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### Hurwitz numbers, moduli of curves, topological recursion, Givental's theory and their relations

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**Publication date**  
2014

[Link to publication](#)

#### **Citation for published version (APA):**

Spitz, L. (2014). *Hurwitz numbers, moduli of curves, topological recursion, Givental's theory and their relations*.

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