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DOI

[10.1016/S0165-1765\(96\)00930-5](https://doi.org/10.1016/S0165-1765(96)00930-5)

Publication date

1996

Published in

Economics Letters

[Link to publication](#)

Citation for published version (APA):

Hommel, C. H., & van Eekelen, A. (1996). Partial equilibrium analysis in a noisy chaotic market. *Economics Letters*, 53, 275-282. [https://doi.org/10.1016/S0165-1765\(96\)00930-5](https://doi.org/10.1016/S0165-1765(96)00930-5)

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A078513286 ISN: 683785

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Verzoeken te behandelen voor: 16-05-2005 **Ingediend door:** 0004/9999
Datum en tijd van indienen: 02-05-2005 18:53 **Datum plaatsen:** 02-05-2005
 18:53 **Type aanvrager:** UKB **I.D.:** UVA KEUR (UB GRONINGEN)

PPN: 831768878

Economics letters 1978 Amsterdam North-Holland

Gewenst: 1996-00-00 **Deel:** 53 **Nummer:** 3 **Electronisch leveren (LH=N)**

Auteur:	Titel van artikel:	Pagina's:
Hommes, C. (ed.)	Partial equilibrium analysis in a noisy chaotic market	275-282

Opmerking:
arno ID: 27023

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Vol. 35(1991)-77(2002)
Vol. 1(1978)-

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Partial equilibrium analysis in a noisy chaotic market

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Received 19 June 1996; accepted 23 September 1996

Abstract

We investigate the validity of partial equilibrium analysis in a nonlinear chaotic market model subject to small noise. Despite the sensitivity to market parameters in a chaotic economy, partial analysis can still detect both qualitative and quantitative features of price fluctuations which are robust against small noise and market interdependencies.

Keywords: Autocorrelations; Chaotic market; Noise; Partial analysis

JEL classification: D40; B41; E32

1. Introduction

Does a partial equilibrium analysis in the tradition of Marshall (1919), in which small 'second-order' effects are disregarded, provide useful insight into economic market behaviour? This is an important question for economic analysis. Schumpeter (1954, pp. 990–991) described partial analysis as a "principle of the negligibility of indirect effects", but also warned that "the postulate, which is so powerful a simplifier, severely restricts the method's range of application". In a recent paper, Currie and Kubin (1995) argue that, if market forces are nonlinear, the answer to the above question would be negative. By means of a simple non-linear cobweb-type two-market model they show that a partial analysis of the first market yields conclusions which are strikingly different when compared with the complete two-market model, when small interdependencies between the two markets are taken into account. For example, the partial analysis of the first market can lead to regular price fluctuations converging to a stable 3-cycle, whereas in the two-market model with extremely small

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interdependencies the stable 3-cycle has disappeared and chaotic price fluctuations occur.¹ Since in general market forces are non-linear, the implications of this simple example for economic analysis are profound; partial analysis would be inappropriate and only a full general equilibrium analysis could yield useful insight into market behaviour. The conclusion of Currie and Kubin (1995, p. 31) is that “the range of application of partial analysis may thus be even more limited than Schumpeter could have appreciated”.

The present letter argues that this conclusion is too quick and that partial equilibrium analysis can yield useful and “robust” conclusions, even when market forces are non-linear and price fluctuations are (close to) chaotic. We shall use the same model as in the work of Currie and Kubin (1995) but add small noise to the demand equations of both markets. In the noisy model, the 3-cycle disappears and price fluctuations are chaotic. We show that these chaotic price fluctuations have both qualitative and quantitative features, such as sample autocorrelations, that are robust against small noise and small market interdependencies. When small noise is taken into account, both the partial analysis of the first market and the complete two-market model yield qualitatively and quantitatively the same results. Even in an unstable market, when interdependencies are weak, partial equilibrium analysis can thus lead to useful economic insight.

2. The model

In order to be self-contained, we briefly recall the cobweb model as described by Currie and Kubin (1995).² The first commodity takes one period to produce. The price p_{1t} adjusts instantaneously and the market clearing price is determined by a linear demand function

$$q_{1t} = a_1 - p_{1t} + s_1 p_{2t} + \epsilon_{1t}, \quad a_1 > 0, \quad (1)$$

where p_{2t} is the price of the second commodity, s_1 is a market interdependency parameter, which is positive (negative) if the two goods are substitutes (complements), and ϵ_{1t} is an independently and identically distributed (IID) random variable. Profit per unit of the first commodity is

$$\pi_{1t} = p_{1t} - c, \quad (2)$$

where c represents the production cost per unit of output. A simple quantity adjustment process is postulated, where the rate of change in production is proportional to the unit profit, i.e.

$$\frac{q_{1,t+1} - q_{1t}}{q_{1t}} = \sigma \pi_{1t}, \quad (3)$$

¹ Traditionally, the cobweb model has played an important role in economic dynamics. In the last decade, several workers have investigated the possibility of complicated chaotic price fluctuations in the cobweb model (e.g., Artstein, 1983; Jensen and Urban, 1984; Chiarella, 1988; Hommes, 1991, 1994; Brock and Hommes, 1995).

² In fact we consider a slightly simplified version without taxes (i.e. their parameters $t_1 = t_2 = 0$) and interest (i.e. $r = 0$). This simplified model contains all essential features for our purposes.

where $\sigma > 0$ is the adjustment parameter. Marshall (1920, p. 345) justified such a mechanism by noting that, when the demand price is greater than the supply price, “there is at work an active force tending to increase the amount brought forward for sale”.

The second market is even simpler. There is no production lag, demand and supply are linear and the market clears instantaneously:

$$\begin{aligned} q_{2t}^d &= a_2 + s_2 p_{1t} - p_{2t} + \epsilon_{2t}, & a_2 > 0, \\ q_{2t}^s &= b p_{2t}, & b > 0, \\ q_{2t}^d &= q_{2t}^s, \end{aligned} \quad (4)$$

where s_2 is the second market interdependency parameter and ϵ_{2t} is another random IID variable. The interdependent two-market model is thus given by (1)–(4). The reader may easily check that solving for the quantity of the first commodity yields the quadratic difference equation

$$q_{1,t+1} = \left(1 + \sigma \frac{(a_1 + \epsilon_{1t})(1+b) + s_1(a_2 + \epsilon_{2t})}{1+b-s_1s_2} - \sigma c \right) q_{1t} - \sigma \frac{1+b}{1+b-s_1s_2} q_{1t}^2. \quad (5)$$

Using the simple linear transformation $x_t = [\sigma(1+b)/\mu(1+b-s_1s_2)]q_{1t}$, yields the familiar logistic equation

$$x_{t+1} = \mu x_t(1-x_t), \quad (6)$$

with

$$\mu = 1 + \sigma \frac{(a_1 - \epsilon_{1t})(1+b) + s_1(a_2 + \epsilon_{2t})}{1+b-s_1s_2} - \sigma c. \quad (7)$$

Throughout the paper we fix the parameters $a_1 = 3$, $a_2 = 2$, $b = 1$ and $c = 1$. Both ϵ_{1t} and ϵ_{2t} will be drawn independently from a uniform distribution on $[-\epsilon, \epsilon]$. We shall investigate how the price–quantity fluctuations depend upon the interdependency strengths $s = s_1 = s_2$, the noise level ϵ and the adjustment parameter σ .

3. Numerical simulations

Fig. 1(a) shows the bifurcation diagram with respect to the adjustment parameter σ , in a partial equilibrium analysis of the first market (i.e. $s = 0$) without noise (i.e. $\epsilon = 0$). The diagram illustrates the well-known period doubling bifurcation route to chaos, as σ increases. Fig. 1(b) shows the diagram with small coupling between the two markets ($s = 0.03$). Increasing the market interdependency $s = s_1 = s_2$ shifts the bifurcation diagram to the left. This is immediately clear by noting that, for $s > 0$, $\epsilon = 0$, for fixed μ in (7) the corresponding σ will be smaller. In particular, without market interdependency, the period 3 window occurs for $1.414 < \sigma < 1.428$ whereas, with small market interdependency $s = 0.03$, the period 3 window occurs for $1.393 < \sigma < 1.406$. Fig. 1(c) shows the bifurcation diagram without coupling

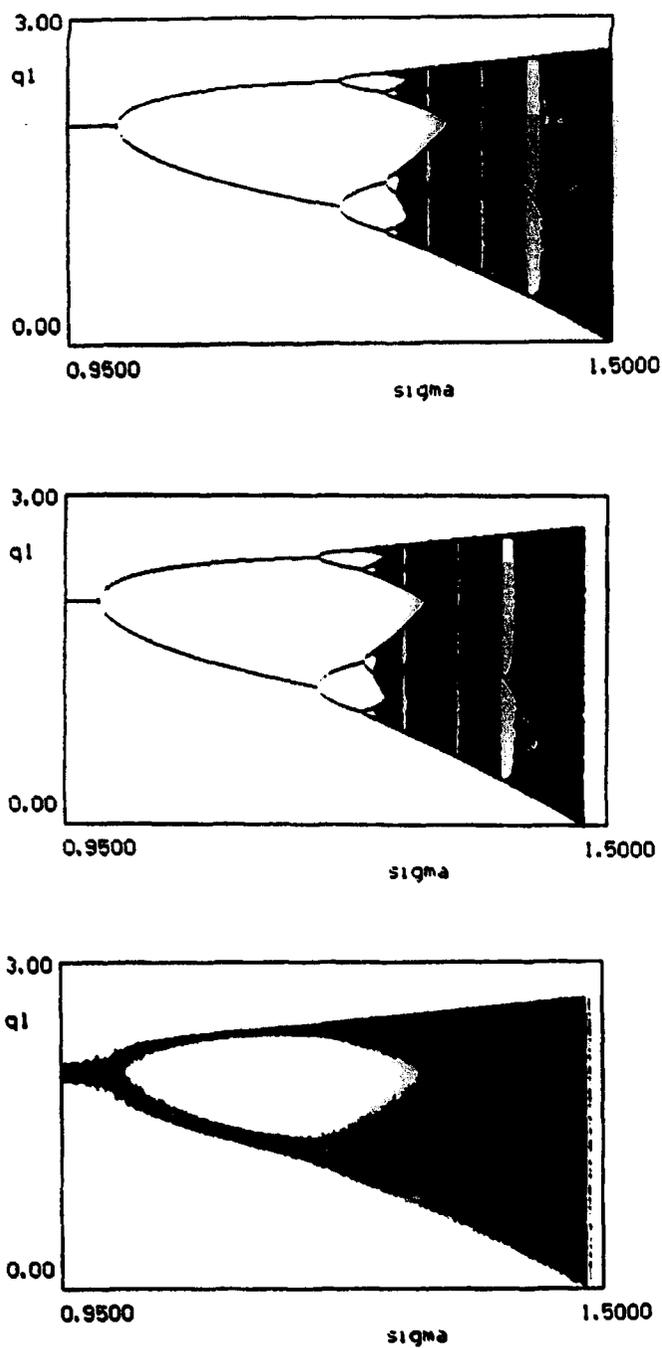


Fig. 1. Bifurcation diagrams with and without noise: (a) no coupling, no noise; (b) coupling ($s = 0.03$), no noise; (c) no coupling, small noise ($\epsilon = 0.01$).

($s = 0$), subject to small noise ($\epsilon = 0.01$). Clearly, small noise destroys the fine detailed structure in the diagram. For example, the period 3 window clearly visible in Figs. 1(a) and 1(b) has been “destroyed” by noise. In fact, in the noisy diagram, only the steady state, the

2-cycle and perhaps the 4-cycle can be distinguished from the chaotic region. All other periodic windows are dominated by the small amount of noise.

Fig. 2 shows some typical time series of the quantity q_1 in the first market, for the parameter $\sigma = 1.42$ in the period 3 window, with and without market coupling and/or noise. In Fig. 2(a), without coupling and noise, prices converge to a stable 3-cycle. Fig. 2(b) shows that small coupling ($s = 0.03$; $\epsilon = 0$) destroys the stable 3-cycle and leads to chaotic price fluctuations. The same is true for the time series in Fig. 2(c) (no coupling; small noise) and Fig. 2(d) (small coupling; small noise). However, in all cases, although the time series are not exactly periodic, they are still characterized by phases of almost period 3 fluctuations. In addition, phases of close to steady state and up and down period 2 oscillations can be identified in all series. These almost periodic phases occur at irregular and seemingly unpredictable time intervals.

In order to check for remaining, statistically significant structure, Fig. 3 plots the autocorrelation function (ACF) for each of the time series of Fig. 2.³ In the case of the stable 3-cycle, the ACF in Fig. 3(a) is periodic. In the case of small coupling (Fig. 3(b) the ACF is significant at lags 1, 3, 4 and 6, The same is true in the case of small noise (fig. 3(c)). In the case of small noise and small coupling (Fig. 3(d)), the same significant lags occur, with an additional significant lag 9.

This numerical evidence shows that, even in a chaotic market, certain qualitative and quantitative features are robust against small noise and small market interdependencies. In the present case, especially the autocorrelations at lags 1 and 3 are highly significant in all time series with small noise and small market interdependencies, thus indicating a “noisy” period 3 cycle.

4. Concluding remarks

In non-linear systems, as a control parameter is varied, highly structured bifurcation routes to chaos arise. In the physical sciences, such detailed bifurcation routes to chaos have actually been observed in fine-tuned controlled physical experiments. Economics is not like physics, however. Fine-tuned experiments are almost impossible and economic time series are probably much noisier than most physical time series. Consequently, economic analysis should not focus on the details of chaos, but only on the qualitative and quantitative global features in non-linear systems that are robust against small noise. Among such robust features are for example autocorrelations of noisy chaotic time series. As long as market interdependencies are weak, partial equilibrium analysis can be very useful in detecting such robust features of nonlinear market forces. Schumpeter's warning concerning the limitations of partial analysis remains, but “the simplifier may be powerful enough to provide a simple model and useful economic insight into erratic price fluctuations”.

³ All sample ACFs are based upon 240 observations. All ACF plots are displayed with “Bartlett bands” of twice the standard deviation if the data were truly random (c.g., Box et al., 1994, pp. 32–34)). Among the few references on ACFs of chaotic processes are the papers by Bunow and Weiss (1979) and Sakai and Tokumaru (1980); ACFs of chaotic price time series in the cobweb model have been investigated by Hommes (1996).

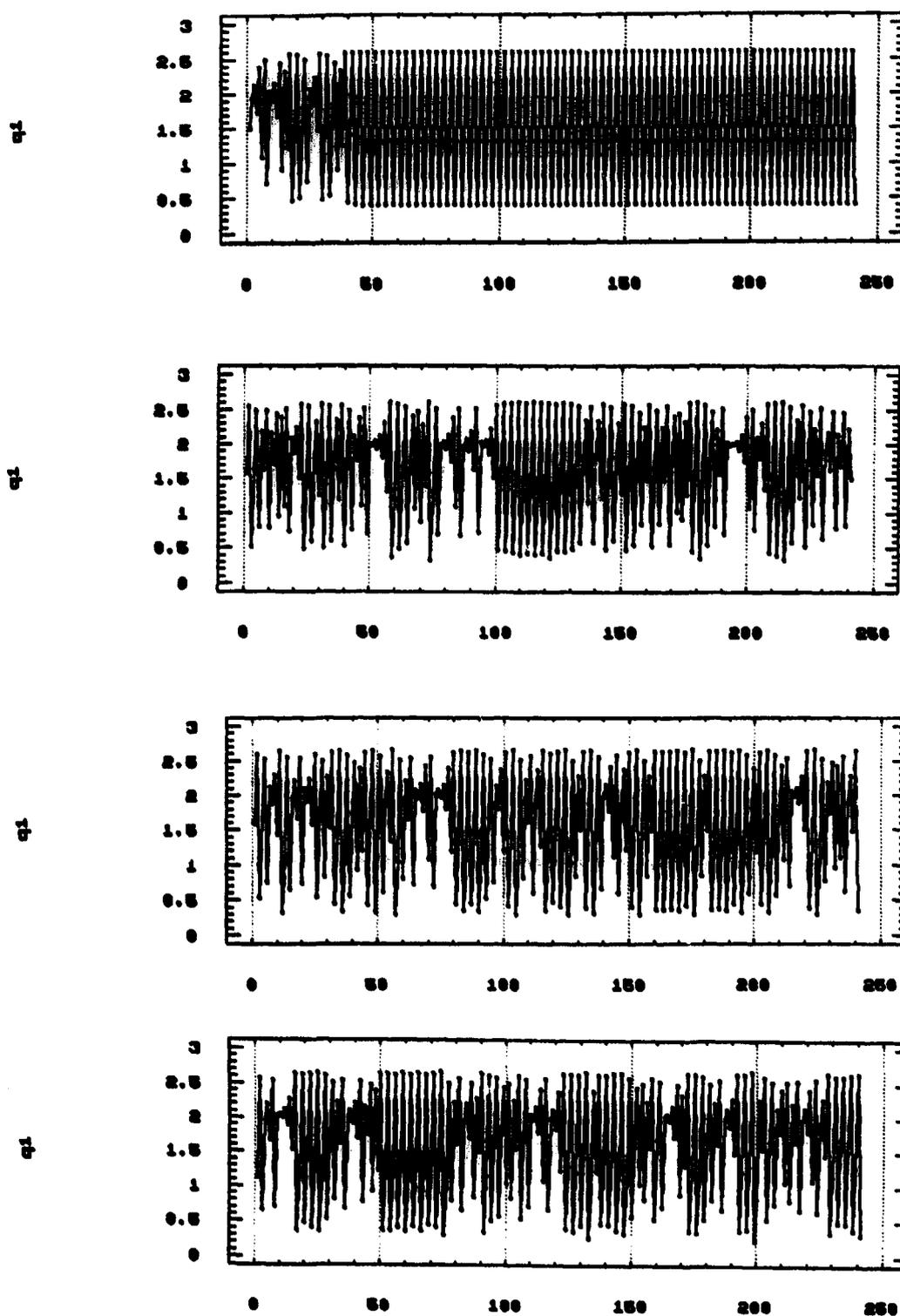


Fig. 2. Time series of q_t for $\sigma = 1.42$, with and without coupling and/or noise: (a) $s = 0$, $\epsilon = 0$; (b) $s = 0.03$, $\epsilon = 0$; (c) $s = 0$, $\epsilon = 0.01$; (d) $s = 0.03$, $\epsilon = 0.01$.

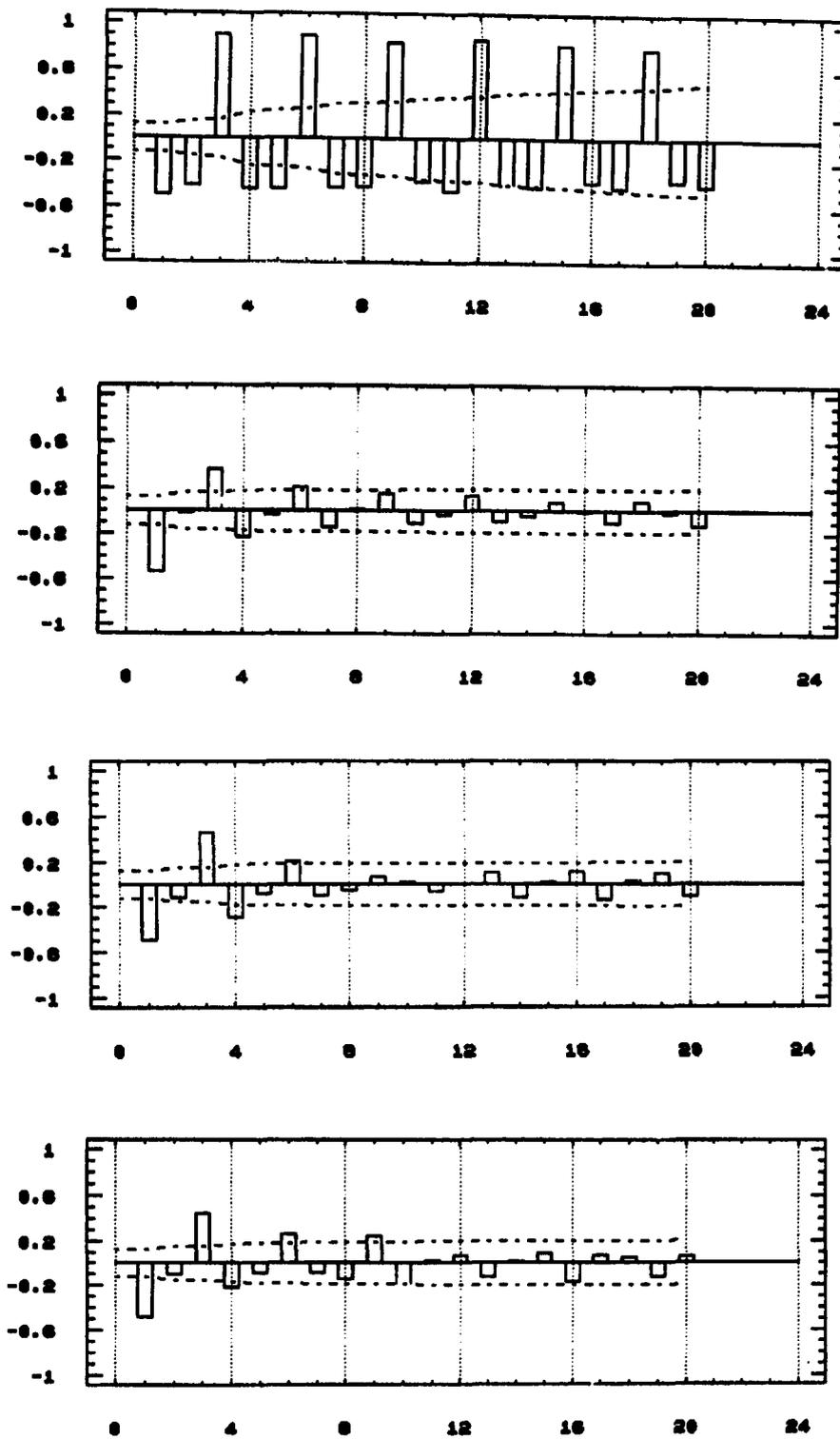


Fig. 3. ACFs corresponding to the time series in Fig. 2.

Acknowledgements

We would like to thank Jacob Goeree, Jan Tuinstra and Claus Weddepohl for stimulating discussions.

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