A developmental research on introducing the quantum mechanics formalism at university level

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Chapter 3

Theoretical framework

The general theoretical background for this research is laid out in this chapter. This forms the background for the designed material. In the final section we discuss some issues concerning the interpretation of quantum mechanics. Subsequent chapters introduce additional, more specific theoretical considerations as needed in the chapter in question.

3.1 Constructivism

The literature review from Chapter 2 shows us two things:

• Students’ prior conceptions are of great influence on learning a new topic (e.g. quantum mechanics).

• Learning quantum mechanics requires a new cognitive model (some call it a paradigm shift). Various authors argue that students have to build this new cognitive framework themselves. In other words, teaching activities should elicit cognitive activity that enables students to build this new cognitive model, taking into account their prior conceptions.

These two statements together might be seen as the backbone of constructivism. Although not using the term constructivism nor mentioning quantum mechanics, Ausubelian learning theory also contains these statements as important ingredients (Ausubel, Novak, & Hanesian, 1978). The epigram neatly expresses the first notion:

If I had to reduce all of educational psychology to just one principle, I would say this: The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly. (p. iv)

Ausubelian learning theory distinguishes between two dimensions to categorize learning. These dimensions might be called the internal cognitive dimension and the external educational dimension. The external educational
dimension qualifies the type of instruction: on the one hand whether the content that is to be learned is presented in its final form (reception learning), or on the other hand, whether this content must be discovered by the learner (autonomous discovery learning). In the latter type of learning, the content is only incorporated into cognitive structure after it is discovered.

The internal cognitive dimension qualifies the type of cognitive activity, or process, within the learner. On one end of the continuum we have rote learning, on the other meaningful learning. Both types of learning are defined in [Ausubel et al. (1978)] as follows:

**meaningful learning** takes place if the learning task can be related in nonarbitrary, substantive (nonverbatim) fashion to what the learner already knows, and if the learner adopts a corresponding learning set to do so.

**rote learning** occurs if the learning task consists of purely arbitrary associations, [...] if the learner lacks the relevant prior knowledge necessary for making the learning task potentially meaningful, and also [...] if the learner adopts a set merely to internalize it in an arbitrary, verbatim fashion [...] (p. 27)

The process of meaningful learning is further described by assimilation theory. Assimilation theory is concerned with explaining what internal cognitive process takes place when learning new content. Meaningful learning is the process where new content interacts with already existing cognitive structures, thereby assimilating old and new meanings, forming a new, more general cognitive structure. Depending on the meaning of the new content relative to the already available cognitive structure, there are different ways in which this interaction takes place. For instance, say there is some established (abstract) idea \( A \), which is related to more concrete pieces of information \( a_1 \ldots a_4 \). Then in *derivative subsumption*, “new information \( a_5 \) is linked to superordinate idea \( A \) and represents another case or extension of \( A \). The critical attributes of the concept \( A \) are not changed, but new examples are recognized as relevant” (p. 68). In other interactions, the new information may result in a change of meaning of the superordinate idea (correlative subsumption). Or some established ideas might be recognized as related and may give rise to a new, more abstract superordinate idea, which becomes linked to the established ideas (superordinate learning). Finally, a new idea \( A \) may be seen as “related to already existing ideas \( B, C, \) and \( D \), but is neither more inclusive nor more specific than ideas \( B, C, \) and \( D \)” (combinatorial learning, p. 68).

Meaningful learning and rote learning can be defined in terms of assimilation theory. Meaningfully learned content is assimilated in existing cognitive structures, thereby linked with already existing, more abstract concepts, which are higher in hierarchy and thus stronger embedded in memory. Contrary to meaningfully learned content, rotely learned content is not strongly anchored in cognitive structure, but exists relatively isolated and unstructured. It is therefore not easily retrieved from memory. If it is forgotten, there is also no
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A more general, abstract concept left that might facilitate future learning.

The question now is what the requirements are for meaningful learning. For instance, we might wonder if reception learning, e.g. lectures in higher education, is suitable to elicit meaningful learning. The effectivity of lectures is often questioned (e.g. \cite{Wilson1994, Zollman1996}). Some claim that reception learning, especially verbal reception learning necessarily provokes rote learning (as reviewed by \cite{Ausubel1978}, p. 117-122). For instance, \cite{NiiChin1996} compare Problem Based Learning (PBL) to the traditional didactic lecture (DL), stating that “PBL offers an advantage over the traditional didactic lecture where students learn through a passive rote method” (p. 162, emphasis added). However, a consequence of the two dimensions of learning defined above, is that reception learning can be meaningful. The key characteristic of meaningful learning is that there is cognitive activity, as described by assimilation theory, and that it is the learner who has to construct new knowledge. We like to define constructivism in this sense. Ausubel is critical towards those equating reception learning with rote learning, and discovery learning with meaningful learning. Some have concluded from the constructivist description of learning that learners should always discover content themselves, sometimes also with minimal guidance. A critical review of such minimal guidance teaching approaches is given by \cite{KirschnerSwellerClark2006}. Arguments against such approaches are given based on human cognitive architecture.

\cite{Mayer2004} similarly notes that there are several periods in history in which there is a renewed interest in (pure) discovery learning. Each time, research shows no clear advantage for (pure) discovery learning, but nevertheless, after time has passed the idea is picked up again and the historical cycle starts over. Mayer concludes that:

In many ways, guided discovery appears to offer the best method for promoting constructivist learning. The challenge of teaching by guided discovery is to know how much and what kind of guidance to provide and to know how to specify the desired outcome of learning. In some cases, direct instruction can promote the cognitive processing needed for constructivist learning, but in others, some mixture of guidance and exploration is needed. (p. 17)

The question remains, as explicitly stated by Mayer, what the conditions are for guided discovery learning. How much guidance is needed? \cite{McDermott1991} poses this same problem and gives a direction for an answer to this question:

A major challenge in curriculum development is to determine just how much guidance is the right amount to achieve the necessary level of interest and involvement. If the instructor, computer, or text assumes either too large or too small a role, there is a danger that students will not become intellectually engaged at a deep enough level to be able to transform the material into a meaningful...
and useful form. Rote memorization may replace the development of both conceptual understanding and scientific reasoning ability. (p. 305)

Another question is under what circumstances discovery learning should be preferred over reception learning? Ausubel has something to say on this, which is based on the Piagetian stages and is thus age dependent:

[...] inductive concept formation based on non-verbal, concrete, empirical problem-solving experience exemplifies early developmental phases of information processes. Concept assimilation through meaningful verbal reception learning exemplifies later stages. (p. 27, original emphasis)

Meaningful verbal reception learning, by definition, enables learners to incorporate abstract notions into their cognitive structure, without the need for concrete-empirical props that support concepts to be learned. However, as meaningful reception learning requires concept assimilation, learners need to already have incorporated an adequate body of higher-order abstractions and transactional terms. Ausubel concludes:

At [the abstract] stage of development, therefore, properly arranged verbal reception learning is highly meaningful. Hence it is unnecessary to introduce concrete-empirical props or time-consuming discovery techniques in order to make possible or to enhance intuitive understanding of abstract propositions. (p. 121)

The problem presents itself as a Möbius strip: the “twist” is now moved to what counts as an “adequate body of higher-order abstractions”. Furthermore, Van Hiele (1986) found that the stages are not only age dependent, but also content dependent. For this he introduced the notions of levels of reasoning. At lower levels of reasoning, concrete-empirical props play an important role, as expressed by the first Van Hiele level, which is called the visual level, or ground level (described in Section 3.1.2). Ausubel’s criteria for meaningful reception learning are actually consistent with this notion of content specific levels of reasoning. After all, meaningful reception learning can only occur if there is already a cognitive structure present containing higher-order abstractions to which newly introduced concepts can be linked (as described by assimilation theory). If this body of higher-order abstractions is lacking, then there is nothing, or not enough, to link to. To still “learn” something (and pass an exam), learners may resort to rote learning. Sometimes new ideas cannot simply and will not simply be acquired by reorganizing existing knowledge. So much we know from the literature on quantum mechanics education we have reviewed.

But what about some of the conclusions from the literature on quantum mechanics education? Are these conclusions in line with the above? Many of the described teaching strategies seem to include some form of discovery
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learning. However, they are all in a way guided, certainly not pure discovery learning. Furthermore, based on Ausubelian learning theory we have indications that in the case of learning quantum mechanics it is to be expected that some concrete-empirical props are necessary, as students’ cognitive structure is expected to contain little abstractions that are useful for learning quantum theory. There is simply very little to link new concepts to. Based on the literature review on quantum mechanics education and the above discussion of Ausubelian learning theory, we consider guided discovery learning a suitable teaching approach for introductory quantum mechanics.

An explanation for the poor results found in traditional lecturing methods (i.e. verbal reception learning) might be found in a study by Hrepic, Zollman, and Sanjay Rebello (2007). They have found that a typical lecture is understood differently by students than by experts. Students were given six questions before and after watching a videotaped lecture in which the topic of the questions was discussed. The lecture was considered a good lecture. The students were asked to find answers to the questions in the video. The same was done with experts. For some questions both students and experts agreed on the extent to which the answer was discussed in the lecture. However, some questions were perceived by the experts as being answered, but not by the students and vice versa. Also, experts perceived all questions answered more frequently than the students did. Furthermore, the experts more often stated that the answer to the questions could be inferred from the information given in the lecture. Experts gave more correct inferences than students. The authors conclude that:

When interviews with experts are examined we find that unlike students, experts (may)

1. have plentiful resources to correctly figure out things they never thought about before by engaging in deductive reasoning and testing their models.

2. […]

3. consider a variety of reasons that would both support or disprove each of the possible answers when they are not sure about the answer.

This conclusion is in accordance with assimilation theory: the experts can relate the content from the lecture more meaningfully to what they already know. However, the learning of the students based on the lecture is not necessarily rote. Surely, there are lectures that are very successful in eliciting rote learning. But it does not follow that the lecture in general cannot result in meaningful learning.

The question now is how we might structure a teaching sequence for introductory quantum mechanics and what guidance is needed. More specifically, we may formulate the following questions:
1. What concrete-empirical props are needed for students to form the concepts needed in quantum theory?

2. What framework might we use that explains how these concepts can be formed based on the concrete-empirical props? From this framework it should follow what activities are required of both students and teachers.

3. Is there a guiding principle that enables us to organize a teaching sequence so that students will work with the materials in a meaningful way?

Question 1 will be a topic investigated throughout this thesis, for which we have developed and tried out teaching materials. A possible answer to question 2 might be given by the Van Hiele level scheme, briefly outlined below in Section 3.1.2. As guiding principle (question 3), we consider the problem posing approach, described in the next section.

3.1.1 Problem posing approach

Constructivism, as defined at the beginning of this chapter, states that meaningful learning requires learners to assimilate new information in existing cognitive structure. The meaning new learners give to this new information depends on this existing cognitive structure. In other words: students’ prior conceptions are relevant for learning. This description, however, does not explain what drives this learning.

Klaassen (1996) argues that for students to “meaningfully engage in an activity there should be a sense in which they know why they are doing it” (p. 87). Furthermore, “they will have to develop some sense of purpose for going to study events they have never witnessed or paid attention to” (p. 87). Science education should make pupils want to add to their “existing conceptual resources, experiential base and belief system . . . in a way that leads to a proper understanding of science” (Klaassen, 1996, p. 106). The problem posing approach aims at all the above.

A teaching sequence that is problem posing is not designed with hindsight, understandable for those who already understand. On the contrary, its purpose is:

to plan pupils’ process of learning the topic in such a way that all along the pupils themselves know what they are doing and why, and that by building on what they already know are given ample chance to further extend what they already know, driven by what they themselves are doing, by their reasons for doing it, and by the conclusions they reach and problems they encounter as a result for doing it. (p. 109)

The notion of, and need for a problem posing approach is also experienced by those teaching physics and chemistry. For instance, Shiland (1995) notes:
Is there a question you hope your students never will ask? Because few of my students appear to be Journal subscribers, here is my question.

Mr. Shiland, what is all this theory (quantum mechanics) good for?

What makes this question so frightening is that it is reasonable. It is so reasonable that a more general version has been proposed to guide general and secondary chemistry curriculum reform:

Is the theory being introduced essential in order to explain some important chemical reaction, experiment, or property?

What is more troubling is that such a reasonable question never is asked. In 10 years of presenting this material to several hundred students, only a handful have said anything that approaches this question. Most students view the presentation of quantum mechanics as consistent with the rest of their school science career. It is more meaningless information that must be memorized. They accept the latest dose of abstract material without question . . . (p. 215)

This is exactly what a problem posing approach wants to prevent.

We raised the following question in the previous section: Is there a guiding principle that enables us to organize a teaching sequence so that students will work with the materials in a meaningful way? As guiding principle, we propose to use the problem posing approach. The teaching design, described in Chapter 6 and for which we draw conclusions in Chapter 10, tries to follow this approach. We operationalize it as follows. Each point in the teaching sequence might give rise to a question that a student could reasonably come up with, and which leads to the next element in the teaching sequence. This question is, so to speak, the problem a student might pose, which expresses the purpose to study the next element in the sequence. When designing our education it remains a question how orthodox we should be. Perhaps it is not always necessary to strictly follow the problem posing approach. Perhaps it is not always possible, in the sense that it might result in a very long teaching design. Making a shortcut might violate the problem posing approach, but it could result in a shorter teaching design.

\[1\text{It is of course possible that at a certain point in the teaching sequence multiple questions are posed, each leading to its own element in the teaching sequence. This will result in parallel paths in the teaching sequence that need to be followed sequentially.}\]
3.1.2 Van Hiele level scheme

One question remains, namely what framework might we use that explains how students form concepts based on concrete-empirical props? In the discussion of Ausubelian learning theory we already stated that if students lack an adequate body of higher-order abstractions, concrete-empirical props may be important for concept formation. However, Ausubel, based on the Piagetian stages, only claims that discovery learning is the primary mode of learning for children, whereas adults primarily learn through reception learning. In the former, concrete-empirical props play an important role. Van Hiele (1986) adds to this a content specific notion of levels of reasoning. Van Hiele experienced difficulties in teaching geometry at high school. It was then (1950s) customary to teach geometry starting from axioms and definitions. In doing so, Van Hiele noticed that his students had difficulties with this approach. He would try his best to teach the subject, but after a while, when they seemed finally to understand, students would remark that it was not so difficult after all, and would ask him why he explained it with so much difficulty. This made Van Hiele think he and his students were initially speaking a different language. Elaborating on this idea, Van Hiele introduced the notion of levels of thinking:

You can say somebody has attained a higher level of thinking when a new order of thinking enables him, with regard to certain operations, to apply these operations on new objects. (p. 39)

In other words, depending on a learner’s cognitive structure, the quality of reasoning, and the language he uses may differ. This is expressed by the levels of reasoning:

visual/ground level Learning starts with an observation of some kind, involving one, or more of our senses (hence concrete-empirical props). Van Hiele focused on visual observations and called the first level of thinking the visual level, also called the ground level. This does not mean only vision is used at this level: in principle all senses are involved. At this level we already use language to define what we see, e.g.: “this is a rhombus”. By looking at the figure of a rhombus, you immediately recognize another rhombus. To check if this use of language is calibrated, you can point at this other rhombus and exclaim: “this is a rhombus as well!” and wait if your discussion partner (e.g. student, or teacher) agrees. This phenomenon is described by Gestalt psychology, by which Van Hiele is said to be influenced. The rhombus in this context is said to be a structure.

In short, on this level we are naming the structures we observe. It is the immediate recognition of what is to be seen as a whole, that plays an important role here.

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2 This section is based on a conference contribution: Koopman, Kaper, and Ellermeijer (2006).
3.2 Conceptual understanding versus procedural fluency

**descriptive level** In a next step we are able to name properties of the things we observe. This brings us to the second level: the *descriptive level*. For instance, we simply know what a rhombus is, but we do not know what distinguishes it from other geometrical figures. By describing its properties we can find out. On this level, we can say to each other: “a rhombus has four sides”. Disputes concerning what is, or what is not a rhombus, can be resolved on the descriptive level. Because now we make explicit what we understand to be a rhombus. It is characteristic of this level to have a network of relations, which was lacking on the first level.

**theoretical level** After we have made a complete enough description of our problem, we might want to reflect on this description. In doing so, we are moving to the third level of thinking: the *theoretical level*. For instance: it might occur to us that some of the relations we have found, depend on each other. We might start to wonder why. Also, we might notice that some relation follows from a set of other relations. Or perhaps some relation is in contradiction to some set of other relations.

Just as the second level is concerned with elements of the first level, the third level is concerned with elements of the second level. On the third level we scrutinize the relations that are laid out on the second level. The level scheme is thus hierarchic. When moving to a higher level, we learn a new language that makes it possible to speak about the previous level. In this sense it is impossible to move to a next level, without learning a new language. In teaching, it might sometimes appear as if a student is at the theoretical level, because he uses words from that level. But this might not be the case. We can only say a student is at the theoretical level when this student is able to *reason* on a theoretical level.

Although much more has been written about the Van Hiele levels, we use the above scheme as a heuristic in designing teaching materials (Chapter 6) in which students are presented with concrete, observable situations (ground level) which they describe in a precise manner (descriptive level). This description should then require a more minimal description, based on axioms and rules (theoretical level).

### 3.2 Conceptual understanding versus procedural fluency

From the literature review for Quantum Mechanics (Section 2.1, Chapter 2) it became apparent that there seems to be a traditional focus on teaching students how to work with the formalism, as formulated by Johnston, Crawford, and Fletcher (1998):

> [...] there is a strong tradition in physics departments for “transmissionist” education strategies – teach them how to do the subject
first, and let them worry about what it all means when they get to research level. Perhaps it is time to query that tradition, especially if [...] there is an increasing need for quantum mechanics to be understood by professionals who will never be researchers. (p. 443)

The literature suggests that the approach described above elicits “surface” learning, resulting in fragmented mental models of students. This suggests rote learning, where concepts are not assimilated in students’ cognitive structure. This “transmissionist” education strategy might best be described by a focus on teaching procedures within the mathematical formalism of the theory to be learned. In mathematics education there is a useful term for this: procedural fluency.[Kilpatrick, Swafford, & Findell, 2001] This is actually one of five strands of mathematical proficiency. In the current context, the first two strands seem relevant: conceptual understanding and the aforementioned procedural fluency. They are called strands because on the one hand they constitute skills that can be trained (and tested) independently, but on the other hand they depend on each other. In other words: on a short term it may be possible to train procedural fluency, but on a longer term this cannot be done efficiently without also improving conceptual understanding.

These concepts (procedural fluency and conceptual understanding) together with Ausubelian learning theory enable us to explain results such as found by [Johnston et al., 1998]. A focus only on procedural fluency contradicts meaningful learning, because it ignores conceptual understanding and thus no linkage is made to already learned content (a requirement for meaningful learning). Such a focus stimulates students to rote learn the procedures taught.

In somewhat other wordings, [Singh, Belloni, and Christian, 2006] make a similar distinction in the context of quantum theory. They introduce the concepts “unintuitive foundational aspects” and “functional understanding”. The unintuitive foundational aspects may be a subset of the broader core of conceptual understanding. Functional understanding seems to map to procedural fluency. It seems that many physicists are of the opinion that one has to first learn how to work with quantum mechanics (thus focusing on procedural fluency) before the unintuitive foundational aspects can be understood, or even studied. The results reported by [Johnston et al., 1998], as well as [Kilpatrick et al., 2001] suggest that it is not possible to make this distinction.

We return to this topic in Chapter 8 where we look at the development of conceptual understanding and procedural fluency for the case of the wave function.

[Knigh, 2004] makes a similar distinction for physics knowledge, which falls into three categories: factual, conceptual, procedural (p. 25-26).
3.3 Interpretation

We finally want to address an issue touched upon in Chapters 1 and 2: the interpretation of quantum mechanics and its relation to learning and understanding quantum mechanics. As noted before, the combination of “interpretation” and “quantum mechanics” seems to have a controversial connotation. From a philosophical standpoint there is nothing strange about an interpretation of a theory. The interpretation defines how the formal system is related to experimental outcomes. Ballentine (1970) defines interpretation as a set of correspondence rules that relate theoretical concepts to experience:

Quantum theory, and indeed any theory, can be divided [...] into:

(a) A mathematical formalism consisting of a set of primitive concepts, relations between these concepts (either postulated or obtainable by given rules of deduction), and a dynamical law.

(b) Correspondence rules which relate the theoretical concepts of (a) to the world of experience. (p. 359)

Cushing (1994) also uses the above definition of theory and makes the same distinction between formalism and corresponding interpretation. However, as will be explained below, Cushing claims that an ontological interpretation of quantum mechanics, instead of an epistemological interpretation, will be able to give us understanding. Bohm and Hiley (1993) explains this distinction between epistemological and ontological interpretation:

...Bohr and Heisenberg have implied, that quantum theory is concerned with our knowledge of reality and especially of how to predict and control the behaviour of this reality, at least as far as this may be possible. Or to put it in more philosophical terms, it may be said that quantum theory is primarily directed towards epistemology which is the study that focuses on the questions of how we obtain our knowledge (and possibly what we can do with it).

It follows from this that quantum mechanics can say little or nothing about reality itself. In philosophical terminology, it does not give what can be called an ontology for a quantum system. Ontology is concerned primarily with that which is and only secondarily with how we obtain our knowledge about this ...(p. 1, original emphasis)

Both Ballentine and Cushing argue that the formalism of quantum mechanics allows multiple interpretations. The orthodox interpretation of quantum mechanics is the Copenhagen interpretation. There seem to be many “definitions” of what the Copenhagen interpretation actually is, but salient in these

4Although not mentioned explicitly by Ballentine, this seems to apply to a physical theory.
descriptions seems to be that a “pure state provides a complete and exhaustive description of an individual system (e.g., an electron)” (Ballentine 1970, p. 360). As long as the correspondence rules yield the same experimental results, we have no means of distinguishing experimentally between different interpretations. The main difference between alternative interpretations then lies in their ontological status. An example of an ontological interpretation is the Bohm interpretation. This interpretation uses the same formalism, but adds to that an ontological interpretation in which the quantum particle always has a certain position and momentum. This theory (i.e. quantum formalism plus Bohm interpretation) is said to be observationally equivalent to the Copenhagen interpretation. However, because it is more restrictive, in the sense that it contains additional ontological claims (i.e. particles with position and momentum) there might be a situation in which the Bohm version of quantum mechanics predicts a certain outcome, whereas the orthodox version remains agnostic. This is the problem posed by Cushing (1994, p. 53-55). He discusses a possible experimental test to rule out the Bohm interpretation. However, nothing has been measured yet. Up to date, the Copenhagen interpretation remains the most popular one. Baily and Finkelstein (2010) explain this popularity by the positivistic aspect of this interpretation, “in that it allows practicing physicists to apply quantum theory without having to worry about what is ‘really going on’ (otherwise known as ‘Shut Up and Calculate!’)” (p. 2).

It is not our intention to add to the discussion on the interpretation of quantum theory, but to signal that there is an ongoing debate about the interpretation, that needs to be addressed, one way or the other, when teaching quantum mechanics. An argument to support this stance, is given by Cushing (1994), who argues that a formalism together with an instrumentalist interpretation is able to give an explanation of physical phenomena. However, it does not necessarily give us understanding. Understanding requires an interpretation that allows one to “grasp the character of and the relations among the phenomena” (p. 11). An interpretation enables one to understand the meaning of the formalism in terms of the underlying physical processes (cf. p. 15-16). A theory (formalism plus interpretation) can thus function on different levels, Cushing describes the following three levels:

**empirical adequacy** is provided by a formula, or algorithm that reproduces observed data, e.g. phenomenological or semi-empirical calculational scheme;

**explanation** is given by a successful formalism (i.e. a set of equations and rules), with emphasis on unification via derivability from a more general framework. The formalism is given an instrumentalist interpretation;

**understanding** is possible when an interpretation is available that allows one to grasp the character of and the relations among the phenomena. (p. 10-11, paraphrased)
Cushing (1994) further explains what he thinks is the main characteristic of understanding:

As a paradigm of an explanation that can (or may) produce understanding for physical processes, I take a causal explanation, consisting either of direct cause-effect between phenomena and events or of a common cause located in the past of the collection of phenomena under consideration. The ultimate goal is to construct a framework that is empirically adequate, that explains the outcomes of our observations, and that finally produces in us a sense of understanding how the world could possibly be the way it is. It remains an open question whether all three of these goals are simultaneously attainable.

My argument here really begins from the intuition, based on experience and on some history of physics, that understanding of physical processes involves a story that can, in principle, be told on an event-by-event basis. This exercise often makes use of picturable physical mechanisms and processes. (p. 11)

An example of these three levels is given by the following. Boyle’s law provides empirical adequacy, the formalism of statistical mechanics gives an explanation, whereas with the kinetic theory (or model) of gases a causal picture story can be generated, which enables us to understand (p. 14).

Understanding as defined above is how an expert views it. It is not how understanding is defined in (cognitive) psychology or in education. However, there are indications from physics education research that this notion of understanding is in accordance with students’ reasoning. An example is given by the bridging analogies of Clement and Brown (Clement, 1993). In the case of the normal force, a sequence of linked models enables students to understand the mechanism behind this force. Another example is given by Frederiksen and White (2002) with a focus on the linkage between models. Their models are chosen such that they enable students to visualize the mechanism on a micro-level, and reason what the macroscopic consequences are.

One of the few studies on quantum mechanics education that explicitly considers the interpretation of quantum mechanics in the teaching design, is Müller and Wiesner (2002), discussed in Section 2.1.5. The authors have chosen the statistical ensemble interpretation of quantum mechanics (Ballentine, 1970), because it “provides a clear and comprehensive way of talking about quantum phenomena” (p. 202).

Baily and Finkelstein (2010) have compared two almost identical introductory courses on quantum mechanics, that differed only significantly in their lecturers stance on the interpretation of quantum mechanics. One of the lecturers would explicitly address the interpretation of quantum mechanics throughout the course, taking on a “quantum” standpoint, while the other lecturer might be said to take on a more “agnostic” standpoint (i.e. refusing an explicit interpretation). Based on an online survey to probe students’ beliefs about quantum mechanics, students in the first course (i.e. with the “quantum” lecturer)
answered mainly consistent with a quantum interpretation (~90%). Students from the second course (with the “agnostic” lecturer) were also found to take on a quantum standpoint in majority (~45%), although a large number was found to take on a realist, i.e. deterministic and local standpoint (~30%). Approximately 25% took on an agnostic standpoint. This shows that the lecturer’s interpretation of quantum mechanics significantly influences the interpretation found among students. More importantly: refusing to discuss the interpretation of quantum mechanics, might result in students adopting an interpretation that is considered inappropriate.

Many note the traditional focus on the formalism of quantum mechanics (as for example Baily & Finkelstein, 2010). According to Cushing (1994), although this focus may give an explanation, it is not enough for understanding. What if this traditional focus on the formalism prohibits understanding? Furthermore, the “unintuitive foundational aspects” of quantum theory, of which Singh et al. (2006) speaks, might be specific to the Copenhagen interpretation. What if this traditional choice for the Copenhagen interpretation is pedagogically speaking not the best way to elicit understanding? Cushing (1994) argues that the Bohm interpretation might be better at “producing a sense of understanding how the world could possible be the way it is” (p. 11). John Bell also came to that conclusion (Bell 2004):

Of course […] the pilot-wave picture5 of de Broglie and Bohm exists. Moreover, in my opinion, all students should be introduced to it, for it encourages flexibility and precision of thought. In particular, it illustrates very explicitly Bohr’s insight that the result of a ‘measurement’ does not in general reveal some preexisting property of the ‘system; but is a product of both ‘system’ and ‘apparatus’. (p. xi-xii)

The de Broglie-Bohm picture is considered impossible due to the use of so-called “hidden variables”, which are considered impossible in a quantum theory that is to describe nature. Bell recalls his encounter with the revival of the de Broglie pilot-wave, in the papers of David Bohm:

But in 1952 I saw the impossible done. […] Bohm showed explicitly how parameters could indeed be introduced, into nonrelativistic wave mechanics, with the help of which the indeterministic description could be transformed into a deterministic one. More importantly, in my opinion, the subjectivity of the orthodox version, the necessary reference to the ‘observer’, could be eliminated. (p. 160)

5The pilot-wave picture was put forth by de Broglie in 1927 and resembles the Bohm version of quantum mechanics. However, at the time, it was not yet developed as a consistent framework. During the 1927 Solvay Conference de Broglie received strong criticism from proponents of the Copenhagen interpretation. This made de Broglie abandon the idea, until Bohm in 1952, independently from de Broglie, worked out a consistent theory.
Bell continues to express his amazement of Bohm’s result, as this idea was already available in 1927 due to de Broglie (i.e. the pilot-wave picture):

Why then had Born not told me [by way of his 1949 book *Natural Philosophy of Cause and Chance*] of this ‘pilot wave’? If only to point out what was wrong with it? Why did von Neumann not consider it? More extraordinarily, why did people go on producing ‘impossibility’ proofs after 1952, and as recently as 1978? When even Pauli, Rosenfeld, and Heisenberg, could produce no more devastating criticism of Bohm’s version than to brand it as ‘metaphysical’ and ‘ideological’? Why is the pilot wave picture ignored in text books? Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show that vagueness, subjectivity, and indeterminism, are not forced on us by experimental fact, but by deliberate theoretical choice? (p. 160, emphasis added)

The emphasized phrase is in fact the same message that Cushing (1994) advocates.

This thesis will not further address, and certainly not resolve the issues raised here. The interpretation of the wave function does play a role in the teaching design, as explained in Chapter 6. However, not on the level as discussed in the current chapter. A choice has been made to first address issues that are less controversial and that will probably have a higher impact. An example of such an issue is the remedial mathematics program for Quantum Chemistry (Chapter 5). However, there are enough arguments to further investigate the role the interpretation of quantum mechanics plays in teaching and learning quantum mechanics in general, and in understanding quantum mechanics in particular. In Chapter 10 we consider possible directions for such future research and propose how a teaching design might address the interpretation of quantum mechanics.

References


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6Impossibility proofs in quantum mechanics, in this context, aim at showing that it is impossible to have a quantum theory with hidden variables.


of Pharmaceutical Education, 60, 162–164.