In a hurry to work with high-speed video at school?

Heck, A.; Uylings, P.

DOI
10.1119/1.3317451

Publication date
2010

Document Version
Author accepted manuscript

Published in
The Physics Teacher

In a hurry to work with high-speed video at school

André Heck, University of Amsterdam, Amsterdam, The Netherlands
Peter Uylings, Bonhoeffercollege, Castricum, The Netherlands

Casio Computer Co., Ltd., brought in 2008 high-speed video to the consumer level with the release of the EXILIM Pro EX-F1 and the EX-FH20 digital camera.® The EX-F1 point-and-shoot camera can shoot up to 60 six-megapixel photos per second and capture movies at up to 1200 frames per second. All this, for a price of about US $1000 at the time of introduction and with an ease of operation that allows high school students to work in 10 minutes with the camera. The EX-FH20 is a more compact, user-friendlier, and cheaper high-speed camera that can still shoot up to 40 photos per second and capture up to 1000 fps. Yearly, new camera models appear and prices have gone down to about US $250 for a decent high-speed camera. For more details we refer to Casio's website.[1]

In this paper we want to illustrate that with the advent of such high-speed video technology at the consumer level, or at least at a level that schools can afford such cameras for use in a science lab, video analysis in education has reached a next stage of effectiveness in understanding science. Students now have the opportunity to work directly with high-quality video data in cases where motion was in the past too quick for recording with a normal digital camera or webcam, e.g., data collection of human and animal locomotion or motion in sports. It allows students to carry out authentic activities in which they record video clips or image sequences of rather fast motions and use them for a detailed investigation of a real-world phenomenon. They can work with software tools to measure on movies or image sequences of real phenomena, to analyze collected data, to build and simulate computer models and to compare results from computer simulations with the obtained video data. In this paper we also want to point out that for this kind of practical investigative work it is convenient that the video analysis system which is used by the student provides tools for perspective correction and tracking of points of interest.

A falling shuttlecock: A classroom setting

Our showcase example concerns a vertical fall experiment where the effects of air resistance are important and measurable. This topic is not new in education: in the popular experiment of dropping coffee filters,[2,3] balls and party balloons,[4-8] or paper cones[9,10] students investigate the movement of an object released at a certain height and they determine the influence of weight, size and shape of the falling object on its motion. The intended audience of students (age 15-16) carries out such experiments as practical classroom work using various data collection techniques. In these experiments, which normally take two lessons, it is stated without support that the drag force acting on the falling objects is approximately proportional to the square of their velocity. Students then use this to explain how a constant terminal velocity is reached.

Many of the objects used in the previously mentioned experiments exhibit a scholastic and artificial character. For a follow-up investigation to be carried out one or two years later, we have selected the motion of a sports object, viz., a badminton shuttlecock. We know from classroom experience that investigating drag forces in a setting of real-life sports constitutes a more interesting challenge to a high-school student.

Another reason for choosing this object is that in the past studies have been published that use the vertical fall of a badminton shuttlecock to investigate various models of air resistance: Mark Peastrel and his colleagues[11] measured the times required for a shuttlecock to fall given distances (up to almost 10 meters) and they compared their data with the predictions of several models of air resistance. Their least-squares analysis of distance-time data favored a resistive force quadratic in velocity. In an attempt to conclusively determine whether the drag force on a feather shuttlecock in vertical fall is proportional to the velocity or the velocity squared, Kathleen McCreary[12] recorded in 2005 parts of the motion of a shuttlecock (immediately after the start of the vertical fall, after a fall of 1.33 m, and after a fall of 1.88 m) with a high-speed camera that could record 250 frames per second and she analyzed her video clips with VideoPoint.[13] She concluded that the resolution of the camera and the fact that she could only record parts of the motion of the shuttlecock instead of a complete trajectory made it impossible to find conclusive evidence of the value or the nature of the resistive force.

This paper will reveal that technology at school level has improved to such an extent that a successful aerodynamics study is currently within reach of high school students. As a pilot case of practical investigative work intended for pre-university students, we studied the motion of the free falling shuttlecock. Next to the Casio high-speed camera, we used the Coach 6 computer learning environment[14] for video analysis and graphical modeling. The simulation results from the computer model were compared with the video data. We did this tryout also to get an idea whether this work is feasible in upper secondary education and how much time it would cost students. We conclude from our study that senior high school students (age 17-18) are able to find in reasonable time strong support for the quadratic model of air drag of the shuttlecock from this experiment. The results are in agreement with results of the PhD study of Alison Cooke[15-17] and of a more recent research study,[18] which illustrates that current technology contributes to the realization of students' practical investigative work that is pretty close to the level of experiments carried out by sports scientists. On the basis of our classroom experiences for many years with video recording, video analysis, and graphical modeling, and on the basis of our pilot study we estimate that students in a pre-university stream can investigate the free fall of a shuttlecock in one afternoon, provided that they are already familiar with the techniques. For novice the estimated amount of time must be doubled.

Video analysis of the experiment

In our experiment, which was performed in the open stairwell of an entrance hall, we used a commonly available synthetic badminton shuttlecock of brand name "Angel Sports", weighing 3.28 g and having a maximum skirt diameter of 6.5 cm. We dropped the shuttlecock from the first floor and we recorded its motion from a height of approximately 4.5 m with the Casio EXILIM Pro EX-F1 digital camera at a resolution of 384×512 and a frame rate of 300 fps. No special arrangements such as extra light sources were set up. Figure 1 shows the experimental setting through a frame from the recorded video clip of one of the trials. The annotation points at the shuttlecock and it is above the meter stick that was positioned on a little cart for calibrating distance.

Fig 1. The experimental arrangement viewed with the high-speed camera.

Figure 1 reveals the small but for the required accuracy substantial problem of perspective distortion, which forced McCreary[12] to record separate parts of the motion of the shuttlecock. The remedy offered by Coach 6, viz., correction of the perspective distortion in the recorded video clip on the basis of a known shape in the scene, is convenient and easy to apply, and has been reported elsewhere.[19,20] Figure 2 shows the result of perspective correction.

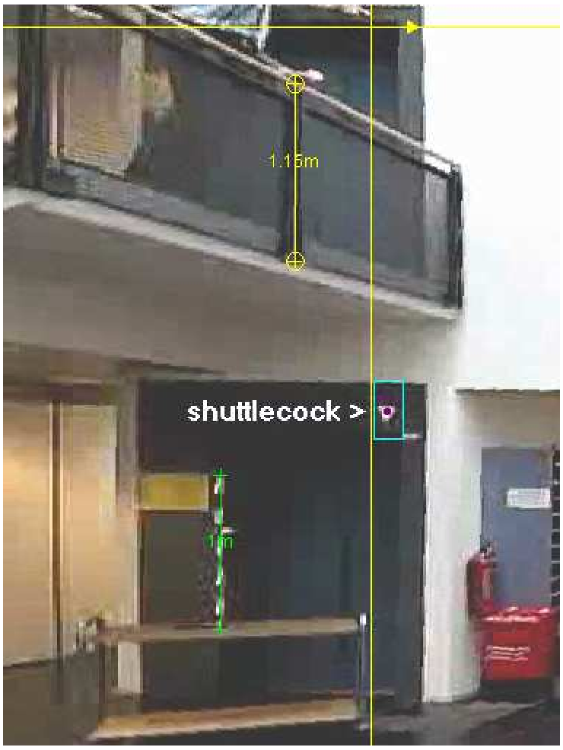

Fig 2. Screen shot of measurement in the video clip obtained after correction of the perspective distortion using the point-tracking facility.

The vertical fall of the shuttlecock from the height of approximately 4.5 m takes about 1.3 second before the object hits the floor. With the frame rate of 300 fps this means that about 390 frames are at our disposal for video analysis. Manual data collection by clicking on points of interest in a video clip that contains so many frames is time-consuming, error prone and boring work to do. Currently this is the only possibility in popular video tools like VideoPoint[13] and Vernier's Logger Pro 3.[21] The Coach 6 video tool[14] provides its user with the facility of automated data collection in the video clip via a technique that is also known as point-tracking.[19,20]

For each point of interest (including the origin of a moving coordinate system), the user specifies at the start of the tracking process a template around this point that will henceforth be automatically matched in subsequent video frames. Matching takes place in a certain moving search area, the size of which is also user-definable. In Figure 2 the search area around the currently measured point (here the center of the skirt of the shuttlecock) is visible as a small rectangle. In this way, the coordinates of the moving object with respect to the user-defined coordinate system, the origin of which was chosen to be the point were the shuttlecock is dropped, have been automatically determined in all frames of the video clip without any difficulty.

The result of the automated data collection in the video clip of the free falling shuttlecock is shown in Figure 3. This diagram also contains the velocity-time graph, which was obtained by the built-in, quintic penalized spline-smoothing based algorithm[22] for computing numerical derivatives. Such advanced algorithms are needed in a computer learning environment if one does not want to be confronted in students' activities with technical obstacles such as noise dominating data after numerical differentiation with finite-difference formulas.

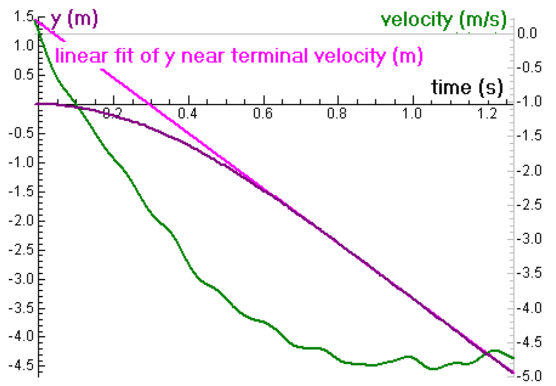

Fig 3. The vertical position-time graph (purple curve) and the velocity-time graph (green curve) of the falling shuttlecock. The linear function fit of the position-time graph shows the approach to linear motion after about 0.8 second.

The velocity-time graph indicates that the falling shuttlecock reached after a short time a constant velocity. In Figure 3 we have used this information about the restricted range in which the velocity is fairly constant to perform a straight-line fit of the position-time graph. This method, which was advocated by Gluck,[7] gives a value of 4.75 m/s for the terminal velocity.

Theory

For reference, we present kinematical formulas describing the falling shuttlecock. We will use the terminal velocity v_T of the shuttlecock as a parameter in the models because this quantity can be estimated in the analysis of the experimental data. We assume that during its fall the following two forces act on the shuttlecock: the gravitational force $F_{grav} = mg$ and the drag force F_{drag}, which depends explicitly on the velocity v of the shuttlecock. Depending on the Reynolds number $\mathcal{R}e$, three cases can be distinguished.

Case 1: linear drag

If the Reynolds number is very small, say $\mathcal{R}e<1$, then the drag force is proportional to the velocity:

$$F_{drag} = -kv \tag{1}$$

When the falling object reaches terminal velocity, v_T, the net force on it is zero, so

$$F_{drag} = mg \quad \text{or} \quad k = -\frac{mg}{v_T}, \tag{2}$$

where m is the mass of the object and g is the gravitational acceleration. Thus, the equation of motion of the falling object is in this case

$$\frac{dv}{dt} = g\left[\frac{v}{v_T} - 1\right] \tag{3}$$

This differential equation can be solved analytically with initial condition $v(0) = 0$:

$$v = v_T\left[1 - e^{\frac{gt}{v_T}}\right] \tag{4}$$

This can be integrated once more, with initial position $y(0) = 0$, to find position as function of time:

$$y = \frac{v_T^2}{g}\left[1 - e^{\frac{gt}{v_T}}\right] + v_T t \tag{5}$$

If the air resistance or the time range is small an approximate solution is

$$y \approx -\frac{1}{2}gt^2\left[1 + \left(\frac{g}{3v_T}\right)t\right] \tag{6}$$

In other words, the time course of the position of the falling object can be approximated by a third degree polynomial with only terms of degree two or higher.

Case 2: quadratic drag

If the Reynolds number is large, say between 10^3 and $2\cdot 10^5$, then the drag force is proportional to the square of the velocity:

$$F_{drag} = kv^2 \tag{7}$$

When the falling object reaches terminal velocity, v_T, the net force on it is zero, so

$$F_{drag} = mg \quad \text{or} \quad k = \frac{mg}{v_T^2}, \tag{8}$$

Thus, the equation of motion of the falling object is in this case

$$\frac{dv}{dt} = g\left[\left(\frac{v}{v_T}\right)^2 - 1\right] \tag{9}$$

This differential equation can be solved analytically with initial condition $v(0) = 0$:

$$v = -v_T \tanh\left(\frac{gt}{v_T}\right) \tag{10}$$

This can be integrated once more, with initial position $y(0) = 0$, to find position as function of time:

$$y = -\frac{v_T^2}{g}\ln\left[\cosh\left(\frac{gt}{v_T}\right)\right] \tag{11}$$

If the air resistance or the time range is small an approximate solution is

$$y \approx -\frac{1}{2}gt^2\left[1 - \left(\frac{g^2}{6v_T^2}\right)t^2\right] \tag{12}$$

In other words, the time course of the position of the falling object can in this case be approximated by a fourth degree polynomial with only terms of degree two and four.

The proportionality constant k for quadratic drag force is often written as

$$k = \frac{1}{2}C_d \rho A\,, \tag{13}$$

where C_d is the drag coefficient (a dimensionless quantity depending on the physical characteristics of the surface of the falling object), ρ is the density of air, and A represents the cross-sectional area of the falling object. For ease of comparison of our results with those from the research literature[15-17] we adopt the convention to take for A the square of the maximum skirt diameter d. From equation (8) follows the relation between the drag coefficient C_d and the terminal velocity v_T:

$$v_T^2 = \frac{2mg}{C_d \rho d^2} \qquad (14)$$

Case 3: moderate Reynolds number

If the Reynolds number is moderate, say between 1 and 1000, we have a combination of linear and quadratic drag

$$F_{drag} = -k_1 v + k_2 v^2 \qquad (15)$$

What is less know, but can easily verified in a computer algebra system like Maple, is that even in this case the equation of motion can be solved analytically. We leave out details because it goes beyond the scope of this paper.

Comparing theory and experiment

Recall that one of our motivations of doing this small research project was to investigate various models of air resistance of a shuttlecock and to find out whether high-speed video recording of a vertical fall of a shuttlecock could give conclusive evidence for one of the models. In particular we were curious whether the experimental data match better with the quadratic drag model than with the linear drag model. We used two strategies: (1) fitting the experimental data to the formulas of the previous section and (2) building a computer model that solves the equation of motion for each aerodynamic model, running a simulation, and finding the best values of the parameters so that the position-time curve of the computer model matches best with the experimental curve shown in Figure 3. In this section we present the results.

1. Nonlinear least-squares curve fitting

There are various software packages available to do a nonlinear fit for our data set. For example, one could export the experimental position-time data to an Excel sheet and do nonlinear curve fitting with Microsoft Excel Solver and non-linear regression statistics.[23] We exported the data set to *CurveExpert*[24] and used this curve fitting system to perform the regression analysis and to find the best estimate for the terminal velocity based on the equations (4) and (10). The results (best estimate and standard deviation σ) are shown in Table 1.

model	v_T (m/s)	σ (m/s)
linear	-7.36	0.08
quadratic	-5.13	0.01

Table 1. Least-squares regression of terminal velocity for linear and quadratic drag force.

Clearly quadratic drag describes the experimental data better than linear drag. This would be even more obvious if we were to draw the regression curves together with the data. We do not show these graphs because the computer models of the next paragraph will lead to the same diagrams and we will show them there.

The terminal velocities (and standard deviations) that would follow from the approximative equations (6) and (12) are –10.19 m/s (σ=0.04) for linear drag and –7.49 m/s (σ=0.12) for quadratic drag, respectively. This shows that they are not applicable in the case of a vertical fall of a shuttlecock.

We can use equation (14) to determine the drag coefficient C_d from the terminal velocity $v_T = -5.13\,\mathrm{m/s}$, mass $m = 3.28\,\mathrm{g}$, maximum skirt diameter $d = 6.5\,\mathrm{cm}$, air density $\rho = 1.20\,\mathrm{kg/m^3}$, and gravitational acceleration $g = 9.81\,\mathrm{m/s^2}$:

$$C_d = 0.48 \qquad (16)$$

This value is in good agreement with reported literature values[15,16] that are constant values ranging from 0.48 to 0.53, for Reynolds numbers that occur in a badminton game (13,000< $\mathcal{Re}$<190,000).

2. Graphical computer models

Coach 6 provides a modeling tool whose graphical modus is similar to the modeling software systems STELLA and Powersim.[25] Whereas the analytical approach of the theory section in this paper may be too advanced for many a high school student, (graphical) computer modeling turns out to be within reach of any student in a few lessons. Figure 4 shows the graphical model for the vertical fall of an object subject to drag force that depends on the velocity of the object with a proportionality constant *k*. Actually the model looks the same for both linear and quadratic drag; only the formulas for the drag force and the terminal velocity differ. This graphical model can be considered as a representation at conceptual level of the system dynamics, where relation arrows indicate dependencies between quantities. Students quickly grasp this idea.

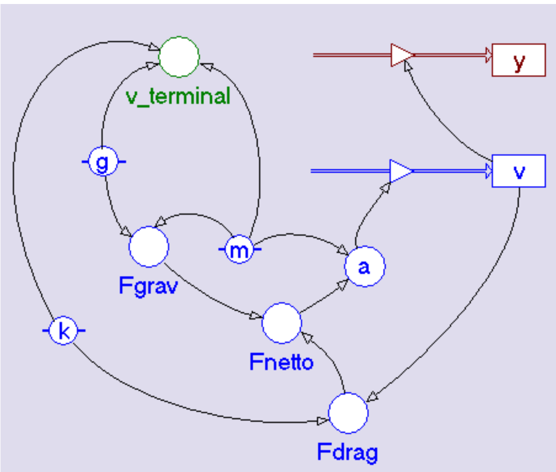

Fig 4. A graphical model representing the equation of motion for an object that falls vertically and that is subject to air resistance.

When we use equation (1) for linear drag, select the best value for the proportionality constant *k*, and run a simulation, we still do not get a good match between the simulation results and the experimental data. Figure 5 and 6 show the best simulation results for the position-time graph and the corresponding velocity-time graph, respectively. The experimental quantities of position and velocity are displayed in these diagrams as background graphs.

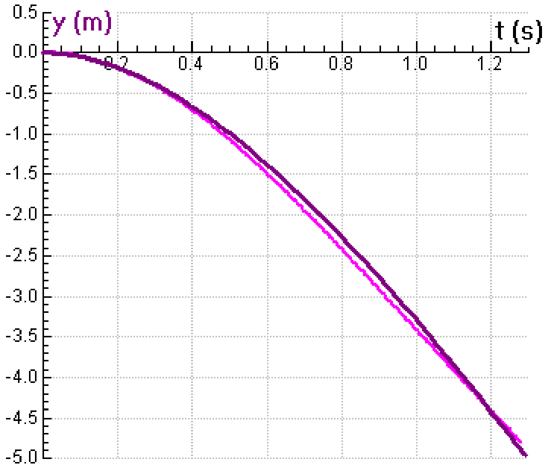

Fig 5. The position-time graph of the best simulation for the linear drag model.

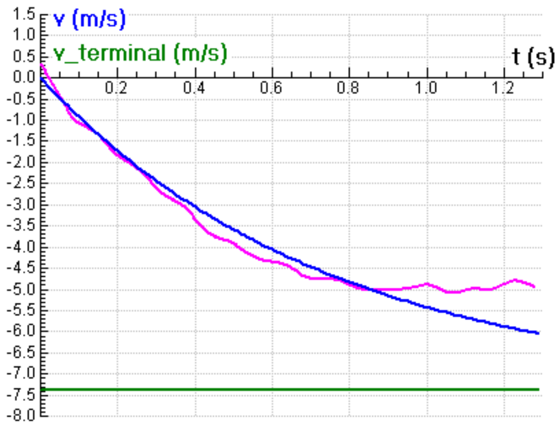

Fig 6. The elocity-time graph of the best simulation for the linear drag model.

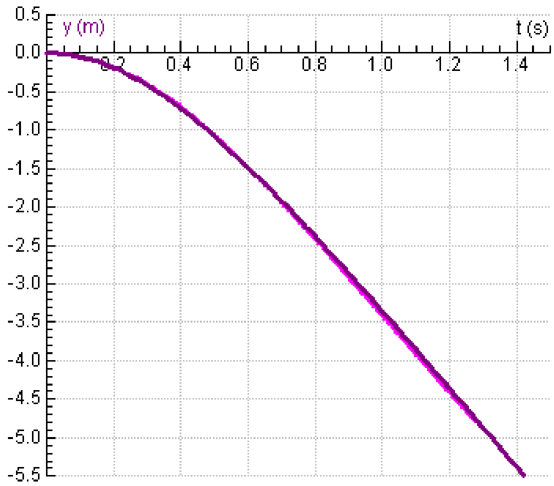

Fig 7. The position-time graph of the best simulation for the quadratic drag model.

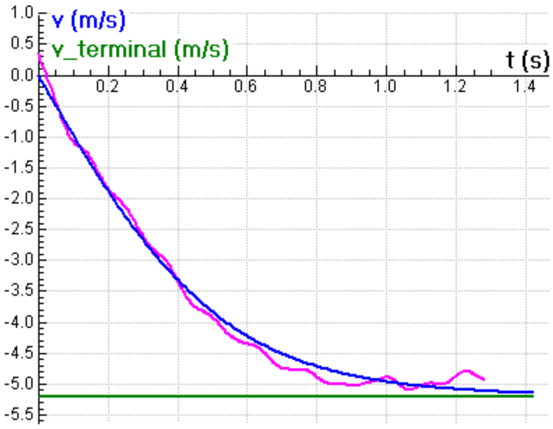

Fig 8. The velocity-time graph of the best simulation for the quadratic drag model.

Figures 7 and 8 illustrate that a much better match between modeling results and experimental data is obtained for the quadratic drag model, using equation (7). The constant k has been specified in the model via equation (13) with $C_d = 0.47$, leading to a terminal velocity of -5.2 m/s.

Conclusion

The example discussed in this paper illustrates that the high-speed video technology that has recently become available at consumer level enables students to record accurately the vertical fall of real sports objects and to study the effects of air resistance. This was often not possible with ordinary video cameras or webcams. When the video analysis system that the students use to measure on the video clip has suitable tools available, such as facilities for correction of perspective distortion and a point-tracking algorithm for automated measurement, then the data collection process becomes a piece of cake. Hereafter, nonlinear regression based on a theoretical model enables students to conclusively determine which model of air resistance is most suitable: linear drag or quadratic drag. They do not have to believe what the textbook says about this, but instead they can figure it out themselves. When the students have a (graphical) modeling tool at their disposal to solve the equation of motion numerically, they can use this to compare experimental data with theory.

When all the mathematical tools, video analysis, and modeling, are provided in a single computer learning environment like the Studio-MV version[14] of Coach 6, then the investigative work of the students is not frustrated by technical problems of computer tools that do not work well together and the students can just continue to work in a system with which they are already familiar. This saves time and enables them to work in much the same way as sports scientists actually do. Other interesting practical investigations and research projects recently carried out by pre-university students with the help of a high-speed camera concern detailed studies of bouncing balls,[26] the start of a sprint,[20] the yoyo motion of a toy,[27] kicking a soccer ball, bungee jumping,[28] human locomotion,[29] and the motion of billiard balls.[30]

Acknowledgements

The authors would like to thank their colleague Cor de Beurs for the acquisition of the high-speed camera in the framework of the ITS Academy.

References

1. http://exilim.casio.com
2. N.F. Derby, R.G. Fuller, P.W. Gronseth, “The Ubiquitous Coffee Filter,” *Phys. Teach.* **24**, 153-159 (March 1997).
3. R.A. Young, “Improving the Data Analysis fro Falling Coffee Filters,” *Phys. Teach.* **39**, 398-400 (Oct. 2001).
4. K. Takahashi, D. Thompson, “Measuring air resistance in a computerized laboratory,” *Am. J. Phys.* **67**, 709-711 (Aug. 1999).
5. M.S. Greenwood, C. Hanna, J.W. Milton, “Air Resistance Acting on a Sphere,” *Phys. Teach.* **35**, 364-368 (March 1986).
6. V. Pagonis et al., “Effects of Air Resistance,” *Phys. Teach.* **35**, 364-368 (Sept. 1997).
7. P. Gluck, “Air Resistance on Falling Balls and Balloons,” *Phys. Teach.* **41**, 178-180 (March 2003).
8. R. Cross, “Aerodynamics of a Party Balloon,” *Phys. Teach.* **45**, 334-336 (Sept. 2007).
9. A. Mooldijk, E. Savelsbergh, “An example of the integration of modeling into the curriculum: a falling cone.” In: R. Pinto, S. Surinach (eds.) *Proceedings of the GIREP: Physics Teacher Education Beyond 2000*, Barcelona (pp. 625-628), Paris:Elsevier, 2001.

10. J. Wooning, A. Mooldijk, T. van der Valk, "Top angle and the maximum speed of falling cones," *Sci. Educ. Int.* **17** (3), 161-169 (2006).
11. M. Peastrel, R. Lynch, A. Armenti, Jr., *Am. J. Phys.* **48**, 511-513 (July 1980).
12. K. McCreary, "A Study of the Motion of a Free Falling Shuttlecock," Wooster Physics Junior Thesis, 2005, www.wooster.edu/physics/jrIS/Files/McCreary.pdf
13. *VideoPoint* is a video-based motion analysis program designed for science education and distributed in the US by PASCO Scientific
14. A. Heck, E. Kędzierska, T. Ellermeijer, "Design and implementation of an integrated computer working environment," *J. Comput. Math. Sci. Teach*. **28** (2), 147-161 (2009.
    Coach 6 is a versatile computer learning and authoring environment that provides integrated tools for MBL-based measurement, control activities, digital image and video analysis, and computer modeling. It has been translated into many languages, it is used in many countries, and it is distributed by the CMA Foundation. The Studio-MV version of Coach 6 only supports video analysis and modeling activities. More information at: www.cma.science.uva.nl
15. A. Cooke, "The Aerodynamics and Mechanics of Shuttlecocks," PhD Thesis, Cambridge University, Cambridge, UK.
16. A. Cooke, "Shuttlecock Aerodynamic," *Sports Eng.* **2** (2), 85-96 (1999).
17. A. Cooke, "Computer simulation of shuttlecock trajectories," *Sports Eng.* **5** (2), 93-105 (2002).
18. L.-M. Chen, Y.-H. Pan, Y.-J. Chen, "A study of shuttlecock's trajectory in badminton," *J. Sports. Sci. Med*., 8 (4), 657-662 (2009).
19. A. Heck, P. Uylings, "Capturing the real world in the classroom," *International Journal for Technology in Mathematics Education*, **13** (3), 107 – 116 (2006).
20. A. Heck, T. Ellermeijer, "Giving students the run of models of sprinting," *Am. J. Phys.* **77**, 1028-1038 (Nov. 2009).
21. Logger Pro 3 is software for data collection and data analysis that has been developed and is distributed by Vernier Software & Technology. www.vernier.com
22. J. Ramsey, B. Silverman, "Functional Data Analysis" (2nd ed.) Springer Series in Statistics, New York: Springer Verlag, 2005.
23. D. Harris, "Nonlinear Least-Squares Curve Fitting with Microsoft Excel Solver," *J. Chem. Educ*. **75**, 119-121 (Jan. 1998).
24. *CurveExpert* is a comprehensive curve fitting system in which many models are built-in and users can add their own regression models. http://curveexpert.webhop.net
25. *STELLA* (www.iseesystems.com) and *Powersim* (www.powersim.com) are popular system dynamics based graphical modeling software packages used in education.
26. A. Heck, T. Ellermeijer, E. Kędzierska, "Striking Results with Bouncing Balls." In: C.P. Constantinou, N. Papadouris (eds.) *Physics Curriculum Design, Development and Validation*, Proceedings of the GIREP 2008. Nicosia, Cyprus, 2009.
27. A. Heck, P. Uylings, "Yoyo Joy." In: F. Olivero, R. Sutherland (eds.) *Technology in Mathematics Teaching*, Proceedings of ICTMT7, Vol. 2, (pp. 380-387), Bristol: University of Bristol, 2005.
28. A. Heck, P. Uyling, E. Kędzierska, "Understanding the physics of bungee jumping," *Phys. Educ.* **45** (1) 63-72 (2010).
29. T. Ellermeijer, A. Heck. "Walk like a physicist: an example of authentic education." In: *Proceedings of the GIREP 2002: Physics in New Fields,* Lund, Sweden, July 2002.
30. D. G. Alciatore, "The Illustrated Principles of Pool and Billiards." New York: Sterling Publishing, 2004.

Key words: Video Analysis, Newtonian Mechanics, Aerodynamics

PACS: 01.50.ff, 01.50.H, 01.80.+b, 5.40.Gj, 47.80.Cb, 47.85.Gj

Biographic information of the authors:

André Heck earned MSc degrees in mathematics and chemistry. He is project manager at the Faculty of Science of the University of Amsterdam. His research area is the application of ICT in mathematics and science education.

AMSTEL Institute, University of Amsterdam, Amsterdam, the Netherlands;
A.J.P.Heck@uva.nl

Peter Uylings graduated in physics and mathematics and he obtained his doctorate in theoretical physics. He works part-time as teacher at a secondary school. His research area is ICT in science education and science curriculum development. Since 2002, he constructs physics computer exams as an innovation in the nationwide curriculum.

AMSTEL Institute, University of Amsterdam, Amsterdam, the Netherlands;
P.H.M.Uylings@uva.nl