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Delayed Capillary Breakup of Falling Viscous Jets

A. Javadi,1,2 J. Eggers,3 D. Bonn,2,4 M. Habibi,1 and N. M. Ribe5

1Institute for Advanced Studies in Basic Sciences, Zanjan 45195-1159, Iran
2Laboratoire de Physique Statistique, École Normale Supérieure, 24 rue Lhomond, 75231 Paris Cedex 05, France
3School of Mathematics, University of Bristol, University Walk, Bristol BS8 1TW, United Kingdom
4Institute of Physics, University of Amsterdam, Science Park 904, 1098 XH Amsterdam, the Netherlands
5Lab FAST, UPMC, Université Paris-Sud, CNRS, Bâtiment 502, Campus Université, 91405 Orsay, France

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Thin jets of viscous fluid like honey falling from capillary nozzles can attain lengths exceeding 10 m before breaking up into droplets via the Rayleigh-Plateau (surface tension) instability. Using a combination of laboratory experiments and WKB analysis of the growth of shape perturbations on a jet being stretched by gravity, we determine how the jet’s intact length $l_b$ depends on the flow rate $Q$, the viscosity $\eta$, and the surface tension coefficient $\gamma$. In the asymptotic limit of a high-viscosity jet, $l_b \sim (g Q^2 \eta^4 / \gamma^4)^{1/3}$, where $g$ is the gravitational acceleration. The agreement between theory and experiment is good, except for very long jets.

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In the literature, the dependence of the breakup length on the flow rate and fluid properties such as the surface tension has been extensively studied for high-speed jets in quiescent or coflowing fluids [6,7]. By contrast, viscous jets falling under gravity have been the subject of only a few experimental [8,9] and theoretical [4,9–11] studies, none of which arrived at a prediction for the breakup length as a function of the flow rate and the fluid properties.

Experiments.—We used silicon oils with densities $\rho = 963–974 \text{ kg m}^{-3}$, surface tension coefficient $\gamma = 0.021 \text{ N m}^{-1}$, and viscosities $\nu = 50–27800 \text{ cS}$. A thin vertical jet was generated by ejecting the oil downward through a nozzle of diameter $2r_0 = 2–4 \text{ mm}$ at a constant flow rate with a range $Q = 0.0036–1.4 \text{ ml/s}$, using either a syringe pump controlled by a stepper motor or an open reservoir with an adjustable valve at the bottom. The reservoir was sufficiently large (14 cm × 14 cm wide and 20 cm deep) that the flow rate was constant to within ±2% during all of the experiments. To eliminate the influence of air drag on longer jets (breakup length $l_b > 2.5$ m), we enclosed the nozzle and the jet in a cylindrical vacuum chamber with inner diameter 19 cm and length $\approx 7.5$ m. The bottom portion (2 m) of the cylinder was transparent to permit observation. A partial vacuum was created inside the cylinder using a Siemens rotary vacuum motor, allowing even the longest jets ($l_b = 7.5$ m) to remain perfectly straight.

We observed three distinct regimes of behavior of the ejected fluid, including the periodic dripping regime at very low flow rates and the $J$ regime at high flow rates. At intermediate flow rates, however, we did not observe the dripping faucet regime but rather an oscillatory “pulsating” regime. Here, the jet had a reasonably steady shape, especially near the nozzle, and broke up at a well-defined distance that greatly exceeded the nozzle diameter.
However, small periodic oscillations of the jet’s shape about the mean diameter occurred, corresponding to the absolute instability identified by Ref. [9].

We measured the breakup length \( l_b \) for a total of 87 experiments, including 67 in the jetting regime and 20 in the pulsating regime. In some experiments, we detected the point of breakup by moving a rapid camera step by step along the jet as it thins. In most cases, however, we first located the breakup point approximately by eye with the help of a stroboscope and then used a rapid camera at this location to make a more precise measurement. Figure 2 shows \( l_b \) as a function of flow rate for three different viscosities. As one expects intuitively, the breakup length is an increasing function of both the flow rate and the viscosity.

**Dimensional analysis.**—The first step toward a more quantitative understanding is a dimensional analysis. The breakup length \( l_b \) depends on the viscosity \( \nu \), the surface tension coefficient \( \gamma \), the density \( \rho \), the flow rate \( Q \), the gravitational acceleration \( g \), and the nozzle radius \( r_0 \). Buckingham’s \( \Pi \) theorem [12] then implies \( \Pi_b = f(\Pi_\eta, \Pi_\gamma, \Pi_\nu) \), where

\[
\begin{align*}
\Pi_b &= l_b \left( \frac{g}{Q^2} \right)^{1/5}, \\
\Pi_\eta &= \eta \gamma (Qg^2)^{-1/5}, \\
\Pi_\gamma &= \frac{\gamma}{\rho} (Q^4 g^3)^{-1/5}, \\
\Pi_\nu &= r_0 \left( \frac{g}{Q^2} \right)^{1/5},
\end{align*}
\]

(1)

where \( \eta = \rho \nu \).

**WKB analysis.**—Because the structure of the jet varies slowly in the axial direction, the growth of perturbations can be treated using a WKB-type approach in which disturbances locally have the form of plane waves [11]. The starting point is the equations governing plug flow in a slender vertical jet of viscous fluid [13]:

\[
\begin{align*}
\partial_t A + (Av)' &= 0, \quad \text{(2a)} \\
\rho A (\partial_t v + vv') &= 3 \eta (Av)' + \rho g A - \gamma A \kappa', \quad \text{(2b)} \\
\kappa &= \frac{1}{r(1 + r^2)^{1/2}} - \frac{r''}{(1 + r^2)^{3/2}}, \quad \text{(2c)}
\end{align*}
\]

where \( r(z, t) \) is the jet’s radius, \( A = \pi r^2 \), \( v(z, t) \) is the axial (vertical) velocity, and \( \kappa \) is the mean curvature of the jet’s outer surface. The primes denote differentiation with respect to the distance \( z \) beneath the nozzle. Equations (2a) and (2b) express conservation of mass and momentum, respectively. The three terms on the right side of (2b) represent the viscous force that resists stretching, the weight of the fluid, and the surface tension force, respectively, all per unit length of the jet.

In the absence of perturbations, the steady flow of the jet is governed by (2) with \( \partial_t = 0 \). A general analytical solution of these equations was obtained by Ref. [3] in the limit of no surface tension (\( \gamma = 0 \)). The corresponding axial velocity \( v(z) \) is shown in Fig. 1 for two values of the normalized ejection speed \( \hat{v}_0 = v_0 / (3g\nu)^{1/3} \). A clear distinction is evident between the jet’s upper part, where the weight of the fluid is balanced primarily by the viscous force that resists stretching, and its lower part, where the weight is balanced by inertia. The boundary between the two is the point where the viscous and inertial terms in (2b) are equal and occurs at a distance \( B(v^2 / g)^{1/3} \) from the nozzle, where \( B = B(\hat{v}_0) \equiv 5.0 \). Because \( l_b = 50-500(v^2 / g)^{1/3} \) in our experiments, breakup always occurs in the inertia-dominated part of the jet. The prefactor \( B \) drops to zero for \( \hat{v}_0 \equiv 1.219 \), meaning that the weight is then balanced primarily (> 50%) by inertia everywhere in the jet.
The analytical solution of Ref. [3] can also be used to estimate the magnitude of the neglected surface tension term in (2b) relative to inertia and the viscous force. For the parameter values of our experiments, it turns out that surface tension is negligible in the inertia-dominated part of the jet but not in the viscosity-dominated part. However, because the viscosity-dominated part of the jet is very short compared to the breakup length, we are justified in using the solution of Ref. [3] as our base state.

To model the fluctuating environment surrounding the jet, we introduce small perturbations with different initial wave numbers \( k_0 \) at different points along the jet, i.e., at different times \( t_0 \), since the fluid element in question exited the nozzle. Each of these perturbations will grow to a point the jet will break. We posit that the observed growth rate at the initial position \( z_b \) is \( \sigma(r(z_b), k_b) = 0 \), whence (3) implies that the optimal initial wave number is \( k_b = 1/r(z_b) \). We can therefore write the integrated growth rate as a function of \( z_0 \) and \( z_b \), viz.,

\[
s(z_0, r^{-1}(z_b), z_b) \equiv S(z_0, z_b). \tag{6}
\]

Next, differentiate the equation \( s(z_0, k_0) = s_{ct} \) with respect to \( k_0 \) to obtain

\[
\frac{\partial s}{\partial k_0} = \frac{\partial s}{\partial z_0} + \frac{\partial s}{\partial z_b} \frac{\partial z_b}{\partial k_0} = 0. \tag{7}
\]

However, the condition of minimal breakup length requires \( \partial z_b/\partial z_0 = 0 \), and so (5) implies \( \partial s/\partial z_0 = 0 \). This derivative can now be evaluated using the definition (4) for \( s \), noting that the dependence on \( z_0 \) enters only through the lower limit of integration. We thereby find that the optimal growth rate at the initial position \( z_0 \) is \( \sigma(r(z_0), k_0) \), whence (3) implies that the optimal initial wave number is \( k_0 = 1/r(z_0) \). We can therefore write the integrated growth rate as a function of \( z_0 \) and \( z_b \), alone, viz.,

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s(z_0, r^{-1}(z_b), z_b) \equiv S(z_0, z_b). \tag{6}
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Now, suppose that breakup occurs when the quantity \( s(z_0, k_0) = s_{ct} \) with respect to \( k_0 \) to obtain

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FIG. 3 (color online). Comparison of observed and theoretically predicted breakup lengths. The circles represent observed dimensionless breakup lengths $\Pi_b = l_b (g/Q^2)^{1/5}$ for 87 experiments, with their values of $\Pi_\eta = (\gamma/\rho)(Q^2 g)^{-1/5}$ indicated by colors. The solid line labeled “$\Pi_\nu$” represents the predicted breakup length for $\Pi_\nu = 1.0$, $\Pi_\phi = 1.5$, and $s_{cr} = 8.86$. The dotted and dashed lines are the same as the solid line, but for $\Pi_\nu = 0.25$ and 8.0, respectively. The dashed line at the upper right represents asymptotic expression (9) with $s_{cr} = 8.86$. The solid line labeled “$\Pi_\nu$” represents the predicted dimensionless distance $\Pi_s = z_0 (g/Q^2)^{1/5}$ at which the perturbation introduced is most dangerous. The vertical bar shows where $z_0$ moves from the inertia-dominated to the viscosity-dominated portion of the jet as $\Pi_\nu$ increases.

The resolution of the paradox pointed out in the introduction is now clear: Viscosity plays completely independent roles in the axial momentum balance of the steady basic state and in the growth of perturbations about that state. The analytical solution of Ref. [3] for the basic state shows that viscous forces are negligible in the inertia-dominated part of the jet's length in most of our experiments. However, this does not imply that the effect of viscosity can be neglected in the expression (3) for the Rayleigh-Plateau growth rate. That expression has two limits, depending on the Ohnesorge number $\Pi_{Oh} = \nu/\rho g/\gamma r$: a viscosity-independent limit $\sigma \sim (\gamma/\rho r)^{1/2}$ for $\Pi_{Oh} \ll 1$ and a (less familiar) viscosity-dominated limit $\sigma \sim \gamma/\eta r$ for $\Pi_{Oh} \gg 1$. To determine which limit is relevant for our experiments, we used the solution of Ref. [3] to calculate $\Pi_{Oh}(r = r_b) = \Pi_{Oh}^0$ for each experiment, where $r_b$ is the jet radius at the distance $z = z_b$ where the jet breaks up. We thereby find that $\Pi_{Oh}^0 \in [0.65, 2160]$ and that 66 of our 87 experiments have $\Pi_{Oh}^0 > 10$. It is therefore not surprising that the viscosity has a strong influence on the breakup lengths we observe.

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