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Perspectives on an integrated computer learning environment

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I conducted various exploratory case studies on the performance of secondary school students and the usability of developed tools in ICT-supported practical investigation tasks in the classroom. In most of these studies, pre-university students carried out quantitative mathematical modeling activities using ICT, that is, they explored mathematical models with the support of ICT tools in order to come to grips with natural phenomena and to interpret real data. The authentic nature of the practical investigations can be interpreted as the opportunity for students to work on real-world problems, with the goal to come to grips with phenomena through scientific methods. Students worked directly with high-quality data in much the same way as scientists and practitioners do and they explored mathematics and science in authentic problem situations, with the teacher helping them to learn to do scientific inquiry of good quality. Data were collected by the students themselves or originated from professional data bases and published research studies.

In this chapter I report on the results of classroom studies on ICT-supported practical investigations. It is organized as follows. In the first introductory section I explain how the reports in subsequent sections have been compiled from papers published in conference proceedings and journals. These papers, together with corresponding instructional materials, can be found on the CD-ROM that is part of the presented research and development work. Because the research settings and methods applied in the case studies had much in common I also briefly discuss these aspects of the case studies in the introduction. The classroom studies have been grouped on the basis of the subject of the students' investigations and on the characteristics of the ICT use. This led to the following categorization:

- Student work with real data, tables, and graphs in the context of human growth.
- Investigation of shapes of real objects through digital image analysis and mathematical modeling.
- Video analysis of human locomotion.
- Video-based practical work at pre-vocational secondary school level.
- Spreadsheet-based investigation in the context of survival analysis of clinical data and in the context of handling weather data.

- Computer-based modeling in the context of quantitative pharmacology, and particularly in the context of alcohol metabolism.
- Combination of video analysis and computer-based modeling in a study of bouncing balls.

The instructional materials of the practical investigation tasks of the first and third category (modeling human growth and gait analysis, respectively) were used by quite a few students as starting point of their profile research projects. In this thesis I mainly report on the student work and the use of ICT in the practical investigation tasks listed above.

2.1 Introduction

In most of the exploratory case studies, the main subject of my research was the student learning of quantitative mathematical modeling using ICT, with a special interest in the usability of the applied ICT tools. The chosen approach in the student practical investigations was mostly guided instruction, in which students were introduced in inquiry work and in the use of particular ICT tools for doing research on the basis of knowledge and skills that they were supposed to have mastered before. A strong motivation for the instructional development was to confront students with real world problem situations and to let them experience that, with the mathematics and science knowledge and skills that they already possessed, they could in principle carry out practical work that resembles what scientists and practitioners do. Students were expected to develop and practice research abilities by carrying out small investigation tasks, using techniques and ICT tools that are common in scientific inquiry. In most cases, the students carried out the practical investigations in small teams.

In short, the exploratory case studies served the following goals: (1) They were meant to gain insight in the needs of secondary school students for doing authentic inquiry work. (2) They helped me specify requirements for an integrated computer learning environment from a mathematical point of view. (3) They served to test the usefulness and scope of (prototypical) implementations of particular tools for collecting, processing, and analyzing data. (4) They gave an impression of the potential of ICT regarding the realization of challenging, cross-disciplinary practical work in which secondary school students were engaged in activities such as experimenting, data collection, and data analysis in much the same way as scientists and practitioners.

I cooperated with mathematics and physics teachers in all classroom studies, partly thanks to the NWO-project ‘Teacher in Research’. The instructional materials were created in close collaboration with these secondary school teachers, who always had more than twenty-five years of experience in teaching and in creating instructional materials (two of them were authors of mathematics textbooks used in Dutch schools). The classroom study always took place at the schools where the collaborating teachers worked. These schools always had adequate or really good ICT facilities for their teachers and students. Naturally, the fact that the teachers had collaborated in the instructional design and had much teaching experience, as well as the fact that the school were well-equipped for computer-based inquiry work by students influenced the research outcomes. But I am of opinion that innovations in education are better

tried out first under conditions that seem optimal, before testing them broadly under regular conditions, for example with less experienced teachers, or at schools with less good facilities. In exploratory case studies, this selection of schools and teachers involved allows to explore the potential of innovations. Naturally, when an innovation seems promising, it must be evaluated more broadly under conditions regularly found in educational practice. But this final phase, which is for example present in the Integrative Learning Design framework for design-based research of Bannan-Rittland (2003), was beyond the scope of the work presented in this thesis.

In most case studies, the teachers involved in the instructional design were also the leading persons in the classroom who guided the students in their practical work. Whenever possible, I took in the classroom the role of observer of the activities in which the teachers and students engaged. This often turned into participatory observation: Then I took on an active role during practical investigations by assisting students in their data collection, answering their questions, and by giving advice. At the same time, by asking questions during practical investigations I could probe the students' understanding of the problem situation and their inquiry skills. In-class impromptu interviews with students occurred while they were engaged in data collection or data handling, usually following up a comment or questions made by a student to his or her teammate(s), other classmates, the teacher, or to the researcher. But I always tried to avoid a strong influence from my part on the students' way of working and reasoning. For this reason, I prepared myself for every classroom meeting with students by making a list of things I aimed to find out about the students' inquiry learning and about their envisioned use of ICT tools. Classroom observations and in-class impromptu interviews were recorded as field notes, to which I also added personal impressions of how the instructional materials had functioned.

Classroom observation, and participatory observation in particular, is a standard research method for data collection in design research. Other classical methods for data collection that I applied in my exploratory case studies were:

- Interviews with teachers after each lesson to record their impressions of the activities and of how the instructional materials and the ICT tools had functioned.
- Audio and video recording of teacher instructions and group discussions of student teams in the classroom, which I afterwards used to compare my classroom observations and the teacher's impressions (but I hardly transcribed them verbatim).
- Computer registration of student activities with screen capture software to find out what the students had really done, for example, when they were unobserved by the teacher and the researcher.
- Completed worksheets and reports of students, which gave an impression of the quality of the students' work and indicated whether the students had in the first place been able to meet the learning goals of the instruction.
- Questionnaires for students, mostly used to find out how they had experienced the lesson activities and whether this was in accordance with their expectations of doing practical investigations. I probed what students thought of the subject of study, the instructional materials, the use and quality of the ICT tool, and so forth.

I did not apply all research methods in each and every classroom study, but always strived for a multi-method approach to data collection in order to increase trustworthiness of the results and findings. The aim of all data collection was to answer my leading questions in the exploratory case studies such as “How did the designed instructional materials and the ICT tools function in school practice?”, “Were learning goals achieved?”, “Did the students use the ICT tools in the envisioned way?”, and “How could the instructional materials and the ICT tools be improved?”

The reports on the outcomes of the classroom case studies, presented in the next sections, have all been structured in the following way:

- Reference to the full publication(s) about the described practical investigation.
- Introduction to the subject of the practical investigation and an outline of the research objectives regarding the students’ learning of doing scientific inquiry by use of ICT tools, and the research instruments.
- Short description of the classroom study, which includes data about the participants, time and place of the experiment, the learning objectives, and the lesson design, the instructional materials, and the envisioned use of ICT.
- Summary of the main results and findings of the classroom study.
- Possible extensions of the subject of study, with references to publications of the author for the interested reader.

The reports about the classroom studies are deliberately held short because of the amount of case studies to discuss (ten studies) and because the interested reader can anyway get more information from the papers published in journals and conference proceedings, which can be found together with corresponding instructional materials on the accompanying CD-ROM. However, three exceptions to this general rule have been made: The classroom studies about human growth and gait analysis are discussed in more detail because the instructional materials of these practical investigation were used by quite a few students as starting point of their profile projects. I elaborate on the classroom study about bouncing balls because classroom experiences were only discussed in detail in an unpublished Dutch manuscript (Heck, 2008f).

The number of classroom studies was large because I wished to cover many different types of data collection and data handling, using various ICT tools. Table 2.1 illustrates the broad range of inquiry activities and ICT tools involved. Regarding the use of the COACH environment, Table 2.1 also shows that a large variety of ICT tools was covered by the classroom case studies: handling given data, image measurement and analysis, video analysis, text-based and graphical modeling, regression analysis, and a combination of measurements and modeling. Measurements with sensors, image measurement, control experiments, and more combinations of ICT tools in version 6 of COACH were explored through sample activities and field experiments. They are discussed in the Chapter 3.

Instead of choosing an in-depth approach investigating a few inquiry techniques and ICT tools, I selected a breadth-first approach in which a broad collection of inquiry activities and a versatile set of ICT tools were explored to get a more holistic view of ICT-supported practical investigations by students. One of the goals of my

work was after all to contribute to the development of an integrated computer learning environment for inquiry-oriented mathematics and science education, which is by definition a general purpose system. It is true that for each implemented component in the envisioned system, and also in COACH, a dedicated computer program may be more sophisticated, but it is the integration of a variety of ICT tools in a single environment that determines the usability of a computer learning environment. This is the key point in the STOLE concept outlined in Section 1.2.

<i>subject</i>	<i>ICT tool</i>	<i>data collection and data handling</i>		
		<i>collection</i>	<i>processing</i>	<i>analysis</i>
human growth	COACH 5 measurement activity without an interface	data from Dutch growth studies	graphical representation; increase diagram	graph sense; basic statistics; regression with simple functions
image analysis of bridges and hanging chains	COACH 5 image measurement and analysis	measurement on given video clips in which all frames are equal	comparison of graphs for different problem situations	regression with quadratic functions
modeling shapes of bridges and hanging chains	COACH 5 text-based modeling	computer generated data	comparison of graphs; use of parameters in a model; length of a curve	comparison of modeling results with graphs of mathematical functions
gait analysis: case study 1	COACH 5 video analysis	measurement on self-recorded video clips in the classroom	position-time, angle-time, and angle-angle graphs	comparison of graphs with gait patterns; sinusoidal regression
gait analysis: case study 2	COACH 5 video analysis	measurement on self-recorded video clips in a sports center; measurement of step frequency	position-time, angle-time, and angle-angle graphs; step frequency vs gait speed	comparison of graphs with gait patterns; regression with simple mathematical functions
video-based practical work at pre-vocational level	COACH 5 video analysis	measurement on self-recorded video clips in the classroom	position-time graphs	graph interpretation
survival analysis	EXCEL spreadsheet	data from a clinical study	handling survival data with censoring; actuarial methods	comparison of methods for estimating survival probabilities
analysis of weather data	EXCEL spreadsheet	weather data from a meteorological institute	moving average	comparison of statistical methods
alcohol metabolism	COACH 6 graphical modeling	data from research studies and measurements with a breath analyzer	interpretation of concentration-time graphs	comparison of model results with data; bi-exponential regression
bouncing balls	COACH 6 video analysis and graphical modeling	measurement on self-recorded video clips in the classroom	position-time, velocity-time, acceleration-time graphs	comparison of the quality of models

Table 2.1: Overview of tool use, data collection and data handling involved in the classroom case studies on practical investigations by secondary school students.

2.2 Student Work with Real Data about Human Growth

References

A short paper appeared in 2002 in the ICTMT 5 Conference Proceedings; a longer, more detailed version appeared in 2004 on the conference CD-ROM.

Heck, A., & Holleman, A. (2002). Modelling human growth. In M. Borovcnik & H. Kautschitsch (Eds.), *Technology in Mathematics Teaching Proceedings of the ICTMT 5 in Klagenfurt, 2001*, Schriftenreihe Didaktik der Mathematik v. 25 (pp. 405–408). Vienna: öbv&hpt Verlagsgesellschaft.

Heck, A., & Holleman, A. (2004). Modelling human growth. Extended paper in M. Borovcnik & H. Kautschitsch (Eds.), *Electronic Proceedings of ICTMT 5, Klagenfurt 2001*. Vienna: öbv&hpt Verlagsgesellschaft (CD-ROM).

Introduction

The Dutch human growth study of 1997 (Wit, 1998; Fredriks, 2004) was used to create ICT-based instructional materials for students who were in their first year of the Second Stage of pre-university education (age 15-16 yr.) and had no experience with practical investigation tasks. Much of the work could have been done with a spreadsheet software like EXCEL, but I chose for COACH because for some assignments I needed a regression tool that could be interactively used and such a tool had been (at that time) newly designed and implemented in COACH 5, ready for local evaluation in practice. The students in this classroom case study had not worked with COACH before. The motivation to work with scientific data was that, although body growth of children is often used in mathematics textbooks as an example for discussing processes of change, statistics, and discrete and continuous models of growth, the real world context is in many exercises only used for dressing up a mathematical problem or as an ideal illustration of a mathematical concept, resulting in unrealistic data presented in textbooks.

In this case study I explored the ICT-supported graph sense of the students and I aimed at getting answers to the following questions:

- Do the instructional materials and the chosen instructional setting enable students, who have no prior practical experience with COACH, to acquire the skills and experience to use the tabular and graphical tools effectively in their study of human body growth?
- What do students think of the use of the software?
- What do students think of the subject, the instructional materials, and the use of real data?
- Since it is the first time that the students do practical work in the context of mathematics all by themselves with a computer, how do they like it and what difficulties do they encounter?

The research instruments used to get answers to these questions were classroom observations, teacher interviews, video recordings (including video capturing of computer work), a questionnaire, the reports of the students about their work, and the underpinning COACH result files.

The Classroom Experiment

Twenty-six vwo-4, Mathematics A students (13 male and 13 female; age 15-16 yr.), who had chosen for the Culture & Society or the Economics & Society profile, participated in the experiment. The practical assignment was not part of the students' examination portfolio because it was their first experience with a practical investigation in the context of mathematics, but instead it was graded as a regular test in the semester. The students worked in pairs for two weeks in March 2001. Estimated study load was 6 to 8 hours. In the first week, work took mainly place in the computer laboratory during the regular mathematics lessons (of 45 minutes). The mathematics teacher and the researcher were present as assistants. In the second week, the students could make use of the computer facilities at school to work autonomously on the assignments.

Instructional materials were developed in accordance with the learning objectives to let students

- work with real data and with diagrams that are actually used in health care;
- experience how much useful information can actually be obtained from diagrams;
- see that the change of a quantity is often as important and interesting as the quantity itself;
- practice ICT skills; and
- carry out practical work in which they can apply much of their mathematical knowledge.

The instructional material consisted of the following three compulsory assignments, of which the first one took place in the regular classroom and the other two in a computer laboratory at school:

1. *The Dutch Growth Study of 1997*. A quiz gives the students an idea of how much they already know about body growth and puberty. A newspaper article introduces the fourth national growth study and illustrates the relevance of growth data.

2. *Mean Height Growth*. Students familiarize themselves with the main tools of COACH 5 for studying human growth. They learn how to make data plots and increase diagrams of height in relation to age, and they learn how to interpret these diagrams in the context of child growth.

3. *Secular Height Growth in 1980-1997*. Secular growth means the changes in the development of children from one generation to another. In this task, students use the data from the Dutch surveys of 1980 and 1997 to study the changes in mean height for Dutch boys and girls in this period. Questions stimulate students to formulate and underpin their conclusions.

Finally, the students selected one of the following subjects to investigate independently in teams of two:

A. *Growth Charts of Native Dutch Children* (no use of ICT in this activity). Students familiarize themselves with the growth charts for native Dutch children and with the mathematical terminology. They learn how these diagrams are made, what they mean, and how they are used. They also compare their own height and weight with their peer group.

B. Mathematical Model of Height Growth for Girls. Students compare the mean height growth for girls with Turner syndrome with the growth for healthy girls and they make a quadratic model of the height growth for girls from age 3 until puberty.

C. Mean Weight Growth. Students investigate the mean weight growth for healthy Dutch boys and girls in relation to age. They look for similarities and differences, and they make a simple mathematical model of weight-for-age for children until puberty.

D. Mathematical Weight-for-Height Model. Students investigate the mean weight growth for Dutch children in relation to height and make a mathematical model of weight-for-stature.

In data handling, representing the data in tabular and graphical way is an essential step to start making sense of the data in relation to the problem situation. For example, data graphs are used to detect and clarify patterns in large numerical data sets and to make trends evident. In the activities of the practical investigation about human growth, the main role of ICT was to help visualize, process, and analyze real growth data. Computer use is convenient in such work because the computer screen is sufficiently large for displaying growth diagrams and increase diagrams such that one can discover differences in mean body growth between boys and girls. The diagrams on the left-hand side of Figure 2.1, which is a screen dump of a COACH activity, are about mean height growth of Dutch children in relation to their age.

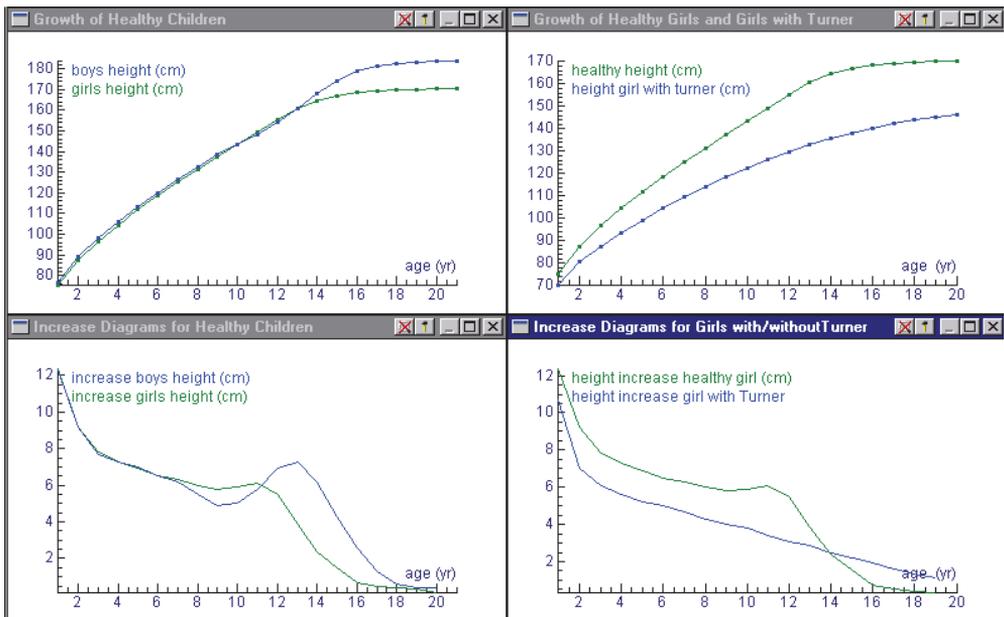


Figure 2.1: Screen shot of a COACH activity about the mean height and the increase in mean height for Dutch boys and girls.

Two things catch the eye in the increase diagrams, which were constructed by a built-in data processing function: (1) the growth spurt during puberty; and (2) the fact that this occurs for boys later than for girls, and with higher intensity. Students were prompted by questions in the compulsory activities to ‘look behind the data or graph’, that is, to connect the graphical information with the context. I use here the terminology of Shaughnessy (2007, pp. 989–991) for levels of graph sense, who ex-

tended earlier work of Curcio (1981, 1987) in which she had distinguished the following three components of graph sense: (1) ‘reading the data or graph’ [i.e., locating presented information and translating it into other form of communication]; (2) ‘reading between the data or graph’ [i.e., interpreting and integrating presented information]; and (3) ‘reading beyond the data or graph’ [i.e., generating information by extending and predicting data and by inferring of data]. These levels of graph sense were also covered in the student activities. Regression analysis is a typical example of ‘reading beyond the data or graph’ and this was applied by students doing assignments B, C, or D via the (at that time) newly developed function fit tool of COACH 5.

At this point I would like to point out that it is also instructive to investigate diagrams for children with growth disorders. The diagrams on the right-hand side of Figure 2.1 are about mean height growth of healthy Dutch girls and girls with Turner syndrome. Two symptoms of Turner syndrome can be found in the diagrams: slow growth and no pubertal growth spurt. For girls with Turner syndrome the growth rate looks after the age of three like a linear function of age. In other words, the mean height of girls with Turner syndrome in relation to age can be described mathematically by a parabola. As a matter of fact, the quadratic model describes it up to a millimeter. This underpins the idea to explore a quadratic model for the growth of healthy girls in their childhood (from age 3 to the onset of puberty), which is a component of the Infancy-Childhood-Puberty (ICP)-model for human growth (Karlberg, 1989). It is then useful that a COACH user can manually fit the data by using the mouse to translate and stretch the graph of a template function selected as the regression curve; see Figure 2.2 (taken from Ellermeijer & Heck, 2002, p. 53).

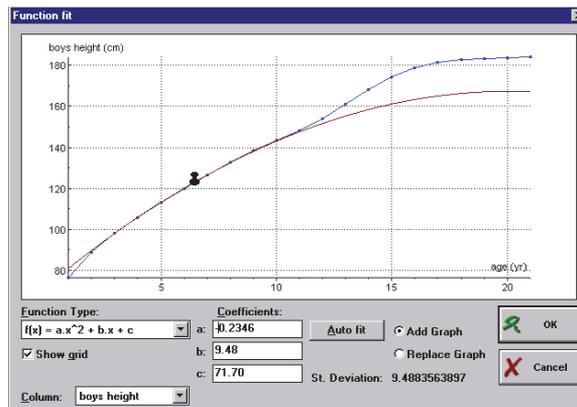


Figure 2.2: Screen shot of a manual function fit of the childhood component of the ICP-model for human growth via the regression tool of COACH 5.1.

The regression tool instantly gives in return to mouse actions in the regression window the updated graph and the updated parameter values of the template function. The dynamics and instant feedback of the regression tool helps the user match a graph with data. In this particular case, students doing assignment B were expected to search for a parabola that on the one hand fits well the mean boys height between 3 and 10 years of age, and that on the other hand reaches its maximum at the age of 20 years, when height growth usually stops. The formula of a parabola, $y = ax^2 + bx + c$, has been selected as function type in the above screen shot; the selected column corresponds with the mean height of Dutch boys. The icon of the pin on the screen shot is such

that the approximation has been fixed at that location. By dragging another point of the parabola with the mouse one can now re-scale the graph. When the fixed pin is released by double clicking, then one can translate the parabola, that is, independently change the parameter c . This quadratic model is one of the regression models that can be described by a formula of type $y = \mu f(\nu x + \rho) + \sigma$, where f is a simple mathematical function and μ , ν , ρ , σ are parameters, and that have been implemented in COACH.

Results and Findings

I summarize the outcomes of the classroom experiment in the same order as the questions in the introductory paragraph.

Performance of the students. The strongest impression made the good quality of the students' work in general. An example is the following description of the difference in height and height increase between healthy girls and girls with Turner syndrome:

“The difference in height for girls with and without Turner syndrome is not so big until the age of two. You can also see that they skip puberty more or less. Where a healthy girl starts growing faster during puberty and you can see a peak in the diagram, this is not the case for girls with Turner syndrome. They grow as it were constant.”

The students who wrote this in their report made good sense of the height diagram and of the increase diagram, regardless of the weaknesses in the formulation from mathematical point of view.

The abilities of students to create and interpret graphs has been focus of numerous research studies (See, for example, Leinhardt, Zaslavsky, & Stein, 1990, and references therein). Classroom observations indicated that the students in this classroom experiment usually had no problems with interpretation tasks up to intermediate level. Local processes such as point reading were easy for them. However, most of the questions in the learning material were interpretation tasks on a global level and could be characterized as interval reading. In general, the students had no difficulties with these tasks. Interval/point confusion was not noticed. Maybe that the authors' attention to unambiguous phrasing of questions paid off or that the focus on interval tasks simply avoided the confusion.

Only one question in the activity about height-for-age, namely “Who grows faster, boy or girl, and at what age?”, put some students on the wrong track. They did not use the increase diagrams to answer the question, but they instead confused the statement “boys growing faster than girls” with “boys being taller than girls” and used the graph of the difference in height between boys and girls for their answer. This is an example of the slope/height confusion, meaning that the students confused the quantity and the change of a quantity. However, it may well illustrate that the students were not used to think of change of a quantity as an interesting quantity itself. This was noticed in assignment C, which is essentially the same as the activity about height growth (weight instead of height). Yet no student team investigated increase diagrams.

Students' opinions about the use of Coach. The students quickly familiarized with COACH. They reported that they liked the quick and easy way of creating graphs from tables, and vice versa. They appreciated that they could copy and paste graphs and tables into their report. They found it convenient that the instructions were not

only presented on paper, but also inside the COACH activity. On the negative side, students mentioned that saving results caused sometimes runtime errors¹ and that explanations were sometimes too vague for them. Some students reported difficulties with getting the graphs exactly in the way they wanted them: Choosing unique names for quantities and labeling axes seemed to be the biggest problems. Name clashing troubled students who followed instructions literally and who forgot that names of quantities must be unique. The students had hardly problems with a second vertical axis in the diagram. They used it for the first time when they were asked to plot the difference in height of boys and girls. For example, one student team explained the use of a second axis as follows: “The second vertical axis is for scaling the difference. This way, you can see the line better and it is not so small.” Fortunately, when students had difficulties with the software, this did not hinder them to get good results in the end as teams often helped each other out.

As was noted in the paragraph about the classroom experiments, COACH has a function fit tool that enables students to work with various regression models. The students could easily work with the tool, even though regression was no subject in the mathematics curriculum, and it was observed that the students felt free to try any function type in the tool. For example, the weight-for-length curve in assignment D was fitted by students as a modified rational function and as a modified exponential curve. When asked a simple function fit, most students interpreted this as a fit with a straight line; a quadratic fit was apparently not simple for them. The students had difficulty in analyzing a curve in pieces, or at least they did not do this if this was not explicitly asked.

I end this paragraph with some general remarks. Girls performed better than boys in this experiment and weak students had a chance to do better. Girls seemed to be more interested in the subject of body growth and they paid more attention to the report (even decorate it with pictures). This is also evident from the population of students who chose the subject as human growth for their profile project and used the instructional materials as starting point for their research: All students were female.

Students' opinions about the subject and the instructional materials. Almost all students found that the quiz and the newspaper article formed a nice introduction to the subject. Half of the students found the 15-minutes demonstration of the software at the beginning too short. From the students' responses in the questionnaire no conclusion could be drawn whether the students rated the COACH instructions as clear and sufficient. They had no complaints about the number of hints. All students reported that the structure of the second activity, which consisted of explicit questions before the task of answering the research question, helped them to formulate their summary of height growth of Dutch children. Half of the students were of opinion that the third activity about secular height growth, in which they got less procedural hints, linked up well with what they had learned about COACH before. For them the gradual move in the sequence of activities from a closed assignment with guiding questions and detailed procedural help for using the software environment to a more open-ended, less pre-structured assignment worked well. But also half of the students found that this assignment did not link up well with their mathematical knowledge.

¹The installation of the software on the computers in the computer laboratory differed from the way the COACH developers had intended. This was the main cause of the problems that students had with saving results.

Students' experiences. All students indicated that they liked it that they were provided the opportunity to choose the last assignment. Many of them admitted that they chose the task that they guessed would be easiest, others (mostly girls) gave more personal reasons. For example, one team wrote: "Turner syndrome looks to us an interesting subject. You have to compute something about girls and then it is nice to see if it matches with yourself." None of the students found the assignments easy, but their biggest complaint was that time was too short.² No surprise that the students' appreciation of the practical investigation ranged from statements like "We do not like it much. We need more time for it." to comments such as "We like it very much. You get a lot of freedom. Only time was too short." Some students preferred the normal mathematics lessons in which the teacher is always around to help them. Others enjoyed the freedom in this kind of activities or simply liked the subject. We quote one student team: "It was fun to do; something different from the textbook and an interesting subject."

Extensions

One can hardly expect that secondary school students discover a mathematical model for human growth by themselves. But it is already nice if they can validate a proposed mathematical model. A fine model to test is the Infancy-Childhood-Puberty (ICP) model developed by Karlberg (1989) for height in relation to age. This model breaks down growth mathematically into three partly superimposed components:

1. *Infancy* (0-3 years). Restricted growth, in which the growth rate is a linear function with respect to height, can be represented by the modified exponential curve:

$$H_1 = a_1 - b_1 e^{-c_1 t}, \quad \text{where the symbol } t \text{ stands for age.}$$

2. *Childhood* (from 3 years of age to the onset of puberty). A simple quadratic function describes growth during this period quite well:

$$H_2 = a_2 t^2 + b_2 t + c_2.$$

3. *Puberty.* The contribution of the pubertal growth spurt to the final height can be modeled using a logistic function:

$$H_3 = \frac{a_3}{1 + e^{c_3 - b_3 t}}.$$

Here, $a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3$ are positive parameters, which must be estimated from the growth data. The mean height for each age, H , is given by $H = H_1 + H_2 + H_3$. One can use the following curve fitting procedure (Heck, 2002b). First, note that the childhood component is the only part of the model that is described by an unrestricted growth process. Therefore, it makes sense to begin searching for a parabola that on the one hand fits well the height between 3 and 10 years of age, and that on the other hand reaches its maximum at the age of 20 years, when height growth usually stops. After subtraction of the extrapolated values of the childhood component from the observed values during the periods before and after this phase, two additional

²The students were right: The authors of the activities had underestimated the time that students need to learn COACH in the first activities, especially when small errors in the instructional materials or students' reading mistakes disturbed the learning process.

components are extracted and modeled. The modified exponential regression model and the logistic model can be applied for these two periods, respectively. To this end, the logistic regression model has been added to the list of template models in version 6 of COACH. Heck (2004b, 2008c) presented a logistic model for mean weight of Dutch boys and girls during adolescence. The above curve fitting procedure is called the least squares method of peeling-off functions (Foss, 1969) and is much applied in scientific data handling. Examples of this approach in the context of human growth have been presented in bilogistic modeling of weight as a function of age (Heck, 2004b; see also, Heck, 2008c). These mathematical models describe the human growth quite well: For example, the difference between computed and measured mean height of Dutch boys turned out to be everywhere less than 0.5 millimeters. This is a beautiful result for a formula that is completely built up from mathematical models that are all subjects in the Dutch pre-university mathematics curriculum. However, one must not think that the ICP model is the only successful mathematical model for height growth of boys and girls. In the literature (e.g., Bock & Thissen, 1976; Jolicoeur, Abidi, & Pontier, 1991; Kanefuji & Shohoji, 1990; Susman, Murphy, Zerbe, & Jones, 1998) one can find a variety of mathematical models that could be explored by students.

But there are many more topics that students could investigate. They suggested themselves the following new research questions for further practical work: “How can it be that humans get taller every generation? Will this process ever stop and does this hold for animals, too?”; “When does the mean height usually begin to decline?”; and “How does your own height and weight or the data of your classmates compare with the reference data?” The last suggestion is probably the most interesting and most instructive. Students will then encounter as a matter of course many problems in doing research, and not only mathematical problems. Some of the questions are: How do you set up the research study? Are you going to measure growth parameters or just ask participants? How do you deal with personal data? What if people refuse to collaborate? How do you present your results? And so on. In this way students can indeed experience many aspects of doing research. This has been realized in several profile projects, in which students used this practical investigation as a starter to familiarize with the subject and have high-quality reference data.

2.3 Computer-Based Investigations of Mathematical Shapes of Real Objects

In this section I present two classroom studies about modeling shapes of bridges and hanging chains: (1) an image analysis activity in a vwo-4, Mathematics B class; and (2) a text-based modeling activity in a vwo-5, Mathematics B class.

2.3.1 Image Analysis of Bridges and Hanging Chains

References

A short paper appeared in 2002 in the ICTMT 5 Conference Proceedings; a longer, more detailed version appeared in 2004 on the conference CD-ROM.

Heck, A., & Holleman, A. (2002). Investigating bridges and hanging chains. In M. Borovcnik & H. Kautschitsch (Eds.), *Technology in Mathematics Teaching Proceedings*

of the *ICTMT 5 in Klagenfurt 2001*, Schriftenreihe Didaktik der Mathematik v. 25 (pp. 409–412). Vienna: öbv&hpt Verlagsgesellschaft.

Heck, A., & Holleman, A. (2004). A practical investigation task with the computer at secondary school: bridges and hanging chains. Extended paper in M. Borovcnik & H. Kautschitsch (Eds.), *Electronic Proceedings of ICTMT 5, Klagenfurt 2001*. Vienna: öbv&hpt Verlagsgesellschaft (CD-ROM).

Introduction

In this classroom study I used the video analysis tool of COACH 5 to let students in their first year of the Second Stage of pre-university education (age 15-16 yr.), who had no prior experience with practical investigations and the video analysis tool, collect real data from a digital image, in particular of a main span of a bridge and a hanging chain. Using the regression tool they had to find out whether a parabola is an appropriate mathematical formula to describe the shape or not. Besides, students had to use their physics knowledge about force as a vector quantity to reason why under certain conditions a parabola describes the shape well and under other conditions not.

In this case study I explored the ICT-supported data handling and reasoning of the students and I aimed at getting answers to the following questions:

- Do the instructional materials and the chosen instructional setting enable students, who have no prior practical experience with the video analysis tool of COACH, to acquire the right skills and experience to use the video analysis tool effectively in their study of shapes of bridges and hanging chains?
- Does COACH, and in particular the video analysis tool, work without technical problems when applied to explore a digital image?
- What do students think of the use of the software?
- What do students think of the subject, the instructional materials, and the use of real data?
- Since it is the first time that the student do practical work in the combined context of mathematics and physics, how do they like it and what difficulties do they encounter?

The research instruments used to get answers to these questions were classroom observations, teacher interviews, video recordings (including video capturing of computer work), a questionnaire, the reports of the students about their work, and the underpinning COACH result files.

The Classroom Experiment

Twenty vwo-4, Mathematics B students (16 male and 4 female; age 14-15 yr.), who had chosen for a Nature profile, participated in the experiment. The practical assignment was not part of the students' examination portfolio because it was their first experience with a practical investigation in the combined context of mathematics and physics, but it was graded instead as a regular test in the semester. The students worked in pairs for two weeks at the end of March 2001; estimated study load was 6 to 8 hours. In the first week, work took mainly place in the computer laboratory during two class periods of 45 and 90 minutes, respectively. The mathematics teacher and the researcher were present as assistants. In the second week, the students could make use

of the computer facilities at school to work autonomously on the assignments and one mathematics lesson was spent to answers questions of students and help them further with their investigation.

Instructional materials were developed in accordance with the learning objectives to let students

- work with real data collected from digital images;
- apply mathematical models to investigate shapes;
- practice ICT skills, in particular use tools to collect data from video clips and images; and
- carry out practical work in which they can apply much of their knowledge of mathematics and physics.

The instructional material consisted of the following four assignments:

1. *Bixby Creek Bridge*. Students get acquainted with some facilities that COACH provides for video analysis, but they only have to add some measurement points instead of setting up the complete video measurement.
2. *Zeeburger Bridge*. Students apply what they learned in the first activity to analyze the shape of the Zeeburger Bridge and find that a quadratic function describes the shape of the main span quite well. They are guided through the whole process of setting up a video measurement, including the choice of the coordinate system, the calibration of length, and so on.
3. *Five weights*. Students explore a related, but simpler physical model, namely a lightweight chain with objects of equal weight attached at equal horizontal distances (See Figure 2.3). By measuring slopes and angles in the video clip, students are expected to discover that the slopes of the right segments of the chain have a fixed ratio, namely 1:3:5. patterns. Students can explain this on the basis of fundamental physics principles: They must only realize that force is a vector quantity and that equilibrium of forces holds at each point of application where a weight is attached to the chain. This simple observation about slopes can then lead to an explanation of the fact that the points of attachment lie on a parabola.

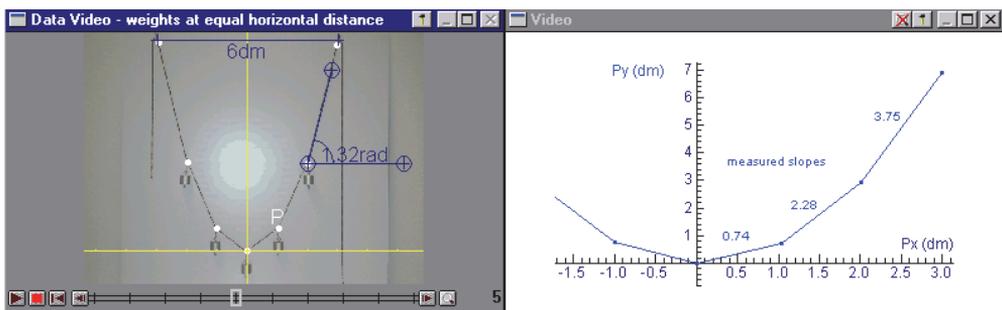


Figure 2.3: Screen shot of a measurement of positions, angles and slopes in an image.

4. *Necklace*. The next step is to realize that from an analysis of slope ratios also follows that the curve is not a parabola in case the weights are attached at equal distances along the chain. Finally, students can utilize the fixed ratio of positive

slopes to approximate the free-hanging ‘ideal’ chain by modeling it in their mind as a string of $2n + 1$ beads, for large n , where the beads are close at equal distance of each other. After the theoretical part, students could verify the non-quadratic shape of a necklace hanging under gravity by measuring a digital image of such an object.

Results and Findings

From the questionnaire, filled out by 8 teams, the classroom observations and the students reports I concluded that the students liked the practical work. Their enthusiasm was greater than in regular lessons. Most students worked without stops and did their best to get accurate data in measurements. They quickly familiarized themselves with COACH. Working with graphs and measuring positions, angles and slopes in video clips caused no insuperable problems. A quotation of a female student team illustrates this: “It is fun and it makes a change. It was not really difficult, not even the theoretical part. You must ‘see it’, and then it is easy. Please, do this more often.” The assignments apparently went well together and the theoretical part about the chain with 5 weights linked up well with the students’ physics knowledge: They had just learned about force as a vector quantity. But the students differed in opinion about the connection with their mathematical background. Half of the students wrote that this part of the assignment did not link up well with their mathematical knowledge. In fact, many of them needed assistance from the teacher, but they managed to understand it and write the explanation in their own words (cf., Heck, 2007d).

In the reports, the presence of units of length for slopes indicated that some students confused slope and increase of a quantity. Maybe this was caused by the difference between mathematics, in which tangents are dimensionless, and science, where slope is treated as a quantity. Another interesting difference between the use of diagrams in mathematics and science popped up in the classroom experiment when students were making graphs invisible in a plot. To their surprise, students got weird diagrams with no coordinate system or no labels near the axes. They were apparently thinking of a graph as a representation of a function, that is, as a representation of a single object, so that it suffices to work with one variable. This is common in mathematics. In science however, a graph represents a relation between quantities. Then one works with at least two variables.

The appearance of ‘weird graphs’ during the computer laboratory session could also be attributed to the fact that at the time of the classroom experiment COACH only allowed measurements on video clips and not on a single digital image. Nevertheless, it could be used to measure on digital images by just converting them into video clips in which all frames are equal and by making the time variable invisible in the diagrams. But students could (and did) overrule this setting and obtain unexpected diagrams. From this experience, the conclusion was drawn that indeed a dedicated digital image analysis tool was needed in COACH. Since it is sometimes in reality difficult, if not impossible, to make a photograph straight in front of the real object, the decision was made to design and implement a perspective correction tool for both digital image and video analysis. It has proven its value in many case studies (cf., Heck, 2004a, 2008a-b; Heck & Ellermeijer, 2009; Heck & Uylings, 2006, 2010a; Heck & Vonk, 2009).

Some students misunderstood the question in the third activity about the pattern in the angles and slopes of the chain segments. They wrote that larger angle implies larger slope, and that the tangent of the angle is equal to the slope. They were right of course, but this was not the authors’ intention. Most students did not find the

pattern from their collected data. They found the pattern first in the task to look at angles and slopes for the points $(0, 0)$, $(1, 1)$, $(2, 4)$ and $(3, 9)$ in the standard parabola $y = x^2$, and then checked if this also occurred more or less in the measured data. Actually, this is not a wrong way of doing.

Extensions

Some students were a bit disappointed about the mathematical contents. They found that the assignments focussed too much on learning COACH or on physics aspects, and that they had less to do with mathematics. In retrospect, I must admit that they were right. I could have gone further into mathematical modeling (as presented in Heck, 2007d) and I could have paid more attention to the role of symmetry, perspective, and the coordinate system in the problems. Especially the effect that a change of the coordinate system has on the data and on the formulas would have been an interesting topic and a natural introduction to the study of invariance of properties of curves under transformations. I also overlooked some opportunities while preparing the instructional material. For example, to convince students that really more is needed than curve fitting for deep understanding of a shape, I could have asked them to apply the sinusoidal regression model $y = a \sin(bx + c) + d$ to the data collected for the Zeeburger bridge. They would have seen, maybe to their surprise, that this model works as good as the quadratic model. But which one is correct? And what is meant by ‘correct’? Can a sinusoidal regression model approximate any piece of a parabola or are there limitations? Similarly, the students could have validated that the ‘ideal’ chain hanging under gravity with suspension points $(-1, 1)$ and $(1, 1)$, and with its minimum at $(0, 0)$ can be approximated very well by the rational formula $y = 9x^2/(11 - 2x^2)$. What are actually the advantages of the exact formula for the catenary? Such tasks and questions would have given the students more food for thought and they would have revealed the dangers of so-called experimental mathematical modeling.

Students understood the universal application of digital image analysis and video analysis. They suggested the following subjects of practical work in which video and image measurement could play a role: the shape of the atrium at school, the streamline of cars, the supporting power of a structure, movements in sports, collisions, and acceleration, deceleration, and movements of objects. Many of them expected that it would be more interesting and fun to work with video clips instead of with still images. I am of opinion that the discrete modeling approach offers an opening to the investigation of other systems of masses acting under gravity on a rope and that the scope of investigation can be broadened to anchor catenaries (Lamb, 2000), shapes of suspension bridges, and to architectural structures. This kind of activities would illustrate the use of common mathematical shapes and functions such as straight lines, parabola, exponential and logarithmic curves, and it would reinforce some of the ideas of calculus. But maybe more important, it would bring the real world into mathematics lessons in an attractive way.

2.3.2 Modeling Shapes of Bridges and Hanging Chains

Reference

Heck, A. (2007). Modelleren van bruggen en bogen [Modeling bridges and chains]. *Nieuwe Wiskrant*, 27(1), 42–52.

Introduction

In this classroom study I explored whether the text-based modeling tool of COACH 5 could be effectively used by students in their second year of the Second Stage of pre-university education (age 16-17 yr.) for discrete modeling of mathematical curves. In previous school years, these students had learned in mathematics and physics lessons that a characteristic property of a quadratic function is that the increase diagram is a straight line. In other words, the increase diagram of the increase diagram of a quadratic function, referred to as the second increase diagram, is constant. In the language of derivatives, it means that the second derivative is constant. But one may wonder whether students really value this characteristic property of a quadratic function and the parabola. This was explored in this case study with students who had no prior experience with the modeling tool. It was in the strict sense not a classroom study because the students obtained a home version of COACH so that they could carry out their practical investigation at home, on the basis of instructional materials with concrete exercises and tasks.

The Classroom Experiment

Twenty-four vwo-5, Mathematics B students (13 male and 11 female; age 16-17 yr.) puzzled at the end of May 2001 in small teams of 2 to 3 persons with the help of the text-based modeling tool of COACH 5 over the problem of how the consecutive ratios 1:3:5:7:... of positive slopes of equal sized segments with kinks at equal horizontal distance lead to a parabolic shape. They started in their practical investigation with the computer program shown in Figure 2.4. The model window is divided into two parts: On the left-hand side is the actual program and on the right-hand side are the initial values. As shown, x and y start at -5 and 25 , respectively. Furthermore, dx equals 1. This means that x each time increases with step size 1. On the left-hand side it is visible that first the value of dy is calculated and hereafter the new values of x and y . Then it starts all over again, that is, on the left-hand side stands the repetition with the code that is repeatedly executed until the condition $x > 5$ is satisfied. The diagram window illustrates that the computer points lie on the standard parabola.

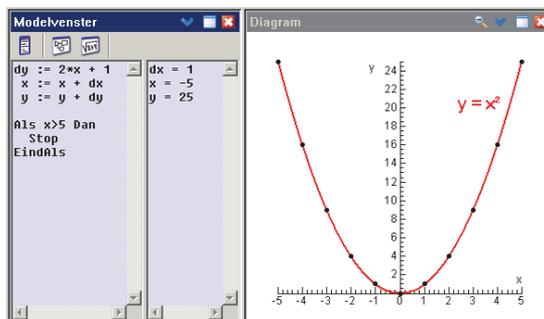


Figure 2.4: Screen shot of a COACH computer program generating points on the standard parabola $y = x^2$.

None of the students had difficulties with understanding the above computer program. Problems only arose when asked to adapt the computer program such that it works for any step size dx , under the restriction that the direct formula $y = x^2$ may not be used. Students were expected to report how they had attacked and hopefully solved the problem. At least, they had to show that their computer program worked well.

Results and Findings

Creativity and perseverance of students were required to arrive at the following line of computer code: $dy = 2*x*dx + dx^2$. Most students began first to study a second concrete case with a particular value of dx such as $dx = \frac{1}{2}$ or $dx = \frac{1}{4}$ in the hope and expectation that they could learn from these cases. In the third assignment, the complexity was increased: Now students had to make a computer program for the case that (0, 10) and (10, 10) were the ‘suspension points’ and (5, 5) was the lowest point. They also had to determine the mathematical formula of the quadratic function going through these three points. In another exercise, students had to extend the computer program such that it also computed the length of the parabola between (0, 10) and (10, 10) via small step sizes. Finally they had to write a computer program that models a chain hanging under gravity. Outcomes of student work were quite good, except that some teams had not been able to successfully extend their computer program to include computation of the length of a curve between two point or successfully write a computer program for a hanging chain. In general, the students appreciated the mathematical puzzle-like problem: It was fun to do and it gave a sense of satisfaction when the puzzle had been solved after some serious effort.

Extensions

In the specified paper I also discussed how a model of modeling, in this case the modeling cycle of Blum and Leiß (2005, p. 1626) shown in Figure 2.5, can be used to structure and analyze quantitative mathematical modeling activities about shapes of bridges and hanging chains.

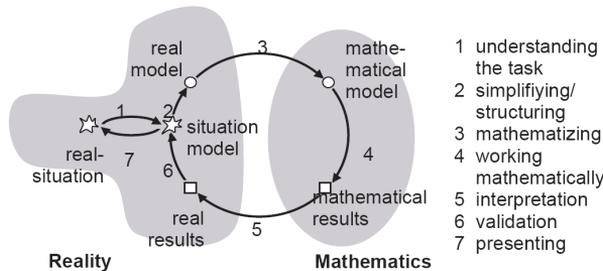


Figure 2.5: The modeling cycle of Blum and Leiß (2005, p. 1626).

In the paper, this idea of using the scheme of Blum and Leiß (2005) to describe a possible route for the discovery and validation of a mathematical formula for the shape of a main span cable of a suspension bridge and the shape of a hanging chain was worked out. Below I outline this approach.

The first transition in the modeling cycle of Blum and Leiß (understanding the task) means that one must come from the real world situation to a situation model (cf., Leiß et al., 2010). This mental image of the situation often comes unconsciously into being through informal knowledge and experience, as well as through knowledge of the discipline. Such a mental image can be visual in the form of graphical representations, but can also be constructed analytically through symbols and verbal descriptions. For a model of a suspension bridge it may be an ideal image of such type of bridge in the form of a schematic picture.

The simplification and structuring continues in the modeling cycle until a ‘real model’ with a concrete presentation of a question has been created. The search for

the essence of the problem posed and the transformation from a situation model to an abstract model of reality that is as simple as possible and that is free from distracting details is often a phase of great creativity. In this case, the mental model of a chain with equal weights hung up symmetrically at equal distances (horizontally or along the chain) for the span of a bridge or a hanging chain serves this purpose.

Mathematization can take place, as in the instructional materials, by first exploring a real model of weights and measuring slopes and angles in a digital image, followed by an explanation that builds upon equilibria of vector forces at points of attachment.

In order to come from a mathematical model to modeling results one can follow various routes, for example the classical route with pencil and paper, in which an algebraic/analytic approach via differential equations is tried, and a route in which a computer model is implemented to find and explore approximations. The second approach was chosen in the practical investigation of the vwo-5 students.

Interpretation and validation of the mathematical model concerns three aspects of quality: (1) the descriptive quality, [i.e., how well the model describes the available observations]; (2) the predictive quality [i.e., whether the model enables making predictions that turn out to match well when verification takes place]; and (3) the explanatory quality [i.e., whether the model explains observations or leads to a better understanding]. Comparison of model results with empirical data covers the first aspect of evaluating the mathematical model. The predictive power of the mathematical model is mainly in the wide applicability of the chosen approach. The explanatory quality is rooted in the application of well-known physics concepts and laws.

I have often applied the model cycle of Blum and Leiß (2005) in the design and evaluation of practical investigations. Another example, which is discussed in this Section 2.8, is the modeling of bouncing balls (Heck, Ellermeijer, & Kędzierska, 2010a).

2.4 Video Analysis of Human Locomotion

In this section I present two classroom studies about modeling human locomotion: (1) a video analysis activity in a vwo-5 Mathematics B class at school; and (2) a motion analysis activity in a masterclass for interested students who were in their penultimate or last year of pre-university education and for which the experimental part took place at the University Sports Center. In both case studies, the authentic nature of the practical investigations played a key role in the design of the student activities. I interpreted in this case the authentic nature of the activities as the opportunity for students to work directly with high-quality, real-time data about human gait in much the same way movement scientists do. Students were guided to use the same theoretical framework, nomenclature, research methods, and techniques as practitioners. In essence, the learning goals were to make their science learning resemble science practice, in which investigations can often be characterized as being challenging, complex, open-ended, and cross-disciplinary, and as requiring a strong commitment of participants plus a broad range of skills. For this purpose, I selected a topic for which it is a priori clear that a definitive model that explains everything is not available, but for which it is already possible to come to grips with phenomena by way of experimenting and modeling on the basis of basic knowledge and skills learned at school, and in which students could learn about commonly used research methods.

2.4.1 Gait Analysis in the Classroom

Reference

Ellermeijer, T., & Heck, A. (2003). Walk like a physicist: An example of authentic education. In: *Physics in New Fields. Electronic Proceedings of GIREP 2002*. Retrieved March 19, 2010, from <http://pinf.fysik.lu.se/abstracts/fullText/148.pdf>

Introduction

In this case study I explored whether ICT-supported gait analysis is a suitable subject to introduce students to authentic scientific inquiry and I aimed at getting answers to the following questions:

- Is the computer learning environment COACH a valuable tool for the students in their practical work, in the sense that it supports them in obtaining, organizing, displaying, manipulating and analyzing data? In particular, does the video analysis tool, which is a kind of research instrument heavily used in empirical work of movement scientists, provide the students the opportunity to engage in activities that scientists and practitioners are involved in as well?
- Does the instructional strategy of first familiarizing students in a classroom setting with the way practitioners work before engaging them in doing their own gait analysis work well, that is, does the chosen setup make authentic science feasible at student level?
- What difficulties do students encounter in their video analysis activities?

The research instruments used to get answers to these questions were classroom observations, teacher interviews, video recordings (including video capturing of computer work), the reports of the students about their work and the underpinning COACH result files.

The Classroom Experiment

Eighteen vwo-5, Mathematics B students (10 male and 8 female; age 16-17 yr.), who already had some practical experience with COACH, participated in the experiment. The practical assignment was not part of the students' examination portfolio because it was the first time that students would do a complete video analysis activity (from recording a video clip, to measuring interesting quantities, data handling, and analyzing/interpreting results), but it was graded instead as a regular test in the semester. The students worked in pairs for three weeks in June 2002; estimated study load was 10 hours. In the first two weeks, work took mainly place in the computer laboratory during the regular physics lessons (two lessons per week of 45 minutes). The teacher and the researcher were present as assistants. In the third week, the students could make use of the computer facilities at school to work autonomously on the assignments or continue working at home via the home version of COACH 5.

Instructional materials were developed in accordance with the learning objectives to let students

- work with real data collected from video clips made by a webcam;
- carry out practical work in which they can apply much of their present knowledge of mathematics and physics in a real life context, in this case human locomotion such as walking, running, hopping, and so on;

- practice ICT skills, in particular making a video clip and carrying out measurements on it with a data video tool;
- experience that diagrams that are used in practice are not just pretty pictures, but convey much information about the real life phenomenon under study;
- be in contact with current research work, in this case movement science, including the nomenclature and research methods used.

These objectives are rooted in the belief that one of the main purpose for doing practical work is to experience authentic mathematics and science, and to enjoy and become competent in it. I chose on purpose a complex, real-life phenomenon for which a complete mathematical and biomechanical description fails and certainly would be out of reach of students, because students can personally experience in this way that simplified mathematical and physical models are nonetheless useful in the sense that they can still yield interesting results and provide qualitative answers to research questions.

The instructional material consisted of the following four assignments, of which the experimental part took place in the computer laboratory at school:

1. *Mathematical analysis of human gait.* Students are introduced into the typical normal walk cycle and the events of gait. They also practice their skills in using the graphical facilities, the video analysis support, and the regression tool of COACH. In the first activity students explore in particular the movements of the arms during normal walking. The arm motion with respect to the shoulder joint is similar to that of a pendulum. In Figure 2.6 is shown a still of the video clip with measured points marked and the diagram window that displays the measured horizontal positions of the shoulder and the hand with respect to the fixed coordinate system.

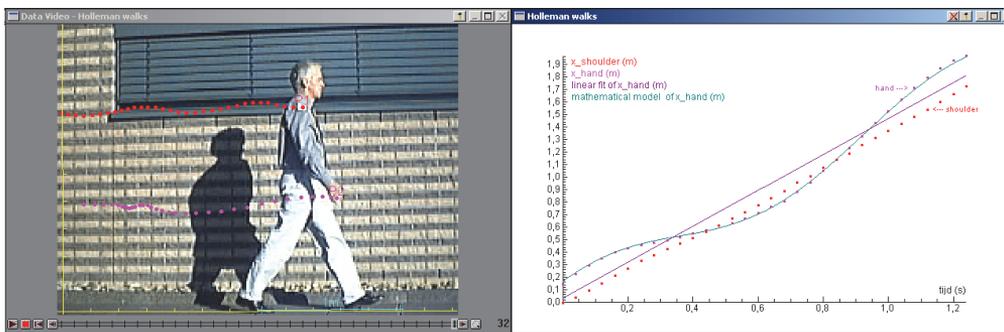


Figure 2.6: Screen shot of COACH activity 1: Arm movement during normal walking.

At first sight, the shoulder moves horizontally with constant velocity. The horizontal position of the hand, $x(t)$, is mathematically modeled as a combination of a straight line and a sinusoid, that is, by the formula $x(t) = at + b\sin(ct + d) + e$, with parameters a , b , c , d , and e that must be estimated. This task prepares students to multi-step regression (first a straight-line fit of the measured data and then sinusoidal regression applied to the residual values) and to the use of a moving reference frame in video measurement.

2. *Swing phase in sauntering gait.* Students analyze the motion of the swing leg during a slow walk. They record the coordinates of the hip joint and the ankle joint

of one leg with respect to the knee joint and they derive from these data the hip joint angle and the knee joint angle of the leg as a function of time. They check via the least squares method of peeling-off functions (Foss, 1969) how well these functions can be approximated by sums of two sine functions: First they approximate the motion by a single sine function, then they subtract this function fit result from the original data and approximate the residual data by a sine function again (cf., the upper-right window of Figure 2.7). Instructions are detailed and guide students through technical steps. It prepares them for further motion analysis.

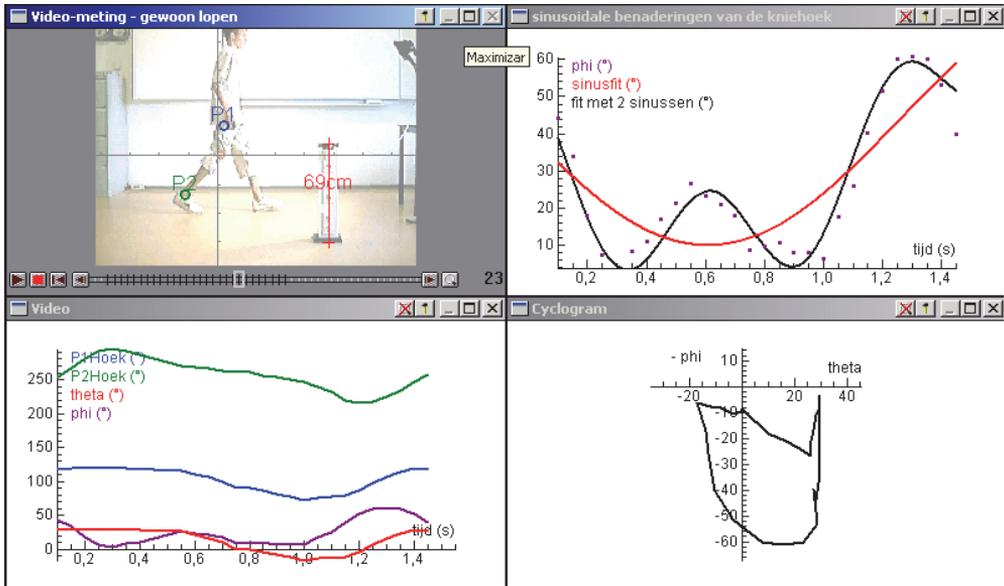


Figure 2.7: Screen shot of a COACH activity with a video analysis of a normal walk of a student in the physics classroom.

3. *The gait cycle in sauntering gait.* This activity is a continuation of the previous one, but now the complete gait cycle is considered, that is, the sequence of motion occurring from heel strike to heel strike of the same foot. On the left-hand side of Figure 2.7 is shown a still from the recorded video clip of a student walking normally in the classroom and the time-graphs of the measured angles of one full gait cycle. In the upper-right window of the screen shot is shown that a sum of two sinusoids with different frequencies still is a rather good description of the knee joint angle as a function of time. This means that the leg movement is well described by a lateral and dynamically coupled oscillator model (cf., Holt, Hamill, & Andres, 1990; Yam, Nixon & Carter, 2004). This holds for many human gait patterns. In addition, students investigate the leg motion via the so-called hip-knee cyclogram, in which the joint angles of knee and hip are plotted against each other. This diagram is a parametric curve with respect to time. Characteristic points on the curve correspond with distinct events during the gait cycle. Cyclograms change when gait conditions alter. For example, cyclograms of uphill, downhill and level walk have different shape; Bartlett (2007, pp. 96–106) discussed the qualitative differences between cyclograms of walking and running. In the lower-right window of Figure 2.7 is shown the cyclogram

obtained by the students who studied a normal walk. The shape of the cyclogram and the points marking important events in the stride such as heel-contact, heel-rise, and toe-off are roughly the same as the diagram that can be found in the research literature (cf., Goswami, 1998; Bartlett, 2007)

4. Investigating one's own motion. The main purpose of the first three activities was to familiarize the students with the theoretical framework, terminology, research methods and techniques of movement scientists and to prepare the students for the fourth assignment, which is a small investigation task that they can carry out independently. In this activity, the students perform a gait of their own choice and record the motion with a webcam. Hereafter they collect and analyze data on their own video clip. To limit the practical work to a rather short assignment, one can let the students only construct the hip-knee cyclogram of their motion, let them compare their result with one obtained by a fellow student, and ask for a short note about their findings. The main mathematical issue in this activity is graph interpretation: students must relate the diagrams that they create with real world events.

Results and Findings

For practical reasons a fixed experimental setup was used for recording the video clips of students. A webcam connected to a notebook computer was placed in the physics classroom and the video capturing and processing software was prepared such that one clip after another could be created without the need to set up everything again. VIRTUALDUB (Lee, 2011) was used for this purpose. At the time of the classroom experiment, COACH had no built-in facilities to capture and process video clips. Video processing tools are often needed to improve the quality of a video clip (e.g., the brightness), to remove superfluous frames from a clip, and to compress it in a manageable format. All this would have been too time-consuming for students to do individually in this practical work.

The first research question can be specialized into the following two subquestions:

1. Can pre-university students by using a webcam, a computer, and the video analysis tool of COACH obtain a useful hip-knee cyclogram that resembles such a diagram found in the research literature?
2. Is the mathematical analysis that the students practiced in the first parts of the practical investigations still applicable when it comes to exploring self-selected gait patterns?

If the screen shot in Figure 2.7 of a student walking normally in the classroom did not yet convince the reader that good results could be obtained by students, below in Figure 2.8 is another screen shot of a successful video analysis of a student skipping in the classroom. Figure 2.9 is a screen shot illustrating the result for a similar skipping movement carried out on a motorized treadmill (taken from a student profile project), which enables the analysis of more than one gait cycle.

Let me use Figure 2.9 to explain how a student collects motion data with COACH. In the upper-left window is shown a video clip of a student skipping on a treadmill. Students gather data simply by clicking on points on the video clip: First they inform the system that they want to reposition the origin of the coordinate system for each frame of the video clip on the knee joint of the right leg and that they want to measure in each frame the positions of two points, namely the hip and ankle joint. By measuring

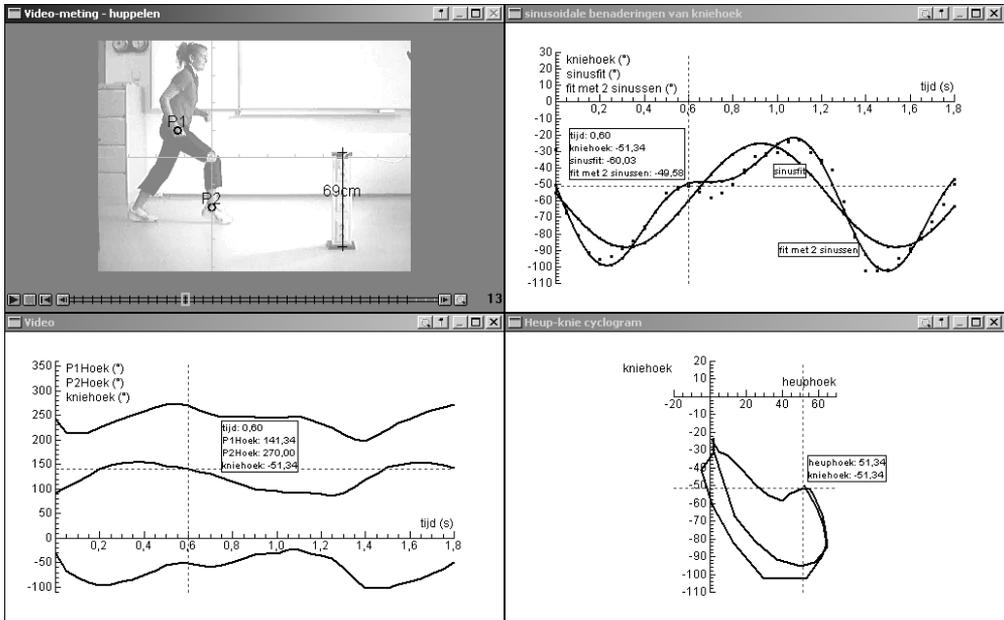


Figure 2.8: Screen shot of a COACH activity with a video analysis of a skipping movement of a student in the classroom.

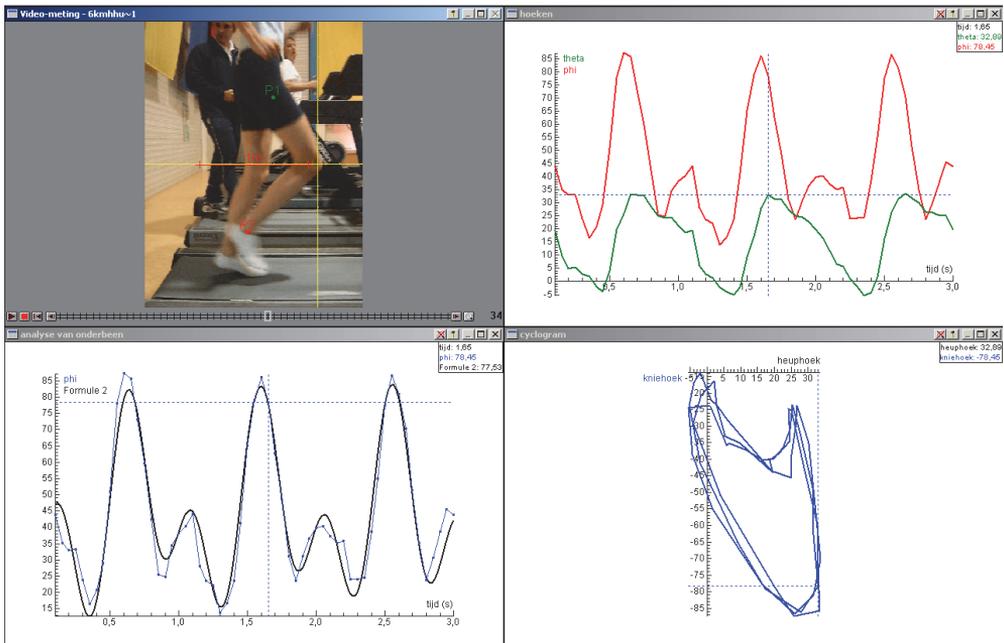


Figure 2.9: Screen shot of a COACH activity with a video analysis of a skipping movement of a student on a motorized treadmill in a fitness center.

positions in polar coordinates, the hip and knee joint angles can easily be computed from the measured quantities (yet this is an interesting geometrical task to derive the right formulas). In the upper-right diagram the hip joint angle and the knee joint angle are plotted against time. Using a treadmill, students can easily record more than one gait cycle. This has the advantage that they can verify periodicity of the movements and can filter irregularities if they wish. In the lower-left diagram is illustrated how well the model of two sinusoids works for the knee joint angle as function of time in skipping gait. The periodicity of the motion is nicely visualized in the cyclogram in the lower-right window. The cross-hairs in the diagrams indicate that COACH is in scanning mode: This means that pointing at a graph or a table entry automatically shows the corresponding video frame and that selecting a particular frame highlights the corresponding points in diagrams. This makes scrubbing, that is, advancing or reversing a video clip manually, an effective means to precisely identify and mark interesting events in the movie and to relate them with graphical features.

All things considered, the body movements during locomotion are complex and only simple mathematical and physical models are applied. Yet, one of the most important messages coming from this classroom study is that it goes surprisingly well, even when one uses simple recording apparatus like a webcam. The above screen shots of COACH activities illustrate this. The hip-knee cyclogram of the skipping motion may look a bit different from the cyclogram of normal walk, but it still agrees with diagrams found in scientific literature. No matter what the motion actually is—normal walking, hopping, jogging, running, walking backwards, walking on your toes, walking on high-heeled shoes, or stair climbing—students almost always got useful cyclograms in the classroom experiment. Very few students in the classroom experiment had insuperable difficulties with using the video tool, with creating the graphs, and with applying a given regression model. This contributed to the students' satisfaction. All of them got results! The fact that students could work well with the video analysis tool did not mean that there were no problems. Some students had conversion problems with recorded video clips because COACH allowed at the time of the classroom study only a limited number of compression and decompression techniques.

However, interpreting graphs in the context of body movements turned out to be different story. For many a student, comparing cyclograms of two different gait patterns just meant writing down the differences between the diagrams without coupling them to the motions in the video clips. I suspect that many a student did not quite grasp the meaning and purpose of a cyclogram, even though the instructional material (after a first adaptation) discussed the diagram extensively. This may also be the reason that students did not bother much about the orientation of hip and knee joint angles, so that their cyclograms occasionally were rotated or mirrored with respect to the ones presented in the instructional material. Especially the convention of negative knee joint angles seemed to be not understood or considered irrelevant. Discussing in class a prototypical example of the use of a cyclogram is probably the recommended remedy here.

Extensions

Multi-step regression is a common data analysis technique when data can be best handled by decomposing phenomena into several parts. For example, Heck and Uylings (2005) applied it in the decomposition of the up- and downward winding of a yoyo into a rotational motion and a vertical displacement. Heck (2004c) successfully applied a

decomposition of the motion of a diver making a $3\frac{1}{2}$ forward somersault dive into a parabolic motion of the center of gravity and a rotational motion of the diver in tuck position (See also p. 104). Regression via the least squares method of peeling-off functions (Foss, 1969) is also a useful data handling technique with many applications. For example, it can also be applied for simple sinusoidal regression analysis of tidal waves (Van Gastel, Heck, & Uylings, 2008; see Section 3.3.3), for exploring a bi-exponential model for the decay of beer foam in a glass (Heck, 2009a,b; see Section 3.5.3), and for bi-exponential modeling in quantitative pharmacology (Heck, 2007a-c; see Section 2.7).

Apart from familiarizing secondary school students with useful data handling techniques, another important goal of the practical investigation was to bring them in contact with motion analysis, that is, with the science of analyzing video clips of body and body part motions in order to study the kinematics and the forces involved. The availability of affordable digital video and computer technology offers students the opportunity to actively engage in motion analysis. Gross (1998) reported about educational projects that meet this learning goal for undergraduate students in a biomechanics course about human movement. Staying closer to the practical investigation about gait analysis, a motion analysis of the arm movement during gait would be a natural extension of the presented work.

2.4.2 Gait Analysis in a Masterclass

Reference

Heck, A., & Holleman, A. (2003). Walk like a mathematician: An example of authentic education. In T. Triandafillidis & K. Hatzikiriakou (Eds.), *Technology in Mathematics Teaching*, Proceedings of ICTMT 6 (pp. 380–387). Athens: New Technologies Publications.

Introduction

The main differences between the classroom study presented in Section 2.4.1 and the masterclass discussed in this section were the following:

- The masterclass was organized for interested students who were in their penultimate or last year of pre-university education, whereas in the classroom experiment vwo-5 students from one class were obliged to do the practical investigation.
- The students in the masterclass spent a full day at the University of Amsterdam to learn about motion analysis of human gait followed by home work, whereas the students' work in the classroom experiment was spread over several lessons during three weeks.
- In the masterclass, experimental work took place in the University Sports Center. At this location experimental conditions could be more controlled through motorized treadmills and more types of gait experiments could be carried out.

The Classroom Experiment

Eight vwo-5 and vwo-6 students (4 male and 4 female; age 17-18 yr.) participated in the masterclass that took place on the 8th of November 2002 partly in a studio classroom at the AMSTEL Institute of the University of Amsterdam, and partly at the University Sports Center. All students used the masterclass as a start of their profile research project.

The instructional materials outlined in Section 2.4.1 were used to introduce the participants to motion analysis of human movement. Optional activities about the following gaits were added to the instructional materials: walking backwards, jogging, sprinting, race walking, and walking on high-heeled shoes. These activities were included to prepare students for comparison studies of various gait patterns and of gait patterns for which the effect of changing one independent experimental variable was explored (for example, comparison of level walk with uphill and downhill walking). In the sports center, students collected data about such gaits by use of a motorized treadmill. Use of this equipment enables the recording of more than one gait cycle in a video clip. This has the advantage that one can verify periodicity of the movements and can filter irregularities if needed. More importantly, use of a motorized treadmill allows a systematic change of conditions: Students walked and ran at different speeds set by the treadmill and with different slope angles of the treadmill.

Using a motorized treadmill with adjustable speed, one can easily investigate the relationship between gait speed v and step frequency f . Humans choose a step frequency or stride length that minimizes metabolic energy consumption. The step frequency has been found empirically to obey the power law $f = cv^b$, where c is a constant and $b \approx 0.52$ for normal gait speeds of adults (Bertram & Ruina, 2001). An alternative formula taken from (Bellemans, 1981) is $f = \frac{v}{av+b}$. It is based on the idea that a person who walks in a relaxed manner automatically adjusts his or her stride length to the gait speed in accordance with a linear relationship. Students can collect data and compare the empirical data with the given mathematical models.

Results and Findings

All participants obtained good motion analysis results. Figure 2.10 illustrates the results of a student running at a speed of 10 kilometers per hour. In this screen shot of a COACH activity, the upper-left window shows a still of a video clip of a running girl. Notice the homemade marker on her knee (expensive markers are not needed!): That is where the origin of the coordinate system is placed in each frame of the video clip. In the sports center, a student discovered that drawing a big dot with a non-permanent marker pen on the knee joint works as well and does not hinder the leg movement. This illustrates that this student was reflecting on what she was doing and how she could improve the ease and quality of data collection. The measured point P1 and P2 are visible on the hip joint and ankle joint, respectively. The polar coordinates of these points have been used to compute the hip joint angle and the knee joint angle. In the lower-left diagram, the hip joint angle and the knee joint angle have been plotted against time. In the lower-right diagram one can see how well the knee joint angle is modeled by a sum of two sinusoids. Two hip-knee cyclograms are visible in the upper-right window: One parametric curve belongs to the running speed of 10 km/hr, the other curve is a background graph for a speed of 15 km/hr. Trying to understand which one belongs to which curve is a good exercise in developing graph sense. Students are able to bring this work to a success. For example, one student team that was comparing hip-knee cyclograms of level walking on a treadmill with walking on a 10% uphill slope reported:

“Full extension of the leg, that is, a knee angle of zero degrees, occurs less in walking against a 10% slope. This is so because firstly you make more steps and consequently your leg has less contact with the ground floor. Secondly, if you have put your foot flat on the floor, you still do not have a

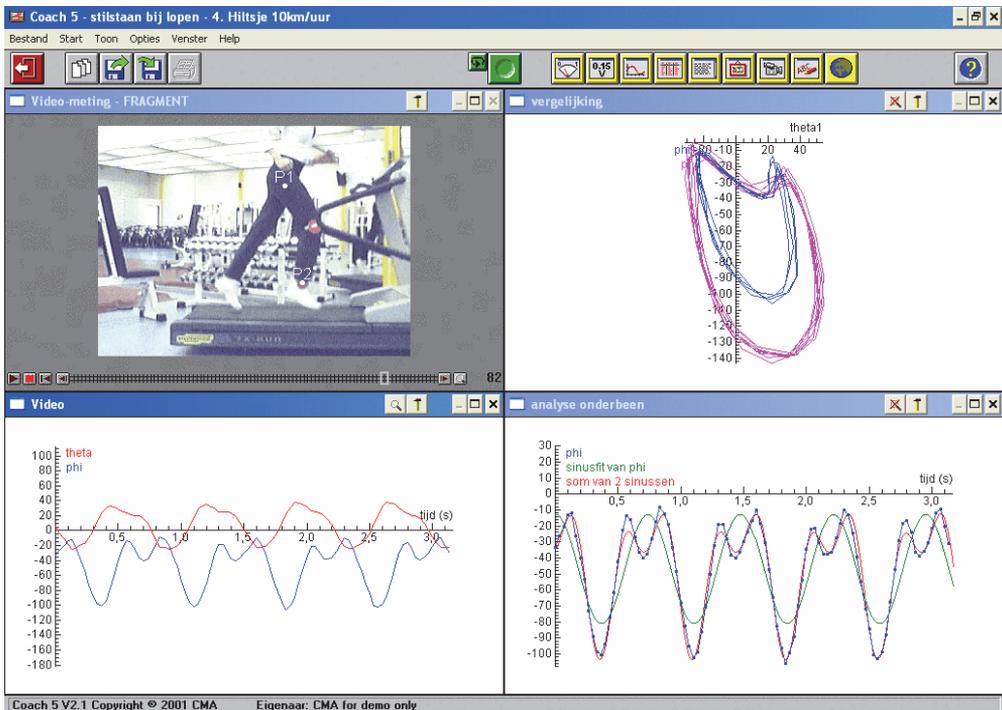


Figure 2.10: Screen shot of a COACH activity with a video analysis of a running movement of a student on a motorized treadmill in a sports center.

knee angle of zero degrees because you walk uphill. The positive hip angle is greater when you walk uphill, because then you have to raise your legs more, with the consequence that the hip angle gets larger. The negative hip angle becomes larger, because you leave your leg behind for a longer time since your front leg needs more time to put enough force for getting uphill.”

Compare this with the following conclusion of Goswami (1998, p. 26):

“The range of hip movement has, in fact, a linearly increasing trend as we go from -13° to $+13^\circ$ slope [...]. The knee angle behaves in an opposite but remarkably symmetric manner. The total range of knee angle is a linearly decreasing function of slope ...”

The students results were quite close to the conclusions in the scientific report. Their terminology lacked precision, but in fact they did very well in their case study. They did more than just writing down an empirical result: They also tried to explain the results obtained from their data in terms of body motion. It is clear that for these students the graphs are about something and that many things read in the diagrams can be connected with the gait process.

The remarkable results of motion analysis of students are not occasional, but have been repeatedly found in students’ profile projects about human and animal locomotion. A nice example was published by Heck and Van Dongen (2008). Figure 2.11 is

a picture taken from the student's profile report and shows the hip-knee cyclogram obtained for a normal walk at a speed of 5 km hr^{-1} . Several gait events have been marked in the diagram, the curve of which is best read counter-clockwise. There is a very good matching between this diagram and Figure 4 in (Goswami, 1998, p. 20) and with similar diagrams in textbooks (e.g., Bartlett, 2007, pp. 96-102). Figure 2.11 also illustrates that students better understand the advantage of scrubbing the video clip and inspecting the synchronized tables and diagrams at the same time. This appears to be something that must be stressed in introductory practical video analysis work because students tend to forget about it.

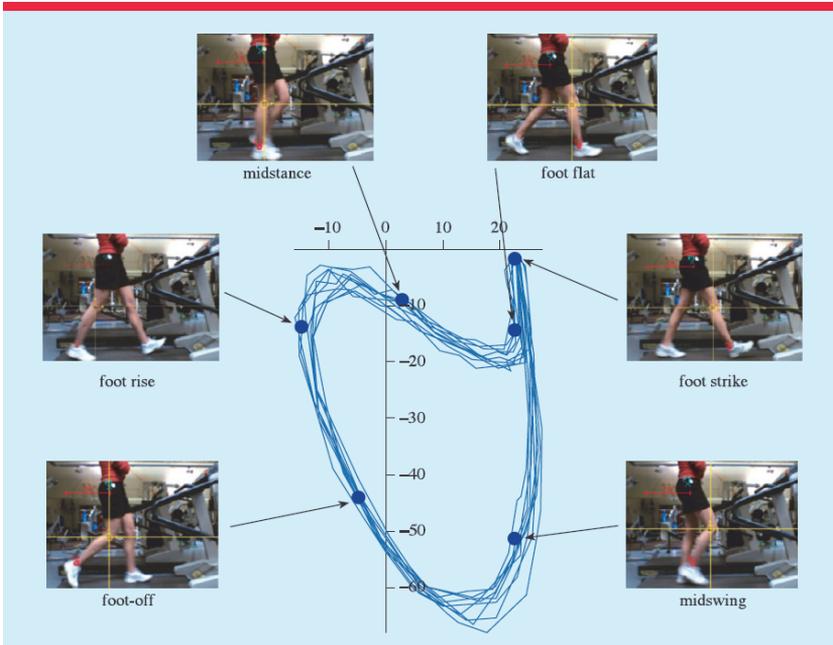


Figure 2.11: The measured hip-knee cyclogram for normal walk at a speed of 5 km hr^{-1} . Several gait events have been marked and the corresponding frame in the video clip is shown (read counter-clockwise).

Improvements in the Computer Learning Environment

Video measurements of several consecutive gait cycles of human locomotion on a motorized treadmill brought to the fore the wish of users to automatically trace the position of a special point in a video clip. Manual measurement in a video clip quickly becomes a time burden in a research project. To get an idea: A measurement of the leg motion during one gait cycle of about 1 second, recorded at 30 frames per second, means that one has to click altogether at least 90 times on a video clip for measuring the position of a single point. Thus, the measurement leading to a hip-knee cyclogram would imply at least 270 mouse clicks. This is too time-consuming, extremely boring, and limits the number of experiments in an investigation enormously. What needs to be implemented in the computer learning environment is the possibility to track the motion of an object (e.g., a marker or an eye-catching point). This has been implemented in version 6 of COACH and I will discuss it in the next chapter on the basis of field experiments and usability studies (See, for example, Section 3.2.3).

Testing of a computer learning environment in educational practice reveals design flaws or limitations caused by design choices that had not been anticipated on before. For example, COACH allows one background graph in a window for reasons of clarity, but this means that a data plot in a measurement activity can only be compared with one background data plot and not with two or more data plots. This hindered students in their practical investigations when they studied the influence of a gait parameter on the shape of a cyclogram. They had to use tricks to overcome this obstacle. For example, one student team solved this problem in their study on the effect of gait speed in the following way: "...The only solution is to let all graphs have the same graph size' ...". The vague term 'graph size' means that the ranges of the plotted quantities were chosen the same for all diagrams.

Another example of a user's need that originated from practical investigations about human locomotion was the availability of a sinusoidal regression tool that allows curve fitting with sums of two or more sinusoids. How instructive the least squares method of peeling-off functions may be, it turns out to be time-consuming and limiting the amount of data analysis in practical investigations when more than one data graph has to be analyzed. Because the number of data points is too low for Fourier analysis, it calls for high-resolution, spectral estimation tools and a sinusoidal regression tool that works well for a small data set. After thorough evaluation of the educational usability of many methods of spectral analysis (Heck, 2004d) on the basis of sample data sets and computer simulations, I came to the conclusion that the following two methods would be appropriate for inclusion in a computer learning environment: the symmetry-adapted, modified Prony method described in detail by Smyth (2000) and the R-ESPRIT method (Mahata, 2003). They have been incorporated in the signal analysis tool of version 6 of COACH and I will discuss this tool in the next chapter on the basis of field experiments and usability studies (See, for example, Section 3.3.3).

Extensions

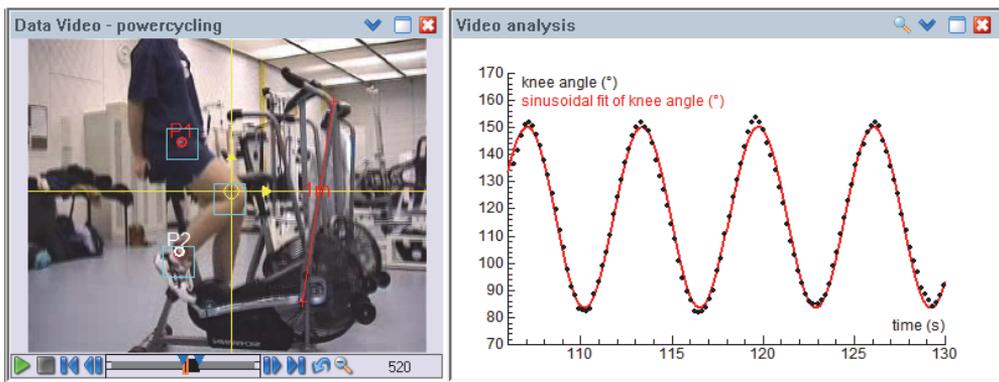


Figure 2.12: A screen shot of a COACH activity about fitness cycling, with automated tracking of three points (including the origin of a moving frame of reference located at the knee joint). The diagram to the right shows the data plot of the knee joint angle calculated from the measured position data of the hip and ankle joint of the right leg, and a sine regression curve that fits the knee angle data.

Many devices in a fitness center such a rowing machines and exercise bikes can be used for practical investigations about periodic human movements, in which the influence

of various parameters can be explored. Figure 2.12, taken from (Heck & Ellermeijer, 2009) shows a screen shot of a COACH 6 activity in which the cycling movement has been measured via automated point tracking and a sinusoidal fit of the time evolution of the knee joint angle has been determined. This type of investigation resembles the gait analysis described in this section.

2.5 Video-Based Practical Work at Pre-Vocational Secondary School Level

Reference

Heck, A., & Uylings, P. (2006). Capturing the real world in the classroom. *The International Journal for Technology in Mathematics Education*, 13(3), 107–116.

Introduction

For successful performance in practical investigations, students need to develop a broad range of abilities that includes amongst others asking good questions, connecting a real world phenomenon with the world of mathematics and science, setting up an investigation or experiment, and collecting, representing, analyzing, and interpreting information. In this particular classroom study, students in pre-vocational secondary education developed and practiced such skills by carrying out a small investigation task using digital video technology.

The video measurement project was part of the examination programme of the vmbo tl-3 classes at the Bonhoeffer College in Castricum, the Netherlands. The acronym vmbo stands for Dutch pre-vocational education. It was introduced in 1999 as education for practically inclined students following their primary education. It takes four years of study and sixty percent of secondary school students in the Netherlands participate in this type of education. Students passing the examination may go on to senior vocational education and prepare for a particular profession. The classroom study took place with students in the theoretical learning pathway (vmbo tl), which prepares them for middle management and for secondary vocational training, or for senior general secondary education. There exists undoubtedly some contradiction between ‘vocational education’ and ‘theoretical learning pathway’. In general, there seems to be a tendency to keep a tight hand on students in the classroom as the only way to make them learn anything. Clarity and discipline in classroom activities are the key words. How appropriate such an approach may be in itself, it can also lead to a situation that practical investigative work by students, which is essential in science education, is offered in the same closed, fixed manner as theory. This is a pity because learning by doing and through lab work will have much more appeal for the students than theory lessons and it will stimulate them to put more efforts in their learning. This case study was done to explore whether it is possible to entice vmbo tl-3 students with practical work to a more active, self-responsible, and self-controlled way of learning. One of the purpose of this study were to get answers to questions such as “How open can a task be for vmbo tl-3 students?” and “With how much responsibility can they deal and when must a teacher bring structure to the activities in the classroom?”

The main objective of this project was to let students carry out a short investigation on a self-chosen motion, at their own educational level. They could show an exploring attitude and adequate practical skills. This concerned the preparation,

implementation, and completion of the investigation, as well as effective collection, processing, and presentation of data that were obtained from self-made video clips

Putting the video measurement project into the examination programme was a sign of a high level of ambition of the teachers' team within the theoretical pathway at the Bonhoeffer College. Other teachers may find it too ambitious. This is why the students' activities and their results were monitored, in order to find answers to the following questions: Are vmbo tl-3 students able to deal with the open task given to them and what performances are turned in? Has the recording of the video clips by students themselves additional value, or should one use prepared video clips instead? Does the investment of time and energy still bring students some understanding of mathematics and science? How could the instructional setting be improved to advance the smooth running of things and to optimize the students results?

The research instruments used to get answers to these questions were classroom observations, teacher interviews, audio recordings of teacher and teaching assistant guiding and helping students, and the short reports of the students about their work.

The Classroom Experiment

Eighty vmbo tl-3 students (58 male and 32 female; age 14-15 yr.) in three classes in the theoretical learning pathway did the video measurement project in March and April 2004. The students did not have much experience with COACH: They had seen it before used by the teacher and they had occasionally carried out themselves measurements with sensors. A school network with sufficient computers and a well-equipped science lab were available at school. At the time of the project, five webcams were available and a teaching assistant helped the students, if needed. The project work was done under the responsibility and guidance of the students' own physics teacher. There were three teachers with different backgrounds involved: One of the teachers had much experience with video analysis activities. Another teacher was familiar with computer supported lab work and demonstration experiments, but had not used video analysis before. The third teacher hardly had experience with COACH and acted more as a fast-follower. The most experienced teacher started with the project in his class and the other teachers followed in their classes, taking advantage of the first findings. The consequences for the students' activities and results in the classrooms are discussed.

Instructional materials were developed in accordance with the learning objective to let students carry out a short video-based investigation on a self-chosen motion, at their own educational level. The student project work, which was planned for five lesson hours of forty-five minutes, consisted of the following five components.

1. *Introduction to video capture and measurement.* The teacher gives a live demonstration of using the video software VIRTUALDUB to capture a video clip with a webcam and process the recorded movie. There are many technicalities and things to take into account, but the idea of the demonstration is to show that any student is able to do this as well; it is not only for 'whiz kids'. When the teacher gives the second example of video capture, the students make their own class notes. They are expected to rely on their own notes when they themselves capture a video clip; no manual is provided. Next, it is the turn of two students to capture a video clip in front of the class. If all goes well, this gives each student the feeling that he or she will have success, too.

The teacher's demonstration and the students' recording of a video clip takes about half an hour. So there is still some time left to practice measuring on a video clip.

After a short demonstration, the students can practice a bit with video activities that have been prepared for this purpose (Students only have to trace a point by mouse clicking; the measurement settings and diagrams are preset). It is important that the sample video activities are appealing. For reasons of attractiveness, especially for the girls, the activity of a toddler on a slide in the playground has been chosen (See Figure 2.13). Video clips with a sweet toddler seem more effective than video clips that are undoubtedly oriented toward physics, such as movies of falling balls, collision of balls, or motion of objects on an air track.

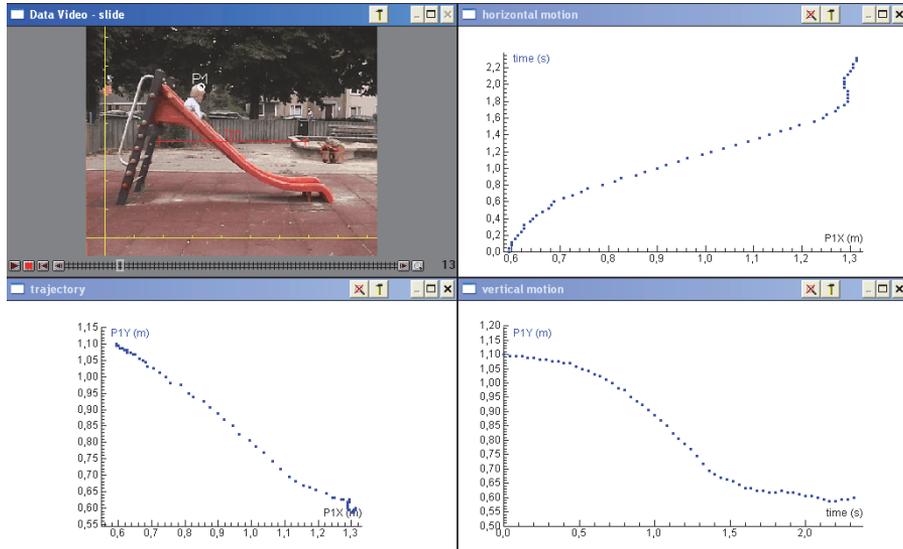


Figure 2.13: Screen shot of a sample video activity to introduce video measurement.

2. *How to make a useful video clip.* Students get five video clips with the following type of problems in capturing and measurement: brightness, occlusion, superfluous frames, and perspective distortion. They must answer the following questions: (1) What is wrong or problematic with the video clip? (2) Can one fix the problem afterwards with the video software? and (3) How could the mistakes have been prevented?

3. *Video measurement explained.* The teacher demonstrates how to create a COACH video activity from scratch, how to embed the captured video clip, and how to carry out a measurement on the movie. Things that pass under review are the coordinate system and calibration, video compression, plane of motion and camera direction, display of collected data in diagrams, and so on. It is intended that the teacher uses a video clip that was recorded before in one of the lessons before as an inspiring example. This demonstration also serves another purpose: It gives the teacher once more the opportunity to explain the students what is actually expected from them.

4. *Video capture.* Students work in pairs and capture their video clips using a webcam. They can get help from the teacher or the teaching assistant if necessary, but they are expected to work as much as possible on their own, using their own class notes or getting help from fellow students.

5. *Collecting data and writing a short report.* Students carry out a measurement on their self-made video clip, create relevant diagrams, annotate graphs with relevant information, and finally write a short report. This report may be a screen shot of the

COACH activity with some words of explanation and conclusions; an example is shown in Figure 2.14.

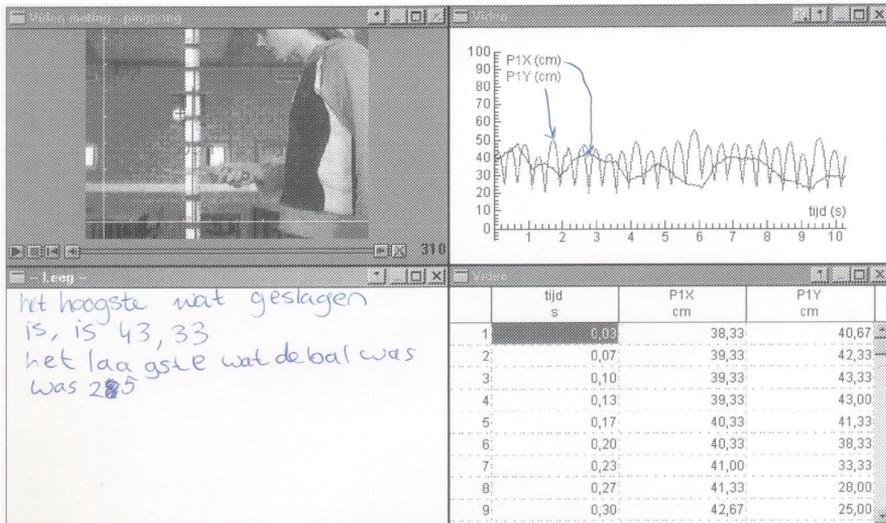


Figure 2.14: The full report of a student team about table tennis.

Results and Findings

The first time that the most experienced teacher did the classroom experiment, unforeseen technical problems popped up. Although every student team succeeded in making a useful video clip, capturing and embedding the video clip in a COACH activity was hindered by the following things:

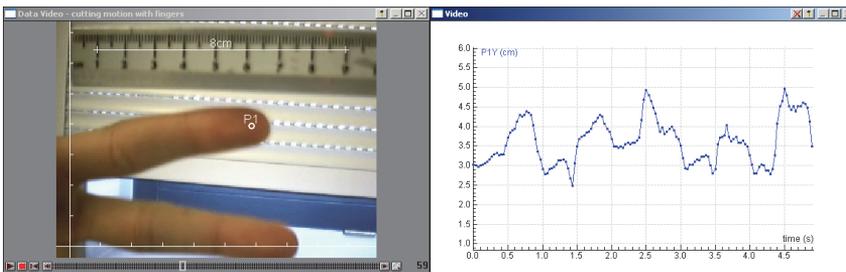
- Because some students forgot to use video compression they flooded the computer network at school with files of 60 MB or larger, which essentially meant a collapse of the network. Without serious video compression (e.g., MPEG-4 Video Codec V2), the files were also too big to use diskettes as transport media.
- The network installation of COACH at school prohibited that students could save video clips in a COACH project; these rights were only reserved for teachers. Transport of video clips from a laptop to the school computer via a memory stick could also be done only by the teacher or teaching assistant.

Each team spent about twenty minutes capturing a video clip. Most of the time was required for finding one's way in the menu items of VIRTUALDUB to specify correct settings of the software. Consequently, students paid less attention to setting up the experiment in an adequate way. Many of them wanted to capture immediately and hardly did a test run to see whether the quality could be improved. Also, students often forgot about a measuring stick in the movie clip for calibration purposes. If the teacher had stressed in the previous lessons that the best camera direction is perpendicular to the plane of motion (in order to avoid perspective distortion), then the students did this well. However, if the teacher had not emphasized this, students sometimes made less useful video clips. While capturing, students seemed to forget their research questions; they simply recorded the phenomenon without consulting their notes. The lesson to be learned from this is that one should require from students a better plan

for their work and that one should not expect that all goes well the first time students produce a video clip for data collection purposes.

The main conclusion about capturing a video clip with VIRTUALDUB by vmbo tl-3 students was that, despite its success, it was a bit too time-consuming to do with the whole class at once. It involved too many technicalities that distracted the students from setting up their experiment in an appropriate way, and it made them rely on teachers and assistants for help. In common practice there is not many staff available to support a lesson or lab session. For this reason, advice was sent to the COACH developers to adjust and extend the video tool for the purpose of making the process of capturing and measuring a video clip easier. This will be further discussed when the design of the video analysis tool for COACH 6 is explained, but at this stage it suffices to remark that for this version of the software the file structure of an activity has been completely changed so that the video clip itself is part of a COACH file, file compression is automated, and capturing is possible within the software environment so that an external program for recording is unnecessary and a recorded video clip can be used immediately after capturing.

The experience of the teacher with the learning environment and with video measurement, as well as the availability of laptop computers and the choice of location to capture the video clips, had a great impact on the scenes recorded. The least experienced teacher chose a setting with which he felt most comfortable, namely the science laboratory and a limited choice of subjects to record: rolling balls (12 teams) and bouncing balls (3 teams). However, one student team got inspired and recorded a cutting motion with their fingers. Figure 2.15 shows part of their report.



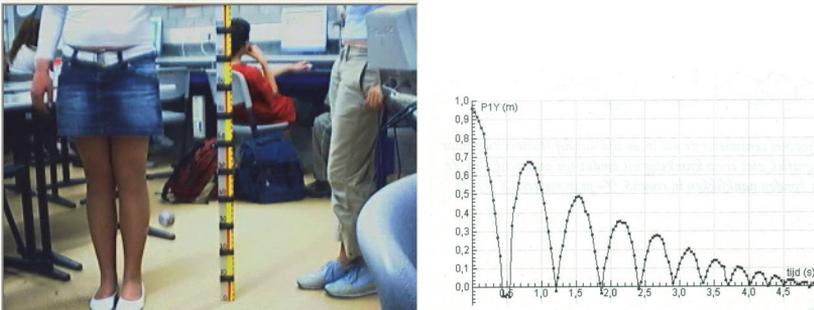
This graph is more readable. You see that the finger goes up and down, because we made a cutting movement. The movement is irregular, because the finger goes sometimes faster. The height is measured on the y-axis, the highest point is 5 cm. The time is measured in (s). The total time of the graph is 5 seconds. The initial point of the graph is (0, 3.0). The end point of the graph is (5, 3.5).

Figure 2.15: Part of the report of a student team on a cutting motion with fingers.

In the class of the teacher who already had experience with COACH, but not with video analysis, work was also done within the science laboratory only. However, students were stimulated to make a subject choice of their own. This led to a variety of video clips: rolling and bouncing balls, springs, a pendulum, jumping from standing position, hitting each other, hand waving, and the moving second hand of the clock in the science laboratory. The subjects in the class of the most experienced teacher, who also used two laptops in order to be able to capture anywhere in school, were the most variable: Flight of a paper airplane, rope-skipping, darts throwing, throw-in of a

soccer ball, projectile motion of a table tennis ball, skipping, and doing a handstand are nice examples. The creativity of the students was certainly switched on.

In all classes, students did their best to obtain accurate data in their video measurement, explain the graphical representations, and link up the graphs with the phenomenon in the real world that they had recorded. They used mathematics and physics beyond their expected level. Note in the fragment of the report shown in Figure 2.15 that the student read out coordinates of points without any mistake and paid attention to units of quantities. The student sought for an explanation for the irregular graph in terms of the real world motion. In Figure 2.16, which shows a fragment of the report of a student on a bouncing ball, there is a similar attempt to explain the form of the graph in physical terms.



In this graph are arcs, these arcs arise because the ball is bouncing all the time. In the beginning the arcs are high because the ball has a high speed because it is released at a height of 1 meter. Hereafter the arcs get smaller because the ball reduces speed. This is a uniformly decelerated movement. $Y=m$ in meters, $X=$ time in seconds

Figure 2.16: Part of the report of a student team on a bouncing ball.

In practice, many vmbo tl-3 students tended to be easily satisfied with one graphical representation of collected data, regardless of its suitability. To make sure that students used all standard diagrams of motion, instructional materials were adapted by inclusion of a COACH activity with three diagrams as shown in Figure 2.13). Students had to explicitly report on all three graphs. Because of the experience in the first classroom experiment that vmbo tl-3 students had difficulty with understanding horizontal displacement on a vertical axis, instructional materials were adapted so that time was plotted against horizontal position, as shown in the upper-right window of Figure 2.13. These are lessons that can almost only be learned by instructional designers in classroom practice.

Extensions

In summary, the involved teachers and I were quite pleased with the enthusiasm and performance of the students in their project work. The students were able to create interesting and useful video clips and they could carry out meaningful measurements. Also, they could link the recorded motion with graphical representations and physics concepts better than usual (if at all) with pencil and paper. The classroom study provided many ideas for improvement of the video analysis module in the computer learning environment and for the design of new video tools. It also provided useful

information for the experimental physics exam with video analysis at school. It confirmed that use of computers in science exams constitutes an interesting development and is a good addition to the traditional written exams (cf., Boeijen & Uylings, 2004). Practical assignments like the one presented in this paper provide powerful training and preparation to the students, next to the exciting and challenging content itself. It is expected that this kind of activity contributes to the quality of profile projects of vmbo students.

2.6 Spreadsheet-Based Data Handling

In this section I present two classroom studies about spreadsheet-based data handling: (1) survival analysis of clinical data by vwo-5, Mathematics A students; and (2) a spreadsheet-based modeling activity in a vwo-3 class. In both case studies, learning objectives were to let students work with realistic data sets and to let them experience that data handling is not just application of standard recipes, but may involve setting up statistical models and underpinning the assumptions herein. The spreadsheet program EXCEL was used to remove the drudgery of data manipulation and to let students focus on the modeling aspects of the assignments.

2.6.1 Survival Analysis of Censored Clinical Data by Students

Reference

Van den Camp, W., & Heck, A. (2003). Survival analysis by students. In T. Triandafilidis & K. Hatzikiriakou (Eds.) *Technology in Mathematics Teaching*, Proceedings of ICTMT 6 (pp. 388–394). Athens: New Technologies Publications.

Introduction

The current presentation of statistics and data analysis in mathematics textbooks at secondary level may give students the wrong impression that data handling is in fact nothing more than applying some standard recipe. They are not confronted with questions such as “Do the data make sense?”, “What statistical model seems applicable and for what reason?”, “Do you need more data for making sound conclusions?”, and “How do you deal with large data sets?” In other words, the development of algorithmic-procedural skills is overemphasized in textbook treatment of probability and statistics at secondary level and not enough attention is paid to modeling and interpretive skills. This lack of focus on how to collect and manipulate data meaningfully, using ICT tools, becomes a serious handicap for students when they collect and explore data in a research project.

Statistical reasoning and thinking are crucial when data sets are incomplete. The handling of so-called censored observations is one of the main issues in survival analysis. This classroom study was about a practical investigation task in which students computed survival probabilities for patients treated after a cardiac arrest. The data set came from a clinical trial of patients. It consisted of censored observations, which means that some of the data were not available during the whole period of the trial. Interpreting such data and drawing conclusions involves assumptions on censoring. The students investigated various points of view and they were invited to come up with alternatives. They used a spreadsheet program to compute survival probabilities

so that they could avoid laborious tabular work and could focus on statistical reasoning, that is, on making sense of statistical information, on constructing models, and on interpreting results, instead of on filling out tables. In this classroom study I explored whether this extracurricular topic of survival analysis provides a nice opportunity for students to apply their knowledge of statistics to a real world problem and to get acquainted with a modern, much used statistical application, and whether statistical thinking, which involves according to Ben-Zvi and Garfield (2004, p. 7) “understanding of why and how statistical investigations are conducted and the ‘big ideas’ that underlie statistical investigations,” becomes an attainable learning goal in this way. I aimed at getting answers to the following questions:

- What do students think of the subject, the instructional materials, and the use of real data?
- Do the instructional materials and the instructional setting enable students to
 - (i) understand the big ideas in survival analysis;
 - (ii) work meaningfully with statistical data, including formulating their own ideas; and
 - (iii) acquire the skills and practice to use a spreadsheet program effectively.

The research instruments used to get answers to these questions were classroom observations, teacher interviews, video recordings (including video capturing of computer work), audio recordings, a questionnaire, the reports of the students about their work and the underpinning spreadsheets.

The Classroom Experiment

Eighteen vwo-5, Mathematics A students (5 male and 13 female; age 16-17 yr.) participated in May 2003 in the classroom experiment. All students had the following learner profile: They had chosen the profile Culture & Society, which prepared them for a university study in the humanities. They were not confident about their mathematical competencies. While teachers of disciplines with a social and cultural context were very pleased with the engagement of the students in their lessons, the mathematics teacher had to deal with a passive attitude.

The practical assignment was not part of the students’ examination portfolio, but it was graded as a regular test in the semester. The students had to report about their work on the basis of their spreadsheets and they had to hand in a diskette with their computer results. Although the learning outcomes were assessed with respect to statistical literacy, reasoning, and thinking, the main criteria for grading the students’ work were: “Did they give a clear account of what they had done in their analysis and why?” and “Did they express a clear statement of their points of view with respect to handling censored observations and of the conclusions drawn?” These criteria were communicated at the beginning of the classroom experiment. The students worked in pairs for three weeks; the estimated workload was 8 hours. Work took mainly place behind the computers in the multimedia center of the school during the regular mathematics lessons and the teacher was acting as an assistant, who occasionally clarified the instructional materials to individual teams. The students could also work autonomously at this location during school hours or at home. This cooperative, student-centered, non-intimidating setting of guided inquiry was chosen in the hope and expectation that this would enhance students’ statistical reasoning and thinking.

Instructional materials were developed in accordance with the learning objectives to let students

- work meaningfully with real data coming from a clinical trial on treatment after a cardiac arrest;
- experience that statistical work is more than applying some cookbook recipe;
- look upon a statistical problem from various points of view and comment on methods;
- develop their own statistical models, explain their choices, and draw conclusions;
- learn and practice skills with respect to data manipulation in the spreadsheet program EXCEL.

The instructional materials were an adaptation of the paper of Borovkova (2002) written for a summer course for mathematics teachers to a lesson design, including instructional materials at secondary student level. Although the instructional materials looked similar to an ordinary mathematics textbook chapter, the students' activities fulfilled the characteristics of open-ended tasks that stimulate students to think critically and require from them explanation of the chosen strategy(ies) at each step. There was scarcely a closed question with a single correct answer. The problem activity was based on a real world situation in order to capture students' interest and to make it relevant and convincing to the students. The students' text consisted of the following four parts:

1. *The problem situation.* Students are briefly introduced to survival analysis and censored observations in the context of a clinical trial of patients treated after a myocardial infarction. Censoring means that because of external causes it is not possible to get the desired information during the whole time of research from all the patients. For instance, while studying survival time after cardiac arrest, some patients might die from another cause or stop their participation to the clinical trial. In such instances it is not clear what to do with this patient information. The students work during most of the investigation task with the data shown in the table below.

Starting point is a clinical trial of 146 patients after cardiac arrest. These 146 patients were kept track of during the next 10 years, as far as this was possible . The table below lists the data known about the patients:					
A	B	C	D	E	F
year since entry into the study	alive at beginning of the year	number dying during the year	cumulative number of deaths	number of censored persons during the year	cumulative number of persons censored
1	146	27	27	3	3
2	116	18	45	10	13
3	88	21	66	10	23
4	57	9	75	3	26
5	45	1	76	3	29
6	41	2	78	11	40
7	28	3	81	5	45
8	20	1	82	8	53
9	11	2	84	1	54
10	8	2	86	6	60

Column E: the number of patients whose situation was no longer kept track of from that year on, i.e., the number of censored individuals.

Data from a clinical trial on myocardial infarction (taken from the students' text).

2. *Examples from ‘familiar’ statistics.* The classical gambler’s ruin example of throwing a coin repeatedly until heads comes up is used to let students recall the systematic method of computing probabilities of events through a tree structure of chances and to let them practice the statistical terminology and notation. The students apply survival analysis techniques to the gambler’s ruin example and to some invented data of a clinical trial in which censoring of observations does not yet play a role. Students who feel not confident about their spreadsheet skills are invited to practice first with a supplementary tutorial note on working with EXCEL.

3. *Handling survival data with censoring; simple ways to estimate survival probabilities.* The data of the clinical trial with censored observations prevent the use of standard methods of statistical summarization and inference already known to the students. The concrete problem in this part of the learning material is to estimate the five year survival probability of a patient on the basis of the given survival data. The students are confronted with three rather naive points of view: (P) Eliminate all individuals who are censored and use the remaining ‘complete’ data; (Q) Eliminate all persons who are censored in the first 5 years and use the remaining data; and (R) Consider all persons who are censored in the first 5 years as survivors and count them as such, that is, treat them as if they did not die from heart disease. The estimated probability of surviving the first 5 years after treatment of the heart failure would range from overly pessimistic to overly optimistic using the above data set and the above points of view (11.6%, 35%, and 48%, respectively). Since these survival probabilities are so different, the importance of refining the points of view becomes quite obvious to the students. They are explicitly asked to reflect on the given points of view, to try to formulate better alternatives, and to calculate the survival probabilities of all methods under consideration. Questions run like “Which of the two points of view is most credible and on what grounds?” and “Using the tabular data, decide which point of view is taken here. How do you get to your conclusions?”

4. *The life-table or actuarial method.* This section gives a more refined technique for computing survival times used by statistical practitioners. Depending on how one looks at the censored data, one will still obtain different survival probabilities, but the differences are smaller than before. The two points of view on censoring in the actuarial method presented in the instructional material are: (A) Anyone censored in a year is immediately censored at the beginning of that year; and (B) Anyone censored in a year is censored at the end of that year. Once more, the students are explicitly asked to reflect on these points of view, to try to formulate an alternative, and to substantiate their own point of view.

Results and Findings

From the questionnaire and some informal conversations with students during the lessons it is clear that the students very much appreciated the use of a spreadsheet program. They felt that they had acquired good skills in working with EXCEL during the tasks. This seemed valuable to them because they thought it useful for work in other disciplines as well. The collaborating teacher and the researcher were also quite pleased with the role that the computer environment had in this experiment, but for other reasons than the students: They found that it provoked the students to work in an active way and to talk extensively and thoughtfully about the statistical and computational problems involved within their team, with the teacher, and with other groups of students. Significantly, their discussion was almost exclusively on the tasks,

a phenomenon the teacher had hardly seen before during the regular paper-and-pencil work in the classroom.

The spreadsheet program allowed the students to work with formulas in an active way, which is less intimidating than the usual way of working with formulas on paper. The most important difference between spreadsheet and pencil-and-paper work is that the variables in a spreadsheet program do not have a name. Instead, the role of variables is played by cells. The cell itself corresponds to the concept of variable, whereas the content of the cell corresponds to the current value of the variable. Combining variables into new values, that is, creating formulas is done by pointing to the cells containing the values needed. Neuwirth (1995) called his technique the gestural description of mathematical formulas. In other words, the spreadsheet program emphasizes the process character of a formula, that is, the use of a mathematical formula as a way to describe a process of computing a result. In pencil-and-paper work however, the students' success in dealing with formulas depends to a large extent on how far they have succeeded in learning to look at formulas as mathematical objects. It is known that this shift in focus from process to object character of formulas and functions is difficult for many students (cf., Tall et al., 2001). Certainly this holds for the participants of the classroom experiment, whose algebraic thinking was weak. The computer results revealed that they had sometimes found good solutions to the exercises, but that they had not been able to express their answers in terms of formulas.

The active students' attitude had substantial impact on the quality of their results. This might easily have been overlooked if the researcher had only read their reports, because the students' understanding of the subject was better than their written reports suggested. Listening to the conversations amongst the students and discussing the answers with them brought this to the fore. Therefore I concluded that the question whether the learning materials and instructional setting enabled students to understand the 'big ideas' in survival analysis can be answered in the affirmative. The main grounds for this conclusion were that some students were able to come up with such alternative points of view on censoring which they could only have achieved because they really had a good idea of what they were doing, felt confident enough to come up with original ideas, and knew what survival analysis is all about.

The above conclusion can also be underpinned with the solutions students gave for the final problem in which they were asked to compare the actuarial method described in Section 3.2 of the instructional materials with methods used before in Section 3.1 of the instructional material and to reflect upon two points of view of censoring in the actuarial method. These points of view were: (A) censoring at the end of the year, with formula $s(t) = d(t)/n(t)$; and (B) censoring right at the beginning of the year, with formula $s(t) = d(t)/(n(t) - w(t))$. Here $n(t)$ denotes the number of patients at the beginning of year t still participating in the study, $d(t)$ is the number of patients dying that year, $w(t)$ is the number of persons censored during that year, and $s(t)$ is the probability to die during that year of the study. One example of students' good reasoning, but weak documentation from a mathematical point of view is the following: "You could say that the values of Section 3.2 are better than those in 3.1. The reason is that the points of view in 3.2 are more clear and logical in the first place. Secondly, the points of view (A) and (B) are derived from viewpoint (Q). In these points of view, the data censored in the year are subtracted and not, like in point of view (P), that none of the censored data are taken into account."

The students also had to formulate a better or intermediate point of view and to express this both in everyday words and in a formula. One team was unable to give a formula related with their point of view. Because of unavoidable interaction between teams, the solution chosen by half of the teams was represented by the formula $s(t) = d(t)/(n(t) - 0.5w(t))$, which is a quite complicated formula for students at this level. In fact, only one team could give this formula in EXCEL without bracketing errors; the other teams described it in words and actions, and seemingly entered the numerical values one by one in their sheet. One team explained their choice as follows: “Point of view (C) must be exactly in the middle, so we take the formula $s = d/(n - 0,5w)$. This formula means that censored data are subtracted midyear, we assume that in the middle of the year censored data are halfway.” Two teams came up with the formula $s(t) = (d(t)/n(t) + d(t)/(n(t) - w(t)))/2$, which was easier to use in EXCEL and was also easier to understand as lying between the given points of view. One team came up with a really original idea: They decided to use the average number of censored persons per year, and this average being six persons for the given data set, proposed the formula $s(t) = d(t)/(n(t) - 6)$. Their explanation was: “We think that this is a better solution because it balances between the two extremes of not including some things at all or counting very accurately.” This kind of statistical reasoning exceeded all expectations that the instructional designers had beforehand.

Extensions

The work climate and the performance of the students in the classroom experiment was unusually good. Probably, the instructional materials and the instructional setting let the students overcome their math anxiety, in particular their fear of using formulas and their lack of confidence in their own answers to mathematical questions. They could discuss problems with peers or the teacher in a non-intimidating setting. The mental blockade that the students could not overcome in regular lessons when they worked with formulas on paper turned out to be absent when they worked with formulas in a spreadsheet program. They were also able to make their own choices for handling censored observations, to explain the methods they had chosen, to compute survival probabilities, and to draw conclusions from the computations. These results of the classroom study indicate that development of statistical literacy for all students at secondary level can be realized through the use of suitable ICT. More classroom experiments, using a variety of real contexts, are needed for exploring the full potential of ICT-supported statistical inquiry by students and for a broader perspective on the embedding of probability theory and statistics in the mathematics curriculum.

2.6.2 Handling Weather Data

Reference

Van den Camp, W., & Heck, A. (2004). Wat heb je aan het gemiddelde? [How useful is an average?]. *Nieuwe Wiskrant*, 24(2), 33–38.

Introduction

In statistics assignments in lower secondary education at havo and vwo level,³ students normally work with small series of observations in simple, often artificial and

³havo is the Dutch acronym for senior general secondary education, which is a 5-years programme that prepares for higher vocational education (e.g., at universities of applied sciences). vwo stands for pre-university education.

invented contexts and without ICT tools. This differs much from the way in which statistics is used outside the school situation, in practice. The risk is that students be introduced to all kinds of statistical notions, computational methods and graphical representations, but that they only get a vague idea how to apply this knowledge and skills to structuring and interpretation of real data sets. This may play tricks on the students when they carry out practical assignments and the profile project in upper secondary education.

One can anticipate this by starting already in lower secondary education practicing meaningful processing and analysis of data sets of medium size. The most appropriate work format may be a small, spreadsheet-based practical assignment. Added value of such assignment in lower secondary education is that the students already become acquainted with such work at an early stage of their school career and can combine this with learning to use a spreadsheet program effectively.

This classroom study addresses the research question whether a spreadsheet-based investigation of a realistic data set in a small practical assignment by students in lower secondary education is viable. With this goal in mind, the assignment “How Useful is an Average?” was developed and tried out in a vwo-3 class. Aim of the classroom study, especially regarding the use of ICT, was to get answers to the following questions:

- Is the assignment for the students realistic and motivating?
- Is a spreadsheet program a helpful tool for students in their the computational work?
- Is working with absolute and relative cell pointers in a spreadsheet program like EXCEL viable for students of this age?
- Can a spreadsheet program help students to come to underpinned conclusions about a posed complex problem?
- Have the students been prepared well through the instructional materials for doing a small investigation on their own?

The research instruments used to answer the research question were a pretest, classroom observations, informal interviews with students, the report of the students and the underpinning EXCEL sheets.

The Classroom Experiment

Twenty vwo-3 students (9 male and 11 female; age 14-15yr.) at gymnasium level participated in June 2004 in the classroom experiment, in which they explored precipitation data originating from the Royal Netherlands Meteorological Institute (KNMI). They used arithmetic means in various ways for the answering of concrete questions about the weather. One of the notions was moving average, but unlike climate scientists do, it was not used in the practical assignment to smoothen sharp fluctuations in the weather observations so that a trend becomes more visible. In this assignment students had to concentrate on computing with averages over various, possibly overlapping, time periods and noticing what was the effect of the choices.

The mathematical notion of moving average is no part of the Dutch mathematics curriculum. The reasons may be that it only makes sense for large data sets. The

data set that the students used in the assignment “How Useful is an Average?” was sufficiently large. It concerned the daily precipitation data of the weather station De Bilt over the year 2000, taken from the Climate Atlas of the Netherlands (Heijboer & Nellestijn, 2002). Students may understand the concept of moving average and be able to do computation, but the thought of using it may simply not occur to them in a contextualized presentation of a question. This gap between arithmetic mean as algorithm and the qualitative properties of the average plus application in concrete situations is confirmed in educational research (cf., Bakker, 2004, section 4.3; Garfield & Ben-Zvi, 2008, ch. 9; Konold & Pollatsek, 2004).

Instructional materials were developed in accordance with the objectives to let students learn to

- view statistics in a more realistic way;
- do investigations at an early stage;
- effectively use ICT tools for the processing and analysis of large data sets; and
- reflect on statistical methods, in this case on the variable use of arithmetic mean in a concrete context.

The instructional materials consisted of a student text with assignments and a hand-out for introducing the use of a spreadsheet in the context of weather data. The student text starts with rather closed questions and tasks (Figure 2.17), moves toward more open questions (Figure 2.18), and ends with suggestions for small investigations (Figure 2.19). The students work from the beginning with the spreadsheet program EXCEL and with files containing daily precipitation data in tabular form. Students who are not so familiar with working in EXCEL may first work through a tutorial on the software and an accompanying spreadsheet. In the original planning, one lesson (of 45 minutes) was meant for brushing up one’s knowledge and skills of using EXCEL, two lessons for the exercises (open and closed questions), and two lessons for a short investigation by students on their own.

The aim of this practical assignment is to place the students’ work with the average of a series of observations in a meaningful context. Focus is not on the algorithm to compute the arithmetic mean, because this is what students are already good in, but instead on the position that this notion has in the processing of data and in statistical reasoning. In the chosen context of Dutch weather, data can be grouped in various ways for computations of mean values in various time periods, depending on the problem posed:

- You need the average of all data (*What is the average daily precipitation in the year 2000?*);
- You need the average of consecutive groups of numbers, for example each time for a period of one week or one month (*What is the most rainy month in the fall of 2000?*);
- You need the average of overlapping series of numbers, for example the average over the last 10 days, for every day in a period of one month or a year (*What is the most rainy period of 10 days in 2000?*);

- You need the average of more than one series of numbers, but the series of numbers are separated by unnneeded data (*What was the most rainy summer month in the last thirty years? Was spring more rainy than autumn in 2000?*).

The above list of different ways of using the arithmetic mean indicates that alternative conceptions of students may be expected.

At the travel agency “*Beautiful Holland*” one can book holidays in places all over the Netherlands. Because customers like good weather they think that their holiday is spoiled when it rains too much. The travel agency wants to advertise an insurance against rainy weather during holidays. They think about the following setup: Because an average precipitation of 0.1 mm/day in De Bilt has been reported 25.5 times over the year 2000, persons who book in the Netherlands a “*holiday with insurance against rainy weather*” get refunding of 50% of the costs of the booked stay if the average precipitation in the booked period is greater than the yearly average. Not knowing whether this promotional action is too expensive or not, the board of the travel agency has asked a meteorologist to prepare a table and graph representing for each day in 2000 the average precipitation **of that day plus the preceding nine days**. This means that you always look at a period of ten days and that you compute on the last day of the period the average precipitation of the time period under consideration. For holidays of eleven days, twelve days, and so on, you can do the same investigation, but at the travel agency “*Beautiful Holland*” they know from experience that a ten-days holiday was most popular in recent years. This is why they look for the moment only at risks for 10-days holidays. The first day for which this calculation can be done with the precipitation data of the year 2000 is the 10th of January.

Exercise 4. Why can't you do the calculation for the 5th of January, using the table in the spreadsheet *Average-precipitation_for_10_days.xls*?

Exercise 5. In case you want a solution regarding the insurance for the first 9 days of January, what suggestion would you make? Come up with a realizable and defensible proposition.

Figure 2.17: Fragment of the instructional material with closed exercises.

We now want to investigate holidays of 15 days that are all within the period of the 6th of July up to and including the 28th of August. The travel agency chooses in favor of the promotional deal: “*50% of your travel costs refunded when the average precipitation during holidays is more than x mm per day.*”

In order to avoid that the costs of this insurance become too high, the agency does not want to pay for more than 10% of the holidays. You may assume that all booked holidays are evenly spread over the given period.

Exercise 13. Do your own investigation and give advice about the values of x that may be selected to meet the requirements of the travel agency. Write the advice down on at most one A4-sized page. Add to your advice as appendix the Excel file in which you worked as much as possible with formulas. Make sure that your files are neatly arranged.

Figure 2.18: Fragment of the instructional material with open exercises.

4. When do we go camping?

Suppose that your class goes camping for 5 days. Which period is then best? First select the criteria that you want to take into account. Is precipitation, temperature, or hours of sunshine important? Or do you want to take other things under consideration? Explain your choices and clarify which period is in your opinion best for camping. Also submit the files that you have used. Log your work.

Figure 2.19: Fragment of the instructional material with investigation tasks.

The instructional materials do not contain the term ‘moving average’, but the nomenclature contains expressions like ‘10-days preceding average’ and ‘17-days ambient average’. This terminology is deliberately used in the student text to specify the precise period over which the average is computed and to specify whether the computed average is connected with the starting-point, midpoint or end point of the time period. It is considered unimportant that these terms do not belong to the common mathematics jargon, as long as the meaning of the terms are clear to students.

The practical assignment has been tried out in a gymnasium-3 class of twenty students, with a good attitude towards studying and with good study results. Because this is not a common situation for third classes, the assignment should be tried out in several other classes. However, it is expected that this assignment is suitable for all classes, even for havo-3.

Results and Findings

The results and finding are presented in three parts: (1) the pretest results; (2) experiences with the closed part of the practical assignment; and (3) experiences with the open part of the practical assignment, which the students carried out in pairs.

Pretest results.

Through a pretest I investigated whether the students in the vwo-3 class perhaps already had a well developed insight in the versatile use of the arithmetic mean at the beginning of the practical assignment. Students were for example asked to choose in various contexts from four ways to group data for computing averages as mentioned in the previous paragraph. The analysis of the answers of the students showed that the pre-knowledge of the students was diverse. For example, in the first context in the pretest—“You have the daily weather data of the last thirty years. You want to know how many years in this period of 30 years had the lowest average temperature in the month of January.”—44% of all students chose a wrong answer.

It took about half of the students more than one lesson to work through the hand-out on working with EXCEL. This revealed that the spreadsheet program was not used much by this group of students after they had learned to work with the tool in earlier study years. A classroom experiment in a vwo-5 class about survival analysis (Van den Camp & Heck, 2003) revealed the same problem. The conclusion is obvious: Once learned, working with a spreadsheet must be maintained, otherwise learning to use the tool is meaningless.

Experiences with the closed part of the practical assignment.

It turned out that the student could manage well the closed part of the practical assignment. Lively discussions arose about the (un)fairness of choices that students had made. The students’ reports were quite acceptable.

Students did not find Exercise 4 (Figure 2.17) very difficult, but their answers revealed that they sometimes forgot that the data were real and that one could look them up and extend them by use of the climate atlas. However, about Exercise 5 the student’ opinions on ‘reasonable’ and ‘doable’ differed much. Many students were of opinion that one could use for the 6th of January the average of January 1 up to and including January 6. When the teacher asked whether customers would think the same they changed their mind. Many students proposed a realistic solution of the problem at hand, such as “just put the last nine days of 1999 in the table, too.” Sometimes the thoughts of the students drifted too much away from the mathematics toward the

context of a travel agency. An example is the following answer: “I would make an arrangement that the insurance does not apply when you go those first 9 days, but that you would get a discount on travel costs. In this way you still attract persons to travel, because it is cheaper.” The teacher had a discussion with four teams about the realistic nature of the problem posed. These students were of opinion that most travel agencies only offer holidays that can be booked during the season for complete weeks. Then bookings from Tuesday to Thursday are not possible. This point of view of the students indicates that the assignment was realistic to them, sometimes perhaps too realistic so that the mathematical thinking suffered from it.

Experiences with the open part of the practical assignment.

The format in which the students were expected to answer Exercise 13 in the more open part of the assignment (Figure 2.18) was unclear for all students. The authors had in mind that the students would for example write a letter to the travel agency, but the students gave short answers in common school language, with most often a spreadsheet as underpinning resource and not much explanation. At hindsight, the presentation of the question was unclear on this point. A better idea would have been to explicitly ask for a advisory letter and pay, together with the teacher of Dutch language, attention to the quality of such letter.

Classroom observations of the more open part of the assignment revealed that students had difficulties in using EXCEL or at least did not use the spreadsheet program effectively: In particular, these problems were about absolute and relative cell references. It would have been wiser to pay attention to these two spreadsheet concepts in the tutorial sheet on working with EXCEL and to explain the advantages and disadvantages of using each type of cell references. In the educational research literature (e.g., Haspekian, 2003, 2005a,b), the versatile use of cells in spreadsheet programs, alternative conceptions of students, and the instrumental genesis of the spreadsheet tool are discussed in depth. In other word, the tasks in the hand-out about working with EXCEL should be more tailor-made for specific problems that occurred or could occur in this practical assignment. In the classroom experiment, students lost much valuable time with solving problems in use of the spreadsheet program. The open part of the assignment before the small students’ investigation had been planned for two lesson hours, but it took more time, namely nearly four lessons. This limited the time for gaining experience with doing an investigation. With a better tutorial about the use of the spreadsheet program, the time needed for the renewed assignment is estimated as: one lesson EXCEL + three lessons for the assignment (+ two lessons for a small investigation task by students). Because of lack of time in the classroom experiment, the last research question “Have the students been prepared well through the instructional materials for doing a small investigation on their own?” could not be answered adequately because only a few students came to work on the open investigation shown in Figure 2.19.

The unease of some students with the spreadsheet program came to the fore when these students did the computational work first with a calculator and then copied the results in the spreadsheet. Small changes in the presentation of the question then resulted in a lot of computational work. A positive effect was however that it motivated the students to make the step to using formulas in the spreadsheet program. The students learned by own experience what a spreadsheet really is and what advantages it offers.

Various groups of students formulated their reasoning in clear words. Figure 2.20 shows the answer of a student team to Exercise 13 (Figure 2.18). This answer is, despite a counting error of 39 days, clear and matches the setting of the spreadsheet. The students had not seen (or they had forgotten), like many students, that the unit for precipitation is 0.1 mm. The students' answer also does not reveal insight in the assumption that one can take the year 2000 as starting-point, provided that this a 'regular' year regarding precipitation.

Exercise 13

The company want to pay back 50% of the costs of 15-days travels in which on average x mm of rain per days falls (period from July 6 up to and including August 28). To make sure that the costs do not become too high, they do not want to pay for more than 10% of the holidays.

In the period from July 6 up to and including August 28 there exist 39 15-days holiday that are completely within this period. 10% of this is therefore $0.1 \times 39 = 3.9$ travels, i.e., 4 travels that must be paid back.

In order to compute how many travels must be paid back you must calculate the 15-days average and then determine a cut-off. When the average is above the cut-off value, the travel is paid back, otherwise not.

Because the company wants to pay back only 4 travels, I advise to set the cut-off value on 30.8 mm, because then they only have to pay back 3 travels. See Exercise 13.xls for data.

Figure 2.20: The answer of a student team to Exercise 13.

A number of students had not quite grasped the idea that the year 2000 was more or less randomly chosen as a kind of 'standard year' for making mathematically underpinned statements. The student text does not emphasize this enough. For a high-performing class it would be interesting to compare the results of 2000 with results for other years and verify whether the choice of the reference year matters much, or not at all.

Extensions

Additional small practical assignments at havo/vwo lower level in the context of weather forecasting were developed and tried out in classroom:

Is the Weather Well Represented? (2 havo/vwo)

Students verify whether the graphs and table about weather in their textbook are in agreement with the data on the website www.knmi.nl of the Royal Dutch Meteorological Institute (KNMI). This promotes a critical look at data in the textbook.

The Wind Rose. (3 havo/vwo)

Students explore wind data at particular locations and make wind roses. A meteorological wind rose is a form of circular diagram in which the frequencies of the eight principal wind directions and strength of the wind at a particular location over a specified time period are represented. In this way, the students get acquainted with a map diagram that is not present in any of their textbooks, but which they can understand and work with on the basis of their existing mathematical knowledge. One could say that this prepares students for the nationwide examination, in which new or less familiar graphical representations pop up regularly. In addition, a link is made between mathematics and earth science.

2.7 Computer-Based Modeling in Quantitative Pharmacology

References

Two aspects of computer-based modeling by students in the context of quantitative pharmacology, and particularly in the context of alcohol metabolism, have been discussed in the papers listed below: (1) compartmental models in quantitative pharmacology; and (2) a blended approach to learning computer-based modeling of discrete dynamical systems at secondary school. In addition, the classroom study was a usability study of the graphical, system dynamics-based modeling tool of COACH 6. This is the reason for including more than one reference.

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Introduction

In May and June 2007, a small team of university lecturers and secondary school teachers jointly developed and piloted an e-class for 4th and 5th grade students (age 15–17 yr.) at both pre-university and general vocational level (vwo and havo, respectively). The goal was to develop and try out innovative ways of teaching mathematics that would enable schools to offer optional courses such as Mathematics D and Nature, Life, and Technology (also named Advanced Science, Mathematics, and Technology) for small numbers of students. The e-class concept can be summarized as web-supported instruction in a blended learning approach. This means that one of the ideas behind the set-up of an e-class was to develop a rich virtual learning environment (henceforth abbreviated as VLE) equipped with study guides, digitized instructional materials, software for learning and doing mathematics and science, video instructions, animations, (self-)assessments, communication tools for students and teachers to chat and discuss, and so forth. Central was the blended approach, considered as a combination of online learning and face-to-face education at school. Actually, the broader definition of Heinze and Procter (2004) for blended learning in higher education—“learning that is facilitated by the effective combination of different modes of delivery, models of teaching and styles of learning, and founded on transparent communication amongst all parties involved with a course.”—seemed more applicable to the e-class setting. This definition of blended learning was adopted by the designers of the e-classes, acknowledging that there is no universally accepted definition (Whitelock & Jelfs, 2003; Bonk & Graham 2006) and that there are some serious doubts about its conceptual integrity (Oliver & Trigwell, 2005).

The set-up of the pilot e-class on discrete dynamical systems was both simple and complex. The instructional materials consisted of the digitized chapter on number sequences, recursive formulas, and discrete dynamical systems from a brand-new mathematics textbook, supplemented by investigative activities. Everything was stored in a master copy of the e-class inside the VLE, from which clones were constructed for use by real classes. Topics for the optional investigative activities were: numeric approximation of roots of quadratic and cubic polynomial equations, exploring mathematical models of bouncing balls, and compartmental modeling of the intake and clearance of drugs in humans, in particular of alcohol. Students could build and simulate models of discrete dynamical systems with the computer learning environment COACH. Instructions for learning to work with software and demonstrations of worked-out examples were given through screencasts created by the teacher to gear with students' needs and made available in the VLE. A screencast, a term coined by Udell (2004, 2005), is "a digital movie in which the setting is partly or wholly a computer screen and in which audio narration describes or explains the story on the on-screen action." Students got weekly on-line assignments, which they submitted digitally. At home they could get assistance in a chat room from peers and the teacher: they could discuss exercises, ask each other for further information or explanation, and so on.

The complexity of the e-class setting lay in the well-known fact that use of ICT in education does not always lead to better quality of teaching and learning. Two major challenges were: (1) How to find the right mix for a blended learning arrangement with regard to content, knowledge construction, and communication within a regular curriculum setting (John & Wheeler, 2008; Kerres & De Witt, 2003; Laurillard, 2002) that leads to meaningful learning? and (2) How to initiate, sustain, and structure interaction, and how to enhance its quality (Hannafin, 1989)? In the design of the pilot e-class, decisions were mainly based on the teachers' experiences, research-based design principles of multimedia learning (cf., Mayer, 2005), and principles of learning sciences for the design of pedagogical arrangements (cf., Sawyer, 2006).

About three hundred students from 4th and 5th grade (age 15-17 yr.) at both pre-university and general vocational level, from fifteen classes at four schools in the region of Amsterdam, participated in the case study. Only a minority of the students belonged to the envisioned group of motivated and mathematically competent students who would choose Mathematics D in their Nature & Technology profile. However, because the new style of teaching and learning was also considered relevant for mathematics courses at other levels of education, the new approach was piloted with a diverse group of students to get experiences from many directions. For example, the investigative activities about alcohol metabolism were tried out in a havo-4 class (with students in the Nature & Technology profile) and two vwo-4 classes, one with students who had chosen for Mathematics A in the Nature & Health profile and one with students taking the Mathematics B programme in the Nature & Technology profile. In this section I only present the experiences with the e-class concept and the outcomes of the e-class about discrete dynamical systems and applications in the context of alcohol metabolism. In this case study I aimed at getting answers to the following questions:

- How do teachers and students experience the e-class?
- Do the instructional materials and the chosen instructional setting enable students, who have no prior practical experience with the COACH, to get started

with mathematical modeling and the graphical system dynamics-based modeling tool of COACH?

- How can a process of repeated modeling cycles be realized in an instructional sequence about quantitative pharmacokinetics and alcohol metabolism?

The research instruments used to get answers to these questions were evaluation reports of the students, interviews with students and teachers, a questionnaire at the end of the pilot project, and student work.

The Classroom Experiment

All course materials were made available online in a SAKAI-based VLE. The choice for SAKAI (<http://sakaiproject.org>) was based on the fact that almost full responsibility for the e-class could be transferred to the individual teacher who could, for example, select his or her own assignments from the master copy of the e-class, adapt the instructional materials to the ability level of his or her students, invite students and colleague teachers, select or deselect ways of communication, write a study planner, add their own screencasts, give the e-class a more personal touch, and so forth. Students were requested to hand in their answers to some of the online assignments through the request-and-delivery system for exercises inside the VLE: This homework was mainly a WORD document, an EXCEL sheet, a COACH result file, or another kind of digital document. Students got partial credit for their homework. The drop box facility of the VLE was convenient for personal transfer of documents between a student and the teacher (for example, to get dedicated help or advice on a task, or for the simple reason that homework could not be delivered within the scheduled time frame). Weekly meetings between teacher and students were scheduled, because face-to-face contact remained highly valued. Sometimes these contacts stimulated the teacher to create a screencast for the purpose of procedural scaffolding. The course ended with a written test and a computer-based assessment. Thus, the assessment of the students was in accordance with the way they had studied the subject contents.

I was the main designer of the supplementary investigative activities, including those about quantitative pharmacology. The subject of modeling intake and clearance of alcohol in humans was a deliberate choice for providing students insight into processes of pharmacokinetics and physiology via compartmental modeling and for discussing a topic that many a student could personally relate to. Graphical modeling software provided the tools for computer modeling. Many of the mathematical models were simple enough so that students with limited experience in modeling could still implement them. From instructional point of view I considered it relevant that there exist several models that students could use to investigate the blood alcohol concentration (BAC)-time profile after consumption of one or more alcoholic drinks. The versatility of the models was expected to contribute to the students' understanding that the quality of a model depends on the requested descriptive, predictive, and explanatory power as well as on the modeler's preferences, background, and goals.

Students were invited to explore with the computer models effects of various scenarios of alcohol consumption, in an attempt to answer questions such as "Does it matter in the long term whether you drink fast or slowly?", "Does it matter for the alcohol intake and clearance whether the consumption is after a meal or not?", and "Are there gender difference in human alcohol metabolism?" Exploration of such questions was expected to give students a broad idea of alcohol pharmacokinetics (cf.,

Greefrath, Siller, & Weitendorf, 2011) and to provide them with examples of compartmental models that can also be applied in investigations of similar processes of change. Results from computer models were compared with measured data obtained with breath analysis equipment. For this purpose, secondary data were mostly used in the classroom. Such data were considered useful in discussions about the quality of various models and reminded students of the fact that understanding of the mathematical models was not the only instructional goal; understanding of the investigated phenomena was also considered important.

The broad range of models for intake and clearance of alcohol in the human body ensured that students had the opportunity to practice evaluation and revision of models, and in this way would get acquainted with the concept of model progression. In principle, they could develop the critical attitude that is necessary for successful modeling of biological, chemical or physical phenomena. All models presented in the instructional material, ranging from the simplest linear elimination model to a rather sophisticated physiology-based compartmental model, are actually used in pharmacokinetics studies (cf., Atkinson et al., 2007; Lands, 1998; Norberg, 2001; Shargel, Wu-Pong, & Yu, 2005). This implied that the students' investigation work was not only fun to do, but also resembled research practice. However, the following comment must be given: In order to be able to start with the modeling activities early in the mathematics curriculum, even before students learn about the exponential and logarithmic function, the mathematical models in the instructional materials were all discrete dynamical systems, even though the modeled physiological processes are continuous. In higher education and in pharmacokinetics research one would use differential equations and a more complete range of mathematical functions. An e-class about continuous dynamical systems, based on the same principles of web-supported instruction in a blended approach, has been developed in the meantime (Heck, Houwing, Val, et al., 2009).

Dealing with a versatile set of models of alcohol metabolism in humans was also meant to give students a good idea of the common method of working in mathematical modeling: First one simplifies the situation to such an extent that a simple model can be constructed. Hereafter one evaluates this model, preferably by comparing it with experimental data, and one adapts it if necessary. In the process of evaluation, parameter estimation plays an important role as well and one better not underestimates the complexity of finding suitable parameter values. Adaptation of the model normally means that one makes the model more complicated by taking more factors that cannot really be neglected into account or by undoing some earlier simplifications (This may also mean that more complicated relations between variables are used instead of increasing the number of variables in the mathematical model). One comes into the process of simplifying first and then adding step-by-step more details to the model, with the purpose of matching the model better with reality.

For the instructional design I considered the progressive aspect of modeling as a pointer to a suitable manner to introduce it to students: In line with the well-documented instructional strategy of gradually introducing learners to increasingly more sophisticated or comprehensive subject matter (cf., Gagné, 1985; Rosenshine, 1995), it seemed me best to let students first work with a simple model and improve it by changing or adding details before they construct models from scratch. I motivated the change of models of alcohol metabolism by letting students compare the results of

computer models with real data. The mathematical models used in the instructional materials were:

- (1) the open 1-compartment model of Widmark (1932) with zero-order elimination;
- (2) a hybrid open 1-compartment model with zero-order elimination and first-order absorption;
- (3) the open 1-compartment model of Wagner and Patel (1972) with clearance described by Michealis-Menten kinetics; and
- (4) the 3-compartment model of Pieters, Wedel, and Schaafsma (1990).

In the first three models, the main point was that the relations between variables became increasingly more complex; in the fourth model, the number of variables in the model was increased by the simple fact that more compartments were involved. In addition to model progression, I also used in the beginning small exercises and closed questions, and gradually moved to more open tasks and reduced support. Other mathematical models of alcohol metabolism such as the 2- and 3-compartment models of Norberg and colleagues (2000, 2001) or the physiology-based model of Umulis, Gürmen, Sing, and Fogler (2005) could have been included in the instructional materials, but this would have been beyond the learning objectives. I only implemented these more advanced model in COACH to explore the scope of the graphical modeling tool. Anyway, 2- and 3-compartment models leading to bi- and tri-exponential models of concentration-time profiles were included in the instructional materials in the context of elimination of pain-alleviating drugs, using real data collected from research literature. In addition, the instructional materials treated modeling of repeated intravenous administration and oral administration of drugs.

Students were guided in the instructional materials to convert the mathematical models into computer models via the graphical, system dynamics-based modeling tool of COACH 6. This modeling tool allows students to create and run numerical models and to compare modeling results with experimental data. A text-based, equations-based, and graphical editor is provided. The first type of modeling is programming in a language that is dedicated to mathematics, science, and technology education. The last two types of modeling support a system dynamics approach that is the basis of a graphical, aggregate-focused software such as STELLA (Steed, 1992). This type of modeling system uses aggregated amounts, that is, quantities (commonly called levels or stocks) that change in time through physical inflows and outflows, as the core elements of a model. Not only physical flow, but also information flow determines the systems behavior over time. Information flow is best understood as an indication of dependencies or influences between variables in the model. These relations are made explicit in the form of mathematical formulas and graphical or tabular relationships. The variables involved can be levels, flows, parameters, and auxiliary variables.

The level/flow modeling language has a graphical representation in which a user can express his or her thoughts about the behavior of a dynamical system, and these ideas are then translated into more formal mathematical representations. The conventional symbol of level and flow is a rectangular box and a double arrow with a valve, respectively. A single arrow commonly represents information flow. Auxiliary variables and parameters appear as circles and circles with small handles on both sides, respectively. An example of a graphical model is shown Figure 2.21.

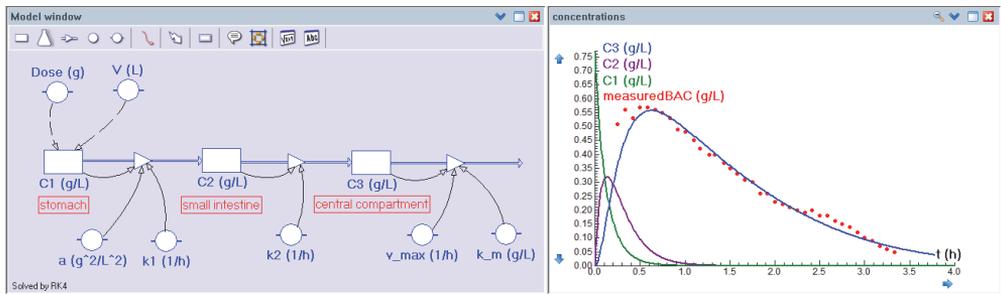


Figure 2.21: Screen shot of a graphical model implementing the Pieters 3-compartment model of alcohol metabolism and the calculated concentration-time curves in a simulation, with a background data plot of measured blood alcohol concentration (BAC) after drinking 3 standard units.

A graphical model represents a computer program that commonly implements an iterative numerical solution of a system of differential equations. In this particular case it represents a computer model for the Pieters 3-compartment model of alcohol metabolism, which is given by the following model equations:

$$\frac{dC_1}{dt} = -\frac{k_1}{1 + a \cdot C_1^2} \cdot C_1, \quad \frac{dC_2}{dt} = \frac{k_1}{1 + a \cdot C_1^2} \cdot C_1 - k_2 \cdot C_2, \quad \frac{dC_3}{dt} = k_2 \cdot C_2 - \frac{v_{max}}{k_m + C_3} \cdot C_3,$$

with initial conditions $[C_1(0), C_2(0), C_3(0)] = [C_0, 0, 0]$, where $C_0 = D_0/V$, that is, the initial amount of alcohol D_0 divided by the so-called volume of distribution V of the central compartment, and where C_1 , C_2 , and C_3 are the alcohol concentrations in the stomach, the small intestine, and the central compartment, respectively, related to the volume of distribution of the third compartment. Figure 2.22 is a common pictorial representation of the compartmental model, in which the volumes of distribution of the compartments are not specified but hidden in the parameters. It strongly resembles the structure of the graphical model shown in Figure 2.21.

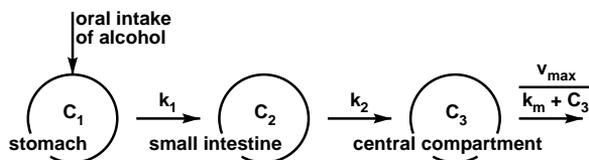


Figure 2.22: Pictorial representation of the Pieters 3-compartment model.

The first differential equation in the Pieters model, which models emptying of the stomach, does not represent a simple first-order process, but a feedback control is built-in that depends on the instantaneous concentration in the stomach, C_1 . In this way, the effect of an empty or full stomach on alcohol clearance can be taken into account mathematically. The clearance of alcohol from the human body has been modeled by Michaelis-Menten kinetics, which would lead to an almost linear elimination for high alcohol concentrations. The diagram on the right-hand side in Figure 2.21 illustrates that the Pieters 3-compartment model describes, for suitable parameter values, rather well the measured BAC data, originating from breath analyzer data of the author after drinking 3 glasses of red wine at once on an empty stomach early in the morning. In

the computer model a negative value has been chosen for the feedback parameter a to get an accelerated intake of alcohol, because drinking happened after fasting.

Results and Findings

From evaluation reports of students, interviews with students, and a questionnaire at the end of the study year (175 responses from 3 schools; 88 male and 87 female students at pre-university level; 53 students who had chosen the Nature & Technology profile) can be concluded that most of the students appreciated that they could

- work with and learn from screencasts;
- consult their peers and the teacher in the chat room;
- plan more or less their working hours;
- learn a lot, in a mixed and attractive approach; and
- build and simulate graphical models in COACH 6. They liked the software, once they got more familiar with it through the screencasts and by helping each other.

The majority of the students considered the course as more demanding and taking more time than usual. About one quarter of the students found the course too difficult, and about half of the students found it difficult, but manageable. About half of the students brought up that they had much more homework than usual. Many students characterized the final computer assignment as difficult. The teachers in the design team had actually been too optimistic about the amount of work for the students.

But also the participating teachers had been busy in the e-class of the pilot project. Especially, the weekly grading of homework handed in by the students was time-consuming and stressful. Also, teachers had to regularly log into the VLE to check if everything was going well and if their students needed a helping hand. Teachers received more emails than usual from their students that needed to be answered. Despite the work it took, almost all teachers looked back at the pilot project as a successful enterprise, with great prospects. Many of the participating teachers continued to run an e-class in the following years, sometimes in a less labor-intensive setting.

From both teachers' and students' point of view, one of the most successful components in the e-class were the screencasts. The aims of screencasting were simple: to allow students to

- see and hear the thinking and explanation of a teacher when presenting a worked-out example;
- watch whenever and as many times they want;
- go step by step through an instruction; and
- look at the whole process and not only at the final result of an exercise or activity.

The main benefits of the screencasts in the e-class were that they supported student learning outside the classroom, functioned in case of software instruction better than user manuals or help pages, and helped the teacher in the sense that (s)he could make optimal use of the face-to-face contact time with students. As Garner (2008) suggested, use of screencasts for feedback to student assignments and for answering

student questions in areas of conceptual difficulty is also a strong option for learning support. But honestly, the screencasts were in the e-class more applied to help students develop procedural fluency in using the modeling tool and doing the given tasks. Experience (e.g., Bennedsen & Caspersen, 2005; Fahlberg-Stojanovska & Stojanovska, 2007) is that screencast-based exercises encourage students and provide them the necessary scaffolds to carry out mathematical tasks and to practice skills. The success of the applied form of screencasting can be underpinned by research-based principles of multimedia learning (cf., Mayer, 2005, 2009) and cognitive load theory (cf., Kalyuga, 2009, ch. 2; Mayer, 2005). In comparison with user manuals, screencasts as demonstrations or tutorials of software packages reduce cognitive load because of the multimedia principle, the modality principle, the redundancy principle, the signaling principle, the personalization, voice and image principles, the self-regulation principle, the split-attention effect, and the worked-out example effect. These and other principles of the cognitive theory of multimedia learning by Mayer (2005, 2009) and cognitive load theory (e.g., Kalyuga, 2009; Plass, Moreno, & Brüncken, 2010) also provide good guidelines for the design of e-classes from usability perspective. They seem to have been applied until now in the e-classes more for promoting acquisition of procedural knowledge than for supporting development of conceptual understanding.

Although the process of learning to use the graphical modeling tool was for both teachers and students quite smooth, many of them had to get used to the fact that, whereas the computer modeling tool has only a limited set of graphical elements to build a model and a small set of rules of combining them, it still allows its user to express a solution to a problem in many ways. As a matter of fact, many a mathematician seems to dislike that graphical modeling often leads to different pictures of a mathematical model, which may reflect different styles of students' thinking and working, whereas there is in their expert opinion one suitable mathematical representation and mode of operation. For example, the word problem of a tank that contains at the beginning a solution of 100 kg salt, that is cleaned with fresh water every hour such that 20% of the remaining quantity of salt is removed, and to which every hour 16 kg salt is added, is modeled by the recurrence relation $S_t = 0.8 \cdot S_{t-1} + 16$, $S_0 = 100$. Once formulated in this mathematical form, it is in the eyes of expert mathematicians just a matter of technique to solve the problem. But they ignore then that the above recurrence relation is just one way of writing down the relationship between consecutive elements in the sequence, and that the graphical modeling activities may assist students in such versatile thinking and form a pathway for the students to a better understanding and appreciation of conventional mathematical expressiveness. During the pilot study of the e-class, vwo-4 students came up in their homework assignments with the three well-functioning types of graphical models shown in Figure 2.23 (the hidden formulas for the flows are written underneath the icons).

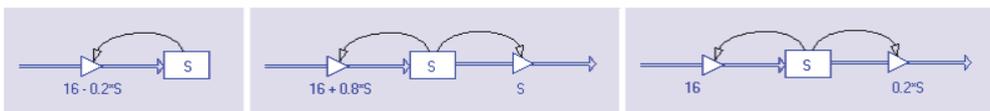


Figure 2.23: Graphical models of the recurrence relation $S_t = 0.8 \cdot S_{t-1} + 16$, $S_0 = 100$.

The graphical model on the left side indicates the student's understanding that there is a certain amount of salt in the tank (the level) and a change of this amount (the flow) that can be specified by a mathematical formula. No distinction has been

made between inflows and outflows of salt, as is done in the other graphical models. This student apparently had realized that an outflow is the same as an inflow with a minus sign in the associated formula and created a picture that actually represents the relationship $\Delta S_t = 16 - 0.2S_t$. In the graphical model on the right side, the difference ΔS_t is decomposed in an addition of 16 and a subtraction of $0.2S_t$. The graphical model in the middle is close to the recurrence relation in the sense that the right arrow represents that the current amount of salt is simply replaced by the formula shown in the left arrow. In the e-class, teachers did not find a need to use some metaphor, such as the commonly applied bathtub-water flow system, for introducing graphical system dynamics-based modeling to their students. They were satisfied with their approach to start with the mathematical notions of sequence and recurrence relations and treat the graphical model as a pictorial representation of a computer program for computing sequences, for which programming details become temporarily visible when the user points with the mouse to an icon and become editable when the information behind the icons is explicitly requested in a dialog window.

Because of limited time for the e-class pilot at the end of the study year, the process of repeated modeling cycles in the context of quantitative pharmacology could not be fully explored: The students only carried out the first part of the instructional materials about the open 1-compartment model of Widmark (1932) with zero-order elimination of alcohol. The introduction of the subject of alcohol metabolism and the graphical implementation of the Widmark model went rather smoothly. Refinements of the so-called Widmark factor as a gender-specific function of human body weight and body height (Seidl, Jensen, & Alt, 2000), as well as refinements of the volume of distribution as a gender-specific function of human body weight and age (Watson, Watson, & Batt, 1980) allowed students to run simulations that addressed questions about the influence of gender and human body composition on alcohol metabolism. The main problems with the graphical modeling tool arose when drinking scenarios had to be implemented using step functions. Students were not familiar with such mathematical functions and how they can be used to construct piecewise-defined constant functions. The notation used in the COACH software for defining step functions and repeated step functions actually worsened the conceptual problems. The usability of the software was in this respect not matching the students' ability level. But most students could construct computer models of various drinking scenarios after explanation of step functions by their teacher, although it remained to a certain extent rather cumbersome. As a matter of fact, at the time of the pilot of the e-class, a drinking scenario could not be implemented via a manually drawn graph. A connection between a model variable and a manually created data graph would have made it easier for students to specify drinking scenarios because step functions could formally have been avoided in this approach. In other words, this classroom experiment gave input for the (re)design or extension of a useful graphical modeling tool.

Extensions

The design of the instructional materials about quantitative pharmacology was done in the expectation that it would not be used for one course in an particular study year, but instead would be spread out over two or three years during which students learn more mathematics. On purpose it only treated discrete dynamical systems. Therefore, an extension towards modeling of continuous dynamical systems in the context of pharmacology or other processes in life sciences would be a logical next step.

Another direction of extending the supplementary activities could be the exploration of compartmental models in other life science applications.

2.8 Video Analysis and Modeling of Bouncing Balls

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Introduction

In this case study I explored how a model of modeling, in this case the modeling cycle of Blum and Leiß (2005) shown in Figure 2.24, could be used to structure and analyze a practical investigation about the behavior of bouncing balls. I investigated the potential role of technology and ICT in the practical investigation based on the steps in the modeling cycle. For example, with a high speed camera students could explore the motion of a bouncing ball in great detail and computer modeling enabled them to relate empirical results to physics theory. The instructional sequence was designed such that students could experience that reality (the measurements) is not automatically in line with the predictions of the theory (the models), but often even strikingly different. This was done to motivate a process of repeated cycles from measurement to interpretations, and in this way it realized a rich laboratory activity. The practical investigation was made possible by the integrated ICT tools in COACH 6 for measurements on videos made by a high speed camera (via automated point tracking), and for modeling, simulations and animations.

I chose the subject of motion of bouncing balls for the following reasons:

- The physics (Newton's laws of motion and concepts of energy) and the mathematics (processes of change and kinematic graphs) are still simple enough to be accessible to most of the students.
- An experiment with a falling object, in which data are collected with a stopwatch and meter stick, using sensors such as a microphone or a sonic ranger, or via webcams and video analysis tools, is rather easily carried out.
- Students are from personal experience aware of aerodynamic effects on falling objects, which can be incorporated in the computer models.
- One can compare measured data with modeling results in various ways (height-time profile, flight times, duration of bouncing, etc.).

A lot of attention went in the instructional activities to the validation of (intermediate) mathematical models in the hope and expectation that looking at various models of one and the same phenomenon would promote a critical attitude of students.

In this case study I explored whether such a modeling approach was possible and appealing for pre-university students with a Nature & Technology profile. From software development point of view I was interested in the question whether the students could easily make use of the hybrid structure of the modeling tool of COACH 6 that combines a classical system dynamics approach with event-based modeling for processes that change abruptly. In short, I aimed in this case study at getting answers to the following questions:

- To what extent is the idealized modeling route in the instructional materials practicable for students?
- Do the instructional materials and the chosen instructional setting, in which both the video analysis tool and the graphical system dynamics-based modeling tool of COACH play an important role, enable students to investigate bouncing balls at a good level of quality?

In the second question I operationalized the quality of the students' work as the level of agreement of their written work with the intended high quality outcome from the author's point of view, which was in the exercises and tasks based on components of the framework of concepts of evidence (Gott, Duggan, & Roberts, 2003) such as: distinguishing the independent variable and the dependent variables; understanding calibration and sampling in measurement; understanding the notion of accuracy, precision and outlier in data collection, also in connection with the choice of the measurement instrument; table and graph comprehension; explicating assumptions connected to steps in the modeling process; making hypotheses about certain aspects of the phenomenon or model; coping with side effects of numerical methods; and understanding that a series of experiments and data collection by other measurement methods can add to the reliability and validity of evidence of a mathematical model.

The research instruments were an informal interview with the collaborative physics teacher, the students' reports, and the underpinning COACH files.

The Classroom Experiment

Eighteen vwo-5 students (age 16-17 yr.), who had chosen the Nature & Technology profile, participated in this case study in June 2007 as a practical investigation in physics. The students were familiar with video analysis and graphical modeling because they had studied falling objects in this way in the previous school year. Earlier in 2007 they had done graphical modeling in biology lessons in an ecological experiment about permeability of soils. The students had also participated in the mathematics e-class that had numeric approximation of roots as supplementary practical investigation. The students worked in pairs in the physics case study, but they formed not necessarily the same teams as in the e-class. They used the STUDIO MV version of COACH 6 so that they could also work at home with the software environment on video analysis and computer modeling, at a distance from the physics teacher. Due to lack of time at the end of the study year, the students only did the modeling of bouncing balls with no aerodynamics involved (exercises 13 up to and including exercise 24 from the instructional materials). This implies that they did not go repeatedly through the modeling cycle and that I only could compare the quality of their answers to the exercises with the responses that I expected or hoped for. In the final lesson in the study year, the physics teacher discussed aerodynamics effects and more elaborated computer models of the behavior of bouncing balls.

The main objectives of the developed student activities were (cf., Hestenes, 1992, 1996, 2008, 2010):

- to engage secondary school students in modeling activities and give them a feel for what models and modeling mean in the practice of mathematicians and physicists; and
- to focus on basic physics models that make the structure and coherence of scientific knowledge more evident.

There are various views on scientific modeling and how students can develop modeling competencies (cf., Van den Berg, Ellermeijer, & Slooten, 2008). But, as Doerr and Pratt (2008, p. 261) pointed out, the collective epistemological perspective is that

“modeling is the activity of mapping from one system to another. This activity is driven by the need to describe, explain, and/or predict some particular phenomena of interest to the modeler. Elements from the real world of the experienced phenomena are selected, organized, and structured in such a way that they can be mapped onto a model world. This model world necessarily simplifies and distorts some aspects of the real world while maintaining other features and allowing for manipulations of these features (or objects) in accordance with the rules of the model world.”

In other words, the model is separate from the world to be modeled. In this view, a model is a constructed system of objects, relationships, and rules whose behavior resembles that of some other system in the real world of phenomena. The rules of the model world come in the presented modeling activities from mathematics and physics.

Another common epistemological underpinning of scientific modeling is that it is a cyclic or iterative process. Doerr and Pratt (2008, p. 261) brought forward that

“the source of this iteration comes from the attempts at validation in the real world of the outcomes of the manipulations of the objects in the model world. This validation can take several different forms. In some cases, the validity of outcomes is simply measured against the criteria of usefulness for some particular purpose. In other cases, validity is determined by comparison to other models or to other experienced phenomena or to predicted data. The outcomes of the validation process result in either a satisfactory model or generate another cycle of modeling activity.”

The cyclic nature of this modeling paradigm is generally presented in variations of moves between the real world and the model world. The model of Blum and Leiß (2005), shown in Figure 2.24, is one of the most popular models of mathematical modeling in mathematics education research. Other models of mathematical modeling have been proposed by Blomhøj and Jensen (2007), Galbraith and Stillman (2006), Hestenes (2008, 2010), Maaß (2006), and Voskoglou (1994, 2007), to name a few.

Acknowledging that models of modeling are mostly used as theoretical framework for educational research to describe the ideal modeling process, to formulate modeling competencies, to classify modeling task, to discuss obstacles of students during the distinct modeling phases and transitions (cf., Gallbraith and Stillman, 2006; Maaß, 2006, 2010), and that these models do not imply that one necessarily steps during

modeling through each and every phase in a modeling cycle in the given ordering, I developed instructional materials based on the model of Blum and Leiß (2005, p. 1626). Below I present the instructional design based on the ideal-typical seven modeling steps for studying the motion of the bouncing balls.

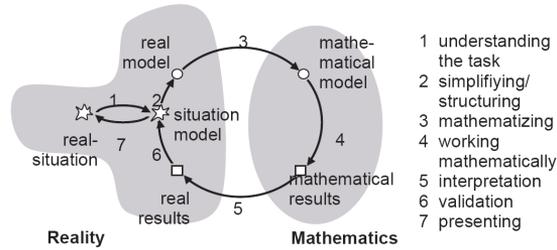


Figure 2.24: The modeling cycle of Blum and Leiß (2005, p. 1626).

The modeling process begins with a reality-based problem: “What is the behavior of a bouncing ball?” When trying to come to grips with this problem situation, one sets up a mental model of the situation, the so-called situational model. This model originates from past experience or by experimenting with balls. Possible driving questions in this step of the modeling cycle are: “What effect does the ground surface have on the bouncing motion?”, “What effect have the choice of material, the size, shape, or temperature of the object on the motion and can this be understood?”, “What effect had the medium in which the ball moves?”, “Can the bounce time, that is, the time it takes for a ball to come to rest, be computed and predicted?”, “How often does a ball bounce?”, and so on. Making sketches of the problem situation often supports the thinking process that confines the problem situation to a manageable situational model: In this case, it is limited to the motion of a hollow ball filled with air, bouncing vertically without spin on a hard, horizontal surface. The ball can be a soccer ball, tennis ball, or a table tennis ball; the surface can be a hard flat floor or table. This can be seen as preparing the second step of conscious simplification, structuring and idealization of the situational model to a real model. Assuming that the motion of a bouncing solid rubber jazz ball dropped from a certain height above a hard ground surface is studied leads to a simpler real model, in which aerodynamic effects can be ignored. Furthermore, a strategic choice of the modeler can be to first study a simpler but related problem; in this case it is the free fall of an object (without aerodynamics). This real model can be more easily mathematized than the real model of a bouncing object.

In the mathematization step, a modeler moves from the real world to the mathematics world. There are different kind of moves possible. For example, in case of a free falling object, one can carry out experimental modeling, that is, data collection via an experiment followed by regression analysis. The adjective ‘experimental’ indicates that the model is not supported by scientific reasoning other than that the mathematical function appears to describe the measured data quite well. In the instructional materials, students are invited to study a free falling object with a video analysis of a recorded experiment and experimental modeling of the motion via regression. Figure 2.25 is a screen shot of the video analysis.

In the lower-left window is a still of the original video clip of the experiment in which a person standing on a ladder lets a ball fall down from a height of 4.5 m.

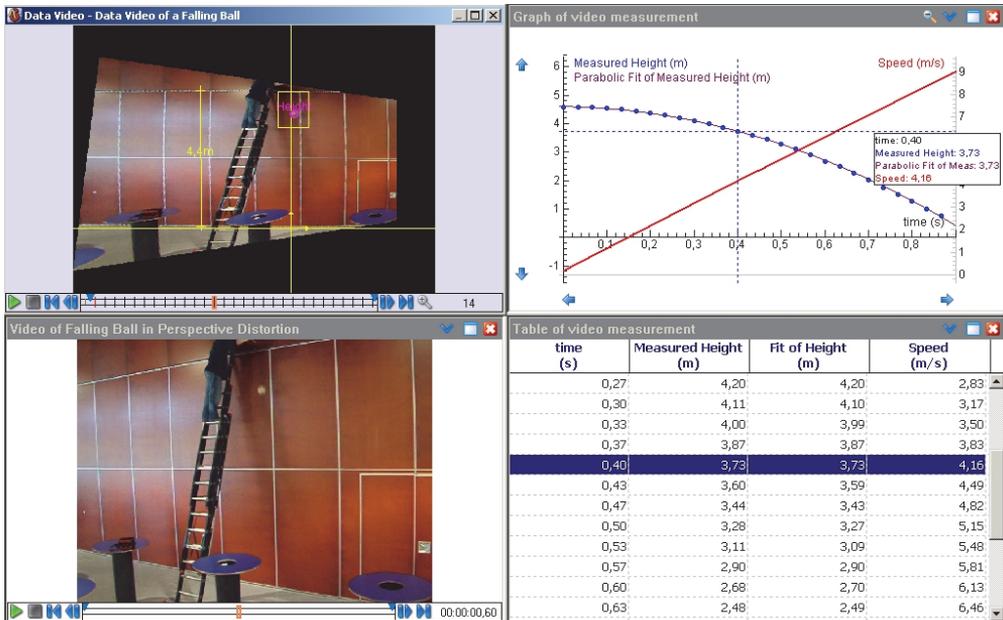


Figure 2.25: Screen shot of a COACH activity with a video analysis of a free falling ball and modeling via a regression curve.

Data collection using this video clip is a bit problematic because of the perspective distortion. To handle this problem, COACH provides a tool to correct the perspective view of an image plane of motion. Details about the software implementation of the perspective correction tool and more inspiring examples of its use can be found in (Heck & Uylings, 2006; see also Section 3.2.2). The upper-left window in the above screen shot shows the result of rectifying the video clip to a fronto-parallel view of the scene. Automated point tracking makes the data collection in the rectified video clip easy (cf., Heck & Ellermeijer, 2009; Heck & Uylings, 2005, 2006, 2010a; Heck & Vonk, 2009; see also Section 3.2.3). The windows on the right-hand side show the tabular and graphical results of measuring the height of the ball with respect to time. In the graph window, the (numerical) derivative of the vertical position has been computed as well: It is a straight line of which the slope is close to the acceleration of gravity. It also motivates a parabolic regression curve to fit the data.

Note that in the above approach, the measured data are not used to validate the mathematical model, but instead are mathematically processed and analyzed so that the mathematical model follows from the data set. This is not unusual in practice. For example, Blomhøj and Jensen (2003) presented an example of a lesson design in which the mathematization of a pharmacological process of clearance of a drug also happens in reverse order: First a suitable regression formula in the form of a sum of two exponential functions is determined for the concentration-time profile and hereafter a two-compartmental model is developed that has the regression formula as its solution. In the instruction sequence of the e-class on quantitative pharmacokinetics, presented in Section 2.7, I sometimes took this approach via the least squares method of peeling-off functions (Foss, 1969).

Experimental modeling via regression is practical, gives excellent results, and may serve as a first run through the cyclic modeling process, but it would be more informative and more satisfactory if one could underpin it with a mathematical model using elementary concepts of kinematics. Here, theory of differential equations and computer modeling come into play. Figure 2.26 is a screen shot of a graphical modeling activity about a free falling object.

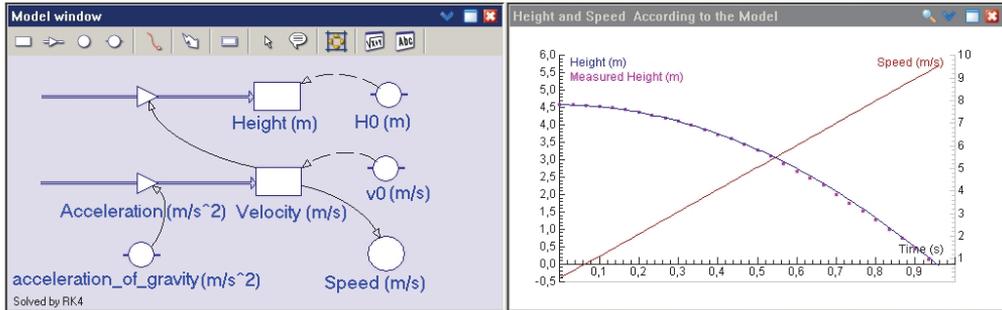


Figure 2.26: Screen shot of a COACH activity with a computer model of a free falling ball and a comparison of modeling results with empirical data.

The graphical model represents a computer program for a system of differential equations that originates from Newton's laws of motion. I refer to Section 2.7 (p. 78) for a brief explanation of the meaning of the common icons in the graphical model. The diagram on the right-hand side right illustrates that the vertical position of the ball computed in the model matches very well with the measured data.

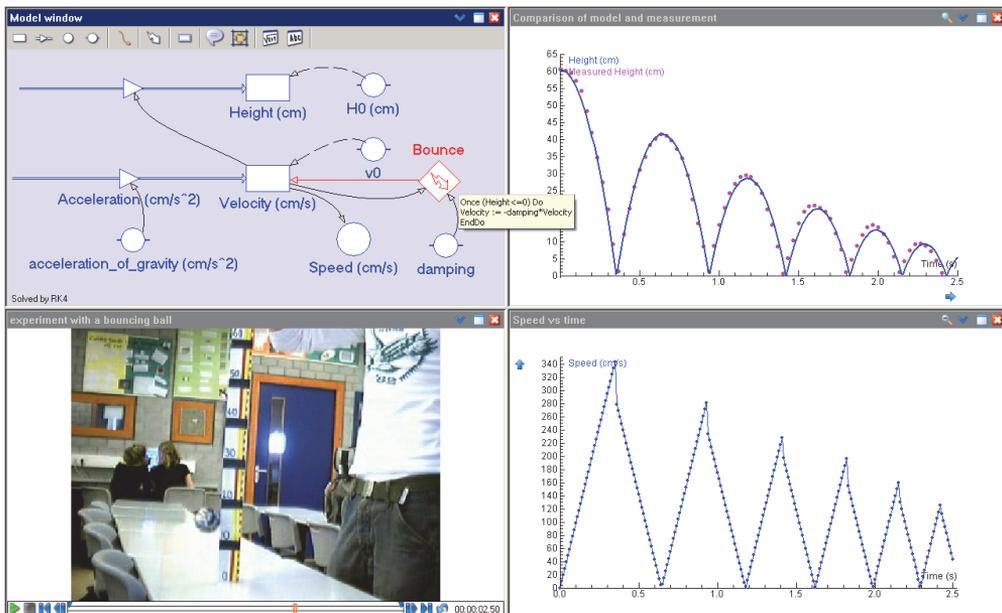


Figure 2.27: Screen shot of a COACH activity with a computer model of a bouncing solid rubber jazz ball and a comparison of modeling results with empirical data.

The instructional sequence continues with the addition of air drag to the mathematical modeling, followed by the discussion of modeling a bouncing jazz ball for which air drag can be ignored. Figure 2.27 illustrates that the graphical model of a bouncing ball compared to the graphical model of a free falling object in Figure 2.26 only differs by the presence of an event icon (the thunderbolt icon labeled ‘Bounce’). At the discrete time event when the ball hits the table one must instantly update the velocity in order to change the direction of motion, that is, to change the velocity. Let us look at the details of this event: It is triggered when the height becomes less than or equal zero. Because of energy loss due to inelastic collision—one notices that the maximum heights of the bouncing ball decrease in time—the downward velocity changes into an upward velocity and its magnitude is decreased by multiplication with a damping factor between 0 and 1. This damping factor is commonly called the coefficient of restitution. The yellow sticker, which pops up when one keeps the cursor for a while on top of the events icon, contains the following computer code: `Once (Height<=0) Do Velocity := -damping*Velocity EndDo`. The choice of the word `Once` in the computer code instead of `If` is important. In the latter case, the condition would be checked each time when the computer program passes this piece of code and it is then possible that the sign of the velocity is forever changing, while the ball position remains negative. In order to avoid such erratic behavior and inconsistencies an event has been implemented in COACH 6 according to the principle of software triggering: As soon as some particular condition is fulfilled, a once-only action is put in running and only at the point that this condition is not satisfied anymore, then the event can occur anew. In this case study I explored whether students could easily understand the event-based modeling.

The graph window in the upper-right corner of the screen shot in Figure 2.27 contains the motion graph computed via the computer model and the measured data points at the background. It shows that the model graph and the measured data match well for the particular type of ball that was used in the experiment (a solid rubber jazz ball, also known as a super ball). For many a teacher and student this is often the point to stop work and continue with another topic. This is a pity, because what does one learn then about the motion of a bouncing ball in general? Can one predict the bounce time, given the initial position and speed of the ball as well as the coefficient of restitution? Do the computed bouncing time and measured bouncing time match? In what sense does bouncing on the moon differ from bouncing on earth? I am of opinion that such questions need to be addressed, too. Mathematics and physics go hand in hand when one attacks such questions. What is more, when one would investigate bouncing table tennis balls through experiments and mathematical modeling, then one would quickly find that empirical results and modeling results differ for simple mathematical models. See, for example, Figure 2.28.

In the rest of the instructional sequence, students gradually study more advanced models of bouncing balls. In increasing order of model complexity, the activities concern:

- The simple model (without air drag) of the motion of a bouncing table tennis ball on the basis of sound registration of bounces [When the flight time between consecutive bounces is plotted against the time of a bounce, a linear data plot is expected (cf., Aguiar & Laudares, 2003; Bernstein, 1977; Stensgaard & Lægsgaard, 2001).].

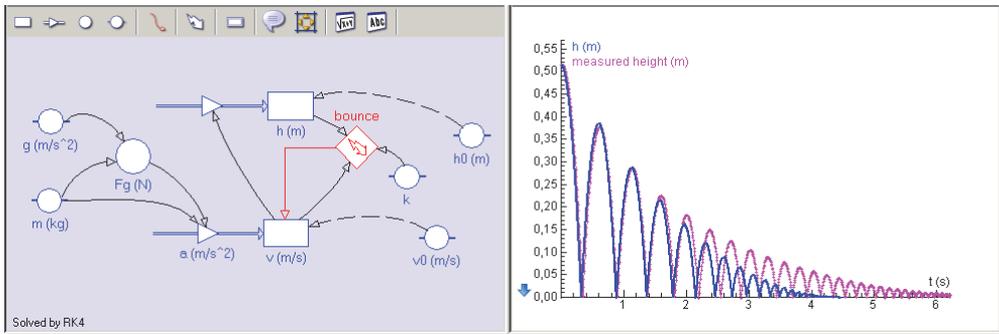


Figure 2.28: A comparison of a model of a bouncing table tennis ball (neglecting air resistance) with measured data (obtained via high speed video recording).

- the simple model (without air drag) of a bouncing table tennis ball on the basis of a high speed video clip.
- A model with air drag included for a bouncing table tennis ball.
- A model with air drag included, but now with a non-constant coefficient of restitution for a bouncing table tennis ball (cf., Hubbard & Stronge, 2001) .

The final activity results in a good match between model and experiment (Figure 2.29).

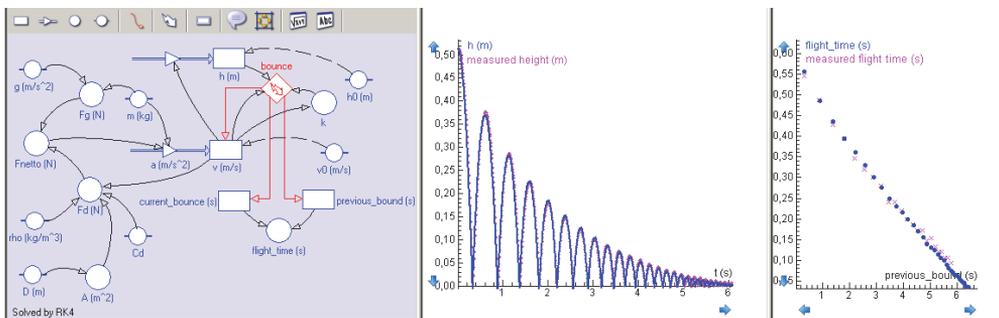


Figure 2.29: A comparison of a model of a bouncing table tennis ball, which includes air resistance and a non-constant coefficient of restitution, with empirical data.

All tasks and exercises in the instructional materials have been designed to promote successful modeling by students. For each step of the modeling cycle, students carry out relevant tasks and exercises that guide them through a particular modeling step. Two examples of such tasks in the structuring step toward a real model are:

Exercise 6 (motion of a falling ball)

- What forces act on the ball as soon as ball drops from the hand of the experimenter?
- Which force do you think plays the most important role in the computation of the height-time curve?
- Mention some factors that could influence the motion of the ball.

Exercise 18, first two subtasks (motion of a bouncing table tennis ball)

When you listen to the sound that a bouncing table tennis ball make, you hear that the time interval between two consecutive ground contacts gets shorter.

- a) What does this shortening of time interval between consecutive bounces mean for the heights that the ball reaches?
- b) What could cause this shortening of time?

Such tasks and exercises are expected to help students come to grips with the real world problem situation, transform it into a manageable real model, move to a mathematical model, solve this mathematical model, interpret and validate the solution. This is the same approach of guiding students through the modeling cycle that was applied in the instructional sequence about quantitative pharmacology, discussed in the Section 2.7. Here, the underlying idea is that students must first gain sufficient experience with the steps in the modeling cycle before they can independently carry out scientific modeling (e.g., in a profile research project). Students are first helped to overcome possible blockages in steps of the modeling cycle. In the above example, the students are explicitly asked to identify relevant factors and make simplifying assumptions. Like Galbraith and Stillman (2006), I used the modeling cycle to identify possible student blockages whilst undertaking modeling tasks during transitions in the modeling process and to design tasks and exercises that help students overcome these blockages. In contrast with the approach taken by Maaß (2006, 2010) and Blomhøj and Jensen (2007), less focus is in the students activities on the development of modeling competencies based on single steps of the modeling process and on metacognitive and reasoning competencies that do not belong to a specific modeling step but are needed throughout the whole modeling process. In the latter approach, tasks in single modeling steps would be directed and rephrased toward modeling competency: “making assumptions” would become “being able to make assumptions” and “simplifying a problem” would be transcribed into “being able to simplify a problem”. My choice for focussing on activities that lead to successful scientific modeling also explains why I consider a good match between modeling results and empirical results very important and why this is stressed in the validation of the mathematical model. Without experimentation and video recording of the motion of the table tennis ball with a high speed camera, and without careful validation of the intermediate models one would never go so many times through the modeling cycle until one arrives at a good description of the bouncing table tennis ball. In my opinion, students should not only understand the modeling process, but also be educated to strive for models of good quality, which describe empirical data, have predictive power, and lead to sensible explanations or better understanding of real phenomena.

Results and Findings

The first research question “To what extent is the idealized modeling route in the instruction materials practicable for students?” could only be answered on the basis of partial information because the students who participated in this case study only did the modeling of bouncing balls with no aerodynamics involved (from Exercise 13 up to and including Exercise 24 in the instructional materials; see Table 2.2). However, they had studied the motion of a free-falling object in the previous study year with the same teacher and in accordance with the first part of the instructional materials, that is, using video analysis and graphical modeling. This is in fact the envisioned way of

using the instructional material: splitting the modeling of the motion of bouncing balls into separate smaller case studies and treating these studies across several school years. The teacher also used COACH activities from the instructional materials for discussion in the classroom of aerodynamic effects on the motion of a bouncing ball. From the teacher's interview and the students' reports I concluded that the idealized route seems practicable for students who already have sufficient physics background knowledge and have already sufficiently practiced video analysis and computer modeling.

<i>Task</i>	<i>Description/Goal</i>
<i>3.2 Experiments and measurements</i>	
13	Video analysis of a bouncing ball on the basis of a given video clip recorded by a student with a webcam. Timings of ground contact and determination of time intervals between bounces. Judgment of the accuracy of the measured data.
14	Simultaneous video- and sound recording of a vertically bouncing table tennis ball with a webcam and a sound sensor. Timings of ground contact, determination of time intervals between bounces and estimation of the bounce time from sound data. Judgment of the accuracy of the measured data.
15	Video analysis of a vertically bouncing table tennis ball. Timings of ground contact, determination of time intervals between bounces and estimation of the bounce time from the video clip. Comparison of results of video analysis and sound analysis.
16	Video analysis of a vertically bouncing table tennis ball on the basis of a given video clip recorded with a high speed camera. Construction of height-time and velocity-time graphs and determination of times of ground contact, length of time between consecutive bounces, and bounce time. Comparison of results from video measurements in movies recorded at different frame rates.
<i>3.3 A simple mathematical model of a bouncing ball</i>	
17	Characterization of height-time graphs in term of the bouncing process: ideal and frictionless collision, inelastic collision, and real bouncing.
18	Qualitative analysis of a table tennis ball that bounces vertically on a table. Explanation of energy loss.
19	Energy loss for an inelastic collision when air resistance is ignored. Experimental determination of the energy loss via a video measurement.
20	Analysis of the velocity-time of a vertically bouncing ball.
21	Text-based computer modeling of a bouncing ball.
22	Graphical modeling of a bouncing ball and the Runge-Kutta method. Comparison of the computer modeling results with empirical results.
<i>3.4 Modeling with mathematical formulas</i>	
23	Proof of mathematical formulas for the coefficient of restitution and bounce time of an ideal bouncing ball without friction. Application of the formulas in case of measured data.
24	Mathematical analysis of a sound registration of a bouncing table tennis ball.

Table 2.2: Overview of the activities from the instructional materials that were carried out by the vwo-5 students participating in this case study.

As I already mentioned before, the second question “Do the instructional materials and the chosen instructional setting enable students to investigate bouncing balls at a good level of quality?” was only answered by retrospective inspection of the students' reports and their underpinning computer files. In cases where students had not been able to solve a problem or carry out a task, they had been invited to write down what they had thought about or what they had attempted, so that at least an impression of their ideas and work was possible. When students' work differed from my own ideal-

ized answers and results, I tried to infer from the language use of the students in their reports and from the mathematical representations (graphs and tables) that they had used what differences actually occurred and whether the source lay in the students' lack of mathematics or physics knowledge and understanding, in possible weaknesses in modeling competencies, or in lacking abilities in working with the computer learning environment. The idealized answer or result was not just a correct answer to a question, but it meant in some cases more that students wrote sensible statements about the accuracy of experimental data and could make suggestions for improvement of experiments. I present in the below paragraphs my findings and follow the ordering of the activities in the instructional materials, as tabulated in Table 2.2.

The first task of analyzing a video clip (*Exercise 13*) caused no serious problems for the students: They apparently had mastered the instrumental skills for using the video analysis tool. Maybe because the students were not fully aware that it would be convenient for comparison of measured data with modeling results to set time $t = 0$ at the frame in which the girl in the video clip drops the ball, no student team calibrated time in this way. Because the computer learning environment offers the opportunity to translate background graphs horizontally one can afterwards match the time scales of the measurement and the computer model. However, the students' work could be a sign that they just started a video measurement without giving much thought about time calibration. For the rest, the students' work was of good quality. For example, they made sensible statements about the accuracy of the measurements and the experimental setting: They identified the rather low frame rate of the video clip as the most important cause of imprecise timings of bounces and of spreading in calculated time intervals between two bounces. The following quote illustrates this: "The time interval that the ball is in contact with the ground is very short. So it is in a video measurement very difficult to find the bounce point, since the video clip is a sequence of pictures. The more frames per second recorded, the better the measurement results will be." This statement illustrates that the students were well aware of the difference between time in the real world and time in the movies. Whereas time is in the real world a continuous, irreversible quantity that cannot be fixated to a particular value, time is in a video clip a notion open to interpretation (Boyd & Rubin, 1996). In a video clip, time is a discontinuous quantity, which is only experienced as continuous and with which one can manipulate: One can play a movie faster or slower, step frame by frame through a video clip, play a movie in forward and backward direction or pause it (for example to have a close look or to do a measurement or calibration). Time is controllable in a video clip and all students know this through the various technologies that they use in their daily lives.

In *Exercise 14*, all students were able to mention the two most important causes of shortening of time intervals between bounces: air resistance and energy loss during the bounce due to deformation of the ball. Because of the closed character of the question, students gave no further motivation of their answers. But students undoubtedly had alternative conceptions of the bouncing process. For example, it seemed not clear to every student that the start of the sound peak indicated the start of ground contact. The following statement illustrates this: "I think that the accuracy of a sound measurement may be better than that of a video measurement, because in case of noise you cannot really 'miss' anything. But it is so that you have a long moment (some ms) of sound, you must guess a bit where the bounce took place (average or something

alike).” This lack of full understanding of the data collection may be caused because the students did not really do the experiment themselves, but instead only replayed and used a pre-recorded synchronized video and sound measurement. If they had in reality carried out the experiment, they might have had a better understanding of the experimental setting. Laboratory experience seems indispensable in science education. It may also explain why one student team wrote: “Video measurement is in this case more accurate, because we cannot determine whether the ball bounces straight when we only have a sound fragment.” This team wanted evidence of the ‘correct’ motion of the bouncing ball and apparently did not realize or had forgotten that video and sound recording were synchronized in the COACH activity. The main reasons for this are in my opinion that secondary school students are in general not provided with many opportunities to familiarize with synchronous recordings in an experimental setting and often do not benefit optimally of the video scrubbing technique for finding information in video clips.

The next two tasks (*Exercise 15* and *16*) are related with the previous one and therefore the findings are the same. Most students indeed considered a video measurement in a video clip with 30 frames per second less accurate than a sound measurement with a higher sampling rate. They also noted that data collection by manual point-clicking in a video clip may lead to noisy signals because one may not always select the same spot on the ball. Questions about accuracy apparently triggered students to think about the data collection process. The almost full responsibility of preparing the video measurement (e.g., making a convenient choice of coordinate system, calibration of distance, and so on) did not cause problems and in the instructional sequence I had already taken care of successful point tracking settings (Note: it is a great advantage of an activity-based computer learning system that an author or teacher can decide upon what to prepare in advance and what to leave to his or her students).

Although the responses to *Exercise 17* indicated that some students were deceived by the meaning of word elastic in common language, most students were able to link the given three graphs to the ideal and friction-less collision, inelastic collision, and real bouncing process. Some students were from physics point of view very accurate in the motivation of their answer, illustrating that they not only possessed physics knowledge, but also knew how to apply it in concrete cases.

The students gave in *Exercise 18* good arguments for the shortening of the time intervals between bounces. The fact that in the video clip recorded at a frame rate of 250 fps it is really visible that the speed of the ball before the bounce is greater than after the bounce brought the students to the idea that loss of energy during the bounce is a more important factor in the bouncing motion of the ball than air resistance while the ball is in the air. Two student teams also showed good understanding of how to find evidence for this. One of these teams wrote: “Because this is a measurement with so many frames per second, it is possible to do an accurate measurement. You can compute in a diagram the derivative of height and hereafter once more take the derivative so that you get acceleration, yes one has thought about this. You can see from the acceleration whether a ball is more slowed down by air friction or by a bounce.” The other team also revealed in their response their understanding that one can use a calculated variable or graphical representation as a means to distinguish between the importance of factors on the motion of the ball.

The focus of *Exercise 19* was on the energy loss in an inelastic bounce of a ball and the students were asked to compute the percentage of loss of kinetic energy at the bounce of a table tennis ball via a video analysis of a movie recorded at a frame rate of 250 fps. I had in mind that the student would do a line fit of the data shortly before and after the bounce, as shown in Figure 2.30. The slopes of the line fits would then give values for the velocity of the ball immediately before and after the bounce.

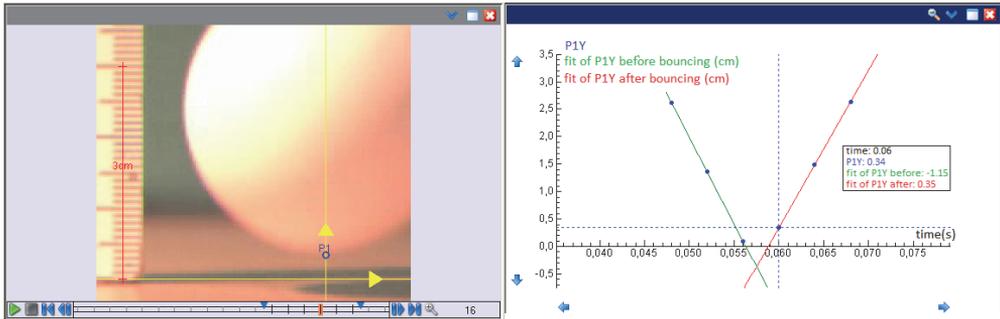


Figure 2.30: Video analysis of a bouncing table tennis ball via a movie recorded at a frame rate of 250 fps.

However, the students gave several incorrect answers and used incorrect formulas for the percentage of loss of kinetic energy. The main cause of the mistakes in velocity estimations from the video clip was that many a student assumed that the ball bounced when the ball was at its lowest point in the video measurement and then they used the neighboring points to estimate the velocities of the ball. Linear regression, as shown in the above figure, makes this implausible. As a consequence of their approach, students underestimated the speed after the bounce. Surprisingly, another problem was that students used wrong formulas such as:

$$\begin{aligned} \text{energy loss} &= \text{speed}_{\text{after}}/\text{speed}_{\text{before}} \times 100, \\ \text{energy loss} &= \text{speed}_{\text{after}}^2/\text{speed}_{\text{before}}^2 \times 100, \\ \text{energy loss} &= \text{speed}_{\text{after}}/\text{speed}_{\text{before}} \times 100 - 100. \end{aligned}$$

This may indicate that students had serious weaknesses in algebraic thinking and algebraic manipulation.

The analysis of the velocity-time graph in *Exercise 20* generally caused no problem, but sometimes the wording of the students was a bit clumsy. Sketching the kinetic energy profile revealed some alternative conceptions such as the assumption of linear increase and decrease of kinetic energy in time interval between two bounces.

In *Exercise 21*, students were asked to adapt a text-based computer program for a free-falling ball to the case of a bouncing ball and they managed to do this without serious problems. Some students could even give a good explanation for the problem with the numerical method that after many bounces, when the height of the ball in the computer model gets small, suddenly it becomes negative and drifts away (See the screen shot of student work in Figure 2.31). An example of a reasonable explanation from a student team: “The model approximates the situation in steps. Therefore it happens sometimes that the ball only bounces under $h = 0$. When the ball after that does not get above $h = 0$ after the bounce, then it will accelerate in the downward

direction, which explains the sudden decrease at the end of the graphs.” Assuming that these students did not get help from a more knowledgeable person, their report showed that they knew what they had been doing in the computer model. From the outcomes of *Exercise 22*, in which students had to do use the graphical mode of the modeling tool in COACH and had to switch from Euler’s numerical method of solving differential equations to a Runge-Kutta method, I came to the same conclusion that most students knew what they had been doing in the modeling activity.

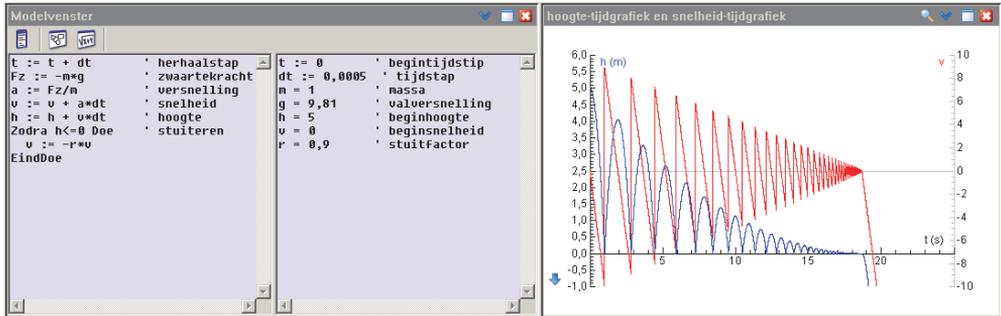


Figure 2.31: A text-based model of a bouncing ball written by a student team, which illustrates numerical problems after a while in the computer simulation.

The outcomes of the last two tasks (*Exercise 23* and *24*) made clear that the students could work well with given algebraic formulas and use empirical data to do calculation based on the given formulas. But proving the correctness of the mathematical formula for the bounce time was beyond their ability level. I guess that I demanded too much from the students with regards to their level of algebraic skills.

In summary, I concluded from the students’ reports and their computer files that the quality of the students’ work matched in general well with the intended high quality outcome from the author’s point of view. The tasks and exercises were practicable: computer modeling went without much trouble or blockages and the students’ answers gave the impression that they knew what they had been doing in the modeling activities. Use of given mathematical formulas in a physics context worked fine, but independent derivation of formulas and the requested algebraic manipulation turned out to be beyond the algebraic skills level of the majority of the students

Extensions

An interesting and challenging problem situation for students is the bouncing ball on an oscillating platform that introduces students also into the concept of chaos (cf., Verhulst, 2009, ch. 6; Cristina Vargas, Huerta, & Sosa, 2009). An animation linked to the computer model can visualize the chaotic motion in an appealing way, which may even help students better understand the behavior of this system. See Figure 2.32, in which modeling results are shown for a ball bouncing on a platform that oscillates as a sine function with amplitude $y_{\text{platform}} = -A \varepsilon \sin(t/\sqrt{\varepsilon})$, where A and ε are positive numbers and ε is small. Figure 2.32 illustrates that for $A = 5$ and $\varepsilon = 0.001$ the ball keeps bouncing periodically, and that for a very small change of the parameter value into $\varepsilon = 0.00099$ the ball lies after some time simply on the oscillating platform.

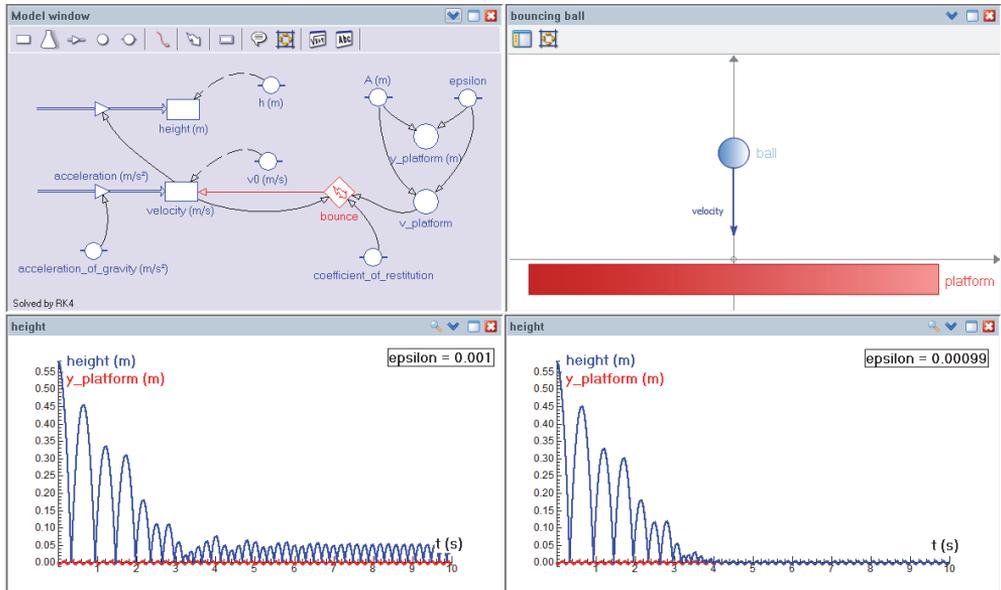


Figure 2.32: Modeling and animation of a bouncing ball on an oscillating platform. The diagrams illustrate different behavior for a very small change in the value of the parameter ε .

Students could also investigate the influence of temperature, ball type and ground surface on the bouncing motion. Alternatively, they could investigate two-dimensional bouncing and include exploration of the effects of spin on the motion of the ball. The interested reader is referred to the many articles in physics education journals such as *The American Journal of Physics* (e.g., Cross, 1999a, 2002a,b, 2010a; Lewis, Arnold, & Griffiths, 2011) and *The Physics Teacher* (e.g., Brody, 1984, 1990; Cross, 2010).

Another interesting approach to modeling bouncing balls that avoids the use of discrete time events has been worked out by Bridge (1998): He assumed that the ball during impact phase can be considered as an ideal spring following Hooke's law. This model can be implemented in COACH and gives a splendid match between model and measurement, but it may be beyond the students' ability level. A computer simulation of this advanced model allows an estimation of the impact time: It is about 1 millisecond and this is in good agreement with empirical results of a high speed camera recording (Hubbard & Stronge, 2001).

Alternatively one could implement the spring-mass-damper model of a bouncing ball, worked out by Nagurka and Huang (2006), to account for energy losses. A contact time in the order of milliseconds would also be predicted from measurements of the time it takes for a table tennis ball to come to rest when dropped from a certain height.