Matrix perturbations: bounding and computing eigenvalues
Reis da Silva, R.J.

Citation for published version (APA):

General rights
It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations
If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: http://uba.uva.nl/en/contact, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

Download date: 02 Dec 2018
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Error in the lower bound (and detail for $q \leq 10$) of the smallest eigenvalue of the perturbed matrix $A$ resulting from the Finite Difference discretization ($n = 32, \xi = 1/33$) of Problem (2.17) with $E$ a rank-one diagonal matrix resulting from Equation (2.18).</td>
<td>49</td>
</tr>
<tr>
<td>2.2</td>
<td>Error in the lower bound (and detail for $q \leq 10$) of the smallest eigenvalue of a random rank-one perturbation $E$ of the matrix $A$ obtained from the discretization of Poisson equation on a $40 \times 40$ grid.</td>
<td>51</td>
</tr>
<tr>
<td>2.3</td>
<td>Error in the lower bound (and detail for $q \leq 10$) of the smallest eigenvalue of the perturbed matrix Poisson with $E$ a rank-2 random matrix.</td>
<td>52</td>
</tr>
<tr>
<td>2.4</td>
<td>Error in the lower bound of the smallest eigenvalue of the perturbed matrix $A$ for the matrix Sherman1 when a multiple eigenvalue is clustered (below) and comparison with the naive approach (above).</td>
<td>53</td>
</tr>
<tr>
<td>2.5</td>
<td>Error in the lower bound (and detail for $q \leq 20$) of the smallest singular value of $B_c$ where $B$ is the matrix ILLC1850 of dimensions $1850 \times 712$ from the Matrix Market and $c$ is a random $n$-dimensional vector.</td>
<td>53</td>
</tr>
<tr>
<td>3.1</td>
<td>Interpolating polynomials of different degree for different values of $\theta$. The seven asterisks represent the eigenvalues.</td>
<td>64</td>
</tr>
</tbody>
</table>
3.2 Eigenvalue perturbation by a rank-one matrix (left) and a rank-$k$
matrix (right). ................................................................. 77

3.3 Illustration of the construction in the proof of Theorem 3.4.7. Three
of the seven eigenvalues of $A$, indicated by the circles, are already
on the quadratic curve $C$, and a rank-4 matrix $Z$ is needed to push
the remaining four eigenvalues onto $C$, after which the augmented
matrix can be formed. ...................................................... 83

3.4 Normality preserving augmentations of a $3 \times 3$ matrix for two dif-
erent values of $\gamma$. ...................................................... 86

3.5 Eigenvalue trajectories of a normality preserving perturbation, and
of the same perturbation written as the sum of non-normality pre-
serving normal perturbations. ...................................... 89

4.1 Arrowhead updates from $\hat{A}_{k-1}$ to $\hat{A}_k$ for consecutive values of $k$. 99

4.2 Approximating a matrix $A$ from structural engineering from below
by a sparser matrix $A_0$ by subtracting from $A$ a definite matrix $H$;
the sparsity plot of $A$ (with 3648 nonzero entries) is on the left and
of $A_0$ (with 968 nonzero entries) on the right. See the experiments
with this pair of matrices in Section 4.4.4] .................................. 116

4.3 Lanczos method (left) with a random start vector, versus SPAM
(right) with the discretized reaction term as approximating matrix
$A_0$, and the largest eigenvalue of $A$ as target. .................. 123

4.4 From left to right: Lanczos and Full SPAM approximating the
largest, the second, the fifth, and the smallest eigenvalue of $A$, us-
ing algebraic rank-12 approximation from below (for the smallest
eigenvalue, we applied both the methods to $6I - A$). Lanczos and
Full SPAM used in each experiment the same start vector] ....... 124

4.5 Lanczos versus Full SPAM and SPAM(1) with diagonal (left), tridi-
agonal (middle left), and with algebraic rank-6 approximation from
below (both middle right and right). In the three leftmost pictures,
the target was the largest eigenvalue, at the right it was the smallest
eigenvalue (i.e., the largest of $\alpha I - A$). ......................... 125
4.6 Lanczos versus Full SPAM and SPAM(1) for bcsstk04, bcsstk07 and bcsstk10, with approximating matrices from below. All three methods used the same start vector.

4.7 Lanczos and SPAM(1) compared with SPAM(1,ℓ) for small values of ℓ. Left: reaction-diffusion problem, smallest eigenvalue. Other pictures: largest eigenvalue. Middle left: banded matrix; middle right: bcsstk07; right: bcsstk10. The graphs for Lanczos and SPAM(1) can also be found in Figures 4.4, 4.5, and 4.6.

4.8 Comparing SPAM(1,ℓ) with JD(1,ℓ) and JD(ℓ). The eigenvalue problems are exactly the same as the corresponding ones in Figure 4.7, and the curves for SPAM(1,ℓ) can be found there as well.

5.1 Two stories building. The elasticity of the walls is denoted by the \( a_j \)s, their displacement by the \( \tilde{x}_j \)s and the mass of the ceilings are denoted by the \( m_j \)s for \( j \in \{1, 2, 3\} \).