Fractional integration and cointegration in financial time series

Stakėnas, P.

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Summary

This thesis is devoted to the study of fractionally integrated and cointegrated time series models. Fractional time series bridge the gap between so-called weakly dependent, or $I(0)$, and integrated, or $I(1)$, time series, thus offering a richer model structure. Fractionally integrated series are considered in modelling many economic, financial, geologic and opinion poll time series (for a review refer to Baillie (1996), Gil-Alana and Hualde (2009)). In the thesis we also provide an empirical application for the U.S. interest rate data in each chapter illustrating theoretical results.

Chapter 2 of this thesis analyzes estimation and inference in univariate fractionally integrated time series via autoregressive approximation. The estimation is based on minimization of conditional sum of squared residuals, where residuals are formed approximating infinite autoregressive structure of a process by a finite-order autoregression and in line with Hualde and Robinson (2010) and Nielsen (2011) we call the resulting estimator a conditional-sum-of-squares (CSS) estimator. Assuming the order of autoregression k increases at a suitable rate along with the sample size T, the CSS estimator of the fractional parameter d is shown to be consistent at a rate $\sqrt{T/k}$ and asymptotically normally distributed with unit variance. Autoregressive coefficients of the linear process are shown also to be consistent at a rate $\sqrt{T/k}$.

Chapter 3 of this thesis considers the issue of testing for fractional cointegration. It has been documented that traditional tests for cointegration may have little power, if observed series is fractionally cointegrated, i.e. if the cointegration errors is a long memory rather than a weakly dependent process. Recent research studies provided a number of tests for cointegration, taking into account possible fractionality of cointegration errors. This chapter investigates most popular regression-based tests for fractional cointegration in terms of their size and power by means of finite sample Monte Carlo simulations and provides some practical guidelines for empirical researchers.

Chapter 4 studies estimation and inference of cointegration vector(s) in a fractionally cointegrated system employing a regression-based approach. In “strongly cointegrated” regressions (when the difference between integration order of observables and cointegration

errors exceeds 1/2) the OLS estimator of the cointegration vector does not have an optimal rate of convergence in a part of parameter space. We use the approach of Saikkonen (1991) appending the regression equation with leads and lags of filtered regressor and estimate cointegration vector with OLS in the appended regression in this way obtaining optimal convergence rate and local asymptotic mixed normal distribution of the estimator. Although the estimator depends on the values of integration order and cointegration strength, we show that use of consistent estimates does not affect asymptotic properties of the estimator. This allows to construct feasible Wald test for linear restrictions on the coefficients with nuisance-free asymptotic null distribution.

Chapter 5 studies estimation and inference in a multivariate fractionally cointegrated system based on the conditional Gaussian likelihood. Asymptotic distribution of an estimator is derived as well as asymptotic distribution of the feasible Wald tests for testing linear restrictions on parameters, which have a standard null χ^2 distribution. Observed time series may be stationary or non-stationary, while cointegration relations may be weak (difference between integration orders of observed time series and cointegration errors being smaller than 1/2) or strong (larger than 1/2). In this sense results are very general and are comparable to the results of semiparametric analysis in Hualde and Robinson (2010).

The thesis analyzes different aspects of fractionally integrated and cointegrated time series models and contributes to the literature by suggesting new asymptotic inference procedures in (co)fractional models. Finite sample Monte Carlo simulations illustrate theoretical results, but as they have shown (Sections 4.4 and 5.4), suggested asymptotic inference procedures may not always be satisfactory in finite samples, hence refined asymptotic inference in fractional models, employing bootstrapping procedures or finite sample corrections, is a possible future research avenue.