Product market competition and investments in cooperative R&D

Hinloopen, J.; Vandekerckhove, J.

DOI
10.2202/1935-1682.2922

Publication date
2011

Document Version
Final published version

Published in
The B.E. Journal of Economic Analysis & Policy

Citation for published version (APA):

General rights
It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations
If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: https://uba.uva.nl/en/contact, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.
Product Market Competition and Investments in Cooperative R&D

Jeroen Hinloopen* Jan Vandekerckhove†

*University of Amsterdam, j.hinloopen@uva.nl
†Maastricht University, j.vandekerckhove@maastrichtuniversity.nl

Recommended Citation
Product Market Competition and Investments in Cooperative R&D*

Jeroen Hinloopen and Jan Vandekerckhove

Abstract

Building on the framework developed by Qiu (1997) we investigate the influence of product market competition on incentives to invest in cooperative R&D. For that we disentangle the three components that make up the combined-profits externality. The strategic component is always negative and the size component is always positive. The spillover component is negative (positive) with Bertrand (Cournot) competition. Cournot competition thus yields more cooperative R&D, which could drive the Cournot-Nash price below the Bertrand-Nash price. Our decomposition also explains why, under Cournot competition, cooperative R&D exceeds non-cooperative R&D only if spillovers are “high enough.”

KEYWORDS: Bertrand competition, Cournot competition, cooperative R&D, efficiency

*Thanks are due to Raymond de Bondt, to seminar participants at the annual meetings of EARIE (Toulouse, September 2008), IIOC (Boston, April 2009) and SAET (Ischia, June 2009), and, in particular, to Stephen Martin for his numerous suggestions to make the writing sing. The usual disclaimer applies.
1 Introduction

R&D cooperatives are a cornerstone of today’s business landscape. These “organizations, jointly controlled by at least two participating entities, whose primary purpose is to engage in cooperative R&D” (Caloghirou et al., 2003), affect every industry. Empirical studies show that they influence members’ research capabilities (Mowery et al., 1996), that they enhance members’ labor productivity and improve their price-cost margins (Benfratello and Sembenelli, 2002), and that they affect the intensity of product market competition (Vonortas, 2000). Little is known, however, about the role of the product market on firms’ incentives to invest in R&D.\(^1\) In this paper we investigate the influence of the type of product market competition on the incentives to invest in cooperative R&D.

An important aspect of R&D is its public good character: parts of newly created knowledge leak to competitors, which reduces the incentive to invest in R&D. In practice, these technological spillovers can be substantial (Bloom et al., 2007). A policy response to the concomitant market failure is to allow firms to cooperate in R&D while remaining competitors on the product market.\(^2\) Internalizing the technological spillover also increases the efficiency of the R&D process; duplication of R&D efforts is reduced and research synergies are better explored. Not surprisingly, the existence of technological spillovers is one of the key motivations for firms to join an R&D cooperative (Hernán et al., 2003).

Incentives to invest in cooperative R&D are determined by two externalities (Kamien et al., 1992): the competitive-advantage externality and the combined-profits externality. Both externalities hinge on the technological spillover. The competitive-advantage externality refers to the impact of a firm’s R&D on its rivals’ cost of production: through technological spillovers every firm benefits from its rivals’ R&D efforts without having to pay for those efforts. The combined-profits externality captures the impact of a firm’s R&D efforts on all other firms’ profits. Strengthening rivals’ position in the product market through technological spillovers reduces a firm’s profits but enhances

\(^1\)To date, the literature has examined only what the effects are of a change in conduct on the product market. If firms collude rather then compete à la Cournot, the incentives to invest in cooperative R&D increase because more of the benefits of any cost reduction accrue to the innovating firms (d’Aspremont and Jacquemin, 1988).

\(^2\)Details of this policy differ between constituencies; see Martin (1997) for a comparison between the US, the EU and Japan. Alternatively, private R&D is subsidized (see e.g. Hinloopen, 1997; 2000). In theory this stimulates R&D investments. However, most of the evidence shows that public funding foremost crowds out private R&D investments (Mamuneas et al., 1996; Wallsten, 2000).
its rivals’ profits. The combined-profits externality can thus be either positive or negative, depending on the size of the technological spillover. The larger is the technological spillover, the more likely it is that the combined-profits externality is positive.

To date, little is known about the role of product market competition on firms’ incentives to invest in cooperative R&D. Intuitively, softening competition in the product market enhances firms’ incentives to invest in R&D. At the same time, lower competition intensities give way to increased mark-ups which could outweigh the social benefits of higher R&D investments. Obviously, the competitive-advantage externality always affects the R&D investment decision. Firms that conduct R&D cooperatively consider in addition the combined-profits externality. Therefore, in order to understand the role of product market competition on firms’ incentives to invest in cooperative R&D, we analyze the combined-profits externality when competition in the product market is in prices (Bertrand) or quantities (Cournot). We also characterize the parameter space where either mode of competition yields a higher total surplus.

We identify three separate components that jointly make up the combined-profits externality: a strategic component, a spillover component, and a size component. The strategic component is always negative. Firms that cooperate in R&D realize that a larger market share comes at their rivals’ expense, which diminishes the incentive to invest in R&D. The size component, on the other hand, is always positive, provided that technological spillovers are strictly positive. Any firm’s cost reduction that spills over to its rival is more to this rival’s benefit the more this rival produces, because the cost reduction applies to more units of production. This enhances the incentive to invest in R&D.

The role of product market competition runs through the spillover component. It carries a negative sign with Bertrand competition while it is positive with Cournot competition. R&D results that spill over are beneficial to joint profits if firms are Cournot competitors. Because quantities are strategic substitutes, any reduction in rivals’ costs increases the efficiency of total production. In contrast, if competition is over price, any reduction in rivals’ production costs that emerges from the technological spillover diminishes total profits, because all firms will lower their price in response to the cost reduction. As a result, R&D cooperatives have a stronger incentive to invest in R&D when competition is over quantities rather than price.

For our analysis we build on the seminal contribution of Qiu (1997), who examines the incentives to invest in R&D non-cooperatively under both Cournot and Bertrand competition. He finds that firms always invest more
in R&D when competition is over quantities. Yet, in spite of the larger concomitant cost reduction, Cournot competition always yields higher prices. At the same time, when R&D costs are moderate, Cournot competition can yield a larger total surplus. The reduction in post-innovation production costs is then so large that the increase in producers’ surplus outweighs the reduction in consumers’ surplus. We show that this result carries over to a setting of cooperative R&D.

The diverging R&D investment incentives open up an interesting possibility: the post-innovation production cost could be so much lower under Cournot competition that the Cournot-Nash price is below the Bertrand-Nash price. We show that this can be case. We show further that with cooperative R&D, Cournot competition always yields a higher producers’ surplus. Accordingly, total surplus with Cournot competition is greater than total surplus with Bertrand competition whenever the Cournot-Nash price is below the Bertrand-Nash price. We provide evidence suggesting that this has been the case in the semiconductor industry.

Our analysis also allows for a unifying explanation of a result that has been reported in various papers: with Cournot competition the level of cooperative R&D exceeds the non-cooperative R&D level in case of relatively ‘large’ spillovers only (d’Aspremont and Jacquemin, 1988; Kamien et al., 1992; Poyago-Theotoky, 1999; Amir and Wooders, 2000; Miyagiwa and Ohno, 2002; Amir et al., 2003; Manasakis and Petrakis, 2009; and others). Absent technological spillovers, only the strategic component of the combined-profits externality remains. Coordinating the R&D investment decision then reduces the level of R&D. With Cournot competition, both the size component and the spillover component oppose the strategic component if spillovers are positive. Hence, some minimum level of spillover is required for the combined profits externality to be positive, and to outweigh the competitive-advantage externality.3

The type of product market competition thus matters for the effectiveness of an R&D-stimulating policy that allows for R&D cooperatives. A lower intensity of competition (Cournot) always yields more R&D investment. At the same time, and perhaps more surprisingly, it is quite possible that this increased R&D activity more than compensates for the higher mark-ups in the product market. A lower intensity of competition in the product market should thus not constitute an argument per se to be more hesitant towards

---

3With Bertrand competition only the size component is positive. Therefore, a level of spillover could exist such that the combined-profits externality is positive and strong enough to outweigh the competitive-advantage externality.
diminishing competition intensity higher up the production chain. Indeed, the superior market performance of Cournot competition is more likely to obtain when the technological spillover is high. With cooperative R&D this is quite likely to apply because firms have an incentive to share information if they conduct R&D cooperatively (Kamien et al., 1992; Poyago-Theotoky, 1999; Tesoriere, 2008).

We proceed as follows. We introduce our model in the next section, and characterize the R&D investment incentives in Section 3. Market performance with Cournot and Bertrand competition is compared in Section 4. There we characterize parameter values for which the Cournot-Nash price is below the Bertrand-Nash price. Section 5 concludes.

2 A model of strategic R&D with technological input spillovers

We consider a two-stage duopoly where firms invest in cost-reducing R&D and then compete on the product market. Inverse demand equations are given by:

\[ p_i = a - (q_i + \theta q_j), \]

\[ i, j = 1, 2, i \neq j, \] and where \( q_i \) and \( p_i \) are the respective quantity and price of product \( i \). The parameter \( \theta \) captures the extent of product differentiation. If \( \theta = 1 \) both firms produce an identical product; if \( \theta = 0 \) each firm is a local monopolist. For the remainder of the paper we focus on intermediate cases: \( \theta \in (0, 1) \). Unless stated otherwise, \( i, j = 1, 2, i \neq j \) holds throughout the paper. Market demand in direct form is:

\[ q_i = \frac{1}{1 - \theta^2} \left[ (1 - \theta) a - (p_i - \theta p_j) \right]. \]

Each firm produces one version of the differentiated product with constant marginal costs \( c \) and no fixed costs, where \( a > c > 0 \). Investments in R&D reduce marginal costs whereby either firm appropriates part of its rival’s research efforts through the technological spillover. In particular, if firm \( i \) invests \( x_i \) in R&D, its effective R&D investment \( X_i \) is given by:

\[ X_i = x_i + \beta x_j. \]

\[ ^4 \text{Here we follow Qiu (1997). Note that a standard quadratic utility function yields these inverse demands (Singh and Vives, 1984).} \]
In (3) \( \beta \in [0, 1] \) is the technological spillover. The reduction in marginal cost brought about by \( X_i \) is determined by an R&D production function \( f(X_i) \) (Kamien et al., 1992). Returns to R&D are diminishing in scale, that is, 
\[
 f' > 0, \quad f'' < 0, \quad \text{and} \quad f(0) = 0.5
\]

We adhere to the criterion stated by Amir et al. (2008): “The R&D technology and the spillover process should be such that any total R&D investment level cannot generate more cost reduction if allocated to \( n \) labs, run independently but with spillovers at their natural rate, than if spent all in one lab.” Put differently, it must be at least as beneficial to a firm to invest in one research laboratory than to invest in several independent research laboratories that possibly benefit from mutual spillovers. This criterion bounds the R&D production function \( f \). In particular, it requires that:
\[
f(X_i) \leq f(x_i) + f(x_j),
\]
which holds for any \( f \) exhibiting decreasing returns to scale. We set (Amir, 2000):
\[
f(X_i) = \frac{\sqrt{X_i}}{\gamma},
\]
where \( \gamma > 0 \) captures the efficiency of the R&D process: a lower value of \( \gamma \) corresponds to more efficient R&D. Firm \( i \)'s profits then equal:
\[
\pi_i = p_i q_i - (c - y_i) q_i - x_i,
\]
with \( y_i = \sqrt{(x_i + \beta x_j)} / \gamma \).\(^6\)

\(^5\)Adams and Griliches (1996) find strong diminishing returns to scale in the production of scientific articles. Madsen (2007) however cannot reject the hypothesis of constant returns to scale in the number of patents applied for. In assuming diminishing returns to scale we follow the bulk of the literature.

\(^6\)Note that in (6) there are technological input spillovers: the free flow of knowledge between rivals occurs during the R&D process (as in Kamien et al., 1992; Amir, 2000; and others). Alternatively, there are technological output spillovers: final R&D results spill over between firms (as in d'Aspremont and Jacquemin, 1988; Qiu, 1997; Gersbach and Schmutsler, 2003; Erkal and Piccinnin, 2010; and others). There are good reasons to believe that technological input spillovers are more appropriate. Geroski (1995) identifies three channels through which the technological spillover flows: (i) through knowledge workers that change employer, (ii) through the deduction of rivals’ line of reasoning by observing their behavior, and (iii) through casual encounters at seminars and the exchange of ideas through publications. All these channels relate to a free flow of knowledge that occurs during the R&D process. Indeed, based on a sample of more than 10,000 firms, Kaiser (2002) finds that technological spillovers occur foremost during the R&D process (see also Fershtman and Gandal, 2011). Moreover, the assumption that R&D outputs are additive can be questioned. Independently obtained R&D results are quite likely to exhibit some overlap if the research
3 R&D investment incentives.

In this section we examine firms’ incentives to invest in cooperative R&D. We closely follow the analysis of Qiu (1997), assuming Cournot competition in the product market (the corresponding analysis with Bertrand competition is part of the proof of Proposition 1 in the Appendix, Section 6.2.1). The first-order conditions for optimal outputs are:

\[
\frac{\partial \pi_i}{\partial q_i} = p_i + q_i \frac{\partial p_i}{\partial q_i} - (c - y_i) = 0. \tag{7}
\]

Throughout we assume stable, interior solutions. The second-order and stability conditions require (Qiu, 1997, p. 225):

\[
\frac{\partial^2 \pi_i}{\partial q_i^2} = 2 \frac{\partial p_i}{\partial q_i} + q_i \frac{\partial^2 p_i}{\partial q_i^2} \leq 0,
\]

\[
\Omega^C = \frac{\partial^2 \pi_i}{\partial q_i^2} \frac{\partial^2 \pi_j}{\partial q_j^2} - \frac{\partial^2 \pi_i}{\partial q_i \partial q_j} \frac{\partial^2 \pi_j}{\partial q_i \partial q_j} > 0,
\]

respectively.

At the R&D stage, firms maximize joint profits:

\[
\Pi^C = \sum_{i=1}^{2} \pi_i^C. \tag{8}
\]

In the terminology of Kamien et al. (1992) we consider an R&D cartel. Maximizing (8) over \( x_i \) yields:\(^7\)

\[
\frac{\partial \Pi^C}{\partial x_i} = \frac{\partial \pi_i}{\partial q_j} \frac{\partial q_j}{\partial x_i} + \frac{\partial \pi_j}{\partial q_i} \frac{\partial q_i}{\partial x_i} + \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi_j}{\partial x_i} - 1 = 0,
\]

or:

\[
\frac{\partial \Pi^C}{\partial x_i} = \frac{1}{\Omega^C} \left( \frac{\partial \pi_i}{\partial q_j} \frac{\partial^2 \pi_j}{\partial q_i \partial q_j} - \frac{\partial \pi_j}{\partial q_i} \frac{\partial^2 \pi_j}{\partial q_i \partial q_j} \right) \tag{9}
\]

\[
+ \frac{\beta y_i}{y_j \Omega^C} \left( -\frac{\partial \pi_i}{\partial q_j} \frac{\partial^2 \pi_i}{\partial q_j \partial q_j} + \frac{\partial \pi_j}{\partial q_i} \frac{\partial^2 \pi_j}{\partial q_i \partial q_j} \right) + q_i + \beta q_j \frac{y_i}{y_j} - 2\gamma y_i = 0.
\]

Efforts were aimed at reducing the production cost of the same product, while the non-overlapping part is unlikely to be a perfect match to rivals’ research results. More generally, differences in research strategies, internal organization, and corporate culture diminish any firm’s ability to appropriate fully rivals’ R&D results (see also Hinloopen, 2003; Hinloopen and Vandenekkhove, 2009; Stepanova and Tesoriere, 2011).

\(^7\)Throughout the paper we follow the related literature in that we consider symmetric equilibria only. See Salant and Shaffer (1998) for an example where R&D cooperation yields asymmetric equilibria.
Hinloopen and Vandekerckhove: Cournot, Bertrand and Cooperative R&D

<table>
<thead>
<tr>
<th></th>
<th>Cournot</th>
<th>Bertrand</th>
</tr>
</thead>
<tbody>
<tr>
<td>strategic component</td>
<td>$- \frac{1}{y_i \Omega_i \sigma_i} \frac{\partial \sigma_i}{\partial x_i} \frac{\partial^2 \pi_j}{\partial x_i^2} &lt; 0$</td>
<td>$- \frac{1}{y_i \Omega_i \sigma_i} \frac{\partial \sigma_i}{\partial x_i} \frac{\partial^2 \pi_j}{\partial x_i^2} &lt; 0$</td>
</tr>
<tr>
<td>spillover component</td>
<td>$\frac{\alpha_{ij}}{y_i \Omega_i \sigma_i} \frac{\partial \sigma_i}{\partial q_i} \frac{\partial^2 \pi_j}{\partial q_i^2} &gt; 0$</td>
<td>$\frac{\alpha_{ij}}{y_i \Omega_i \sigma_i} \frac{\partial \sigma_i}{\partial q_i} \frac{\partial^2 \pi_j}{\partial q_i^2} &gt; 0$</td>
</tr>
<tr>
<td>size component</td>
<td>$\frac{\beta_{ij}}{y_i \Omega_i \sigma_i} \frac{\partial \sigma_i}{\partial q_i} \frac{\partial^2 \pi_j}{\partial q_i^2} &gt; 0$</td>
<td>$\frac{\beta_{ij}}{y_i \Omega_i \sigma_i} \frac{\partial \sigma_i}{\partial q_i} \frac{\partial^2 \pi_j}{\partial q_i^2} &gt; 0$</td>
</tr>
</tbody>
</table>

Table 1: The components of the combined-profits externality

If firms were to compete in the R&D stage they would maximize $\pi^C_i$ over $x_i$ conditional on (7). This gives (Qiu, 1997, p. 225):

$$\frac{\partial \pi^C_i}{\partial x_i} = \frac{\partial \pi_i}{\partial q_j} \frac{\partial q_j}{\partial x_i} + \frac{\partial \pi_i}{\partial x_i} - 1 = 0,$$

or:

$$\frac{\partial \pi^C_i}{\partial x_i} = \frac{1}{\Omega_i} \frac{\partial \pi_i}{\partial q_j} \frac{\partial^2 \pi_j}{\partial q_j^2} - \frac{\beta y_i}{y_j \Omega_j} \frac{\partial \pi_j}{\partial q_j} \frac{\partial^2 \pi_j}{\partial q_j^2} + q_i - 2\gamma y_i \equiv 0. \quad (10)$$

Comparing then (10) with (9) gives us the components of the combined-profits externality:

$$\sum_{j \neq i} \frac{\partial \pi^C_j}{\partial x_i} = \frac{\partial \Pi^C_i}{\partial x_i} = \text{strategic component (--) \ spillover component (--) \ size component (+)}$$

We are now in a position to state our central result (Section 6.2 of the Appendix contains the proofs of all propositions):

**Proposition 1** (i) The combined-profits externality of investing in cooperative R&D can be decomposed into three components: a strategic component, a spillover component, and a size component. (ii) With Cournot competition, the sign of the strategic component is negative, while the signs of the spillover component and size component are positive; with Bertrand competition, the sign of the size component is positive, while the signs of the strategic component and the spillover component are negative.

The three components of the combined-profits externality are summarized in Table 1. Jointly, they are responsible for what drives the comparison between cooperative and non-cooperative R&D. Several papers find that cooperation in R&D reduces R&D investments if there are no technological
spillovers (d’Aspremont and Jacquemin, 1988; Kamien et al., 1992; Poyago-Theotoky, 1999; Amir and Wooders, 2000; Miyagiwa and Ohno, 2002; Amir et al., 2003; Manasakis and Petrakis, 2009; and others). Absent technological spillovers, all that remains is the strategic component. The only additional consideration of cooperating firms is then that their R&D efforts are detrimental to rivals’ profits as they reduce the rival’s market share. That is:

**Corollary 1** Absent technological spillovers, cooperation in R&D yields a reduction of R&D efforts, independent of the type of product market competition.

Further, R&D activities increase rivals’ size through the technological spillover as it allows them to acquire R&D inputs for free. This is beneficial to all rivals’ profits. Accordingly, the size component is always positive. With Cournot competition, the spillover component is also positive. Hence, some level of spillover is required for the combined-profits externality to be positive, and to outweigh the competitive-advantage externality.

**Corollary 2** With Cournot competition there is a level of spillover such that cooperative R&D efforts exceed non-cooperative R&D efforts.

In this vein, specific models yield specific threshold technological spillovers (see d’Aspremont and Jacquemin, 1988; Kamien et al., 1992; Miyagiwa and Ohno, 2002; and others). Indeed, if spillovers are maximal, the result is all the more likely to obtain. For instance, Kamien et al. (1992) show that with Cournot competition, an R&D cartel with maximal spillovers yields the highest level of R&D investments, a result that carries over to the case with Bertrand competition for the larger part of the parameter space.

With Bertrand competition the spillover component is negative. Therefore, a level of spillover could exist such that the combined-profits externality is positive and strong enough to outweigh the competitive-advantage externality.8

### 4 Market performance

Qiu (1997) shows that levels of non-cooperative R&D under Cournot competition exceed those under Bertrand competition. In this section we show that this result also holds for cooperative R&D. Note that the diverging R&D

---

8Kamien et al. (1992) report for their model that such a threshold technological spillover exists, and that it is larger than the threshold value with Cournot competition.
investment incentives open up an interesting possibility: the post-innovation production cost could be so much lower under Cournot competition that the concomitant equilibrium price is below the Bertrand-Nash price. We characterize the region of the parameter space where this occurs.

### 4.1 Second-stage Bertrand competition

Maximizing (6) with respect to price yields equilibrium prices conditional on effective R&D efforts:\(^9\)

\[
\hat{p}_i(X_i, X_j) - c = \frac{(a - c)(2 + \theta)(1 - \theta) - 2y_i - \theta y_j}{4 - \theta^2}.
\]  

(11)

Inserting (11) into (6) and maximizing the resulting sum of firms’ profits over R&D investments results in the following cost reductions:\(^10,11\)

\[
\hat{y}^B = \frac{(a - c)(2 + \theta)^2(1 - \theta)^2(1 + \beta)}{\gamma(4 - \theta^2)^2(1 - \theta^2) - (2 + \theta)^2(1 - \theta)^2(1 + \beta)}.
\]  

(12)

Equilibrium output then equals:

\[
\hat{Q}^B = \frac{2\gamma(a - c)(2 + \theta)(1 - \theta)(4 - \theta^2)}{\gamma(4 - \theta^2)^2(1 - \theta^2) - (2 + \theta)^2(1 - \theta)^2(1 + \beta)}.
\]

Single-firm profits are given by:

\[
\hat{\pi}^B = \frac{\gamma(1 - \theta^2)(4 - \theta^2)^2 - (2 + \theta)^2(1 - \theta)^2(1 + \beta)}{\gamma(4 - \theta^2)^2} (\hat{q}^B)^2,
\]  

(13)

where \(\hat{q}^B = \hat{Q}^B / 2\). Consumers’ surplus and total surplus respectively equal:

\[
\hat{CS}^B = (1 + \theta) (\hat{q}^B)^2,
\]  

(14)

and

\[
\hat{TS}^B = \frac{\gamma(4 - \theta^2)^2(1 + \theta)(3 - 2\theta) - 2(2 + \theta)^2(1 - \theta)^2(1 + \beta)}{\gamma(4 - \theta^2)^2} (\hat{q}^B)^2.
\]  

(15)

\(^9\)A hat refers to a conditional equilibrium expression.

\(^10\)A tilde refers to an unconditional equilibrium expression; superscript \(B \) (\(C \)) stands for second-stage Bertrand (Cournot) competition.

\(^11\)The second-order and stability conditions are examined in Section 4.3.
4.2 Second-stage Cournot competition

Maximizing (6) over quantities leads to equilibrium outputs conditional on R&D investments:

\[ \bar{q}_i(X_i, X_j) = \frac{(a - c)(2 - \theta) + 2y_i - \theta y_j}{4 - \theta^2}. \]  

Maximizing the sum of first-stage profits with respect to R&D investments gives:

\[ \gamma^C = \frac{(a - c)(2 - \theta)^2(1 + \beta)}{\gamma(4 - \theta^2)^2 - (2 - \theta)^2(1 + \beta)} , \]  

and

\[ \bar{Q}^C = \frac{2\gamma(a - c)(4 - \theta^2)(2 - \theta)}{\gamma(4 - \theta^2)^2 - (2 - \theta)^2(1 + \beta)}. \]  

Single-firm profits then equal:

\[ \bar{\pi}^C = \frac{\gamma(4 - \theta^2)^2 - (1 + \beta)(2 - \theta)^2}{\gamma(4 - \theta^2)^2} (\bar{q}^C)^2 , \]

with \( \bar{q}^C = \bar{Q}^C/2 \). Consumers’ surplus and total surplus under second-stage Cournot competition then equal:

\[ \bar{C}S^C = (1 + \theta) (\bar{q}^C)^2 , \]

and

\[ \bar{T}S^C = \frac{\gamma(4 - \theta^2)^2(3 + \theta) - 2(1 + \beta)(2 - \theta)^2}{\gamma(4 - \theta^2)^2} (\bar{q}^C)^2 , \]

respectively.

4.3 Regularity conditions

The admissible parameter space is bounded by four conditions that emerge from the R&D stage: post-innovation costs have to be positive and the equilibrium must be an interior solution. Under Bertrand and Cournot competition, the second-order conditions respectively require that:

\[ \gamma \geq \frac{(1 + \beta)[(2 - \theta^2 - \theta \beta)^2 + (2\beta - \theta^2 \beta - \theta)^2]}{(4 - \theta^2)^2(1 - \theta^2)(1 + \beta^2)} = \gamma_{RL} , \]  

(R1)
and
\[ \gamma \geq \frac{(1 + \beta)[(2 - \theta \beta)^2 + (2\beta - \theta)^2]}{(4 - \theta^2)^2(1 + \beta^2)} = \gamma_{R2}. \] (R2)

The requirement that post-innovation costs are positive under Bertrand and Cournot competition implies that:
\[ \gamma > \frac{a(1 - \theta)(1 + \beta)}{c(1 + \theta)(2 - \theta)^2} = \gamma_{R3}, \] (R3)

and
\[ \gamma > \frac{a(1 + \beta)}{c(2 + \theta)^2} = \gamma_{R4}. \] (R4)

respectively. It is straightforward to see that condition R3 is redundant; the parameter space is therefore bounded by regularity conditions R1, R2 and R4.

### 4.4 Cooperative R&D

In line with Proposition 1 we observe:

**Proposition 2** For any \( \theta \in (0, 1) \) and \( \beta \in [0, 1] \) we have that \( \tilde{y}^C > \tilde{y}^B \) under R1, R2 and R4.

That is, the level of cooperative R&D efforts under second-stage Cournot competition exceeds the level of cooperative R&D with Bertrand competition. Qiu (1997) reports the same result for non-cooperative R&D.

### 4.5 Price

For comparing equilibrium prices we introduce an assumption:

\[ \beta > \gamma(4 - \theta^2) - 1 = \beta^*. \] (A1)

Note that \( \beta^* < 0 \) whenever \( \gamma < 1/(4 - \theta^2) \), while \( \beta^* > 1 \) for \( \gamma > 2/(4 - \theta^2) \). That is, assumption A1 always holds if the R&D process is relatively cost efficient while it cannot hold if the R&D process is relatively expensive. For intermediate cases the size of the technological spillover determines whether it is met. In general, the larger is the technological spillover, the more likely it is that A1 holds, all else equal. The assumption itself does not rule out the existence of equilibria:

**Lemma 1** The set where regularity conditions R1, R2, R4 and assumption A1 hold is not empty.
Note from the proof of Lemma 1 (see Section 6.1 of the Appendix) that whenever assumption A1 holds, the admissible parameter space is bounded in addition by conditions R1 and R4 only.

Equilibrium prices compare as follows:

**Proposition 3** For any $\theta \in (0,1)$ and $\beta \in [0,1]$ we have that $\bar{p}^C < \bar{p}^B$ under $R1$, $R4$ and $A1$.

According to Proposition 3, with cooperative R&D the Cournot-Nash price can be below the Bertrand-Nash price.\(^{12}\) As a result, whenever assumption A1 holds, consumers’ surplus is larger under second-stage Cournot competition than with second-stage Bertrand competition. Post-innovation costs are then so much lower under second-stage Cournot competition that in equilibrium lower prices obtain. This is due to the different magnitude of the combined-profits externality for the two types of product market competition. Recall that this difference is more pronounced the larger is the technological spillover as the spillover component is increasing in the technological spillover (see Table 1). In that case it is also more likely that assumption A1 holds. The subset in the admissible parameter space where Proposition 3 holds, is illustrated in Figure 1.\(^{13}\)

The conditions in Proposition 3 are likely to hold for the semiconductor industry, in particular for the production of memory chips. Capacity constraints suggest that in this industry competition is over quantities (De Bondt, 1989), technological spillovers are significant (Bernstein and Nadiri, 1989; Gruber, 1998), and “Memory chips are very much a commodity with little scope for differentiating products of the same generation.” (Gruber, 2000). Moreover, in 1987 a consortium of 14 US semiconductor firms formed an R&D cooperative (Sematech) that increased the efficiency of members’ R&D efforts by eliminating duplicative research efforts (Irwin and Klenow, 1996). The industry was very R&D intensive (Irwin and Klenow, 1996) and “By any measure, price deflators for semiconductors fell at a staggering pace over much of the last decade.” (Aizcorbe 2002). This all suggests that in the semiconductor industry Cournot competition has been quite instrumental in bringing down prices through the resulting large (cooperative) R&D efforts.

---

\(^{12}\)This contrasts Proposition 2 in Qiu (1997) for non-cooperative R&D, according to which the Cournot-Nash price is always above the Bertrand-Nash price.

\(^{13}\)Cellini et al. (2004) and Mukherjee (2005) also document a case where consumers’ surplus under Cournot competition exceeds that with Bertrand competition. They focus on markets with free entry. The number of firms entering the market with Cournot competition exceeds that with Bertrand competition. The resulting increase in the number of product varieties can more than compensate for the higher price that always obtains under Cournot competition. In contrast, Proposition 3 holds for an exogenous market structure.
4.6 Profits

Because firms enjoy more market power when they compete over quantities, all else equal, the joint implications of Propositions 2 and 3 for profits are not immediately obvious. The following proposition clarifies:

**Proposition 4** For any \( \theta \in (0, 1) \) and \( \beta \in [0, 1] \), we have that \( \bar{\pi}^C > \bar{\pi}^B \) under R1, R2 and R4.

Despite the fact that under Cournot competition firms always incur higher R&D costs and that for a subset of the parameter space the equilibrium price is below what obtains with Bertrand competition, producers' surplus is always higher when firms compete over quantities. For non-cooperative R&D Qiu (1997) reports an identical result.

4.7 Total surplus

Combining the message of Propositions 3 and 4 implies that total surplus with Cournot competition exceeds that with Bertrand competition whenever Assumption A1 holds. Competition over quantities could also yield higher
total surplus in combination with a higher equilibrium price if the difference in producers’ surplus is large enough. The complete ranking of total surplus is along the lines of Proposition 5 in Qiu (1997):

**Proposition 5** For any \( \theta \in (0, 1) \) and \( \beta \in [0, 1] \) the following holds under R1, R2, and R4:

(i) \[ 0 < \gamma < (1 + \beta)/(4 - \theta^2) : \quad \tilde{T}S^C > \tilde{T}S^B \]

(ii) \[ (1 + \beta)/(4 - \theta^2) < \gamma \] there exists a unique \( \gamma^*(\theta) \) such that:

(a) \[ \gamma > \gamma^*(\theta), \forall \beta \in [0, 1] : \quad \tilde{T}S^B > \tilde{T}S^C \]

(b) \[ (1 + \beta)/(4 - \theta^2) < \gamma < \gamma^*(\theta) \exists ) \beta^*(\gamma) \in [0, 1] \] such that:

\[ \beta < \beta^*(\gamma) : \quad \tilde{T}S^C < \tilde{T}S^B \]

\[ \beta = \beta^*(\gamma) : \quad \tilde{T}S^C = \tilde{T}S^B \]

\[ \beta > \beta^*(\gamma) : \quad \tilde{T}S^C > \tilde{T}S^B . \]

According to Proposition 5 there are two situations where Cournot competition leads to higher total surplus than Bertrand competition. First, as in case (i), where both consumers’ surplus and producers’ surplus are higher, and second, as in case (ii, b) with strong technological spillovers, where the lower consumers’ surplus is offset by the higher producers’ surplus.

For an R&D cooperative it is profitable to increase the technological spillover (Kamien *et al.*, 1992; Poyago-Theotoky, 1999; Tesoriere, 2008). According to Proposition 5, it is then more likely that total surplus is maximized with Cournot competition. To the extent that the intensity of product market competition plays a role in sustaining R&D cooperatives, this finding suggests that a lower intensity of competition should not be a reason *per se* for the competition authorities to be less inclined to sustain cooperation in R&D.

### 5 Conclusions

Our decomposition of the combined-profits externality explains the influence of the type of product market competition on firms’ incentives to invest in cooperative R&D. It also explains why levels of cooperative R&D exceed non-cooperative R&D levels in case of relatively large spillovers only, and why this threshold technological spillover is larger with Bertrand competition than with Cournot competition. Perhaps more remarkable is that the post-innovation production cost under Cournot competition can be so much lower than under Bertrand competition, that the Cournot-Nash price is below the Bertrand-Nash price.
An obvious policy implication is that sustaining R&D cooperatives can be particularly beneficial in markets where the intensity of competition in the product market is relatively low. Not only would this trigger higher R&D investments, it could also lead to a drop in consumer prices below the level that would obtain under more intense product market competition. This is all the more likely because firms have an incentive to increase the technological spillover once they form an R&D cooperative.

6 Appendix Proofs

6.1 Proof of lemma 1

For A1 and R4 to hold jointly it must be that $1 < a/c < (2+\theta)/(2-\theta)$, or $2(a-c) < \theta(a+c)$. Indeed, a and c can always be chosen such that this inequality holds. For A1 and R1 to hold jointly it must be that $1 > [(2-\theta^2-\theta\beta)^2 + (2\beta - \theta^2\beta-\theta^2)/(4-\theta^2)(1-\theta^2)(1+\beta^2)]$, or $\beta > \left(\frac{2 - \theta^2 - \sqrt{(4 - \theta^2)(1 - \theta^2)}}{2 - \sqrt{(4 - \theta^2)}}\right)/\theta = g(\theta)$.

Note that $f(\theta)$ is continuous and strictly decreasing in $\theta \in (0,1)$, that $\lim_{\theta \to 0} f(\theta) = 0$, and that $\lim_{\theta \to 1} f(\theta) = 1$. For A1 and R2 to hold jointly it must be that $1 > [(2 - \theta\beta)^2 + (2\beta - \theta\beta - \theta^2)/(4 - \theta^2)(1 + \beta^2)]$, or $\beta > \left(\frac{2 - \sqrt{(4 - \theta^2)}}{2 - \sqrt{(4 - \theta^2)}}\right)/\theta = g(\theta)$. Note that $g(\theta)$ is continuous and strictly increasing in $\theta \in (0,1)$, that $\lim_{\theta \to 0} g(\theta) = 0$, and that $\lim_{\theta \to 1} g(\theta) = 2 - \sqrt{3} > 0$. Finally, note that $f(\theta) - g(\theta) > 0 \\forall \theta \in (0,1)$. That is, whenever A1 holds, the admissible parameter space is bounded by conditions R1 and R4.

QED

6.2 Proofs of propositions

6.2.1 Proof of proposition 1

Here we follow Qiu (1997). Totally differentiating first-order conditions (7) with respect to $x_i$ gives:

$$
\begin{pmatrix}
\frac{\partial^2 \pi_i}{\partial \omega_i^2} & \frac{\partial^2 \pi_i}{\partial \omega_j \partial \omega_i} \\
\frac{\partial^2 \pi_j}{\partial \omega_i \partial \omega_j}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial q_i}{\partial x_i} \\
\frac{\partial q_j}{\partial x_i}
\end{pmatrix} = -\frac{1}{2\gamma} \begin{pmatrix}
\frac{1}{y_i} \\
\beta \frac{1}{y_j}
\end{pmatrix}.
$$
From this we obtain:
\[
\frac{\partial q_i}{\partial x_i} = \frac{1}{2 \gamma y_i y_j \Omega^C} \left( \beta y_i \frac{\partial^2 \pi_i}{\partial q_i \partial q_j} - y_j \frac{\partial^2 \pi_j}{\partial q_i^2} \right),
\]
and
\[
\frac{\partial q_j}{\partial x_i} = \frac{1}{2 \gamma y_i y_j \Omega^C} \left( y_j \frac{\partial^2 \pi_j}{\partial q_i \partial q_j} - \beta y_i \frac{\partial^2 \pi_i}{\partial q_j^2} \right).
\]
Observe that \(\partial \pi_i/\partial q_j = q_i(\partial p_i/\partial q_j)\), which is negative if products are demand substitutes. Note further that \(\partial \pi_i/\partial x_i = q_i/(2\gamma y_i)\), and that \(\partial \pi_j/\partial x_i = \beta q_j/(2\gamma y_j)\). The signs of the strategic and spillover component respectively follow:
\[
\begin{align*}
\frac{\partial \pi_i}{\partial q_j} + \frac{\partial^2 \pi_j}{\partial q_i \partial q_j} - \frac{\partial \pi_j}{\partial q_i} \frac{\partial^2 \pi_i}{\partial q_j^2} &= \frac{\partial \pi_i}{\partial q_j} \left( \frac{\partial^2 \pi_j}{\partial q_j \partial q_j} - \frac{\partial^2 \pi_j}{\partial q_j^2} \right), \\
\frac{\partial \pi_i}{\partial q_j} + \frac{\partial^2 \pi_i}{\partial q_i \partial q_j} - \frac{\partial \pi_i}{\partial q_j} \frac{\partial^2 \pi_j}{\partial q_i^2} &= \frac{\partial \pi_i}{\partial q_j} \left( \frac{\partial^2 \pi_i}{\partial q_i \partial q_i} - \frac{\partial^2 \pi_i}{\partial q_i^2} \right).
\end{align*}
\]
The second-stage equilibrium prices follow from:
\[
\frac{\partial \pi_i}{\partial p_i} = q_i + (p_i - c + y_i) \frac{\partial q_i}{\partial p_i} \equiv 0.
\]
An interior solution is assumed. The second-order condition and the stability condition require:
\[
\begin{align*}
\frac{\partial^2 \pi_i}{\partial p_i^2} &= 2 \frac{\partial q_i}{\partial p_i} + (p_i - c + y_i) \frac{\partial^2 q_i}{\partial p_i^2} \leq 0, \\
\Omega^B &= \frac{\partial^2 \pi_i}{\partial p_i^2} \frac{\partial^2 \pi_j}{\partial p_j^2} - \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} \frac{\partial^2 \pi_j}{\partial p_j \partial p_j} > 0.
\end{align*}
\]
Totally differentiating the first-order condition with respect to \(x_i\) yields
\[
\begin{pmatrix}
\frac{\partial^2 \pi_i}{\partial p_i^2} & \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} \\
\frac{\partial^2 \pi_j}{\partial p_i \partial p_j} & \frac{\partial^2 \pi_j}{\partial p_j^2}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial p_i}{\partial x_i} \\
\frac{\partial q_i}{\partial x_i}
\end{pmatrix}
= -\frac{1}{2 \gamma} \begin{pmatrix}
\frac{1}{y_i} & \frac{\beta}{y_j} \\
\frac{\beta}{y_i} & \frac{1}{y_j}
\end{pmatrix}.
\]
From this we obtain:
\[
\frac{\partial q_i}{\partial x_i} = \frac{1}{2 \gamma y_i y_j \Omega^B} \left( \beta y_i \frac{\partial^2 \pi_i}{\partial q_i \partial q_j} - y_j \frac{\partial^2 \pi_j}{\partial q_i^2} \right),
\]
and
\[
\frac{\partial q_i}{\partial x_i} = \frac{1}{2\gamma y_i y_j \Omega_B} \left( y_j \frac{\partial^2 \pi_j}{\partial q_i \partial q_j} - \beta y_i \frac{\partial^2 \pi_i}{\partial q_i^2} \frac{\partial q_j}{\partial p_j} \right).
\]
Observe that \( \frac{\partial \pi_i}{\partial p_j} = (p_i - c - y_i) (\partial q_i/\partial p_j) \), which is positive if products are demand complements. Note further that \( \frac{\partial \pi_i}{\partial x_i} = q_i/(2\gamma y_i) \), and that \( \frac{\partial \pi_j}{\partial x_i} = \beta q_j/(2\gamma y_j) \). It then follows that
\[
\frac{\partial \Pi^B}{\partial x_i} = \frac{\partial \pi_i}{\partial p_j} \frac{\partial p_j}{\partial x_i} + \frac{\partial \pi_j}{\partial p_i} \frac{\partial p_i}{\partial x_i} + \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi_j}{\partial x_i} - 1 = 0,
\]
or:
\[
\frac{\partial \Pi^B}{\partial x_i} = \frac{1}{\Omega_B} \left( \frac{\partial q_i \partial \pi_i \partial^2 \pi_j}{\partial p_i \partial p_j \partial p_i \partial p_j} - \frac{\partial q_i \partial \pi_j \partial^2 \pi_i}{\partial p_i \partial p_j \partial p_i \partial p_j} \right) + \frac{\beta y_i}{y_j \Omega_B} \left( - \frac{\partial q_j \partial \pi_i \partial^2 \pi_i}{\partial p_j \partial p_j \partial p_i^2} + \frac{\partial q_j \partial \pi_j \partial^2 \pi_i}{\partial p_j \partial p_i \partial p_j \partial p_i} \right) + q_i + \frac{\beta q_j y_i}{y_j} - 2\gamma y_i = 0
\]
(22)

For non-cooperative R&D, the equivalent expression to (10) is:
\[
\frac{\partial \pi_i^B}{\partial x_i} = \frac{1}{\Omega_B} \frac{\partial q_i \partial \pi_i \partial^2 \pi_j}{\partial p_i \partial p_j \partial p_i \partial p_j} - \frac{\beta y_i}{y_j \Omega_B} \frac{\partial q_j \partial \pi_i \partial^2 \pi_i}{\partial p_j \partial p_i \partial p_j \partial p_i} + q_i - 2\gamma y_i \equiv 0.
\]
(23)

Comparing (23) with (22) yields the components of the combined-profits externality with Bertrand competition:
\[
\sum_{j \neq i} \frac{\partial \pi_j^B}{\partial x_i} = \frac{\partial \Pi^B}{\partial x_i} - \frac{\partial \pi_i^B}{\partial x_i} = \underbrace{- \frac{1}{\Omega_B} \frac{\partial q_i \partial \pi_j \partial^2 \pi_j}{\partial p_i \partial p_i \partial p_j}}_{\text{strategic component (-)}} + \underbrace{\frac{\beta y_i}{y_j \Omega_B} \frac{\partial q_j \partial \pi_j \partial^2 \pi_i}{\partial p_j \partial p_i \partial p_j \partial p_i}}_{\text{spillover component (-)}} + \underbrace{\beta q_j y_i}{y_j} \quad .
\]

The signs of the separate components of the combined-profits externality are unambiguous.

6.2.2 Proof of Proposition 2
\[\tilde{y}^C > \tilde{y}^B \iff 2\gamma (a - c) (1 + \beta) (4 - \theta^2)^2 (1 - \theta) \theta^3 > , \text{ or } \beta > -1.\]  
QED
6.2.3 Proof of Proposition 3

Prices are lower under second-stage Cournot competition than under second-stage Bertrand competition if, and only if $\bar{Q}^C > \bar{Q}^B \iff \gamma < (1 + \beta)/(4 - \theta^2)$. $\quad \text{QED}$

6.2.4 Proof of Proposition 4

First note that

$$\tilde{\pi}^C = \frac{\gamma(a - c)^2(2 - \theta)^2}{\Delta^C}$$

and

$$\tilde{\pi}^B = \frac{\gamma(a - c)^2(2 + \theta)^2(1 - \theta)^2}{\Delta^B},$$

where $\Delta^C = \gamma(4 - \theta^2)^2 - (1 + \beta)(2 - \theta)^2$ and $\Delta^B = \gamma(4 - \theta^2)^2(1 - \theta^2) - (1 + \beta)(1 - \theta)^2(2 + \theta)^2$. The result then follows as:

$$\tilde{\pi}^C - \tilde{\pi}^B = \frac{2\gamma^2(a - c)^2(4 - \theta^2)^2(1 - \theta)\theta^3}{\Delta^C \Delta^B} > 0.$$  

$\quad \text{QED}$

6.2.5 Proof of Proposition 5

Here we follow the proof of Proposition 5 in Qiu (1997). First note that $\widetilde{TS}^B - \widetilde{TS}^C = \gamma(a - c)^2 F(\gamma, \theta, \beta) / [(\Delta^B)^2(\Delta^C)^2]$, where $\Delta^B = \gamma(1 - \theta^2)(4 - \theta^2)^2 - (2 + \theta)^2(1 - \theta)^2(1 + \beta)$, $\Delta^C = \gamma(4 - \theta^2)^2 - (2 - \theta)^2(1 + \beta)$, and $F(\gamma, \theta, \beta) = [\gamma(2 + \theta)^2(1 - \theta)^2(4 - \theta^2)^2(3 - 2\theta)(1 + \theta) - 2(2 + \theta)^4(1 - \theta)^4(1 + \beta)] [\Delta^C]^2 - [\gamma(2 - \theta)^2(4 - \theta^2)^2(3 + \theta) - 2(2 - \theta)^4(1 + \beta)] [\Delta^B]^2$.

Define $G(\gamma, \theta, \beta) = F(\gamma, \theta, \beta) / (\gamma^2(4 - \theta^2)^2)$. Obviously,

$$\text{sign} \left( \frac{\widetilde{TS}^B - \widetilde{TS}^C}{\Delta^C \Delta^B} \right) = \text{sign}(G(\gamma, \theta, \beta)).$$

Note that $G(\gamma, \theta, \beta) = \gamma^2 g_1 + \gamma g_2 + g_3$, where $g_1 = (4 - \theta^2)^4(1 - \theta)^2(1 + \theta)(4 - 2\theta - \theta^2)$, $g_2 = -2(4 - \theta^2)^2(1 + \beta)(16 - 32\theta + 6\theta^2 + 20\theta^3 - 15\theta^4 + 3\theta^5 - \theta^6 + \theta^7)$, and $g_3 = (1 - \theta)^2(2 + \theta)^2(2 - \theta)^2(1 + \beta)^2(4 - 2\theta + \theta^2 - \beta^2)$. It follows that $G(\gamma, \theta, \beta)$ is strictly convex in $\gamma$ as $\partial^2 G(\gamma, \theta, \beta) / \partial \gamma^2 = 2g_1 > 0$ (note: $\min_{\{\theta\}} g_1 = \lim_{\theta \to 1} g_1 = 0$).

Moreover, $g_2^2 - 4g_1g_3 > 0 \forall \theta \in (0, 1)$. Hence, given any $\theta \in (0, 1)$, there are two real solutions to $G(\gamma, \theta, \beta) = 0$, in particular:

$$\gamma_1(\theta) = -\frac{g_2 - \sqrt{g_2^2 - 4g_1g_3}}{2g_1}, \quad \text{and} \quad \gamma_2(\theta) = \frac{-g_2 + \sqrt{g_2^2 - 4g_1g_3}}{2g_1}.$$
The larger root is to be considered as

\[
\begin{align*}
\min_\theta \{ \gamma^* - \bar{\gamma}_1(\theta) \} &= \lim_{\theta \to 0} \{ \gamma^* - \bar{\gamma}_1(\theta) \}_{\beta=1} = 0,
\end{align*}
\]

where \( \gamma^* \) is the threshold value induced by R2. Label the larger root \( \bar{\gamma}(\theta) \).
Then observe that \( \min_{\theta, \beta} \{ \partial \bar{\gamma}(\theta) / \partial \beta \} = \lim_{\theta \to 0} \partial \bar{\gamma}(\theta) / \partial \beta |_{\beta=0.5} = 0.25 \). This gives rise to the different lines in Figure 2 for different values of \( \beta \). Obviously, for any \( \gamma > \bar{\gamma}(\theta) \) we are in situation (i) while situation (ii) emerges for any \( \gamma > \bar{\gamma}(\theta) \). The rest of the proof then follows. QED

Figure 2: \( G(\gamma; \theta, \beta) \) for different sizes of the technological spillover, whereby \( a = 100, c = 10; \) and \( \theta = 0.9 \).
References


