Quantifying systemic risk using Bayesian networks

Sumit Sourabh, Markus Hofer and Drona Kandhai develop a novel framework using Bayesian networks to capture distress dependence in the context of counterparty credit risk. Then, they apply this methodology to a wrong-way risk model and stress-scenario testing. Their results show that stress propagation in an interconnected financial system can have a significant impact on counterparty credit exposures.

Since the global financial crisis of 2007–8, modelling counterparty credit risk (CCR) has gained significant attention (see, for example, de Graaf et al 2018; Green et al 2014; Gregory 2015; Simaitis et al 2016). One of the major challenges in CCR modelling is including distress dependence between financial institutions. The financial crisis of 2007–8 has shown that the default of an institution can have a cascading effect on the whole economy. Therefore, it is crucial to quantify stress propagation as a consequence of the distress dependence between financial institutions.

Network-based models are increasingly being used for modelling systemic risk due to the complex interconnected nature of financial institutions (see, for example, Battiston et al 2016). They are being applied in diverse areas such as financial contagion, interbank networks and stress propagation as well as being used to identify early-warning signals. However, methods from network theory have not yet been incorporated into financial derivatives pricing and risk management.

Bayesian networks (BNs) provide a powerful framework to capture complex relationships between financial institutions in a holistic manner. In this article, we develop a methodology using BNs (Koller & Friedman 2009), which are well-known class of models in machine learning, to calibrate an entity’s probability of distress, conditional on the distress of a different entity, using credit default swap (CDS) data. The main novelty of our approach is that we use importance sampling to identify significant upward jumps in CDS time series as an indicator of distress. As a result, we do not need to make any assumptions about the distribution of the underlying CDS data. We apply the calibration of conditional probabilities from BNs to wrong-way risk (WWR) modelling and stress-scenario analysis.

WWR in the context of counterparty risk occurs when exposure to a counterparty is adversely correlated with its credit quality. A typical example of this is a portfolio with an emerging market counterparty where the exposure to the counterparty is adversely correlated with its credit quality. A typical example of this is a portfolio with an emerging market counterparty, where the exposure to the counterparty is adversely correlated with its credit quality.

Turlakov (2013) proposes a WWR model using the likelihood of a systemic event – which is defined as the distress of the sovereign or a stressed market event – co-occurring with the distress of a counterparty. Turlakov suggests a qualitative approach to calibrate the conditional probability parameter based on expert opinion and selective historical data. Nonetheless, a quantitative method for calibration is missing. We show that, using BNs, we can calibrate this parameter empirically to real market data.

As a second application, we can use our framework to analyse stress scenarios of distress of a single entity or multiple entities using a European CDS dataset. For instance, stress-scenario calculations have an application in the 2020 European Banking Authority (EBA) stress test, where a part of the CCR losses is quantified by the distress of the two most vulnerable counterparties in the portfolio. Based on our model, we can also include the effect these distresses have on other counterparties in the region.

Model description

In this section, we briefly introduce BNs and describe our quantitative methodology to calibrate the probability of distress of an entity, conditional on the distress of another entity. We learn the network using CDS data, where the nodes of the network represent financial entities and the edges represent the distress dependence between them. Once the network structure and parameters have been learnt, we use a sampling algorithm to calibrate the conditional probability parameter for each entity.

Bayesian networks. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a directed, acyclic graph, where $\mathcal{V}$ is a finite vertex set and $\mathcal{E} \subseteq \{(i, j) : i, j \in \mathcal{V}, i \neq j\}$ is a set of edges without any self-loops. For the graph $\mathcal{G}$, a collection $\{X_i : i \in \mathcal{V}\}$ of random variables forms a BN (Koller & Friedman 2009) over $\mathcal{G}$ if:

$$P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i | Pa_{X_i})$$

(1)

where $Pa_{X_i}$ is the set of parents of the node $X_i$ and $|\mathcal{V}| = n$. The condition in (1) allows us to calculate the joint probability distribution of random variables $X_1, \ldots, X_n$ in a tractable manner.

Learning a BN involves both structure and parameter learning, which refer to the topology and edge weights of the BN, respectively. The structure learning of a BN is an optimisation problem where we maximise the Bayesian information criterion (BIC) (Koller & Friedman 2009) score over the search space of possible configurations of the network using a hill-climbing algorithm (Koller & Friedman 2009). Starting with an empty network, edges are added, deleted or reversed in $\mathcal{G}$ until there is no further improvement in the BIC score. The structure of the network is chosen to maximise the likelihood score, using the maximum likelihood estimation (MLE) parameters:

$$\max_{\mathcal{G}} L(\mathcal{G}, \theta_{\mathcal{G}}) : \mathcal{D} = \max_{\mathcal{G}} L(\mathcal{G}, \hat{\theta}_{\mathcal{G}}) : \mathcal{D}$$

1 For a graph $\mathcal{G}$ and dataset $\mathcal{D}$, the BIC is defined as:

$$\text{score}_{\text{BIC}}(\mathcal{G}) : \mathcal{D} = \log P(\mathcal{G} | \mathcal{D}) - \frac{1}{2} |\mathcal{G}| \log N$$

where $|\mathcal{G}|$ is the number of independent parameters in the network and $N$ is the number of samples in the data. In our case, the loglikelihood term $P(\mathcal{G} | \mathcal{D})$ can be expressed as $\prod_{i=1}^{n} P(C_i | Pa_{C_i})$, where $C_i$ corresponds to an entity in a credit network.
The parameters corresponding to the edge weights that denote conditional probability are estimated based on (1) assuming a prior distribution over the parameters and (2) updating this with each instance of the data to obtain the posterior distribution: the Bayes rule. We choose a Dirichlet distribution as our prior distribution since it is the conjugate prior for the multinomial distribution, which is the distribution of our sample data.2

Learning a BN from CDS data. The sample data used for the construction of the network consists of CDS spreads for a five-year tenor. This consists of daily CDS liquid spreads of Russian entities from September 14, 2010 until August 15, 2015. Our methodology can be used on proxy CDS quotes (Sourabh et al 2018) as well. Figure 1 shows the CDS spreads for the Russian entities in the portfolio.

We use the notion of modified ε-drawups (Anagnostou et al 2018) to detect distress in CDS data. Modified ε-drawups build upon the notion of ε-drawups (Kaushik & Battiston 2013) and can identify instances in the time series where significant upward jumps have occurred. The co-movement of significant jumps implies distress dependency between two time series, which we use to learn the network structure.

A modified ε-drawup is defined as an upward movement in the time series at a local minimum, in which the amplitude of the movement, that is, the difference between the subsequent local maxima and minima, is greater than a threshold ε. Figure 2 shows an example of a modified ε-drawup. The ε parameter at time t is set to be the standard deviation in the time series between days t − n and t, where n is chosen to be 10 days; this is consistent with the choice in Kaushik & Battiston (2013). The advantage of this approach is that it is scale-free and well-adapted to the underlying volatility of the time series.

After obtaining the modified ε-drawups for the CDS time series of entity i, we define a discrete random variable $X_i^t$ such that $X_i^t = 1$ if entity i has a modified ε-drawup on day t; $X_i^t = 0.5$ if entity i has a modified ε-drawup on day $t + 1$, $t + 2$ or $t + 3$; and $X_i^t = 0$ otherwise. Using the time lag in the definition allows us to capture the fact that entities can cause each other a slight delay.

Bayesian inference. Once the BN has been learnt, we can evaluate the queries for conditional probabilities $P(i \mid j)$ for entities i and j in the network. We use the logic sampling algorithm (Koller & Friedman 2009) based on Monte Carlo simulations to perform these queries, which follows these steps. First, it samples the random variables corresponding to entities one by one in the topological order implied by the structure $G$ of the BN. This means that the variables with no parents are sampled first, followed by their children. Each sample consists of a vector of instances (either a 0 or a 1 in our case) for all the random variables in the network. Once we have generated a sufficiently large number of samples, we can estimate $P(i \mid j)$ by $n_{i,j} / n_j$, where $n_{i,j}$ is the number of samples in which random variables corresponding to both i and j take a value 1, and $n_j$ is the number of samples in which the random variable corresponding to entity j takes a value 1.

Applications.

Wrong-way risk model. For WWR in counterparty credit, Turlakov (2013) proposes the following model to calculate WWR-adjusted expected positive exposure (EPE):

$$EPE_{WWR} = P(sov \mid cpty) \cdot EPE_{est} + (1 - P(sov \mid cpty)) \cdot EPE $$

where $P(sov \mid cpty)$ is the conditional probability of sovereign distress given the entity’s distress, which we refer to as the CountryRank of the entity; $EPE_{est}$ is the EPE calculated under a stressed market; and EPE is calculated using the normal market. We calculate $EPE_{est}$ based on Turlakov (2013), but in general we can use other methods of calculating stressed exposure (see.

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2 If the prior distribution of the multinomial parameters is Dirichlet, then the posterior distribution is also a Dirichlet distribution (Koller & Friedman 2009).
for example, Hofer 2016). Given exposures EPE and EPE\_WWR, CVA and WWR, CVA can be calculated as follows:

\[
CVA = LGD \cdot \sum_{i=0}^{n} EPE(t_i) \cdot P(t_i, t_{i+1}) (cpty) \tag{3}
\]

\[
CVA_{WWR} = LGD \cdot \sum_{i=0}^{n} EPE_{WWR}(t_i) \cdot P(t_i, t_{i+1}) (cpty) \tag{4}
\]

where LGD is the loss given default, \(t_0, t_1, \ldots, t_n\) are time points until the maturity of the trade and \(P(t_i, t_{i+1}) (cpty)\) is the marginal probability of distress of the counterparty in the interval \((t_{i-1}, t_i)\).

**Stress-scenario analysis.** The calibration of conditional distress probabilities can be used to analyse the impact of distress of one or more entities on the network. For instance, one of the scenarios in the 2020 IBA stress test for banks is the distress of multiple counterparties in the portfolio. With our approach, we can also measure the direct impact these stresses have on the rest of the portfolio and therefore create a more realistic stress.

**Numerical results**

**CountryRank for Russian entities.** Figure 4 shows the structure of a BN learnt from CDS data of Russian counterparties. The entities that are more central in the network are those which have a higher CountryRank than the entities on the periphery. In figure 4, node C6 (located at the top of the tree) has a CountryRank value which is smaller than that of node C2; this is because it has a weaker direct link than node C2. Using the learned structure and parameters, we can infer the probabilities of the sovereign distress conditional on the distress of the entity, defined as the CountryRank of the entity (Sourabh et al 2020, table 2) shows the CountryRank values for the two periods. We observe that, as a result of there being a denser network due to the Russian crisis, the CountryRank values are 20% higher on average during the second period. This shows that stress propagation as a result of the distress of an entity is higher during a stressed period than during a calmer period.

**Impact on WWR model.** We test the impact of incorporating CountryRank in a WWR model using two synthetic trades. The first trade consists of a ruble payer interest rate swap (IRS) maturing in 10 years. We calculate the CVA for this trade using the default probabilities of 17 CDS quotes of Russian entities. Figure 6 shows the impact of the CountryRank parameter on the EPE\_WWR profile of the IRS. The EPE\_WRR profile is generated using a
A shock of 1,000 basis points to the Russian interest rate (IR), which is consistent with the historical change in the IR during the Russian financial crisis of 2014–15. The EPE profile, which is a weighted average of EPE and stressed EPE profiles, is directly proportional to the conditional probability of distress of the sovereign given the entity distress. In order to give a fair comparison of the impact of CountryRank on the WWR CVA, we use the same CDS level for all counterparties. The CDS spread is set to be 335bp for the sovereign and 500bp for the other counterparties. Figure 7 shows that the WWR CVA, which includes the systemic risk of an entity as opposed to CVA, increases by approximately 105% to 175% compared with CVA, depending on the CountryRank of the entity.

Our second example consists of a floating-floating USD/RUB cross-currency (XCCY) swap with five-year maturity, with notional exchange at the start and at maturity of the swap. We calculate CVA for this trade with each of 17 Russian entities, using a flat CDS spread of 335bp for the sovereign and 500bp for the other counterparties. Figure 8 shows the impact of the CountryRank parameter on the EPE profile of the XCCY swap, where the stressed EPE profile is generated using a shock of 50% depreciation in the RUB/USD exchange rate. Figure 9 shows that the WWR CVA for the cross-currency swap trade increases by approximately 75% to 140%, depending on the CountryRank of the entity. Moreover, the impact on the WWR CVA for an entity is linearly dependent on its CountryRank, which is a direct consequence of the WWR model of Turlakov (2013). Our analysis shows that WWR can lead to a significant increase in the CVA due to the highly interconnected nature of the financial system. Our network-based approach is a first attempt at quantifying systemic risk in WWR CVA in a quantitative manner. This will allow financial institutions to protect themselves against extreme market scenarios and account for entity-related credit costs more accurately.

**Stress scenarios.** We can use our framework for the calibration of conditional probabilities to analyse stressed scenarios. To demonstrate this, we considered a European CDS dataset from August 15, 2010 to March 15, 2018 consisting of entities from financials and government sectors. In our first scenario, we stressed the CDS of the Republic of Greece; its impact on selected entities is summarised in table B. Our results show that, as a consequence of financial stress on the Greek sovereign, the entities that are affected the most are financial institutions from Greece, Portugal and Spain. The impact on sovereigns and financials in the rest of Europe is limited. In the second scenario, we restrict our dataset to financial institutions only, and we stressed two Spanish banks at the same time. Table C

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4 The IR in Russia rose from 5.5% at the beginning of 2014 to 16.5% in December 2014.

5 The RUB/USD exchange rate dropped from 0.03 at the beginning of 2014 to 0.014 in December 2014.
shows the impact of the joint distress of these two Spanish financial institutions on selected other European financials. For more information on this impact on the complete list of entities, we refer the reader to Sourabh et al. (2020). In this case, we also observe that entities in southern Europe have a higher impact.

Future work
The presence of WWR in a financial system can lead to high mutual credit exposures between banks, so it is important to analyse its impact on funding collateral. The framework we have developed for calibrating conditional probabilities can also be used for collateralised debt obligation pricing. From a risk management perspective, it would be interesting to investigate whether the credit risk can be redistributed through the network via regulation or policy recommendations.

<table>
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<tr>
<th>Entity</th>
<th>Country</th>
<th>Sector</th>
<th>Conditional distress probability</th>
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<tbody>
<tr>
<td>C2</td>
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<td>Financials</td>
<td>0.728</td>
</tr>
<tr>
<td>C3</td>
<td>Portugal</td>
<td>Financials</td>
<td>0.716</td>
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<td>C4</td>
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<td>Financials</td>
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</tr>
<tr>
<td>C5</td>
<td>Portugal</td>
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<td>0.679</td>
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<tr>
<td>C6</td>
<td>Greece</td>
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<td>Financials</td>
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<tr>
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<tr>
<td>C15</td>
<td>Germany</td>
<td>Financials</td>
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</tr>
</tbody>
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Sumit Sourabh is a quantitative analyst at ING Bank in Amsterdam, Markus Hofer is a quantitative analyst at Bayerische Landesbank in Munich, and Drona Kandhai is the head of quantitative analytics at ING in Amsterdam and a professor of computational finance at the University of Amsterdam. The authors are thankful to Ioannis Anagnostou, Javier Rivero and Marcel Boersma for their valuable feedback on early results of this work. This project was partially supported by the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie Grant Agreement 675044 (http://bigdatafinance.eu): Training for Big Data in Financial Research and Risk Management. The authors report no conflicts of interest and declare that they have no relevant or material financial interests related to the research in this paper. The authors alone are responsible for the content and writing of the paper. The views expressed here are their personal views and do not necessarily reflect the position of their employer.

Email: s.sourabh@uva.nl, Markus.Hofer@bayernlb.de, B.D.Kandhai@uva.nl.

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