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Modelling in Cross-Disciplinary Authentic Student Research Projects

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In the Dutch secondary education system, students must carry out at the end of their school career a rather large research or design project to demonstrate their ability to apply acquired knowledge and skills while pursuing a research question or design goal in some depth. They are encouraged to choose the topic themselves and they are to some extent free in setting up their work. Ideally, the students do not only see it as a compulsory subject but also enjoy the stimulating aspects of doing their own research or design. Challenging and authentic projects, which are representative for actual research and design work done by professionals, seem effective in this respect. The focus of this paper is on how the use of ICT can contribute to the realisation of such projects in mathematics and science education and on how it can give students opportunities to take the nature and level of their work close to the characteristics of work of experts in the field. Three examples of students' inquiry work are analysed with regard to ICT usage, authenticity and resemblance with an expert's approach.

1 INTRODUCTION

In the Dutch examination programme of upper secondary education, students are obliged to build up a portfolio consisting of several small practical investigation tasks (4-10 hrs) and one rather large cross-disciplinary research or design project (80 hrs). The main instructional purpose of the small investigation tasks is to give students opportunities to (1) build up general competencies such as research skills, ICT skills, communication skills, and so on; (2) deepen or enlarge existing mathematical and scientific knowledge; and (3) become more proficient in applying knowledge and skills in practice. In the final project, usually done in the last year before the final central examination, students must demonstrate their achieved level of knowledge, skills, and attitude by the creation of a masterpiece of independent work on a topic of their own choice. For example, in a research project students must demonstrate their research abilities, ranging from choosing a manageable problem, formulating a good research question and structuring their work to drawing conclusions and presenting the research results. The final project is part of the examination programme and a sufficient mark is needed for obtaining a secondary school diploma.

Notwithstanding that the final project is an assessment, it is seen by many curriculum developers and by many teachers as a chance to expose students to the real world of research and design, and as an opportunity to let students enjoy doing research and development on a subject of their own choice. 'Challenge' and 'authenticity' seem to be useful keywords for describing activities that really turn students on. Because contemporary R&D work makes heavy use of ICT and is often done in multidisciplinary teams, it is natural to raise the following educational research questions: How can the use of ICT contribute to the realisation of challenging cross-disciplinary authentic research projects in mathematics and science education, which are representative for actual research done by real scientists, and how can it give students opportunities to take the nature and level of their work close to the characteristics of work of experts in the field?

In order to be able to answer the above questions, the meaning of the adjectives 'cross-disciplinary' and 'authentic' is first explicated in section 2 in the context of student inquiry work and mathematical modelling. This is especially required because there exist competing views on authentic and realistic student work in research in mathematics and science education. The adjective 'challenging' depends very much on the level of expertise and motivation of the individual student doing his or her research project and is discussed only in an exemplary way. In sections 3 to 5, three illuminating examples of students' inquiry work are analysed with regard to ICT usage, authenticity and resemblance with an expert's approach. Two of these examples are secondary school students' final research projects, one about bungee jumping and another one about gait analysis. A third example, about the chaotic motion of a spring pendulum, comes from a CAS-supported investigation of a first-year university physics student, only three months after entering the university. The analysis of all three projects together leads to conclusions about the use of ICT for realising student research projects that resemble actual research practice.

2 AUTHENTIC INQUIRY BY STUDENTS: WHAT DOES IT MEAN?

Researchers on mathematics and science education use terms like authentic science, authentic practice, authentic learning, authentic inquiry, authentic modelling, and so on (see, for example, Edelson & Reiser, 2006; Palm, 2002, 2006; Roth, 1995; Roth et al., 2008; Schwartz & Crawford, 2004; Vos, 2009; Weiss et al., 2009; Wellington, 1998; Woolnough, 2000). The meaning of the adjective 'authentic' is so diverse, even when connected with a single noun, that it is wise to first explicate the meaning of 'cross-disciplinary authentic student research project' that is used in this paper and to compare it with other uses of this term in inquiry-based mathematics and science education.

I refer to an 'authentic student research project' as a students' investigation having the following characteristics:

- Students work on a self-selected, challenging, messy, ill-defined, open-ended problem that is rooted in a real life situation instead of a more abstract or ideal situation;

- Students do not follow some standard recipes, but they examine their problem from different perspectives, using a variety of resources and high-order skills.
- A broad range of competencies is required to make the project a success, and one of these abilities is making good use of ICT for information gathering, data acquisition, data processing and analysis, problem-solving, reporting, and communication with team members;
- Students' work is open-ended in the sense that there exist multiple methods or approaches to obtain many possible or even competing results and answers. The student researchers actually decide whether the investigation can be finished for whatever reason;
- It offers students the opportunity to be in contact with contemporary, cross-disciplinary research and to learn about the nature of mathematics and science;
- Students disclose their own understanding through a portfolio and/or a more or less polished product like a report, paper, or presentation.

A project is called 'cross-disciplinary' when more than one discipline contributes in an essential way to the process of coming to an understanding of the problem situation. The student research projects presented in this paper are mainly rooted in applied mathematics and physics. The term 'cross-disciplinary' is used, and not a term like 'interdisciplinary,' to emphasise that all disciplines are required to get satisfactory results: The whole is more than the sum of the parts.

In science education literature much has been written about authentic inquiry that gives students the opportunity to learn science by doing science like 'real' scientists, as opposed to carrying out practical work that follows cookbook recipes and has hardly any resemblance with experimental work done in real laboratories (Roth, 1995; Schwartz & Crawford, 2004; Woolnough, 2000). It is noted that an inquiry approach is effective in developing core skills in students, improves their attitudes towards science and technology, and allows students to get an honest picture of the nature of science (Schwartz & Crawford, 2004). Authentic scientific work is linked with the real practice of doing science rather than with the methods narratives of published research that keep up the myth of the 'scientific method.' Educational research literature (cf. Chinn & Malhotra, 2002) often emphasises the differences between the work of experts and novices, and it questions whether authentic science is feasible in educational practice because of its complexity and cross-disciplinary nature, the problems with assessing student work, and its requirements regarding content knowledge, research abilities, engagement, motivation, and time availability. However, according to Roth et al. (2008) the problem of how to organise authentic science lessons does not lie with the level of agreement between school science and laboratory science but rather with the levels of control, authority, mastery, and authorship that students are enabled to exercise. Following this line of thinking, the emphasis in this paper is on the students' acts, that is, on what the students really did, which decisions they made, how they used ICT to put their ideas into action, and how their work can be characterised as authentic work, rather than on the details of the mathematical and scientific knowledge that the students applied.

In research literature on mathematics education, the adjective 'authentic' is mostly associated with applications of mathematics and with mathematical modelling. Common ground seems to be Lebow's (1993) description of an authentic activity as experiences of personal relevance that permit learners to practise skills in environments that have considerable resemblance to those in which the skills are used. It is derived from socio-constructivist and RME principles of locating learning in the context of reality and of looking upon mathematics as a human activity in which sense making by students through participation in the mathematical representation process is highly valued. For a review of the different meanings attached to a words like '(in)authentic' in mathematics education in general, and in the teaching and learning of mathematical modelling in particular, see for example (Palm, 2002, 2006) and (Vos, 2009), respectively.

3 PHYSICS OF BUNGEE JUMPING

The work of the student researchers who carried out the bungee jumping project resembles the work of 'real' scientists regarding affective and cognitive processes in authentic inquiry. To begin with, there were (1) the students' intrinsic motivation to select the subject of their investigation; and (2) the generation of their own research question(s). The students wrote in their report that they teamed up to investigate the physics of bungee jumping, triggered by their own interest and the article of Menz (1993) on www.bungee.com. In particular, they were intrigued by the alleged 'greater-than-*g* acceleration' of a bungee jumper. The students formulated the following research question: "How large is the acceleration at a bungee jump and to what extent is this acceleration influenced by the relative mass of the rope and the jumper?" They hypothesised that the acceleration would be greater than *g* and that this effect would be more dramatic in case the rope is relatively heavier compared to the jumper.

In order to find quantitative support for their hypothesis the students designed an experiment in which they recorded with a webcam the motion of dropped wooden blocks of various weight attached to ropes of various stiffness and collected position-time data through video measurements. The students realised that working with a scale model of a bungee jumper or a wooden block is not the same as investigating real bungee jumping, but that it would provide them with enough information on what happens in reality and could lead to a good understanding. This indicates a research design in which procedures to address a problem are determined, variables to investigate are selected, control of variables is thought of, and measurements are planned. Note that not the measured position is the variable of interest, but a derived quantity, viz., acceleration. The students had learned in previous video analysis activities that adequate numerical derivates could be obtained from measured position data.

After some trials, the students realised that the stiffness of the rope plays only a minor role and that the mass ratio between rope and objects needs to be large enough to notice an outstanding result. Therefore they repeated the experiment with objects of larger mass ratio. This is another aspect of authentic inquiry: Researchers are responsible for detecting flaws in their experimental set-up and must decide how to adjust their original plans. In this case, the students decided to change the mass ratio and to concentrate on the moment when the object has fallen a distance equal to the rest length of the elastic (because they had observed that the acceleration is greatest at this point of the motion). They collected more data and determined the graph of the acceleration at this moment as a function of the mass ratio of elastic and block. They compared their data plot with the graph of a theoretical relationship taken from (Kagan & Kott, 1996). The students noted that these graphs were alike, with the theoretical values just a bit higher. They attributed the difference mainly to the development of heat during the motion. Again, this indicates a behaviour of the students that resembles the attitude of competent researchers: Comparing own results with work of others and trying to explain differences by scientific reasoning.

The student researchers not only investigated the acceleration of a bungee jumper by quantitative methods, but also tried to explain the greater-than-*g* acceleration both on qualitative, theory-driven grounds, as well as through a more detailed mathematical model of the phenomenon. The students followed the approach of Kagan and Kott (1996), in which it is noted that here one does not deal with a falling rigid body, but instead with a system consisting of the falling object and the rope. The interested reader is referred to (Heck et al., 2010) for more ICT-supported experimentation and a detailed description of a mathematical model of the system that can be implemented in a modelling and simulation tool to compare experimental results with theory. The main point to make here is that the students' work contains clear signs of their understanding that observation, experiment and theory are related tools for scientific thinking.

The students wrote an article about their work and got it published (Dubbelaar & Brantjes, 2003). This article triggered quite a number of reactions in the journal and for almost a year on Internet. There were complaints about the quality of physics teaching in the Netherlands, arguing that obviously acceleration of a bungee jumper could clearly not exceed the acceleration of gravity and that the students' work proved that the level of physics education in the Netherlands had decreased in the last decades. The editorial commentary on this was subtle, but to the point: "The students who wrote the paper may consider it a compliment that scepticism overcame professional physicists and physics teachers. That's how (or maybe it is just the point that) experienced intuition can be wrong." In other words, the students' work revealed that in practice, scientists are not as rationally, non-algorithmic thinking and having an open mind as commonly pictured in literature about science and scientists.

Regarding the use of ICT in this particular student research project, the following main roles can be identified:

- Internet gave the students access to many of the same resources that professionals use in their research. They could gather information and get in contact with experts;
- The students used video analysis software, in particular the video tool of the learning and authoring environment named Coach (Heck et al., 2009), to collect, visualise, process, and analyse data. Numerical differentiation, computing the area under the curve, and function fitting were the main mathematical tools used in the data processing and data analysis.
- The students used a standard text processor and presentation tool to report their results.

4 GAIT ANALYSIS

In this project, a high school student collected and analysed gait data in much the same way movement scientists do, namely, via recording and measurement of motions with a video tool and via electromyography, i.e., measurement of muscle activity. The authenticity of the student project was rooted (1) in the use of inexpensive tools (a webcam, a simple EMG/ECG set and Coach software), which are on the one hand fit for educational practice and on the other hand in essence close to the techniques used by biomechanists; and (2) in the student's use of the theoretical framework, nomenclature, and research methods of practitioners. That is to say, the student conducted many aspects of motion analysis herself: She formulated research questions about a self-selected gait pattern; gathered information; designed and carried out experiments; processed, analysed and interpreted data; and finally published results (Heck & van Dongen, 2008).

The main research question in this project was: "How goes human gait?" It was specialised as follows:
(1) "Which phases are distinguished in the gait cycle?"
(2) "What muscle activity happens during gait?"
(3) "How do bones and joints make gait possible?"

The first and the third sub question were addressed by video measurement with a webcam of the planar motion of the leg around the hip and knee joint during normal walk on a motorised treadmill in a sports centre. The recorded data were analysed in a knee angle vs. hip angle diagram and the periodic leg motion was modelled as a force-driven harmonic oscillator (Holt et al., 1990). The angular motion curve of the knee joint is described in this model by a sum of two sine functions, in which one frequency is nearly twice as large as the other one. This can be verified via a curve stripping regression technique (i.e., repeated sine regression), or via automated sinusoidal regression based on advanced, high-resolution methods. Figure 1 shows the knee joint angle vs. time point plot for normal walking at a speed of 5 km/h. The diagram also shows the graph of the best function fit with a sum of two sine functions. For a description of the experimental setting of the video recording of the normal walk, for a picture of the measured knee angle vs. hip angle diagram, and for the connection between points on this parametric curve and the phases of the gait cycles, the reader is referred to (Heck & van Dongen, 2008).

Muscle activity is typically studied using dynamic electromyography (EMG). In case of measurement of activity of superficial muscles, surface electrodes are placed on the skin surface to detect the electric activity responsible for contraction of muscles (Kamen & Gabriel, 2010). EMG recording is rather difficult because correct placement of electrodes is critical. Processing and interpreting an EMG for a muscle is also not easy. This research project offered the student the opportunity to personally experience such challenges faced in gait analysis. She not only learned science content by doing science, but also got a valid picture of how scientific knowledge is obtained and what the nature and status of scientific knowledge is. These are in fact commonly identified aims of practical work ('knowing that,' 'knowing how,' and 'knowing about;' cf. Wellington, 1998; Woolnough, 2000).

Although the student found it difficult to process and interpret the recorded EMG data, she managed to read off from the EMG signal when muscle activity was on and off in normal walking and to interpret the EMG signal in terms of phases and events in the gait cycle. Essential for successful analysis is that the data acquisition system allows simultaneous measurement with sensors and video capture. In Coach, the video clip and the measured data are synchronised: This means that clicking with the mouse on a point on the graph automatically shows the corresponding video frame and that selecting a particular frame highlights the corresponding points in diagrams, when scanning mode is on. Scrubbing, i.e., advancing or reversing the video clip manually and simultaneously watching the points in the data graphs that correspond with the video frame in view, is then an effective means to reveal that maxima and minima in the clearly periodic, raw and processed EMG signal are consistently linked to certain gait events. The lower diagram in Figure 2 shows the graph of the processed EMG signal during one gait cycle: 'Processing' means here that the absolute value of the raw signal is taken with respect to the zero level of the signal (so-called full-wave rectification) and that the rectified signal is smoothed via a moving average. These techniques exemplify the use of basic mathematics in biomechanical practice. Peaks in the diagram correspond with muscle activity and corresponding gait events have been marked in the diagram. The most general conclusion of the student, based on the processed EMG diagram, was that the gastrocnemius is predominantly active during stance phase.

The role of ICT in the gait analysis project did not differ much from that in the bungee jumping project, except that the hardware and software offered the student here the opportunity to collect and work with high-quality, real-time data about human gait in much the same way movement scientists do. Regarding mathematical content, emphasis was put in the project on data processing, regression, visualisation and interpretation of graphs in a real world context.

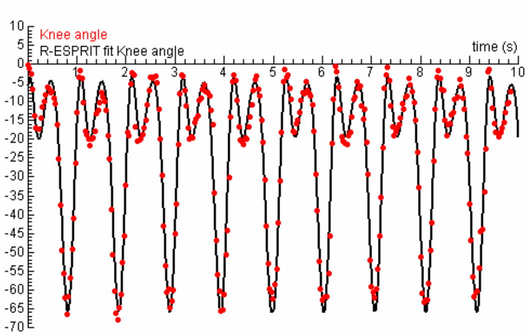

Figure 1 Knee joint angle vs. time: collected data and a regression curve.

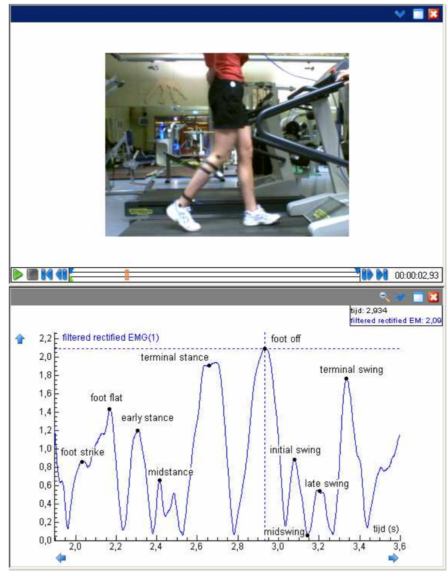

Figure 2 Processed EMG signal of the gastrocnemius muscle for normal walking at low speed

5 MOTION OF THE SPRING PENDULUM

A learning activity can also make use of authentic software tools, i.e., computer environments that are used by professional mathematicians and scientists on the workplace. Computer algebra systems belong to this category of tools. A popular system, *Mathematica* (www.wolfram.com), is advertised as "an environment for doing anything technical, that is, not for computing only, but also for modelling, simulation, visualisation, development, documentation, and deployment." All of these purposes are relevant for doing research and hardly anyone ignores the advantages of using ICT for this type of work. At the Science Faculty of the University of Amsterdam freshmen get the opportunity to experience this in a short (10 hrs) CAS-supported investigation. Students can choose a project out of a given list, work it out in groups of two, and finally report on it through a *Mathematica* notebook. Other aims of the student projects are to motivate the freshmen in their first steps in the discipline and to let them experience that their present knowledge of mathematics and science, empowered by a computer algebra system, already suffices for investigating a phenomenon at a rather deep level. The authentic nature of the activity also lies in ample choice of exploratory pathways: the students themselves select the research methodology and the level of complexity to which they feel themselves challenged; a general research question is already given to direct the research. The fact that all projects concern well-known, solved problems does not matter much: the students are unfamiliar with the problems, they get the opportunity to explore and solve them in their own way, or they can follow in the footsteps of researchers.

I discuss the work of a physics student who chose to study the motion of a spring pendulum, for which the given research question was: "Is the orbit of a given spring pendulum predictable and to what extent is the motion influenced by small changes in initial conditions?" The spring pendulum is an understandable and yet intriguing example of a nonlinear physics problem that shows chaotic behaviour (van der Weele & de Kleine, 1996). Although its mathematical modelling is often used in mechanics courses to exemplify the Euler-Lagrange formalism, the model can also be derived from first principles of Newton mechanics. Figure 3 shows the Cartesian and polar coordinate systems used in studying the motion of the pendulum bob with mass *m* that is attached to a massless spring of rest length *l*. The spring can make on its turn a two-dimensional pendular motion around a fixed point that is taken as origin of the coordinate system.

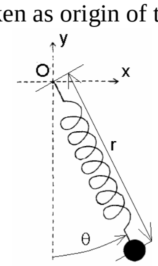

Figure 3 The spring pendulum configuration

There are two basic forces in the problem when dissipation is not taken into account: a constant gravity force (where *g* denotes the acceleration of gravity), and a spring force that is assumed to obey Hooke's law, which means that the spring force is proportional to the extension of the spring (the proportionality constant is denoted by *k*). In Cartesian coordinates, Newton's second law of motion and decomposition of the forces acting on the mass in the coordinate directions leads to the following system of differential equations:

$$\ddot{x} = -\frac{kx}{m}\left(1-\frac{l}{\sqrt{x^2+y^2}}\right), \quad \ddot{y} = -g-\frac{ky}{m}\left(1-\frac{l}{\sqrt{x^2+y^2}}\right)$$

In order to increase the authenticity of the research project, the student must derive the equations of motion. This is uncommon in secondary school practice: There, mathematical equations are often given without much explanation.

Hereafter, the student determined the equilibrium positions: there are a stable equilibrium point below the origin and an unstable point above the origin under reasonable conditions. The student implemented the equations of motion in *Mathematica* and verified that the numerical solution of the differential equations with the spring pendulum bob initially at rest in one of the equilibrium points remains constant. He chose for this and for further computations the following rational parameter values (which we take as well):

$m = 1/20\,\mathrm{kg}$, $g = 981/100\ \mathrm{m/s^2}$, $k = 1\,\mathrm{kg/s^2}$, $g = 981/1000\ \mathrm{m}$

He motivated his choice of fractions instead of floating-point number by the intention to work as much as possible in exact arithmetic, trying to avoid in this way problems with numerical precision and reducing errors introduced by numerical computation. The noticeable point here is that the student apparently realised that he should distinguish between phenomena that inevitably occur in reality and those that are caused by inexact, numerical computations. This is a good sign of reflection about the quality of research. The student's choice of parameter values actually implies that a remarkable behaviour of the spring pendulum can be found, ranging from periodic to chaotic motion (cf., van der Weele & de Kleine, 1996). Figure 4 shows rather stable oscillatory movements in case the bob starts at position (1.45, -1.45) and (0.4, 0.4), respectively, with zero speed. Figure 5 shows the periodic orbit of the bob when it starts at (0.1, 0.281058) without speed and at a point nearby, viz., (0.1, 0.281058). This figure illustrates what is meant by chaos: A small change in the initial condition can lead to a dramatic change in behaviour of the solution. This is not a consequence of numerical computations in a given working precision, but a fundamental property of the dynamical system.

Being confronted with regular and chaotic behaviour of solutions of the motion of a spring pendulum, the student set himself the task to systematically analyse in what regions chaos occurs. For example, he looked at various starting points on the *x*-axis. Figure 6 shows how close periodic and chaotic motions can be: The starting points for the orbits are (1.496, 0) and (1.495, 0), respectively.

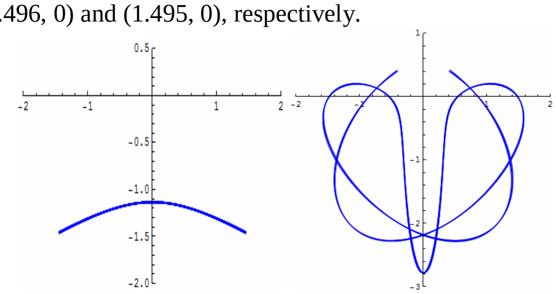

Figure 4 Two oscillatory motions of a spring pendulum

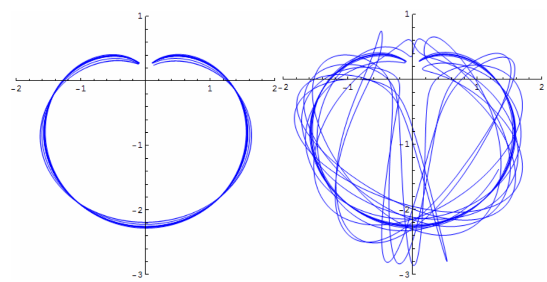

Figure 5 Motions starting at nearby points close to origin

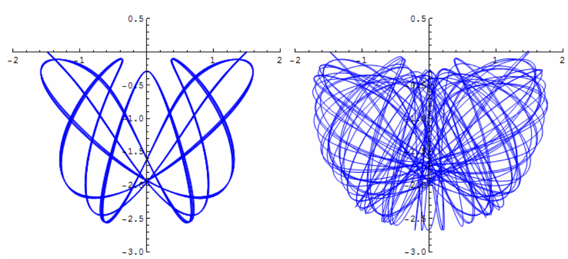

Figure 6 Motions starting at nearby points along the *x*-axis

The student explored the motion systematically and discovered that small deviations from the stable equilibrium point and large extensions of the spring pendulum do not lead to chaotic behaviour. He also concluded that there are regions in space where chaos occurs and where chaos hardly occurs. Of course he did not have the background knowledge to get advanced results. But this was not the aim of the project and lab assistants better make not many suggestions in such directions. Instead, they should give students ample opportunity and encourage them to discover things on their own, explore ideas, and enjoy doing investigations.

6 CONCLUSION

The educational issue in the described students' investigative work is the ICT-supported interaction between experimental work and mathematical modelling, in which the obtainment and interpretation of results is based on methods from mathematics and science. The fact that students must apply their mathematical and scientific knowledge in a meaningful way in a concrete context leads at the same time to consolidation and deepening of this knowledge. The examples in this paper show that ICT can contribute to the realisation of authentic inquiry and raise its level by allowing students to gather information about the subject and be in contact with experts; to collect real-time data of good quality; to process, analyse and visualise data; to do computations that are otherwise impossible; to build and use computer models of dynamic systems in much the same way as professionals do; to compare results from experiments, models, and theory; and to report results. Besides, students can develop, practise and demonstrate the following research abilities:

- formulate good research questions that guide the work;
- design and implement an experiment for collection of relevant data;
- apply mathematical knowledge and techniques, and science concepts in new situations;
- effectively use a computer in research work;
- construct, test, evaluate, and improve computer models, and have insight in their role in science;
- interpret, theoretically underpin and reflect on results;
- collaborate with others in an investigation task.

The student-driven experimental design, the modelling process, the underlying thinking processes, the discussions with peers, the effective use of ICT, and the improvement of students' mathematical and scientific literacy in the research are considered more important than the obtained results.

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BIOGRAPHICAL NOTES

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