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SUBSTITUTING COMPLEMENTS

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ABSTRACT

The presence of multiple sellers in the provision of (non-substitutable) complementary goods leads to outcomes that are worse than those generated by a monopoly (with a vertically integrated production of complements), a problem known in the economic literature as complementary oligopoly and recently popularized in the legal literature as the tragedy of the anticommons. We ask the following question: how many substitutes for each complement are necessary to render the presence of multiple sellers preferable to a monopoly? Highlighting the asymmetries between Cournot (quantity) and Bertrand (price) competition and their dual models, we show that the results crucially depend on whether firms compete by controlling price or quantity. Two substitutes per component are sufficient when firms choose price. However, when firms choose quantity, the availability of substitutes, regardless of their number, is ineffective. Considering more complex cases of multi-complementarity, we ask the related question of how many complements need to be substitutable and offer comments on equilibrium prices and quantities under different scenarios.

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Keywords: Anticommons, complementary inputs, oligopoly, antitrust, competition.

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1. Introduction

The presence of multiple sellers in the provision of non-substitutable complementary goods leads to outcomes that are worse than those generated by a monopoly with a vertically integrated production of complements. This problem is known in the economic literature as complementary oligopoly[1] and recently popularized in the legal literature as the tragedy of the anticommons. In this article we ask the question of whether the presence of several substitutes for each complement may improve the problem and the related question of how many such substitutes are necessary.

Highlighting the asymmetries between Cournot (quantity) and Bertrand (price) competition and their dual models, we show that the outcome crucially depends on whether parties compete by choosing price or quantities. In the interaction between the competitive forces operating in the market for complements and those in the market for substitutes, the forces producing the most unfavorable non-cooperative outcome for the parties prevail, moving away from the cooperative (monopolistic) outcome. When considered independently, markets for complements and markets for substitutes produce outcomes that either approach or depart from the social optimum depending on the control variable and the number of firms in the market. However, our analysis shows that when the two dimensions of substitutability and complementarity interact, the outcomes are invariably extreme – social welfare is either maximized or minimized, independent of the number of parties.

Further, considering more complex cases of multi-complementarity, we ask the related question of how many complements must be substitutable and offer comments on equilibrium prices and quantities under different scenarios. Although we tackle these problems on a theoretical level, it is also worthwhile to make a connection to some recent cases.

The issue of complementarity between inputs of production played a central role in some recent cases. In the U.S. Supreme Court case *Kelo v. New London*,[2] a city planned an integrated

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[1] Often the term complementary monopoly is used instead. We prefer the term complementary oligopoly as it emphasizes the interaction between the two firms.

[2] U.S. Supreme Court, *Kelo v. New London*, 545 U.S. 469 (2005) affirming the decision of the Supreme Court of Connecticut (268 Conn. 1, 843 A. 2d 500). The court affirmed that a city's decision to take property for the purpose of economic development fulfills a public purpose and thus satisfies the "public use" requirement of the Fifth Amendment to the U.S. Constitution.

development project designed to revitalize its stagnant economy. In the process of acquiring the necessary land, it purchased most of the property from willing sellers. However, some property owners held onto their property and refused to sell at the going market prices. The Supreme Court affirmed that the municipality legitimately initiated takings proceedings against property owners who refused to sell. The premise of such a decision is that the parcels of property necessary for the development represent complementary inputs for the project, all of which had to be secured before the project could be implemented. The court found that forcing a city to acquire private property exclusively through voluntary market transactions could create major disadvantages. In fact, the presence of fragmented land creates the conditions for strategic behavior by property owners, a potential impediment to the success of the economic development plan.[3]

The issue of complementarity was also central to the cases brought against Microsoft before American[4] and European[5] courts for violation of antitrust laws.[6] This interesting aspect of the case emerged when Judge Jackson of the District Court ordered the break-up of the firm as a way to reduce its market dominance and enhance competition.[7] The proposed break-up would have taken place along vertical lines, with one firm holding the Windows operating system and the other firm incorporating the remaining branches of Microsoft, including applications such as Office and Internet Explorer. The break-up would have created two firms selling two complementary goods – an operating system and applications – as a structural remedy to prevent the adoption of anticompetitive practices aimed at extending market dominance to software applications and, in turn, to facilitate entry and competition in the operating systems market.[8]

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[3] For a discussion of the problems of fragmentation of property, see Parisi (2002a and 2002b).

[4] See, e.g., United States v. Microsoft Corp., 84 F. Supp. 2d. 9, 87 F. Supp. 2d. 30, 97 F. Supp. 2d. 59 (D.D.C. 2000), *aff'd in part, rev'd in part*, 253 F.3d 34 (D.C. Cir. 2001); United States v. Microsoft Corp., 231 F.Supp. 2d 144 (D.D.C. 2002), aff'd Massachusetts v. Microsoft Corp., 373 F.3d 1199, (D.C. Cir. 2004); In re South Dakota Microsoft Antitrust Litigation, 657 N.W.2d 668 (S.D. 2003); New York v. Microsoft Corp., 297 F.Supp. 2d 15 (D.D.C. 2003); Barnhill v. Florida Microsoft Anti-Trust Litigation, 2005 WL 766971 (Fla. App.); In re Microsoft Corp. Antitrust Litigation, 2005 WL 906364 (D.Md); Comes v. Microsoft Corp., 696 N.W.2d 318 (Iowa 2005).

[5] COMP/37.792 MICROSOFT/ W2000; C 179 2004/C 179-0018 Case T-201/04, Microsoft Corp. v. Commission.

[6] Gans and King (2000); David A. Balto, Standard Setting in a Network Economy §§I-II (Feb. 17, 2000), http://www.ftc.gov/speeches/other/standardsetting.htm (last visited June 19, 2005); Gilbert and Katz (2001, pp. 40-42).

[7] In June 2000, Judge Thomas Penfield Jackson ordered Microsoft to be divided into an operating systems company and a company that would hold the other branches of its business, including MS Office, Internet Explorer and other products. United States v. Microsoft Corp., 97 F. Supp. 2d. 59 (D.D.C. 2000). A year later the US Court of Appeals for the District of Columbia vacated Jackson's remedy of breaking up the company. United States v. Microsoft Corp., 253 F.3d 34 (D.C. Cir. 2001).

[8] Joel I. Klein, Assistant Atty. General, Antitrust Division, U.S. Dept. of Justice, Rethinking Antitrust Policies for the New Economy, delivered at the Haas/Berkeley New Economy Forum, Haas School of Business, University of California at Berkeley, Portola Valley, California (May 9, 2000), *available at* http://justice.gov/atr/public/speeches/4707.htm; Jürgen Mensching, The Microsoft Decision - Promoting Innovation,

These cases have drawn attention to an issue raised in the past: the presence of multiple sellers in the provision of complementary goods may be worse than a monopoly. The situation of independent supply of complementary inputs that emerged in these recent cases resembles the scenario first discussed by Cournot (1838), studying zinc and copper as complements to produce brass, and Ellet (1839), in his study on internal improvements in the United States, showing that divided ownership on the segments of a road or railway creates a complementary oligopoly. Similar problems were subsequently considered by Edgeworth (1925). Divided ownership leads to higher prices and lower quantities than unified (monopolistic) ownership, because of the complement's cross-price effect.

The cross-price effect is the result of an externality problem that generates a prisoner's dilemma among the suppliers of complementary goods. Analogous to the recently popularized anticommons problem, this framework is a particularly powerful tool for the analysis of issues related to the fragmentation of physical and intellectual property rights.[9] From this perspective, fragmenting a single resource into complementary portions creates the conditions for the complementary oligopoly and anticommons problems formalized in Buchanan and Yoon (2000) and Parisi, Schulz and Depoorter (2005).[10]

This result is driven by the fact that when complements are at stake, an increase in the price of a fragment causes a negative externality to the sellers of other complementary goods as the quantity demanded plummets for everyone. Each seller earns the full benefit (additional revenue) of an increase in the price of his good, but bears only part of the cost associated with his pricing decisions (reduction in the quantity demanded). As a result, suppliers of complements will overprice their products relative to a monopolist's profit-maximizing price, a problem also known

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delivered at the 4th Annual Competition Law Review Conference, London, UK (October 22, 2004), *available at* http://europa.eu.int/comm/competition/speeches/text/sp2004_017_en.pdf.

[9] Michelman (1982) coined the term "anticommons," referring to the fact that in a normal commons problem there is an externality in use (each owner can use the common resource), while in anticommons property there is an externality in exclusion (each owner can exclude others from use), a problem also discussed in Demsetz (1967, p. 357); Heller (1998), discussing the location of small shops on the Moscow storefront, brought the problem to the attention of a legal audience; Heller and Eisemberg (1998) show that patents may result in an anticommons problem, as each holder of a patent on a segment of a gene has a veto power on the development of derivative innovations based on that gene (see Epstein and Kuhlik, 2004, for a different view). See also O'Rourke (2000). Parisi (2002a) applies anticommons theory to explain that property tends to increase fragmentation (entropy) and derive an economic rationale for the *numerus clausus* doctrine (see also Depoorter and Parisi, 2003, and, for a different view, Gordley, 2003); Depoorter and Parisi (2002) show that *fair use* doctrines in copyright law help counter anticommons problems.

[10] Owners of complementary goods will impose externalities on each other by exercising hold-up and exclusion rights, as in the examples mentioned above. These externalities in exclusion are symmetrical to the externalities in use created in the traditional commons problem.

as double marginalization (Spengler, 1950).

Ellet (1839) first pointed out that unified ownership of complements raises social welfare. Allen (1938) subsequently extended and confirmed Ellet's observation. A monopolist (or a cartel of individual producers) bears, in fact, the entire costs and benefits of any price decision and thus has incentives to choose a lower, profit-maximizing price. Put differently, if the producers of the complementary inputs are allowed to merge or coordinate their pricing, they can lower their prices and raise their profits, with a Pareto improvement for consumers and producers alike. The gains obtainable through such a merger or price coordination can be viewed as the horizontal equivalent of vertical integration to avoid double marginalization (Nalebuff, 2006).

It is important to note that the legal problems to which the complementary oligopoly and anticommons theories have been applied have the common characteristic of the uniqueness of the complements. Fragmented owners face an anticommons problem to the extent that the complementary rights that they seek to acquire cannot easily be substituted with other rights. Only in such cases does each fragment owner have an opportunity to exercise hold-up strategies against the owners of complementary products. There is a wide range of situations where uniqueness or quasi-uniqueness can be found. Unique and non-substitutable are the votes of the five members of the United Nations Security Council (as long as unanimity is the rule); (quasi)unique are the patents that a developer of a derivative innovation needs to hold in order to proceed with his research; (quasi)unique are the parcels of land that a builder may need to acquire to consolidate fragmented property and develop the land for a large-scale construction. In *Kelo*, the parcels of land that the city of New London forcedly acquired were essential to the implementation of the development plan. In the *Microsoft* case, the substitutability was imperfect, although not impossible like in *Kelo*. The point that the Microsoft break-up solution and its proponents have raised is that when the complements are not unique, there could be competition in the production of each of them, thus counteracting (or solving) the complementary oligopoly and anticommons problems. Several firms may produce competing applications compatible with the operating system, or there may be several interchangeable operating systems compatible with a given application. To put it in Ellet's rhetoric, there could be several alternative routes for each segment of a fragmented line of transportation.

The next question is: How much competition in the supply of each complement do we need in order to counter the anticommons problem? Previous studies have considered different dimensions of the problem, especially concerning bundling.[11] Although most of this literature focuses on substitute products, some contributions focus on complements as we do. Lewbel (1985) studies bundling of complements and of substitutes under monopoly, also considering the case in which one of the products is sold in a competitive market. Economides and Salop (1992) compare the equilibria reached under joint ownership and independent ownership of the complementary inputs when firms set prices, examining the outcome under different market structures. Anderson and Leruth (1993) study whether complements produced by the same firm will be sold separately or bundled under monopoly and oligopoly. Venkatesh and Kamakura (2003) unveil the optimal bundling strategy of a monopolist depending on whether its products are complements, substitutes or independent. Nalebuff (2000) examines competition between integrated producers and independent producers, showing the interesting tradeoff between the gains from coordinated pricing and the effect of competition from the independent producers.

Studying firms' divestiture strategies, Tan and Yuan (2003) consider a market with imperfect substitutes and two firms competing in price. They demonstrate that firms may have incentives to divest along complementary lines. The price competition between producers of complements tends to raise the equilibrium price of the bundled good, thereby countering the incentives to lower the price generated by Bertrand competition in the market for imperfect substitutes.

In this paper, we consider the case in which input producers do not merge and do not price coordinate, but face the competition of other input producers. We will proceed as follows: Section 2 contains the analysis, Section 3 discusses some extensions of the model and Section 4 concludes.

2. Analysis: How many substitutes for each complement?

The question set forth in the introduction can be understood in more precise terms by asking: *How many substitutes of each complement do we need to avoid the anticommons problem*? The answer suggested by this study is that the presence of two substitutes per complementary component is sufficient to avoid the anticommons problem when producers compete in price, while the outcome of quantity competition invariably results in a near disappearance of the

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[11] For a review of the literature on the issue of bundling, see Kobayashi (2005).

market, regardless of the number of available substitutes.[12]

In order to provide a simple and easily interpretable answer, we use a linear demand function, as in Buchanan and Yoon (2000), for a good consisting of a bundle of several inputs. For simplicity we consider the case where the good is composed of two complementary inputs, which can be supplied to the consumer by a monopolist or separately by two different suppliers. Assuming that there is no alternative use for the individual inputs unless combined in the fixed ratio 1:1 (as in the *Microsoft* case), the demand function for the final good is:

(1) $P = \alpha - \beta Q$

To keep the analysis simple – and without loss of generality – let us further assume that the marginal cost of production is equal to zero. It is known that a monopolist would maximize his profit PQ by selling a quantity $Q^M = \alpha / 2\beta$ at a price $P^M = \alpha / 2$.[13] The problem we analyze here instead concerns the case in which consumers need to buy the components separately from the two firms. In order to assess the optimal pricing or production behavior of the two producers, it is preliminarily important to ascertain whether they set the price or the quantity of the components. Next we will ask the question of how many substitutes for each complementary component are needed in order to eliminate the anticommons problem, that is, to do no worse than under a monopoly.

[FIGURE 1]

2.1. Price-setting complementary oligopoly

This is the case described by Cournot (1838), Ellet (1839) and Buchanan and Yoon (2000), although with different applications. Each individual firm i = 1, 2 sets the price p_i for its component, which results in a final total price of the bundle $P = \Sigma p_i$. Each of the individual firms maximizes its profit:

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[12] Along a similar line of reasoning, Motta (2004, p. 309, note 10), analyzes the classical case of double marginalization (Spengler, 1950) due to the fact that the unique producer and the unique retailer of a good can set prices independently, a situation that is analogous to the anticommons case discussed in the text. He notes that if at least two retailers compete by reducing their prices the producer will be able to set a monopoly price.

[13] The monopolist problem is $\max_Q [\alpha Q - \beta Q^2]$, whose first order condition yields $\alpha - 2\beta Q = 0$. Alternatively, the problem may be characterized in terms of the price as $\max_P [(\alpha / \beta)P - (1 / \beta)P^2]$, whose first order condition yields $\alpha - 2P = 0$.

(2) $\text{Max}_{pi}\ [p_i Q]$ or $\text{Max}_{pi}\ [p_i\,(\alpha - p_i - p_{j \neq i}) / \beta]$

As noted in Sonnenschein (1968), this problem is the dual of the ordinary Cournot oligopoly, in which firms set quantities instead of prices as here. The first order conditions yield:

$$(3)\qquad \begin{aligned} (\alpha - p_1 - p_2) / \beta - p_1 / \beta = 0 \\ (\alpha - p_1 - p_2) / \beta - p_2 / \beta = 0 \end{aligned}$$

from which it follows that firms will set the price of the two components at $p_1 = p_2 = \alpha / 3$, with a total price for the bundle $P^A = p_1 + p_2 = 2\alpha / 3 > P^M$ (where the superscript "^{A}" stands for "anticommons"). Consequently, the quantity sold will be $Q^A = \alpha / 3\beta < Q^M$. The firms' total profits are $p_1Q^A + p_2Q^A = P^AQ^A = 2\alpha^2 / 9\beta < \alpha^2 / 4\beta = P^MQ^M$. Both producers *and* consumers are therefore better off under a monopoly or a collusive cartel.

What would happen, however, if there were available substitutes for the two complementary components of the bundle? Imagine that Component 1 is now produced by two competing firms, Firm 1x and Firm 1y, instead of being produced by a single firm. Each of these firms produces perfectly substitutable versions of Component 1.

Remembering that we are considering a model in which sellers set the price, note that firm 1x and 1y compete against each other in the substitutes market for component 1 as in a Bertrand duopoly, in addition to competing in the complements market. Since there is a substitute for component 1, each firm knows that consumers will buy *everything* from the firm with the lowest price. Therefore, each firm has an incentive to lower its price below that of the other.

Within the anticommons game, a Bertrand-duopoly subgame takes place, in which competitors price at marginal costs. It follows that if there is a substitute on the market for Component 1, this component will be sold at a price equal to zero (as we have assumed for simplicity that the marginal cost is zero). The seller of Component 2 would then be in a position to extract monopoly rents, pricing its component at the price of the full bundle, $P^M = p_1 + p_2 = \alpha / 2$. Although the return for Component 2 would be higher in this case, the total price of the bundle and the resulting consumer welfare would be the same as under a monopoly.

If substitutes are available for both components, Bertrand-type competition in each sub-market leads to a result where both components are priced at marginal costs, as in perfect competition.

2.2. Quantity-setting complementary oligopoly

Consider now the case where the producers of the complementary goods set the quantity rather than the price of the components they sell. As shown by Singh and Vives (1984) this case is the dual of the traditional Bertrand oligopoly model, in which producers of substitute goods set prices. In this case, if a firm produces more than its competitor, the components produced in excess will not be sold (recall that only the bundle is valuable for consumers and there is no market for spare components). In other words, each firm controls the maximum total production: it can choose to produce less, but would not rationally produce more than the other.

Each of the firms has incentives to reduce its quantity below the quantity produced by its competitor. This quantity-undercutting strategy can be easily understood considering that firms in this setting are price-takers. If a firm produces more than its competitor, the excess portion of its production will necessarily remain unsold for any positive price. The market clears only if such products are sold at a price equal to zero. This implies that, by reducing its quantity, each firm can drive the price of its competitor to zero capturing the entire monopolist's surplus for the bundle of goods. Similar to a Bertrand duopoly for substitute goods in which each firm, by reducing its price, can drive the quantity sold by its competitor to zero, in the case of complements, the firm selling the lower quantity is rewarded with a price equal to the price of the whole bundle, since the price of the other component falls to zero.

Such incentive to 'steal the market' from the competitor has diametrically opposite effects in the two cases considered. In the case of substitutes with price (Bertrand) competition, the result is a race to the marginal-cost pricing with the maximization of social-welfare. In the case of complementary goods produced by firms that compete in quantity, the result is a race to the lowest quantity: the market supplies a quantity equal to zero, with a total erosion of any consumer or producer surplus from trade. From society's standpoint, this outcome is least desirable, as it minimizes social welfare.

How would such an equilibrium change if there were substitutes for the complementary inputs? Consider the case where there are two firms producing Component 1, Firms 1x and 1y. If setting quantity, these firms would compete against each other as in a standard Cournot duopoly, which in itself would lead to a larger production than that of a single monopolist. However, in our case such an increase in output will not take place. If only one of the components is substitutable, the producer of the other component will be able to control the maximum quantity produced and will have incentives to lower it below the Cournot outcome. Cournot incentives to increase production above the monopolistic level are counteracted by the fact that if either Firm 1x or 1y increases their production, the market price for Component 1 does not decrease continuously as in the Cournot model but drops discontinuously to zero due to the fact that the excess supply of products unmatched by the complementary Component 2 will remain unsold.

Given the incentives of the producer of Component 2, in equilibrium we could not observe Component 1 being produced in the same or a smaller quantity than Component 2. Also, the alternative outcome in which the quantity of Component 1 exceeds the quantity of Component 2 is not a possible equilibrium, because either Firm 1x or 1y would be induced to reduce the total quantity of Component 1 below that of Component 2. This would lead to a race to the bottom, where firms would eventually reduce production to zero.[14]

The competition engendered by the availability of substitutes in the market of one complementary good will not affect the overall outcome: In equilibrium firms will in fact produce a quantity equal to zero.

In the scenario where both components have a substitute, the solution will be the same. In fact, an outcome in which one component is produced more than the other is not sustainable for the reasons just given. An outcome in which both components are produced in the same positive quantity is also unlikely to be sustainable, because any of the firms can, by a marginal reduction in quantity, obtain a discontinuous increase in its profits also to the benefit of its Cournot competitor. Given that any very small deviation is sufficient, free-riding between the Cournot competitors with respect to which should reduce the quantity are not important.

3. Extensions

3.1. Mixed Cases

The results above can be extended to consider situations where the producers of one component compete with one another, setting prices in a Bertrand-type fashion, while the producers of the other component compete by setting quantity in a Cournot-type fashion. Given the complementarity of the two components in the consumer market, the outcome of these mixed cases will be determined by the subset of producers best capable of restricting quantity: components produced in larger quantity than the others will not be sellable. In Sections 2.1 and 2.2, we concluded that the producers of substitutable components, when competing by setting

[14] In our simple setting in which marginal costs are assumed to be zero, a firm is indifferent between producing a positive quantity and producing nothing. With positive marginal costs, a firm strictly prefers not producing (and making zero profits) than producing (and making negative profits).

quantity, will be able to restrict output more effectively than those competing by setting prices. Thus the outcome of mixed cases will be similar to the case where both pairs of producers set quantity. The total quantity produced in these mixed cases will therefore be zero.

3.2. N Complements

The results developed in Section 2 can also be extended to the case where the final bundled good consists of more than two complementary components. We can consider the general case where there are *n* complements, *m* of which are substitutable. In this case as well, the result depends on whether producers compete by setting price or quantity.

3.2.1. Price Competition

Let us begin by considering the case of price competition. The *m* substitutable components will be priced at marginal costs, while the remaining $n - m$ components will be priced at the anticommons price. The game is thus solved as an anticommons game of the Buchanan-and-Yoon type with $n - m$ players and a resulting price equal to $P^A = \alpha (n - m) / (n - m + 1)$ and quantity $Q^A = \alpha / (n - m + 1) \beta$. With $m = n - 1$, the producer of the only non-substitutable component will act like a single monopolist capturing the entire monopolist's rent, charging price P^M and selling quantity Q^M. In the limit case of $n = m$, each of the *n* components is substitutable and consequently will be priced at marginal cost. The solution will be the same as that in a perfect competition game. In short, the outcome improves towards perfectly competitive (Bertrand-type) pricing as the number *m* of substitutable components approaches the total number *n* of components. The presence of more than two substitutes for each component does not affect the outcome.

3.2.2. Quantity Competition

In the case of quantity competition, the solution is a straight extension of the solution found in Section 2.2. Recall that, given complementarity, a firm that produces in excess of other producers will have unsold excess supply. If $m < n$, one or more components are not substitutable. The producer(s) of the non-substitutable good(s) will be able to control the maximum quantity produced. These firms will have incentives to set quantity below any quantity set by the competitors. The presence of $n > 2$ components does not change the outcome already derived for the duopoly case: the quantity produced will be zero. The same result holds in the limiting case $m = n$: all components are substitutable. Concluding, the outcome is driven by the least substitutable component. Availability of substitutes for some or all of the components does not affect the result.

3.2.3. Other considerations

It is interesting to note the different relevance of price control versus quantity control in the various settings analyzed here. When considering the supply of the complementary components, we have observed that when producers control quantity, production will be zero. If the producers of the complementary goods instead control price, the equilibrium price will be higher than the monopolist's price and the resulting quantity will be less than the monopolist's profit-maximizing quantity, but not zero. Thus when looking only at this stage of the game, price control leads to socially (and for this matter also privately) preferable equilibria when compared to quantity control.

The same holds when all components are substitutable, but here the outcome is even more extreme than in the standard Bertrand and Cournot cases. Not only does price competition lead to socially preferable equilibria compared to quantity competition, but also such outcomes are diametrically opposed from the point of view of social welfare. While price competition maximizes social welfare, quantity competition minimizes it.

In intermediate cases, when only some of the components are substitutable, or when more than two substitutes per component are available, quantity and price control also differ. In price control the number of substitutes per component is irrelevant (two substitutes per component are enough to create a Bertrand subgame for each component). However, the number of substitutable components is important. The outcome continuously improves from anticommons price and quantity towards the monopoly outcome as the number of substitutable components increases. When all but one component are substitutable, the outcome is exactly the same as it would be under a monopoly. When all components are substitutable, the outcome is the same as under perfect competition.

In quantity control, the number of substitutes per component is unimportant, because the Cournot-subgames for individual components are weaker than the incentives to reduces quantity provided by competition in the complements market. The number of substitutable components is also irrelevant as the outcome is driven by the least substitutable component, which controls the maximum quantity produced.

4. Conclusions

Much of the industrial organization literature and much of the conventional wisdom in antitrust policy focus on models of competition in which firms compete in the supply of substitutes, whether perfect or imperfect. The emphasis on substitutes is traditionally explained by the fact that in most markets complements tend to be produced and/or sold in bundles, leaving little practical relevance to models of complementary competition. This factual reality has undergone substantial change in recent years due to the intellectual property regimes that govern new technologies and drug manufacturing. The production of high-tech products, as well as the production of most new-generation drugs, requires the use of a large number of inputs of production that are often covered by separately-owned patents. A producer of any such final product must acquire intellectual property rights from different input sellers to produce his final good. Similarly, several technological products must be used in combination with others (e.g., software applications must be used in combination with an operating system). The interrelationship between these factors of production and/or consumption, has given new practical relevance to the issue of complementarity in both industrial organization theory and antitrust policy. As is to be expected, many of the results obtained in models involving substitutes are changed – and often entirely reversed – when complements are involved. Mergers of firms that produce substitutes may undermine competition, thus harming consumer welfare; but when firms supply complements, mergers may not necessarily be detrimental to consumers. A similar analysis should be applied when evaluating structural break-up remedies in monopoly cases.

Real world markets are characterized by complex relationships where a wide range of substitutes and complements are produced. When evaluating these complex real-life conditions, competing economic frameworks come into operation, often pointing to different policy results. In this paper we have addressed the tradeoffs and boundaries between these competing paradigms of analysis. Our analysis identifies that the minimum amount of substitutability needed to overcome the problems generated by complementarities in production and/or consumption.

The general results suggest that the problems and cross-price effects occasioned by the presence of complements are easily corrected when substitutability exists between alternative complements, especially when the firms offer substitutes and compete on price. In general, having two substitutes for each complementary input is sufficient to minimize deadweight losses. In our example of a vertical break-up of firms, we show that minimal substitutability is needed when firms compete on price. In general, having two producers for each input avoids the anticommons problem and leads to outcomes that are never worse – and generally better – than under monopoly. When all but one of the complements are substitutable, the equilibrium outcome coincides with that of a monopoly. On the contrary, when firms compete by choosing quantity, the outcome invariably leads to no production at all, as incentives to undercut the quantity at work in the complements market dominates the Cournot incentives to increase quantity in the substitutes market.

Our findings are qualitatively similar, but quantitatively different in the various settings considered in the paper. The policy implications of our results obviously depend on the specific industry application. Assumptions of price versus quantity competition may be reasonable in some context but not in others. In some industries, prices are announced and published in advance and firms can adjust quantities, but have little flexibility to adjust prices in the short term. In other industries, production and planning take place well in advance of resale, while price can be easily adjusted in reaction to competition and/or market changes. Furthermore, policy analysis should consider both the short- and long-term effects of structural interventions. A vertical break-up of a monopoly may generate beneficial effects in the long term, but may create transitional losses in the short and medium term. Our analysis sheds some light on these problems, showing how the availability of substitutes to complementary goods affects both the magnitude and the resilience of transitional losses in various market settings.

Further extensions of our findings may offer valuable insights for a vast array of heterogeneous problems, ranging from modern land development policy and competition policy to the design of appropriate structural remedies by antitrust authorities and intellectual property. These problems are traditionally treated in isolation by different strands of economic theory, which have thus far used a different set of analytical categories to examine problems that share similar structural features: anticommons in some cases, complementary duopoly or complementary monopoly in others. Problems like those that emerged in the *Kelo* or *Microsoft* cases can be usefully appraised with this framework. More generally, legal problems that involve the correction of anticommons problems, the break-up of firms and the approval of mergers can be usefully evaluated in light of our findings. Intellectual property also offers an interesting field of application, since the development of derivative innovation are not always necessarily linked to the purchase of a specific patent. To the extent that different patents can be substitutable for some use, our findings may apply. Our analysis considers a market in which choices are made simultaneously. The study of the sequential case would constitute an interesting extension. Moreover, we consider pure complements and perfect substitutes, therefore our model yields quite extreme results: pricing at marginal costs or no output at all. Taking into account lesser degrees of complementary / substitutability would yield quantitatively very different outcomes, while the main thrust of the paper would remain valid: the effects of substitutability of complements depends on the type of competition and not only on the number of substitutable complements and the number of substitutes per complement.

References

Allen, R. G. D. (1938), *Mathematical Analysis for Economists*, New York: St. Martin's Press.

Anderson, S. P., and Leruth, L. (1993), 'Why firms may prefer not to price discriminate via mixed bundling,' 11 *International Journal of Industrial Organization,* 49-61.

Buchanan, J., and Yoon, Y. J. (2000), 'Symmetric Tragedies: Commons and Anticommons, 43 *Journal of Law and Economics*, 1-13.

Cournot, A. A. (1838), *Researches into the Mathematical Principles of the Theory of Wealth*, (translation 1897), New York: Macmillan.

Demsetz, H. (1967), 'Towards a Theory of Property Rights,' 57 *American Economic Review*, 347-359.

Depoorter, B., and Parisi, F. (2002), 'Fair Use and Copyright Protection: A Price Theory Explanation,' 21 *International Review of Law and Economics*, 453-473.

Depoorter, B., and Parisi, F. (2003), 'Fragmentation of Property Rights: A Functional Explanation of Servitude Law,' 3 *Global Jurist Frontiers,* Article 2.

Economides, N. and Salop, S. C. (1992), 'Competition and Integration Among Complements, and Network Market Structure,' 40 *Journal of Industrial Economics*, 105-123.

Edgeworth, F.Y. (1925), 'The Pure Theory of Monopoly,' in F.Y. Edgeworth (ed.), *Papers Relating to Political Economy*, vol. I, London: Macmillan.

Ellet, C. jr. (1839), *An Essay on the Laws of Trade, in Reference to the Works of Internal Improvement in the United States*, Richmond: P. D. Bernard.

Epstein, R. A., and Kuhlik, B. (2004), 'Is there a Biomedical Anticommons?,' 27 *Regulation*, 54-58.

Gans, J.S., and King, S.P. (2000), 'Mobile Network Competition, Customer Ignorance and Fixed-to-Mobile Call Prices,' 12 *Information Economics and Policy*, 301-328.

Gilbert, R.J., and Katz, L.M. (2001), 'An Economist's Guide to *U.S. v. Microsoft*,' 15 *Journal of Economic Perspectives*, 25-44.

Gordley, J.R., (2003), 'Servitudes,' 3 *Global Jurist Frontiers*, Article 3.

Heller, M. A. (1998), 'The Tragedy of the Anticommons: Property in the Transition from Marx to Markets,' 111 *Harvard Law Review*, 621.

Heller, M. A., and Eisemberg, R. S. (1998), 'Can Patents Deter Innovation? The Anticommons in Biomedical Research', 280 *Science*, 698-701.

Kobayashi, B. H. (2005), 'Not Ready for Prime Time? A Survery of the Economic Literature on Bundling,' *George Mason University Law and Economics Working Paper series no. 05-35*.

Lewbel, A., (1985) 'Bundling of Substitutes or Complements,' 3 *International Journal of Industrial Organization*, 101-107.

Michelman, F. I. (1982), 'Ethics, Economics and the Law of Property,' in J. R. Pennock and J. W. Chapman (eds.), *Ethics, Economics and the Law*, 24 *Nomos*, New York: New York University Press.

Motta, M. (2004), *Competition Policy, Theory and Practice*, Cambridge: Cambridge University Press.

Nalebuff, B. (2000), 'Competing Against Bundles,' in P. Hammond and G. Myles (eds.), *Incentives, Organization, and Public Economics*, Oxford: Oxford University Press.

Nalebuff, B. (2006), 'Bundling,' in L. Blume and S. Durlauf (eds.) *New Palgrave Dictionary of Economics*, 2^{nd} ed., London: Macmillan.

O'Rourke, M. A., (2000), 'Toward a Doctrine Fair Use in Patent Law', 100 *Columbia Law Review*, 1177-1250.

Parisi, F. (2002a), 'Entropy in Property,' 50 *American Journal of Comparative Law*, 595-632.

Parisi, F. (2002b), 'Freedom of Contract and the Laws of Entropy,'10 *Supreme Court Economic Review*, 65-90.

Parisi, F., Schulz, N., and Depoorter, B. (2005), 'Duality in Property: Commons and Anticommons,' 25 *International Review of Law and Economics*, 578-591.

Singh, N., and Vives, X. (1984), 'Price and Quantity Competition in a Differentiated Monopoly,' 15 *Rand Journal of Economics*, 546-554.

Sonnenschein, H. (1968), 'The Dual of Duopoly Is Complementary Monopoly: or, Two of Cournot's Theories Are One,' 76 *Journal of Political Economy*, 316-318.

Spengler, J. J. (1950), 'Vertical Integration and Antitrust Policy,' 58 *Journal of Political Economy*, 347-352.

Tan, G. and Yuan, L. (2003), 'Strategic Incentives of Divestitures of Competing Conglomerates,' 21 *International Journal of Industrial Organization*, 673-697

Venkatesh, R. and Kamakura, W. (2003), 'Optimal Bundling and Pricing Under a Monopoly: Contrasting Complements and Substitutes from Independently Valued Products,' 76 *Journal of Business*, 211-231.

FIGURES

FIGURE 1: *Substituting Complements*