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## Inelastic decay processes in a gas of spin-polarized triplet helium

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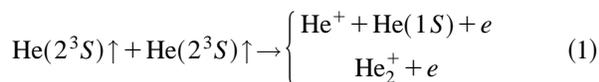
We calculate the rates of elastic and inelastic collisions in a gas of spin-polarized triplet helium ( $^4\text{He}^*\uparrow$ ) in a wide range of temperatures ( $0 < T < 0.5$  K) and magnetic fields ( $0 < B < 100$  kG), which includes all values of  $T$  and  $B$  relevant for trapped  $^4\text{He}^*\uparrow$ . At densities  $n \leq 10^{13}$  cm $^{-3}$ , where three-body recombination is not important, the main inelastic processes are spin relaxation and relaxation-induced ionization, both induced by the spin-dipole interaction in pair collisions. For ultralow temperatures  $T \leq 10$  mK and low magnetic fields  $B \leq 100$  G, the leading decay channel is relaxation-induced ionization. If either  $T$  or  $B$  is higher, the gas decays through spin relaxation. The ratio of elastic to inelastic rate is found to be large even in the nanokelvin regime, which is promising for evaporative cooling of  $^4\text{He}^*\uparrow$ .

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### I. INTRODUCTION

Spin-polarized metastable triplet helium ( $^4\text{He}^*\uparrow$ ), a gas of helium atoms in the  $2^3S$  state with spins aligned, is a promising system for magnetostatic trapping [1] and studying Bose-Einstein condensation (BEC) and related macroscopic quantum phenomena [2]. The uniquely large radiative lifetime of an individual He ( $2^3S$ ) atom,  $\tau_0 \approx 8 \times 10^3$  s [3,4], enables active experimental studies of metastable triplet helium. For *in situ* diagnostics and manipulation of magnetically trapped  $^4\text{He}^*\uparrow$  there are a number of convenient optical transitions. Furthermore, an efficient detection method can be based on the ionization decay of He( $2^3S$ ) atoms due to interatomic collisions or interaction with the walls, an advantage over stable atoms. The growing interest in the gaseous phase of metastable triplet helium is stimulated by recent successful experiments on three-dimensional subrecoil cooling of He( $2^3S$ ), where temperatures  $\sim 100$  nK have been reached [5].

Spin polarization and surface-free confinement are crucial requirements for the stability of a gas of metastable triplet helium. Magnetostatic trapping provides us with both. Spin polarization strongly reduces the rate of Penning ionization in pair atomic collisions in the bulk, and surface-free confinement prevents the quenching which would result from collisions of He( $2^3S$ ) atoms with the walls. Unpolarized metastable triplet helium gas is very unstable [6,7]: the Penning ionization rate is so fast (the rate constant  $\sim 10^{-9}$  cm $^3$ /s [8]) that it easily dominates over elastic collisions at low temperatures. In the *fully* polarized gas the spin conservation rule forbids Penning ionization via the ordinary mechanism in pair collisions, since the total spin  $S$  of colliding particles in the final state cannot exceed 1, whereas in the initial state  $S=2$ . A theoretical analysis of the decay kinetics of  $^4\text{He}^*\uparrow$  at ultralow temperatures [2] shows that the ionization process



is mainly caused by the spin-dipole interaction and is five orders of magnitude slower than Penning ionization in the

unpolarized gas. The leading mechanism of the process (1)—relaxation-induced ionization—involves virtual spin-relaxation transitions in the colliding pair of atoms from the initial  $S=2$  quasimolecular state  $^5\Sigma_g^+$  to the  $S=0$  state  $^1\Sigma_g^+$ , which autoionizes through the ordinary Penning mechanism. In low magnetic fields ( $B \leq 100$  G) relaxation-induced ionization is field independent, with a rate constant  $\alpha_{\text{ri}} \sim 10^{-14}$  cm $^3$ /s, and is the main two-body decay channel of the  $^4\text{He}^*\uparrow$  gas. In higher fields  $\alpha_{\text{ri}}$  decreases and relaxation-induced ionization becomes slower than the process of spin relaxation, which is also induced by the spin-dipole interaction in pair collisions. The relaxation rate constant  $\alpha_{\text{rel}}$  decreases with increasing  $B$  for  $B > 750$  G, but even in fields  $B \sim 100$  kG it remains larger than the low-field value of the rate constant of relaxation-induced ionization. These two-body inelastic processes dominate over three-body recombination for gas densities  $n \leq 10^{13}$  cm $^{-3}$ , corresponding to BEC at temperatures  $T \leq 2$   $\mu$ K. In low fields, which are more promising for achieving BEC, the characteristic decay time of the gas,  $\tau \sim 1/\alpha_{\text{ri}}n$ , will be of order seconds or larger at these densities.

As noted in [2], the scattering length  $a$  for the interaction between two  $^4\text{He}^*\uparrow$  atoms, following from the recently calculated interaction potential  $U_2(R)$  of the  $S=2$  quasimolecular state  $^5\Sigma_g^+$ , [9] is positive and large. This ensures the stability of a Bose condensate with respect to elastic interaction between particles. Moreover, the comparatively large value of  $a$  (two orders of magnitude larger than in spin-polarized atomic hydrogen) should provide efficient evaporative cooling and condensate formation [10,11], both processes being determined by the characteristic time of elastic collisions,  $\tau_c = (n\sigma v_T)^{-1}$ , where  $\sigma = 8\pi a^2$  is the cross section of elastic scattering and  $v_T$  is the thermal velocity.

Various schemes for trapping and cooling metastable triplet helium are being pursued. For instance, one can first load a magneto-optical trap (MOT), either from an atomic beam [7] or by using a cryogenic buffer gas of ground state He atoms [1], and then switch to magnetostatic trapping by extinguishing the light. Between the MOT and the trapping phases, the gas could be optically precooled even to subrecoil temperatures [5]. Another scheme under active investi-

gation is the loading of a magnetostatic trap directly from an atomic beam by optical methods [12]. A promising trap loading method, which is well suited to attain high densities, is direct loading of spin-polarized metastable triplet helium from a cryogenic discharge into a deep magnetic trap, with the  $^4\text{He}^{*\uparrow}$  thermalized, before reaching the surrounding walls, by diffusion in ground-state He vapor at  $T \approx 0.4$  K [13]. After removing the He vapor much of the  $^4\text{He}^{*\uparrow}$ , isolated from the walls, remains trapped. Then, evaporative and optical cooling may be used to reach ultralow temperatures with densities relevant for BEC. The possibilities of cooling  $^4\text{He}^{*\uparrow}$  to the BEC regime are fundamentally determined by the ratio of the elastic collision rate to the rate of inelastic decay processes. Therefore, the temperature dependence of these processes is a question of principal importance for future investigations of spin-polarized triplet helium.

In this paper we calculate the rates of elastic and inelastic collisions in  $^4\text{He}^{*\uparrow}$  gas in a wide range of temperatures ( $0 < T < 0.5$  K) and magnetic fields ( $0 < B < 100$  kG) which includes all values of  $T$  and  $B$  relevant for trapped  $^4\text{He}^{*\uparrow}$ . The character of the temperature dependence of inelastic processes is predetermined by which interparticle distances they occur and is influenced by the presence of a weakly bound  $s$  level, with a binding energy  $\varepsilon_0$  of several millikelvin, in the potential  $U_2(R)$ . Spin relaxation in low fields ( $B \leq 100$  G) occurs at large distances and increases with  $T$  either due to increasing density of states for the relative motion of atoms (for quasielastic relaxation in fields where  $2\mu_B B \ll T$ ) or due to increasing number of partial waves contributing to the transition amplitude (in fields  $B \sim 10$  G corresponding to a dip in  $\alpha_{\text{rel}}$  at ultralow temperatures). Spin relaxation in high fields ( $B \geq 100$  G) and relaxation-induced ionization occur at distances smaller than the characteristic radius of interaction in the potential  $U_2(R)$  and, in fact, are resonantly enhanced at ultralow temperatures by the presence of the weakly bound  $s$  level. But already at temperatures above  $500 \mu\text{K}$  the rates of these processes decrease with increasing  $T$ . Similar arguments apply to the rate constant of direct dipole-exchange Penning ionization, introduced in [14], which always remains much smaller than  $\alpha_{\text{in}} = \alpha_{\text{rel}} + \alpha_{\text{ri}}$ . The rate constants of spin relaxation and relaxation-induced ionization caused by the spin-orbit interaction in pair collisions are also much smaller than  $\alpha_{\text{in}}$ . Our calculations show that at temperatures  $T > 10$  mK in all fields investigated the decay of the  $^4\text{He}^{*\uparrow}$  gas is determined by spin relaxation, with a rate constant in the range  $10^{-13} - 10^{-14} \text{ cm}^3/\text{s}$ . For lower  $T$  the situation is similar to that in the zero temperature limit: in fields  $B \leq 100$  G the leading decay channel is relaxation-induced ionization, with the rate constant  $\alpha_{\text{ri}} \sim 10^{-14} \text{ cm}^3/\text{s}$ , and in higher fields the decay is governed by spin relaxation,  $\alpha_{\text{rel}}$  being maximum ( $\sim 5 \times 10^{-13} \text{ cm}^3/\text{s}$ ) at  $B \sim 1$  kG.

The presence of a weakly bound  $s$  level in the potential  $U_2(R)$  also leads to a resonance enhancement of the elastic cross section  $\sigma$  at ultralow energies. At collision energies  $E > 500 \mu\text{K}$ , i.e., energies comparable with the binding energy of this level and larger,  $\sigma$  acquires an energy dependence and decreases with increasing  $E$ , as one may expect from the theory of elastic scattering of slow particles (see, e.g., [15]). Accordingly, the elastic rate constant  $\alpha_{\text{el}} = \sigma v_T$  is characterized by a maximum at  $T \approx 2$  mK, with the low tem-

perature behavior,  $\alpha_{\text{el}} \propto \sqrt{T}$ , at smaller  $T$ . For  $T \geq 1$  mK, in any field, the ratio of elastic to inelastic rate constant  $\alpha_{\text{el}}/\alpha_{\text{in}} > 10^4$ . At lower temperatures this ratio decreases as  $\sqrt{T}$ , but remains large even in the nanokelvin regime.

## II. SPIN RELAXATION

We begin with the process of spin relaxation in pair collisions, induced by the spin-dipole interaction

$$\hat{H}_{\text{int}} = \frac{4\mu_B^2}{R^5} [(\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2)R^2 - 3(\hat{\mathbf{S}}_1 \cdot \mathbf{R})(\hat{\mathbf{S}}_2 \cdot \mathbf{R})] \quad (2)$$

( $\hat{\mathbf{S}}_1, \hat{\mathbf{S}}_2$  are the spin operators of the colliding atoms and  $\mathbf{R}$  is the internuclear distance). This interaction can change not only the spin projection  $M$  of the quasimolecule, but also the total spin  $S$ . Representing the spin wave function of the initial state of the quasimolecule ( $S=2, M=2$ ) as  $\Phi_{22} = \Phi_1^{(1)}\Phi_1^{(2)}$ , one finds that  $\hat{H}_{\text{int}}$  causes transitions to the following final spin states  $\Phi_{SM}$ :

$$\begin{aligned} \Phi_{21} &= [\Phi_1^{(1)}\Phi_0^{(2)} + \Phi_0^{(1)}\Phi_1^{(2)}]/\sqrt{2}, \\ \Phi_{20} &= [\Phi_1^{(1)}\Phi_{-1}^{(2)} + \Phi_{-1}^{(1)}\Phi_1^{(2)} + 2\Phi_0^{(1)}\Phi_0^{(2)}]/\sqrt{6}, \\ \Phi_{00} &= [\Phi_1^{(1)}\Phi_{-1}^{(2)} + \Phi_{-1}^{(1)}\Phi_1^{(2)} - \Phi_0^{(1)}\Phi_0^{(2)}]/\sqrt{3}. \end{aligned} \quad (3)$$

Here  $\Phi_{M_1}^{(1)}$  and  $\Phi_{M_2}^{(2)}$  are the spin states of the colliding spin-1 atoms, with spin projections  $M_1$  and  $M_2$ , respectively.

Atoms with opposite or zero spin projection on the direction of the magnetic field, produced in the relaxation process, escape from the trap. In the case of transitions  $\Phi_{22} \rightarrow \Phi_{2M}$  ( $M=0,1$ ), changing only the spin projection  $M$ , the quasimolecule remains in the  $S=2$  electronic state  $^5\Sigma_g^+$  from which the ordinary Penning ionization is impossible. The interaction potential in this state,  $U_2(R)$ , is characterized by a fairly deep potential well ( $\sim 1500$  K [16,9]) containing many bound states, with the highest  $s$  level having a binding energy  $\varepsilon_0$  of several millikelvin. For this potential the characteristic radius of interaction  $R_e$  (the range of action of the field) can be estimated from the condition  $|U_2(R_e)| = \hbar^2/mR_e^2$ , which gives  $R_e \approx 70a_0$  (in the Van der Waals tail of the potential) and the characteristic energy  $E_e = |U_2(R_e)| \approx 10$  mK. Accordingly, collision energies  $E_i \ll E_e$  should correspond to the  $s$ -wave scattering limit in elastic collisions between  $^4\text{He}^{*\uparrow}$  atoms.

The transitions  $\Phi_{22} \rightarrow \Phi_{00}$  lead to the formation of a quasimolecule in the  $S=0$  electronic state  $^1\Sigma_g^+$ , which is autoionizing via the ordinary Penning mechanism and is characterized by the interaction potential  $U_0(R)$  having an even deeper well ( $\sim 8000$  K [16]) than  $U_2(R)$ . As the Penning ionization occurs with probability close to unity at interparticle distances  $R \lesssim 7a_0$  ( $a_0$  is the Bohr radius) [16], and the spin-dipole transitions occur at larger  $R$ , we use a simple model for finding the wave function of the relative motion of atoms in the  $^1\Sigma_g^+$  state: we put a perfectly absorbing boundary at a distance  $R_0 \approx 7a_0$  and consider the potential to be purely elastic and equal to  $U_0(R)$  at larger  $R$ .

The spin-dipole interaction is much weaker than the elastic interaction between particles and, hence, the relaxation

transition rates can be calculated within first-order perturbation theory. The scattered wave of atoms for each spin state (3) can be written as

$$\Psi_{SM}(\mathbf{R}) = \frac{m}{\hbar^2} \int G_S(\mathbf{R}, \mathbf{R}') H_{SM}(\mathbf{R}') \Psi_{22}(\mathbf{R}') d\mathbf{R}'. \quad (4)$$

Here  $\Psi_{22}(\mathbf{R})$  is the wave function of the relative motion of atoms in the initial state, with energy  $E_i = \hbar^2 k_i^2/m$ ;  $G_S(\mathbf{R}, \mathbf{R}')$  is the Green function of the Schrödinger equation for the relative motion in the final-state potential  $U_S(R)$  with energy  $E_f = E_i + E_M$ , where

$$E_M = 2\mu_B B(2-M) \quad (5)$$

is the change of the Zeeman energy in the transition. The quantity

$$H_{SM}(\mathbf{R}) = \left( \frac{384\pi A_{SM}}{5} \right)^{1/2} \frac{\mu_B^2}{R^3} Y_{2,2-M}(\theta, \phi) \quad (6)$$

is the transition matrix element over the spin variables for the operator (2), with angles  $\theta, \phi$  specifying the orientation of the internuclear axis with respect to the direction of the magnetic field, and  $Y_{2,2-M}(\theta, \phi)$  a spherical harmonic. The coefficients  $A_{SM}$  take the following values:  $A_{21} = 1$ ,  $A_{20} = 2/3$ ,  $A_{00} = 1/3$ .

The expansion of  $G_S(\mathbf{R}, \mathbf{R}')$  in spherical harmonics reads

$$G_S(\mathbf{R}, \mathbf{R}') = \sum_{l,m} \frac{g_{Sl}(R, R')}{RR'} Y_{lm}\left(\frac{\mathbf{R}}{R}\right) Y_{lm}^*\left(\frac{\mathbf{R}'}{R'}\right), \quad (7)$$

where  $g_{Sl}(R, R')$  is the Green function of the Schrödinger equation for the radial motion in the potential  $U_S(R)$  with (orbital) angular momentum  $l$  and energy  $E_f$ . At  $R=R'$ ,  $g_{Sl}(R, R')$  is continuous and the discontinuity in the derivative  $\partial g_{Sl}(R, R')/\partial R$  is equal to 1. For  $R \rightarrow \infty$  the function  $g_{Sl}(R, R')$  should contain only an outgoing spherical wave, and we have

$$g_{Sl}(R, R') = \frac{\chi_{Sl}(k_f, R')}{2k_f R R'} \exp(ik_f R), \quad (8)$$

where the momentum of the scattered wave  $\hbar k_f = \sqrt{mE_f}$ . The function  $\chi_{Sl}(k, R)$  describes elastic  $l$  scattering of an incident plane wave  $\exp(i\mathbf{k} \cdot \mathbf{R})$  in the potential  $U_S(R)$ . This function is a solution of the Schrödinger equation for the radial motion with orbital angular momentum  $l$  and energy  $E = \hbar^2 k^2/m$ :

$$\left( -\frac{\hbar^2}{m} \frac{d^2}{dR^2} + U_S(R) + \frac{\hbar^2 l(l+1)}{mR^2} - E \right) \chi_{Sl}(k, R) = 0. \quad (9)$$

When calculating  $\chi_{0l}(k, R)$  one should account for the Penning ionization in the  $S=0$  state  $^1\Sigma_g^+$ . Since, in accordance with our model for the interaction in this state, there is a perfectly absorbing boundary at a distance  $R_0 \approx 7a_0$  and the potential  $U_0(R)$  [16] at larger  $R$ , we have a boundary condition  $\chi_{0l}(k, R) \sim \exp[-ik_0(R-R_0)]$  at  $R \rightarrow R_0$ , with  $k_0 = \hbar^{-1} \{m[E - U_0(R_0) - l(l+1)\hbar^2/mR_0^2]\}^{1/2}$ . The precise value of  $R_0$  is not important because the radial motion at  $R \sim R_0$  is quasiclassical. For  $R \rightarrow \infty$  the function

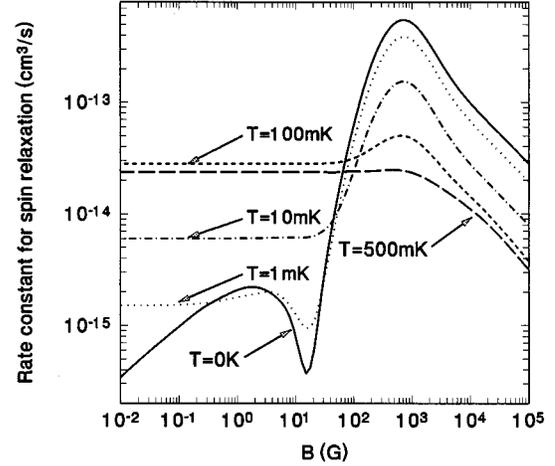


FIG. 1. Rate constant for spin relaxation ( $\alpha_{\text{rel}}$ ) as a function of  $B$  for several values of  $T$  (0, 1, 10, 100, and 500 mK).

$$\chi_{0l}(k, R) = (1 + \mathcal{S}_{0l}) \sin(kR + \delta_{0l}) + i(1 - \mathcal{S}_{0l}) \cos(kR + \delta_{0l}),$$

where  $\mathcal{S}_{0l}(k)$  is the modulus of the  $\mathcal{S}$ -matrix element for elastic  $l$  scattering of an incident plane wave in the potential  $U_0(R)$ . As the probability of the Penning ionization in the  $S=2$  state  $^5\Sigma_g^+$  is negligibly small, the function  $\chi_{2l}(k, R)$  at  $R \rightarrow \infty$  takes the form  $\chi_{2l}(k, R) = 2 \sin(kR + \delta_{2l})$ .

The spin relaxation rates are determined from the radial flux of particles in the scattered wave  $\Psi_{SM}(\mathbf{R})$ ,

$$J_{SM} = \frac{\hbar}{mi} \left( \Psi_{SM}^* \frac{d}{dR} \Psi_{SM} - \Psi_{SM} \frac{d}{dR} \Psi_{SM}^* \right), \quad (10)$$

at  $R \rightarrow \infty$ . Outside the BEC regime, representing the event rate per unit volume as

$$\nu_{\text{rel}} \equiv \alpha_{\text{rel}} n^2/2 = (\alpha_{21} + \alpha_{20} + \alpha_{00}) n^2/2, \quad (11)$$

for the relaxation rate constant  $\alpha_{SM}$  in each relaxation channel (3), using the expansion of  $\Psi_{22}(\mathbf{R})$  in spherical harmonics, from Eqs. (4), (7), (8), and (10) we obtain

$$\alpha_{SM} = \left\langle \int J_{SM}(\mathbf{R}, \mathbf{k}_i) R^2 \Big|_{R \rightarrow \infty} \frac{d\Omega_i d\Omega_R}{4\pi} \right\rangle = A_{SM} \alpha_* \left\langle \left( \frac{E_i + E_M}{E_e} \right)^{1/2} F_S(E_i, E_M) \right\rangle, \quad (12)$$

where  $d\Omega_i$ ,  $d\Omega_R$  are the elements of the solid angle associated with the vectors  $\mathbf{k}_i$  and  $\mathbf{R}$ , and the symbol  $\langle \rangle$  stands for the thermal average over  $E_i$ . The quantity

$$\alpha_* = (512\pi/15) (\mu_B^2 m/\hbar)^2 (E_e/m)^{1/2} \approx 1.3 \times 10^{-14} \text{ cm}^3/\text{s},$$

and

$$F_S(E_i, E_M) = \sum_{l_i, l_f} (2l_i + 1)(2l_f + 1) \begin{pmatrix} l_i & 2 & l_f \\ 0 & 0 & 0 \end{pmatrix}^2 \times \left| \frac{3}{4k_i k_f} \int_0^\infty \chi_{2l_i}(k_i, R) \chi_{2l_f}(k_f, R) \frac{dR}{R^3} \right|^2. \quad (13)$$

The same result is obtained in the distorted-wave approximation. Due to identity of particles, in Eq. (13) the angular momenta of the partial waves in the incoming and outgoing channel,  $l_i$  and  $l_f$ , can take only even values. The angular dependence of the quantity  $H_{SM}(\mathbf{R})$  given by Eq. (6) implies the following selection rule:  $l_f = l_i \pm 2$  or  $l_f = l_i$  (except  $l_f = l_i = 0$ ).

Our numerical results for the field dependence  $\alpha_{\text{rel}}(B)$  at various temperatures, obtained on the basis of Eqs. (11), (12), and (13), are presented in Fig. 1. The thermal average was performed assuming a Boltzmann distribution. The functions  $\chi_{Sl}$  were found by numerical integration of (9), using potential  $U_2(R)$  [9] for  $\chi_{2l}(k_i, R)$  and  $\chi_{2l}(k_f, R)$  and potential  $U_0(R)$  [16] for  $\chi_{0l}(k_f, R)$ .

The temperature dependence of spin relaxation is determined by which interparticle distances the transitions occur. For fields and temperatures satisfying the condition

$$E_M, T \ll E_e, \quad (14)$$

due to the  $1/R^3$  dependence of the spin-dipole interaction the relaxation transitions occur at distances  $R \sim 1/k_f$  and the scattered wave can contain many partial waves. At these distances the relative motion of atoms can be considered as free, and the expressions for the rate constants  $\alpha_{SM}$ , following from Eqs. (12) and (13), may be approximated as [2]

$$\alpha_{SM} \approx A_{SM} \alpha_* \left( \frac{T + 4\pi E_M}{4\pi E_e} \right)^{1/2}. \quad (15)$$

In the limiting cases  $T \ll E_M$  and  $T \gg E_M$ , this expression is exact. For  $T \gg E_M$  the rate constants  $\alpha_{SM} \propto \sqrt{T}$ . This dependence originates from the increase of the density of states for the relative motion of colliding atoms with increasing temperature. For intermediate values of  $T/E_M$  our numerical results reveal that Eq. (15) is accurate to within 20% in fields  $B \leq 1$  G at temperatures  $T \leq 200$   $\mu\text{K}$ . In these fields  $\alpha_{\text{rel}}$  continues to increase with temperature up to  $T \sim 100$  mK (see Fig. 1), due to increasing density of states. For  $T \geq 100$  mK the relaxation transitions already occur at smaller distances where the interaction between atoms is important. This circumstance leads to a slight decrease of  $\alpha_{\text{rel}}$  with increasing  $T$ .

For low temperatures  $T \ll E_e$ , in fields satisfying the inequality

$$E_M \gg T \quad (16)$$

the transitions predominantly occur at distances  $R \ll 1/k_i$ , and the initial-state wave function of the relative motion of atoms,  $\Psi_{22}(\mathbf{R})$ , may be represented by the  $s$ -wave contribution. Hence, in Eq. (13) one may keep only the term with  $l_i = 0$  and, in accordance with the above selection rule, the angular momentum of the scattered wave  $l_f = 2$ , i.e., only  $s$ - $d$  relaxation transitions are important. One may also ne-

glect the initial energy  $E_i$  compared to  $E_M$  and put  $E_f = E_M$ . As the highest  $s$  level in the potential  $U_2(R)$  has a binding energy  $\varepsilon_0$  of several millikelvin, at  $E_i \ll E_e$  the energy dependence of the initial-state wave function is rather close to that characteristic for the case of resonance  $s$ -wave scattering of slow particles. In this case, which requires the condition  $\varepsilon_0 \ll E_e$  [15], at distances  $R \ll 1/k_i$  we have

$$\chi_{20}(k_i, R) = 2 \left( \frac{\varepsilon_0}{\varepsilon_0 + E_i} \right)^{1/2} k_i R \xi(R), \quad (17)$$

where  $R\xi(R)$  is the solution of Eq. (9) for the potential  $U_2(R)$  with  $l=0$  and  $E=0$  ( $\xi \rightarrow 1$  for  $R \rightarrow \infty$ ). Then, only in the limit of ultralow temperatures  $T \ll \varepsilon_0$  will the relaxation rate constants  $\alpha_{SM}$  (and hence  $\alpha_{\text{rel}}$ ) be temperature independent.

In fields  $B \geq 10$  G ( $E_M \geq E_e$ ) our numerical calculations reproduce the zero temperature results of Ref. [2] for temperatures below 500  $\mu\text{K}$ . In high fields,  $B \geq 100$  G, the main contribution to the amplitude of spin relaxation comes from interparticle distances  $R \leq R_e$ . Therefore, in the limit  $\varepsilon_0 \ll E_e$  the temperature dependence of the relaxation rate constant at  $T \ll E_e$  would be determined by the energy dependence of  $\chi_{20}(k_i, R)$  given by Eq. (17). This leads to  $\alpha_{\text{rel}} \propto \langle (E_i + \varepsilon_0)^{-1} \rangle$ , resonantly enhanced at  $T \leq \varepsilon_0$  and decreasing with increasing temperature at higher  $T$ . Our numerical calculations show qualitatively the same behavior:  $\alpha_{\text{rel}}$  is temperature independent at  $T \leq 500$   $\mu\text{K}$  and decreases with increasing  $T$  at higher temperatures (see Fig. 1). It is interesting that the decrease of  $\alpha_{\text{rel}}$  continues up to  $T \sim 500$  mK and is not significantly influenced by the  $d$ -wave contribution in the initial state wave function.

At temperatures  $T \leq 500$   $\mu\text{K}$  in fields  $B \sim 10$  G, where  $k_f a \sim 1$ , the relaxation transitions occur at distances  $R \sim 1/k_f$  and there is a strong cancellation of the contributions of the two terms in  $\chi_{20}(k_i, R) \propto 1 - a/R$  to the amplitude of the  $s$ - $d$  transition, since the scattering length  $a$  is large and positive. Accordingly, the field dependence of  $\alpha_{2M}$  and, hence,  $\alpha_{\text{rel}}$  shows a dip (cf [2]). With increasing  $T$  the cancellation is less pronounced and, in addition, more partial waves in the initial and final states contribute to the transition amplitude. Therefore  $\alpha_{\text{rel}}$  increases with  $T$  (see Fig. 1).

The results of this section show that the maximum rate of spin relaxation ( $\alpha_{\text{rel}} \approx 5 \times 10^{-13}$   $\text{cm}^3/\text{s}$ ) is reached in the zero temperature limit for  $B \approx 750$  G. For any other temperatures and fields  $\alpha_{\text{rel}}$  is only smaller.

### III. RELAXATION-INDUCED IONIZATION

In pair collisions of spin-polarized He( $2^3S$ ) atoms spin-dipole transitions to the  $S=0$  quasimolecular state  $1^1\Sigma_g^+$  [ $\Phi_{22} \rightarrow \Phi_{00}$  in Eq. (3)] also involve ‘‘relaxation-induced’’ ionization [2], because this state is autoionizing through the ordinary Penning mechanism. The rate of relaxation-induced ionization is determined by the radial flux of atoms,  $J_{00}$ , in the scattered wave  $\Psi_{00}(\mathbf{R})$  onto the absorbing boundary at  $R_0$ . For  $R \rightarrow R_0$  the Green function  $g_{0l}(R, R')$  can contain only an incoming spherical wave, and we have

$$g_{0l}(R, R') = \frac{\tilde{\chi}_{0l}(k_f, R')}{2(k_f k_{0f})^{1/2}} \exp[-ik_{0f}(R - R_0)], \quad (18)$$

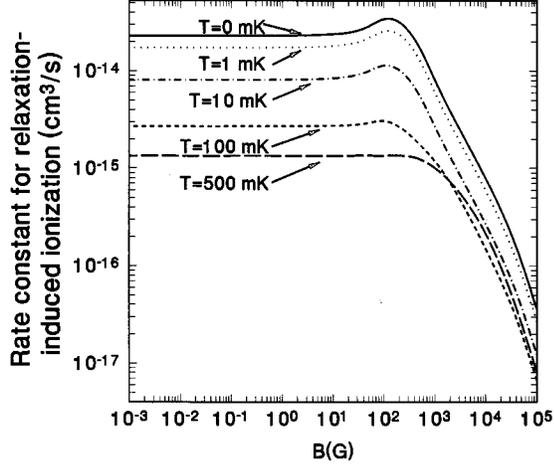


FIG. 2. Rate constant for relaxation-induced ionization ( $\alpha_{\text{ri}}$ ) as a function of  $B$  for several values of  $T$  (0, 1, 10, 100, and 500 mK).

where the quantity  $k_{0f} = k_0(E_f)$ . The function  $\tilde{\chi}_{0l}(k_f, R)$  describes elastic scattering of an outgoing spherical  $l$  wave, “starting” at  $R = R_0$ , by the potential  $U_0(R)$ . For  $R \rightarrow R_0$  this function contains the outgoing spherical wave and its reflection from the centrifugal barrier. For  $R \rightarrow \infty$ ,  $\tilde{\chi}_{0l}(k_f, R) = \sqrt{1 - \mathcal{S}_{0l}^2(k_f)} \exp(ik_f R)$ . The function  $\tilde{\chi}_{0l}(k_f, R)$  is connected with  $\chi_{0l}(k_f, R)$  by the relation

$$\tilde{\chi}_{0l}(k_f, R) = \frac{\chi_{0l}^*(k_f, R) - \mathcal{S}_{0l}(k_f) \chi_{0l}(k_f, R)}{\sqrt{1 - \mathcal{S}_{0l}^2(k_f)}} i e^{-i\delta_{0l}}. \quad (19)$$

Representing the event rate per unit volume for relaxation-induced ionization by  $\nu_{\text{ri}} = \alpha_{\text{ri}} n^2/2$ , from Eqs.(4), (7), (10), and (18) we obtain

$$\alpha_{\text{ri}} = \left\langle \int J_{00}(\mathbf{R}, \mathbf{k}_i) R^2 \Big|_{R=R_0} \frac{d\Omega_i d\Omega_R}{4\pi} \right\rangle = \frac{1}{3} \alpha_* \left\langle \left( \frac{E_i + E_0}{E_e} \right)^{1/2} \tilde{F}_0(E_i, E_0) \right\rangle, \quad (20)$$

where the quantity  $\tilde{F}_0(E_i, E_0)$  is given by Eq. (13) with  $\chi_{0l}(k_f, R)$  replaced by  $\tilde{\chi}_{0l}(k_f, R)$ .

The wave function  $\tilde{\chi}_{0l}(k_f, R)$  was found numerically for the potential  $U_0(R)$  [16]. The field dependence of  $\alpha_{\text{ri}}$  at various temperatures is presented in Fig. 2. Irrespective of the ratio between  $E_0$  and  $T$  the main contribution to the amplitude of relaxation-induced ionization comes from interparticle distances  $R \lesssim R_e$ , and the temperature dependence of  $\alpha_{\text{ri}}$  is similar to that of spin relaxation in high fields. For  $T \ll E_e$  the initial state wave function  $\Psi_{22}(\mathbf{R})$  is determined by the  $s$ -wave contribution, with  $\chi_{20}(k_i, R)$  given, in the limit  $\varepsilon_0 \ll E_e$ , by Eq. (17). Accordingly, the rate constant  $\alpha_{\text{ri}} \propto \langle (E_i + \varepsilon_0)^{-1} \rangle$  is resonantly enhanced and temperature independent at ultralow temperatures  $T \lesssim \varepsilon_0$  and decreases with increasing  $T$  at higher temperatures. For temperatures below 500  $\mu\text{K}$  our numerical calculations reproduce the zero temperature result of Ref. [2]. Above 500  $\mu\text{K}$  the rate constant  $\alpha_{\text{ri}}$  decreases with increasing  $T$  up to the highest temperatures investigated.

In low fields,  $B \lesssim 100$  G, relaxation-induced ionization is field independent, since in this case for orbital angular momenta  $l$  giving the main contribution to the transition amplitude the  $\mathcal{S}$ -matrix element  $\mathcal{S}_{0l}(k_f)$  is close to unity, the function  $\tilde{\chi}_{0l}(k_f, R) \propto \sqrt{k_f}$ , and, hence,  $\tilde{F}_0 \propto 1/\sqrt{E_0}$ . In high fields, where  $E_0$  exceeds the height of the centrifugal barrier for  $l$  contributing to the transition amplitude, we have  $\mathcal{S}_{0l}(k_f) \ll 1$  and  $\tilde{\chi}_{0l}(k_f, R) \approx i \exp(-i\delta_{0l}) \chi_{0l}^*(k_f, R)$ . Hence,  $\alpha_{\text{ri}} \approx \alpha_{00}$ , i.e.,  $\alpha_{\text{ri}}$  rapidly decreases with increasing  $B$ .

As one can see from Figs. 1 and 2, relaxation-induced ionization dominates over spin relaxation in low fields  $B \lesssim 100$  G at sufficiently low temperatures  $T \lesssim 10$  mK, where  $\alpha_{\text{ri}} \sim 10^{-14}$  cm<sup>3</sup>/s.

#### IV. OTHER DECAY MECHANISMS

Another ionization channel present in pair collisions of He( $2^3S$ ) $\uparrow$  atoms is Penning ionization via the direct dipole-exchange mechanism [14]. The physical picture of this process at  $R \gg a_0$  is as follows. The excited electron of one of the atoms approaches the excited electron of another atom and forces it out into the continuous spectrum. By changing the direction of spin because of the spin-dipole interaction with this electron, it takes the latter’s place, but already in the ground state. The ratio of the spin-dipole interaction between two electrons to the Coulomb interaction at a distance of the Bohr radius is  $4\mu_B^2/e^2 a_0^2 = \gamma^2$ , where  $\gamma = 1/137$  is the fine structure constant. Consequently, the characteristic ratio of the dipole-exchange ionization rate to the rate of ordinary Penning ionization of unpolarized atoms via the two-electron exchange mechanism will be of order  $\gamma^4$  [14]. Explicitly calculating the two-electron exchange mechanism for both dipole-exchange and ordinary Penning ionization we find that the dipole-exchange ionization width  $\Gamma(R)$  (the probability of ionization per unit time for the quasimolecule state with  $S=2$  at a fixed distance  $R$  between atoms) is related to the ionization width  $\Gamma_0(R)$  for the quasimolecule state with spin  $S=0$  as

$$\Gamma(R) \approx 2.0 \gamma^4 \Gamma_0(R). \quad (21)$$

Kinetic energies of nuclei in the final state of the Penning ionization process are much smaller than the energy of the ejected electron (15 eV) and hence may be dropped from the energy conservation law. Then, representing the ionization rate per unit volume as  $\nu_i = \alpha_i n^2/2$  and making a summation over the final states of the motion of nuclei, in first-order perturbation theory we obtain for the ionization rate constant

$$\alpha_i \approx \left\langle \int |\Psi_{22}(\mathbf{R})|^2 \Gamma(R) d\mathbf{R} \right\rangle = \left\langle \frac{2\pi}{k_i^2} \sum_l \int_0^\infty (2l+1) |\chi_{2l}(k_i, R)|^2 \Gamma(R) dR \right\rangle, \quad (22)$$

where the summation is performed only over even  $l$ . As one can easily see from this expression,  $\alpha_i$  is field independent.

The rate constant  $\alpha_i$  was found numerically, with  $\Gamma(R)$  from Eq. (21) and  $\Gamma_0(R)$  from the data of Ref. [16]. The widths  $\Gamma_0(R)$  and  $\Gamma(R)$  decrease exponentially with increasing  $R$ . Therefore the main contribution to the integral in Eq. (22) comes from distances in the vicinity of the turning point

in the repulsive part of the potential  $U_2(R)$ . As this distance is much smaller than  $R_e$ , the temperature dependence of dipole-exchange ionization is similar to that of spin relaxation in high fields and relaxation-induced ionization. Our calculations show that  $\alpha_i \approx 10^{-16}$  cm<sup>3</sup>/s at temperatures  $T \lesssim 1$  mK (cf. [2]) and decreases with increasing  $T$ . For any fields and temperatures  $\alpha_i$  is much smaller than either  $\alpha_{\text{rel}}$  or  $\alpha_{\text{ri}}$ .

Two-body spin relaxation and relaxation-induced ionization in the  $^4\text{He}^*\uparrow$  gas can also occur due to the spin-orbit interaction. As both initial and final quasimolecular states in these processes are  $\Sigma$  states, the corresponding interaction Hamiltonian  $V_{SO}(\mathbf{R})$  (the matrix element of the spin-orbit interaction over the electron coordinates) is nonzero only in second-order perturbation theory [17]. At large interparticle distances, corresponding to the van der Waals tail of the potentials  $U_2(R)$  and  $U_0(R)$ , the quantity  $V_{SO} \sim (A/\Delta E)^{1/2} C/R^6$ , where  $C \approx 3300$  a.u. is the van der Waals constant,  $A \sim 1$  K is the fine structure splitting for the  $2P$  state of helium, and  $\Delta E \sim 1$  eV is the spacing between  $2S$  and  $2P$  states. With these numbers one can see that  $V_{SO}$  is much smaller than the spin-dipole interaction  $\mu_B^2/R^3$ . As  $V_{SO}$  does not exceed  $\sim A^2/\Delta E$ , it is much smaller than  $\mu_B^2/R^3$  also at smaller distances. Therefore, in metastable triplet helium the contribution of spin-orbit interaction to spin relaxation and relaxation-induced ionization can be neglected.

A further decay mechanism of  $^4\text{He}^*\uparrow$  is three-body recombination. As there are bound states in the potential well  $U_2(R)$ , elastic interaction between  $^4\text{He}^*\uparrow$  atoms in the course of three-body collisions can lead to the formation of a spin-polarized  $\text{He}_2^*$  molecule, which then decays through Penning ionization due to spin-dipole interaction. The rate constant of three-body recombination,  $L$ , is defined so that  $Ln^3/3$  is the recombination event rate per unit volume. In the limit of ultralow temperatures  $T \ll \varepsilon_0$  the rate constant  $L$  is resonantly enhanced by the existence of the weakly bound  $s$  level in the potential  $U_2(R)$ . Most important is the recombination to this particular level. For the potential  $U_2(R)$  [9] direct calculation yields  $L \approx 2 \times 10^{-27}$  cm<sup>6</sup>/s [18]. However, the value of  $L$  is very sensitive to the shape of  $U_2(R)$ . Possible improvement of this potential is likely to make it deeper and increase the binding energy  $\varepsilon_0$ , and can decrease  $L$  within an order of magnitude. In low fields  $B \lesssim 100$  G the rate at which atoms are lost due to recombination,  $Ln^3$  is smaller than  $\alpha_{\text{in}} n^2$  for densities  $n \lesssim 10^{13}$  cm<sup>-3</sup>. In higher fields spin relaxation is faster than relaxation-induced ionization and dominates over three-body recombination, even for higher densities. Because  $L$  is resonantly enhanced, it will decrease rapidly with increasing temperature.

## V. ELASTIC AND INELASTIC RATES

At sufficiently low densities ( $n \lesssim 10^{13}$  cm<sup>-3</sup> at ultralow temperatures) the decay of the  $^4\text{He}^*\uparrow$  gas is determined by two-body inelastic processes. The results of the previous sections indicate that for most of the fields and temperatures investigated spin relaxation is the dominant decay channel, and only at sufficiently low temperatures  $T \lesssim 10$  mK in low fields  $B \lesssim 100$  G is the decay determined by relaxation-induced ionization. The temperature dependence of the total rate con-

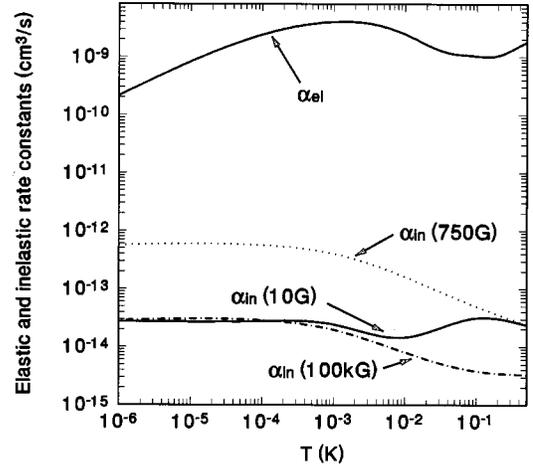


FIG. 3. Rate constants of elastic ( $\alpha_{\text{el}}$ ) and inelastic ( $\alpha_{\text{in}}$ ) collisions as functions of  $T$ . The upper curve corresponds to  $\alpha_{\text{el}}$ , and the three lower curves to  $\alpha_{\text{in}}$  in fields  $B=10$  G,  $B=750$  G, and  $B=100$  kG.

stant of inelastic collisions,  $\alpha_{\text{in}} = \alpha_{\text{rel}} + \alpha_{\text{ri}}$ , is presented in Fig. 3 for three values of the magnetic field:  $B=750$  G where the inelastic rate is maximum, a very high field  $B=100$  kG, and a low field  $B=10$  G ( $\alpha_{\text{in}}$  is field independent for  $B \lesssim 10$  G). The inelastic rate  $\nu_{\text{in}} = \alpha_{\text{in}} n^2/2$  should be compared with the rate of elastic collisions,  $\nu_{\text{el}} = \alpha_{\text{el}} n^2/2$ , where  $\alpha_{\text{el}} = \langle \sigma v \rangle$  is the elastic rate constant. The latter determines the rates of evaporative cooling and Bose condensate formation, and it is the ratio  $\alpha_{\text{el}}/\alpha_{\text{in}}$  that places limitations (minimum achievable temperature, maximum condensate density) on these processes [10,11].

We calculated  $\alpha_{\text{el}}$  for a Boltzmann distribution of particles, by using the relation

$$\alpha_{\text{el}} = \langle \sigma v \rangle = \left\langle \frac{16\pi\hbar}{mk} \sum_l (2l+1) \sin^2 \delta_{2l}(k) \right\rangle, \quad (23)$$

where the orbital angular momentum  $l$  can take only even values. The phase shifts of elastic  $l$ -wave scattering,  $\delta_{2l}(k)$ , were found numerically for the potential  $U_2(R)$  [9]. For collision energies  $E \ll E_e$ , the  $s$ -wave contribution is dominant and we are rather close to the case of resonance  $s$ -wave scattering of slow particles, where  $\sigma \propto (E + \varepsilon_0)^{-1}$  [15]. Accordingly, the rate constant  $\alpha_{\text{el}}$  displays a maximum at  $T \sim \varepsilon_0$ . For lower temperatures  $\alpha_{\text{el}} \propto \sqrt{T}$ , and at higher temperatures  $\alpha_{\text{el}} \propto 1/\sqrt{T}$ . The results of our numerical calculations, presented in Fig. 3, show qualitatively the same picture. Moreover, even at temperatures  $T \geq E_e$  the elastic rate constant continues to decrease until  $T \approx 100$  mK, in spite of the fact that at such collision energies, comparable with the height of the centrifugal barrier for  $l=2$ , the contribution of  $d$ -wave scattering is important.

The temperature dependence of the ratio  $\alpha_{\text{el}}/\alpha_{\text{in}}$  is presented in Fig. 4 for the same field values as  $\alpha_{\text{in}}$  in Fig. 3. At temperatures  $T \geq 1$  mK this ratio is larger than  $10^4$  in any field. For lower temperatures  $\alpha_{\text{el}}/\alpha_{\text{in}} \propto \sqrt{T}$ . But even at  $B=750$  G, where  $\alpha_{\text{el}}/\alpha_{\text{in}}$  is minimum, it is larger than 10 for  $T \geq 10$  nK.

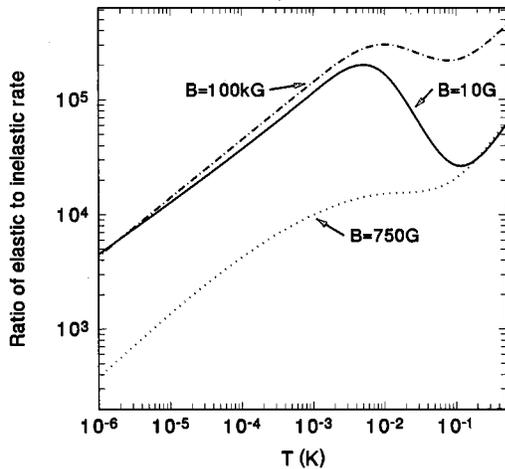


FIG. 4. Temperature dependence of the ratio of elastic to inelastic rate constant ( $\alpha_{el}/\alpha_{in}$ ), in fields  $B=10$  G,  $B=750$  G, and  $B=100$  kG.

Our calculations rely on the interaction potential  $U_2(R)$  [9]. The existence of a weakly bound  $s$  level in this potential influences the low temperature values and the temperature dependence of elastic and inelastic rates. Slight variations of  $U_2(R)$  within the accuracy of its calculation can change the binding energy  $\varepsilon_0$ , for example, making it larger. In this case the zero temperature values of  $\alpha_{in}$  can decrease by a factor of 2 or 3 (see [2]). But, importantly, the elastic rate should experience approximately the same decrease because the elastic cross section  $\sigma \propto (E + \varepsilon_0)^{-1}$ , as well as  $\alpha_{in} \propto (E + \varepsilon_0)^{-1}$ . Hence, the ratio  $\alpha_{el}/\alpha_{in}$  should not significantly change. In the high temperature regime both elastic and inelastic rates are less sensitive to the variations of the potential  $U_2(R)$ , and  $\alpha_{el}/\alpha_{in}$  also remains unchanged.

## VI. CONCLUDING REMARKS

Our analysis of the temperature dependence of the inelastic rates in spin-polarized metastable triplet helium shows that the stability of this gas is not compromised by temperature up to the highest temperatures investigated (0.5 K). In fact, the gas is least stable in the limit of ultralow temperatures, due to the resonant enhancement of  $s$ -wave scattering. Thus, cryogenic techniques for loading  $^4\text{He}^*\uparrow$  into a magnetostatic trap [13] remain promising with respect to the intrinsic stability of the gas. The large values of the ratio  $\alpha_{el}/\alpha_{in}$  are good for evaporative cooling of  $^4\text{He}^*\uparrow$  from high temperatures to the microkelvin regime. They are of the same order of magnitude as those for alkali atom gases and one or two orders of magnitude larger than in hydrogen, which has been successfully cooled below 1 mK [19,20]. In view of recent success in achieving BEC in trapped rubidium [21] and lithium [22] at nanokelvin temperatures by using evaporative cooling, one may also think of cooling  $^4\text{He}^*\uparrow$  to  $T \lesssim 10$  nK. These temperatures correspond to critical BEC densities  $n \lesssim 10^{10} \text{ cm}^{-3}$  where the decay of the gas is already determined by the radiative lifetime  $\tau_0 \approx 8 \times 10^3$  s.

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