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The Circulation of Ideas in Firms and Markets

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This paper models the generation, circulation, and completion of new ideas, showing how markets and innovative firms complement each other in a symbiotic relationship. Novel ideas are initially incomplete and require further insight before yielding a valuable innovation. Finding the complementary piece requires ideas to circulate, which creates appropriation risks. Circulation of ideas in markets ensures efficient completion, but because ideas can be appropriated, market entrepreneurs underinvest in idea generation. Firms can establish boundaries that guarantee safe circulation of internal ideas, but because firms need to limit idea circulation, they may fail to achieve completion. Spin-offs allow firms to benefit from the market’s strength at idea completion, whereas markets benefit from firms’ strength at generating new ideas. The model predicts diverse organizational forms (internal ventures, spin-offs, and start-ups) coexisting and mutually reinforcing each other. The analysis provides new insights into the structure of innovation-driven clusters such as Silicon Valley.

Key words: innovation; market for ideas; start-ups; spin-offs

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1. Introduction

Schumpeter (1926, 1942) thought of innovations as novel combinations of existing factors. The novelty often pertains to boundary spanning insights that reconfigure existing technologies and customer needs (Weitzman 1998). The process of generating an innovation often starts with a “half-baked concept.” Incomplete ideas need to be circulated unprotected, to be combined with insight by others before a valuable novel factor combination emerges. This paper studies what economic environments are best suited for the generation and elaboration of early-stage ideas. In the tradition of Coase (1937), we focus on the distinction between firms and markets. The central finding of this paper is that firms and markets form a symbiotic relationship in the development of novel ideas, rather than constituting alternative ways of organizing invention.

We provide a formal economic theory of the creation and elaboration of early-stage ideas. Ideas are costly to develop. They are incomplete concepts that require a complementary agent (Teece 1986) with the “missing piece of the puzzle.” Because these ideas are novel, it is unknown what the missing expertise is. Ideas have to be shared openly to be completed. This creates appropriation risk. We assume that ideas are too preliminary to be patentable and that there is no effective contractual protection for the exchange of incomplete ideas (Arrow 1962, Anton and Yao 1994).

Whenever an idea bearer finds a listener with complementary insight (a so-called “complementor”), it is optimal to develop the concept jointly. Idea theft arises when the listener grasps the idea but cannot complement it, so the two agents are pure substitutes. In a market environment any agent can freely share his ideas with any other agent. Ideas circulate through a sequence of agents until they find a complementary match. The free circulation of ideas enhances the rate of elaboration. However, in a large market agents do not expect to meet again, so individual reputation fails. Ideas may be stolen, even multiple times, and be implemented by agents who were not the inventors. The model shows that in equilibrium too few market agents (and possibly none) seek to generate new ideas.

What alternative arrangements may support costly idea generation, when individual reputation fails? Established firms have long been recognized for their unique role in the innovation process (Rosenberg 1994). In our approach, firms provide a governance mechanism to overcome the opportunism in market exchanges (Williamson 1975), creating an environment for internal idea generation. They must be able to claim ownership on internal ideas to prevent unauthorized leakages (Liebeskind 1997). Innovative firms also require internal transparency so that employees can observe which employees generate ideas and which ones receive rewards. This
ensures that employees with ideas can safely circulate them internally because they would be protected from appropriation by either the firm or other employees. So an innovative firm needs to maintain a reputation to ensure that employees are willing to disclose their ideas (Kogut and Zander 1992). We use a local reputation model where the firm owner incurs some costs to ensure visibility among a finite set of agents, the firm’s employees (Kreps 1986). Under those conditions, the model shows how firms can provide effective incentives for idea generation and initial circulation. However, the size of the firm limits the set of possible matches. The optimal innovation policy is to empower employees to spin out ideas after establishing that no internal complementors could be found. Firms benefit by spinning out ideas that could not be completed internally.

The symbiotic relationship between firms and markets is built on two factors. A market failure for sufficient idea generation calls for innovative firms, whereas firm failure to elaborate all internal ideas creates a market role for idea completion. Markets thus benefit from innovative firms as a source for idea spawning, and firms benefit from the ability of markets to complete incomplete concepts. In equilibrium, markets and firms complement each other, with firms generating more ideas, and markets completing many. When the cost of generating ideas is relatively low, both markets and firms generate ideas. An increase in idea-generation costs causes a substitution from market- to firm-generated ideas, and the number of firms increases. Once the cost passes a critical level, markets cease to generate ideas. Further increases in idea-generation costs reduce the number of innovative firms. It follows that firm density is highest for intermediate idea-generation costs.

The model suggests a range of organizational structures for idea completion. Figure 1 provides a simplified graphical description. First, some ideas are generated and complemented within firms (internal ventures). Second, some ideas are generated within firms but find no internal completion, so the inventor seeks to find a complementor in the market (spin-offs). In the third case, the spin-off idea is stolen in the market and completed by entrepreneurs other than the inventor (start-up with other entrepreneur’s idea). In the fourth case, a market idea is stolen (start-up with other firm’s idea). In the fifth case, the inventor is a market agent who finds a complementor in the market (a “classic” start-up). The last two cases only occur when markets support idea generation.

Our notion of the firm is most relevant in fast changing industries where the future direction is unpredictable and where incomplete ideas require elaboration that cannot be easily planned. It only applies to highly innovative firms that actively manage their employees’ ideas. We also distinguish between established multiproject firms on one hand and newly created single-project start-ups or spin-offs on the other.

The model predictions are consistent with the history of Silicon Valley, which includes a long list of people leaving established firms to start new firms. They challenge the view that highly innovative environments are solely driven by the actions and interactions of independent entrepreneurs (Saxenian 1994). In Porter’s (1998) view, economic clusters depend on the confluence of mutually sustaining factors—in our case, an interplay between established firms generating and market entrepreneurs elaborating novel ideas. Our analysis suggests that standard empirical measures, such as patent citations or spin-offs counts, understate the full extent of the interlinkages between established and entrepreneurial firms. These measures cannot trace the circulation of early-stage ideas that are stolen, possibly multiple times. An interesting empirical implication from the model is that markets excel at idea generation when the cost of generating ideas is low; firms play their biggest role when generation costs are intermediate; for high generation costs private sector solutions fail, presumably leaving room for government and academic research.

2. The Model

2.1. Base Assumptions

There is a continuum of agents that live indefinitely, are risk-neutral, and use a discount factor of \( \delta \). Agents either work as independent market agents or as employees within a “large firm”—defined below. At the beginning of each period, agents decide whether to spend time and effort to generate a novel idea or to meet with another agent to elaborate ideas. Active agents may also choose to operate a firm. We first
discuss a market environment before considering the activities of firms and their employees.

Ideas are incomplete and require elaboration before they become valuable innovations. We assume that agents are unable or unwilling to sign binding nondisclosure agreements (NDAs) prior to listening to an idea.\(^1\) So idea elaboration requires sharing them without contractual protection. We denote idea generators by \(G\). Any agent generating an idea incurs some private cost \(\psi\) as well as the opportunity cost of not interacting with other agents for one period. Elaboration occurs by pairwise random matching of active agents. Agents with an idea (whether their own or stolen in previous periods) are denoted by \(I\) (for “idea bearers”). Agents that discuss ideas without contributing ideas themselves are denoted by \(O\) (for “opportunists”).

Successful elaboration of a novel idea requires a specific fit of complementary skills, which is idea-specific and cannot be identified prior to a match. Elaboration can be thought off as validating an idea and/or augmenting it with additional insight. The probability of an idea-specific fit is given by \(\phi\).\(^2\) If a listener fits, we call him a “complementor,” otherwise he is a “substitute.”

When an idea finds a match, it can be implemented by a cooperative effort, generating an expected net payoff \(z\).\(^3\) If both agents seek to implement the idea with someone else in a later period, competition is such that the sum of their individual returns \(z_0\) is less than the cooperative return (i.e., \(z > 2z_0\)). This ensures that once two agents are a good match, cooperation is the efficient strategy. The two agents have symmetric bargaining power ex post, and they split the value of the idea equally, both receiving \(\frac{1}{2}z\).

In a market environment, all active agents are randomly matched pairwise each period. \(I\)s and \(O\)s are indistinguishable prior to matching, and agents cannot observe each other’s prior history. When a listener is not a match, the idea bearer and the listener are perfect substitute, and each could pursue the idea further. For sufficiently low values of \(z_0\), it is never optimal to have more than one agent circulating the idea.\(^4\) The bargaining solution is that either of the two agents pursues the idea further. Although ideas can be carried across periods, each agent can remember one idea at most. Note that our model abstracts from any interactions across ideas; i.e., different ideas do not compete with each other. However, simultaneous implementation of one idea would be highly competitive. These assumptions ensure a tractable steady-state model of idea circulation.

Consider how a firm may create an environment where new ideas can be exchanged in an orderly and protected manner. The firm monitors that internal ideas circulate freely internally but are not leaked. The firm also needs to ensure that employees invest in idea generation and so needs to be credible in rewarding it. Besides a boundary, the firm thus needs an internal system for inventors to be recognized and compensated. We assume that each firm has a finite number \(q\) of employees. We justify this assumption with the impossibility of creating a global reputation mechanism; i.e., firm owners can maintain internal transparency only among a limited number of employees.\(^5\)

We assume employees face the same idea-generation costs \(\psi\) and the same probability of fit \(\phi\) within the firm. We deliberately assume a level playing field, where the coexistence of firms and markets emerges not because of differences across agent types but because of strategic complementarities between firms and markets.\(^6\)

The model endogenizes the density of firms. We assume free entry, where any new firm owner is required to make a sunk investment. Establishing external boundaries, internal transparency, and a reputation among employees is not a trivial task, so the i\(\text{th}\) entrant faces a sunk fixed cost \(K_i\), where \(K_i\) is distributed according to a cumulative distribution \(\Omega(K_i)\) with density \(\omega(K_i)\) over the range \(K_i \in [K_{\min}, K_{\max}]\). This assumption ensures an upward-sloping supply curve and can be justified by the presence of some scarce factor. We denote firm owners by \(F\) and employees by \(E\). As long as an owner operates a firm, he cannot generate or complement ideas.

In the employment contract, the firm owner claims ownership of employees’ ideas. This enables the owner to prevent employees from taking out internal ideas

\(^1\) See Hellmann and Perotti (2011) for a more detailed discussion of this assumption. Agents involved in assessing new ideas, such as venture capitalists, academic researchers, and Hollywood producers, routinely refuse to sign NDAs. If used at all, NDAs are employed at much later stages of idea elaboration, to formalize commitments to a project that is well defined and requires execution (Bagley and Dauchy 2008).

\(^2\) For simplicity, we assume that \(\phi\) is constant across all possible matches, irrespective of whether agents are in the market or inside firms, irrespective of what type the agent is himself, and irrespective of how many others previously looked at the particular idea.

\(^3\) A complementor can also prove that an idea is worthless, in which case the return is zero. Net payoff \(z\) measures the expected return to ideas, including the valuable and worthless ones.

\(^4\) Formally, the two agents prefer not to pursue the idea simultaneously whenever \(\phi z > \phi z_0 + 2(1 - \phi)z \Leftrightarrow z < (2\phi - 1)z/\phi\).

\(^5\) In addition, there may be diseconomies of scale in terms of monitoring the firm boundary, which also implies finite-sized firms.

\(^6\) In reality, the probability of fit may obviously differ between firms and markets because of comparative advantages and/or self-selection among heterogeneous agents. Still, it is difficult to find the right skill composition ex ante for new ideas whose elaboration cannot be planned.
(that is, ideas that had been reported to the firm). Under trade secret law, agents signing an employment contract agree not to steal the firm’s ideas. Trade secret law therefore supports a legal boundary that enables safe internal circulation along with controls on external leakages. To establish the trade secret, the firm needs to record in some verifiable form the inventor’s idea as an internal project. Some bureaucratic procedures and a paper trail are required for the firm’s internal reward system and for establishing the trade secret. We assume that with such a system the firm can fully prevent idea stealing. Once an idea is reported, the generator is assigned the task to implement it via internal matching. In managerial terms, he becomes the “internal project champion” or what we will call an “intrapreneur.” Because no employee can leak the idea outside the firm, the generator can count on cooperation from all internal listeners. The firm uses an internal rotation system that corresponds to random matching in markets. Employees may leave the firm at will, but they need permission from the firm to pursue any idea protected by trade secrets. Firm can be selectively porous, allowing some but not all ideas to go outside the firm boundaries.

Overall there are five distinct types of agents: idea generators (G), idea bearers (I), opportunists (O), employees (E), and firm owners (F). We denote the fraction of types by \( n_G, n_I, n_O, n_E, \) and \( n_F, \) so that \( n_G + n_I + n_O + n_E + n_F = 1. \) Figure 2 provides a graphical representation of the evolution of types, showing how and why agents may change their type from one period to the next.

### 2.2. Idea Circulation within Markets

In this section, we discuss how ideas circulate within markets. For expositional convenience, we temporarily ignore firms (i.e., we set \( n_E = n_F = 0 \)) and focus on an environment where all agents act as entrepreneurs. Central to the analysis is the density of ideas in circulation, which can be measured by the fraction of agents carrying an idea, given by \( \hat{\zeta} = \frac{n_I}{n_I + n_O}. \) This measure of idea density is determined endogenously and reflects individual agents’ choices to either develop an idea or act opportunistically.

To determine the equilibrium we ask how many agents choose to pursue a \( G, I, \) and \( O \) strategy. Agents not carrying an idea from last period will choose among a \( G \) and an \( O \) strategy. We denote all life-time utilities with \( U. \) The utility of an opportunist is given by \( U_O = (1 - \theta)\delta U_O + \theta (\frac{1}{2} z + \delta U_O) + \theta (1 - \phi)(\frac{1}{2} \delta U_O + \frac{1}{2} \delta U_I). \) The first term reflects the case where the agent is matched with another opportunist, so the immediate return is zero and the agent gets the discounted utility of being an opportunist next
The second term reflects the case where the opportunity we use an idea bearer and there is a fit, so the agent gets \( \frac{1}{2}z \) and then comes back next period as an opportunist. The third term reflects the case where the agent is matched with an idea bearer but there is no fit. In this case, the efficient solution is that the two flip an even coin. With probability one-half the agent goes back without an idea, else the agent steals the idea and becomes an idea bearer next period.

The utility of an idea bearer is independent of whether the idea has been self-generated or stolen and is given by

\[
U_I = (1 - \theta)\phi_1(z + \delta U_O) + (1 - \phi_1) \cdot \left( \frac{1}{2} \delta U_O + \frac{1}{2} \delta U_I \right) + \theta \phi_2(z + \delta U_O) + 2\phi(1 - \phi_1)(\frac{1}{2}z + \frac{1}{2} \delta U_O + \frac{1}{2} \delta U_I) + (1 - \phi_1)^2 \delta U_I,
\]

The first bracketed term reflects the case where the agent is matched with an opportunist. With chance \( \phi \) there is a fit and the pair implement the agent’s idea, after which the next expected period payoff equals \( \delta U_O \). If there is no fit, with probability one-half the agent retains the idea for the next period, otherwise the opportunist takes away the idea. The second bracket term reflects the case where two idea bearers are matched. When both ideas fit, each agent gets \( z \). When only one fits, the payoff is \( \frac{1}{2}z \) plus a half chance to take the idea further as before. When none of the ideas fit, each agent carries his idea forward.

The utility of a generator is given by

\[
U_G = \delta U_I - \psi
\]

which equals its expected payoff of an idea bearer minus the cost of developing the idea. Obviously, \( U_G < U_I \), because it is more profitable to already have an idea than to incur some generation cost to produce one.

Entrepreneurs may have insufficient incentives for generating an idea or excessive incentives to be an opportunist stealing other ideas. Any agent without an idea can choose between generating an idea versus listening to others’ ideas, implying \( U_G = U_O \). This condition determines the density of ideas, measured by \( \theta \).

Intuitively, there is an equilibrating mechanism in the market, so if there are too few (many) idea generators, the return to being an opportunist becomes relatively low (high), encouraging more (fewer) agents to generate ideas.

The following proposition describes a pure market equilibrium, where variables are indexed by \( M \). We compare the market outcome to the benchmark of a socially efficient equilibrium (indexed by \( S \), defined as the allocation that maximizes the sum of all utilities.

We define

\[
\Delta = \frac{\phi z}{2 - \delta + \phi \delta}, \quad \psi^M = \delta \Delta, \quad \text{and} \quad \psi^S = \frac{\delta \phi z}{1 - \delta + \delta \phi}.
\]

Note that he may also come back as a generator. As shown below, in equilibrium we always have \( U_C = U_O \), so that with loss of generality we use \( U_C \) in the utility expressions.

### Proposition 1 (Market Equilibrium)

If \( \psi \geq \psi^M \), then no ideas are generated in the market. If \( \psi < \psi^M \), then the following applies:

(i) The equilibrium fraction of idea bearers is given by

\[
\theta^M = \frac{\delta - \psi}{\Delta} < 1,
\]

(ii) The utilities are given by

\[
U_G = U_O = \frac{\theta^M \Delta}{1 - \delta} = \frac{\delta \Delta - \psi}{1 - \delta} \quad \text{and} \quad U_I = \frac{\Delta - \psi}{1 - \delta},
\]

which are all increasing in \( z \) and \( \phi \) and decreasing in \( \psi \).

(iii) The equilibrium number of generators is given by

\[
n_G^M = \theta^M \phi_1/(1 + \theta^M \phi),
\]

which is increasing in \( z \) and \( \phi \) and decreasing in \( \psi \). The equilibrium number of opportunists is given by

\[
n_O^M = (1 - \theta^M)/(1 + \theta^M \phi),
\]

which is increasing in \( z \) and \( \phi \) and decreasing in \( \psi \). The equilibrium number of idea bearers is given by

\[
n_I^M = \theta^M / (1 + \theta^M \phi),
\]

which is increasing in \( z \) and decreasing in \( \psi \). It is also increasing in \( \phi \) for larger values of \( \psi \) but decreasing in \( \phi \) for smaller values of \( \psi \).

(iv) In comparison to the socially efficient outcome, the market equilibrium has a smaller feasible range (i.e., \( \psi^M < \psi^S \)), fewer generators (\( n_G^M < n_G^S \)), fewer idea bearers (\( n_I^M < n_I^S \)), more opportunists (\( n_O^M > n_O^S = 0 \)), a lower utility for generators (\( U_G < U_G^S \)), and a lower utility for idea bearers (\( U_I < U_I^S \)).

Proposition 1 shows idea generation in the market is feasible for a limited range of idea-generation costs. For any \( \psi \in [\psi^M, \psi^S] \), idea generation is socially desirable but will not be achieved in a market exchange. Even if idea generation is feasible in the market, some agents participate in elaborating ideas without contributing any, whereas the efficient equilibrium contains no opportunists (i.e., \( \theta^M < 1 = \theta^S \)).

The comparative statics are largely intuitive, but the effect of \( \phi \) on \( n_I \) is more subtle. A higher likelihood of fit encourages idea generation (higher \( n_I \)), which increases the flow of new idea bearers (leading to higher \( n_I \)). The steady-state fraction of idea agents (\( n_I \)) is also affected by the speed at which ideas are completed. Higher values of \( \phi \) imply faster idea completion, which decreases \( n_I \). The net effect can go either way. Intuitively, the idea-generation effect becomes more important at higher levels of idea costs, where entrepreneurs require stronger incentives to generate ideas. The appendix formally shows that there exists a critical value \( \psi^* \in (0, \psi^M) \) such that the “old” idea effect (more idea generation) dominates the “old” idea effect (faster idea completion) so that \( n_I \) is increasing in \( \psi \) if and only if \( \psi > \psi^* \).

### 2.3 Optimal Firm Policies

In this section, we examine how firms devise optimal policies for managing their employees’ ideas. Once an employee has generated an idea, he can talk to any of the firm’s employees. We denote the number of agents...
that an idea bearer talks to within a firm by $Q$. $Q$ differs from $q - 1$ because in equilibrium there is some employee turnover, giving an idea bearer additional agents to talk to. The probability that an agent finds no match inside the firm is given by $(1 - \phi)^q$, and the probability of internal completion is $1 - (1 - \phi)^q$.

Idea generation and circulation within firms require several conditions. Employees must be willing to join the firm. They must have an incentive to generate ideas. Once ideas are generated, employees must have an incentive to disclose them to the firm. The firm must be able to commit to rewarding idea generation and ensure that employees do not take ideas outside the firm.

The firm compensates employees for generating ideas by paying a bonus $B$ to any employee who generated an idea that was internally implemented.\(^9\) In our model it is optimal for the firm not to provide any compensation for idea completion. Providing feedback is costless in this model, and the firm does not want to create incentives that detract employees from idea generation (i.e., it does not want to encourage employee turnover, giving an idea bearer additional matches, he leaves the firm and becomes an idea bearer in the market). Because each agent must first generate an idea, the ante utility of joining a firm is given by $U_k = -\psi + \delta U_{f, j}$. Firm profits, denoted by $\Pi$, are the sum of per employee profits, i.e., $\Pi = qU_k$, where $U_k$ is the firm's lifetime profit from one employee's position. It behaves very similarly to $U_{f, j}$ above, namely, $U_f = \delta U_{f, j}$, $U_{f, j} = \phi(z - B + \delta U_t) + (1 - \phi)\delta U_{f, j+1}$, and $U_{f, Q+1} = U_{f}$.

Consider now the firms' entry decisions. Let $\Pi$ denote firm profitability, which is the same for all firms. Free entry implies that agents create firms until the marginal benefit equals their outside opportunity cost, i.e., until $1 - K > U_0$. In equilibrium, the number of firms is thus given by $n_f = \Omega(\Pi - U_0)$. The fraction of agents working in firms as employees is given by $n_f = qn_f$.

Firms are never viable if the entry cost of the first entrant, denoted by $K_{min}$, is too high nor if the cost of generating ideas $\psi$ is too high. We denote $\psi^F$ as the highest value for which there can be idea creation within firms. The appendix shows that for $K_{min}$ sufficiently small, there exists a range of values $\psi \in (\psi^M, \psi^F)$ where firms can generate ideas whereas markets cannot.

A reputation condition ensures that firm owners prefer to maintain their reputation over a deviation where they refuse to pay out bonuses. The maximal deviation gain would occur in the rare event that all employees implemented an idea at the same time, warranting total bonuses of $qB$. If an owner were to refuse to pay these bonuses, he would cash them for himself and then start all over again as a normal agent that obtains a utility $U_0$. The reputation condition is therefore given by $\delta \Pi > qB + dU_0$, which ensures that the ongoing profits from operating a firm exceed the one-time benefit from refusing to pay out bonuses. It is easy to show that this condition is always satisfied for $\delta$ sufficiently close to 1.

We are now in a position to state the properties of the firm's optimal compensation policies. For this, it is useful to define $\bar{\phi} = \sum_{i=0}^Q (1 - \phi)^{-1} \delta^i$, $\bar{\phi} = \sum_{i=0}^Q (1 - \phi)^{-1}$, $B^* = [\psi + \delta \Lambda - (1 - \phi)^{2}\delta^{Q+1}\Lambda]/\phi^2$, and $\bar{U} = [\phi^2 z + (1 - \phi)^{2}\delta^{Q+1}\Lambda - \psi]/(1 - \delta)$.

**Proposition 2.** (i) The firm's optimal compensation is given by $B^*$, which ensures that employees have an incentive to generate and disclose ideas rather than leaving the firm without reporting them. If $\psi < \psi^M$, then $U_f = \bar{U}_0$ and $U_{f, i} = U_i, \forall i = 1, \ldots, J$. If $\psi \in [\psi^M, \psi^F)$, then $U_f = \bar{U}_0$ and $U_{f, i} > U_i, \forall i = 1, \ldots, J$.

(ii) The firm's profits per employee are given by $\Pi/q = \bar{U} - \bar{U}_0$.

(iii) The fraction of employees that generate versus elaborate ideas is given by $f_{el} = 1/(1 + \bar{\phi})$ and $f_{l} = \bar{\phi}/(1 + \bar{\phi})$.

Within a firm, employees always have an incentive to generate ideas because this is the only way of receiving any compensation. Part (i) shows that the optimal compensation ensures that an idea generator has an incentive to disclose his idea within the firm. Whenever $\psi < \psi^M$, the incentive constraint is satisfied with equality (i.e., $U_{f, j} = U_t$). For $\psi \in [\psi^M, \psi^F)$, there is slack in the incentive constraint (i.e., $U_{f, j} > U_t$) so that employees strictly prefer disclosing over leaving.
Part (i) also shows that in equilibrium the employees’ ex ante participation is always binding; i.e., agents are indifferent between becoming an employee or being an opportunist in the market \((U_c = U_o)\). This condition allows us to solve for the closed-form solution for \(B^*\).

Part (ii) expresses the firm’s steady-state profits, which can be expressed as the total value of ideas implemented in the firm (denoted by \(\bar{U}\)) minus the employees opportunity costs \(U_o\). An important insight for the analysis of the coexistence equilibrium is that the firm’s profits are negatively affected by the returns to opportunism in the market.

Part (iii) derives the steady-state task allocation within the firm. We denote the fraction of generators by \(f_G\) and the fraction of idea bearers by \(f_I\). The fraction of generators \(f_G\) is technologically determined and does not depend on the market equilibrium payoff. It is increasing in \(\phi\) so that if idea implementation becomes easier, there is more time to generate ideas.

Finally note that, conditional on finding a match, there are no differences in the expected speed of completing ideas inside firms and markets. However, the probability of completion is given by \(1 - (1 - \phi)^G < 1\) inside firms, yet it equals to 1 in the market, where ideas circulate until they get resolved. Naturally, the probability of a generator implementing his own idea in the market is strictly smaller than 1—see appendix.

### 3. Coexistence Equilibria

#### 3.1. Coexistence When Only Firms Generate Ideas

In this section, we examine the full model where firms and markets interact. We divide our discussion into two parts. In §3.1, we consider the case where markets fail to generate ideas, i.e., \(\psi > \psi_M\). In §3.2, we consider the case where \(\psi < \psi^M\) so that markets generate innovations.

We now characterize a coexistence equilibrium where all ideas are created inside firms, but markets play a role circulating and elaborating ideas. Agents choose to belong to either the firm sector as owners or employees, or they join the market sector where they generate ideas or participate in the circulation of ideas. Note that for \(\psi > \psi_M\), the market fails to generate new ideas and firms are necessary to create a protected local environment for idea generation.

Figure 2 shows how at the end of each period, employees can leave their firm, and market agents can become employees. Indeed, for every employee that leaves, the firm hires a new employee. The fraction of employees leaving the firm sector at the end of each period is given by \((1 - \phi)^C f_I\), which is the fraction of idea bearers who did not find an internal match. The total number of employees leaving firms is thus given by \(n_F(1 - \phi)^C f_I\). For \(\psi > \psi_M\), departing employees are the only idea generators. The density of ideas in the market (the fraction of idea bearers) is thus given by \(n_I = n_F(1 - \phi)^C f_I / \phi\). Using \(n_F + n_E + n_I + n_O = 1\), straightforward calculations reveal that

\[
\theta^M = \frac{q_n F}{1 - (q + 1) n_F} \frac{(1 - \phi)^Q}{\phi} \frac{\gamma \hat{\phi}}{1 + \gamma \hat{\phi}}.
\]

The higher the density of firms, the higher the fraction of idea bearers in the market. In this case, the utility of being an opportunist in the market is given by \(U_o = \theta^M \Delta / (1 - \delta)\), which can be expressed as

\[
U_o = \frac{q n_F}{1 - (q + 1) n_F} \frac{(1 - \phi)^Q}{\phi} \frac{\tilde{\phi} - \Delta}{1 + \tilde{\phi} - 1 - \delta}. \quad (\text{b})
\]

This market equation (b) expresses the utility of market agents as a function of the firm density \(n_F\). The following summarizes the key properties of the \(\tilde{m}\) curve.

**Market Equilibrium (Part 1).** For \(\psi \in [\psi^M, \psi^F]\), the \(\tilde{m}\) curve is upward sloping; i.e., \(U_o\) is increasing in \(n_F\). For a given \(n_F\), \(U_o\) is increasing in \(z\), independent of \(\psi\).

Clearly, the utility of independent agents increases with the number of firms. More firms means that more ideas leak out into the market, increasing the likelihood that an opportunist encounters an idea. Note also that \(U_o\) is independent of generation costs \(\psi\) because ideas are not generated in the market.

Our next step is to solve for the equilibrium firm density. The firm’s entry condition is given by

\[
n_F = \Omega(q U_F - U_O) = \Omega(q \bar{U} - (q + 1) U_O). \quad (\text{c})
\]

This firm equation (c) expresses the firm density \(n_F\) as a function of market utility \(U_o\). The \(\bar{F}\) curve is essentially a measure of firm profitability, which under free entry determines the number of firms. The following summarizes its key properties.

**Firm Equilibrium.** The \(\bar{F}\) curve is downward sloping; i.e., \(n_F\) is decreasing in \(U_o\). For a given \(U_o\), \(n_F\) is increasing in \(z\) but decreasing in \(\psi\).

The main insight is that a higher utility for market agents increases the firm’s employment costs and thus reduces the density of firms. The number of firms is higher when ideas are more valuable (higher \(z\)) and generation is easier (lower \(\psi\)).

Because \(\tilde{m}\) is upward sloping and \(\bar{F}\) is downward sloping, there exists a unique equilibrium.

**Proposition 3.** (i) For \(\psi \in [\psi^M, \psi^F]\), there exists an equilibrium such that all ideas are generated inside firms,
but a fraction \((1 - \phi)^2\) is implemented in the market. The organizational structures for idea completion are internal ventures, spin-offs, and start-ups where the idea was originally developed by some firm employee.

(ii) An increase in \(\psi\) decreases \(U_O\) and \(n_F\).

(iii) An increase in \(z\) increases \(U_O\) and increases \(n_F\) provided \(n_F\) is not too large.

Proposition 3 says that for \(\psi \in [\psi^M, \psi^F]\) firms hire employees to generate ideas, whereas market agents wait for firm ideas that cannot find an internal fit. In equilibrium we observe three organizational structures for idea completion—see Figure 1. In the first case, the idea is generated and complemented internally. In the second case, there is no internal completion, but the employee is allowed to spin off his idea, and he finds a complementor in the market. In the third case, the employee also leaves the firm, but his idea is stolen in the market and gets completed by a team of entrepreneurs that no longer includes the original idea generator.

The equilibrium of Proposition 3 occurs at the intersection of the \(\mathcal{M}\) and \(\mathcal{T}\) curves. Higher generation costs \(\psi\) decrease the number of firms and the utility of market agents. This is because a higher \(\psi\) shifts the \(\mathcal{T}\) curve down without affecting the \(\mathcal{M}\) curve. Increasing the value of ideas \(z\), the utility of market agents is always increased, but the effect on the density of firms is ambiguous. Intuitively, a higher value of ideas increases firm profits and the density of firms \(n_F\), as reflected in the outward shift of the \(\mathcal{T}\) curve. However, a higher value of ideas also increases the utility of market agents and thus the cost of hiring employees. As shown in Proposition 4, this generates more opportunity for firms, explaining why the density of firms \(n_F\) is increasing in \(\psi\).

Proposition 4 has two additional organizational structures for idea completion, compared to Proposition 3. One is a “classic” start-up where one of the founders generated the idea and the second founder complements it. The other is a start-up where the original idea was taken from another market agent—see Figure 1.

Propositions 3 and 4 have different comparative statics for \(\psi\). Figure 3 shows how the number of firms depends on \(\psi\). Proposition 4 shows that for lower values of \(\psi\), the number of firms is increasing in \(\psi\). The intuition is that higher generation costs discourage idea creation in both firms and markets. Markets are more affected because of the stealing problem, so market idea generation declines rapidly with \(\psi\), as shown in Figure 3. This rapid decline reduces the utility of market agents and thus also the cost of hiring employees. As shown in Proposition 4, this generates more opportunity for firms, explaining why the density of firms \(n_F\) is increasing in \(\psi\).

For \(\psi > \psi^M\), higher generation costs discourage idea generation within firms, and \(n_F\) decreases with \(\psi\). Firms cease to exist beyond \(\psi^F\). Overall, we note that the number of firms is highest for intermediate values of \(\psi\) reaching its maximum at \(\psi^M\). The key intuition is that for higher values of \(\psi\), fewer firms can afford idea generation, but that for lower values of \(\psi\), markets replace firms as the main source of idea generation.
The introduction of firms does not lead to a socially efficient allocation. All the coexistence equilibria have some opportunists in the market, and firms incur fixed cost, neither of which occurs in a social first-best equilibrium. However, compared to an equilibrium without firms, the introduction of firms is socially beneficial. If some agent chooses to start a firm, others are either indifferent or better off, such as when they benefit from an idea that is spun off the new firm. Hence firm entry is socially desirable.

4. Discussion
This section discusses the main insights of the model. Our theoretical model establishes the limits of free idea exchange for idea elaboration and predicts a symbiotic relationship between firms and markets. It explains the simultaneous occurrence of alternative organizational structures for innovation. The results relate to a variety of literatures that examine innovation in firms or markets.

The model provides some new insights into understanding high technology clusters such as Silicon Valley. Rather than viewing them solely as a hotbed of entrepreneurial activity, our theory emphasizes the importance of established firms in an interdependent innovation process. This view builds on the seminal work of Saxenian (1994), which emphasizes the open exchange of ideas and “cross-pollination” as the main causes of Silicon Valley’s innovative success. Yet a careful read of the history of Silicon Valley, and similar clusters elsewhere, also emphasizes the contribution of other factors, notably the presence of established firms (Porter 1998, Bresnahan et al. 2001). On Wikipedia’s 2009 list of the 50 largest technology firms globally, 12 are U.S.-based firms, half of which are located in Silicon Valley.

The notion of a symbiotic relationship between new and established firms is related to Agarwal and Cockburn (2003), who use patent citations to show that large R&D intensive firms play an anchor role in regional innovation systems. Their results are highly consistent with our model. From our perspective, patent citations only reveal the tip of the iceberg. They only measure those ideas that were successfully completed and could be patented, whereas we are concerned with earlier stages of the innovation process where ideas are too preliminary to be patented. Thus, there may be many more ideas that flow from the anchor firms to the local start-ups than those captured by patent citations.

At these earlier stages, ideas are mainly protected by trade secret laws, and its close cousin, non-compete covenants. Hyde (1998) and Gilson (1999) argue that Silicon Valley’s success stems in part from the loose enforcement of trade secret and non-compete laws. Stuart and Sorensen (2003) and Marx et al. (2009) provide empirical support, showing that new firm formation and labor mobility are higher when enforcement of non-competes is weaker. Our model also suggests that labor mobility is required for the symbiotic relationship between large innovative and small entrepreneurial firms. However, our model also suggests a boundary condition for the above argument. If firms could not protect any trade secrets at all, incentives for innovation would be severely stunted. Our model shows that innovative firms need to maintain a delicate balance between open and close firm boundaries, authorizing some but not all leakages. That is, they need to be selectively porous. Furthermore, our model suggests that what matters is that firms have a right of first refusal on internally generated ideas. Such a right may ultimately be enforced through trade secret laws, but in practice it also requires some employee loyalty. An interesting case is Gene Amdahl, who pleaded for a long time to implement his ideas within IBM, before finally starting Amdahl Computers.

Our model accounts for the coexistence of distinct organizational structures for idea completion (see Figure 1): internal ventures, spin-offs, and three distinct types of start-ups, where the idea was generated by one of the founders, by some other entrepreneur, or by some employee inside an established firm. This departs from much of the prior literature that presumes “classic” start-ups that commercialize their founders’ ideas. Because idea stealing occurs in equilibrium, our model generates “nonclassic” types of start-ups where none of the founders generated the original idea (although one of them contributed to its elaboration).

Following Teece (1986), there is a strategy literature that looks at how and when start-ups align with complementary asset providers to commercialize their innovation (Stuart et al. 1999, Gans and Stern 2003, Hsu 2006). So far this literature has focused on a later stage of innovation where it is known who owns the complementary asset. In our model of early-stage ideas, it is the process of finding a complementor itself that creates appropriation risk.

Our model shows that spin-offs play a central role in the codependency of firms and markets. A large

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12 One important difference is that in our model, there is no conflict around employee departures, whereas the above literature presumes such a conflict. See Rajan and Zingales (2001), Cassiman and Ueda (2006), Hellmann (2007), and Klepper and Thompson (2010) for theories that address conflicts between firms and employees.
13 One conjecture is that this balance requires an intermediate level of intellectual property enforcement. If enforcement is too tight, market institutions for idea circulation may remain underdeveloped, but if it is too lax, firms lose their ability to protect themselves and their employees from excessive appropriation.
14 Related theory models include Anton and Yao (1994, 2002), Baccara and Razin (2007), and Biais and Perotti (2008).
prior literature examines the importance of spin-offs for innovation clusters. Indeed, any history of Silicon Valley comprises a long list of talented people leaving large firms with novel ideas. In the semiconductor industry, each generation of new firms was started by employees leaving their parent firms, and similar experiences occurred in the laser and computer storage industry. Consistent with our model, Klepper and Sleeper (2005) find that lack of internal fit is an important determinant of spin-offs activity. Agarwal et al. (2004) and Gompers et al. (2005) provide further evidence on the role that large corporations play in entrepreneurial spawning. Note that the empirical measures for spin-offs are likely to underestimate the full impact of the parent companies. This is because, as shown in the third case of Figure 1, some of the ideas that get spun out of firms get stolen in the market. By the time the idea finds its complemenitor, it no longer involves any of the firm’s employees and can therefore no longer be traced back to the true parent firm.

Many employee ideas are implemented internally. Companies such as Google or 3M pride themselves of continually generating new ideas in house (The Economist 2009, Bartlett and Mohammed 1995). Using patent citation data, Almeida (1996), Singh (2005), and Branstetter (2006) all find that knowledge diffuses more easily within than across firms, especially across geographic distances. Our model has a parsimonious description of knowledge management in firms, focusing only on innovative ideas, not knowledge routines (Cyert and March 1963, Kogut and Zander 1992, Grant 1996, Garicano 2000). Monitoring of firm boundaries plays a central role in our model (Liebeskind 1996, Chou 2007). Consistent with this, Azoulay (2004) finds that pharmaceutical firms, while actively outsourcing other projects, maintain strong firm boundaries around knowledge intensive projects.

One open question in this literature is why innovative people ever choose to work for established firms, where they do not own their ideas and where the innovator’s returns appear to be much lower. Our model provides several clues. First, similar to Lewis and Yao (2003) we show that allowing employees to spin off their ideas helps firms to attract creative employees in the first place. Second, the firm’s commitment to allow the idea generator to become an intrapreneur (and possibly the spin-off entrepreneur) preserves good incentives for idea generation. Third, our model provides a new perspective on the returns to idea creation in firms and markets. A common perception is that returns for entrepreneurs are greater than for intrapreneurs. This perception is anecdotal, largely shaped by looking at successful outcomes and therefore prone to selection biases. In our model, entrepreneurs and intrapreneurs achieve the same utility, but the structure of their payoffs is quite distinct. Specifically, intrapreneurs receive a lower compensation in case of success, but they have a higher probability of being involved with the implementation. Entrepreneurs by contrast capture a larger value of their ideas if they succeed to hold on to them, but they are less likely to be part of the team that implements the idea.

An interesting empirical implication concerns the costs of generating new ideas. For low generation costs, we find that markets work relatively well, although firms coexist and also generate some new ideas. For intermediate costs, firms perform relatively well because of their ability to manage employee incentives, although markets coexist and also generate some new ideas. For high generation costs, neither firms nor markets are able to sustain idea generation. We might expect government and academia to subsidize this type of research. Our prediction about when markets, firms, and academia perform best does not depend on financial constraints. An alternative hypothesis is that entrepreneurs are better at generating cheaper ideas merely because they are financially constrained, lacking the established firms’ access to capital. Empirically the two hypotheses can be distinguished by comparing idea generation across clusters with different degrees of financing constraints, possibly using the availability of venture capital as a proxy for financial constraints. Our model predicts that the comparative advantages of markets, firms, and academia hold across all clusters, irrespective of the availability of venture capital, whereas the financial constraints hypothesis suggests that these comparative advantages should diminish or vanish in environments where venture capital is abundant.

In our model idea generation is costly, but elaboration is costless. Moreover, each idea is unique, but different agents can complement it. Incentives are therefore needed for idea generation but not for elaboration. A large literature examines the balance between early and late innovators in the context of sequential innovation, focusing in particular on the role of patents in favoring early innovators (Gallini and Scotchmer 2002). Our model is not geared to assess the desirability of patenting itself because it assumes that elaboration is costless and because it

15 There are several ways of interpreting idea-generation costs (each suggesting different empirical proxies): finding new ideas involves direct costs (capital and expenses), implies opportunity costs (forgone production/salaries), and may require prolonged times of attention (research time horizons).

16 Interestingly, academic institutions and government also tend to be the strongest proponents for “open science,” the free circulation of scientific ideas through publications (Aghion et al. 2008, Stephan 2010).
focuses on novel ideas that are too preliminary to be patented.\textsuperscript{17}

Our model has other limitations. Modeling idea diffusion (where more agents may be carrying the same idea over time) is complex, so we focus on the case of idea circulation. Moreover, ideas do not interact with each other, so we ignore how ideas compete with each other for establishing new dominant designs or how they augment each other in complementary systems. Our model of firms assumes a fixed size for all firms and simplifies the complexity of managing internal innovation. We focus on the generation and circulation of ideas, leaving aside how markets and firms may interact for the commercialization or production of the resulting innovations. Finally, the paper establishes the symbiosis of markets and firms, ignoring alternative organizational forms such as social networks.

5. Conclusions

This paper examines how different economic environments enable the elaboration of early-stage ideas. It identifies a fundamental codependency between firms and markets in the development of innovations. Ideas are inherently incomplete and require some complementary elaboration. At the outset it is difficult to know who could provide the complementary piece. Ideas therefore need to circulate, but this exposes them to appropriation risk. A free circulation of ideas in a market setting is efficient for elaboration but fails to fully reward generation efforts. Creative individuals may voluntarily join firms with reputational capital to ensure that their ideas receive feedback without being stolen. Firms create legal boundaries, based on trade secret law, which enable internal circulation and prevent idea theft. Yet firms have limited capacity to elaborate ideas internally and may therefore allow some employees to spin off their ideas. The model identifies a natural symbiosis between the ability of firms to sustain idea generation and the comparative advantage of markets in elaborating ideas.

Our result on the coexistence of firms and markets, and the simultaneous occurrence of alternative models of idea completion, challenges some of the traditional thinking in the strategy literature. A typical explanation for diversity in strategies relies on a “contingency logic” (Lawrence and Lorsch 1967): Different types of firms (or different organizational structures) pursue different approaches because they each face a different institutional context. In our model, all agents are identical, and they all face the same institutional context, yet they pursue distinct innovation strategies. Our explanation of organizational diversity does not depend on a contingency logic but instead emerges from the symbiotic interplay of two alternative ways of structuring economic activity—markets and firms. This suggests new ways of thinking about the diversity of organizational structures, focusing on their endogenous interactions rather than their contextual differences.

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Appendix

Proof of Proposition 1. Throughout the appendix, we define $D = 1/(1 - \delta)$. We conveniently rewrite $U_0 = (1 - \theta)\delta U_0 + \theta(2\phi z + \delta U_0) + \theta(1 - \phi)(\frac{1}{2}\delta U_0 + \frac{1}{2}\delta U_1)$ as $U_0 = \delta U_0 = \theta(2\phi z + \theta(1 - \phi)(\frac{1}{2}\delta U_0 + \frac{1}{2}\delta U_1)$, and $U_1 = (1 - \theta)(\frac{1}{2}\delta U_0 + (1 - \phi)(\frac{1}{2}\delta U_0 + \frac{1}{2}\delta U_1) + \theta(1 - \phi)(\frac{1}{2}\delta U_0 + \frac{1}{2}\delta U_1) + (1 - \phi)^2\delta U_1)$ as $U_1 - \delta U_0 = (1 - \theta)(\frac{1}{2}\delta U_0 + (1 - \phi)(\frac{1}{2}\delta U_0 + \frac{1}{2}\delta U_1) + \theta(1 - \phi)(\frac{1}{2}\delta U_0 + \frac{1}{2}\delta U_1)$.

We obtain after transformations $U_1 - U_0 = \phi(\frac{1}{2}z + \phi z) + (1 - \phi)(\frac{1}{2}z + \phi z)U_0$ so that $U_1 - U_0 = \phi z/(2 - \delta + \phi\delta) = \Delta$. For future reference, we note that $d\Delta/z = \phi/(2 - \delta + \phi\delta) > 0$ and $d\Delta/d\phi = (2 - \delta)\omega/(2 - \delta + \phi\delta) > 0$. Using $\Delta$ in the above, we obtain after transformations $U_0 = D\Delta$ and $U_1 = D + D\Delta = D(1 + \theta - \theta\Delta)$. Moreover, $U_0 = \delta U_0 - \psi = D\Delta = \sum\Delta + \psi = \Delta - \psi$.

Suppose for now that $U_0 \geq 0$. Equilibrium requires that $U_0 = U_0$ or else no agent would be willing to generate ideas. Using the above expressions for $U_0$ and $U_0$, we obtain $d\Delta/d\psi = \psi/(2 - \delta - \psi\Delta)$ such that $\psi = \psi(\Delta)$. Note that $\theta < 1$ because $1 > \delta > \psi + \Delta$. We rewrite the market utilities as $U_0 = U_0 = D\Delta - D\psi$ and $U_1 = D\Delta - D\psi$. For future reference, we note that $d\theta/d\psi = -1/\Delta < 0$, $d\theta/dz = \psi/\Delta\psi > 0$ and $d\theta/d\phi = \psi(2 - \delta) > 0$.

Note that $\theta \geq 0$ whenever $\delta - \psi/\Delta \geq 0 \Leftrightarrow \psi \leq \delta = \psi^M$. At $\psi = \psi^M$ we have $U_0 = U_0 = 0$. For $\psi > \psi^M$, there is no idea generation in markets, but for $\psi < \psi^M$, markets allow for idea generation.

For the comparative statics, we have $dU_0/d\psi = -D < 0$, $dU_0/dz = D\phi/(2 - \delta + \phi\delta) > 0$, and $dU_0/d\phi = D\phi/(2 - \delta + \phi\delta)^2 > 0$.

\textsuperscript{17} See Jaffe and Lerner (2004) and Boldrin and Levine (2008) for an extensive discussion of the patenting system.
Because \( U_0 = U_C \) we have the same results for \( \dot{U}_C \). For \( U_t \), we use \( U_t = \Delta + U_0 \), so that
\[
\frac{dU_t}{dp} = \frac{dU_t}{dp} < 0, \quad \frac{dU_t}{dz} = \frac{dU_t}{dz} + \frac{d\Delta}{dz} > 0 \quad \text{and} \quad \frac{dU_t}{dp} = \frac{dU_t}{dp} + \frac{d\Delta}{dp} > 0.
\]

To derive the fractions \( n_G, n_I, \) and \( n_O \) (where \( n_C + n_I + n_O = 1 \)), note that the basic flow equation for idea bearers is given by \( n_{I} = (1 - \phi)n_{I,1} + n_{G,1} \), where the subscript \( t \) denotes periods. In steady state, we have \( n_t = n_G/\phi \). Using \( n_C + n_I + n_O = 1 \), we obtain \( \phi n_I = 1 - n_O - n_I \). From \( \theta = n_I/(n_O + n_I) \), we obtain \( n_O = 1/(1 - \theta) n_I \), which we use to obtain \( \phi n_I = 1 - (1 - \theta) n_I \). Thus, \( n_O = (1 - \theta)/(1 + \theta \phi) \) and \( n_C = \phi/(1 + \theta \phi) > n_O \) and \( n_I = 0 < n_O \).

For each match, the probability of a fit is given by \( \theta(\phi + 1 - \phi) + (1 - \theta) \phi = (\phi) \), but the probability that the idea bearer keeps his idea is given by \( \theta(\phi + 1 - \phi) + (1 - \theta) \phi = (\phi) \). Thus, the probability of a generator implementing his idea is given by \( \phi + \xi \phi + \xi^2 \phi + \cdots = 5 \sum_{n=0}^{\infty} \xi^n = \phi/(1 - \xi) = 2\phi/[2 - (1 - \phi)(1 + \theta - (1 - \theta))] \).

**Proof of Proposition 2.** Let \( U_{E1} \) denote the utility of a newly starting employee (same for an old employee without ideas) and let \( U_{E1} \) be the utility of an employee that is about to talk to the \( j \)th internal match. We have \( \dot{U}_E = -\psi + \delta U_E, \dot{U}_{E1} = \phi(B + \delta U_E, (1 - \phi)\delta U_{E1,j+1}) \) for any \( j = 1, \ldots, Q \) and \( U_{E1,0} = U_E \), which is the utility of leaving the firm. Using the above equations, we obtain after transformations \( \dot{U}_E = -\psi \delta U_{E1} = (1 - \phi)\delta U_{E1,j+1} \) for all \( j \). Define \( \delta = \sum_{j=0}^{\infty} \delta_j \) as the utility that \( U_E = (1 - \phi)\delta U_{E1,0} \). Let \( \delta = \dot{U}_E, \delta = \dot{U}_{E1} = \psi \phi B + (1 - \phi)\delta U_{E1,0} \). Thus, \( \delta = \dot{U}_E \), implying that \( \delta = \dot{U}_E \). In a market equilibrium we must have \( \dot{U}_E = U_E \) so that \( U_E = \dot{U}_E \) is a market equilibrium. Thus, we have \( \delta = \dot{U}_E \), implying that \( \delta = \dot{U}_E \).

The social efficient outcome, agents without ideas should always generate ideas rather than being an opportunist, so \( n_I = 0 \) and \( \theta = 1 \). Consider any split \( s \) of the idea value \( z \), then \( \dot{U}_E = \psi (z + \delta U_{E1}) = \phi (1 - \phi)(z + \delta U_{E1}) + (1 - \phi)\phi (z + \delta U_{E1}) + (1 - \phi)\phi (z + \delta U_{E1}) + (1 - \phi)\phi (z + \delta U_{E1}) + (1 - \phi)\phi (z + \delta U_{E1}) + (1 - \phi)\phi (z + \delta U_{E1}) \), \( U_E = \dot{U}_E \), and \( U_E = \psi n_I = \psi \phi (z + \delta U_{E1}) \), which are independent of \( s \). We thus obtain \( \dot{U}_E = U_E = \phi (z + \delta U_{E1})/(1 + \delta \phi) \equiv \Delta \delta \) and therefore \( \dot{U}_E = \Delta \delta - D\psi \) and \( U_E = \Delta \delta - D\psi \). Idea creation is socially efficient whenever \( \dot{U}_E = \Delta \delta - D\psi \geq 0 \), where \( \psi \leq \Delta \delta = \phi \). Comparing these to the market equilibrium, we note that \( \Delta = \phi z/(2 - \delta + \phi) < \phi z/(1 - \delta + \phi) = \Delta \delta \) so that \( \psi^M = \Delta \delta < \Delta \delta = \phi \). Moreover, \( U_E = \dot{U}_E = D\phi \Delta - D\psi < D\Delta \delta - D\psi = U_E^1 \) and \( U_E = D\Delta \delta - D\psi < D\Delta \delta - D\psi = U_E^2 \). The socially efficient equilibrium fractions are given by \( n_I = n_G/\phi \) and \( n_C = 1 - n_I \) so that \( n_I = 1/(1 + \phi) > n_I, n_C = \phi/(1 + \phi) > n_O \) and \( n_I = 0 < n_O \).

For each match, the probability of a fit is given by \( \theta(\phi + 1 - \phi) + (1 - \theta) \phi = (\phi) \), but the probability that the idea bearer keeps his idea is given by \( \theta(\phi + 1 - \phi) + (1 - \theta) \phi = (\phi) \). Thus, the probability of a generator implementing his idea is given by \( \phi + \xi \phi + \xi^2 \phi + \cdots = 5 \sum_{n=0}^{\infty} \xi^n = \phi/(1 - \xi) = 2\phi/[2 - (1 - \phi)(1 + \theta - (1 - \theta))] \).
Let $f_{E,i}$ be the fraction of employees that are at the $j$th stage of creating an idea. We have $f_{E,i} = f_G$ and $f_{E,i+1} = (1-\delta)f_{E,i}$ for all $j = 1, \ldots, Q$. The total number of idea bearers inside the firm is $f_i = \sum_{j=0}^{Q} f_{E,i} = \sum_{j=0}^{Q} f_G (1-\delta)^j = f_G \delta^{-1}$, where we define $\delta = \sum_{j=0}^{Q} (1-\delta)^j$. Using $f_{E,i+1} = f_E$, we immediately obtain $f_{E,i} = \frac{f_E}{(1-\delta)}$.

For the upper bound $\psi^U$, consider the condition $\psi^U \geq K_{\min} + U_{O}$. We have $\psi^U = qU_{Q}(D(\phi \mathbf{z} + (1-\delta)(\delta^{Q+1} - \psi)) - U_{Q})$. Suppose $\psi > \psi^U$, then the first entrance set a complete absence of ideas so that $U_{O} = 0$. Thus, the first entrants’ condition simplifies to $qU_{Q}(D(\phi \mathbf{z} + (1-\delta)(\delta^{Q+1} - \psi)) \geq K_{\min} \Rightarrow \psi \leq \phi \mathbf{z} + (1-\delta)(\delta^{Q+1} - (1-\delta)(K_{\min}/q)) = \phi^*$. The condition $\psi^U > \psi^M$ requires $\delta \phi \mathbf{z} + (1-\delta)(\delta^{Q+1} - (1-\delta)(K_{\min}/q)) \geq \delta \sum_{j=0}^{Q} (1-\delta)^j > \delta/(2-\delta + \phi \delta)$.

**Proof of Proposition 3.** As a preliminary step, we show how the equilibrium fractions $n_1$ and $n_\ell$ depend on the density of firms $n_q$. Every period, there are some employees leaving with ideas, which we denote by $n_1 = n_q(1-\delta)^j f_E = n_q(1-\delta)^j \phi^* (1+\delta)$. Idea bearers in the market are either newly departed employees or else preexisting idea bearers that either store an idea or generated it as an employee and circulated it already in the market. Formally, $n_{1,i} = n_{1,i-1} + (1-\delta)n_{1,i-1} \Leftrightarrow n_{1,i} = n_q/\phi \Leftrightarrow n_{1,i} = n_q(1-\delta)^j \phi^* (1+\delta)$. Using $n_{1,i} + n_{\ell,i} + n_q = 1$, we get $n_q = n_1(1-\delta + \phi \delta)$, where $\Phi = (1-\delta)^j \phi^* (1+\delta)$. Using $\theta = n_q/(n_1 + n_q)$, we get $\theta = \Phi^*/[1 - (1+\delta)]$.

We now derive the $\alpha$ curve, which shows how the market utility varies with the density of firms. The market utility is given by $U_{D} = \theta D \Delta a$, where $\theta$ is now given by the above expression. The $\alpha$ equation is thus defined by $U_{D} = D \Delta a \theta \Phi^*/[1 - (q + 1)n_q]$. Clearly, $\partial U_{D}/\partial \theta = D \Delta a \Phi^*/[1 - (q + 1)n_q]^2 > 0$. For the comparative statics, we have $\partial U_{D}/\partial q = qU_{Q} \Phi D \Delta a [2 - \delta + \phi \delta] < 0$ and $\partial U_{D}/\partial \delta = D \Delta a \Phi^*/(2 - \delta + \phi \delta) > 0$. The comparative statics is ambiguous and analytically difficult to trace, so we do not examine it here.

For the $\alpha$ curve, we examine the number of firms, given by $n_f = (\Omega - (q + 1)U_{O})$. We immediately note that $\partial n_f/\partial U_{O} = -(q+1)\Omega < 0$. Moreover, for a given $U_{O}$, we have $\partial n_f/\partial q = qU_{Q} \Phi D a > 0$. The total effect of increasing $\psi$ is thus given by

$$\frac{dn_f}{d\psi} = \frac{\partial n_f}{\partial \psi} + \frac{\partial n_f}{\partial U_{O}} \frac{dU_{O}}{d\psi} = \frac{dn_f}{d\psi} < 0,$$

so an increase in $\psi$ leaves the $\alpha$ unaffected and shifts the $\alpha$ curve backward, resulting in a lower $n_f$ and a lower $U_{O}$.

**Proof of Proposition 4.** For $\psi < \psi^M$, there are generators in the market. This means that there are five types of agents: firm owners, firm employees, market idea bearers, market opportunists, and market idea generators. We have $n_q + n_f + n_1 + n_0 + n_\ell = 1$. The number of employees leaving in any period is again given by $n_{1,i} = n_q(1-\delta)^j f_E = n_q(1-\delta)^j \phi^* (1+\delta)$. Ideas are also generated by market agents, so the number of idea bearers in the market is now given by $n_{1,i} = n_{1,i-1} + (1-\delta)n_{1,i-1} + n_\ell \Leftrightarrow n_\ell = (n_1 + n_q)/\phi \Leftrightarrow n_\ell = n_q(\Phi + n_\ell)/\phi$. From $\theta = n_q/(n_1 + n_q)$, we get $n_\ell = n_q/(1-\theta)/\theta$, which we use in $(q + 1)n_f + n_\ell + n_1 + n_0 = 1 \Rightarrow \theta = (1-n_q - (q + 1)n_f)$. Thus, $\theta(1-n_q - (q + 1)n_f) = n_q(\Phi + n_\ell)/\phi$, which we solve to obtain $n_\ell = (\Phi - \Phi(q + 1)n_f - qU_{Q})/(1 + \Phi)$. This expresses $n_\ell$ as a function of $\Phi$ and $\theta$. We can do the same for $n_1$ using $n_1 = \theta(1-n_q - n_\ell)$ and for $n_0$ using $n_0 = n_q/(1-\theta)/\theta$. This implies that we can express all of the equilibrium fractions as a function of $n_\ell$ and $\theta$. We now show how these two variables are determined in equilibrium.

A market equilibrium requires $U_{O} = U_{C} = U_{O}$. From Proposition 1, we know that $U_{O} = U_{C} \Leftrightarrow \theta = \theta^M = \delta - \psi/\Delta$ and that as a result we have $U_{O} = D \Delta a$. This is the $\alpha$ curve. The market utility is now established by market exchange, and the number of firms does not affect it. Formally, $\partial U_{O}/\partial n_f = 0$, which makes the $\alpha$ flat. Moreover, we have $\partial U_{O}/\partial q = -D < 0$ and $\partial U_{O}/\partial \psi = D \Delta a (2 - \delta + \phi \delta) > 0$. The $\alpha$ curve is again given by $n_f = (\Omega U_{O} - (q + 1)U_{O})$, so that its comparative statics are identical to those of Proposition 3.

**References**


