A developmental research on introducing the quantum mechanics formalism at university level
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Citation for published version (APA):
Koopman, L. (2011). A developmental research on introducing the quantum mechanics formalism at university level

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Chapter 10

Conclusions

Developmental research has two main aims, as described in Chapter 1. First, developing prototype materials in a real teaching environment. Second, making a scientific contribution by formation of a local model that explains the structure of the developed prototype. The prototype materials have been described throughout this thesis. The aim of the current chapter is to draw conclusions based on the teaching experiments. These conclusions fall into three categories. First, conclusions related to the interventions that aimed at the prerequisites for learning (quantum mechanics): remediating the observed mathematics deficiencies and improving students’ study strategies (Chapter 5). These interventions mainly contain adjustments to the form in which the course is given. Second, conclusions concerning interventions related to the physics content: the introductory sessions, described in Chapter 6. These interventions mainly contain adjustments to the content of the course. As described in Chapter 1, the outcomes of developmental research may be formulated in terms of a didactical structure. For the introductory sessions we describe this didactical structure. The third type of conclusion relates to the methodology used in this research.

In the teaching experiments only the content of the first part of the course was adjusted (i.e. the introductory sessions). Based on our experiences and the research literature, we argue how one might proceed after the introductory sessions that were designed in this research. In the last section of this chapter we consider possible directions for future research and development.

10.1 Conclusions related to the interventions

For each type of intervention we first summarize the findings as presented earlier. Next, we give more general reflections on the process of development and implementation that have not yet been discussed. Based on these reflections we draw general conclusions and give recommendations.
10.1.1 Warming-up, cooling-down, and remedial mathematics

Summary of findings

The warming-up, cooling-down, and remedial mathematics interventions (referred to “study supporting activities” in the following) were generally found to be effective. The success of the remedial mathematics program has been motivated in Chapter 5. In short: students’ mathematical fluency improved and during lectures less time had to be spent on explaining (basic) mathematics. The warming-up and cooling-down questions resulted in students spending time on studying for the course, focusing them on topics that were considered important for the corresponding lectures.

Conclusion 1. Remedial mathematics activities have a positive effect on students’ mathematical skills; during lectures and tutorial sessions, mathematics becomes less of an obstacle. Furthermore, warming-up and cooling-down activities help students adopt a more successful study strategy.

The extent to which students took the study supporting activities seriously, was found to depend strongly on how good these activities were integrated in the lectures. Students were not easily motivated to complete activities such as these. What was found to work: making the activities obligatory (in case of the remedial mathematics), giving a bonus for students’ final grade (in case of the cooling-down questions), or tightly integrating the activities in the lectures (in case of the warming-up questions). Students should see the relevance of their effort. This can be achieved by giving some form of feedback. For instance, direct feedback on quizzes can possibly be given either by telling students which answers were correct/incorrect, or by providing (short) answers. Another form of feedback can be given by referring to the activities in lectures, or tutorial sessions. Most successful is probably a combination of all of the above (Mazur, 1997).

Conclusion 2. To stimulate participation, there should be a clear incentive for students to complete study supporting activities. What is found to work: providing different forms of feedback, giving a bonus, and making activities obligatory.

An important conclusion from Chapter 5 is that despite the improvement of mathematical skills, students still do not perform as expected in the final exam. This shows that quantum mechanics is more than only abstract mathematical manipulations. Students also need to develop new physics concepts, which have to become part of a conceptual framework. In other words, the problem is not just that students cannot perform the required (mathematical) calculations, but they have to be able to choose an appropriate solution strategy based on their conceptual understanding of quantum mechanics. Furthermore, this also requires them to transfer the calculus they have learned to the physics context.
Conclusion 3. Proper mathematical skills are indispensable, but are not sufficient for understanding quantum chemistry.

Reflection on development and implementation

The lecturer plays an important role in the development and implementation process; a result of the developmental research approach chosen. All interventions were designed in close cooperation with the initial lecturer of the Quantum Chemistry course (during 2005–2007). It was this lecturer who experienced difficulties motivating the need for quantum mechanics to the students (hence the introductory sessions), who observed that students lacked the needed mathematical skills (hence the remedial mathematics activities), and who observed that students could study more effectively (hence the JiTT activities and diagnostic tests). In 2008 a new lecturer took over who had not been involved in this development process. This lecturer was less motivated to use the remedial activities developed in earlier rounds. In fact, the lecturer agreed to continue these interventions, mainly because of their importance for the research, not because he was convinced of their necessity. As a result it was found that the activities were poorly integrated in the lectures and students took them less seriously. There was still a bonus for the cooling-down questions, but the remedial mathematics program was no longer obligatory. Overall, it was found that students’ participation dropped. This showed itself during the tutorial sessions. In discussions with the researcher, some students experienced the same difficulties with the mathematics as before. As these students had not made the remedial mathematics tests, the researcher could not refer to these tests in his explanation, which had proved to be effective in earlier research rounds.

Conclusion 4. Remedial and JiTT activities require a lecturer, or teacher to be closely involved and motivated. He should know the material, study student responses, and act accordingly, giving feedback or providing tailored instruction.

This conclusion confirms some best practices of Just-In-Time Teaching (Novak, Gavrin, Christian, & Patterson, 1999).

Recommendation 1. Developing remedial and JiTT activities takes a serious amount of time. Lecturers and teachers that want to use such methods should be supported by their educational organization, and not have to reinvent the wheel. On the other hand, given the fact that the benefit of these activities has been established, lecturers who do not want to use them should give arguments for doing so. The activities should not easily be discarded. Training and support for these lecturers, provided by the educational organization, may help to raise the acceptance.

When implementing these activities it was found that students had some difficulties with the online grading system used for the remedial mathematics
program. For example, students had to learn how to input mathematical expressions. A correct answer that is input incorrectly, results in an item graded as incorrect. This causes frustration amongst students and undermines their motivation to participate in the activities. Furthermore, other parallel courses had similar activities. Some students were found to experience all these different activities as an overload. It sometimes confused them, in the sense that they seemed to lose track of what assignments had to be done for which course.

**Recommendation 2.** If several courses choose to use remedial and JiTT activities, then preferably a uniform system should be used (e.g. same kind of online quizzes, same kind of feedback, etc.). This way students are not overloaded with a plethora of quizzes, tasks, bonuses, and so on, each using a different interface. Furthermore, parallel courses should not compete. If one course gives a bonus, other parallel courses should as well. Students often set priorities according to what is most economical, i.e. they focus on activities that most directly contribute to their grade. Thus, a strategy on supporting activities, such as remedial and JiTT, is required from the educational organization.

**Recommendation 3.** There should be a progression, or development in the activities as students progress in the curriculum. In the end, students should be educated such that they know how to prepare for lectures, know when to re-read relevant preparatory materials, etc. Ultimately, making study supporting activities redundant. In other words, students should know what is expected from them, and learn what it means to be an academic.

### 10.1.2 Introductory sessions

Before drawing any conclusions concerning the introductory sessions, we need to define more precisely what we mean by “introduction”, or “introductory sessions”. Up to this point, emphasis has been put on the introduction as an organizational unit: four introductory sessions before the regular course. Apart from this organizational definition, we can give a definition based on the aim of the introduction, which is to guide students to quantum theory, by discussion of experiments that students are able to interpret with their prior knowledge. To explain the experimental outcomes, students are stimulated to formulate hypotheses and introduce model terms. These hypotheses, together with the descriptive rules found in the experiments require a logical system with axioms and rules. From this logical system (theory), the descriptive rules

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1. With “experiments” we also mean more generally “phenomena”.
2. With “interpret with prior knowledge” we mean that certain terms need to function as observable terms. This means that students do not question their applicability. In the photoelectric effect assignment, for example, the kinetic energy of the (fastest) electrons needs to be (or become) an observable term. Without it, students will not be able to find the relation between (maximum) kinetic energy and the frequency $f$ of the incident light. This relation is needed for them to hypothesize that light comes in packets of energy $hf$. 

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must follow, and it should make additional predictions. This logical system is, ultimately, quantum mechanics. The end point of the introduction is not yet this logical system, but at least the Schrödinger equation, along with an idea how to define observables and relate them to measurement outcomes (i.e. operators and expectation values). This is not yet quantum theory, but it is expected that enough ingredients are available to introduce students to the axioms of quantum theory. These axioms can be related to activities from the introduction and their choice can thus be motivated. We define “introduction” first and foremost in the above sense, apart from how it might be implemented and organized in an educational setting.

Summary of findings

The introductory sessions have been described in Chapter 6. The core of these sessions was formed by the analogical reasoning, described in Chapter 7. This part of the design succeeded in letting students introduce the concept wave function to describe the electron interference pattern in a double-slit experiment. Students considered the wave function as a useful description of electrons in this context. The amplitude of the wave function was related to the probability of finding electrons in a certain region.

Conclusion 5. The double-slit analogy is appropriate to let students introduce the concept of wave function.

The analogical reasoning, however, did not yet result in the scientific concept that was expected. For example, students do not identify a superposition of two wave functions — each describing an electron emerging from either of two slits — with one electron. Students do come up with possible hypotheses how it is possible that one electron at a time can still result in an interference pattern. In an additional teaching activity, students should get the opportunity to express these hypotheses explicitly, such that they can be tested against what they know about electrons. In such an assignment, students might also be challenged to take a stance on whether a wave function, which is an addition of two (circular) waves, describes one, or two electrons. As this is part of phase 3 of the analogical reasoning, we may consider this phase as unfinished in our teaching experiment.

Conclusion 6. Phase 3 of the analogical reasoning (evaluation) was not yet completed as students did not give the term wave function an appropriate interpretation.

Because students were found to use the term wave function tentatively, and its meaning was still changing, it can be said to function as a model term. This is consistent with our conclusion that phase 3 of the analogical reasoning, in which new terms are evaluated, was not yet finished. For a model term to become an observable terms there must at least be agreement

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3 This is an implementation of the Van Hiele level scheme (Chapter 3).
on the meaning of the term. Another property of an observable term is that it is used to formulate experiences. However, it seems strange to use the wave function to formulate experiences. After all, it is always emphasized that the wave function itself cannot be observed. In this sense, the wave function is something different from the position of a (classical) particle as a function of time. This in fact touches upon questions of interpretation. At the very least, it can function as an observable term in the sense that we can agree upon how the wave function is related to experimental outcomes. For students this means that they will have to become more confident using this term.

Recommendation 4. To let students become more confident using the term wave function, activities where students can apply this new term to predict experimental outcomes might be useful.

In Chapter 8 we looked into the relation between students’ conceptual understanding of the wave function and their procedural fluency when working with the wave function. The main course is very much focused on working with the formalism, and thus on procedural fluency. For example, students use the wave function mainly in calculations for which they can apply the rule that $|\psi|^2$ is a probability density. It is unnecessary for students to worry about the interpretation, they only need to apply this rule. Based on the interview results, this appears not to have contributed to students’ conceptual understanding. The concept wave function remains a hybrid concept: on the one hand students know how to use this term in calculations, on the other hand, they still associate it with the trajectory of a particle. The meaning of density plots of the wave function, as well as the sign in those plots, was not clear to all students. The retention test, described in Chapter 9, also revealed the lack of conceptual understanding, supporting the above conclusions.

Conclusion 7. A focus on procedural fluency seems insufficient for the development of students’ conceptual understanding.

In explaining chemical bonding, no use was made of the idea that overlapping wave functions (atomic orbitals) interfere and thus give rise to a higher or lower electron density between two nuclei. Instead, students used self distilled, inappropriate rules to identify the bonding and anti-bonding molecular orbitals: a plus (minus) combination of atomic orbitals is though to be always bonding (anti-bonding). This could in part be related to the examples given to students during the course, which did not require them to adopt a more sophisticated explanation of chemical bonding based on quantum mechanics.

Conclusion 8. Students show spontaneous, inductive reasoning when presented with examples (of chemical bonding for example): they formulate rules based on these examples and generalize these rules to situations in which they do not necessarily apply.

Recommendation 5. It is important to discuss various examples with students, for instance in a Peer Instruction format (Mazur, 1997). When inap-
appropriate rules are created by the students, they will then be quickly noticed and counter examples can then be deliberately chosen.

Under the circumstances of the last research round, the effect of the introductory sessions on the remainder of the course was found to be limited (Chapter 5). In the post-course interview students expressed that they saw little connection between the introduction and the remainder of the course. We return to this finding in the next section.

Reflection on development and implementation

Many of the developed activities require classical physics and mathematics that students (should) have learned earlier, but for which they lack procedural fluency, or which they do not master at a level that is required. Regarding the mathematics, the remedial program is appropriate to let students brush up on their prior knowledge. The physics knowledge needed is in part addressed by the JiTT approach. For subject matter that is required on a more abstract level, we have found that this approach is not always sufficient. Several of the designed activities that required such more abstract knowledge (e.g. the photoelectric effect, interference of waves) gave students the opportunity to review their prior knowledge. However, because the content of these assignment was somehow familiar to students, they did not always work on the materials seriously. In particular, items where no calculations were needed, but where students were required to give an argumentation, were found to evoke this behavior. Students might not be used to giving argumentations, but instead are trained to perform calculations. Another explanation might be that the questions were not “problem posing” enough. In other words: the questions might not be questions students would have come up by themselves. Finally, because some of the questions are very open, with different answers possible, students might become insecure of the “correctness” of their answer, or, conversely, they might have been uncertain what was expected of them.

Conclusion 9. Prior knowledge that is required at a higher, or more abstract level, is best taught during contact hours, in a challenging way such that students experience that they learn something new. Brushing up on prior knowledge is best done outside contact hours using materials that differentiate between students’ need to brush up.

As explained above, a new lecturer took over in 2008. This lecturer had not been involved in the development process of the interventions. Four lectures were made available in which the introductory sessions could be given. These sessions were prepared by the researcher, a teacher, and a teaching assistant from the problem solving sessions. The lecturer was not closely involved. As a result, there was little integration between the introduction and the remainder of the course. This partly explains why students did not use the concepts from the introduction (most notably interference) in explaining chemical bonding. More could have been done in the remainder of the course with the results
of the introduction. In particular, students spent considerable time on understanding the double-slit experiment and describing the interference pattern. This could have been used to explain chemical bonding, as this also is an interference phenomenon. The lack of integration also showed itself in the written exams. No questions were included that referred to the topics covered in the introduction. For students this might be seen as a signal that the topics covered in the introduction are unimportant.

Conclusion 10. As isolated unit, the learning outcomes of the introductory sessions for the course as a whole are limited.

Recommendation 6. The activities from the introduction can play a role in the application of quantum mechanics to a chemistry context (e.g. in case of interference phenomena). Furthermore, the exam should reflect that the content of the introduction is also part of what is to be learned, not merely a motivation to start talking about quantum mechanics. Of course, this requires a lecturer who fully supports the approach taken in the introduction and is motivated to use the materials himself.

In all research rounds in which we tried out the introductory sessions, we used four lectures of two hours each. Each year we focused on a particular part of the introduction, which often resulted in skipping other parts. Therefore, not all parts have been tried out in one single research round. This would simply require more time. When planning a course that uses the materials designed, this should be taken into account.

10.2 Didactical structure of the introduction

To explicate the outcome of this research we use the concept of a didactical structure, explained in the Introduction (Chapter 1). The didactical structure defines the characteristics of the teaching-learning sequence, in particular:

- The physics content (knowledge) of each step;
- The motive for students to study certain knowledge content (this stems from the problem posing approach).

By motive we mean the logic that leads from one step to the next from the student’s perspective.

In this section we formulate the didactical structure of the introduction. We consider the introduction itself suitable both for physics and chemistry. However, teaching activities after the introduction should address the specific needs for physics and chemistry students. In Section 10.3 we explain how the introduction fits in a larger scheme that includes quantum theory and focuses on either quantum chemistry, or quantum physics.

The didactical structure for the introduction is graphically represented in Figure 10.1. This structure is explained in detail in Chapter 6. We here make
10.2. Didactical structure of the introduction

some concluding remarks. The didactical structure described here might be seen as the design of the next research round. Most parts have been tested, some have not. This is indicated in the figure. For those parts that have not yet been tested, we can give a motivation for their design and their position in the didactical structure.

What is missing from the structure in Figure 10.1 is the motivation for students to look at the photoelectric effect and the hydrogen spectrum in the first place. The orientation that can be given is that through interaction of matter and radiation (e.g. visible light) we can learn much about the properties of substances. For chemistry students this might be illustrated through examples where spectroscopy is used. Physics students might be more motivated by the use of spectra of for instance stars to determine their composition. But also photosynthesis and vision are contexts in which matter and radiation interact, that might be used as motivation. A reason for the teacher to particularly discuss the photoelectric effect and the hydrogen spectrum is that they are often treated in high school physics. By reviewing these, we can connect to students prior knowledge and also give them the opportunity to express their conceptions.

Based on this research, as well as on the literature review (Chapter 2) this design has the following prerequisites:

- Students need to understand that oscillating charges (e.g. electrons) emit radiation and thus energy. This needs to be addressed before we can expect students to meaningfully engage in a search for an alternative mechanism to explain the structure of the atom (hydrogen in particular). This prerequisite was identified as a learning deficiency by Taber (2005), using the topology of learning impediments (Chapter 2, page 35).

- A large part of the design depends on students’ understanding of wave phenomena in general and interference phenomena in particular. Vokos, Shaffer, Ambrose, and McDermott (2000) identified various learning difficulties among students in first-year, second-year, and third-year physics courses. This indicates that students do not find this content easy to comprehend. In our research we have also found that students need to be carefully guided when working on the double slit assignment. Although our students have worked on wave phenomena and interference during high school, these results nevertheless show that it is perhaps wise to schedule a preparatory course on classical wave phenomena prior to discussing quantum mechanics. In particular it would be useful if students already have learned what is now covered in the water domain of the double slit assignment.

- Various concepts from classical mechanics are needed to work on the designed materials. Students should in particular understand: the relation between potential energy and kinetic energy, how to calculate the potential energy of a charge in an electric field (for the photoelectric effect and interference pattern buildup assignments), and how a magnetic
Chapter 10. Conclusions

Quantization

Matter waves

Probability

Towards a new theory

Photoelectric effect

light comes in packets with energy

light has a particle-like property

If light has particle-like properties,
might electrons conversely have
wave-like properties? A
characteristic of waves is that
they interfere. First study double-
slit interference for waves.

Double-slit experiment

Compare interference of water waves,
light, electrons:
- we need a wave, to describe electrons;
- addition of waves gives interference;
- $|\Psi|$ (amplitude) determines number of
electrons detected in region.

Figure 10.1: Didactical structure of the introduction. The rectangles show the
physics content (or knowledge) which is connected by motives shown in gray,
rounded rectangles. The dashed rectangles are activities that have not been
tested in this research. The numbers 1–6 refer to notes in the main text on
page 220. Note that there are points from which multiple arrows emerge; there
is thus no unique sequence in which the structure should be passed through.
As a rule, the structure is meant to be traversed top down, finishing a topic
(indicated by the large rectangles) first.
10.2. Didactical structure of the introduction

- **Quantization**
- **Matter waves**
- **Probability**
- **Towards a new theory**

**Photoelectric effect**

- Light comes in packets with energy
- Light has a particle-like property

If light has particle-like properties, might electrons conversely have wave-like properties? A characteristic of waves is that they interfere. First study double-slit interference for waves.

**Double-slit experiment**

- We need a wave, \( \psi \), to describe electrons;
- Addition of waves gives interference, describing pattern;
- \( |\psi|^2 \) determines number of electrons detected in region.

What is interfering with what? Are electrons interfering with each other?

**Interference pattern build-up**

- Fire electrons one by one:
  - Interference pattern still visible, build-up appears to be random;
  - Electrons do not interfere with each other;
  - \( |\psi|^2 \) describes single electron, not two;
  - \( |\psi|^2 \) relates to probability.

What is the correspondence between wave and particle properties (i.e. momentum \( p \) versus position \( x \) and energy \( E \) versus \( \hbar \omega \))? **De Broglie postulate**

- Electron diffraction on graphite:
  - Newton rings;
  - \( \hbar \) postulate for electron.

How can we describe electrons statistically? **Classical probabilities**

- First try to describe a classical system with probabilities:
  - For continuous variable \( x \) we need probability density \( \rho(x) \):
    \[
    \rho(x) = \lim_{h \to 0} \frac{I(x, x + h)_x / I_{tot}}{h} 
    \]
  - \( P(a, b) = \int_a^b \rho(x) \, dx \)

What is the relation between \( |\psi|^2 \) and \( \rho \)?

**An equation for \( \psi \)**

- Search for an equation analogues to Newton's second law, satisfying:
  1) solution for free electron (assumption from double-slit experiment);
  2) superposition principle (linearity);
  3) \( \psi \) with wave number \( k \) describes electron with momentum \( p = \hbar k \);
  4) \( \psi \) with angular frequency \( \omega \) describes electron with total energy \( E = \hbar \omega \).

This should lead to the Schrödinger equation.

We know how to describe electrons with \( \psi \), but how do we find \( \psi \) for a given situation? **Operators**

- For each observable we need an operator. Idea of eigenvalues as possible measurement outcomes.

**Expectation values**

- A superposition has multiple possible measurement outcomes, each with a certain probability; expectation value

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Figure 10.1 (continued): Didactical structure of the introduction.
dipole (e.g. a compass needle) behaves in a homogeneous and inhomogeneous magnetic field. Again, these are topics treated in high school, but our experience shows that they need at least to be brushed up.

- Finally, as Chapter 5 has shown, some mathematical proficiency is required to follow more advanced course like Quantum Physics and Quantum Chemistry. The use of (more advanced) mathematics in this design is actually limited to the use of complex numbers, goniometric functions and relations, conceptual use of integrals, and second order, linear differential equations.

We motivate some of the choices made in the didactical structure. The numbers below refer to the numbers given in Figure 10.1:

1. Motivation following from the hydrogen spectrum

The outcome of the assignment on the hydrogen spectrum is the idea of energy levels and transitions between energy levels. An energy level corresponds to a stationary electron state. As we cannot understand from classical physics how an electron can stationary “revolve” around a nucleus, this provides a motivation for looking at another theoretical framework. Thus, here the question can be posed how we can account for the stationary electron states. When a quantum theory is formulated, a first good test is to see whether the states follow from this formalism. Furthermore, it is interesting to let students compare the predicted spectral lines to the experimentally found spectrum of hydrogen.

2. De Broglie postulate

The sequence seems to stop here. However, the outcomes of considering electron diffraction on graphite are needed to find the Schrödinger equation.

3. Which-way experiment

A natural question to ask after studying the single electron build-up of the double-slit interference pattern, is which slit the electron has passed through. This is commonly known as a “which-way experiment” and the possibility to determine this “path information”. In such an experiment, the interference pattern would disappear in all cases where the path is known. This result is interesting because it gives the opportunity to reinterpret the meaning of the superposition of two circular waves that was used to describe the interference pattern. Because an electron passing through one of the two slits should be described by one circular wave, it is understandable that the interference pattern disappears. Whereas, to acquire an interference pattern, we need (at least) two circular waves. Hence, we do not know which slit the electron has passed through. A superposition may then also be seen as a combination of possible measurement outcomes.
4. Stern-Gerlach experiment

The discussion of the which-way experiment has revealed that a superposition reflects the fact that we do not know the outcome of a measurement, only that it will correspond to a limited set of possible outcomes. The Stern-Gerlach experiment is interesting to study this further. The step from “which-way experiment” to “Stern-Gerlach experiment” might not be problem posing enough, as it is not clear why one would look at a (seemingly) completely different experiment. However, the physics underlying this experiment is not very difficult, and should be known to first year physics and chemistry students.

5. Operators

This assignment is meant to let students realize that in quantum mechanics observables require operators, instead of “normal” numbers. In finding the Schrödinger equation they have in fact already made use of this, by finding a differential operator that is able to “extract” the momentum from the wave function of a free electron. It is, however, not to be expected that students are able to formulate this finding in terms of operators, eigenvalues and corresponding eigenstates. It is expected that this assignment will give enough motivation to introduce this to students.

6. Expectation values

By now it should be clear that there are states that correspond to different measurement outcomes (i.e. superpositions of eigenstates of the observable under consideration). As each of these outcomes occurs with a certain frequency, we may wonder what their average is. Because the wave function is considered to describe these possible outcomes, we should be able to determine the expectation value of the observable of interest. This is the subject of this assignment. Together with the previous assignment, it is expected that students appreciate the formulation of a logical framework that is able to answer all such questions as posed here.

10.3 After the introduction

The main aim of the courses that were the subject in this research is for students to acquire a coherent conception of the wave function that enables them to understand basic quantum phenomena. In case of Quantum Chemistry this implies understanding of chemical bonding in terms of quantum mechanics. Furthermore, students should be able to work with the formalism of quantum mechanics. The introductory sessions that were designed in this research only form the first part of these courses. These sessions attempted to guide students to quantum theory starting from experiments. In this section we
sketch a more complete design of these two courses, including such an introduction. This design is based on our own experiences, available research literature (Chapter 2), and the theoretical framework (Chapter 3). The design is split up in three parts: the introduction, axioms of quantum theory, and a part that applies these axioms. This third part will be different for Quantum Physics and Quantum Chemistry. Figure 10.2 shows a graphic representation of the design.

The introduction In Section 10.1.2 we have defined the introduction as a sequence that guides students to quantum theory by formulating hypotheses that explain meaningful experiments. Together, these hypotheses form a basis for the formulation of the logical system we call quantum theory. The introduction uses a guided discovery approach, a choice motivated in Chapter 3: meaningful reception learning can only occur if there is already a cognitive structure present containing higher-order abstractions to which newly introduced concepts can be linked (as described by assimilation theory). If this body of higher-order abstractions is lacking, then there is nothing, or not enough, to link to. (p. 52)

Because in the case of learning quantum mechanics it is to be expected that these higher-order abstractions are initially lacking, (guided) discovery learning seems a more appropriate method of instruction in this phase. The outcome of this introductory phase may thus be defined in yet other terms: it should result in an appropriate cognitive structure of higher-order abstractions. We conclude that these higher-order abstractions should include a conception of the wave function (with an interpretation that relates the concept to experimental outcomes), the Schrödinger equation as new dynamical law for the wave function, and the notion that operators are needed instead of classical dynamical variables to define observables. However, with these higher-order abstractions, students do not yet know the formalism of quantum mechanics. This is the topic of the following part in the sequence we describe here: axioms of quantum theory.

After the wave function has been hypothesized, students wonder what this term actually means. In this sense, the interpretation of the wave function already starts to play a role. Discussion about the interpretation of the wave function is actually part of phase 3 of the analogical reasoning (Chapter 7). In Chapter 3 we motivated that the interpretation of quantum mechanics should be addressed when introducing quantum mechanics. The idea that the hypothesized term wave function needs an interpretation to connect it to measurement outcomes emerges rather naturally in this design. We here see what role the interpretation plays in meaningful learning: students try to connect the wave function to what they already know about electrons and classical particles. This opportunity for meaningful learning should be seized. At this point it may be useful to make a distinction between different kinds of interpretations: a minimalistic, instrumentalist (epistemological) interpretation or
Figure 10.2: After the introduction, it is suggested that a course proceeds with a unit on the axioms of quantum theory based on the findings of the introduction. After this, a course diverges for Quantum Physics and Quantum Chemistry, as the focus for both is different.
an ontological interpretation. In the former the wave function is a theoretical concept which is given an interpretation in order to relate the formalism to experimental outcomes. In addition, an ontological interpretation relates the formalism to what is. Similarly, the atom has long been considered merely a useful theoretical concept with empirical consequences, not something that actually exists. A definite answer, however, need not be given; it might be even impossible to give a definite answer. We return to the interpretation of quantum mechanics in the next part: axioms of quantum theory.

**Axioms of quantum theory** From the introduction it should become clear that a new logical system, or formalism, is needed. The descriptive rules that were found should follow from this logical system. Thus, the next part in this sequence, axioms of quantum theory, aims at formulating a theory based on the findings of the introduction. It is our hypothesis that the introduction gives a good basis for students to appreciate the possible axiomatic choices that can be made for such a logical system. However, should we let students discover the axioms of quantum mechanics themselves? In other words, should we proceed with a guided discovery approach?

The difficulty is that a theory does not logically follow from a limited set of experiments. Instead it is an inductive process, where something “magically” happens. For example, Heisenberg (1925) shows this clearly in case of the development of matrix mechanics. He starts out with classical mechanics, and based on an empirical rule (the Rydberg–Ritz rule for the addition of spectral frequencies), he realizes that the dynamical variables from classical mechanics need to be adjusted such that they in fact become matrices. In the introduction something similar happens in formulating the Schrödinger equation (see Chapter 6).

Because of the inductive steps involved, there is some freedom in choosing the axioms for a new theory. Because we want students to end up with a specific set of axioms (those of quantum mechanics), we consider it impractical to let students choose there own axioms. This is an argument against the use of (mainly) guided discovery learning at this stage. Moreover, when higher order abstractions are available (as we expect them to be after the introduction), Ausubel, Novak, and Hanesian (1978) argue that (verbal) reception learning is more effective than (guided) discovery learning. This does not necessarily mean that lectures only are sufficient, or that the way in which they are given is unimportant:

> It is true that much potentially meaningful knowledge taught by verbal exposition results in rote learned verbalisms. This rote

4A translation of the 1925 Heisenberg paper can be found in Van der Waerden (1967), which is a collection of papers that have played a role in the development of matrix mechanics. Aitchison, MacManus, and Snyder (2004) “dissects” Heisenberg’s paper and actually calls it “magical” for the reason that something happens in this paper that is difficult to follow. This magic is what we call the inductive step that Heisenberg made to reach matrix mechanics.
outcome, however, is not inherent in the expository method, but rather in such abuses of this method as fail to satisfy the criteria of meaningful learning. (Ausubel et al. 1978, p. 28)

For the remainder of this sequence, we thus choose a more traditional approach with lectures and problem solving sessions, provided that they are well organized and given such that they stimulate meaningful learning.

The interpretation of the wave function has already been discussed in the introduction. When discussing what the logical system is of quantum mechanics, we can return to the question what the interpretation is of this formalism. As we now have opted for (verbal) reception learning to introduce students to the “definite” set of axioms, it is here possible to present to students different views on the interpretation of quantum mechanics. As became clear in the study of Baily and Finkelstein (2010), it matters whether or not the interpretation of quantum mechanics is discussed (refer to Chapter 2). Because students have already given this some thought, we can relate the canonical interpretations to the ideas they have formed themselves.

Quantum Chemistry and Quantum Physics  After a theory is “in place”, a course will proceed differently for Quantum Chemistry and Quantum Physics, because of the different goals these courses have. Quantum Chemistry is directed more towards explaining chemical bonding using quantum mechanics. Quantum Physics focuses more on understanding the theory on a higher level. By which we mean for instance: formulating the Schrödinger equation as an eigenvalue problem, using commutators, understanding what they have to do with symmetries, time development of the wave function, and so on. A promising approach to explore this (i.e. mainly the formalism of quantum mechanics), is the use of Quantum Interactive Learning Tutorials (QuILTs) as proposed by Singh (2008a) and described in Chapter 2 (page 31).

The main criterion of meaningful learning is that “symbolically expressed ideas are related in a nonarbitrary and substantive (nonverbatim) fashion to what the learner already knows” (Ausubel et al. 1978, p.41). This means that “ideas are related to some specifically relevant existing aspect of the learner’s cognitive structure” (p. 41, original emphasis). Also, the learner must have the intention to learn meaningfully, instead of memorizing the content arbitrarily and verbatim. (Ausubel et al. 1978) formulate the following criteria for reception learning that are likely to result in meaningful learning:

1. The central unifying ideas of a discipline are learned before more peripheral concepts and information are introduced.
2. The limiting conditions of general developmental readiness are observed.
3. Precise and accurate definition is stressed, and emphasis is placed on delineating similarities and differences between related concepts.
4. Learners are required to reformulate new propositions in their own words.

(p. 124, original emphasis)

These should be seen as very general criteria. It still remains a question how they are implemented in a lecture (verbal reception learning). Many techniques exist that stimulate students’ cognitive activity during lectures (e.g. Mazur’s peer instruction).
The literature review for Quantum Chemistry revealed the role of models in explaining bonding. There are indications that the role of modeling, as well as the relation between multiple models, needs to be addressed. One such approach is described by Nahum, Mamlok-Naaman, Holstein, and Kronik (2008), described in Chapter 2, page 40.

In the outline sketched in this section, the interaction between theory and experiment (back and forth) plays an important role. Students need to become confident that the theory works. In the introduction we started with such an activity, mostly working from experiment towards theory (inductively), but in the case of the Schrödinger equation assignment also from theory to experiment (deductively). However, this interaction between theory and experiment should not stop when a theory is available. On the contrary: the strength of a theory shows itself in being able to describe experimental outcomes accurately and, in addition, in its predictive power. This is certainly the case for quantum mechanics. Furthermore, there are many applications in which quantum mechanics shows its value (e.g. quantum computers and quantum information).

For Quantum Chemistry the application of quantum theory is incorporated in the design: quantum mechanics is applied to explain chemical bonding. However, for Quantum Physics, more attention may be given to contexts in which quantum mechanics has shown its value.

10.4 Reflection on the research methodology

In this section we reflect on the developmental research approach used in this research. In developmental research there is a close cooperation with practitioners in the field. The interaction between researchers and practitioners influences the direction of the research. This has become apparent in the current research, where the remedial activities, described in Chapter 5, as well as the introductory sessions, described in Chapter 6, were developed in close cooperation with the lecturer of the Quantum Chemistry course. These developments followed upon choices made after the zeroth, inventory research round. As explained (Chapter 1, Figure 1.1), the idea of developmental research is that an intervention is designed in and motivated by a thought experiment. In this view, the intervention is based mainly on findings of earlier research rounds and relevant research literature. It seems as if the steps taken in this process entirely follow from the observations in earlier research rounds. However, in reality an intervention is the result of a delicate interaction between all those involved in the research: researchers, developers, teachers, lecturers, etc. In this design process tastes, opinions, and prior experiences, among other factors, play a role as well. For instance, in this research it was found that in the Quantum Physics course, not all students were actively engaged during the lecture. It was observed that students were busy with other things (e.g. reading newspapers, chatting, doing homework for other courses). Therefore, it was advised to make the lectures more interactive, for instance by using a
Studio Course setup. However, the lecturer was opposed to the idea of the Studio Course. In this particular example, we see that an important factor is the lecturer as a teacher with a certain teaching style. This will influence the design process and, stemming from the character of developmental research, also the possible theoretical outcomes.

To be able to better understand this process, it might be clarifying to discern the different roles in a developmental research. Van den Akker (1999) mentions two different roles (i.e. researcher and developer) and two types of developmental research approaches. In type I, the roles of designer and researcher coincide, in type II, the researchers are not involved in the design and thus the roles are separated. Based on the above discussion, this classification in designer and researcher roles seems too primitive. What about those responsible for the course, the lecturer, teachers, and teaching assistants? Is the researcher also involved in teaching the designed materials? It thus seems more accurate to discern three roles: that of the researcher, the developer, and, additionally, the teacher. A priori, these roles need not be limited to different persons. For instance, in earlier rounds of this research the lecturer of the Quantum Chemistry course also took on the role of developer. The researcher took on both the role of developer and researcher. In later research rounds, after a new lecturer took over, we have failed to let the lecturer take part in the developmental process, nor in the implementation (i.e. teaching) of the interventions. As a result, during the introductory sessions the researcher effectively took on the role of a teacher as well as that of a researcher, after having developed most of the materials. Earlier in this chapter, we have concluded that this setup has not been beneficiary for the research process, nor for the educational outcomes.

If we think of the aims of developmental research (prototype development and theory construction), then it seems advisable that both teacher and researcher take part in the development process. This way experience and expertise of both teacher and researcher can contribute to the developed teaching materials. This will more likely result in a successful intervention that can be translated from developed prototype to an actual implementation in an educational setting.

**Recommendation 7.** In developmental research both teacher and researcher should take on the role of developer. The roles of teaching and researching should be separated.

Another perspective is to view those involved in the developmental research as stakeholders. Each of the stakeholders might have another aim, stemming from the role one has. These aims inevitably influence the developmental process. Also, stakeholders are important for the research, as they can help support the project and secure the dissemination of its results. It is therefore important to know who the stakeholders are and to make their aims explicit. In this research, the researcher and the respective lecturers were the main stakeholders, as they were those who were most involved in the developmental
research. However, other stakeholders, who could have been more involved in
the research process, are the teachers of the tutorial sessions and the coordi-
nators of the two bachelor programs involved (physics and chemistry). In
our setting, it has not always been possible to get the teachers of the prob-
lem solving sessions involved in the developmental process, as they were most
often PhD students, with limited time. The bachelor coordinators have been
informed of, but have not taken part in the development activities of this re-
search. However, because developmental research somehow tries to contribute
to the teaching practice, it would seem natural to get them more strongly
involved. In this research it might have prevented the change of lecturer, or
ensured that a new lecturer is sought who is motivated to continue the research
project.

**Recommendation 8.** For a developmental research it is advisable to form
a development team consisting of the relevant stakeholders. The aims and
expectations of the stakeholders need to be made explicit. Most important,
this development team needs a clear aim that can guide the development and
research activities.

This recommendation is particularly relevant if an aim of the developmental
research is to contribute to the local teaching practice. It is to be expected
that as a result of the composition of the development team, more authentic
problems will be addressed and that results will more likely be disseminated
in the local teaching practice.

Forming a development team is in itself no guarantee for success. There
are at least two important issues to address. First of all, some familiarity with
developmental research is beneficial. In this research, the lecturers are first
and foremost researchers (in physics, or chemistry) who also have a lecturing
task. One might thus wonder to what extent they are able to take on a role
of developer and if they are familiar with educational research. After all,
educational research methods are very different from the methods used in the
sciences. But also the researcher in this project, being a PhD student, was at
the outset not an expert educational researcher. Therefore, in the development
team it is important that an experienced researcher is participating who can
also take on the role of coach, or project leader.

Secondly, the stakeholders should be involved enough to see the develop-
ment process as a combined effort, where it is clear what is to be expected from
those involved. This might even be seen as a defining property of developmen-
tal research. Otherwise, why would we burden ourselves with a setting in which
a prototype is developed in a “real life context”? Stated differently, the teach-
ers (or lecturers), designers, and researchers should really be stakeholders, for
whom something is at stake. This combined effort of the stakeholders might
for instance be expressed by co-authoring publications in which the results of
the research are reported.

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6At the Faculty of Science, of the University of Amsterdam, each bachelor has its own
coordinator, among other things responsible for the bachelor curriculum.
Recommendation 9. It is advisable that the members of the development team (Recommendation 8) are familiar with educational research and, if not, are supported by an experienced science education researcher. Furthermore, the members should feel committed to the development effort.

The question of who the stakeholders of the research are is also a question of who actually “ordered” the research. What is the problem that is to be addressed by doing a developmental research (i.e. the aim in Recommendation 8)? And, importantly, is there a need amongst the stakeholders to change the educational practice? These all are questions that need to be addressed before starting a developmental research. Developmental research in general, and the development team in particular, requires an initial research question that expresses the need for the developmental research and is able to give focus to the developmental process. As the research progresses, this initial research question evolves in more specific research questions.

Despite these complications, for the possible impact it might have in a teaching environment we consider developmental research the best method available.

We finally want to return to the choice to follow two courses, as motivated in Chapter 1. There we gave the following three reasons to select both courses for this research: 1) to be able to compare the effect of interventions for students with different backgrounds, 2) to make generalizations possible over different student populations, and 3) spread the risk in case the cooperation with a course ends. To start with the latter motivation, with hindsight this has proved to be a relevant motivation, as the cooperation with the QP course ended upon a change of lecturer. For the QC course this change of lecturer has had a significant effect on the choices made in the research, however, fortunately the cooperation did not end. It has not been possible to generalize over different student populations, as the interventions differed too greatly. Finally, regarding the first argument, it was sometimes useful to try out an intervention in the QP first, as these students had better mathematical skill on average. This enabled us to see if an intervention could in principle be successful. However, doing this research in these two, rather different courses also has a drawback. Due to the differences between course content and student population, the needs of the lectures differed resulting in different development questions. Thus, it was not possible to create one development team for both courses. Instead two development teams were formed, one for each course. This all results in more resources required from the researchers. It would perhaps have been more practical to select three similar courses at different universities. Then the above three reasons still hold, but any development effort is more likely to be used in all courses.
Chapter 10. Conclusions

10.5 Future research and development

There are several possible directions that are interesting for future research and development of quantum mechanics education. We consider these possibilities from lower level (the course) to a higher level (the curriculum).

The aim of the introduction sketched in Section 10.2 is to let students formulate enough hypotheses such that most important ingredients of a quantum theory are available. In organizing the activities, a problem posing approach is followed. However, not all parts of the sequence as described have been tried out in full, and not all interventions have been tried out together in one round. It would be interesting to see whether this sequence has the desired effect, and if phase 3 of the analogical reasoning can be completed successfully. In particular, can we let students identify a superposition of two wave functions, each describing a possible state, as a wave function describing one electron?

At a course level, it would be a challenge to integrate the vast amount of research literature available on this subject and use it to design a complete course for introductory quantum mechanics. A rough outline for such a course has been given in Section 10.3, but many details need to be filled in. This again may be done using a developmental research approach. However, it might be advisable to involve other universities as well, as the context at Dutch universities is comparable. Developing a course, or even curriculum, together with other faculties gives the opportunity to share resources and exchange ideas and experiences. If well organized, this may contribute to the quality of the design. We will return to this in the next chapter.

At a curriculum level, it is interesting as well as relevant for the teaching practice to know how students’ understanding of quantum mechanics develops throughout the bachelor curriculum. An issue that should be addressed is the relation between procedural fluency and conceptual understanding, a distinction made in Chapter 8. Introductory courses often focus on procedural fluency: learning how to do quantum mechanics first. The idea is that understanding will come later. Oddly enough, such an approach might be seen as pure discovery learning, as students are expected to discover for themselves what quantum mechanics means after they have learned how it works. We have given arguments in Chapter 3 against the effectiveness of this type of learning. Therefore, this assertion should be tested by following students as they progress throughout their bachelor career. Research literature is available on the development of students during their bachelor career. Research literature is available on the development of students during their bachelor career. Some of which are longitudinal. There are also tests available that can be used for this aim. To name a few, for quantum mechanics there is the QMVI (Cataloglu & Robnett, 2002) and the test developed by Singh (2008b). For quantum chemistry, the test developed by Birk and Kurtz (1999) is useful in a cross-sectional, or longitudinal study. These, as well as other tests have been discussed in Chapter 2.

Another topic for research touched upon, is the interpretation of quantum mechanics. In Section 10.3, we have already indicated what role the interpreta-
tion can play in introducing quantum mechanics. In Chapter 3 we have argued that the discussion of the interpretation of quantum mechanics can possibly play a role in understanding quantum mechanics. The study by Wuttiprom, Sharma, Johnston, Chitaree, and Soankwan (2009) suggested that understanding of the interpretation is not necessary when performing more procedural type of operations (Chapter 2, p. 27). However, it is not clear if and how a discussion of the interpretation might result in better conceptual understanding. In any case it remains questionable whether a focus on procedures (working with the formalism), will eventually lead to a deeper understanding of quantum mechanics. In this research we have concluded that a focus on procedural fluency does not necessarily result in conceptual understanding (Conclusion 7). Therefore, future research might try to improve students’ conceptual understanding and study how conceptual understanding develops during multiple years in which follow-up courses on quantum mechanics are given. In such a research, the role that the interpretation plays in (conceptual) understanding should be considered.

A question that is natural to ask, but which has not been answered in this research, is whether a guided discovery approach, as followed here, is more successful than a more traditional approach (i.e. based on verbal reception learning) when introducing quantum mechanics. Although we have given arguments why we think a guided discovery approach will be more effective, we have not been able to quantitatively compare the learning outcomes of these two approaches. This is only possible when two different, functioning approaches are available. In a sense, an aim of this research has been to develop teaching materials following a guided discovery approach. However, we have concluded that our design is not yet finished, nor complete. There are still parts that do not function as intended. Furthermore, a comparative research would require a different research methodology, as well as a larger number of students. The latter cannot be realized at our university. It would thus also require cooperation with other universities.

Recommendation 10. It is recommended to conduct a developmental research on higher education in cooperation with other universities (that have a similar context) to share experiences, expertise, and resources. Furthermore, especially for studies with a small number of students, such a cooperation has the advantage that the outcomes are better generalizable.

References


