Essays on bargaining and strategic communication

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Chapter 2  Power and the Privilege of Clarity

2.1  Introduction

Clarity seems to be a privilege of the powerful. The less fortunate among us are typically vaguer about their desires and need to think harder about what they say or do not say. Social psychologists have found that workers are more assertive in communicating their desires towards lower ranked co-workers than towards higher ranked co-workers (Kipnis, Schmidt & Wilkinson (1980), Yukl & Falbe (1990)). Gender studies point to a similar pattern in patriarchal societies, where women are found to be more hesitant in stating their wishes and interests than men. The relation between power and clarity could be shaped primarily by history and culture. In the communication literature, the link between power and communication is widely recognized (Keating, 2009) and believed to be strongly mediated by culture (Gudykunst & Lee, 2003). High status individuals are typically approached with more respect and too clear a message by a lower ranked individual about her preferences might simply be seen as ‘disrespectful.’ Similarly, direct communication of preferences may result in the loss of face of the powerful person if it openly contradicts her wishes or of the less powerful person if her wishes are ignored. In contrast to the above disciplines, in economics the relation between power and communication is a largely untouched research area.

In this chapter, we explore the possibility that there is a fundamental strategic foundation to the relation between power and clarity. In particular, we are interested in the communication between members belonging to different groups of a society (or organization or community) with different levels of power. We

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2 This chapter is based on De Groot Ruiz, Offerman & Onderstal (2011a).
3 When discussing the source of miscommunication between men and women, some authors emphasize the role of power relations while others stress the role of culture (Baer (1976), Butler (1976), Maltz & Borker (1982), Heuley & Kramaroe (2001)).
4 As far as we know, in economics the only research touching on this subject concerns how the level of connectedness in a network affects the bargaining power of individuals in bilateral negotiations (Calvó-Armengol, 2001).
focus on bargaining under asymmetric information, as this is a common type of social interaction where clarity matters. One can think of divorce negotiations between men and women, managers and workers discussing the worker’s tasks, members of different castes in India bargaining over the provision of a service or competition authorities discussing merger remedies with multinationals.

How does power come into play in such situations? Importantly, the consequences of disagreement differ among individuals coming from groups with different levels of power. Simply put, people in a more powerful position have better outside options. This can firstly be due to the fact that people who belong to a powerful group benefit from institutional or cultural rules. For instance, in countries with Islamic law, men have more rights than women at divorce. Secondly, people with more power tend to have more social, political or economic resources. Even in communities where women have equal legal rights but do not perform (much) paid work, men tend to have a superior economic position when filing for divorce. In sum, power affects the costs of disagreeing for agents in bargaining settings.

We think about clarity as informational clarity: how much does someone learn about the state of the world from a message? The informational clarity of a message can firstly depend on its literal clarity: the indirectness, inexplicitness, vagueness or ambiguity of the words used (Cheng & Warren (2003), Agranov & Schotter (2010)). In a single interaction with a stranger from a culture one does not know, the literal meaning is all one can go by. Secondly, if people share a cultural and social history, the information messages convey also depends on how messages are used. For instance, the precise statement “I’ll be there at seven o’clock” is in some cultures not at all informative, because people use it under a wide range of intentions as when to come. By contrast, in some countries the ambiguous phrase “I may prefer if you stopped making noise” can be very informative if such a formulation is only used when people are really upset. The more history people share, the more the clarity of messages will depend on their use. In equilibrium, informational meaning is completely determined by use: what message is used in what state of the world?

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5 One may of course be mainly interested in literal clarity, for instance for linguistic purposes. We are chiefly interested in informational clarity as this determines the actions people take.
Language and its interpretation evolve and as such are subject to strategic forces. Hence, in the long run, there will be a tendency towards some strategic equilibrium. This tendency is strongest in dynamic settings where members of one power-group frequently interact with different members of the other group and reputational concerns play a small role. One implication is that in a stable culture, the informational clarity of messages is largely determined by their use. Hence, one should be careful when providing purely cultural explanations for a lack of informational clarity on the side of individuals with little power. If politeness requires vague messages, then a group may start using several polite (and vague) messages. Over time, these messages may evolve to encode harder information if there is a strategic pressure – such as efficiency gains – to do so. For example, consider a wife and husband who can go to the theater or a concert. The wife knows they both prefer the theater. In a Western society, the wife may reply to her husband’s question “Shall we go to the concert” with “Nah, let’s go to the theater.” In a more patriarchal society where it is impolite for the wife to contradict her husband openly, she may instead say “Well, Sir, are you sure you do not feel like going to the theater?” with the same outcome. Another example is the Iranian practice of Taarof civility, which requires among other things that a shopkeeper says his products are ‘worthless’ when asked for the price. Still, despite such politeness, the price is revealed in the end. Customers have learned they should repeat the question a couple of times to get a real answer and cannot just walk away without paying.

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6 Hence, we focus on one-shot situations where people have social information. Other fascinating possibilities are to study the relation between power and clarity in one-shot interactions without social information or in repeated interactions. Although an equilibrium analysis may be informative here, we have to be careful when generalizing our findings to such settings. In interactions without social information, out-of-equilibrium behavior in cheap talk games cannot be ruled out. In such cases, approaches based on rationalizability and some focal meaning of messages, such as Rabin (1990), may be more appropriate. This means that one needs to look at the literal meaning to derive its information content in such cases. One problem is that in these settings we are as of yet only able to theoretically predict only rather conservative lower bounds on information transmission. Another problem is that it is not always clear what the relation between the literal and focal meaning of messages is. Analyzing repeated interactions between the same players is even more challenging. Strictly speaking, it is just a very complex one-shot game, possibly involving an endogenous form of reputation building (Sobel, 1985). In real life, myopic strategic reasoning may give it a dynamic flavor, so that messages may acquire a consistent meaning justifying some equilibrium concept.

7 For more on Taarof and the power-language relationship in Iran, see Beeman (1986).
It is now possible to translate our original question of how power influences clarity into a precise game-theoretic one: how does bargaining power influence information transmission in equilibrium? We study this question in an elementary bargaining setting. A Sender with private preferences and a Receiver with commonly known preferences bargain over a one-dimensional issue.\(^8\) The Sender sends a costless message to the Receiver, after which they play an ultimatum game in which the Sender can reject or accept the proposal of the Receiver.

We find that bargaining power is a key determinant of how much information can be transmitted: information transmission is increasing in the Sender’s power and decreasing in that of the Receiver. In other words, the higher the relative power of an informed agent, the clearer she will be. There is one exception in which full revelation is possible. Senders who are closely aligned with the Receivers or have no bargaining power can fully reveal their type since they will be offered the Receiver’s preferred outcome anyway in equilibrium. The range of Senders who can reveal their type without costs decreases with the relative power of the Sender.

We see our results primarily as a proof-of-principle, as many power relations and strategic settings are more complex in practice. At the same time, we believe that the intuition behind our results holds more generally. If you hold little power, it is not in your best interest to reveal too much information, because that can be exploited. Hence, you better be kind of vague and strategic about what you communicate. If you are powerful, the potential for exploitation is limited and you can afford to be clear.

In addition to shedding light on power relations, this chapter contributes to the theoretical literature on bargaining and information transmission. Our model differs from previous models in that the private information of the Sender does not determine her bargaining power. This allows us to capture the power individuals have due to the social, political or economic position of the group they belong to. Our model is close to that of Matthews (1989), who was the first to study veto threats. In Matthews’ model, however, the Sender’s type determines her disagreement payoff, whereas in our model the disagreement payoff is an exogenous variable which is the same for all Sender types. Hence, we can

\(^8\) We will refer to the Sender as a ‘she’ and the Receiver as ‘he.’
model members of the same group having commonly known and similar power levels but differing in their private preferences. For example, consider men filing for divorce in Saudi Arabia and entering into custody negotiations. They all have the same legal position but differ in their preferences to see their children.

Our modeling choice has profound implications for information transmission. In Matthews’ model, information transmission is limited: the maximum equilibrium size is two. In our setup, a full range of Crawford-Sobel-like partition equilibria exists, potentially allowing for more refined communication. In particular, the role of power in our model mirrors the role of interest-alignment in the Crawford-Sobel game.

The literature on economic bargaining and information transmission has mostly focused on buyer-seller situations, where the outcome-set is zero sum conditional on trade (e.g. Matthews & Postlewaite (1989) and Farrell & Gibbons (1989)). In these models, the bargaining power of the other party is typically unknown, so that power and private information again coincide.\(^9\)

Finally, our model applies to various interesting situations. Consider, for example, custody negotiations between lawyers of divorcing parents. A common situation is that the mother would like to see the children as much as possible, whereas the father’s preferences are not precisely known. If they do not manage to agree, they will have to go through a costly court procedure. In these situations, one can ask how the power of the father relative to the mother affects the ability of his lawyer to communicate the preferences of his client. In the conclusion, we discuss testable economic implications for labor contracts and for remedies merging firms propose to competition authorities.

The remainder of this chapter has the following structure. Section 2.2 presents a simple example of our model that serves to illustrate our set-up and results. Section 2.3 presents the model and the results. Section 2.4 concludes.

\(^9\) For a literature-review on bargaining with incomplete information see Ausubel, Cramton & Deneckere (2002).
2.2 Example

2.2.1 Game

Consider a cheap talk game with veto threats between an informed Sender and an uninformed Receiver. The outcome of the game $x$ is a point on the interval $[0,1]$ or the disagreement point $\delta \not\in [0,1]$. The Sender’s payoff on the interval depends on the state of the world $t$ (her type): $U^S(x, t) = -|x - t|$. The larger the distance between the outcome $x$ and her type $t$, the lower the Sender’s payoff. Her type $t$ is private information of the Sender and it is common knowledge that $t$ is drawn from the uniform distribution on $[0,1]$. The Receiver’s payoff on the interval, $U^R(x) = -x$, is independent of $t$: he always prefers smaller outcomes to larger ones. We vary the payoff of the disagreement point to the players: $U^R(\delta) = -d^R$ and $U^S(\delta) = -d^S$ with $d^R, d^S > 0$. (Note that $d^R$ and $d^S$ are the size of the “harm” if bargaining breaks down.) In particular, we have:

\[
U^R(x) - U^R(\delta) = d^R - x \text{ for all } x \in [0,1]
\]
\[
U^S(x, t) - U^S(\delta) = d^S - |x - t| \text{ for all } x \in [0,1]
\]

Observe that the Receiver prefers $\delta$ to all outcomes more than $d^R$ away from the origin and that the Sender prefers $\delta$ to all outcomes on the line more than $d^S$ away from her type $t$.

The game proceeds as follows. First, the Sender is informed of her type $t$. Subsequently, she sends a costless message $m \in M$ to the Receiver, where $M$ is some sufficiently rich message set. Then, the Receiver proposes an action $a \in [0,1]$ to the Sender. Finally, the Sender accepts or rejects $a$. If she accepts, $a$ is the outcome and if she rejects, $\delta$ is the outcome.

The game is an elementary bargaining setting under asymmetric information and models some important aspects of real interactions. The one-dimensional bargaining set allows us to capture partially aligned and partially conflicting
interests of the players. The one-sided information asymmetry and single round of ‘pre-play communication’ captures the essence of biased information transmission. The game is similar to Matthews’ (1989), except that the disagreement point lies on the real line in Matthews’ model. The disagreement point being now outside of the line allows us to model differences in bargaining power independent of player’s preferences on the line. Hence, the disagreement payoff reflects power individuals have due to their (commonly known) social, political or economic position and which they share with other members of their group.10

The bargaining power of the players in our game is determined by how attractive the disagreement point $d$ is to them. As $d^S$ [$d^R$] becomes smaller, the Sender’s [Receiver’s] payoff of the disagreement point increases and the interval of points that the player prefers to $d$ narrows. Hence, the larger $d^S$ [$d^R$], the smaller the bargaining power of the Sender [Receiver].

### 2.2.2 Equilibria

We look at a refinement of perfect Bayesian equilibria that restricts the Receiver to pure strategies and lets the Sender consider that she may tremble at the veto stage. Hence, for the Receiver we need to specify which message elicits which action, and for each Sender a probability distribution over messages she sends. In equilibrium, the Receiver best responds to his correctly updated posterior beliefs and each Sender type induces the action(s) that give her the highest payoff, even if she plans to veto anyway (since she might tremble). From now on we refer to a perfect Bayesian equilibrium satisfying these two requirements simply as an equilibrium.

It turns out that all equilibria are partition equilibria. In a partition equilibrium, types separate into disjunct intervals. A partition equilibrium can be characterized by the finite set of actions $a_1 < a_2 < ... < a_n$ the Receiver proposes in equilibrium.11 The number of equilibrium actions $n$ is called the size of the

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10 In theory, the disagreement point could be determined both by social position and private preferences. We assume for simplicity that it only depends on a player’s social position, as this is sufficient to address the relationship between power and information transmission.

11 To be precise, the action set characterizes an (infinite) class of essentially equivalent equilibria that induce the same equilibrium outcome and differ only in the messages that are used.
equilibrium. Each type then simply induces that action that is closest to her type, so that the type space can be partitioned into intervals of types who induce the same action. The main intuition for this result is that full separation is impossible in any type-interval contained in \((d^3, 1]\): a revealed type would get zero, such that she has an incentive to mimic a higher type. In fact, there is a minimum distance between any two equilibrium actions, so that the equilibrium action set is finite. Together with the fact that Senders elicit the action closest to their type, this results in a well-ordered partition.

In games with only partition equilibria, the size of the equilibrium provides a natural measure of information transmission, which is invariant to scaling (of the payoff, action or type space). Equilibrium size will correlate well with other measures, such as conditional variance, prediction error, ex-ante efficiency or ex-post efficiency – with the suitability of each depending on the context.\(^{12}\) In all cheap talk games, a size-1 (pooling) equilibrium exists in which no information is transmitted. We say that more information is transmitted as the size of the equilibrium increases.

In this chapter we focus on the relation between power and the maximum information transmission possible in equilibrium. In particular, we look at how power affects the maximum equilibrium size. Like in other continuous cheap talk games, such as Crawford & Sobel (1982) and Matthews (1989), the maximum size equilibrium seems the most plausible one although the equilibrium set is actually hard to refine.\(^{13}\) Chapters 3 and 4 provide a theoretical and experimental justification to focus on the maximum size equilibrium. In Chapter 3, we introduce the Average Credible Deviation Criterion (ACDC). This criterion

\(^{12}\) A problem of these measures is that they lack a natural dimension, such that they are typically not invariant to immaterial transformations of the game. For instance, the average prediction error or the conditional variance is not invariant to scaling of the type space and ex-ante utility is not invariant to immaterial transformation of payoffs of subsets of the type set. The fraction of outcomes that is ex-post efficient does not suffer from this invariance problem, but is a rather crude measure.

\(^{13}\) Traditional signaling refinements have no bite in cheap talk games, as messages are costless. In Chapter 3, we show that in the current game also the cheap talk refinements neologism proofness (Farrell, 1993), announcement proofness (Matthews, Okuno-Fujiwara & Postlewaite, 1991), communication proofness (Blume & Sobel, 1995), the recurrent mop (Rabin & Sobel, 1996) and NITS (Chen, Kartik & Sobel, 2008) are not selective and the non-equilibrium concepts of Credible Message Rationalizability (Rabin, 1990) and Partial Common Interest (Blume, Kim & Sobel, 1993) are not predictive.
generalizes credible deviations approaches as neologism proofness (Farrell, 1993) and announcement proofness (Matthews, Okuno-Fujiwara & Postlewaite, 1991). ACDC does not suffer from non-existence and organizes data from previous experiments successfully. We show in Chapter 3 that, under some additional assumptions, ACDC selects in the model we present in this chapter a unique maximum-size equilibrium. In Chapter 4, we conduct an experiment with games belonging to the current model and find that the maximum size equilibrium indeed predicts behavior best.

As an illustration of what the equilibria look like, consider the case where \( d^S = \frac{1}{2} \) and \( d^R = \frac{1}{4} \). This game has two equilibria. In the pooling equilibrium, the Receiver always proposes action \( \frac{3}{8} \). In this equilibrium, the Receiver ignores all messages and best responds to his prior beliefs. The optimal action of the Receiver always involves a trade-off between maximizing the probability that the proposal is accepted and maximizing the payoff of the proposal conditional on acceptance. Senders all have the same message strategy, which is optimal for each type as it does not matter what they send. The game also has a size-2 equilibrium with actions \( a_1 = 0 \) and \( a_2 = \frac{1}{2} \). Senders in \([0, \frac{1}{4})\) could, for instance, send the message “My type is low,” inducing the Receiver to propose 0 and those in \([\frac{1}{4}, 1]\) the message “My type is high,” inducing the Receiver to propose \( \frac{1}{2} \). When the Receiver receives the message “My type is low” he correctly infers that types are in the interval \([0, \frac{1}{4})\) and proposes 0, as all types in \([0, \frac{1}{4})\) accept 0. If he receives the message “My type is high,” he infers \( t \in [\frac{1}{4}, 1] \). Her expected payoff from proposing action \( a \) now is \( EU^R(a) = \Pr\{a \text{ is accepted}\} \times (U^R(a) - U^R(\delta)) = (\min\{1,a + \frac{1}{2}\} - \max\{\frac{1}{4},a - \frac{1}{4}\}) \times (\frac{1}{4} - a) \), which is maximized at \( a = \frac{1}{2} \). We can characterize the equilibrium set as follows for general \( d^S, d^R > 0 \).
Equilibrium Characterization Example

Let $\bar{\pi} = \max\{0, \min\{d^R - 2d^S, 1 - d^S\}\}$. Any equilibrium of the game described in section 2.2.1 is a partition equilibrium that can be described by a natural number $n \in \{1, \ldots, \bar{\pi}\}$ and a set of equilibrium actions $\{a_1, \ldots, a_n\}$, such that

1. $a_1 = \max\{0, \min\{1 - d^S, d^S, \frac{1}{2}(d^R - d^S)\}\}$ if $n = 1$
2. $a_1 = \min\{d^S, \max\{0, a_2 - 2d^S\}\}$ if $n \geq 2$
3. $a_2 = \min\{\frac{d^R}{2}(d^R - d^S), 2d^S, 1 - d^S\}$ if $n = 2$ and $a_1 = 0$
4. $a_k = a_{k-1} + 2d^S$ if $a_{k-1}$ exists and $a_{k-1} > 0$
5. $a_n \leq \bar{\pi}$ if $d^R \geq 4d^S$

The maximum size $\bar{\pi}$ is equal to 1 if $d^S \geq 1$. If $d^S < 1$, $\bar{\pi} = \max\left\{2, \left\lfloor \frac{d^R}{2d^S} \right\rfloor \right\}$ if $d^R \leq d^S + 1$ and $\bar{\pi} = \max\left\{2, \left\lfloor \frac{3}{2 + \frac{1}{2d^S}} \right\rfloor \right\}$ otherwise, where $\left\lfloor . \right\rfloor$ is the ceiling function.\(^4\)

2.2.3 Power and Clarity

We can now turn to the central question of this chapter: how does power affect information transmission in equilibrium? To determine this, we vary $d^S$ and $d^R$. For instance, suppose that relative to $d^S = \frac{1}{4}$ and $d^R = \frac{1}{4}$, we increase the bargaining power of the Sender to $d^S = \frac{1}{4}$. This results in more information transmission that can be supported in equilibrium: the maximum equilibrium size goes from 2 to 3. This illustrates the key result that the maximum information transmission possible in equilibrium increases with the power of the Sender and decreases with that of the Receiver:

**Maximum Equilibrium size.** $\bar{\pi}$ is decreasing in $d^S$ and increasing in $d^R$.

\(^4\) We provide the proof at the end of the appendix.
This follows directly from the characterization of the equilibria. If the Sender has no power \( d^S \geq 1 \) and all types would accept the Receiver’s optimal action, only a pooling equilibrium exists. If the Sender has some power \( d^S < 1 \), at least a size 2 equilibrium exists. If the Receiver has very little power \( d^R > d^S + 1 \), the maximum size only depends on (and is increasing in) the power of the Sender. If the Receiver has some power as well \( d^R \leq d^S + 1 \), then the maximum size \( \bar{n} \) increases in the relative power of the Sender:

\[
\bar{n} = \max\left\{ 2, \left\lfloor \frac{d^R}{2d^S} \right\rfloor \right\}. \quad \text{Therefore, } \bar{n} \text{ jumps to a higher level if } \left\lfloor \frac{d^R}{2d^S} \right\rfloor \text{ increases sufficiently. As the power of the Sender relative to that of the Receiver becomes large, the maximum equilibrium size also becomes large.}
\]

The intuition for this result is the following. The highest action the Receiver prefers to \( \delta \) (in our game \( d^R \)) and the type density close to this point impose an upper limit on the highest action. The fact that the Receiver always wants to offer a lower action than the Sender imposes a lower bound on how close equilibrium actions can be together. A Sender prefers \( t \), whereas the Receiver would offer \( \max\{0, t - d^S\} \), if he would know \( t \). In our example, the smallest distance between two positive equilibrium actions is \( 2d^S \).

If the Sender is sufficiently powerful (and the Receiver has some power as well), we can construct the maximum size equilibrium as follows. We set \( a_n \) equal to the highest action possible in an equilibrium, which is \( d^R - 2d^S \). From there, we create the tightest partition by iteratively setting \( a_{i-1} \) as close as possible to \( a_i \) as long as \( a_{i-1} > 0 \). In this example, we need to set \( a_{i-1} = a_i - 2d^S \). Finally, we set \( a_i = 0 \). We can show that this tightest partition is in fact an equilibrium. As a consequence, the maximum equilibrium size is

\[
\left\lfloor \frac{d^R}{2d^S} \right\rfloor; \text{ namely 1 for } a_i = 0, \text{ plus } \left\lfloor \frac{d^R - 2d^S}{2d^S} \right\rfloor \text{ positive actions minus 1 if } d^R - 2d^S \text{ is divisible by } 2d^S.
\]

Hence, an increase of the Receiver’s power (decreasing \( d^R \)) leads to a decrease of information transmission by lowering the highest possible equilibrium
action \((d^S - 2d^R)\). An increase of the Sender’s power (decreasing \(d^S\)) increases information transmission firstly by increasing the highest possible equilibrium action. Secondly and more importantly, it lowers the minimum distance between equilibrium actions \((2d^S)\). In the next section, we show that this result holds for a broader class of payoff functions and type distributions. The underlying intuition is twofold. First, the power of a Receiver limits the range of potentially mutually profitable actions. Second, the conflict of interest between Sender and Receiver puts a fundamental upper bound on information transmission, as the Receiver has an incentive to exploit the Sender’s information. However, the bargaining power of the Sender raises this upper bound by limiting how much a Receiver can exploit the Sender.

2.3 Theory

2.3.1 Model

A Sender and Receiver play the following game with an outcome in \(X = \mathbb{R} \cup \{\delta\}\). First, the Sender privately observes her one dimensional type \(t\). It is common knowledge that \(t\) is drawn from the uniform distribution on the interval [0,1].\(^{15}\) Second, the Sender sends a message \(m \in M\), where \(\mathbb{R} \subset M\). Third, the Receiver receives \(m\) and proposes an action \(a \in \mathbb{R}\). Finally, the Sender accepts or rejects \(a\). If the Sender accepts [rejects], the proposed action \(a [\delta]\) is the outcome of the game.

Let \(U^R : X \rightarrow \mathbb{R}\) be the utility function of the Receiver and \(U^S : X \times T \rightarrow \mathbb{R}\) that of the Sender. We model the players’ bargaining power as the payoff of the disagreement point \(U^R(\delta)\) and \(U^S(\delta)\), where we assume \(U^S(\delta, t) = U^S(\delta)\) does not depend on \(t\). \(U^R\) and \(U^S\) satisfy the following assumptions:

\(^{15}\) Given that types \(t\) are drawn from a smooth distribution function \(F\), we can make the assumption of uniformly distributed types without further loss of generality: \(t\) can be replaced by \(\tilde{t} \equiv F(t)\), which is uniformly distributed. Of course, all other variables should be redefined accordingly.
(A1) $U^R$ is twice continuously differentiable, unimodal with a peak at 0 and concave on $\mathbb{R}$.

(A2) $U^S(\cdot, \cdot)$ is continuous, unimodal at $t$ for each $t$ on $\mathbb{R}$; $U^S(x, t)$ is strictly increasing [decreasing] in $x$ for $x < t$ [x > t];

(A3) If a Sender type $t$ is indifferent between outcomes $x_1$ and $x_2 > x_1$, then higher types than $t$ prefer $x_2$ and lower types prefer $x_1$.

Let the outcomes $\lambda(t, U^S(\delta)) < t$ and $\rho(t, U^S(\delta)) > t$ be the indifference points to the left respectively right of $t$ with respect to the disagreement point, i.e.

$$U^S(\lambda(t, U^S(\delta)), t) = U^S(\rho(t, U^S(\delta)), t) = U^S(\delta, t).$$

Let $\lambda^{-1}(x, U^S(\delta))$ and $\rho^{-1}(x, U^S(\delta))$ be the inverse functions of $\lambda$ and $\rho$ with respect to $t$. Finally, we assume that

(A4) $\lambda$ and $\rho$ exist and are twice continuously differentiable and strictly increasing in $t$.

(A5) $\frac{\partial}{\partial x} \lambda^{-1} \geq \frac{\partial}{\partial t} \rho^{-1}$, $\frac{\partial^2}{\partial x^2} \lambda^{-1} \leq 0$ and $\frac{\partial^2}{\partial x \partial U^S(\delta)} \lambda^{-1} > 0$.

If it is clear that we talk about a particular game with fixed $U^S(\delta)$, we will suppress the dependency on $U^S(\delta)$ and write $\lambda(t)$, $\rho(t)$, $\lambda^{-1}(x)$ and $\rho^{-1}(x)$. A simple condition on the Sender’s preferences such that they satisfy (A2)-(A5) is the following:
$U^S(x,t)$ can be written as a function $f(t-x)$, for all $x$ in $\mathbb{R}$, $t$ in $[0,1]$, where $f$ is continuously differentiable, strictly increasing in $\mathbb{R}_-$, strictly decreasing in $\mathbb{R}_+$ and for all $y \in \mathbb{R}$ there is a $z > 0$ such that $f(-z) < y$ and $f(z) < y$. Finally, we require $U^S(0) < f(0)$.\footnote{Due to the invariance of games to affine payoff transformations, it is actually only required that $U^S(x,t)$ can be written as a function $a + b \cdot f(t-x)$, where $b > 0$ and $f$ should adhere to the conditions specified (with $U^S(0) < a + b \cdot f(0)$).}

Our results will often hold even when the Receiver’s utility is not concave and (A5) does not hold, but these assumptions will greatly facilitate the construction of equilibria in Proposition 2.2. In particular, let $\rho^R$ be the point $x$ in $\mathbb{R}$ where the Receiver is indifferent between $x$ and $\delta$ and define the function

$$h(x) \equiv \lambda^{-1}(x)(U^R(x) - U^R(\delta)) + \left(\lambda^{-1}(x) - \rho^{-1}(x)\right)U'^R(x)$$

where a prime (‘) denotes a derivative with respect to $x$.

(A4) and (A5) imply that $h$ is strictly decreasing on $[0, \rho^R)$ for all values of $U^R(\delta)$ and $U^S(\delta)$.\footnote{Under (A4) and (A5), all terms in $h'(x) = \lambda^{-1}(x)(U^R(x) - U^R(\delta)) + \lambda^{-1}(x)U'^R(x) + \left(\lambda^{-1}(x) - \rho^{-1}(x)\right)U''^R(x) + \left(\lambda^{-1}(x) - \rho^{-1}(x)\right)U'''^R(x)$ are negative on $[0, \rho^R)$.} Since $h(\rho^R) < 0$, this implies that for all $U^R(\delta)$ and $U^S(\delta)$ there is an $\bar{\rho} \in [0, \lambda(1)]$ such that

\begin{equation}
\begin{aligned}
&h(x) > 0 \text{ for all } x \in [0, \bar{\rho}) \\
&h(x) < 0 \text{ for all } x \in (\bar{\rho}, \lambda(1)]
\end{aligned}
\end{equation}

(2.2) imposes regularity on the Receiver’s best response. Suppose the Receiver infers from a message that a type lies in the interval $[\underline{t}, \overline{t}]$. As long as $a < \min\{\lambda(\overline{t}), \rho(\underline{t})\}$, increasing $a$ involves the tradeoff between decreasing $U^R(a)$ if $a$ is accepted and increasing the acceptance probability by increasing the
highest type that accepts $a$. (2.2) will ensure that there is a point $\bar{x}$ independent of $t$ and $\bar{t}$ such that (in a few important cases) for $a < \min\{\bar{x}, \lambda(\bar{t}), \rho(t)\}$ it pays to increase $a$ and for $a > \min\{\bar{x}, \lambda(\bar{t}), \rho(t)\}$ it does not.

We follow Matthews (1989) in the refinement of the perfect Bayesian equilibrium that we employ. First, we restrict the Receiver to play pure strategies. Second, we require that also Sender types who plan to veto any equilibrium action send a message inducing an action $a$ that maximizes $U^S(a, t)$. This refinement is motivated on the basis of Selten’s (1975) trembling hand perfection argument: the Sender considers that she might tremble with a small probability and accept the Receiver’s proposed action.

Let for any set $S$, $\Delta S$ denote the set of probability distributions on $S$. A strategy for the Sender consists of a message function $\mu : T \rightarrow \Delta M$ and an acceptance probability function $\nu : \mathbb{R} \times T \rightarrow [0, 1]$. A strategy of the Receiver is a function $\alpha : M \rightarrow \mathbb{R}$. Let $\sigma \equiv \{\mu, \nu, \alpha\}$ be a strategy profile and $\Sigma$ the set of all strategy profiles. The Receiver has correct prior beliefs $\beta^0$. Finally, the Receiver has posterior beliefs $\beta : M \rightarrow \Delta T$. An equilibrium $\sigma^* = \{\mu^*, \nu^*, a^*, \beta^*\}$ is characterized by the following four conditions:

(i) $\alpha^*(m) \in \arg\max_{\alpha \in \mathbb{R}} E^s\{(U^R(a) - U^R(\delta))\nu(a, t) | \beta^*\}$ for all $m$

(ii) $m \in \arg\max_{m \in M} U^S(\alpha^*(m), t)$ for all $m$ in the support of $\mu^*(t)$ for all $t$

(2.3)

(iii) $\nu^*(a, t) = 1$ if $U^S(a, t) > 0$ and $\nu^*(a, t) = 0$ if $U^S(a, t) < 0$

(iv) $\beta^*$ is derived from $\mu^*$ and $\beta^0$ using Bayes’ rule whenever possible

We say that a type $t$ induces action $\hat{a}$, and write $a(t) = \hat{a}$, if $\alpha(t) = \hat{a}$ for all messages $m$ that type sends.

### 2.3.2 Results

As in Matthews’ (1989) and Crawford and Sobel’s (1982) cheap talk game, all equilibria of our game are partition equilibria.
Definition 2.1 An equilibrium $\sigma^*$ is called a partition equilibrium if there is a partition $0 = t_0 < t_1 < \cdots < t_{n-1} < t_n = 1$ of the type space such that each type in $(t_{i-1}, t_i)$ induces action $a_i$ with $a_1 < a_2 < \cdots < a_n$.

Note that any partition equilibrium outcome can be characterized by the actions $a_1 < a_2 < \cdots < a_n$ the Receiver proposes in equilibrium. Due to (A3), the partition is then fixed by $U^S(a_k, t_k) = U^S(a_{k+1}, t_k)$. The number of actions $n$ is called the size of the equilibrium.

Proposition 2.1 Any equilibrium of the cheap talk game is a partition equilibrium.

The intuition behind the proof is that the conflict of interest between the Sender and the Receiver puts a limit on information transmission. After all, the Receiver would prefer to propose $\max\{0, \lambda(t)\}$ if he knew the Sender’s type, whereas the Sender prefers to receive proposal $t$. This means that the highest type (supremum) inducing an action $a_i > 0$ must get zero payoff in equilibrium, because otherwise the Receiver can do better by proposing a lower action instead of $a_i$. This means that $a_{i+1}$ cannot be smaller than $\rho(\lambda^{-1}(a_i))$, because otherwise the highest type inducing $a_i$ would prefer $a_{i+1}$ over $a_i$. Hence, there is a minimum distance between any two strictly positive equilibrium actions. Since the Receiver never makes a proposal higher than $\rho$, this means that the equilibrium action set is finite. As Senders elicit the action closest to their type, this results in a well-ordered partition.

We model an increase in a player’s bargaining power as an increase in her disagreement point payoff. Power influences maximum information transmission:

Proposition 2.2 (i) If the Receiver’s bargaining power increases, the maximum size of the equilibrium decreases. (ii) If the Sender’s bargaining power increases, the maximum size of the equilibrium increases.
Hence, the higher the bargaining power of the Sender relative to that of the Receiver, the more information that can be transmitted in equilibrium. In the proof, we construct a maximum size equilibrium. Using a similar reasoning as in the proof of Proposition 2.1, we show that in equilibrium \( \lambda(t_{k-1}) < a_k \leq \lambda(t_k) \) for all \( k = 2, \ldots, n \) (Lemma 2.1) and \( \rho(t_{k-1}) \leq a_k \) for \( k = 3, \ldots, n \) (Lemma 2.2). In the most interesting case, when \( \lambda^{-1}(0) < \rho^{-1}(\bar{x}) \), at least a size-3 equilibrium exists. Furthermore, the highest possible value an equilibrium action can take is then \( \bar{x} \), the point where \( h(x) \) is zero. We can in this case construct a maximum size equilibrium by

- Setting the highest equilibrium action \( a_n \) equal to \( \bar{x} \);
- Iteratively adding equilibrium actions \( a_{k-1} = \lambda(\rho^{-1}(a_k)) \) until it ‘does not fit anymore’ \( (\lambda(\rho^{-1}(a_k)) < 0) \);
- Setting \( a_1 = 0 \).

Increasing the Receiver’s bargaining power decreases \( \bar{x} \), so that the number of equilibrium actions that ‘fit’ in this equilibrium becomes smaller (or remains the same). Increasing the Sender’s bargaining power firstly increases \( \bar{x} \). In addition, increasing the Sender’s power decreases the distance between equilibrium actions because the function \( a - \lambda(\rho^{-1}(a)) \) becomes smaller.

There are two features of the relation between power and information transmission that deserve further attention. First, Senders who induce 0 could reveal their type. We can use the number of equilibrium actions as a proxy for information transmission in so far as all Sender types who induce an equilibrium action \( a \) send the same message. For each equilibrium outcome, always an equilibrium exists where this holds for all types. However, also pay-off equivalent equilibria exist where types inducing \( a_i = 0 \) reveal their type. The reason is that they are protected by the fact that the Sender does not want to propose an action below 0. This means that Senders who are closely aligned to the Receiver can fully separate. In addition, this implies that if the Sender has no power (if \( \lambda(1) \leq 0 \)), then an equilibrium exists with full separation. In this case the Receiver always proposes 0 and Senders can send any message they want in
equilibrium. Observe that they do not have an actual incentive to reveal their type.

Second, complete power in the Sender’s hands may lead to less information transmission. If the Sender becomes very powerful \((U^S(\delta) \text{ approaches } U^S(t,t))\), her incentive to communicate becomes small. In our model this is not relevant as long as \(U^S(\delta) \leq U^S(t,t)\). However, one can imagine a model where sending a message has a small positive cost. In this case, increasing the power of the Sender will increase information transmission until the costs of communicating outweigh the benefits.

2.4 Conclusion

In this chapter, we examined how power shapes communication under information asymmetry. Our interest was to explore the levels of clarity that are likely to arise between members of groups with different levels of power. We expect cultural patterns to evolve to a strategic equilibrium over time and, hence, we investigated whether there is a strategic relationship between power and clarity. In a game-theoretic bargaining model, we showed that clarity is indeed a privilege of the powerful. When negotiating an outcome, an informed bargainer with (relatively) little bargaining power cannot afford to reveal too much information, as that can be used against her. How much information can be transmitted depends crucially on the relative power of the informed party, the Sender: less information can be transmitted if either the Sender’s power decreases or the Receiver’s power increases.

We see our one-dimensional model with one-sided asymmetric information as a proof of principle providing a more general intuition. The conflict of interest between the two parties imposes an upper bound on information transmission in equilibrium; the informed party cannot reveal too much information, as information allows the uninformed party to exploit her. Crucially, the informed party’s bargaining power limits how much she can be exploited and hence enhances information transmission in equilibrium. In contrast, bargaining power of the uninformed party limits the range of mutually attractive actions where
information transmission is meaningful. Crawford & Sobel (1982) found that information transmission is determined by the alignment of interests when the Sender has no influence on the outcome. We have identified another key determinant of information transmission if the Sender does have some influence on the outcome: bargaining power.

In addition to providing a proof-of-principle, our analysis has testable implications for communication and outcomes. In Chapter 3, we test our predictions in a controlled laboratory experiment and find that the relative power of the Sender increases information transmission. Furthermore, the type of bargaining situations we study often occur in the field.

One application concerns contract negotiations between employers and employees. Here, asymmetric information and bargaining power play a significant role. One can think of an employee’s preference for the work-life balance (salary versus flexibility) or the type of activities she is required to do. For instance, when a department negotiates with a potential new professor about her administrative and teaching duties, the preference of the professor for administration versus teaching are typically unknown. One implication of our analysis is that when the employee has more bargaining power, she will be able to convey her preferences more precisely. As a measure of bargaining power one could use the level of skill of employees or the unemployment rate in a given sector and/or year. Our model predicts that as information transmission increases, the variety of outcomes also increases. As a consequence, our model predicts that the variety of labor contracts in a specific job market should be increasing with the employment rate (in the sector) and the skill-level required for the job.

Another application where our model has testable implications is negotiations between a competition authority and two firms planning a merger. If the merger creates or strengthens market power in the relevant market, the competition authority can demand remedies, such as requiring the firms to sell some production-lines to a third party. Firms always want as few remedies as possible. Preferences of competition authorities are less clear, as they have to weigh economies of scale against market power.\(^{18}\) Competition authorities provide

\(^{18}\) Another possible trade-off for competition authorities concerns collusion (Compte, Jenny & Rey, 2002). A merger reduces the number of competing firms, which can make collusion easier.
information about their preferences to firms before they submit their final proposal, often already in the pre-notification phase. Our model predicts that the variability of the proposals a competition authority receives is increasing in her power. A competition authority with little power always gets the same kind of proposal (across comparable cases), for instance a proposal without remedies. A competition authority with more power can expect to receive proposals that sometimes include remedies and sometimes not. Indicators exist about the strength of competition authorities, such as the OECD’s Competition Law and Policy (CLP) indicators (Høj, 2007) or those developed by Voigt (2006) for a broader set of countries. Such indicators include the formal and factual independence of competition authorities. These proxies for bargaining power could be related to how often final proposals include remedies (or even to the variety of remedies included). A relevant comparison would be between competition authorities in (old) EU member states and the US with those in Latin America or Eastern Europe.19

2.5 Appendix: Proofs

Proof of Proposition 2.1 Let \( \sigma^* \) be an equilibrium. First, suppose the Receiver plays at least three actions in equilibrium and let \( a < a' \) be two strictly positive equilibrium actions. Let \( \bar{t}(a) \) be the supremum of types that induce action \( a \). Then \( 0 < a \leq \lambda(\bar{t}(a)) \), because otherwise the Receiver would be better off by playing \( \lambda(\bar{t}(a)) \) instead of \( a \). Furthermore, \( a' \geq \rho(\bar{t}(a)) \), because otherwise \( U^S(a',\bar{t}(a)) > U^S(a,\bar{t}(a)) \). This means that \( a' - a \geq \rho(\bar{t}(a)) - \lambda(\bar{t}(a)) \). Consequently, an upper bound on the number of equilibrium actions is \( 1 + 1/\eta \), where \( \eta = \min_{t \in [0,1]} \{ \rho(t) - \lambda(t) \} \). This means that the set of equilibrium actions, \( A^* \), is finite. Hence, we can write \( A^* = \{a_1, a_2, \ldots, a_n\} \) with \( a_1 < a_2 < \ldots < a_n \).

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19 For comparative work on competition authorities in Latin America see Schatan & Rivera (2008) and Qaqaya & Lipmile (2008); For competition authorities in Europe see Cseres (2010).
Second, (A2)-(A3) imply that for each consecutive action pair \( a_k, a_{k+1} \) a triple of types \( t_{k-1} < t_k < t_{k+1} \) exists such that

\[
U^S(a_k, t_k) = U^S(a_{k+1}, t_k) \quad \text{and} \quad a_l \in \arg \max_{a \in A} U^S(a, t) \text{ iff } t \in [t_{l-1}, t_l] \text{ for } l = k, k+1
\]

Consequently, a partition of the type space \( 0 = t_0 < t_1 < \cdots < t_{n-1} < t_n = 1 \) exists such that for each \( k \) all types in \( (t_{k-1}, t_k) \) induce the same action \( a_k \) and that \( k \neq l \) implies that \( a_k \neq a_l \). Q.E.D.

The following two lemmas are useful for the proof of Proposition 2.2.

**Lemma 2.1** In equilibrium, \( \lambda(t_{k-1}) < a_k \leq \lambda(t_k) \) for all \( k = 2, \ldots, n \).

**Proof.** Note that by construction, \( a_1 < a_2 < \cdots < a_n \). It must be the case that \( a_k > a_1 \geq 0 \) for all \( k = 2, \ldots, n \). Moreover, \( a_k > \lambda(t_k) \) cannot occur in equilibrium, because if the Sender type is in the interval \([t_{k-1}, t_k]\), the Receiver is better off by offering \( a = \lambda(t_k) \) instead of \( a = a_k \). Finally, \( a_k > \lambda(t_{k-1}) \) because otherwise none of the types in \([t_{k-1}, t_k]\) will accept \( a_k \). Q.E.D.

**Lemma 2.2** \( \lambda(t_{k-1}) \leq a_k \) for \( k = 3, \ldots, n \).

**Proof.** The proof is by contradiction. Lemma 2.1 shows that \( a_{k-1} \leq \lambda(t_{k-1}) \), so that type \( t_{k-1} \)'s utility is equal to the utility of the disagreement point if she induces \( a_{k-1} \). Suppose that \( \rho(t_{k-1}) > a_k \). Then type \( t_{k-1} \) is strictly better off inducing \( a_k \) instead of \( a_{k-1} \). This constitutes a contradiction, because types just below \( t_{k-1} \) would strictly prefer sending \( a_k \) instead of \( a_{k-1} \), while they induce \( a_{k-1} \) in equilibrium. Q.E.D.

**Proof of Proposition 2.2** We first prove \((i)\). If \( \lambda(1) \leq 0 \), the game only has a pooling equilibrium in which \( t_0 = 0, t_1 = 1 \), and \( a_i = 0 \). Otherwise, if
\[ \lambda^{-1}(0) \geq \rho^{-1}(x), \] the maximum size equilibrium has size 2 with \( t_0 = 0, t_2 = 1, \) \( a_1 = 0, \) and \( t_1 \) and \( a_2 \) simultaneously solve

\[
\begin{align*}
    a_2 & \in \arg\max_{a \in [0,1]} \left( U^R(a) - U^R(\delta) \right) \left( \lambda^{-1}(a) - \max\{t_1, \rho^{-1}(a)\} \right) \\
    U^S(t_1, a_2) & = U^S(t_1, 0). 
\end{align*}
\]

Finally, if \( \lambda^{-1}(0) < \rho^{-1}(x), \) the maximum size equilibrium has at least size 3 and can be constructed using the following algorithm:

1. Let \( \pi \) be some natural number. Define \( t_\pi = 1, t_{\pi-1} = \rho^{-1}(x), a_\pi = x, \) and assign value \( \bar{n} - 2 \) to counter \( k. \)
2. Define \( t_k \in (0,1] \) such that \( \rho(t_k) = \lambda(t_{k+1}) \). If such a \( t_k \) does not exist, go to step 3. Otherwise, \( a_{k+1} = \rho(t_k), k \leftarrow k - 1 \) and return to step 2.
3. Relabel \( t_{k+1}, \ldots, t_\pi \) and \( a_{k+1}, \ldots, a_\pi \) such that \( k \leftarrow 1, k+1 \leftarrow 2, \ldots, \bar{n} \leftarrow n. \)
   Let \( t_0 = 0, a_1 = 0, a_2 = \lambda(t_2) \) and define \( t_1 \) such that
   \[ U^S(t_1, 0) = U^S(t_1, a_2). \]

In step 2, \( a_{k+1} \) follows from the requirement that actions maximize expected utility \( (U^R(x) - U^R(\delta)) P \{ U^S(x, t) > U^S(\delta, t) \mid t \in [t_{k-1}, t_k] \}, \) which in our case implies

\[
a_{k+1} \in \arg\max_{a \in [0,1]} \left( U^R(a) - U^R(\delta) \right) \left( \min\{t_{k+1}, \lambda^{-1}(a)\} - \max\{t_k, \rho^{-1}(a)\} \right)
\]

Due to Lemma 2.1, for all \( k = 2, \ldots, n-1 \) it must hold that
\[
a_{k+1} \leq \lambda(t_{k+1}) = \rho(t_k), \]
and hence the condition reduces to

\[
a_{k+1} \in \arg\max_{a \leq \lambda(\delta)} \left( U^R(a) - U^R(\delta) \right) \left( \lambda^{-1}(a) - t_k \right).
\]

---

\[ \text{These two equations have a solution. Let } \tilde{t}_k(a) \text{ be the point where } U^S(a, t_k) = U^S(0, t_k). \]
Then there exists a continuous function \( a_k(a) \) such that \( a_k(a) \in \arg\max_{a \in [0,1]} \left( U^R(a) - U^S(\delta) \right) \left( \lambda^{-1}(a) - \max \left\{ \tilde{t}_k(a), \rho^{-1}(a) \right\} \right). \] Observe that \( a_k(0) \geq 0 \) and \( a_k(1) \leq \lambda(1). \) Hence \( a_k(a) \) has a fixed point on \([0,1].\)

30
If \( a_{k+1} = \lambda(t_{k+1}) \) for \( k \geq 2 \), \( a_{k+1} \) maximizes expected utility if

\[
\lambda^{-1}(a)\left(U^R(a) - U^R(\delta)\right) + \left(\lambda^{-1}(a) - t_k\right)U^{R'}(a) \geq 0 \text{ for } a \leq \lambda(t_{k+1})
\]

From (2.2) it follows that

\[
\lambda^{-1}(a)\left(U^R(a) - U^R(\delta)\right) + \left(\lambda^{-1}(a) - t_k\right)U^{R'}(a) = h(a) + \left(\rho^{-1}(a) - t_k\right)U^{R'}(a) \geq h(a) > 0 \text{ for } a < \rho(t_k) = \lambda(t_{k+1}) < \bar{x}. \]

Similarly, we can justify setting \( a_\pi = \bar{x} \) in step 1 by observing that \( \bar{x} \leq \lambda(1) \) and setting \( a_2 = \lambda(t_2) \) in step 3 by observing that \( a_{t+1} \leq \lambda(t_{k+1}) < \rho(t_k) \).

In addition, no equilibrium exists with an \( a_n > \bar{x} \). Suppose, to the contrary that \( a_n > \bar{x} \). Observe that \( a_n \leq \lambda(1) = \lambda(t_n) \). Furthermore, \( \rho(t_{n-1}) \leq a_n \) by Lemma 2.2, since the equilibrium size is at least 3. We can show that the first order condition for \( a_n \) to be optimal cannot be met, since \( h(a) < 0 \) for \( a \in (\bar{x}, a] \):

First, let \( \rho(t_{n-1}) < a_n \). Now, \( (\lambda^{-1}(a) - \rho^{-1}(a)) \)

\[
(U^R(a) - U^R(\delta)) + (\lambda^{-1}(a) - \rho^{-1}(a))U^{R'}(a) = h(a) - \rho^{-1}(a)(U^R(a) - U^R(\delta)) < 0 \text{ for } a \in (\max\{\bar{x}, \rho(t_{n-1})\}, a_n]. \]

(Observe that \( -\rho^{-1}(a)(U^R(a) - U^R(\delta)) < 0 \) for \( a \in (\max\{\bar{x}, \rho(t_{n-1})\}, a_n)] \). Second, let \( \rho(t_{n-1}) = a_n \). In this case, the derivative for the expected utility for \( a \in [\lambda(t_{n-1}), a_n] \) is \( \lambda^{-1}(a) \)

\[
(U^R(a) - U^R(\delta)) + (\lambda^{-1}(a) - t_{n-1})U^{R'}(a) = h(a) + (\rho^{-1}(a) - t_{n-1})U^{R'}(a). \]

Since the first term is strictly negative and the second term is 0 for \( a = a_n = \rho(t_{n-1}) \), there is an interval \( (a_n - \varepsilon, a_n] \) where the derivative is negative. Hence, \( a_n \) cannot be optimal if it is larger than \( \bar{x} \).

Because \( \lambda(t) \) and \( \rho(t) \) are monotonic by assumption, this algorithm results in the tightest partition which satisfies the equilibrium properties from Lemma 2.1 and Lemma 2.2 so that it implements an equilibrium of the highest possible size.

Suppose the Receiver’s bargaining power \( U^R(\delta) \) increases from \( \Pi \) to \( \bar{\Pi} \). Now
\[ \lambda^{-U}(\overline{x})\left(U^R(\overline{x}) - \Pi\right) + \left(\lambda^{-1}(\overline{x}) - \rho^{-1}(\overline{x})\right)U^{R'}(\overline{x}) \]
\[ = \lambda^{-U}(\overline{x})\left(U^R(\overline{x}) - \Pi\right) + \left(\lambda^{-1}(\overline{x}) - \rho^{-1}(\overline{x})\right)U^{R'}(\overline{x}) + \lambda^{-U}(\overline{x})(\Pi - \overline{\Pi}) < 0. \]

Hence, for the new threshold \( \overline{x} \) following from (2.2), it holds that \( \overline{x} < \overline{x} \). If \( \lambda^{-1}(0) \geq \rho^{-1}(\overline{x}) \), observe that \( \rho^{-1}(\overline{x}) \leq \rho^{-1}(\overline{x}) \leq \lambda^{-1}(0) \). If \( \lambda^{-1}(0) < \rho^{-1}(\overline{x}) \), then with the above algorithm we can get a maximum size equilibrium with partition \( 0 = \overline{t}_0 < \overline{t}_1 < \cdots < \overline{t}_n = 1 \). Furthermore, when running the above algorithm, all new threshold types will be lower than for the original game (\( \overline{t}_{n-k} < \overline{t}_{n-k} \) for all \( 1 \leq k < n \)). Therefore, the maximum size of the equilibrium decreases if the Receiver’s bargaining power increases.

Now we proof \((ii)\). Suppose that the Sender’s bargaining power \( U^S(\delta) \) increases from \( \Sigma \) to \( \hat{\Sigma} \). If \( \lambda(1, \Sigma) \leq 0 \), observe that \( \lambda(1, \hat{\Sigma}) > \lambda(1, \Sigma) \). If \( \lambda^{-1}(0, \Sigma) \geq \rho^{-1}(\overline{x}, \Sigma) \), note that \( \lambda^{-1}(0, \hat{\Sigma}) - \rho^{-1}(\overline{x}, \hat{\Sigma}) < \lambda^{-1}(0, \Sigma) - \rho^{-1}(\overline{x}, \Sigma) \), and \( \rho^{-1}(\overline{x}, \hat{\Sigma}) > \rho^{-1}(\overline{x}, \Sigma) \), since \( \lambda^{-1}(0, \hat{\Sigma}) < \lambda^{-1}(0, \Sigma) \) and \( \rho^{-1}(\overline{x}, \hat{\Sigma}) > \rho^{-1}(\overline{x}, \Sigma) \).

Finally, let \( \lambda^{-1}(0, \Sigma) < \rho^{-1}(\overline{x}, \Sigma) \). Analogous to the proof of \((i)\), we can show that increasing the Sender’s bargaining power results in a higher \( \overline{x} \):

\[ \lambda^{-U}(\overline{x}, \hat{\Sigma})\left(U^R(\overline{x}) - U^R(\delta)\right) + \left(\lambda^{-1}(\overline{x}, \hat{\Sigma}) - \rho^{-1}(\overline{x}, \hat{\Sigma})\right)U^{R'}(\overline{x}) \]
\[ = \lambda^{-U}(\overline{x}, \Sigma))\left(U^R(\overline{x}) - U^R(\delta)\right) + \left(\lambda^{-1}(\overline{x}, \Sigma)) - \rho^{-1}(\overline{x}, \Sigma))\right)U^{R'}(\overline{x}) \]
\[ + \left(\lambda^{-1}(\overline{x}, \hat{\Sigma}) - \rho^{-1}(\overline{x}, \Sigma)\right)U^R(\overline{x}) \]
\[ + \left(\lambda^{-1}(\overline{x}, \hat{\Sigma}) - \rho^{-1}(\overline{x}, \Sigma) + \rho^{-1}(\overline{x}, \Sigma)\right)U^{R'}(\overline{x}) \]
\[ > 0 \]

Observe for the third term in the middle expression that \( \lambda^{-U}(\overline{x}, \hat{\Sigma}) > \lambda^{-U}(\overline{x}, \Sigma) \) by \((A5)\); and for fourth term that \( \lambda^{-1}(\overline{x}, \hat{\Sigma}) < \lambda^{-1}(\overline{x}, \Sigma) \) and \( \rho^{-1}(\overline{x}, \hat{\Sigma}) > \rho^{-1}(\overline{x}, \Sigma) \).
due to the shrinking interval of points the Sender accepts when her power increases.

Hence, with the above algorithm we can get a new maximum size equilibrium partition \( 0 = \hat{i}_0 < \hat{i}_1 < \cdots < \hat{i}_n = 1 \). There are now two reasons why all threshold types will be higher than for the original game \( (\hat{i}_{n-k} > t_{n-k} \text{ for all } 1 \leq k < n) \), possibly resulting in extra equilibrium actions. First, the highest equilibrium action can be higher, since \( \hat{\bar{\pi}} > \bar{\pi} \). Second, the equilibrium actions will be closer together, since for each type it holds that \( \lambda(t, \Sigma) < \lambda(t, \hat{\Sigma}) \) and \( \rho(t, \Sigma) > \rho(t, \hat{\Sigma}) \). Hence, the number of equilibrium actions in the maximum size equilibrium is (weakly) higher when \( U^S(\delta) = \hat{\Sigma} \) than when \( U^S(\delta) = \Sigma \). Q.E.D.

**Proof Equilibrium Characterization Example section 2.2.** Observe that in the example assumptions (A2)-(A5) are satisfied. By Proposition 2.1, all equilibria are partition equilibria. Condition (iv) can be shown to follow Lemma 2.1 and Lemma 2.2, and the other conditions follow from the proof of Proposition 2.2. The characterization of the maximum equilibrium-size is a direct result from conditions (i)-(v). Observe that if \( d^S < 1, \bar{n} = \max \left\{ 2 \left[ \frac{\bar{\pi}}{2d^S} + 1 \right] \right\} \).