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APPENDIX A: ERROR CALCULATION FOR RMS–FLUX POINTS

Here we derive the formula used to estimate the standard deviation of the estimates of the rms used in the rms–flux analysis. We have calculated them in a more general manner to that of Gleissner et al. (2004). Each time series x_t, sampled at a rate Δt, is broken into segments of length $N_{\rm seg}$ and for each segment a periodogram is computed, and the mean count rate estimated, $\langle x^i \rangle$. The periodogram values are calculated at Fourier frequencies $f_j = j/N_{\rm seg}\Delta t$ (for $j = 1, 2, \ldots, N_{\rm seg}/2$), and the periodogram value at the jth frequency is y^i_j for segment i. The frequency resolution of such a periodogram is defined by the length of the segments: $\Delta f = 1/N_{\rm seg}\Delta t$.

The power spectrum at a given flux level is estimated by arranging the periodograms in order of the corresponding average count rate $\langle x^i \rangle$, and averaging them in groups of M periodograms. For each count rate bin we calculate the average periodogram:

$$\langle y_j \rangle = \frac{1}{M}\sum_{i=1}^{M} y^i_j. \quad \text{(A1)}$$

It is well known that, in the absence of sampling distortions, the periodogram ordinates are distributed about the true spectrum following a χ^2 distribution (see Jenkins & Watts 1969; Groth 1975; Priestley 1981; Leahy et al. 1983; van der Klis 1989; Press et al. 1992; Percival & Walden 1993; Timmer & König 1995; Bloomfield 2000; Chatfield 2003). Specifically, $y^i_j \sim P_j \chi^2_2/2$ where $P_j = P(f_j)$ is the true power density at frequency f_j and χ^2_2 is a random variable with a χ^2 distribution (with two d.o.f.).

The expectation and variance of the χ^2 random variable are $\mathrm{E}[\chi^2_2] = 2$ and $\mathrm{V}[\chi^2_2] = 4$. Using the rules for manipulating expectations and variances (e.g. Eadie et al. 1971, p. 23), we have $\mathrm{E}[y^i_j] = P_j$ and $\mathrm{V}[y^i_j] = P^2_j$, and so

$$\mathrm{E}[\langle y_j \rangle] = \frac{1}{M}\sum_{i=1}^{M} \mathrm{E}[y^i_j] = P_j, \quad \text{(A2)}$$

$$\mathrm{V}[\langle y_j \rangle] = \frac{1}{M^2}\sum_{i=1}^{M} \mathrm{V}[y^i_j] = \frac{P^2_j}{M}, \quad \text{(A3)}$$

where we have assumed that the process is stationary, such that $\mathrm{E}[y^i_j] = P_j$ is the same for all segments i (within the given count rate bin), and the individual y^i_j values are distributed independently for all i. The above results show that the averaged periodogram is an unbiased estimator of the true spectrum $P(f_j)$ and its fractional 'standard error' is $M^{-1/2}$.

An estimate of the variability power, S, contained within a specific frequency range, say f_J to $f_J + (W-1)\Delta f$, is given by summing the powers in the averaged periodogram $\langle y_j \rangle$ over the appropriate frequencies:

$$S = \sum_{j=J}^{J+W-1} \langle y_j \rangle \Delta f. \quad \text{(A4)}$$

Given the above information it is straightforward to calculate the expectation and variance of the power estimate S:

$$\mathrm{E}[S] = \Delta f \sum_j \mathrm{E}[\langle y_j \rangle] = \Delta f \sum_j P_j, \quad \text{(A5)}$$

$$\mathrm{V}[S] = \Delta f^2 \sum_j \mathrm{V}[\langle y_j \rangle] = \frac{\Delta f^2}{M} \sum_j P^2_j, \quad \text{(A6)}$$

where we have assumed that the periodogram estimates, and their averages $\bar{y}_j$, are independently distributed for all j, which is asymptotically true (as $N_{\rm seg} \to \infty$). These results show that S is an unbiased and consistent estimator of the integrated power spectrum over the required frequency range.

The estimated power S includes a contribution from the Poisson noise which is expected to contribute a flat spectrum with density $P_{\rm N}$. Over the frequency interval $W\Delta f$ it will therefore have a total power of $P_N W \Delta f$. We may then calculate the source (i.e. noise-subtracted) rms amplitude as

$$\hat{\sigma} = (S - P_N W \Delta f)^{1/2} \quad \text{(A7)}$$

(assuming that the noise level P_N is well determined so we can neglect any uncertainty in this). Applying the usual rules for transformation of variances we find that the standard deviation of the estimator is

$$\mathrm{sd}[\hat{\sigma}] = \frac{1}{2\hat{\sigma}} \left(\frac{\Delta f^2}{M} \sum_j P_j^2 \right)^{1/2}. \quad \text{(A8)}$$

Of course, we do not know the values of P_j, only their estimates $\bar{y}_j$, so we obtain a reasonable estimate of the uncertainty on the rms with the formula

$$\mathrm{sd}[\hat{\sigma}] \approx \frac{\Delta f}{2\hat{\sigma} M^{1/2}} \left(\sum_j \langle y_j^2 \rangle \right)^{1/2}. \quad \text{(A9)}$$

This is essentially just the 'propagation' (by adding in quadrature) of the standard deviations of each periodogram value, which are themselves $\sim y_j^i$, through the averaging over M segments and summation over W frequencies (each of which is independently distributed).

This is different from the prescription used by Gleissner et al. (2004). Their formula is only valid when the power is the same in all frequency bins, i.e. locally white noise. When this is not the case, e.g. when most of the power comes from a few dominant frequencies (strong power laws or peaked QPOs), both the power estimate and the variance on the power estimate are dominated by the frequencies that contribute the most power. One way to see this is to consider adding more frequencies that contribute negligible power to the sum. Using the Gleissner et al. (2004) formula the relative error on the total power is $(MW)^{-1/2}$ (from their equation 2), whereas the above equations predict this as $(\sum P_j^2/M)^{1/2}/\sum P_j$ (combining equations A5 and A6). The former decreases as W increases even if the additional frequencies contribute no power, whereas the latter formulation leaves the relative error unchanged, as expected if the additional data contribute nothing. In general the Gleissner et al. (2004) formula will underestimate the errors, i.e. produce confidence intervals that are smaller than the expected scatter in the rms data.

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