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Vertical relations in cartel theory: managerial incentives, buyer groups & antitrust damages

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Appendices

Appendix A

This appendix contains the proofs of Chapter 2.

A.1 Proofs of Section 2.4

A.1.1 Proofs of Lemmas 2.1 and 2.2

We determine the expected transfers by solving for the schedule of transfers associated with the optimal contract that induces or prevents breaches, respectively.

Contract inducing breaches. If the shareholder wants to induce breaches of the law, then the optimal contract, given action $i \in \{N, C, R\}$, is defined as the solution of

$$\max_{t_{\pi,\sigma}} \left\{ \rho_{\pi} - \sum_{\pi=0}^1 \sum_{\sigma=0}^1 p_{\pi,\sigma}^{b,i} t_{\pi,\sigma} - E_i [F] \right\}, \quad \text{subject to}$$

$$t_{\pi,\sigma} \geq 0, \quad \forall \{\pi, \sigma\}, \quad (LL_b)$$

$$\sum_{\pi=0}^1 \sum_{\sigma=0}^1 p_{\pi,\sigma}^{b,i} t_{\pi,\sigma} - E_i [f] + G \geq 0, \quad (PC_b)$$

$$\sum_{\pi=0}^1 \sum_{\sigma=0}^1 \left(p_{\pi,\sigma}^{b,i} - p_{\pi,\sigma}^{n,i} \right) t_{\pi,\sigma} - E_i [f] + G \geq 0. \quad (IC_b)$$

By limited liability (LL_b), the participation constraint (PC_b) is satisfied whenever the incentive compatibility constraint (IC_b) is satisfied. Now, if $E_i [f] \leq G$, then (IC_b) is satisfied by setting $t_{\pi,\sigma} = 0$, $\forall \{\pi, \sigma\}$, and, thus, $E_i [t^b] = 0$ for $i \in \{N, C, R\}$.

If $E_i [f] > G$, the cheapest way to satisfy (IC_b) is to make a positive transfer only in

- (i) state $\{\pi, \sigma\} = \{1, 0\}$ if $i = N$ (there is no CP, so evidence never comes available and profit realization $\pi = 1$ is most informative of a breach having occurred), and

- (ii) any state in which $\sigma = 1$ if $i \in \{C, R\}$ (signal $\sigma = 1$ is a sufficient statistic), for example, state $\{\pi, \sigma\} = \{1, 1\}$.

Consider first the case in which $i = N$. Then, setting $t_{\pi, \sigma} = 0$ for every state of the world $\{\pi, \sigma\} \neq \{1, 0\}$, while setting t_{10} to bind the incentive compatibility constraint, gives $t_{10} = \frac{E_N[f] - G}{\rho_\pi - (1 - \rho_\pi)}$, which is paid out with probability ρ_π in equilibrium and, thus, $E_N[t^b] = \gamma^N (\beta f^N - G)$, where $\gamma^N = \frac{\rho_\pi}{2\rho_\pi - 1}$.

Consider now the cases in which $i \in \{C, R\}$. Then, setting $t_{\pi, \sigma} = 0$ for every state of the world $\{\pi, \sigma\} \neq \{1, 1\}$, while setting t_{11} to bind the incentive compatibility constraint, gives $t_{11} = \frac{E_i[f] - G}{\rho_\pi \rho_\sigma}$, which is paid out with probability $\rho_\pi \rho_\sigma$ in equilibrium and, thus, $E_i[t^b] = E_i[f] - G$ if $i \in \{C, R\}$.

Contract preventing breaches. If the shareholder wants to prevent breaches of the law by action $i \in \{N, C, R\}$, the optimal employment contract is obtained by solving

$$\begin{aligned} \max_{t_{\pi, \sigma}} \left\{ 1 - \rho_\pi - \sum_{\pi=0}^1 \sum_{\sigma=0}^1 p_{\pi, \sigma}^{n, i} t_{\pi, \sigma} \right\}, \quad & \text{subject to} \\ t_{\pi, \sigma} \geq 0, \quad & \forall \pi, \sigma, \quad (LL_n) \\ \sum_{\pi=0}^1 \sum_{\sigma=0}^1 p_{\pi, \sigma}^{n, i} t_{\pi, \sigma} \geq 0, \quad & (PC_n) \\ \sum_{\pi=0}^1 \sum_{\sigma=0}^1 \left(p_{\pi, \sigma}^{n, i} - p_{\pi, \sigma}^{b, i} \right) t_{\pi, \sigma} + E_i[f] - G \geq 0. \quad & (IC_n) \end{aligned}$$

Again, by limited liability (LL_n), the participation constraint (PC_n) is satisfied whenever the incentive compatibility constraint (IC_n) is satisfied. Now, if $E_i[f] > G$, then (IC_n) is satisfied by setting $t_{\pi, \sigma} = 0, \forall \{\pi, \sigma\}$, and, thus, $E_i[t^n] = 0$ for $i \in \{N, C, R\}$.

If $E_i[f] \leq G$, the cheapest way to satisfy (IC_n) is to make a positive transfer only in the state of the world that is most informative about the law not having been breached, i.e., state $\{\pi, \sigma\} = \{0, 0\}$. Setting $t_{\pi, \sigma} = 0$ for every state of the world $\{\pi, \sigma\} \neq \{0, 0\}$, while setting t_{00} to bind the incentive compatibility constraint, gives

- (i) $t_{00} = \frac{G - E_N[f]}{\rho_\pi - (1 - \rho_\pi)}$ if $i = N$, which is paid out with probability ρ_π in equilibrium and, thus, $E_N[t^n] = \gamma^N (G - \beta f^N)$, and
- (ii) $t_{00} = \frac{G - E_i[f]}{\rho_\pi - (1 - \rho_\pi)(1 - \rho_\sigma)}$ if $i \in \{C, R\}$, which is paid out with probability ρ_π in equilibrium and, thus, $E_i[t^n] = \gamma^C (G - E_i[f])$ if $i \in \{C, R\}$, where $\gamma^C = \frac{\rho_\pi}{\rho_\pi - (1 - \rho_\pi)(1 - \rho_\sigma)}$.

Combining the results. Thus, if $i = N$, we have

$$E_N [t^b] = \begin{cases} 0 & \text{if } \beta f^N \leq G \\ \gamma^N (\beta f^N - G) & \text{if } \beta f^N > G, \end{cases}$$

$$E_N [t^n] = \begin{cases} \gamma^N (G - \beta f^N) & \text{if } \beta f^N \leq G \\ 0 & \text{if } \beta f^N > G, \end{cases}$$

while if $i \in \{C, R\}$, we have

$$E_i [t^b] = \begin{cases} 0 & \text{if } E_i [f] \leq G \\ E_i [f] - G & \text{if } E_i [f] > G, \end{cases}$$

$$E_i [t^n] = \begin{cases} \gamma^C (G - E_i [f]) & \text{if } E_i [f] \leq G \\ 0 & \text{if } E_i [f] > G, \end{cases}$$

which boils down to Lemma 2.1 and 2.2, respectively. \square

A.1.2 Proof of Proposition 2.1

By Lemmas 2.1 and 2.2, for any $i \in \{N, C, R\}$, increasing f^i weakly increases $E_i [t^b]$ and weakly decreases $E_i [t^n]$, thereby weakly relaxing constraint (2.5). Thus, the authority optimally sets all managerial fines as high as possible, i.e.,

$$f^N = f^C = f^R = \bar{f}.$$

Increasing F^N and F^C also weakly relaxes constraint (2.5). The authority optimally sets

$$F^N = F^C = \bar{F}.$$

We derive the authority's optimal choice of F^R . Noting that $\rho_\sigma f^R + (1 - \rho_\sigma) \beta f^C > \beta f^C$ (because $f^C = f^R = \bar{f}$), we have $E_R [t^b] > E_C [t^b]$ and $E_R [t^n] < E_C [t^n]$ by Lemma 2.2.

Shareholder prevents breach. Suppose the shareholder adopts a CP and prevents a breach. Since $E_R [t^n] \leq E_C [t^n]$, the shareholder pays a lower salary if she can credibly commit to blow the whistle whenever she finds evidence. Such a commitment also relaxes constraint (2.5) and is *ex post* credible if and only if the authority sets $F^R \leq \beta F^C$, because the shareholder then pays a lower fine if she reports (F^R) than if she does not report (βF^C).

Shareholder induces breach. Suppose now the shareholder adopts a CP and induces a breach. If the authority sets $F^R > \beta F^C$, the shareholder will not report evidence when she finds it. If instead $F^R < \beta F^C$, the shareholder cannot help but report evidence whenever she finds it. Finally, if the authority sets $F^R = \beta F^C$, the shareholder is *ex post* indifferent between reporting evidence or not. However, *ex ante* she prefers to commit to not reporting evidence, because that reduces her expected transfer since $E_C [t^b] < E_R [t^b]$. Thus, her expected payoff is

$$\Pi_i^b = \begin{cases} \Pi_R^b = \rho_\pi - E_R [t^b] - \rho_\sigma F^R - (1 - \rho_\sigma) \beta F^C & \text{if } F^R < \beta F^C \\ \Pi_C^b = \rho_\pi - E_C [t^b] - \beta F^C & \text{if } F^R = \beta F^C \\ \Pi_C^b = \rho_\pi - E_C [t^b] - \beta F^C & \text{if } F^R > \beta F^C. \end{cases}$$

Now, decreasing $F^R = \beta F^C$ to $F^R < \beta F^C$ entails (i) a discrete downward jump from Π_C^b to Π_R^b , because $E_R [t^b] > E_C [t^b]$ and $\rho_\sigma F^R + (1 - \rho_\sigma) \beta F^C = \beta F^C$ if $F^R = \beta F^C$, while (ii) continuously increasing Π_R^b , because Π_R^b increases as becomes F^R smaller. Thus, the authority optimally sets F^R slightly under βF^C so as to “impose” the discrete downward jump on the shareholder with a minimal effect of the continuous increase. Therefore, $F^R = \beta F^C - |\epsilon|$, where ϵ is arbitrarily small. \square

A.1.3 Proof of Proposition 2.2

The proof consists of three steps: (i) we derive the optimal investigation probability β^* when CPs are available; (ii) we derive the optimal investigation probability $\tilde{\beta}^*$ when CPs are not available; and (iii) we compare the relative sizes of β^* and $\tilde{\beta}^*$.

A.1.3.1 Optimal Investigation Probability When CPs Are Available (β^*)

Lemma A.1 *The optimal investigation probability β^* , as a function of \bar{F} and \bar{f} , is*

$$\text{If } \bar{f} \in [0, G]: \beta^* = \begin{cases} \emptyset & \text{if } \bar{F} < F_0 \\ \frac{2\rho_\pi - 1 + \gamma^C (G - \rho_\sigma \bar{f})}{\gamma^C (1 - \rho_\sigma) \bar{f} + \bar{F}} & \text{if } \bar{F} \geq F_0, \end{cases} \quad (\text{A.1})$$

$$\text{If } \bar{f} \in \left(G, \frac{G}{\rho_\sigma}\right): \beta^* = \begin{cases} \emptyset & \text{if } \bar{F} < F_1 \\ \frac{2\rho_\pi - 1 + G - \rho_\sigma \bar{f}}{(1 - \rho_\sigma) \bar{f} + \bar{F}} & \text{if } F_1 \leq \bar{F} < F_2 \\ \frac{2\rho_\pi - 1 + \gamma^N G}{\gamma^N \bar{f} + \bar{F}} & \text{if } F_2 \leq \bar{F} < F_3 \\ \frac{2\rho_\pi - 1}{\bar{F}} & \text{if } F_3 \leq \bar{F} < F_4 \\ \frac{2\rho_\pi - 1 + \gamma^C (G - \rho_\sigma \bar{f})}{\gamma^C (1 - \rho_\sigma) \bar{f} + \bar{F}} & \text{if } \bar{F} \geq F_4, \end{cases} \quad (\text{A.2})$$

$$\text{If } \bar{f} \in \left[\frac{G}{\rho_\sigma}, \infty\right): \beta^* = \begin{cases} \emptyset & \text{if } \bar{F} < F_1 \\ \frac{2\rho_\pi - 1 + G - \rho_\sigma \bar{f}}{(1 - \rho_\sigma) \bar{f} + \bar{F}} & \text{if } F_1 \leq \bar{F} < F_2 \\ \frac{2\rho_\pi - 1 + \gamma^N G}{\gamma^N \bar{f} + \bar{F}} & \text{if } F_2 \leq \bar{F} < F_3 \\ \frac{2\rho_\pi - 1}{\bar{F}} & \text{if } \bar{F} \geq F_3, \end{cases} \quad (\text{A.3})$$

where $F_0 = 2\rho_\pi - 1 + \gamma^C (G - \bar{f})$, $F_1 = (2\rho_\pi - 1)\pi + G - \bar{f}$, $F_3 = \frac{(2\rho_\pi - 1)\bar{f}}{G}$,
 $F_2 = \frac{((2\rho_\pi - 1)\pi + \gamma^N G)(\gamma^N - (1 - \rho_\sigma))\bar{f} - (\rho_\sigma \bar{f} + (\gamma^N - 1)G)\gamma^N \bar{f}}{\rho_\sigma \bar{f} + (\gamma^N - 1)G}$, and $F_4 = \frac{(2\rho_\pi - 1)(1 - \rho_\sigma)\bar{f}}{G - \rho_\sigma \bar{f}}$.

Proof. By Corollary 2.2, if the shareholder prevents a breach, she adopts a CP and the expected transfer is

$$E_R[t^n] = \max \left\{ \gamma^C (G - \rho_\sigma \bar{f} - (1 - \rho_\sigma) \beta \bar{f}), 0 \right\}. \quad (\text{A.4})$$

Conversely, if the shareholder induces a breach, she may or may not adopt a CP, resulting

in expected transfer, respectively,

$$E_R [t^b] = \max \{ \rho_\sigma \bar{f} + (1 - \rho_\sigma) \beta \bar{f} - G, 0 \}, \tag{A.5}$$

$$E_N [t^b] = \max \{ \gamma^N (\beta \bar{f} - G), 0 \}. \tag{A.6}$$

The authority minimizes β subject to constraint (2.5), which then simplifies to

$$1 - \rho_\pi - E_R [t^n] \geq \rho_\pi - \min \{ E_N [t^b] + \beta \bar{F}, E_R [t^b] + \beta \bar{F} - \rho_\sigma |\epsilon| \},$$

where we neglect ϵ for notational convenience, yielding

$$1 - \rho_\pi - E_R [t^n] \geq \rho_\pi - \min \{ E_N [t^b], E_R [t^b] \} - \beta \bar{F}. \tag{A.7}$$

The expressions for the expected transfers $E_R [t^n]$ and $\min \{ E_N [t^b], E_R [t^b] \}$ depend on how managerial benefit G compares to the expected managerial fine—see (A.4), (A.5), and (A.6). These expressions are stated in the following table as a function of β .

INVESTIGATION PROB. β	PREVENTING BREACH: $E_R [t^n]$	INDUCING BREACH: $\min \{ E_N [t^b], E_R [t^b] \}$
$\beta \in \left[0, \frac{G - \rho_\sigma \bar{f}}{(1 - \rho_\sigma) \bar{f}} \right)$	$\gamma^C (G - \rho_\sigma \bar{f} - (1 - \rho_\sigma) \beta \bar{f})$	$E_N [t^b] = E_R [t^b] = 0$
$\beta \in \left[\frac{G - \rho_\sigma \bar{f}}{(1 - \rho_\sigma) \bar{f}}, \frac{G}{\bar{f}} \right)$	0	$E_N [t^b] = 0$
$\beta \in \left[\frac{G}{\bar{f}}, \frac{\rho_\sigma \bar{f} + (\gamma^N - 1)G}{(\gamma^N - (1 - \rho_\sigma)) \bar{f}} \right)$	0	$E_N [t^b] = \gamma^N (\beta \bar{f} - G)$
$\beta \in \left[\frac{\rho_\sigma \bar{f} + (\gamma^N - 1)G}{(\gamma^N - (1 - \rho_\sigma)) \bar{f}}, 1 \right]$	0	$E_R [t^b] = \rho_\sigma \bar{f} + (1 - \rho_\sigma) \beta \bar{f} - G$

Depending on the relative sizes of \bar{f} and G , some of the rows of this table violate $\beta \in [0, 1]$ and, thus, need to be disregarded. We consider all possible cases in turn.

Suppose $\bar{f} \in [0, G]$. Only the first row is then relevant, because $\frac{G - \rho_\sigma \bar{f}}{(1 - \rho_\sigma) \bar{f}} \geq 1$. We then have $E_R [t^n] = \gamma^C (G - \rho_\sigma \bar{f} - (1 - \rho_\sigma) \beta \bar{f})$ and $\min \{ E_N [t^b], E_R [t^b] \} = E_N [t^b] = 0$. Substituting these transfers in constraint (A.7) and solving for β gives equation (A.1), where $\bar{F} \geq F_0$ ensures that $\beta^* \leq 1$.

Suppose $\bar{f} \in \left(G, \frac{G}{\rho_\sigma} \right)$. All rows of the table are then relevant. For each region of β , i.e., for each row of the table, substituting the associated transfers in constraint (A.7) and

solving for β gives equation (A.2), where the conditions on \bar{F} ensure that the derived solution indeed lies within the relevant region of β .

Suppose $\bar{f} \in \left[\frac{G}{\rho_\sigma}, \infty \right)$. Only the last three rows of the table are then relevant, because $\frac{G - \rho_\sigma \bar{f}}{(1 - \rho_\sigma) \bar{f}} \leq 0$. For each of the last three rows, substituting the associated transfers in constraint (A.7) and solving for β gives equation (A.3), where the conditions on \bar{F} ensure that the derived solution indeed lies within the relevant region of β . \square

A.1.3.2 Optimal Investigation Probability When CPs Are Not Available ($\beta^{\tilde{*}}$)

By Lemma 2.1, the expected transfer is $E_N [t^b] = \max \{ \gamma^N (\beta f^N - G), 0 \}$ when inducing a breach, and $E_N [t^n] = \max \{ \gamma^N (G - \beta f^N), 0 \}$ when preventing a breach. Thus, we have (i) if $G \leq \beta \bar{f}$ then $E_N [t^b] = \gamma^N (\beta f^N - G)$ and $E_N [t^n] = 0$, while (ii) if $G > \beta \bar{f}$ then $E_N [t^b] = 0$ and $E_N [t^n] = \gamma^N (G - \beta f^N)$. In both cases, we have $E_N [t^n] - E_N [t^b] = \gamma^N (G - \beta f^N)$.

The authority's problem is to minimize β subject to $\Pi_N^n \geq \Pi_N^b$, that is, subject to

$$\begin{aligned} 1 - \rho_\pi - E_N [t^n] &\geq \rho_\pi - E_N [t^b] - \beta \bar{F}, \\ \Leftrightarrow \beta \bar{F} &\geq 2\rho_\pi - 1 + E_N [t^n] - E_N [t^b]. \end{aligned}$$

Substituting for $E_N [t^n] - E_N [t^b] = \gamma^N (G - \beta f^N)$ and solving for β gives

$$\tilde{\beta}^* = \begin{cases} \emptyset & \text{if } \bar{F} < F_5 \\ \frac{2\rho_\pi - 1 + \gamma^N G}{\gamma^N \bar{f} + \bar{F}} & \text{if } \bar{F} \geq F_5, \end{cases}$$

where $\bar{F} \geq F_5 = 2\rho_\pi - 1 + \gamma^N (G - \bar{f})$ ensures that $\tilde{\beta}^* \leq 1$.

A.1.3.3 Comparison of β^* and $\beta^{\tilde{*}}$

Assuming that \bar{F} is large enough for $\beta^*, \tilde{\beta}^* \leq 1$ to exist, we have by straightforward algebra

- (i) $\beta^* < \tilde{\beta}^*$ if (a) $\bar{f} \leq G$; or (b) $\bar{f} \in \left(G, \frac{G}{\rho_\sigma} \right)$ and $\bar{F} \geq F_3$; or (c) $\bar{f} \geq \frac{G}{\rho_\sigma}$ and $\bar{F} \geq F_3$;
- (ii) $\beta^* > \tilde{\beta}^*$ if (a) $\bar{f} \in \left(G, \frac{G}{\rho_\sigma} \right)$ and $\bar{F} < F_2$; or if (b) $\bar{f} > \frac{G}{\rho_\sigma}$ and $\bar{F} < F_2$; and
- (iii) $\beta^* = \tilde{\beta}^*$ if (a) $\bar{f} \in \left(G, \frac{G}{\rho_\sigma} \right)$ & $F_2 \leq \bar{F} < F_3$; or if (b) $\bar{f} > \frac{G}{\rho_\sigma}$ & $F_2 \leq \bar{F} < F_3$,

which is equivalent to Proposition 2.2, where we define $F' = F_2$ and $F'' = F_3$. \square

A.2 Proofs of Lemmas 2.3 and 2.4

This appendix derives the expected transfers. Denote by $i = Z$ the shareholder's action of "not adopting a CP and blowing the whistle when the manager shows evidence to her." Similar to the results in Section 2.4, we anticipate that the authority optimally (i) sets the managerial fines f^N , f^C and f^R to their legal maximum \bar{f} ; (ii) sets the corporate fines F^N and F^C to their legal maximum \bar{F} ; and (iii) grants partial corporate leniency when the shareholder reports evidence to the authority, i.e., $F^R = \beta\bar{F} - |\epsilon|$, thereby ensuring that the shareholder always reports when she has evidence of a breach. *We ex post* verify that this anticipation is indeed correct; the proof is long and available on request.

A.2.1 Proof of Lemma 2.3

If the shareholder prevents a breach, she optimally implements a CP to monitor the manager, while paying him a positive transfer if and only if $\pi = 0$, $\sigma = 0$, $r_a = 0$ and $r_p = 0$. Denoting transfers by t_{π,σ,r_a,r_p} , the shareholder minimizes $t_{0,0,0,0} \geq 0$, s.t.

$$\rho_\pi t_{0,0,0,0} \geq (1 - \rho_\pi) (1 - \rho_\sigma) t_{0,0,0,0} + G - \rho_\sigma f^R - (1 - \rho_\sigma) \beta f^C, \quad (\text{A.8})$$

$$\rho_\pi t_{0,0,0,0} \geq G - f^r, \quad (\text{A.9})$$

$$\rho_\pi t_{0,0,0,0} \geq G - f^R, \quad (\text{A.10})$$

$$\rho_\pi t_{0,0,0,0} \geq 0, \quad (\text{A.11})$$

where (A.8) ensures that the shareholder does not "breach and not show evidence to the shareholder and not blow the whistle," (A.9) ensures that the shareholder does not "breach and blow the whistle," (A.10) ensures that the manager does not "breach and show evidence to the shareholder," and (A.11) is the participation constraint. Anticipating that $f^C = f^R = \bar{f}$, we then have

$$\begin{aligned} E_R [t^n] &= \rho_\pi t_{0,0,0,0} & (\text{A.12}) \\ &= \max \left\{ \gamma^C (G - \rho_\sigma f^R - (1 - \rho_\sigma) \beta f^C), G - f^r, 0 \right\}. & \square \end{aligned}$$

A.2.2 Proof of Lemma 2.4

When the shareholder induces a breach, she may do so by (i) adopting a CP or not, and (ii) requesting evidence from the manager or not, while (iii) ensuring that the manager does not blow the whistle.¹⁵⁵ We consider the four possible cases in turn.

¹⁵⁵We assume that \bar{F} is sufficiently large such that the shareholder never induces a breach by requiring the manager to blow the whistle. Anticipating that the authority sets maximum corporate fines, this strategy would mean being imposed the corporate fine \bar{F} for sure, which is irrational if \bar{F} is large enough.

A.2.2.1 Case I: CP and No Request for Evidence

Suppose the shareholder adopts a CP and does not request evidence from the manager. She will then use signal $\sigma = 1$ to induce a breach, because $\sigma = 1$ is a perfectly informative signal of a breach having occurred. The realization of π is then irrelevant. Moreover, the shareholder must ensure that the manager does not blow the whistle or shows evidence to the shareholder. Thus, transfers are contingent on the realization of σ , r_a and r_p . We denote them by t_{σ, r_a, r_p} , such that $t_{\sigma, r_a, r_p} = 0$ if $r_a = 1$ and/or $r_p = 1$.

Interim stage 5'. Suppose the manager has breached and signal σ has been realized. If $\sigma = 1$ the shareholder blows the whistle and the game ends. However, if $\sigma = 0$ the shareholder must ensure that the manager does not blow the whistle or reports to the shareholder, which is the case if she compensates him by $t_{0,0,0} \geq \beta f^C - \min \{f^r, f^R\}$.

Ex-ante stage 4'. From the interim stage 5', we know that the shareholder will set $t_{0,0,0} = \max \{\beta f^C - \min \{f^r, f^R\}, 0\}$. The shareholder uses signal $\sigma = 1$ to induce a breach and solves for $t_{1,0,0}$ by minimizing $t_{1,0,0}$, subject to

$$\rho_\sigma t_{1,0,0} + (1 - \rho_\sigma) t_{0,0,0} + G - E_R[f] \geq t_{0,0,0}, \quad (\text{A.13})$$

$$\rho_\sigma t_{1,0,0} + (1 - \rho_\sigma) t_{0,0,0} + G - E_R[f] \geq G - \min \{f^r, f^R\}, \quad (\text{A.14})$$

$$\rho_\sigma t_{1,0,0} + (1 - \rho_\sigma) t_{0,0,0} + G - E_R[f] \geq 0, \quad (\text{A.15})$$

where (A.13) ensures that the manager does not “not breach the law,” (A.14) ensures that the manager does not “breach and blow the whistle or report to the shareholder,” and (A.15) is the participation constraint, which is implied by (A.13). We then have by (A.13) and (A.14), respectively,

$$t_{1,0,0} \geq \frac{E_R[f] - G}{\rho_\sigma} + t_{0,0,0},$$

$$t_{1,0,0} \geq \frac{E_R[f] - \min \{f^r, f^R\} - (1 - \rho_\sigma) t_{0,0,0}}{\rho_\sigma}.$$

Thus, the authority optimally sets

$$t_{1,0,0} = \max \left\{ \frac{E_R[f] - G}{\rho_\sigma} + t_{0,0,0}, \frac{E_R[f] - \min \{f^r, f^R\} - (1 - \rho_\sigma) t_{0,0,0}}{\rho_\sigma}, 0 \right\}.$$

The expected transfer is then

$$\begin{aligned}
 E_R [t^b] &= \rho_\sigma t_{1,0,0} + (1 - \rho_\sigma) t_{0,0,0} \\
 &= \max \{ E_R [f] - G + \rho_\sigma t_{0,0,0}, E_R [f] - \min \{ f^r, f^R \} - (1 - \rho_\sigma) t_{0,0,0}, 0 \} \\
 &\quad + (1 - \rho_\sigma) t_{0,0,0} \\
 &= \max \{ E_R [f] - G + t_{0,0,0}, E_R [f] - \min \{ f^r, f^R \}, (1 - \rho_\sigma) t_{0,0,0} \},
 \end{aligned}$$

which, when substituting for $E_R [f]$ and $t_{0,0,0}$, rewrites as

$$\begin{aligned}
 E_R [t^b] &= \max \{ \rho_\sigma f^R + (1 - \rho_\sigma) \beta f^C - G + \max \{ \beta f^C - \min \{ f^r, f^R \}, 0 \}, \\
 &\quad \underbrace{\rho_\sigma f^R + (1 - \rho_\sigma) \beta f^C - \min \{ f^r, f^R \}}_C, \underbrace{(1 - \rho_\sigma) (\beta f^C - \min \{ f^r, f^R \})}_B, 0 \},
 \end{aligned} \tag{A.16}$$

where labels A – C are used later in this proof.

A.2.2.2 Case II: No CP and No Request for Evidence

Suppose the shareholder does not adopt a CP and does not request evidence from the manager. She will then use signal $\pi = 1$ to induce a breach. Moreover, the shareholder must ensure that the manager does not blow the whistle or reports evidence to the shareholder. Thus, transfers are contingent on the realization of π , r_a and r_p . We denote them by t_{π, r_a, r_p} , such that $t_{\sigma, r_a, r_p} = 0$ if $r_a = 1$ and/or $r_p = 1$.

Interim stage 5'. Suppose the manager has breached and profit π has been realized. The shareholder then ensures that the manager does not blow the whistle or reports evidence to the shareholder by paying him $t_{\pi, 0, 0} \geq \beta f^N - \min \{ f^r, f^Z \}$ for both profit realizations $\pi \in \{0, 1\}$.

Ex-ante stage 4'. To induce the manager to breach the law in the first place, the shareholder must create a wedge, say $\Delta > 0$, between $t_{1,0,0}$ and $t_{0,0,0}$. She optimally does so by setting $t_{0,0,0} = \max \{ \beta f^N - \min \{ f^r, f^Z \}, 0 \}$ and minimizing $t_{1,0,0} = t_{0,0,0} + \Delta$,

subject to

$$\rho_\pi (t_{0,0,0} + \Delta) + (1 - \rho_\pi) t_{0,0,0} + G - \beta f^N \geq (1 - \rho_\pi) (t_{0,0,0} + \Delta) + \rho_\pi t_{0,0,0}, \quad (\text{A.17})$$

$$\rho_\pi (t_{0,0,0} + \Delta) + (1 - \rho_\pi) t_{0,0,0} + G - \beta f^N \geq G - \min \{f^r, f^Z\}, \quad (\text{A.18})$$

$$\rho_\pi (t_{0,0,0} + \Delta) + (1 - \rho_\pi) t_{0,0,0} + G - \beta f^N \geq 0, \quad (\text{A.19})$$

where (A.17) ensures that the manager does not “not breach the law,” (A.18) ensures that the manager does not “breach and blow the whistle or report to the shareholder,” and (A.19) is the participation constraint, which is implied by (A.17). Noting that (A.18) is always satisfied, we have by (A.17) that

$$\Delta = \max \left\{ \frac{\beta f^N - G}{2\rho_\pi - 1}, 0 \right\},$$

resulting in expected transfer

$$\begin{aligned} E_N [t^b] &= \rho_\pi (t_{0,0,0} + \Delta) + (1 - \rho_\pi) t_{0,0,0} \\ &= \max \{ \gamma^N (\beta f^N - G), 0 \} + \max \{ \beta f^C - \min \{f^r, f^Z\}, 0 \}. \end{aligned}$$

A.2.2.3 Case III: CP and Request for Evidence

Suppose the shareholder adopts a CP and requests evidence from the manager. She can optimally induce a breach by paying the manager a positive transfer if and only if $r_a = 0$ and $r_p = 1$: we denote the transfers by t_{r_a, r_p} , where $t_{r_a, r_p} = 0$ if $(r_a, r_p) \neq (0, 1)$.

The shareholder can request for evidence before or after profit π and signal σ are realized. Since both pieces of information do not affect the transfers, it does not matter when the shareholder requests for evidence. She minimizes $t_{0,1}$, subject to

$$t_{0,1} + G - f^R \geq 0, \quad (\text{A.20})$$

$$t_{0,1} + G - f^R \geq G - \rho_\sigma f^R - (1 - \rho_\sigma) \beta f^C, \quad (\text{A.21})$$

$$t_{0,1} + G - f^R \geq G - f^r, \quad (\text{A.22})$$

$$t_{0,1} + G - f^R \geq 0, \quad (\text{A.23})$$

where (A.20) ensures that the does not “not breach,” (A.21) ensures that the manager does not “breach and not report evidence to the shareholder,” (A.22) ensures that the manager does not “breach and blow the whistle,” and (A.23) is the participation constraint. The

expected transfer is then

$$E_R [t^b] = \max \{ f^R - G, (1 - \rho_\sigma) (f^R - \beta f^C), f^R - f^r, 0 \}.$$

A.2.2.4 Case IV: No CP and Request for Evidence

Suppose the shareholder does not adopt a CP, but requests to see the evidence. Again, she optimally induces a breach by paying the manager a positive transfer if and only if $r_a = 0$ and $r_p = 1$. Thus, we denote the transfers by t_{r_a, r_p} , where $t_{r_a, r_p} = 0$ if $(r_a, r_p) \neq (0, 1)$.

Again, the shareholder can request for evidence before or after profit π and signal σ are realized. Since both pieces of information do not affect the transfers, it does not matter when the shareholder requests for evidence. She minimizes $t_{0,1}$, subject to

$$t_{0,1} + G - f^Z \geq 0, \tag{A.24}$$

$$t_{0,1} + G - f^Z \geq G - \beta f^N, \tag{A.25}$$

$$t_{0,1} + G - f^Z \geq G - f^r, \tag{A.26}$$

$$t_{0,1} + G - f^Z \geq 0, \tag{A.27}$$

where (A.24) ensures that the manager does not “not breach the law,” (A.25) ensures that the manager does not “breach and not show evidence to the shareholder,” (A.26) ensures that the shareholder does not “breach and blow the whistle,” and (A.26) is the participation constraint. The expected transfer is then

$$E_Z [t^b] = \max \{ f^Z - G, f^Z - \beta f^N, f^Z - f^r, 0 \}.$$

A.2.2.5 Summary of Expected Transfers to Induce a Breach

Substituting $f^N = f^C = f^R = f^Z = \bar{f}$ into the expected transfers $E_i [t^b]$ derived above, we arrive at the following table for $E_i [t^b]$, where $E_R [f] = \rho_\sigma \bar{f} + (1 - \rho_\sigma) \beta \bar{f}$. We specifically note that term B in expression (A.16) is redundant, because by $f^C = f^R = \bar{f}$ we have $C > A$ and, therefore, $C > B$.

	REQUEST FOR EVIDENCE	NOT REQUEST FOR EVIDENCE
CP	$\max \{ \bar{f} - G, (1 - \rho_\sigma) \times (1 - \beta) \bar{f}, \bar{f} - f^r, 0 \}$	$\max \{ E_R [f] - G + \max (\beta \bar{f} - f^r, 0), E_R [f] - f^r, 0 \}$
No CP	$\max \{ \bar{f} - G, (1 - \beta) \bar{f}, (*) \bar{f} - f^r, 0 \}$	$\max \{ \gamma^N (\beta \bar{f} - G), 0 \} + \max \{ \beta \bar{f} - f^r, 0 \}$

(*) Noting that the strategy “no CP, request for evidence” entails a weakly higher expected transfer than the strategy “CP, request for evidence,” we eliminate the former from the problem. The remaining expected transfers are stated in Lemma 2.4. \square

A.3 Proof of Proposition 2.3

This appendix solves for the optimal managerial leniency policy f^r , which the authority sets so as to maximize the wedge $E[t^b] - E[t^n]$, where we omit the i in $E_i[t^a]$ for notational convenience. Subsections A.3.1 and A.3.2, respectively, determine this wedge if $f^r = \bar{f}$ and $\bar{f} = 0$. We compare those wedges in Subsection A.3.3 so as to derive the optimal f^r . Throughout the analysis, we assume $\rho_\sigma > \frac{1-\rho_\pi}{\rho_\pi}$ so as to reduce the number of cases in this proof.

A.3.1 No Managerial Leniency

Suppose the authority provides no managerial leniency, that is, $f^r = \bar{f}$.

Preventing a breach. If $f^r = \bar{f}$ the expected transfer to prevent a breach (A.12) becomes

$$E[t^n] = \max \left\{ \underbrace{\gamma^C (G - \rho_\sigma \bar{f} - (1 - \rho_\sigma) \beta \bar{f})}_A, \underbrace{G - \bar{f}}_B, 0 \right\},$$

where constraint B is irrelevant, because (i) if $\bar{f} < G$ then $B < A$, and (ii) if $\bar{f} \geq G$ then $B < 0$. Therefore,

$$E[t^n] = \max \{ \gamma^C (G - \rho_\sigma \bar{f} - (1 - \rho_\sigma) \beta \bar{f}), 0 \}. \quad (\text{A.28})$$

Inducing a breach. Substituting $f^r = \bar{f}$ in the expressions in Subsection 7.2 yields

$$E[t^b] = \min \left\{ \max \{ \rho_\sigma \bar{f} + (1 - \rho_\sigma) \beta \bar{f} - G, 0 \}, \max \{ \gamma^N (\beta \bar{f} - G), 0 \} \right\}. \quad (\text{A.29})$$

The wedge. From (A.28) and (A.29) we have that the wedge $E[t^b] - E[t^n]$ is the same as in the case if manager does not possess evidence, yielding the table on page 150.

A.3.2 Managerial Leniency

Suppose the authority provides managerial leniency, that is, $f^r = 0$.

Preventing a breach. If $f^r = 0$ the expected transfer to prevent a breach (A.12) becomes

$$E[t^m] = \max \{ \gamma^C (G - \rho_\sigma \bar{f} - (1 - \rho_\sigma) \beta \bar{f}), G, 0 \},$$

and the reporting constraint G binds if and only if $G > \gamma^C (G - \rho_\sigma \bar{f} - (1 - \rho_\sigma) \beta \bar{f})$, that is, if and only if

$$\beta > \hat{\beta} = \frac{(\gamma^C - 1)G - \gamma^C \rho_\sigma \bar{f}}{\gamma^C (1 - \rho_\sigma) \bar{f}},$$

where we note that $\hat{\beta} > 0 \Leftrightarrow \bar{f} < \frac{(\gamma^C - 1)G}{\gamma^C \rho_\sigma}$ and $\hat{\beta} < 1 \Leftrightarrow \bar{f} > \frac{(\gamma^C - 1)G}{\gamma^C}$. The following table then summarizes the expected transfer needed to prevent a breach depending on \bar{f} and β .

CAP ON MANAGERIAL FINE \bar{f}	INV. PROB. β	EXPECTED TRANSFER $E[t^m]$
$\bar{f} \in \left[0, \frac{(\gamma^C - 1)G}{\gamma^C} \right)$	$\beta \in [0, 1]$	$\gamma^C (G - \rho_\sigma \bar{f} - (1 - \rho_\sigma) \beta \bar{f})$
$\bar{f} \in \left[\frac{(\gamma^C - 1)G}{\gamma^C}, \frac{(\gamma^C - 1)G}{\gamma^C \rho_\sigma} \right)$	$\beta \in [0, \hat{\beta})$	$\gamma^C (G - \rho_\sigma \bar{f} - (1 - \rho_\sigma) \beta \bar{f})$
	$\beta \in [\hat{\beta}, 1]$	G
$\bar{f} \in \left[\frac{(\gamma^C - 1)G}{\gamma^C \rho_\sigma}, \infty \right)$	$\beta \in [0, 1]$	G

Inducing a breach. Substituting $f^r = 0$ in the expressions in Subsection 7.2 yields

$$E[t^b] = \min \left\{ \bar{f}, \max \left\{ E_R[f] - G + \beta \bar{f}, E_R[f] \right\}, \max \left\{ \gamma^N (\beta \bar{f} - G), 0 \right\} + \beta \bar{f}, 0 \right\} \quad (\text{A.30})$$

If either $\bar{f} < G$ or $\bar{f} > G$ and $\beta < \frac{G}{\bar{f}}$, then (A.30) becomes

$$E[t^b] = \min \{ \bar{f}, \rho_\sigma \bar{f} + (1 - \rho_\sigma) \beta \bar{f}, \beta \bar{f} \} = \beta \bar{f}. \quad (\text{A.31})$$

If $\bar{f} > G$ and $\beta > \frac{G}{\bar{f}}$, then we have

$$E[t^b] = \min \left\{ \underbrace{\bar{f}, \rho_\sigma \bar{f} + (1 - \rho_\sigma) \beta \bar{f} - G + \beta \bar{f}}_A, \underbrace{\gamma^N (\beta \bar{f} - G) + \beta \bar{f}}_B \right\},$$

where we note that

$$\begin{aligned} B < A &\Leftrightarrow \beta < \tilde{\beta} = \frac{(\gamma^N - 1)G + \rho_\sigma \bar{f}}{(\gamma^N - (1 - \rho_\sigma))\bar{f}}, \\ B < \bar{f} &\Leftrightarrow \beta < \tilde{\beta} = \frac{\gamma^N G + \bar{f}}{(\gamma^N + 1)\bar{f}}, \text{ and} \\ A < \bar{f} &\Leftrightarrow \beta < \tilde{\beta} = \frac{(1 - \rho_\sigma)\bar{f} + G}{(2 - \rho_\sigma)\bar{f}}, \end{aligned}$$

where $0 < \tilde{\beta} < \tilde{\beta} < \tilde{\beta} < 1$, because $\bar{f} > G$ and by assumption $\rho_\sigma > \frac{1 - \rho_\pi}{\rho_\pi}$. Thus,

$$\begin{aligned} E[t^b] = B &\Leftrightarrow \left\{ \beta < \tilde{\beta} \text{ and } \beta < \tilde{\beta} \right\} \Leftrightarrow \beta < \tilde{\beta}, \\ E[t^b] = A &\Leftrightarrow \left\{ \beta < \tilde{\beta} \text{ and } \beta > \tilde{\beta} \right\}, \text{ which cannot hold, and} \\ E[t^b] = \bar{f} &\Leftrightarrow \left\{ \beta > \tilde{\beta} \text{ and } \beta > \tilde{\beta} \right\} \Leftrightarrow \beta > \tilde{\beta}. \end{aligned}$$

The following table summarizes.

CAP ON MANAGERIAL FINE \bar{f}	INV. PROB. β	EXPECTED TRANSFER $E[t^b]$
$\bar{f} \in [0, G)$	$\beta \in [0, 1]$	$E[t^b] = \beta \bar{f}$
$\bar{f} \in [G, \infty)$	$\beta \in [0, \frac{G}{\bar{f}})$	$E[t^b] = \beta \bar{f}$
	$\beta \in [\frac{G}{\bar{f}}, \tilde{\beta})$	$E[t^b] = \gamma^N (\beta \bar{f} - G) + \beta \bar{f}$
	$\beta \in [\tilde{\beta}, 1]$	$E[t^b] = \bar{f}$

The wedge. Noting that $\frac{(\gamma^C - 1)G}{\gamma^C \rho_\sigma} < G$ by assumption $\rho_\sigma > \frac{1 - \rho_\pi}{\rho_\pi}$, the two tables above yield the wedge $E[t^b] - E[t^n]$ outlined in the following table.

CAP ON MANAGERIAL FINE \bar{f}	INV. PROB. β	WEDGE $E[t^b] - E[t^n]$
$\bar{f} \in \left[0, \frac{(\gamma^C-1)G}{\gamma^C}\right)$	$\beta \in [0, 1]$	$\beta\bar{f} - \gamma^C (G - \rho_\sigma\bar{f} - (1 - \rho_\sigma)\beta\bar{f})$
$\bar{f} \in \left[\frac{(\gamma^C-1)G}{\gamma^C}, \frac{(\gamma^C-1)G}{\gamma^C\rho_\sigma}\right)$	$\beta \in [0, \hat{\beta})$	$\beta\bar{f} - \gamma^C (G - \rho_\sigma\bar{f} - (1 - \rho_\sigma)\beta\bar{f})$
	$\beta \in [\hat{\beta}, 1]$	$\beta\bar{f} - G$
$\bar{f} \in \left[\frac{(\gamma^C-1)G}{\gamma^C\rho_\sigma}, G\right)$	$\beta \in [0, 1]$	$\beta\bar{f} - G$
$\bar{f} \in [G, \infty)$	$\beta \in [0, \frac{G}{\bar{f}})$	$\beta\bar{f} - G$
	$\beta \in [\frac{G}{\bar{f}}, \tilde{\beta})$	$\gamma^N (\beta\bar{f} - G) + \beta\bar{f} - G$
	$\beta \in [\tilde{\beta}, 1]$	$\bar{f} - G$

A.3.3 Deriving the Optimal f^r

Combining the last table above with that on page 150, we have the following table. Comparing the wedge $E[t^b] - E[t^n]$ if $f^r = 0$ and if $f^r = \bar{f}$, the last column states the optimal managerial leniency policy by maximizing this wedge. The cells containing numbers in brackets are not straightforward to determine and are derived in more detail below.

FINE $\bar{f} \in$	PROB. $\beta \in$	WEDGE IF $f^r = 0$	WEDGE IF $f^r = \bar{f}$	OPTIMAL f^r
$\left[0, \frac{(\gamma^C-1)G}{\gamma^C}\right)$	$[0, 1]$	$\beta\bar{f} - \gamma^C (G - E_R[f])$	$-\gamma^C (G - E_R[f])$	0
$\left[\frac{(\gamma^C-1)G}{\gamma^C}, \frac{(\gamma^C-1)G}{\gamma^C\rho_\sigma}\right)$	$[0, \hat{\beta})$	$\beta\bar{f} - \gamma^C (G - E_R[f])$	$-\gamma^C (G - E_R[f])$	0
	$[\hat{\beta}, 1]$	$\beta\bar{f} - G$	$-\gamma^C (G - E_R[f])$	0 (1)
$\left[\frac{(\gamma^C-1)G}{\gamma^C\rho_\sigma}, G\right)$	$[0, 1]$	$\beta\bar{f} - G$	$-\gamma^C (G - E_R[f])$	$f^r = \bar{f}$ if $\beta < \beta''$, $f^r = 0$ if $\beta > \beta''$ (3)
$[G, \infty) (*)$	$\left[0, \frac{G-\rho_\sigma\bar{f}}{(1-\rho_\sigma)\bar{f}}\right)$	$\beta\bar{f} - G$	$-\gamma^C (G - E_R[f])$	\bar{f} (2)
	$\left[\frac{G-\rho_\sigma\bar{f}}{(1-\rho_\sigma)\bar{f}}, \frac{G}{\bar{f}}\right)$	$\beta\bar{f} - G$	0	\bar{f}
	$\left[\frac{G}{\bar{f}}, \tilde{\beta}\right)$	$\gamma^N (\beta\bar{f} - G) + \beta\bar{f} - G$	$\gamma^N (\beta\bar{f} - G)$	0
	$\left[\tilde{\beta}, \tilde{\tilde{\beta}}\right)$	$\bar{f} - G$	$\gamma^N (\beta\bar{f} - G)$	0 (4)
	$\left[\tilde{\tilde{\beta}}, 1\right)$	$\bar{f} - G$	$E_R[f] - G$	0

(*) The row with $\beta \in \left[0, \frac{G-\rho_\sigma\bar{f}}{(1-\rho_\sigma)\bar{f}}\right)$ is irrelevant if $\bar{f} > \frac{G}{\rho_\sigma}$, because then $\frac{G-\rho_\sigma\bar{f}}{(1-\rho_\sigma)\bar{f}} < 0$.

(1) Granting managerial leniency is optimal if and only if

$$\beta \bar{f} - G > -\gamma^C (G - \rho_\sigma \bar{f} - (1 - \rho_\sigma) \beta \bar{f})$$

iff. $(\gamma^C (1 - \rho_\sigma) - 1) \beta \bar{f} < (\gamma^C - 1) G - \gamma^C \rho_\sigma \bar{f}$, (A.32)

where the LHS is negative by assumption $\rho_\sigma > \frac{1-\rho_\pi}{\rho_\pi}$ and the RHS is positive by $\bar{f} < \frac{(\gamma^C - 1)G}{\gamma^C \rho_\sigma}$. Therefore, (A.32) holds and, thus, $f^r = 0$.

(2) Rewriting (A.32) gives

$$\beta > \beta'' = \frac{\gamma^C \rho_\sigma \bar{f} - (\gamma^C - 1) G}{\gamma^C \rho_\sigma \bar{f} - (\gamma^C - 1) \bar{f}},$$

where $\beta'' > 1$ if $\bar{f} > G$ and it can never be the case that $\beta > \beta''$. Thus, $f^r = \bar{f}$.

(3) If $\bar{f} < G$, then $\beta'' < 1$ and, thus, we have $f^r = 0$ if $\beta > \beta''$, while $f^r = \bar{f}$ if $\beta < \beta''$.

(4) Granting managerial leniency is optimal if and only if $\bar{f} - G > \gamma^N (\beta \bar{f} - G) \Leftrightarrow$

$$\beta < \beta''' = \frac{(\gamma^N - 1) G + \bar{f}}{\gamma^N \bar{f}}.$$

In this region of β , we have

$$\beta < \tilde{\beta} = \frac{(\gamma^N - 1) G + \rho_\sigma \bar{f}}{(\gamma^N - (1 - \rho_\sigma)) \bar{f}} = \frac{(\gamma^N - 1) G + \bar{f} - (1 - \rho_\sigma) \bar{f}}{\gamma^N \bar{f} - (1 - \rho_\sigma) \bar{f}},$$

from which we see that $\tilde{\beta} < \beta'''$ and we have $\beta < \beta'''$ in this region. Thus, $f^r = 0$.

Conclusion on managerial leniency. From the table we observe the following:

1. If $\bar{f} \in [0, \tilde{f})$, where $\tilde{f} = \frac{(\gamma^C - 1)G}{\gamma^C \rho_\sigma}$, then $f^r = 0$;
2. If $\bar{f} \in [\tilde{f}, G)$, then when $f^r = \bar{f}$, we determine β^* and F^r by solving

$$\pi - \beta \bar{f} - \beta F^r < 1 - \pi - G, \quad (\text{A.33})$$

which is most easily satisfied when $F^r = \bar{F}$ (no leniency) and rewrites as

$$\beta > \frac{2\pi - 1 + G}{\bar{F} + \bar{f}} \Rightarrow \beta^* = \frac{2\pi - 1 + G}{\bar{F} + \bar{f}}, \quad (\text{A.34})$$

provided that

$$\beta^* < \beta'' \Leftrightarrow \bar{F} > F^* = \frac{2\pi - 1 + G}{\beta''} - \bar{f}; \text{ and}$$

3. If $\bar{f} \in [G, \infty)$, then when $f^r = \bar{f}$, we determine β^* and F^r by solving (A.33), which is most easily satisfied when $F^r = \bar{F}$ (no corporate leniency) and rewrites as (A.34), provided that

$$\beta^* < \frac{G}{\bar{f}} \Leftrightarrow \bar{F} > F^{**} = \frac{(2\pi - 1)\bar{f}}{G}.$$

These results are the technical equivalent of Proposition 2.3, where we denote

$$\tilde{F} = \begin{cases} F^* & \text{if } \bar{f} \in [\tilde{f}, G) \\ F^{**} & \text{if } \bar{f} \in [G, \infty). \end{cases} \quad \square$$

Appendix B

This appendix contains the proofs of Chapter 3.

B.1 Proof of Proposition 3.3

(1) Shifting $F_D(x)$ to the left (right) increases (decreases) $G_D(\underline{\pi})$, which, in turn, increases (decreases) the LHS of condition (3.5). (2) Suppose $\underline{\pi} < E_D(\pi_t)$. Making $F_D(x)$ more (less) dispersed increases (decreases) $G_D(\underline{\pi})$, which, in turn, increases (decreases) the LHS of condition (3.5). (3) Suppose $\underline{\pi} \geq E_D(\pi_t)$. Making $F_D(x)$ more (less) dispersed decreases (increases) $G_D(\underline{\pi})$, which, in turn, decreases (increases) the LHS of condition (3.5). \square

B.2 Proof of Lemma 3.4

The manager has no incentive to defect in stage 1 if and only if

$$\frac{2E_C}{1 - \delta G_{CC}(\underline{\pi})} \geq E_D + E_N + \delta G_{DN}(\underline{\pi}) \frac{2E_N}{1 - \delta G_{NN}(\underline{\pi})},$$

while, given the realization of π_t^1 , he has no incentive to defect in stage 2 if and only if

$$E_C + \delta G_C(\underline{\pi} - \pi_t^1) \frac{2E_C}{1 - \delta G_{CC}(\underline{\pi})} \geq E_D + \delta G_D(\underline{\pi} - \pi_t^1) \frac{2E_N}{1 - \delta G_{NN}(\underline{\pi})}.$$

These two conditions rewrite as

$$\min \left\{ E_C - E_N + 2\delta \left(\frac{G_{CC}(\underline{\pi})}{1 - \delta G_{CC}(\underline{\pi})} E_C - \frac{G_{DN}(\underline{\pi})}{1 - \delta G_{NN}(\underline{\pi})} E_N \right); \right. \quad (\text{B.1})$$

$$\left. 2\delta \left(\frac{G_C(\underline{\pi} - \pi_t^1)}{1 - \delta G_{CC}(\underline{\pi})} E_C - \frac{G_D(\underline{\pi} - \pi_t^1)}{1 - \delta G_{NN}(\underline{\pi})} E_N \right) \right\} \geq E_D - E_C \quad (\text{B.2})$$

for every profit realization $\pi_t^1 \geq 0$. Subcondition (B.2) can be rewritten as

$$2\delta \left[\frac{G_C(\underline{\pi} - \pi_t^1) - G_D(\underline{\pi} - \pi_t^1)}{1 - \delta G_{NN}(\underline{\pi})} E_N + G_C(\underline{\pi} - \pi_t^1) \left(\frac{E_C}{1 - \delta G_{CC}(\underline{\pi})} - \frac{E_N}{1 - \delta G_{NN}(\underline{\pi})} \right) \right] \geq E_D - E_C,$$

where we note that (i) $G_C(\underline{\pi} - \pi_t^1)$ is increasing in π_t^1 , and (ii) $G_C(\underline{\pi} - \pi_t^1) - G_D \times (\underline{\pi} - \pi_t^1)$ is also increasing in π_t^1 by regularity condition (3.9). Thus, Subcondition (B.2) is most difficult to satisfy if $\pi_t^1 = 0$, that is, if

$$2\delta \left(\frac{G_C(\underline{\pi})}{1 - \delta G_{CC}(\underline{\pi})} E_C - \frac{G_D(\underline{\pi})}{1 - \delta G_{NN}(\underline{\pi})} E_N \right) \geq E_D - E_C. \quad (\text{B.3})$$

Is (B.3) more difficult to satisfy than Subcondition (B.1)? Subtracting the LHS of (B.1) from the LHS of (B.3) yields

$$A := E_C - E_N + 2\delta \left(\frac{G_{CC}(\underline{\pi}) - G_C(\underline{\pi})}{1 - \delta G_{CC}(\underline{\pi})} E_C - \frac{G_{DN}(\underline{\pi}) - G_D(\underline{\pi})}{1 - \delta G_{NN}(\underline{\pi})} E_N \right),$$

where I rewrite

$$G_{CC}(\underline{\pi}) - G_C(\underline{\pi}) = \int_0^\infty \Pr(\pi_t^1 = x | a = C) G_C(\underline{\pi} - x) dx - G_C(\underline{\pi}) \quad (\text{B.4})$$

$$= \int_0^\underline{\pi} \Pr(\pi_t^1 = x | a = C) G_C(\underline{\pi} - x) dx \quad (\text{B.5})$$

$$+ \int_\underline{\pi}^\infty \Pr(\pi_t^1 = x | a = C) G_C(\underline{\pi} - x) dx - G_C(\underline{\pi})$$

$$= \int_0^\underline{\pi} \Pr(\pi_t^1 = x | a = C) G_C(\underline{\pi} - x) dx \quad (\text{B.6})$$

$$+ \int_\underline{\pi}^\infty \Pr(\pi_t^1 = x | a = C) dx - G_C(\underline{\pi})$$

$$= \int_0^\underline{\pi} \Pr(\pi_t^1 = x | a = C) G_C(\underline{\pi} - x) dx + G_C(\underline{\pi})$$

$$- G_C(\underline{\pi}) \quad (\text{B.7})$$

$$= \int_0^\underline{\pi} \Pr(\pi_t^1 = x | a = C) G_C(\underline{\pi} - x) dx, \quad (\text{B.8})$$

where (B.4) follows by definition, (B.5) follows by splitting up the integral, (B.6) follows

by noting that $G_C(\underline{\pi} - x) = 1$ for every $x \in [\underline{\pi}, \infty)$, (B.7) follows by definition, yielding (B.8). Similarly,

$$\begin{aligned} G_{DN}(\underline{\pi}) - G_D(\underline{\pi}) &= \int_0^\infty \Pr(\pi_t^1 = x | a = D) G_N(\underline{\pi} - x) dx - G_D(\underline{\pi}) \\ &= \int_0^{\underline{\pi}} \Pr(\pi_t^1 = x | a = D) G_N(\underline{\pi} - x) dx + G_D(\underline{\pi}) \\ &\quad - G_D(\underline{\pi}). \end{aligned}$$

Therefore, $G_{CC}(\underline{\pi}) - G_C(\underline{\pi}) \geq G_{DN}(\underline{\pi}) - G_D(\underline{\pi})$, because by stochastic dominance we have (i) $\int_0^{\underline{\pi}} \Pr(\pi_t^1 = x | a = C) dx \geq \int_0^{\underline{\pi}} \Pr(\pi_t^1 = x | a = D) dx$, and (ii) $G_C(\underline{\pi} - x) \geq G_N(\underline{\pi} - x)$ for every $x \in [0, \underline{\pi}]$. Thus, noting that $E_C > E_N$ and $\frac{E_C}{1 - \delta G_{CC}(\underline{\pi})} > \frac{E_N}{1 - \delta G_{NN}(\underline{\pi})}$, we have $A > 0$ and, thus, (B.3) is the binding constraint. \square

B.3 Proof of Lemma 3.5

The manager has no incentive to defect in stage 1 if and only if

$$\frac{E_C + G_C(\tau) E_C + (1 - G_C(\tau)) E_N}{1 - \delta P_C(\underline{\pi}, \tau)} \geq E_D + E_N + \delta G_{DN}(\underline{\pi}) \frac{2E_N}{1 - \delta G_{NN}(\underline{\pi})},$$

where $P_C(\underline{\pi}, \tau) = \int_\tau^\infty \Pr(\pi_t^1 = x | a = C) G_C(\underline{\pi} - x) dx + \int_0^\tau \Pr(\pi_t^1 = x | a = C) \times G_N(\underline{\pi} - x) dx$.

Given the realization of π_t^1 , he has no incentive to defect in stage 2 if and only if

$$\begin{aligned} E_C + \delta G_C(\underline{\pi} - \pi_t^1) \frac{E_C + G_C(\tau) E_C + (1 - G_C(\tau)) E_N}{1 - \delta P_C(\underline{\pi}, \tau)} &\geq E_D + \delta G_D(\underline{\pi} - \pi_t^1) \\ &\quad \times \frac{2E_N}{1 - \delta G_{NN}(\underline{\pi})}, \end{aligned}$$

which rewrites as

$$\delta \left(\frac{G_C(\underline{\pi} - \pi_t^1)}{1 - \delta P_C(\underline{\pi}, \tau)} K_C(\tau) - \frac{G_D(\underline{\pi} - \pi_t^1)}{1 - \delta G_{NN}(\underline{\pi})} 2E_N \right) \geq E_D - E_C$$

for every profit realization $\pi_t^1 \geq \tau$. By the regularity condition, this boils down to combined conditions (3.11) and (3.12) in Lemma 3.5.

Noting that $P(\underline{\pi}, \tau) < G_{CC}(\underline{\pi})$ and $E_C < K_C(\tau)$, we have that (3.11) is more difficult to satisfy than (B.1). However, constraint (B.1) is not the binding constraint for the ‘‘always collusion strategy’’ to be stable; constraint (B.2) with $\pi_t^1 = 0$ is the binding

constraint. Thus, constraint (3.11) does not make the “ τ -conditional collusion strategy” less stable than strategy “always collusion strategy” as long as τ is such that (3.11) is easier to satisfy than (B.2) with $\pi_t^1 = 0$, which depends on the precise specification of the density functions.

Fixing $\pi_t^1 \geq 0$, constraint (3.12) is more difficult to satisfy than constraint (B.2), because $P(\underline{\pi}, \tau) < G_{CC}(\underline{\pi})$ and $E_C < K_C(\tau)$. However, depending on the precise specification of the density functions, constraint (3.12) may be easier to satisfy than constraint (B.2), because (i) both constraints are more difficult to satisfy the lower is π_t^1 ; and (ii) constraint (B.2) needs to be satisfied for every $\pi_t^1 \geq 0$; while (iii) constraint (3.12) needs only to be satisfied for every $\pi_t^1 \geq \tau$.

Thus, depending on the precise specification of the density functions, choosing an appropriate τ potentially results in both constraints (3.11) and (3.12) to be satisfied for a larger set of discount factors than constraint (B.2). \square

B.4 Proof of Lemma 3.6

Given the realization of π_t^1 , the manager has no incentive to defect in stage 2 iff.

$$E_C + \delta G_C(\underline{\pi} - \pi_t^1) \frac{E_N + G_N(\tau) E_C + (1 - G_N(\tau)) E_N}{1 - \delta P_N(\underline{\pi}, \tau)} \geq E_D + \delta G_D(\underline{\pi} - \pi_t^1) \times \frac{2E_N}{1 - \delta G_{NN}(\underline{\pi})},$$

which rewrites as

$$\delta \left(\frac{G_C(\underline{\pi} - \pi_t^1)}{1 - \delta P_N(\underline{\pi}, \tau)} K_N(\tau) - \frac{G_D(\underline{\pi} - \pi_t^1)}{1 - \delta G_{NN}(\underline{\pi})} 2E_N \right) \geq E_D - E_C,$$

for every profit realization $\pi_t^1 \geq \tau$. By the regularity condition, we have that the above constraint is most difficult to satisfy if $\pi_t^1 = \tau$; thus, stability is determined by condition (3.13) in Lemma 3.6. \square

B.5 Proof of Proposition 3.6

First claim: price wars in equilibrium. The collusive strategies described in Lemmas 3.5 and 3.6 entail competition in stage 2 after the realization of profit $\pi_t^1 < \tau$ in stage 1; also, the collusive strategy described in Lemma 3.6 always entails competition in stage 1. Moreover, the collusive strategies described in Lemmas 3.5 and 3.6 are potentially more stable than collusion in both stages. Thus, contracts that span multiple managerial interactions potentially entail price wars in equilibrium.

Second claim: if managers adopt a strategy entailing price wars in equilibrium, then cartel stability is increased. The collusive strategies described in Lemmas 3.5 and 3.6 are potentially more stable, but entail a lower profitability—see the proof below. Thus, if managers adopt such a strategy, it means that they are not patient enough to collude in both stages, and, thus, adopt a strategy entailing equilibrium price wars, while compromising on cartel profitability.

Third claim: price wars reduce cartel profitability. The profitability of (i) collusion in both stages; (ii) collusion in stage 1 and τ -conditional collusion in stage 2; and (iii) competition in stage 1 and τ -conditional collusion in stage 2 is, respectively,

$$\frac{2E_C}{1 - \delta G_{CC}(\underline{\pi})} > \frac{K_C(\tau)}{1 - \delta P_C(\underline{\pi}, \tau)} > \frac{K_N(\tau)}{1 - \delta P_N(\underline{\pi}, \tau)},$$

because $2E_C > K_C(\tau) > K_N(\tau)$ and $G_{CC}(\underline{\pi}) > P_C(\underline{\pi}, \tau) > P_N(\underline{\pi}, \tau)$. \square

Appendix C

This appendix contains the derivations of Chapter 4.

C.1 Benchmarks of Subsection 4.3.1

Outcome (4.3) is straightforwardly obtained as the static Nash equilibrium when both owners independently maximize $\pi_i = (p - c) q_i$, while outcome (4.4) is obtained when owners jointly maximize $\sum_{i=1}^2 \pi_i$. When owner j produces $q_j^C = \frac{a-c}{4b}$, owner i 's optimal defection quantity is $q_i^D = \arg \max_{q_i} \left\{ \left(a - b \left(q_i + q_j^C \right) - c \right) q_i \right\} = \frac{3(a-c)}{8b}$, leading to profit $\pi_i^D = \frac{9(a-c)^2}{64b}$. Thus, collusion is stable if and only if

$$\delta_o \geq \frac{\pi_i^D - \pi_i^C}{\pi_i^D - \pi_i^N} = \frac{9}{17}.$$

Consider FJS's one-shot Cournot delegation game. In stage 3, both managers independently maximize $M_i = (p - \alpha_i c) q_i$, leading to quantities as a function of incentives

$$q_i(\alpha_i, \alpha_j) = \frac{a - 2\alpha_i c + \alpha_j c}{3b}. \quad (\text{C.1})$$

In stage 2, both owners substitute these into $\pi_i = (a - b(q_i + q_j) - c) q_i$ to independently maximize profit, yielding outcome (4.2), provided that both owners indeed delegate in stage 1.

If both owners keep control, they each earn the Cournot Nash profit $\pi_i^N = \frac{(a-c)^2}{9b}$. If owner i delegates, while owner j keeps control, then quantities as a function of incentives α_i become $q_i(\alpha_i, 1)$ and $q_j(1, \alpha_i)$ by (C.1). In stage 2, owner i then maximizes $\pi_i(\alpha_i) = (a - b(q_i(\alpha_i, 1) + q_j(1, \alpha_i)) - c) q_i(\alpha_i, 1)$, yielding $\alpha_i = \frac{5c-a}{4c}$ and

$$\pi_i = \frac{(a-c)^2}{8b}, \pi_j = \frac{(a-c)^2}{16b}. \quad (\text{C.2})$$

Since owner i is better off by delegating if her rival keeps control, while owner j is worse off if she keeps control and her rival delegates compared to when both owners delegate, owners indeed delegate in stage 1.

C.2 Equilibrium Incentives With Delegation

In stage 3, managers jointly maximize $\sum_{i=1}^2 M_i$, yielding $q_1 + q_2 = \frac{a-\alpha_1 c}{2b} = \frac{a-\alpha_2 c}{2b}$. Focusing on symmetric equilibria, both managers set the same quantity as a function of incentives, $q_1 = q_2 = \frac{a-\alpha_2 c}{2b} = \frac{a-\alpha_1 c}{2b}$, which holds for symmetric incentives $\alpha_1 = \alpha_2 = \alpha$, resulting in $q_1 = q_2 = \frac{a-\alpha c}{2b}$. Substituting these in the owners' profit functions gives $\pi_i(\alpha) = \frac{[a-(2-\alpha)c](a-\alpha c)}{8b}$, which is maximized at $\alpha_1^{dC} = \alpha_2^{dC} = \alpha = 1$ in stage 2, resulting in outcome (4.5).

C.3 Optimality of “Not Delegating Control” As the Punishment Strategy

This appendix shows that not delegating control is indeed the best strategy for owners to punish a deviant manager. First, suppose owners instead punish by reverting to “delegation and compete in setting incentives.” We then get FJS's static delegation outcome (4.2) with managerial payoff $M_i^{dN} = \frac{4(a-c)^2}{25b}$, which is actually higher than managerial payoff in the collusive delegation equilibrium $M_i^{dC} = \frac{(a-c)^2}{8b}$, thereby making collusion fully unstable in the first place.

Second, suppose owners punish by reverting to “delegation and collude in setting incentives.” In stage 3, managers set quantities as outlined in (C.1). In stage 2, owners substitute these into their joint profit function $\sum_{i=1}^2 \pi_i$, which is maximized with symmetric incentives $\alpha_i = \frac{3}{4} + \frac{a}{4c}$, yielding $\pi_i = \frac{(a-c)^2}{8b}$ and $M_i = \frac{(a-c)^2}{18b}$. If owner i deviates by setting different incentives, straightforward algebra leads to the optimal deviating incentive being $\alpha_i = \frac{21}{16} - \frac{5a}{16c}$ with profit $\pi_i = \frac{25(a-c)^2}{128b}$. This triggers punishment by FJS's static Nash equilibrium with $\pi_i^{dN} = \frac{2(a-c)^2}{25b}$. Thus, owners can commit to punishment iff.

$\delta_o \geq \frac{\frac{25(a-c)^2}{128b} - \frac{(a-c)^2}{8b}}{\frac{25(a-c)^2}{128b} - \pi_i^{dN}} = \frac{25}{41}$, and managers do not defect in the first place iff. $\delta_m \geq \frac{\frac{9(a-c)^2}{64b} - \frac{(a-c)^2}{8b}}{\frac{9(a-c)^2}{64b} - \frac{(a-c)^2}{8b}} = \frac{9}{49}$. These stability conditions are more difficult to satisfy than $\delta_o \geq \frac{25}{97}$, $\delta_m \geq \frac{1}{9}$.

C.4 Owner's Commitment to Avoid Delegation

Suppose owners punish a deviant manager by keeping control, while *competing on the product market*. Owner i then earns $\pi_i^N = \frac{(a-c)^2}{9b}$. If she defects from the punishment

scheme by delegating control, then in stage 3 manager i and owner j compete with respective payoffs $M_i(q_i, q_j) = (a - b(q_i + q_j) - \alpha_i c) q_i$ and $\pi_j(q_i, q_j) = (a - b(q_i + q_j) - c) q_j$, yielding quantities $q_i(\alpha_i) = \frac{a + (1 - 2\alpha_i)c}{3}$, $q_j(\alpha_i) = \frac{a + (\alpha_i - 2)c}{3}$ and profit

$$\pi_i(\alpha_i) = \left(a - b \left(\frac{a + (1 - 2\alpha_i)c}{3} + \frac{a + (\alpha_i - 2)c}{3} \right) - c \right) \frac{a + (1 - 2\alpha_i)c}{3},$$

which owner i maximizes at $\pi_i = \frac{(a-c)^2}{8b}$ with $\alpha_i = \frac{5}{4} - \frac{a}{4c}$. Since defection triggers punishment by FJS's one-shot delegation Nash equilibrium with profit $\pi_i^{dN} = \frac{2(a-c)^2}{25b}$ (see equations (4.2)), owners can commit to punishment if and only if $\delta_o \geq \frac{\frac{(a-c)^2}{8} - \frac{(a-c)^2}{9}}{\frac{(a-c)^2}{8} - \frac{2(a-c)^2}{25}} = \frac{25}{81}$.

Now suppose owners punish a deviant manager by keeping control, while *colluding on the product market*. Owner i then earns $\pi_i^C = \frac{(a-c)^2}{8b}$, while defection from the punishment scheme by delegating control results in competition between manager i and owner j with defection profit $\pi_i = \frac{(a-c)^2}{8b}$ (see equation (C.2)). Since defection profit equals collusive profit, owners will not defect from punishment through delegation.

Appendix D

This appendix contains the proofs of Chapter 5.

D.1 Proof of Proposition 5.5

Without a buyer group, competitive equilibrium wholesale contracts are $(w_i^c, \tau_i^c) = (c, 0)$, resulting in Cournot per-firm quantity, price and per-firm profit of

$$q_i^c = \frac{1-c}{n+1}, p_i^c = \frac{1+nc}{n+1}, \pi_i^c = \frac{(1-c)^2}{(n+1)^2}, \quad \forall i.$$

Below, we determine the optimal buyer group contracts (Lemma D.1) and deviation payoff (Lemma D.2) when the buyer group reduces marginal cost to $c - \alpha$.

Lemma D.1 *With buyer group specific cost savings α , optimal buyer group contracts are*

$$\left(w_i^{bg}(\alpha), \tau_i^{bg}(\alpha) \right) = \left(\frac{n-1+(n+1)(c-\alpha)}{2n}, -\frac{(n-1)(1-(c-\alpha))^2}{4n^2} \right), \quad \forall i$$

resulting in per-firm quantity, price and profit of

$$q_i^{bg}(\alpha) = \frac{1-(c-\alpha)}{2n}, p_i^{bg}(\alpha) = \frac{1+(c-\alpha)}{2}, \pi_i^{bg}(\alpha) = \frac{(1-(c-\alpha))^2}{4n}.$$

Proof. The buyer group sets linear fees $w_i^{bg}(\alpha)$ that induce retailers to jointly produce the monopoly output with marginal cost $c - \alpha$, i.e.,

$$\begin{aligned} Q^m &= \arg \max_Q Q [P(Q) - (c - \alpha)] \\ &= \frac{1 - (c - \alpha)}{2}. \end{aligned}$$

Denote by $Q_{-i} = \sum_{j \neq i} q_j$ the sum of all quantities produced, except for retailer i . The retailers produce symmetric per-firm monopoly outputs if linear wholesale prices $w_i^{bg}(\alpha)$ satisfy Cournot reaction functions

$$q_i(Q_{-i}) = \frac{1 - Q_{-i} - w_i}{2}, \quad \forall i \quad (\text{D.1})$$

with $q_i(Q_{-i}) = Q^m/n$, $Q_{-i} = (n-1)Q^m/n$ and $w_i = w_i^{bg}(\alpha)$. This is the case if

$$w_i^{bg}(\alpha) = \frac{n-1 + (n+1)(c-\alpha)}{2n}.$$

The per-unit revenue made by suppliers on the linear wholesale fees is $w_i^{bg}(\alpha) - (c-\alpha)$, which is refunded to retailers via fixed fees, i.e.,

$$\begin{aligned} \tau_i^{bg}(\alpha) &= -q_i^{bg}(\alpha) \left(w_i^{bg}(\alpha) - (c-\alpha) \right) \\ &= -\frac{(n-1)(1-(c-\alpha))^2}{4n^2}. \end{aligned} \quad \square$$

Lemma D.2 *The optimal deviating contract is $(w_i^d, \tau_i^d) = (c, 0)$, resulting in deviating quantity, price and profit of*

$$\begin{aligned} q_i^d(\alpha) &= \frac{(n+1)(1-c) - (n-1)\alpha}{4n}, \\ p^d(\alpha) &= \frac{n+1 + (3n-1)c - (n-1)\alpha}{4n}, \\ \pi_i^d(\alpha) &= \frac{((n+1)(1-c) - (n-1)\alpha)^2 + 4(n-1)(1-(c-\alpha))^2}{16n^2}. \end{aligned}$$

Proof. Retailer i optimally defects by giving a best response to the per-firm monopoly quantities produced by all other retailers. The optimal deviating output is characterized by reaction function (D.1) with $w_i = c$ and $Q_{-i} = (n-1)q_i^{bg}(\alpha)$, i.e.,

$$q_i^d(\alpha) = \frac{(n+1)(1-c) - (n-1)\alpha}{4n},$$

from which price and profits are determined as $p^d(\alpha) = 1 - (n-1)q_i^{bg}(\alpha) - q_i^d$ and $\pi_i^d(\alpha) = q_i^d(p^d - w_i^d) - \tau_i^{bg}(\alpha) - \tau_i^d$, respectively. \square

Now, if the linear wholesale fee negotiated through the buyer group is lower than or equal to marginal cost c , then a retailer will never defect, i.e., $\delta_{bg}^*(\alpha) = 0$. This is the case if and only if

$$w_i^{bg}(\alpha) \leq w_i^d = c \Leftrightarrow \alpha \geq \frac{(n-1)(1-c)}{n+1}.$$

For $\alpha < (n-1)(1-c)/(n+1)$, the critical discount factor is

$$\begin{aligned} \delta_{bg}^*(\alpha) &= \frac{\pi_i^d(\alpha) - \pi_i^{bg}(\alpha)}{\pi_i^d(\alpha) - \pi^c} \\ &= \frac{(n+1)^2 \left\{ [(n+1)(1-c) - (n-1)\alpha]^2 - 4[1 - (c-\alpha)]^2 \right\}}{(n+1)^2 \left\{ [(n+1)(1-c) - (n-1)\alpha]^2 + 4(n-1)[1 - (c-\alpha)]^2 \right\} - 16n^2(1-c)^2}. \end{aligned}$$

We have

$$\frac{\partial \delta_{bg}^*(\alpha)}{\partial \alpha} = -\frac{16(1-c)n^2(n+1)^2 K_1}{(n-1)K_2},$$

where

$$\begin{aligned} K_1 &= (1-c) [4\alpha(n+1) + (1-c)(n^2 + 2n + 5)] - \alpha^2(n+1)^2, \\ K_2 &= \left\{ \alpha(n+1)^2 [\alpha(n+3) - 2(1-c)(n-3)] \right. \\ &\quad \left. + (1-c)^2 (n^3 + 9n^2 + 3n + 3) \right\}^2 > 0. \end{aligned}$$

To show that $\partial \delta_{bg}^*(\alpha) / \partial \alpha < 0$ for $\alpha \in [0, (n-1)(1-c)/(n+1)]$, we need to show that $K_1 > 0$ in this domain. Therefore, substitute $\alpha = \beta(n-1)(1-c)/(n+1)$, with $\beta \in [0, 1]$, for α in K_1 to get

$$K_1 = (1-c)^2 \left[\underbrace{(1-\beta^2)n^2}_{\geq 0} + \underbrace{(2+2\beta^2+4\beta)n+5}_{>0} - \underbrace{\beta^2-4\beta}_{\geq 0} \right] > 0,$$

thus showing that $\partial \delta_{bg}^*(\alpha) / \partial \alpha < 0$ for $\alpha \in [0, (n-1)(1-c)/(n+1)]$. \square

D.2 Proof of Proposition 5.6

Retailer i 's benefit from forming a buyer group can be decomposed into the gain resulting from the cost savings and the gain resulting from the collusion effect,

$$\begin{aligned} \Delta\pi_i &= \pi_i^{bg}(\alpha) - \pi_i^* \\ &= \underbrace{\pi_i^{bg}(\alpha) - \pi_i^m}_{\text{cost savings}} + \underbrace{\pi_i^m - \pi_i^*}_{\text{collusion}}, \end{aligned}$$

where $\pi_i^m = \max_Q Q(p(Q) - c)/n = (1 - c)^2 / (4n)$ is the collusive (monopoly) per-firm profit when marginal cost is c . Substituting for the relevant profits gives the per-firm benefit from the buyer group,

$$\begin{aligned} \Delta\pi_i &= \frac{(n - 1)^2 (1 - c)^2 + \alpha (\alpha + 2 (1 - c)) (n + 1)^2}{4n (n + 1)^2} \\ &= \underbrace{\frac{\alpha (\alpha + 2 (1 - c))}{4n}}_{\text{cost savings} > 0} + \underbrace{\frac{(n - 1)^2 (1 - c)^2}{4n (n + 1)^2}}_{\text{collusion} > 0} > 0. \end{aligned}$$

The consumers' benefit from the buyer group can be decomposed into the gain resulting from the pass-on of cost savings and the loss resulting from the collusion effect,

$$\begin{aligned} \Delta CS &= CS^{bg}(\alpha) - CS^* \\ &= \underbrace{CS^{bg}(\alpha) - CS^m}_{\text{cost savings}} - \underbrace{(CS^* - CS^m)}_{\text{collusion}}, \end{aligned}$$

where $CS^{bg}(\alpha)$, CS^* , and CS^m are consumer surplus under the buyer group regime, without the buyer group and with a monopoly (cartel) without cost savings, respectively,

$$CS^{bg}(\alpha) = \frac{(1 - (c - \alpha))^2}{8}, CS^* = \frac{n^2 (1 - c)^2}{2(n + 1)^2}, CS^m = \frac{(1 - c)^2}{8}.$$

Therefore, the increase in consumer surplus resulting from the buyer group is

$$\begin{aligned}\Delta CS &= \frac{\alpha(\alpha + 2(1 - c))(n + 1)^2 - (3n^2 - 2n - 1)(1 - c)^2}{8(n + 1)^2} \\ &= \underbrace{\frac{\alpha(\alpha + 2(1 - c))}{8}}_{\text{cost savings} > 0} - \underbrace{\frac{(3n^2 - 2n - 1)(1 - c)^2}{8(n + 1)^2}}_{\text{collusion} < 0},\end{aligned}$$

which is positive if and only if $\alpha > \alpha^{CS} = (n - 1)(1 - c) / (n + 1)$.

Similarly, the increase in total welfare resulting from the buyer group is

$$n\Delta\pi_i + \Delta CS = \frac{3(n + 1)^2(1 - (c - \alpha))^2 - 4n(n + 2)(1 - c)^2}{8(n + 1)^2},$$

which is positive if and only if

$$\alpha > \alpha^{TW} = \frac{(1 - c) \left[\sqrt{\frac{4}{3}n(n + 2)} - (n + 1) \right]}{n + 1}.$$

We have that

$$\alpha^{TW} < \alpha^{CS},$$

because

$$\alpha^{TW} - \alpha^{CS} = \frac{(1 - c)K_3}{n + 1},$$

where $K_3 = \sqrt{\frac{4}{3}n(n + 2)} - 2n < 0$ for all $n \geq 2$. □

Appendix E

This appendix contains the proofs and additional illustrations of Chapter 6.

E.1 Decomposition of Harm Illustrated

Figure AP.1 illustrates the various typical effects decomposed in Section 6.2.2 in a simple model with three production layers ($K = 3$) and final consumers.

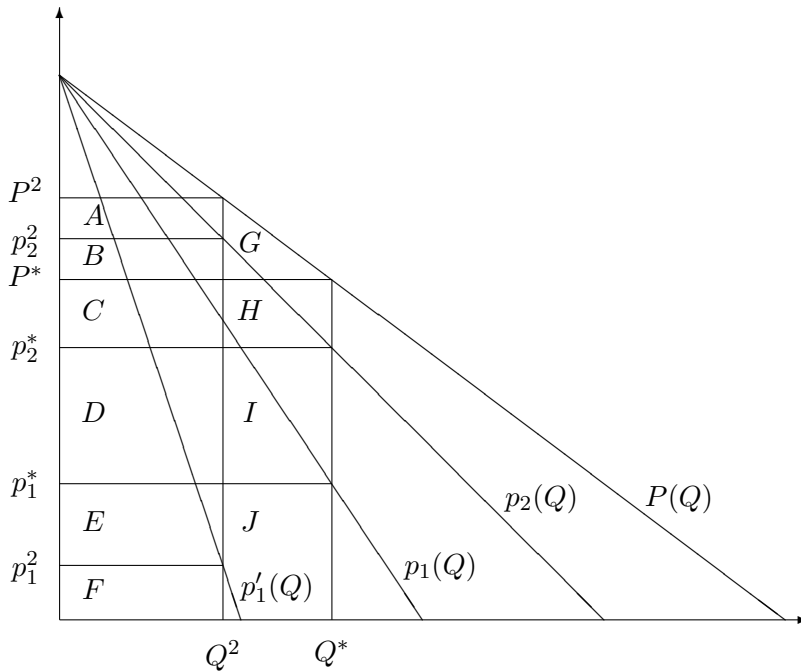


FIGURE AP.1 *Decomposition of antitrust harm in a three layer model.*

We assume that marginal own production costs are constant and equal to zero for all firms, i.e., $c_{j1}(q) = c_{j2}(q) = c_{j3}(q) = 0$, for all j and every q . Given inverse consumer demand $P(Q)$, the implied inverse demand for the product of firms in layer 2 is given by $p_2(Q)$. Competition in layer 2 results in inverse demand function $p_1(Q)$ for the firms in layer 1. Their competitive benchmark is given by the quantity Q^* and prices P^* , p_2^* and p_1^* , respectively. Now suppose firms in the second layer collude. This leads to a reduction in the quantity they supply. That is, for every price p_1 , layer 2 demands less of the input supplied by layer 1, resulting in an inwards shift of $p_1(Q)$ to $p_1'(Q)$. Under the cartel regime, output decreases to Q^2 , and equilibrium prices become P^2 , p_2^2 and p_1^2 , respectively. Note that in this illustration we assume that p_1 decreases under the cartel, which need not be the case.

It is insightful to identify some of the areas in the graph. The loss in profits of the direct purchasers of the cartel (layer 3) equal

$$\Delta\pi_3 = \pi_3^* - \pi_3^2 = C + H - A = (B + C) - (A + B) + H = \xi_3 - \omega_3 + \sigma_3,$$

with $\xi_3 = (p_2^2 - p_2^*)Q^2 = B + C$ being the amount by which the firms in layer 3 are overcharged, $\omega_3 = (P^2 - P^*)Q^2 = A + B$ the amount passed-on to the final consumers, and $\sigma_3 = H$ the output effect. The loss in consumer surplus is

$$\Delta CS = CS^* - CS^2 = (A + B) + G = \xi_C + \sigma_C,$$

where $\xi_C = A + B = \omega_3$ is the overcharge imposed by layer 3, and $\sigma_C = G$ is the output effect for consumers. Profits of the (direct) suppliers to the cartel change by

$$\Delta\pi_1 = \pi_1^* - \pi_1^2 = E + F + J - F = E + J = -\omega_1 + \sigma_1,$$

where $\omega_1 = -E$ is the pass-on from layer 1 to layer 2, which is negative since p_1 has increased. Note that there is no overcharge for layer 1. Its output effect is given by $\sigma_1 = J$. Finally, consider the colluding layer 2. Profits of the cartel members change by

$$\Delta\pi_2 = \pi_2^* - \pi_2^2 = D + I - (B + C + D + E) = -E - (B + C) + I = \xi_2 - \omega_2 + \sigma_2,$$

where $\xi_2 = -E = \omega_1$ is the overcharge from their direct suppliers, $\omega_2 = B + C$ is the pass-on to the next layer and σ_2 is the output effect. The sum total of these effects is negative or otherwise it would not be profitable for the cartel to form. Notice also that

$$\Delta\pi_1 + \Delta\pi_2 + \Delta\pi_3 + \Delta CS = G + H + J + I = \sigma_1 + \sigma_2 + \sigma_3 + \sigma_C,$$

that is, the sum of all effects combined reduces to the sum of output effects.

E.2 Proof of Proposition 6.2

For $\gamma = 1$, we have

$$\lambda_D = \frac{Q^* + Q^g}{Q^g} \left(1 - \frac{1}{2} \prod_{i=g+1}^K \frac{n_i}{n_i + \vartheta_i} \right).$$

It follows that we have $\frac{1}{2} \frac{Q^* + Q^g}{Q^g} \leq \lambda_D < \frac{Q^* + Q^g}{Q^g}$. The lower bound is reached if all downstream layers are perfectly competitive and the upper bound is reached with an infinite number of imperfectly competitive layers downstream. Since the cartel reduces production, we have $Q^g < Q^*$ and hence $\lambda_D > 1$. In addition, it follows from $\frac{Q^*}{Q^g} = \frac{n_g + \vartheta_g^c}{n_g + \vartheta_g}$ that $Q^* \leq 2Q^g$, with $Q^* = 2Q^g$ if the pre-cartel industry was perfectly competitive ($n_g \rightarrow \infty$ or $\vartheta_g = 0$) and the cartel sets the full cartel quantity ($\vartheta_g^c = n_g$). In this case $\frac{Q^* + Q^g}{Q^g} = 3$ and, therefore, $\lambda_D < 3$. \square

E.3 Intermediate Results

This appendix derives intermediate results needed for the proofs of Propositions 6.3 (see E.4), 6.4 (see E.6), and 6.5 (see E.7).

Our first intermediate result characterizes the implied inverse demand function for each layer k .

Lemma E.1 *Given final consumer demand $P(Q) = a - bQ^\gamma$, constant marginal costs c_k in layer k and conjectural variations parameter ϑ_k in layer k , the implied inverse demand function faced by firms in layer k is*

$$p_k(Q) = a - \left(b \prod_{i=k+1}^K \frac{n_i + \gamma \vartheta_i}{n_i} Q^\gamma + \sum_{l=k+1}^K c_l \right), \quad (\text{E.1})$$

for $k = 1, \dots, K - 1$. Furthermore, $p_K(Q) = P(Q)$.

Proof. We will show that for a given K , (E.1) holds for all $1 \leq k \leq K - 1$. Consider firm i in layer k . It maximizes profits $p_k(Q) q_i - (c_k - p_{k-1}) q_i$, where $p_k(Q)$ is the implied inverse demand function that the industry in layer k faces. Using symmetry ($q_i = Q/n_k$) and conjectural variations ($\vartheta_k = dQ/dq_i$) the first-order condition equals

(6.1). Solving for p_{k-1} then gives

$$p_{k-1}(Q) = p_k(Q) + \frac{\vartheta_k}{n_k} Q p'_k(Q) - c_k. \quad (\text{E.2})$$

Given $p_k(Q)$, the implied inverse demand for layer $k-1$, $p_{k-1}(Q)$ can, therefore, be determined recursively. We need to show that (E.1) satisfies the recursive relation (E.2) for all $k = 2, \dots, K$. First consider $k = K$. Using $p_K(Q) = P(Q) = a - bQ^\gamma$ (E.2) reduces to $p_{K-1}(Q) = a - b \frac{n_K + \gamma \vartheta_K}{n_K} Q^\gamma - c_K$, which is equivalent to (E.1) for $k = K-1$. Now we proceed by induction. Assuming that (E.1) holds for k we want to show that it also holds for $k-1$. We have

$$\begin{aligned} p_{k-1}(Q) &= p_k(Q) + p'_k(Q) \frac{\vartheta_k}{n_k} Q - c_k \\ &= a - \left(b \prod_{i=k+1}^K \frac{n_i + \gamma \vartheta_i}{n_i} Q^\gamma + \sum_{l=k+1}^K c_l \right) - \left(\gamma b \prod_{i=k+1}^K \frac{n_i + \gamma \vartheta_i}{n_i} \right) \\ &\quad \times Q^\gamma \frac{\vartheta_k}{n_k} - c_k \\ &= a - \left(b \frac{n_k + \gamma \vartheta_k}{n_k} \prod_{i=k+1}^K \frac{n_i + \gamma \vartheta_i}{n_i} Q^\gamma + c_k + \sum_{l=k+1}^K c_l \right) \\ &= a - \left(b \prod_{i=k}^K \frac{n_i + \gamma \vartheta_i}{n_i} Q^\gamma + \sum_{l=k}^K c_l \right). \end{aligned}$$

Therefore equation (E.1) holds for all k with $1 \leq k \leq K-1$. Finally, it is easily checked that $\gamma > 0$ is a sufficient condition for the second-order condition for an optimum to be satisfied for every individual firm at the equilibrium. \square

Knowing the implied inverse demand functions (E.1), we can now determine equilibrium quantities and prices.

Proposition E.1 *In this model, equilibrium prices and quantities are given by*

$$Q^* = \left[\frac{1}{b} \left(\prod_{i=1}^K \frac{n_i}{n_i + \gamma \vartheta_i} \right) \left(a - \sum_{j=1}^K c_j \right) \right]^{\frac{1}{\gamma}}, \quad (\text{E.3})$$

$$p_k^* = \left(1 - \prod_{i=1}^k \frac{n_i}{n_i + \gamma \vartheta_i} \right) \left(a - \sum_{j=1}^K c_j \right) + \sum_{l=1}^k c_l \quad \forall k \in \{1 \dots K\}. \quad (\text{E.4})$$

Aggregate profits of firms in layer k and consumer surplus are then given by

$$\pi_k^* = \left(\frac{1}{b}\right)^{\frac{1}{\gamma}} \frac{\gamma \vartheta_k}{n_k + \gamma \vartheta_k} \prod_{i=1}^{k-1} \frac{n_i}{n_i + \gamma \vartheta_i} \left(\prod_{i=1}^K \frac{n_i}{n_i + \gamma \vartheta_i}\right)^{\frac{1}{\gamma}} \left(a - \sum_{j=1}^K c_j\right)^{\frac{\gamma+1}{\gamma}} \quad (\text{E.5})$$

$$CS = \frac{\gamma}{\gamma+1} \left(\frac{1}{b}\right)^{\frac{1}{\gamma}} \left(\prod_{i=1}^K \frac{n_i}{n_i + \gamma \vartheta_i}\right)^{\frac{\gamma+1}{\gamma}} \left(a - \sum_{j=1}^K c_j\right)^{\frac{\gamma+1}{\gamma}}. \quad (\text{E.6})$$

Proof. Using (E.1) the first-order condition (6.1) for $k = 1$ reduces to

$$a - \left(b \prod_{i=1}^K \frac{n_i + \gamma \vartheta_i}{n_i} Q^\gamma + \sum_{l=1}^K c_l\right) = 0.$$

Solving for Q gives (E.3). Substituting Q^* into (E.1) gives (E.4). Profits and consumer surplus follow from substituting (E.3) and (E.4) in $\pi_k^* = (p_k^* - p_{k-1}^* - c_k) Q^*$ and $CS = \int_0^{Q^*} P(Q) dQ - p_K^* Q^* = \frac{\gamma}{\gamma+1} b (Q^*)^{\gamma+1} = \frac{\gamma}{\gamma+1} (a - p_K^*) Q^*$, resp. \square

We are interested in how the equilibrium quantity Q^* and equilibrium prices p_k^* depend upon the underlying model parameters. It is easily checked that Q^* increases with an increase in a or n_i and with a decrease in ϑ_i , b or the number of (imperfectly competitive) layers K . The equilibrium price for layer k , p_k^* , increases with an increase in a , γ , c_i or ϑ_i , for $i \leq k$, and with a decrease in n_i for $i \leq k$, with a decrease in c_i for $l > k$.

The dependence of Q^* on γ , however, is ambiguous and Q^* could either decrease or increase with γ .¹⁵⁶ Moreover, we have

$$\lim_{\gamma \rightarrow \infty} p_k^* = a - \sum_{j=k+1}^K c_j \quad \text{and} \quad \lim_{\gamma \rightarrow 0} p_k^* = \sum_{j=1}^k c_j.$$

Firms in layer 1 have all the market power, that is, they extract the entire surplus by setting price p_1^* equal to a minus the marginal costs of the other layers. The other layers price competitively, in the sense that they only recover their marginal costs.

Using $\lim_{\gamma \rightarrow \infty} \left(1 + \frac{\vartheta_i}{n_i} \gamma\right)^{\frac{1}{\gamma}} = 1$ and $\lim_{\gamma \rightarrow 0} \left(1 + \frac{\vartheta_i}{n_i} \gamma\right)^{\frac{1}{\gamma}} = \exp\left[\frac{\vartheta_i}{n_i}\right]$, respectively,

¹⁵⁶Take, for example, the case with $\vartheta_i = 0$ for all i . Then Q^* increases (decreases) with γ if $a - \sum_{k=1}^K c_k$ is larger (smaller) than b .

we find

$$\lim_{\gamma \rightarrow \infty} Q^* = 1 \text{ and } \lim_{\gamma \rightarrow 0} Q^* = \begin{cases} 0 & \text{if } b > a - \sum_{j=1}^K c_j \\ \exp \left[- \sum_{i=1}^K \frac{\vartheta_i}{n_i} \right] & \text{if } b = a - \sum_{j=1}^K c_j \\ \infty & \text{if } b < a - \sum_{j=1}^K c_j. \end{cases}$$

Hence, when demand becomes infinitely concave ($\gamma \rightarrow \infty$), the equilibrium quantity is 1, independent of all other parameters. Therefore, when demand becomes infinitely concave ($\gamma \rightarrow 0$), the equilibrium price is 0 for all layers, independent of all other parameters. The equilibrium quantity is 0 (∞) when $b > (<) a - \sum_{j=1}^K c_j$; only when this condition holds with equality equilibrium quantity is positive and finite.

Collusion in layer g is modeled as an increase in the conjectural variations parameter from ϑ_g to $\vartheta_g^c \in (\vartheta_g, n_g]$. Cartel quantity and prices are

$$Q^g = \left[\frac{1}{b} \frac{n_g + \gamma \vartheta_g}{n_g + \gamma \vartheta_g^c} \left(\prod_{i=1}^K \frac{n_i}{n_i + \gamma \vartheta_i} \right) Z \right]^{\frac{1}{\gamma}}, \tag{E.7}$$

$$p_k^g = \begin{cases} \left(1 - \frac{n_g + \gamma \vartheta_g}{n_g + \gamma \vartheta_g^c} \prod_{i=1}^k \frac{n_i}{n_i + \gamma \vartheta_i} \right) Z + \sum_{l=1}^k c_l & k \geq g \\ p_k^* & k < g, \end{cases} \tag{E.8}$$

where $Z = \left(a - \sum_{j=1}^K c_j \right)$.

The next lemma gives expressions for pass-on & output effects (recall that $\xi_{k+1} = \omega_k$).

Lemma E.2 *The pass-on effect ω_k for layer $k \geq g$ is given by*

$$\begin{aligned} \omega_k &= \left(\frac{1}{b} \right)^{\frac{1}{\gamma}} \left(\frac{n_g + \gamma \vartheta_g}{n_g + \gamma \vartheta_g^c} \right)^{\frac{1}{\gamma}} \left(\frac{\gamma \vartheta_g^c - \gamma \vartheta_g}{n_g + \gamma \vartheta_g^c} \right) \left(\prod_{i=1}^k \frac{n_i}{n_i + \gamma \vartheta_i} \right) \\ &\quad \times \left(\prod_{i=1}^K \frac{n_i}{n_i + \gamma \vartheta_i} \right)^{\frac{1}{\gamma}} \left(a - \sum_{j=1}^K c_j \right)^{\frac{\gamma+1}{\gamma}}, \end{aligned} \tag{E.9}$$

and $\omega_k = 0$ for every $k \leq g - 1$.

The output effect for layer k and the output effect for consumers are given as

$$\sigma_k = \left(\frac{1}{b}\right)^{\frac{1}{\gamma}} \frac{\gamma\vartheta_k}{n_k} \left(1 - \left(\frac{n_g + \gamma\vartheta_g}{n_g + \gamma\vartheta_g^c}\right)^{\frac{1}{\gamma}}\right) \left(\prod_{i=1}^k \frac{n_i}{n_i + \gamma\vartheta_i}\right) \quad (\text{E.10})$$

$$\times \left(\prod_{i=1}^K \frac{n_i}{n_i + \gamma\vartheta_i}\right)^{\frac{1}{\gamma}} \left(a - \sum_{j=1}^K c_j\right)^{\frac{\gamma+1}{\gamma}},$$

$$\sigma_C = \left(\frac{1}{b}\right)^{\frac{1}{\gamma}} \left(\frac{\gamma + \left(\frac{n_g + \gamma\vartheta_g}{n_g + \gamma\vartheta_g^c}\right)^{\frac{\gamma+1}{\gamma}}}{\gamma + 1} - \left(\frac{n_g + \gamma\vartheta_g}{n_g + \gamma\vartheta_g^c}\right)^{\frac{1}{\gamma}}\right) \quad (\text{E.11})$$

$$\times \left(\prod_{i=1}^K \frac{n_i}{n_i + \gamma\vartheta_i}\right)^{\frac{\gamma+1}{\gamma}} \left(a - \sum_{j=1}^K c_j\right)^{\frac{\gamma+1}{\gamma}}.$$

Proof. Straightforward computations show that (for $k \geq g$) we have

$$p_k^g - p_k^* = \frac{\gamma\vartheta_g^c - \gamma\vartheta_g}{n_g + \gamma\vartheta_g^c} \left(\prod_{i=1}^k \frac{n_i}{n_i + \gamma\vartheta_i}\right) \left(a - \sum_{j=1}^K c_j\right),$$

$$p_k^* - p_{k-1}^* - c_k = \frac{\gamma\vartheta_k}{n_k} \left(\prod_{i=1}^k \frac{n_i}{n_i + \gamma\vartheta_i}\right) \left(a - \sum_{j=1}^K c_j\right),$$

$$Q^* - Q^g = \left(1 - \left(\frac{n_g + \gamma\vartheta_g}{n_g + \gamma\vartheta_g^c}\right)^{\frac{1}{\gamma}}\right) \times$$

$$\left[\frac{1}{b} \left(\prod_{i=1}^K \frac{n_i}{n_i + \gamma\vartheta_i}\right) \left(a - \sum_{j=1}^K c_j\right)\right]^{\frac{1}{\gamma}}.$$

Equations (E.9)–(E.10) follow immediately from substituting the above expressions into $\omega_k = Q^g (p_k^g - p_k^*)$ and $\sigma_k = (Q^* - Q^g) (p_k^* - p_{k-1}^* - c_k)$.

Furthermore, we have

$$\begin{aligned} \sigma_C &= \int_{Q^g}^{Q^*} [P(Q) - P(Q^*)] dQ \\ &= \frac{1}{\gamma + 1} b \left[(Q^g)^{\gamma+1} - (Q^*)^{\gamma+1} \right] + b (Q^*)^\gamma (Q^* - Q^g). \end{aligned}$$

Equation (E.11) then follows from substituting $Q^g = rQ^*$, with $r = \left(\frac{n_g + \gamma\vartheta_g}{n_g + \gamma\vartheta_g^c} \right)^{\frac{1}{\gamma}}$. \square

Using Lemma E.2, we can express the measures of harm discussed in Section 6.2.3 in terms of the parameters of the model.

Proposition E.2 *Denote by $r = \frac{Q^g}{Q^*} = \left(\frac{n_g + \gamma\vartheta_g}{n_g + \gamma\vartheta_g^c} \right)^{\frac{1}{\gamma}}$ the fraction by which the cartel reduces output. The damages measures (6.7)–(6.9) are equal to*

$$\lambda_D = \frac{1 - r^{\gamma+1}}{r(1 - r^\gamma)} \left(1 - \frac{1}{\gamma + 1} \prod_{i=g+1}^K \frac{n_i}{n_i + \gamma\vartheta_i} \right), \tag{E.12}$$

$$\lambda_U = \frac{(1 - r)}{r(1 - r^\gamma)} \frac{n_g + \gamma\vartheta_g}{n_g} \left(\prod_{i=1}^{g-1} \frac{n_i + \gamma\vartheta_i}{n_i} - 1 \right), \tag{E.13}$$

$$\lambda_g = \frac{\gamma\vartheta_g}{n_g} \frac{(1 - r)}{r(1 - r^\gamma)} - 1, \tag{E.14}$$

$$\lambda_{g+1} = \frac{\gamma\vartheta_{g+1}}{n_{g+1} + \gamma\vartheta_{g+1}} \frac{1 - r^{\gamma+1}}{r(1 - r^\gamma)}, \tag{E.15}$$

$$\lambda_C = \frac{\gamma(1 - r^{\gamma+1})}{(\gamma + 1)r(1 - r^\gamma)} \prod_{i=g+1}^K \frac{n_i}{n_i + \gamma\vartheta_i}, \text{ and} \tag{E.16}$$

$$\begin{aligned} \lambda_W &= \left(\frac{n_g + \gamma\vartheta_g^c}{\gamma\vartheta_g^c - \gamma\vartheta_g} \right) \left[\frac{r^\gamma - r^{-1}}{\gamma + 1} \left(\prod_{i=g+1}^K \frac{n_i}{n_i + \gamma\vartheta_i} \right) + (r^{-1} - 1) \right. \\ &\quad \left. \times \left(\prod_{i=1}^g \frac{n_i + \gamma\vartheta_i}{n_i} \right) \right]. \end{aligned} \tag{E.17}$$

Proof. Substituting equations (E.9)–(E.11) into (6.7) and using $\left(\frac{n_g + \gamma\vartheta_g}{n_g + \gamma\vartheta_g^c} \right)^{\frac{1}{\gamma}} = r$ and

$\frac{\gamma\vartheta_g^c - \gamma\vartheta_g}{n_g + \gamma\vartheta_g^c} = 1 - r^\gamma$ we obtain

$$\lambda_D = 1 + \frac{1-r}{r(1-r^\gamma)} \sum_{k=g+1}^K \frac{\gamma\vartheta_k}{n_k} \prod_{i=g+1}^k \frac{n_i}{n_i + \gamma\vartheta_i} + \frac{\gamma + r^{\gamma+1} - (\gamma+1)r}{(\gamma+1)r(1-r^\gamma)} \times \prod_{i=g+1}^K \frac{n_i}{n_i + \gamma\vartheta_i}.$$

Equation (E.12) then follows from using

$$1 - \sum_{k=l+1}^K \frac{\gamma\vartheta_k}{n_k} \left(\prod_{i=l+1}^k \frac{n_i}{n_i + \gamma\vartheta_i} \right) = \prod_{i=l+1}^K \frac{n_i}{n_i + \gamma\vartheta_i}, \quad (\text{E.18})$$

which can straightforwardly be shown to hold by induction. Similarly, (6.8) reduces to

$$\lambda_U = \frac{(1-r)}{r(1-r^\gamma)} \sum_{k=1}^{g-1} \frac{\gamma\vartheta_k}{n_k} \prod_{i=k+1}^g \frac{n_i + \gamma\vartheta_i}{n_i}.$$

Equation (E.13) then follows from using

$$\sum_{k=1}^{g-1} \frac{\gamma\vartheta_k}{n_k} \prod_{i=k+1}^g \frac{n_i + \gamma\vartheta_i}{n_i} = \frac{n_g + \gamma\vartheta_g}{n_g} \left(\prod_{i=1}^{g-1} \frac{n_i + \gamma\vartheta_i}{n_i} - 1 \right).$$

Equations (E.14)–(E.16) are derived analogously. \square

Corollary E.1 For linear inverse demand ($\gamma = 1$), the measures of harm are given as

$$\begin{aligned} \lambda_D &= \frac{Q^* + Q^g}{Q^g} \left(1 - \frac{1}{2} \prod_{i=g+1}^K \frac{n_i}{n_i + \vartheta_i} \right), \quad \lambda_U = \frac{n_g + \vartheta_g^c}{n_g} \left(\prod_{i=1}^{g-1} \frac{n_i + \vartheta_i}{n_i} - 1 \right), \\ \lambda_g &= \frac{\vartheta_g Q^*}{n_g Q^g} - 1, \quad \lambda_{g+1} = \frac{\vartheta_{g+1}}{n_{g+1} + \vartheta_{g+1}} \frac{Q^* + Q^g}{Q^g}, \quad \text{and} \\ \lambda_C &= \frac{1}{2} \frac{Q^* + Q^g}{Q^g} \prod_{i=g+1}^K \frac{n_i}{n_i + \vartheta_i}. \end{aligned}$$

Proof. This follows immediately from substituting $\gamma = 1$ and $\frac{1-r^2}{r(1-r)} = \frac{1+r}{r} = \frac{Q^* + Q^g}{Q^g}$ and $\frac{1-r}{r(1-r)} = \frac{1}{r} = \frac{Q^*}{Q^g}$ into equations (E.12)–(E.16). \square

E.4 Proof of Proposition 6.3

First note that we have $\lim_{\gamma \rightarrow \infty} r = 1$ and, using $\lim_{\gamma \rightarrow 0} (1 + \alpha\gamma)^{\frac{1}{\gamma}} = \exp[\alpha]$, we have $\lim_{\gamma \rightarrow 0} r = \exp\left[\frac{\vartheta_g - \vartheta_g^c}{n_g}\right]$. Moreover, the following results are useful

$$\begin{aligned} \lim_{\gamma \rightarrow \infty} r^{\gamma+1} &= \lim_{\gamma \rightarrow \infty} \left(\frac{1 + \gamma \frac{\vartheta_g}{n_g}}{1 + \gamma \frac{\vartheta_g^c}{n_g}} \right) \lim_{\gamma \rightarrow \infty} r = \frac{\vartheta_g}{\vartheta_g^c}, \\ \lim_{\gamma \rightarrow 0} r^{\gamma+1} &= \lim_{\gamma \rightarrow 0} \left(\frac{1 + \gamma \frac{\vartheta_g}{n_g}}{1 + \gamma \frac{\vartheta_g^c}{n_g}} \right) \lim_{\gamma \rightarrow 0} r = \exp\left[\frac{\vartheta_g - \vartheta_g^c}{n_g}\right]. \end{aligned}$$

Now it follows immediately that

$$\lim_{\gamma \rightarrow \infty} \lambda_D = \lim_{\gamma \rightarrow \infty} \frac{1 - r^{\gamma+1}}{r - r^{\gamma+1}} \lim_{\gamma \rightarrow \infty} \left(1 - \frac{1}{\gamma + 1} \prod_{i=g+1}^K \frac{n_i}{n_i + \gamma \vartheta_i} \right) = 1.$$

In order to evaluate $\lim_{\gamma \rightarrow 0} \lambda_D$ first notice that λ_D can be rewritten as

$$\begin{aligned} \lambda_D &= \frac{1 - r^{\gamma+1}}{r(1 - r^\gamma)} \left(1 - \frac{1}{\gamma + 1} \prod_{i=g+1}^K \frac{n_i}{n_i + \gamma \vartheta_i} \right) \\ &= \frac{1 - r^{\gamma+1}}{r} \frac{n_g + \gamma \vartheta_g^c}{\gamma(\gamma + 1)(\vartheta_g^c - \vartheta_g)} \left(\gamma + 1 - \prod_{i=g+1}^K \frac{n_i}{n_i + \gamma \vartheta_i} \right) \\ &= \frac{1 - r^{\gamma+1}}{r} \frac{n_g + \gamma \vartheta_g^c}{\gamma(\gamma + 1)(\vartheta_g^c - \vartheta_g)} \left(\gamma + \frac{\prod_{i=g+1}^K (n_i + \gamma \vartheta_i) - \prod_{i=g+1}^K n_i}{\prod_{i=g+1}^K (n_i + \gamma \vartheta_i)} \right) \\ &= \frac{1 - r^{\gamma+1}}{r} \frac{n_g + \gamma \vartheta_g^c}{\gamma(\gamma + 1)(\vartheta_g^c - \vartheta_g)} \times \\ &\quad \left(\gamma + \frac{\gamma \left[\sum_{i=g+1}^K (\vartheta_i \prod_{j=g+1, j \neq i}^K n_j) + f(\gamma) \right]}{\prod_{i=g+1}^K (n_i + \gamma \vartheta_i)} \right), \end{aligned}$$

where $f(\gamma)$ is a function with the property that $\lim_{\gamma \rightarrow 0} f(\gamma) = 0$. Taking the limit,

$$\lim_{\gamma \rightarrow 0} \lambda_D = \left(\frac{1 - \exp\left[\frac{\vartheta_g - \vartheta_g^c}{n_g}\right]}{\exp\left[\frac{\vartheta_g - \vartheta_g^c}{n_g}\right]} \right) \frac{n_g}{\vartheta_g^c - \vartheta_g} \left(1 + \frac{\sum_{i=g+1}^K (\vartheta_i \prod_{j=g+1}^K n_j / n_i)}{\prod_{i=g+1}^K n_i} \right)$$

$$= h\left(\frac{\vartheta_g^c - \vartheta_g}{n_g}\right) \left(1 + \sum_{i=g+1}^K \frac{\vartheta_i}{n_i}\right),$$

with $h(x) = \frac{\exp(x)-1}{x}$. Note that $\lim_{x \rightarrow 0} h(x) = 1$, $h'(x) > 0$ and $h(1) = e - 1$. Clearly, the second part of the above expression, $\left(1 + \sum_{i=g+1}^K \frac{\vartheta_i}{n_i}\right)$, is bounded from above by $K - g + 1$. Taken together we find that $\lim_{\gamma \rightarrow 0} \lambda_D \leq (1 + K - g)(e - 1)$, with the upper bound being exact when all downstream layers are monopolized (implying $\vartheta_i = n_i$ for all i), and there is full collusion in the colluding layer and perfect competition otherwise ($\frac{\vartheta_g^c - \vartheta_g}{n_g} = 1$). \square

Alternatively, one can consider the relation between total downstream harm and total profit loss (i.e., overcharge plus output effect) of the direct purchasers. That is,

$$\tilde{\lambda}_D = \frac{d_D}{\xi_{g+1} + \sigma_{g+1}} = \frac{\lambda_D}{\Xi},$$

where $\Xi = \frac{\xi_{g+1} + \sigma_{g+1}}{\xi_{g+1}}$. Obviously, $\Xi \geq 1$. Moreover, if layer $g + 1$ is perfectly competitive (that is, $\vartheta_{g+1} \rightarrow 0$ and/or $n_{g+1} \rightarrow \infty$) there is no output effect for this layer of direct purchasers, $\sigma_{g+1} = 0$. This implies $\Xi = 1$ and $\tilde{\lambda}_D = \lambda_D$. Obviously, the downstream damage multiplier $\tilde{\lambda}_D$ can, therefore, also take on any value. This holds even if the indirect purchaser layer is imperfectly competitive as the next lemma shows.

Lemma E.3 *For any $\bar{M} > 0$ and any value of $\vartheta_{g+1}/n_{g+1} \leq 1$, there exists a market structure such that $\tilde{\lambda}_D > \bar{M}$.*

Proof. First, using Lemma E.2 we find that

$$\Xi = 1 + \frac{1-r}{r} \frac{\vartheta_{g+1}}{n_{g+1} + \gamma \vartheta_{g+1}} \frac{n_g + \gamma \vartheta_g^c}{\vartheta_g^c - \vartheta_g}.$$

From the proof of Proposition 6.3 we know that $\lim_{\gamma \rightarrow 0} r = \exp\left[\frac{\vartheta_g - \vartheta_g^c}{n_g}\right]$. Using $h(x) = \frac{\exp(x)-1}{x}$ again we can write

$$\lim_{\gamma \rightarrow 0} \tilde{\lambda}_D = \lim_{\gamma \rightarrow 0} \frac{\lambda_D}{\Xi} = \frac{h\left(\frac{\vartheta_g^c - \vartheta_g}{n_g}\right)}{1 + \frac{\vartheta_{g+1}}{n_{g+1}} h\left(\frac{\vartheta_g^c - \vartheta_g}{n_g}\right)} \left(1 + \sum_{i=g+1}^K \frac{\vartheta_i}{n_i}\right).$$

Since $h(x)$ is maximized at $x = 1$ and $h(1) = e - 1$ we find an upper bound for $\tilde{\lambda}_D$

by taking $\gamma \rightarrow 0$, $\vartheta_g = 0$, $\vartheta_g^c = n_g$ and $\vartheta_k = n_k$ for $k = g + 1, \dots, K$. This upper bound is given by

$$\frac{e - 1}{e} (1 + K - g).$$

Clearly, any level of $\tilde{\lambda}_D$ can be reached by choosing $K - g$ appropriately. □

Moreover, it is easily verified that for the case with $\vartheta_{g+1} = n_{g+1}$, and a perfectly competitive benchmark in the colluding layer, $\vartheta_g = 0$, we have that Ξ equals 2 for $\gamma = 1$, and Ξ goes to $e(1)$ for $\gamma \rightarrow 0$ ($\gamma \rightarrow \infty$).

E.5 An Example of Upstream “Undercharges”

Following *Mandeville Island Farms*, direct suppliers to a buyers cartel that colluded to decrease input prices can maintain a treble-damages action for the “undercharge.”¹⁵⁷ In this paper, we abstract from buyer power effects. Yet, as argued in Section 6.3.2, variations in demand or the cost of production can generate upstream price effects as well. Input prices may increase when the cartel reduces demand. Direct suppliers may also obtain a lower price for their inputs by the cartel members than they would under competition, however, in which case they suffer a straightforward “undercharge.”

Figure AP.2 illustrates an example of input prices in layer $g - 1$ decreasing as a result of a cartel forming in layer g when the marginal upstream costs of production increase in production.

The derived demand for inputs under downstream competition, $p_{g-1}(q)$, turns inwards to $p_{g-1}^g(q)$. Profit maximization given $c_{g-1}(q)$ results in lower input prices to the purchaser cartel, $p_{g-1}^g < p_{g-1}$. As a result, the upstream industry sustains an undercharge of size $(p_{g-1} - p_{g-1}^g) q_{g-1}^g$ on its actual sales—or area U in the figure.

Given linear demand, upstream prices increase after downstream collusion when costs are concave, and decrease when costs are convex.¹⁵⁸ The intuition for the latter becomes clear from the case of perfect competition in the upstream market, in which prices are

¹⁵⁷ See Blair and Page (1996), pp. 131–132.

¹⁵⁸ Analytical results quickly become intractable in longer supply chains. From tedious but straightforward computations it follows that in a chain with two layers of production the upstream Cournot equilibrium price decreases with an increase in ϑ_2 , whenever

$$(\Psi_P + \Psi_c) P'(Q) > \left((n_2 + \vartheta_2) P'(Q) + \vartheta_2 Q P''(Q) - c_2'' \left(\frac{Q}{n_2} \right) \right) ((n_1 + 1) P'(Q) + Q P''(Q)).$$

where $\Psi_P = (n_1 + 1)(n_2 + \vartheta_2) P'(Q) + (n_2 + (n_1 + 3)\vartheta_2) Q P''(Q) + \vartheta_2 Q^2 P'''(Q)$ and $\Psi_c = -n_2 c_1'' \left(\frac{Q}{n_1} \right) - (n_1 + 1) c_2'' \left(\frac{Q}{n_2} \right) - \frac{Q}{n_2} c_2''' \left(\frac{Q}{n_2} \right)$. If consumer demand is linear and marginal

equal to marginal costs. Decreasing returns to scale result in lower marginal costs of production in equilibrium when the quantity of inputs demanded is reduced. These results carry over to the other forms of imperfect competition upstream in our model.

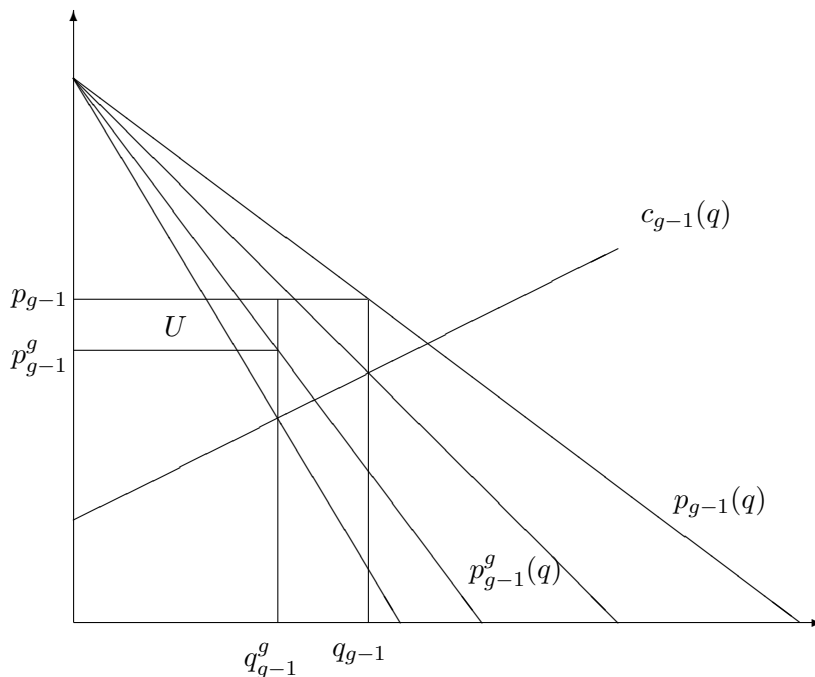


FIGURE AP.2 *Direct sellers undercharged by a purchaser's cartel.*

costs downstream are constant, this condition reduces to $c_1'' \left(\frac{Q}{n_1} \right) > 0$. Furthermore, the class of demand specifications for which there are no upstream price effects when the marginal costs of production are constant both up- and downstream is characterized by

$$P''(Q)P'(Q) + Q \left\{ P'''(Q)P'(Q) - [P''(Q)]^2 \right\} = 0.$$

Obviously, whereas (6.10) does, many nonlinear demand functions do not satisfy this condition.

E.6 Proof of Proposition 6.5

It is sufficient to prove the proposition for the linear case with $\gamma = 1$. Then we have

$$\lambda_U = \frac{n_g + \vartheta_g^c}{n_g} \left(\prod_{i=1}^{g-1} \frac{n_i + \vartheta_i}{n_i} - 1 \right).$$

In the extreme case where all upstream layers are monopolized ($\vartheta_i = n_i$ for $i = 1, \dots, g-1$) and there is full collusion in the colluding layer ($\vartheta_g^c = n_g$) we obtain

$$\lambda_U = 2(2^{g-1} - 1),$$

which can reach any finite level as the number of upstream layers $g-1$ increases. \square

E.7 Proof of Proposition 6.6

We show the effects of the location of the colluding layer on total welfare and the different measures of harm. The change in welfare is

$$\begin{aligned} \Delta W_g &= \sum_{k=1}^K \sigma_k + \sigma_C = (Q^* - Q^g) \left(P^* - \sum_{k=1}^K c_k \right) + \sigma_C \\ &= \Phi \left[\left(1 - \left(\frac{n_g + \gamma \vartheta_g}{n_g + \gamma \vartheta_g^c} \right)^{\frac{1}{\gamma}} \right) \right. \\ &\quad \left. - \frac{1}{\gamma + 1} \left(1 - \left(\frac{n_g + \gamma \vartheta_g}{n_g + \gamma \vartheta_g^c} \right)^{\frac{\gamma+1}{\gamma}} \right) \prod_{i=1}^K \frac{n_i}{n_i + \gamma \vartheta_i} \right], \end{aligned}$$

where $\Phi = \left(\frac{1}{b}\right)^{\frac{1}{\gamma}} \left(\prod_{i=1}^K \frac{n_i}{n_i + \gamma \vartheta_i}\right)^{\frac{1}{\gamma}} \left(a - \sum_{j=1}^K c_j\right)^{\frac{\gamma+1}{\gamma}}$. The change in welfare is independent of the location of the cartel. The direct purchaser overcharge is given by

$$\xi_{g+1} = \omega_g = \Phi \left(\frac{n_g + \gamma \vartheta_g}{n_g + \gamma \vartheta_g^c} \right)^{\frac{1}{\gamma}} \left(\frac{\gamma \vartheta_g^c - \gamma \vartheta_g}{n_g + \gamma \vartheta_g^c} \right) \prod_{i=1}^g \frac{n_i}{n_i + \gamma \vartheta_i},$$

which does decrease in g . \square