the slope of the yield curve (in deviation from the level, respectively slope, of the risk-free rate), while controlling for the level of the risk-free rate itself (as in Gagnon et al., 2011). The advantage of using the above level and slope definitions is that in our regression the coefficient on the level measures the impact of a parallel shift in the yield curve, while the coefficient on the slope measures the effect of an increase in the slope, keeping the average of the yields constant. Moreover, we control for potential fluctuations in the risk-free rate over time. Table 2 reports the theoretically expected signs of the regression coefficients, conditional on the shock that is at play.

Table 3 reports the estimates of \((13)\) for the full sample period. For all the countries the coefficient on the level is negative, while that on the slope is negative in half of the cases and positive in the other half. The fact that the individual country estimates for the level are of identical sign, is an argument to estimate the model also as a panel. The differences in the signs of the slope coefficients motivate us to also estimate the model for sub-panels of countries. We consider Germany, The Nether-
lands, France and Belgium (GNFB) as a separate group and Italy and Spain (IS) as a separate group. The rationale behind the split into these two sub-groups is that the former group of countries is generally considered to belong to the euro-area core, while the other two countries belong to the periphery of the euro area.

The (sub-) panel estimates are also found in Table 3. For both the full panel and the two sub-panels the estimates of the coefficient on the yield curve level are highly significantly negative. The estimates of the coefficient on the yield curve slope are also negative for the full panel and the two sub-panels, although the estimate is insignificant for the core group. The negative estimates for the yield curve slope are in line with what the literature tends to find.\footnote{For example, see Arellano and Ramanarayanan (2012), Broner et al. (2013), Bai et al. (2015) and Perez (2017).} The coefficient estimates are also significant in economic terms. For example, based on the full-panel estimates the effect of a one-percentage point upward shift in the yield curve is associated with a reduction in the WAM by about 1 year, while an increase in the spread between the 10-year and the 1-year yield by one percentage point is also associated with a reduction in the WAM by about 1 year. These magnitudes may seem rather large. However, one needs to realize that the effects of changes in the yield curve on the maturity structure of the full debt stock can only be relatively small, because the existing debt stock can only be rolled over gradually. In fact, if a government intends to meaningfully adjust the maturity structure of its debt stock in response to a change in the yield curve, then it is forced to substantially change the maturity structure of its new debt issues. Finally, we observe that the estimate of the coefficient on the level of the OIS is negative for the full panel and the two sub-panels, although it is not significant for the Italy-Spain sub-panel. We also estimate the model adding the slope of the OIS on the right-hand side of (13). To save space, the numbers are not reported here, but this variable is never significant in the (sub-) panels, while the signs and significance of the coefficients on the yield curve variables remain unchanged. This is also the case for the estimates of the shortened sample discussed below.

The negative coefficient of the yield curve level is consistent with the prediction of Proposition 1 that fluctuations in investor risk aversion generate a negative association between the WAM and the yield curve level, but runs counter to Proposition 3’s prediction of fluctuations in the demand for the liquidity services of short debt driving a positive association between the WAM and the yield curve level. By contrast, the negative coefficient on the slope of the yield curve is consistent with Proposition 3, but at odds with Proposition 2 that fluctuations in expected repayment probability produce a positive association between the WAM and the yield curve slope.

We explore the robustness of the estimates by adding controls to model (13). The estimates for the full panel and for the two sub-panels are presented in Table 4. First, we add the annual amount of debt issued, followed by the joint inclusion of GDP growth and inflation and, finally, the slope of the OIS. Generally, the estimates are numerically and in terms of significance quite close to the original estimates without any of these controls. The significant positive coefficient of GDP growth for the full panel is driven by its positive effect on the average maturity of new debt issues by the highly-rated countries. GDP growth annuls the significance of the level of the risk-free rate for the full panel. The other additional controls are insignificant in all instances.

We also estimate our baseline regression (13) for the sub-sample 2006Q1 to 2017Q4. We consider this subsample period, because given data availability it constitutes the maximum period over which we can estimate the panel VAR in the next subsection. The subsample period coincides largely with the crisis period and, limiting ourselves to the crisis periods 2007Q3 to 2017Q4 would give essentially the same results. For the (sub-) panels the signs and significance of the coefficients on the yield curve level and the slope are in all cases identical to those for the full sample period (see Table 3).

4.2. Panel vector auto regression estimates

The previous subsection provides strong evidence of a negative relationship between the WAM and the level of the yield curve and weaker evidence of a negative relationship between the WAM and the slope of the yield curve. The former would suggest a relatively important role for the presence of shocks to risk-aversion, while the latter would suggest a relatively important role for liquidity-preference shocks. However, because the theoretical model treats the WAM and the yield curve as endogenous, while, moreover, different shocks can hit the economy simultaneously, these estimates cannot immediately be used to test the validity of Propositions 1–3. In this subsection we set up a panel VAR in which the WAM and the level and the slope of the yield curve feature as endogenous variables, while shocks originate from variables that proxy for risk aversion, the expected probability of repayment and the demand for the liquidity services of short debt. By using a panel VAR, we also allow for the possibility of multiple shocks hitting the economies at the same time and that the shocks feed only gradually into the economy, so we can model the dynamic responses of the endogenous variables over time. This subsection ends with a variance decomposition, which allows us to gauge the relative importance of the various shock sources.

Concretely, the baseline panel VAR is set up as follows. The variables to proxy for the shocks to risk aversion, the demand for the liquidity services of short debt and expected repayment probability are, respectively, changes in the VSTOXX, the 10-year KfW-Bund spread and the 5-year EDF.\footnote{Using the CDS spread as an alternative to the expected default frequency yields qualitatively essentially the same results. However, since the CDS is the product of the expected default frequency and the price of this risk, we view the expected default frequency as the conceptually more appropriate variable to capture stochastic repayment risk.} As exogenous variables we always include the level and one lag of OIS, seasonal dummies, country fixed effects and country-specific time trends. We use a Cholesky identification scheme, in which we enter the shocks first. We confine ourselves to a quarterly panel. The vector of endogenous variables, and their ordering,
We order the variables that are included. Notice that, because the VAR is formulated in levels, it also encompasses potential cointegrating relationships. The ordering of relevance for the entire Eurozone bond market. The EDF, which is a country-specific variable, is ordered third. The ordering of the other endogenous variables following the shocks is irrelevant with a Cholesky identification scheme, as shown by Christiano et al. (1999). Notice that, because the VAR is formulated in levels, it also encompasses potential co-integrating relationships among the variables that are included.23

Notes: Estimation method is Ordinary Least Squares (OLS) with Newey-West adjusted standard errors at the country level and OLS with fixed effects, country-specific trends, quarter dummies and panel-corrected standard errors with cross-section weights. “Panel” is the full panel, “Panel GNFB” is the sub-panel formed by Germany, Netherlands, France and Belgium, and “Panel IS” is the sub-panel formed by Italy and Spain. Further, *, ** and *** denote a significant difference from zero at the 10% -, 5% -, and 1% - levels, respectively.

Notes: See Notes of Table 3. Estimation method is Ordinary Least Squares (OLS) with fixed effects, country-specific trends, quarter dummies and panel-corrected standard errors with cross-section weights. QUANTt is the total amount of debt issued in period t divided by 100,000. Further, Xt captures the control(s).

in the VAR is \[VSTOXX, KfW, EDF, Y\_LEVEL, Y\_SLOPE, WAM\_it\], where VSTOXX stands for the VSTOXX, KfW for the 10-year KfW-Bund spread and EDFt for country t’s EDF over the coming 5 years. Under the baseline, we always include one lag of the vector of endogenous variables. The chosen ordering is motivated by the presumed degree of exogeneity of the variables. We order the VSTOXX, first, because a within-period feedback of the VSTOXX, to the other variables is unlikely, as it is based on the pan-European stock market and this market is at most partially integrated with the European sovereign bond markets. That is, the sets of traders differ between those markets and, hence, capital does not flow perfectly from one market to the other. We order the KfW-Bund-spread second. Even though it measures the difference between two German variables, we consider it of relevance for the entire Eurozone bond market. The EDF, which is a country-specific variable, is ordered third. The ordering of the other endogenous variables following the shocks is irrelevant with a Cholesky identification scheme, as shown by Christiano et al. (1999). Notice that, because the VAR is formulated in levels, it also encompasses potential co-integrating relationships among the variables that are included.23

23 Incidentally, unit root tests on the weighted average maturity, the level of the yield curve and the slope of the yield curve generally reject the presence of a unit root.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Theoretically expected signs of the regression coefficients in (13).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicted sign of regression coefficient due to risk factor</td>
<td>Level</td>
</tr>
<tr>
<td>Increase in investors’ risk aversion</td>
<td>–</td>
</tr>
<tr>
<td>Reduction in expected probability of repayment</td>
<td>?</td>
</tr>
<tr>
<td>Increase in preference for monetary services</td>
<td>+</td>
</tr>
</tbody>
</table>

Note: this table states the expected signs of the coefficients in regression Eq. (13) based on the theoretical framework set out in Section 2. A “?” indicates that no unambiguous sign can be obtained from the theory.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>WAM and yield curve.</th>
</tr>
</thead>
<tbody>
<tr>
<td>WAMt = c + δt + μS1:t-1, D1:t + β1Y LEVEL1:t + β2Y SLOPE1:t + γOIS LEVEL1:t + ε1t</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>Netherlands</td>
</tr>
<tr>
<td>(a) Full sample period: 1999Q1 – 2017Q4</td>
<td></td>
</tr>
<tr>
<td>β1</td>
<td>–1.95*</td>
</tr>
<tr>
<td>β2</td>
<td>0.18*</td>
</tr>
<tr>
<td>γ</td>
<td>–0.28*</td>
</tr>
<tr>
<td>Adj. R²</td>
<td>0.34</td>
</tr>
<tr>
<td>Obs.</td>
<td>76</td>
</tr>
<tr>
<td>(b) Period: 2006Q1 – 2017Q4</td>
<td></td>
</tr>
<tr>
<td>β1</td>
<td>–1.29</td>
</tr>
<tr>
<td>β2</td>
<td>0.95*</td>
</tr>
<tr>
<td>γ</td>
<td>–0.013</td>
</tr>
<tr>
<td>Adj. R²</td>
<td>0.23</td>
</tr>
<tr>
<td>Obs.</td>
<td>48</td>
</tr>
</tbody>
</table>

Notes: Estimation method is Ordinary Least Squares (OLS) with Newey-West adjusted standard errors at the country level and OLS with fixed effects, country-specific trends, quarter dummies and panel-corrected standard errors with cross-section weights. Panel GNFB is the sub-panel formed by Italy and Spain. Further, *, ** and *** denote a significant difference from zero at the 10% -, 5% -, and 1% - levels, respectively.

<table>
<thead>
<tr>
<th>Table 4</th>
<th>WAM and yield curve – robustness: adding controls.</th>
</tr>
</thead>
<tbody>
<tr>
<td>WAMt = c + δt + μS1:t-1, D1:t + β1Y LEVEL1:t + β2Y SLOPE1:t + γOIS LEVEL1:t + δXt + ε1t</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>Netherlands</td>
</tr>
<tr>
<td>(a) Full sample period: 1999Q1 – 2017Q4</td>
<td></td>
</tr>
<tr>
<td>β1</td>
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</tr>
<tr>
<td>β2</td>
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</tr>
<tr>
<td>γ</td>
<td>–0.33*</td>
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<tr>
<td>QUANTt</td>
<td>1.90</td>
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<tr>
<td>GROWTHt-1</td>
<td>0.18***</td>
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<tr>
<td>INFlt-1</td>
<td>0.097</td>
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<tr>
<td>QR SLOPEt-1</td>
<td>0.33</td>
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<tr>
<td>Adj. R²</td>
<td>0.33</td>
</tr>
<tr>
<td>Obs.</td>
<td>455</td>
</tr>
</tbody>
</table>

Notes: See Notes of Table 3. Estimation method is Ordinary Least Squares (OLS) with fixed effects, country-specific trends, quarter dummies and panel-corrected standard errors with cross-section weights. QUANTt is the total amount of debt issued in period t divided by 100,000. Further, Xt captures the control(s).
4.2.1. Full sample panel VAR estimation

The size of the shock is always one standard deviation of the variable that is the source of the shock. The impulse responses will always be presented with an error band with two standard deviations on either side of the central line.

A positive risk-aversion shock as captured by an increase in the VSTOXX raises both the level and the slope of the yield curve. The impulse responses are depicted in the first column of Fig. 4. The level jumps on impact, while the slope takes a quarter to become significant. Also in line with Proposition 1, and in line with the estimates in the previous subsection, the WAM falls. The responses have some economic significance: a one-standard deviation shock in the VSTOXX raises the level of the yield curve by about 7 basis points and the difference in yields between 10- and 1-year debt by about 5 basis points after half a year. The WAM falls by roughly a month on impact and reaches a minimum of roughly one- and-a-half months after half a year.

The second column of Fig. 4 shows the impulse responses to an increase in the demand for the liquidity services of short debt as captured by a one standard deviation increase in the KfW-Bund spread. The response pattern is similar to that for the shock to the VSTOXX. The level and the slope of the yield curve exhibit a significant positive response (for the slope after a quarter), while the WAM exhibits a significant decline after a quarter. The effect on the yield curve level is slightly smaller than for the shock to the VSTOXX, while the effect on the slope is of virtually equal size. The maximum fall in the WAM is between two and three months. The level response contradicts Proposition 3, but the responses of the slope and the WAM are in line with this proposition.

The third column of Fig. 4 exhibits the impulse responses following a one standard deviation increase in the EDF. Both the level and the slope of the yield curve exhibit highly-significant increases that are larger than for the other two shocks, while the response of the WAM is comparable in size, but more drawn out. The maximum decrease in the WAM is reached after a few quarters. While the effect on the yield curve slope is in line with Proposition 2, the effect on the WAM contradicts this proposition.

In summary, the results offer support for the theoretical predictions of an increase in investors’ risk aversion, but contradict some of the predictions of an increase in the government’s probability of non-repayment and the investors’ preference for liquidity services.

4.2.2. Panel VAR estimates for country groups

In the above single-equation regression for the WAM we distinguished sub-panels for the groups of the core and periphery countries. We now make the same distinction in our panel VAR analysis. Fig. 5 and Fig. 6, respectively, show the results for the core and periphery countries for the respective shock sources. Overall, the results are reasonably similar for the two country groups and to those for the full sample. Generally the significance of the estimates weakens somewhat, which is not surprising, because the number of observations in each country group is smaller than was the case for the full country sample. Again, an increase in the VSTOXX raises the yield curve level significantly for both country groups. The slope increases significantly for the core group, but in the case of the periphery group its change is not significant. A rise in the KfW-Bund spread generates significant increases in the level and the slope of the yield curve for both country groups, although the magnitude of these increases is larger for the periphery than for the core group. Finally, an increase in the EDF also raises the level and slope of the yield curve significantly for both country groups. These increases are larger than for the other two shocks.

The main difference between the two country groups concerns the responses of the WAM. For the core group, with a slight delay it exhibits a significant drop in response to shocks to the VSTOXX and the KfW-Bund spread, while for the periphery the WAM stays essentially unaltered for both shocks. By contrast, in response to a one-standard deviation increase in the EDF the WAM remains basically unchanged for the core group, while it drops significantly for the periphery. With a fall of the WAM by half a year, the drop is also material. The comparison between the two groups of the response to the WAM is interesting. Apparently, countries that are considered relatively “safe” pay little attention to the expected probability of repayment in setting the maturity structure of new debt issues, while for the “risky” countries, developments in the expected probability of repayment are the main determinant of maturity of new debt issuance. However, rather remarkably, instead of lengthening the WAM so as to reduce roll-over risk, they lower the WAM, presumably to reduce the cost impact of the higher yield curve slope.

4.2.3. Further robustness investigation of panel VAR estimates

As a first further robustness check on the baseline regression, we re-estimate the panel VAR with a different proxy for each individual shock. The impulse responses are shown in Fig. A.1 in the Additional Appendix (not for publication). Replacing the VSTOXX with the PVS measure, neither the yield curve level nor the slope increase anymore, which may be the result of the smaller number of observations. The WAM still falls, but not significantly. The responses to a positive shock to the BBB – AAA spread closely resemble those following a positive shock to the KfW-Bund spread, while the responses to a negative credit rating shock are very similar to those for a positive shock to the EDF.24

As an alternative to the 10-year KfW-Bund spread, we also estimate the baseline panel-VAR with the 3-year (the shortest non-artificially constructed maturity available) KfW-Bund spread kindly made available via UBS Delta and with the spread between the one-year German bond yield and the one-year swap rate, which are both effectively free from default risk, but differ in terms of liquidity. Both variants produce results qualitatively identical to those for the 10-year KfW-Bund spread. The variant with the 3-year KfW-Bund spread yields results that are also quantitatively very similar, while the other variant yields results that are slightly weaker in terms of their negative effect on the WAM.

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24 As an alternative to the 10-year KfW-Bund spread, we also estimate the baseline panel-VAR with the 3-year (the shortest non-artificially constructed maturity available) KfW-Bund spread kindly made available via UBS Delta and with the spread between the one-year German bond yield and the one-year swap rate, which are both effectively free from default risk, but differ in terms of liquidity. Both variants produce results qualitatively identical to those for the 10-year KfW-Bund spread. The variant with the 3-year KfW-Bund spread yields results that are also quantitatively very similar, while the other variant yields results that are slightly weaker in terms of their negative effect on the WAM.
Our second robustness check is based on including four lags, instead of one, of the vector of endogenous variables in the panel-VAR. The results are shown in Fig. A.2 in the Additional Appendix (not for publication). While the confidence bands tend to become wider, because of the larger number of parameters to be estimated, the results remain qualitatively very much in line with the baseline results.

Our third robustness check adds as exogenous variables the same set of controls, and their first lag, that we used earlier for (13). Fig. A.3 in the Additional Appendix (not for publication) shows that the impulse responses are qualitatively unaltered, although the responses of the WAM to the VSTOXX shock and the KfW-Bund spread shock weaken somewhat, likely because of the larger number of parameters to be estimated in the presence of a rather limited sample size.

Our penultimate robustness check assesses the robustness of the results to the ordering of the variables in the VAR system. Therefore, we deploy the generalized impulse response approach of Koop et al. (1996) and Pesaran and Shin (1998), which is invariant to the ordering of variables, as it does not require orthogonalization of the shocks. Fig. A.4 in the Additional Appendix (not for publication) depicts the generalized impulse response functions. These are very similar to the responses shown in Fig. 4.

The final robustness check drops the period of the global financial crisis from the sample, as this may have constituted a structural break in the relationships we study. Fig. A.5 in the Additional Appendix (not for publication) shows the estimates.
for the period as of 2010Q1. Again, they are very similar to the full sample estimates in Fig. 4. Significance is slightly weaker in some instances, most likely due to the smaller number of observations than under the baseline.

4.2.4. The relative importance of the shocks: Variance decomposition

The final step in our analysis is to explore the relative importance of the different sources of shocks: risk aversion, demand for the liquidity services of short debt and the expected probability of repayment.

Tables 5–7 report the forecast error variance decomposition of the yield curve level, its slope and the WAM, respectively. To the yield curve level the main contributor after one quarter is the EDF with about 47%, followed by the level itself with 44%. The VSTOXX contributes about 7% and the KfW-Bund spread about 2%. Over longer horizons, the importance of the EDF rises even further to reach almost 77% after 10 quarters at the cost of a shrinkage in the importance of the level. The VSTOXX and the KfW-Bund shrink marginally over this horizon. The influences of the slope and the WAM are essentially negligible.

Turning to the decomposition of the slope, we see that, at the one-quarter horizon, by far the most important factor is the slope itself with 89% of the total contribution, followed by the level with 9.5% contribution. The other variables provide negligible contributions. At longer horizons, the contribution of the slope quickly falls in favor of the EDF, which after 10 quarters contributes about 38%, roughly the same as the contribution of the slope itself. The VSTOXX and the KfW-Bund spread also gain in importance, together making up about 16% after 10 quarters. The yield curve level declines in importance to about 4% after 10 quarters. Finally, turning to the forecast error variance decomposition of the WAM, by far the largest contribution after one quarter comes from the WAM itself with more than 99%. Over time, the contribution of the WAM shrinks,
but it remains by far the largest factor with 87% after 10 quarters. It is followed by the EDF, which gains in importance over time and contributes 7% after 10 quarters. The VSTOXX and the KfW Bund spread each reach slightly more than 2%, while the slope remains at less than one percent at this horizon.

Table 5
Forecast error variance decomposition of yield curve level at various horizons.

<table>
<thead>
<tr>
<th>Period</th>
<th>VSTOXX</th>
<th>KFW</th>
<th>EDF</th>
<th>Y_LEVEL</th>
<th>Y_SLOPE</th>
<th>WAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.26</td>
<td>2.26</td>
<td>46.59</td>
<td>43.89</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(3.07)</td>
<td>(1.73)</td>
<td>(4.25)</td>
<td>(3.86)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>2</td>
<td>7.20</td>
<td>2.03</td>
<td>58.91</td>
<td>31.10</td>
<td>0.01</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>(3.22)</td>
<td>(1.65)</td>
<td>(4.36)</td>
<td>(3.42)</td>
<td>(0.21)</td>
<td>(0.87)</td>
</tr>
<tr>
<td>5</td>
<td>6.41</td>
<td>1.89</td>
<td>73.06</td>
<td>18.06</td>
<td>0.063</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>(3.52)</td>
<td>(1.88)</td>
<td>(4.903)</td>
<td>(3.08)</td>
<td>(0.56)</td>
<td>(0.79)</td>
</tr>
<tr>
<td>10</td>
<td>6.03</td>
<td>1.83</td>
<td>77.23</td>
<td>14.36</td>
<td>0.096</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>(3.70)</td>
<td>(2.11)</td>
<td>(5.30)</td>
<td>(3.03)</td>
<td>(0.75)</td>
<td>(0.69)</td>
</tr>
<tr>
<td>15</td>
<td>5.96</td>
<td>1.82</td>
<td>78.00</td>
<td>13.72</td>
<td>0.10</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>(3.77)</td>
<td>(2.19)</td>
<td>(5.51)</td>
<td>(3.17)</td>
<td>(0.81)</td>
<td>(0.68)</td>
</tr>
</tbody>
</table>

Notes: the entries in the table report the contributions in percent of the total. The numbers in brackets report the corresponding standard errors.

Fig. 6. Impulse responses for periphery countries Italy and Spain. Notes: see Notes of Fig. 4.
So, overall, of the different shock sources, the EDF has by far the largest effect on the impulse responses after some time. This may not be too surprising in view of the fact that the EDF is a country-specific variable, while the VSTOXX is a common variable and the KfW-Bund spread is a common variable for all countries other than Germany.

5. Concluding remarks

The euro-area debt crisis has brought public debt management to the forefront of the media and the public debate, as it showed the risks associated with high amounts of sovereign debt to be rolled over. In this paper we have investigated the determinants of the maturity structure of euro-area sovereign debt over the period since the inception of the EMU. Using a unique and comprehensive database of sovereign bond issues of six euro-area countries over the period 1999–2017, we focused on the maturity structure of new debt issues, which can be more easily steered into the direction preferred by the Treasury than that of the full stock of outstanding debt, of which the maturity structure is only a slow-moving variable.

We started by constructing a theoretical framework with a maturity choice driven by the trade-off between the liquidity services provided by safe short-term debt, the danger of a debt roll-over crisis and price risk from holding long-term debt. Univariate regressions exhibited a strong negative relationship between the average maturity of new debt and the level of the yield curve, as well as a weaker negative relationship between the average maturity and the yield curve slope. This was followed by a panel VAR analysis that showed that positive shocks to risk aversion, the risk of non-repayment and the demand for the liquidity services of short debt all raise the level and the slope of the yield curve, while reducing the average maturity of new debt. These effects tend to be statistically as well as economically significant. The responses following a positive shock to risk aversion are consistent with our theory, while the responses induced by an increase in the risk of non-repayment and an increase in the demand for the liquidity services of short debt are partially in line with our theory. Using a forecast error variance decomposition, we observe that generally the most important shock source driving the responses is a change in the expected default frequency, which proxies for the probability of non-repayment. We also observe that the expected default frequency appears to be a more relevant driver of the average debt maturity of the periphery than of the core of the Eurozone.

The analysis of this paper may give rise to some potentially useful policy implications. First, in choosing the maturity structure, there is a tradeoff between reducing roll-over risk by increasing the average maturity and the resulting higher interest costs. In recent years the yield curve has flattened, implying that this trade-off has become more favorable. Given the reduction in the cost of issuing long instead of short debt, governments may be well advised to seize this opportunity of reducing roll-over risks at relatively low cost, in particular since debt ratios have increased so much following the global financial crisis, the Eurozone debt crisis and the current Covid-19 crisis.

### Table 6
Forecast error variance decomposition of yield curve slope at various horizons.

<table>
<thead>
<tr>
<th>Period</th>
<th>VSTOXX</th>
<th>KfW</th>
<th>EDF</th>
<th>Y_LEVEL</th>
<th>Y_SLOPE</th>
<th>WAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.78</td>
<td>0.00</td>
<td>0.50</td>
<td>9.46</td>
<td>89.25</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(1.17)</td>
<td>(0.52)</td>
<td>(0.93)</td>
<td>(3.21)</td>
<td>(3.49)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>2</td>
<td>2.85</td>
<td>4.40</td>
<td>5.37</td>
<td>7.49</td>
<td>79.85</td>
<td>0.042</td>
</tr>
<tr>
<td></td>
<td>(2.07)</td>
<td>(1.87)</td>
<td>(1.84)</td>
<td>(2.66)</td>
<td>(3.73)</td>
<td>(0.48)</td>
</tr>
<tr>
<td>5</td>
<td>8.56</td>
<td>7.70</td>
<td>26.39</td>
<td>4.88</td>
<td>52.43</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td>(3.64)</td>
<td>(3.07)</td>
<td>(5.33)</td>
<td>(1.90)</td>
<td>(5.06)</td>
<td>(0.40)</td>
</tr>
<tr>
<td>10</td>
<td>9.09</td>
<td>6.96</td>
<td>38.27</td>
<td>4.14</td>
<td>41.51</td>
<td>0.036</td>
</tr>
<tr>
<td></td>
<td>(4.06)</td>
<td>(3.17)</td>
<td>(7.31)</td>
<td>(1.87)</td>
<td>(5.90)</td>
<td>(0.41)</td>
</tr>
<tr>
<td>15</td>
<td>8.94</td>
<td>6.71</td>
<td>40.75</td>
<td>4.02</td>
<td>39.54</td>
<td>0.038</td>
</tr>
<tr>
<td></td>
<td>(4.11)</td>
<td>(3.18)</td>
<td>(8.12)</td>
<td>(1.93)</td>
<td>(6.43)</td>
<td>(0.43)</td>
</tr>
</tbody>
</table>

Notes: see Notes to Table 5.

### Table 7
Forecast error variance decomposition of the WAM at various horizons.

<table>
<thead>
<tr>
<th>Period</th>
<th>VSTOXX</th>
<th>KfW</th>
<th>EDF</th>
<th>Y_LEVEL</th>
<th>Y_SLOPE</th>
<th>WAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25</td>
<td>0.027</td>
<td>0.095</td>
<td>0.027</td>
<td>0.25</td>
<td>99.35</td>
</tr>
<tr>
<td></td>
<td>(0.83)</td>
<td>(0.51)</td>
<td>(0.63)</td>
<td>(0.52)</td>
<td>(0.78)</td>
<td>(1.44)</td>
</tr>
<tr>
<td>2</td>
<td>0.45</td>
<td>1.18</td>
<td>0.50</td>
<td>0.47</td>
<td>0.55</td>
<td>96.84</td>
</tr>
<tr>
<td></td>
<td>(1.00)</td>
<td>(1.05)</td>
<td>(0.87)</td>
<td>(0.91)</td>
<td>(0.95)</td>
<td>(1.86)</td>
</tr>
<tr>
<td>5</td>
<td>1.75</td>
<td>2.15</td>
<td>3.87</td>
<td>0.50</td>
<td>0.69</td>
<td>91.04</td>
</tr>
<tr>
<td></td>
<td>(1.40)</td>
<td>(1.51)</td>
<td>(2.12)</td>
<td>(1.00)</td>
<td>(0.99)</td>
<td>(3.16)</td>
</tr>
<tr>
<td>10</td>
<td>2.18</td>
<td>2.24</td>
<td>7.10</td>
<td>0.53</td>
<td>0.68</td>
<td>87.27</td>
</tr>
<tr>
<td></td>
<td>(1.55)</td>
<td>(1.56)</td>
<td>(3.41)</td>
<td>(0.97)</td>
<td>(0.96)</td>
<td>(4.48)</td>
</tr>
<tr>
<td>15</td>
<td>2.21</td>
<td>2.24</td>
<td>7.91</td>
<td>0.54</td>
<td>0.68</td>
<td>86.42</td>
</tr>
<tr>
<td></td>
<td>(1.58)</td>
<td>(1.56)</td>
<td>(3.96)</td>
<td>(0.97)</td>
<td>(0.95)</td>
<td>(5.03)</td>
</tr>
</tbody>
</table>

Notes: see Notes to Table 5.
Second, an important question is to what extent the incidence or size of the shocks that we treat as exogenous here can be influenced by following the prudent budgetary policies so often advocated by the European Commission and other international institutions. The Eurozone sovereign bond market is an integrated market, implying that shocks to risk aversion and to the demand for the liquidity services of short debt will be largely determined by the mood and circumstances in the market as a whole. Still, it is conceivable that budgetary prudence in the form of building up fiscal buffers during favorable economic times reduces the vulnerability of an individual country to these shocks, resulting into a more stable yield curve and, hence, more stable costs associated with issuing debt at the various maturities.

Appendix A

A.1. Derivation of (10) starting from (9)

Using a first-order Taylor approximation of \( u'(G_{2g}) \) around the point \( \pi = \pi_0 \), we can write:

\[
u'(G_{2g} (\pi)) = u'(G_{2g} (\pi_0)) + (\pi - \pi_0)u''(G_{2g}(\pi_0))G_{2g}'(\pi_0)\]

Hence,

\[
\frac{u'(G_{2g})}{E_0[u'(G_{2g})]} = 1 + (\pi - \pi_0)\frac{u''(G_{2g}(\pi_0))G_{2g}'(\pi_0)}{u'(G_{2g}(\pi_0))}
\]

Substituting this expression into the covariance term in (9), this term can be written as:

\[
Cov_t \left( \frac{u'(G_{2g})}{E_0[u'(G_{2g})]}, \pi_0 \right) = Var_t(\pi) \left( \frac{u''(G_{2g}(\pi_0))G_{2g}'(\pi_0)}{u'(G_{2g}(\pi_0))}\right)
\]

Substitute this expression into (9) and assume CARA utility, i.e. \( u(x) = -\exp(-\alpha x) \). The result follows immediately.

A.2. Proof of (11)

Take (7) and insert \( G_t' = G_{2g} \) so as to give:

\[
(1 + \pi)G_{2g} = \pi \left[ y - \overline{B_{02}} + \frac{(1 + \nu'(B_{01})) (B_{01} - \overline{B_{01}})}{\sigma \pi_0} \right] - B_{01}
\]

Hence,

\[
G_{2g} = \frac{\pi}{1 + \pi} \left[ y - \overline{B_{02}} + \frac{(1 + \nu'(B_{01})) (B_{01} - \overline{B_{01}})}{\sigma \pi_0} \right] - \frac{1}{1 + \pi} B_{01}
\]

Differentiating with respect to \( \pi \), holding constant \( \pi_0 \), and then imposing \( \pi = \pi_0 \), yields

\[
\frac{1}{\pi_0} G_{2g}'(\pi_0) = \frac{1}{\pi_0 (1 + \pi_0)^2} \left[ y + B_{01} - \overline{B_{02}} + \frac{(1 + \nu'(B_{01})) (B_{01} - \overline{B_{01}})}{\sigma \pi_0} \right]
\]

A.3. Second-order condition

We differentiate the left-hand side of (8) with respect to \( B_{01} \). Applying \( B_{01} = \overline{B_{01}} \), this yields:

\[
E_0 \left\{ \pi u'(G_{2g}) \frac{dG_{2g}}{dB_{01}} \left[ \frac{1 + \nu'(B_{01})}{\sigma \pi_0} - \frac{1}{\pi} \right] + \pi u(G_{2g}) \frac{2\nu'(B_{01})}{\sigma \pi_0} \right\}
\]

Since \( \frac{dG_{2g}}{dB_{01}} > 0 \), a sufficient, but by no means necessary, condition is that \( \pi > \sigma \pi_0 / (1 + \nu'(B_{01})) \), hence, if \( \pi \) is bounded from below at a not too low value.

A.4. Intermediate results

Differentiating (11) with respect to \( B_{01} \) and then imposing \( B_{01} = \overline{B_{01}} \) yields:

\[
\frac{1}{\pi_0} \frac{dG_{2g}}{dB_{01}}(\pi_0) = \frac{1}{\pi_0 (1 + \pi_0)^2} \left[ 1 + \frac{1 + \nu'(B_{01})}{\sigma \pi_0} \right] > 0
\]
Hence,
\[-\left[1 + \nu' (B_{01})\right] z \text{Var}_0(\pi) \frac{1}{n_0} \frac{dC_2(\pi_0)}{dB_{01}} = -\left[1 + \nu' (B_{01})\right] z \text{Var}_0(\pi) \left[\frac{1 + \sigma n_0 + \nu' (B_{01})}{\sigma(n_0(1 + n_0))} \right] < 0.\]

Further, differentiating (10) and imposing $B_{01} = \overline{B}_{01}$ yields:
\[
\frac{1}{n_0} \frac{dC_2(\pi_0)}{d\sigma} = 0,
\]
\[
\frac{d\left(\frac{C_2(\pi_0)}{n_0}\right)}{dn_0} = -\frac{(1 + 3n_0)(y + B_{01} - B_{02})}{n_0^2(1 + n_0)^3}
\]
\[
\frac{1}{n_0} \frac{dC_2(\pi_0)}{d\gamma} = 0,
\]
where the last expression is obtained for the case in which we can write $\nu' (B_{01}) = \gamma f' (B_{01})$.

A.5. The effect of $\sigma$

Differentiating (10) and evaluating at $B_{01} = \overline{B}_{01}$ yields:
\[
\left\{2 \left[1 - z \text{Var}_0(\pi) \left(\frac{y + B_{01} - B_{02}}{n_0(1 + n_0)^2}\right)\right] \nu' (B_{01}) - \left[1 + \nu' (B_{01})\right] z \text{Var}_0(\pi) \left[\frac{1 + \sigma n_0 + \nu' (B_{01})}{\sigma(n_0(1 + n_0))} \right]\right\} dB_{01} = d\sigma
\]

The term in the first pair of square brackets is positive, hence $\frac{db_{01}}{d\sigma} < 0$.

A.6. The effect of $n_0$

Differentiating (10) and evaluating at $B_{01} = \overline{B}_{01}$ yields
\[
\left\{2 \left[1 - z \text{Var}_0(\pi) \left(\frac{y + B_{01} - B_{02}}{n_0(1 + n_0)^2}\right)\right] \nu' (B_{01}) - \left[1 + \nu' (B_{01})\right] z \text{Var}_0(\pi) \left[\frac{1 + \sigma n_0 + \nu' (B_{01})}{\sigma(n_0(1 + n_0))} \right]\right\} dB_{01}
+ \left[1 + \nu' (B_{01})\right] z \text{Var}_0(\pi) \left(\frac{1 + 3n_0)(y + B_{01} - B_{02})}{n_0^2(1 + n_0)^3}\right) \frac{dn_0}{d\sigma} = 0
\]

Since the term preceding $dB_{01}$ is negative and that preceding $dn_0$ is positive, $\frac{db_{01}}{dn_0} > 0$.

A.7. The effect of $\gamma$

Let $\nu' (B_{01}) = \gamma f' (B_{01})$ and differentiating (10) with respect to $\gamma$:
\[
\left\{\left[-\frac{\sigma}{1 + \nu'(B_{01})}\right]2\nu'' (B_{01}) - \left[1 + \nu'(B_{01})\right] z \text{Var}_0(\pi) \left[\frac{1 + \sigma n_0 + \nu' (B_{01})}{\sigma(n_0(1 + n_0))} \right]\right\} dB_{01} + \left[\frac{\sigma}{1 + \nu'(B_{01})}\right] f (B_{01}) d\gamma = 0,
\]

or
\[
2\sigma\gamma f'' (B_{01}) - (1 + \nu' (B_{01}))^2 z \text{Var}_0(\pi) \left[\frac{1 + \sigma n_0 + \nu' (B_{01})}{\sigma(n_0(1 + n_0))} \right] dB_{01} + \sigma f' (B_{01}) d\gamma = 0
\]

Hence, as $f' (B_{01}) > 0$ and $f'' (B_{01}) < 0$, we find $\frac{db_{01}}{d\gamma} > 0$.

Appendix B. Supplementary material

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jimonfin.2020.102293.

References


