Essays on bargaining and strategic communication

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Chapter 5 Formal versus Informal Legislative Bargaining[65]

5.1 Introduction

The outcome of a legislative bargaining process is usually a result of both formal and informal bargaining. When parliament is in session, parliamentary procedures strictly govern what members can do at what time; hence, bargaining is highly formalized. After official sessions have been adjourned, however, bargaining often continues informally in offices, corridors and backrooms, where formal rules barely exist. That bargaining occurs at different levels of formality likely has historical and functional reasons: informal bargaining is arguably faster, whereas formal bargaining provides transparency and legitimacy to the democratic process. The question we address in this chapter is whether the formality of bargaining also systematically affects the bargaining outcome. This is important for understanding institutional choice and parliamentary procedures.

That the bargaining procedure can drastically affect the outcome has been recognized at least since the research boom on spatial voting in the late 1970s. If the procedure favors specific negotiators (e.g., through the order of voting, agenda-setting power, or proposal and voting rights), the outcome may crucially depend on it (e.g., McKelvey (1976; 1979), Schofield (1978), McCarty (2000)). The effect of formality seems different on at least two accounts, however. First, the difference between formal and informal bargaining cannot be captured in terms of changing the agenda or proposal or voting rights. Second, moving from a formal to informal bargaining or vice versa does not *prima facie* favor specific negotiators in any obvious way. The difference between the two is that informal bargaining provides much more flexibility to the bargaining parties. It does not give more flexibility to some parties than to others, however.

[65] This chapter is based on De Groot Ruiz, Ramer & Schram (2011).

Intuitively, the choice of how much weight to put on formal versus informal procedures may be determined by strategic considerations (Elster (1998), Stasavage (2004)). For instance, parties with a strong bargaining position may prefer backrooms and wish to reserve formal voting for well negotiated deals. On the other hand, parties with more extreme positions might prefer to avoid backrooms and follow the more formal procedures in order to allow their proposals to have a chance of success. This study intends to help us better understand such preferences.

More specifically, we compare two bargaining procedures, which we believe are representative for formal and informal bargaining in the field. To obtain a clean comparison, in both cases the bargaining procedure is 'fair' in the sense that it does not *prima facie* favor any negotiator. In this important way, our study differs from the legislative bargaining literature of the 1970s discussed above. The main question we address is whether the increased flexibility of the informal compared to the formal procedure affects the legislative outcome. In addition, if it does, does it do so for purely strategic reasons or do psychological effects play a role? To provide an answer to these questions we analyze legislative bargaining both theoretically and in a controlled laboratory experiment.

In the *informal procedure*, players can freely make and accept proposals at any time.[66] We did not choose for a completely unstructured face-to-face setting, but instead opted for a computerized setting where players can make and accept proposals in continuous time. This allows us to analyze the procedure as a non-cooperative game and to collect data on the bargaining process. We believe that the procedure is sufficiently unrestricted to be representative for informal bargaining like that which takes place in parliamentary backrooms. As we will see, the procedure is also not restrictive in the sense that it imposes no strategic constraints on the players. In the *formal procedure*, proposals and voting are regulated by a finite, closed-rule Baron-Ferejohn (1987) alternating offers

[66] In the 1970s, several experiments used informal bargaining procedures to compare the many cooperative solution concepts that had been proposed. Amongst the first were Fiorina & Plott (1978). The procedures used tend to be rather different from ours, however. More importantly, these studies do not compare their informal procedure to a formal procedure, nor do they model it as a non-cooperative game.

scheme.[67] Though there are potentially very many fair formal procedures, the Baron Ferejohn framework is widely taken to be a suitable model for studying formal legislative bargaining.[68] Our procedure is an elementary Baron-Ferejohn scheme.

We study the effects of formality in the context of a three-player legislative bargaining setting. The game is a straightforward extension of the standard one-dimensional median voter setting (Black, 1948; 1958) and has the following motivation. In the standard setting, the median player's ideal point is the unique (strong) core outcome irrespective of the location of others' ideal points (as long as they are on the same dimension). However, intuitively one may expect that the outcome of a legislative bargaining process or the coalition supporting this outcome is less stable if preferences are far apart −i.e., if polarization is strong−, even if the policy space seems unidimensional. One explanation is that the disagreement point may well lie outside of the line on which all policy proposals are defined. This is a point, we believe, that has hardly been appreciated in the literature.[69] Such a situation may occur for various reasons. First, a decision often involves a new type of policy or project so that the status quo may not fall in the space under consideration. Second, if the disagreement point consists in the termination of a project or a coalition, then it may involve significant transaction costs (e.g., involving new elections). If so, the disagreement point will be of a qualitatively different nature than the issue under negotiation.

[67] Baron & Ferejohn (1987) compare open and closed amendment rules and find distinct equilibria. Note that both settings constitute formal bargaining procedures.

[68] See, in addition to the work by Baron and Ferejohn, amongst many others, Banks & Austen Smith (1988), Merlo & Wilson (1995), McCarty (2000), Diermeier, Eraslan & Merlo (2003), Battaglini & Coate (2008) and Banks & Duggan (2000; 2006). These models tend to reach similar conclusions about agenda setting power. The first proposal is typically always accepted in equilibrium, since players know which proposals would subsequently be accepted or rejected. This gives a great advantage to the player chosen to make the first proposal (Palfrey 2006). Experiments, however, only partly corroborate these theoretical findings (McKelvey (1991), Diermeier & Morton (2005), Frechette, Morelli & Kagel (2005)). The first proposer does indeed have an advantage, but this is not as large as theoretically predicted and, in fact, the first proposal is often rejected, leading to 'delay.'

[69] The only exception we are aware of is Romer & Rosenthal (1978), who make a similar observation when they compare competitive majority rule to a controlled agenda setting mechanism. They do not consider polarization.

An example serves to illustrate the environments we are thinking of. Imagine a legislature that consists of three factions (doves, moderates and hawks) and is deciding on the renewal of a budget for an ongoing war. No single faction holds a majority and any coalition of two does. Doves prefer a reduction of the current war budget, moderates want no change and hawks would like an increase. Preferences are single-peaked with respect to budget revisions. The option to end the war ('retreat') serves as a disagreement point, which cannot simply be represented as a budget revision. (Retreating is qualitatively distinct from a reduction of the budget to zero and, furthermore, spending 5 billion on retreating is quite different from spending this amount on war efforts.) Preferences are such that all parties prefer some revisions to retreating. Polarization is then defined as the distance between the ideal revisions of the factions (relative to the attractiveness of retreating[70]) and captures the extent of divergence of interests. Polarization will most likely drive the stability of the outcome. If polarization is weak, then the factions' preferences lie close together and retreating is a relatively unattractive agreement. Hence, all coalitions will prefer the median ideal to retreating. In this case, we can use Black's Median Voter Theorem (1948, 1958) to predict that the moderates' ideal point will prevail. If the ideal revisions are very far apart, then polarization is strong. In this case retreating is relatively attractive and there are no revisions that any coalition prefers to retreating. With moderate polarization, we get a cyclical pattern. Both doves and hawks prefer retreating to the unaltered budget; however, moderates and doves prefer some negative revisions to retreating; and moderates and hawks prefer the unaltered budget to negative revisions. In this case, it is not clear what the legislature may decide. Intuitively, one may expect the moderates to have the highest bargaining power, as doves and hawks can only coordinate on retreating.

Our model and experimental design take the same basic form as this example. The point of departure is a bargaining *problem* in a median voter setting that is modified to have an exterior disagreement point. Then, we introduce the formal bargaining procedure to obtain the *formal* bargaining game and the

[70] In particular, relative to the distance between a player's ideal budget and the budget(s) she finds equally attractive as retreating.

informal bargaining procedure to obtain the *informal* bargaining game. We analyze cooperative solutions for the bargaining problem and for both games we derive non-cooperative equilibrium predictions.

The bargaining outcome will typically depend on specific characteristics of the bargaining problem (*i.c.*, the extent of polarization). As a consequence, the effect of the procedure may also depend on these characteristics. More specifically, the relevance of formality may arguably be dependent on whether or not the bargaining problem has a core.[71] Our setup allows us to obtain distinct outcomes with respect to the core by varying the level of polarization. When polarization is weak, the core consists of the median ideal and with strong polarization the core is the disagreement point. With moderate polarization, the core is empty.

The non-cooperative predictions depend on the procedure.[72] For the formal game, we derive a unique (refined) subgame perfect equilibrium (SPE) that converges (with the number of bargaining rounds) to the core element when this exists. It typically does not converge at all when the core is empty. In the informal game the disagreement point and all points between the players' ideal points can be supported as an SPE-outcome, irrespective of the extent of polarization. In addition, the equilibrium set cannot be refined in any standard way.

[71] When the core is empty, all outcomes can be supported by some agenda-setting institution (McKelvey (1976; 1979), Schofield (1978)). This is important, because the core will be empty unless extreme symmetry conditions are satisfied (Gillies (1953), Plott (1967), Riker (1980), Le Breton (1987), Saari (1997)). If the core is non-empty, the (non-cooperative) equilibrium outcome for many procedures tends to lie in it (Perry & Reny (1993; 1994), Baron (1996), Banks & Duggan (2000)). There is indeed experimental evidence on the stability of core-outcomes (Fiorina & Plott (1978), McKelvey & Ordeshook (1984), Palfrey (2006)). On the other hand, the outcome is sometimes sensitive to fairness considerations (Isaac & Plott (1978), Eavey & Miller (1984)). Structure may matter even if there is a core, for instance, if some procedures are considered fairer than others (Bolton, Brandts, & Ockenfels, 2005).

[72] Two theoretical breakthroughs have allowed us to overcome challenges in studying the effects of formality. First, for a long time many cooperative solution concepts have been advanced for situations in which the core is empty but none found broad theoretical and empirical support. Miller's (1980) uncovered set as a generalization of the core drew theoretical support (Shepsle & Weingast (1984), Banks (1985), Cox (1987), Feld, Grofman, Hartley, Kilgour & Miller (1987)). However, systematic empirical tests were problematic, as it was impossible to compute the uncovered set in most cases. By developing an algorithm to find the uncovered set, Bianco, Lynch, Miller & Sened (2006; 2008) managed to find solid empirical support using data from many old and new experiments. Second, continuous time bargaining has made the non-cooperative analysis of low-structure settings possible (Simon & Stinchcombe (1989), Perry & Reny (1993; 1994)).

Our interpretation is that informality offers so much strategic flexibility that strategic considerations alone cannot identify an outcome.

From our experiments, we have two main findings: polarization matters and formality matters. Polarization has a strong impact on the outcome. In accordance with theory, the median player is significantly worse off with moderate than with weak polarization. However, we find that increased polarization hurts the median player and does so even at weak levels when her most preferred outcome remains the unique core element. Our experimental findings suggests this is due to intra-coalitional fairness considerations. Such considerations become less important as negotiators gain more experience, however. After players have repeatedly played the game (in ever-changing groups), competition between coalitions is strengthened and the position of the median player also becomes stronger.

Our second result is that the formality of the bargaining procedure matters. The median player is significantly better off with the informal than with the formal procedure. One plausible cause seems to be that flexibility in making proposals at any time increases her ability to exploit her superior bargaining position, as observed by Drouvelis, Montero & Sefton (2010) in a different setting. This points to the more general idea that parties in a superior bargaining position will prefer institutions that impose less structure on the bargaining procedure.

The remainder of this chapter is organized as follows. Section 5.2 models the bargaining problem as a cooperative game and derives solutions for it. Section 5.3 describes and solves the non-cooperative games for the formal and informal bargaining procedures. Our experimental design is presented in section 5.4 and the experimental results are presented in section 5.5. Section 5.6 concludes.

5.2 The Bargaining Problem and Cooperative Solutions

Formally, the bargaining problem is represented by $\Gamma = \Gamma(N,\ Z, u_i, W)$ and consists of a finite set N of players, thought of as factions in a legislature; a collection W of subsets of N, thought of as winning coalitions; a set Z of alternatives; and utility functions u_i, one for each player $i \in N$ representing i's preferences over Z. Note that although winning coalitions have been specified, nothing as yet has been said about the decision making process itself. Procedures governing this process will be described and formalized in the next section.

In the bargaining problems studied here, three players (N = {1,2,3}) bargain over the set of alternatives represented by $Z \equiv \mathbb{R} \cup \delta$, with $\mathbb{R}$ denoting the set of real numbers and δ the disagreement point. Each player $i \in N$ has an ideal point $z_i \in \mathbb{R}$. Without loss of generality we normalize by setting $z_1 = -a < 0$, $z_2 = 0$, and $z_3 = b > 0$, with $b \geq a$. Hence, the ideal point of player 2, the *median* player, is $z = 0$. For players 1 and 3, the *wing* players, z_1 is normalized to lie closer to 0 than z_3. The distance, a, between the closer wing player and the median player will be interpreted as a measure of the *polarization* of players' preferences. We shall distinguish three cases of respectively, weak ($a \leq 1$), moderate ($1 < a < 2$), and strong polarization ($a \geq 2$).

Preferences of all players are single-peaked on $\mathbb{R}$ and represented by piecewise linear von Neumann-Morgenstern utility functions $u_i(z) = 1 - |z - z_i|$. We further assume that the utility attributed to the disagreement point is normalized at 0, that is, $u_i(\delta) = 0$, for all $i \in N$. Hence, each player has an open interval of outcomes with strictly positive values; to wit, $(-a - 1, -a + 1)$ for player 1, $(-1, 1)$ for player 2, and $(b - 1, b + 1)$ for player 3. Note that the endpoints of these intervals yield utility of 0, while the outcomes outside of these intervals are strictly worse for the respective players than the disagreement point δ. Figure 5.1 depicts this payoff structure.

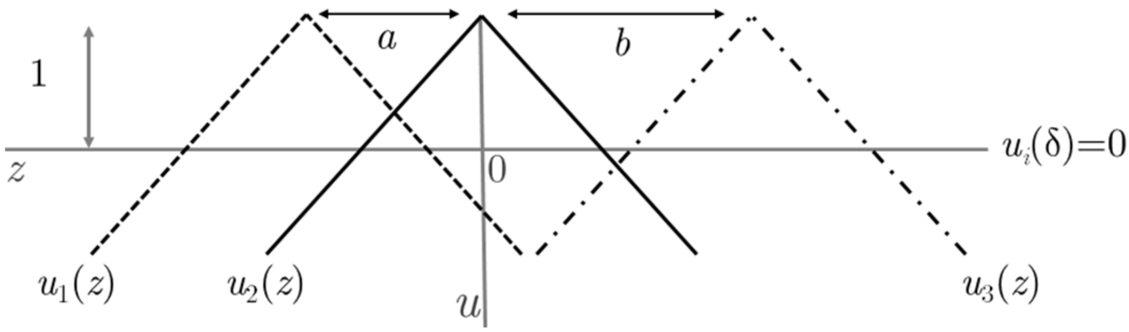


Figure 5.1
The figure shows the payoff structure of Γ.

As for winning coalitions W, we assume that any majority of two players can implement any $z \in Z$ as the outcome. This can be achieved in various ways, determined by the structure of the bargaining process (see section 5.3).

Γ can be regarded as a cooperative game and more precisely as a coalitional game without transferable payoff.[73] We start by defining the dominating and covering relations for any given $\Gamma(N, Z, u_i, W)$.

Definition 5.1 *Let $z, z' \in Z$. We say that*

(i) *z' dominates z, and write $z' \succ z$, if there is a winning coalition $M \in W$ such that all members in M strictly prefer z' to z;*

(ii) *z' covers z, and write $z'Cz$ if $z' \succ z$ and $z'' \succ z' \Rightarrow z'' \succ z$, for all $z'' \in Z$.*

The assumption that the disagreement point δ does not coincide with a point on the line $\mathbb{R}$ is important. If δ did lie in $\mathbb{R}$, then we would have a standard median voter setting and the median ideal 0 would dominate all other possible outcomes.

The counter-positive equivalent of Definition 5.1 reads as follows.

[73] Note that individual players cannot achieve any outcome by themselves and hence the payoffs available to singleton coalitions are not independent of the actions of the complementary coalition. Hence, under some definitions it would fall outside of the class of coalitional games with non-transferable utility.

Definition 5.2 *For an alternative $z \in Z$ we say that*

(i) *z is undominated if for every z' the set of players who strictly prefer z' over z is not a winning coalition;*

(ii) *z is uncovered if for every z' which dominates z there is z'' which dominates z' and does not dominate z.*

To obtain a solution we look at the core and, when the core is empty, at the uncovered set, which is a generalization of the core. These are defined by:

Definition 5.3

(i) *The core $\mathcal{C}(\Gamma)$ of Γ is the set of all points in Z that are undominated;*

(ii) *The uncovered set $\mathcal{U}(\Gamma)$ of Γ is the set of all points in Z that are uncovered.*

Intuitively, the uncovered set is a 'two step core.' If an outcome z is uncovered, there might be an outcome z' that dominates it, but this outcome z' will itself be dominated by an outcome z'' that does not dominate z. This means for instance, that forward-looking negotiators might be hesitant to move away from a point in the uncovered set. The uncovered set has several appealing theoretical properties. It is never empty, is equal to the core if the latter is nonempty and strict (Miller, 1980), contains all Von Neumann Morgenstern sets (McKelvey, 1986) and it subsumes the Banks set (Banks, 1985).[74] McKelvey argued that the uncovered set could be seen as a "useful generalization of the core when the core does not exist" (1986). Recently, this concept has also attracted significant empirical support (Bianco, Lynch, Miller & Sened 2006; 2008).

Whereas the uncovered set is typically large and difficult to calculate, in our bargaining problem it is small and simple.[75] Table 5.1 gives the core and uncovered set for our bargaining problems.

[74] These different papers prove these relations under slightly differing conditions.

[75] In addition, in our game the uncovered set is refined in a nice way by the *von Neumann Morgenstern set* and the *bargaining set*, both of which are unique. More details are given in Appendix 5.7.1.

TABLE 5.1

COOPERATIVE SOLUTIONS

Polarization		$\mathcal{C}(\Gamma)$	$\mathcal{U}(\Gamma)$
weak: $a \leq 1$		0	0
moderate: $1<a<2$	$a < b$	$\varnothing$	$\{-a + 1, 0, \delta\}$
	$a = b$		$\{-a + 1, 0, b - 1, \delta\}$
strong: $a \geq 2$		δ	δ

Notes: For $\Gamma=\Gamma$ (N, Z, u_i, W), the table gives the elements in the core ($\mathcal{C}(\Gamma)$)and uncovered set ($\mathcal{U}(\Gamma)$) for the levels of polarization distinguished in the first column.

When polarization is weak or strong, the core is nonempty and coincides with the uncovered set. It always holds that players 1 and 2 prefer the median ideal to all points right of it, whereas players 2 and 3 prefer the median ideal to all points left of it. In addition, when polarization is weak, players 1 and 2 also prefer the median ideal to the disagreement point. Hence it dominates all points, and, as a consequence, also covers them all. Thus, if polarization is weak, the median ideal is the singleton core and uncovered set. When polarization is strong, no point on the line exists that two players prefer to the median ideal. Hence, the disagreement point dominates all points on the line and constitutes the core and uncovered set. From a behavioral perspective, this case seems less interesting, as no alternative outcome to the disagreement point seems viable.

When polarization is moderate, we get a circular dominance pattern and the core is empty: The median ideal dominates all other points on the line, but is dominated by the disagreement point. The disagreement point itself is in turn dominated by some points on the line (which are dominated by the median ideal etc.). This also means that the median ideal and the disagreement point do not cover each other; they are uncovered. Furthermore, the point closest to the median player that is not dominated by the disagreement point, $-a + 1$, is uncovered as well. (The same holds for $a - 1$ if $a = b$). Consequently, in this case the uncovered set consists of three or four elements: the median ideal, the disagreement point, $-a + 1$ and, if $a = b$, $a - 1$.

For technical details on the dominance relations and the cooperative solutions, see Appendix 5.7.1.

5.3 Formal and Informal Procedure

We now impose procedures on the bargaining problem described in section 5.2 and analyze the resulting non-cooperative games. One may think of this as the legislature selecting exactly one element of the set of feasible alternatives Z by means of a procedure established (or agreed upon) in advance. Formally, such a procedure can be regarded as an extensive game. We shall present and discuss two such games, exemplary for two important frameworks for legislative bargaining, voting and open bargaining. We do so by introducing a 'formal' and an 'informal' bargaining procedure to $\Gamma = \Gamma(N, Z, u_i, W)$.

5.3.1 Formal procedure

We begin with a *formal procedure* for the selection of an outcome in Z, represented by a sequential voting game Γ_T^F. The game – similar to that in Baron & Ferejohn (1989) – consists of multiple rounds, with a predetermined maximum of T rounds. Each round comprises of three stages.

At stage 1, one player is randomly selected with equal probability across players. At stage 2, the selected player i submits her proposal. At stage 3, players vote independently on this proposal. It becomes the final choice if it is accepted by at least two players. Because the player who submitted it supports her own proposal (by assumption), support by one other player suffices to pass the proposal and end the game. Whenever the proposal is voted down by the two other players, it is off the table and the game proceeds to round $t + 1$, where a player is selected to submit a new proposal, and so on. If the game reaches round T and the final proposal is also rejected, the game ends and the disagreement point δ is implemented.

For any given T and bargaining problem Γ, the game Γ_T^F is an extensive form game of finite length with random moves by nature at the first stage and simultaneous moves by all three players at the third stage of each round.[76] Actions played at any stage are observed before the next stage or round begins. In general, players' best responses will not be unique so one can expect multiple Subgame Perfect Nash Equilibria (SPE), possibly with distinct outcomes. In order to select a single best response at each stage, and ultimately to select consistently a single SPE for every given T, we assume players vote as if they are pivotal and shall adopt a number of tie-breaking rules known from the literature on voting games (Baron & Ferejohn (1989), Baron (1996), Banks & Duggan (2006)):

(i) A player accepts a proposal submitted in round t if it provides to her a payoff equal to her expected equilibrium payoff in the subgame beginning at stage 1 of round $t+1$.

(ii) Whenever a player has two best proposals, one that will be accepted and one that will be rejected, she submits the proposal that will be accepted.

(iii) Whenever $-c$ and c are both best proposals for player 2 she submits each of them with an equal probability.

(iv) Whenever δ and $c \in \mathbb{R}$ are both best proposals for a player she submits δ.

The first assumption guarantees that an SPE exists, the remaining assumptions imply that it is unique.[77] From now on, we will refer to this equilibrium simply as 'the equilibrium' of Γ_T^F. Note that equilibrium strategies in a round depend neither on what happened in previous rounds nor on the total number of rounds (T), but only on the number of rounds left before the game ends.

The equilibrium outcome of Γ_T^F can be characterized by the probability distribution of the equilibrium outcomes $\mu^T : Z \to [0,1]$.[78] If all equilibrium proposals are accepted in the first round, μ^T simply allots equal probability to each of the players' equilibrium proposals in the first round. There is, however, the

[76] Though the player that made a proposal has an action set consisting of one element ('accept') at the 3rd stage.

[77] Assumption 4 is particular to our game (as $\delta \notin \mathbb{R}$) and is a convenient tie breaking rule for the case $a = 2$. For other parameter values it is not essential which rule one assumes for these ties.

[78] μ^T is a probability mass function. As we show in Appendix 5.7.2, μ^T has countable support.

possibility of delay. Though for some values of parameters *a, b* and T, all three equilibrium proposals are immediately accepted, for other values an equilibrium proposal, in particular that of player 3, will be rejected. The equilibrium outcome of Γ_T^F may depend in complicated ways on the number of bargaining rounds, T.[79] Hence, our approach is to look at whether the equilibrium outcome converges as T increases. We say that for given values of a and b the equilibrium outcome *converges* if there exists a probability distribution $\mu_{a,b}^*$ on Z such that $\lim_{T\to\infty} \mu^T = \mu_{a,b}^*$ in the sense of weak convergence of probability measures (Billingsley, 1999). If no such limit exists, then we say that the outcome does not converge. The equilibrium outcome converges to some single $z \in Z$, if $\mu_{a,b}^*$ is concentrated at $z \in Z$, i.e. $\mu_{a,b}^*(z) = 1$. This allows us to summarize the SPE of Γ_T^F in the following proposition:

Proposition 5.1

(i) *If* $0 \leq a < 1$ *or* $a = b = 1$, *the equilibrium outcome converges to* 0. *For T sufficiently large, the first round proposals are accepted without delay.*

(ii) *If* $1 \leq a \leq b < 2$ *and* $b > 1$, *the equilibrium outcome does not converge except for some patches of the values of a and b, and never to a single outcome in Z.*

(iii) *If* $a \geq 2$, *the equilibrium outcome is* δ *for sufficiently large T.*

Proof: The proof is given in Appendix 5.7.2. It is a terse and long exercise in backward induction, since the non-convexity of the outcome set precludes the use of standard techniques and results.

[79] Numerically, the equilibrium outcome can be calculated for each value of a, b and T. Simulations show that it always appears to converge to a cycle in T, the length of which depends in erratic ways on a and b. To illustrate, Appendix 5.7.2 provides simulation results that show how the period of the cycles depends on a and b.

Comparing Proposition 5.1 to Table 5.1, we conclude that the equilibrium outcome converges to the single element of the core for $a < 1$ and $a \geq 2$, and that it does not converge to a single outcome if the core is empty.[80]

Proposition 5.1 specifies the parameter configurations for which we have robust predictions. Note that legislative bargaining in the field can go through a substantial number of rounds, but that the number is often finite (due to time constraints, for example). If the outcome converges as the number of rounds increases, this allows for a stable prediction for such cases. If there is a sufficient number of rounds, the prediction will not depend significantly on the exact number, on who gets to propose first or on the precise values of a and b. If the outcome does not converge, however, then the outcome will depend on all these parameters, and typically in a very sensitive and non-linear way.[81] For instance, in our experiment we will use $T = 10$. For this case, we can derive a unique prediction, whether the core is empty or not (See Table 5.3 below). When the core is nonempty, it does not matter much whether T is 9, 10, 11, 20 or 100 or whether a=.4 or a=.41 or whether player 1 or player 3 starts.[82,83] When the core is empty, however, then the outcome depends crucially on all these parameters. Though we have predictions for the specific parameters of our experiment, they will be strongly affected by small changes. This makes the outcome hard to predict in practice. In addition, this arguably makes it more difficult for players to coordinate on the equilibrium. Hence, Proposition 5.1 helps us understand when specific predictions for finite T are robust (i.e., this is the case for (i) and (iii)).

5.3.2 Informal Procedure

Now, we turn to the informal bargaining procedure where all players can make and accept proposals in continuous time, which we denote by Γ_T^I. The

[80] In the non-generic case of $a = 1$ and $b > 1$ the core consists of 0 but the equilibrium outcomes do not converge as T goes to infinity.

[81] See footnote 79.

[82] In addition, in the field (as in our experiment) the outcome set will not be exactly continuous but discrete and very fine-mazed. In this case, the outcome even converges in a finite number of rounds if the core is non-empty.

[83] Parameter values exist (especially those close to values where the core is empty), where more than 10 rounds are required to see converging behavior.

basic tenet of Γ_T^I is a triple of proposals $\left(p_1^t, p_2^t, p_3^t\right)$ on the table at all times $t \in [0, T]$ until one of the proposals is accepted lest the game ends with the disagreement point δ at time T. It has by now been well established that such a game with continuous time cannot be solved without further assumptions, however (Simon & Stinchcombe (1989), Perry & Reny (1993; 1994)). Drawing on Perry & Reny's game (1993), we introduce a reaction and waiting time. Our game requires different rules than those used by Perry & Reny (1993; 1994), nonetheless, as it concerns a more complex bargaining problem. In contrast to Perry & Reny (1993), Γ is not a two-player game; Γ is different from the bargaining problems Perry & Reny (1994) consider in that Γ can have an empty core and does not have transferable payoffs.

The rules of the game are as follows. Player i can either be silent (ς), $p_i^t = \varsigma$, have a proposal on the table, $p_i^t = z, z \in Z$, or accept the proposal of another player $j, p_i^t = a_j$. For each player i, p_i^t as a function of time is assumed to be piecewise constant and to be right-continuous. We say that a player moves at time t when $p_i^t \neq p_i^{t-} \equiv \lim_{s \uparrow t} p_i^s$. It is natural to only allow only such discrete changes in proposals, since actual negotiations (face-to-face or computerized) consist of discrete actions ('I propose x', 'I accept', 'I withdraw y') in a continuous time.

Players start with no proposal on the table: $p_i^{0-} \equiv \varsigma$. We introduce a uniform reaction and waiting time, ρ. In particular, if some player moves at time t, no player can move at $t \in (s, s+\rho)$.[84] This models the fact that players cannot react (or act again) immediately after a player has moved and that the time it takes to process information, make a decision and execute it is roughly the same for all players at all times. Essential is that we allow ρ to be arbitrarily small.[85]

[84] We also allow a player to resubmit her old proposal and induce the reaction time ρ. Intuitively, this is the strategic move "I still propose z." Technically, if $p_i^{t-} = z$, then we define the move that resubmits the same proposal as $p_i^t = z^*$. (We set $z^{**} \equiv z$, such that if $p_i^{t-} = z^*$, then resubmitting z is $p_i^t = z$.) If accepted, z^* just induces z as outcome.

[85] In our model the waiting time is exactly equal to the reaction time, unlike in Perry and Reny (1993). What is important is that we exclude the possibility of making a proposal and then withdrawing it before it can be accepted. For this purpose, any reaction time smaller than

Player i accepts j's proposal by setting $p_i^t = a_j$. In order to ensure that a player knows which proposal she accepts, if player i plays $p_i^t = a_j$ she accepts p_j^{t-}. To ensure a unique, well-defined outcome a player i can only accept a proposal at time t if she is silent herself ($p_i^{t-} = \varsigma$).[86] In addition, one can (naturally) not accept a proposal from a player who is silent. As soon as a proposal has been accepted, the accepted proposal is the outcome of the game. If no proposal has been accepted before or at $t = T$, then the outcome is the disagreement point δ. After a proposal has been accepted or when $t > T$, no player can move anymore. Formally, we always let the game end at $t = T + \rho$.[87]

To define strategies and derive equilibria, we need to introduce some further definitions.

Definition 5.4

(*i*) A *history, h,* consists of a specification of $\{p_1^t, p_2^t, p_3^t\}$ for $t \in [0, \tau(h))$, where $\tau(h) \in [0, T + \rho]$ is the history's end.

(*ii*) $\tilde{t}(h) \equiv \sup\{t < \tau : p_i^t(h) \neq p_i^{t-}(h)\}$ is the *last moment* before $\tau(h)$ that any *player moved* (if no player has moved, we conveniently set $\tilde{t}(h) \equiv -\rho$).

(*iii*) A proposal function is *right-continuous and piecewise linear* if for all $\tau < T$ and each i, there is an $\varepsilon > 0$ such that $p_i^s(h) = p_i^t(h) \forall s \in [t, t + \varepsilon)$. As discussed above, we will only consider such proposals.

(*iv*) History h is an active history if $\tilde{t}(h) + \rho \leq \tau(h) \leq T$ and no proposal has been accepted.

the waiting time would suffice; since there is no time-discounting, this would make the analysis unnecessarily more complex.

[86] If a player were not required to be silent when accepting a proposal, her proposal could be accepted while she were accepting another. Essentially, we are requiring that a player removes her own proposal before accepting another. Given that ρ can be arbitrarily small, this assumption is not behaviorally restrictive.

[87] This is because at time t, it is not yet known what happens at t itself. Any time after T would do.

(*v*) The set of admissible proposals at each history for player i is called $Z_i(h)$, with $Z_i(h_\tau) \subset \hat{Z}$, where $\hat{Z} = Z \cup \{\varsigma, a_1, a_2, a_3\}$.[88] $Z_i(h)$ is subject to the following restrictions:

a. $Z_i(h_\tau) = \{p_i^{t-}\}$ if h_τ is not an active history

b. $Z \cup \{\varsigma, p_i^{\tau-}(h_\tau)^*\} \subset Z_i(h_\tau)$ if h_τ is an active history

c. $a_j \in Z_i(h_\tau)$ if h_τ is an active history $i \neq j$, $p_j^{\tau-}(h_\tau) \neq \varsigma$ and $p_i^{\tau-}(h_\tau) = \varsigma$

(*vi*) The *outcome* of a history h is $z(h)$:

$$z(h) \equiv \begin{cases} \text{The unique element of } \{p_j^{t-}(h)\text{: } p_i^t(h) = a_j \text{ for some } t < \tau(h)\} \text{ if it exists} \\ \varnothing \text{ if } \{p_j^{t-}(h)\text{: } p_i^t(h) = a_j \text{ for some } t < \tau(h)\} = \varnothing \text{ and } \tau(h) \leq T \\ \delta \text{ if } \{p_j^{t-}(h)\text{: } p_i^t(h) = a_j \text{ for some } t < \tau(h)\} = \varnothing \text{ and } \tau(h) > T \end{cases}$$

(*vii*) Let $\overline{H}$ be the set of histories in which all proposals are right-continuous and piecewise linear and admissible and have $\tau(h) = T + \rho$. Call any $\overline{h} \in \overline{H}$ a *resolved history.*

(*viii*) Let H be the set of histories in which all proposals are right-continuous, piecewise linear and admissible, and that have no outcome ($z(h_\tau) = \varnothing$). Any $h_\tau \in H$ is called an *unresolved history.*

(*ix*) h is a *subhistory* of h', or $h \subseteq h'$, if $\tau(h) \leq \tau(h')$ and $p_i^t(h) = p_i^t(h')$ for each i and for all $t \in [0, \tau(h))$ Furthermore, h is a '*proper subhistory*' of h', or $h \subset h'$, if $h \subseteq h'$ and $\tau(h) < \tau(h')$.

(*x*) $h_s(h)$ is the unique history such that $\tau(h_s(h)) = s$, $h \subseteq h_s(h)$ and no player moves in $[\tau(h), s)$.

(*xi*) $H_{\tau'} \equiv \{h \in H : \tau(h) = \tau'\}$ is the set of *histories ending at* τ'.

(*xii*) $\tilde{H} \equiv \{h \in H : \tau(h) \geq \tilde{t}(h) + \rho\}$ is the set of *active histories* in *H.*

[88] Because we allow players to repeat their previous proposal, we actually have $\hat{Z} = Z \cup Z^* \cup \{\varsigma, \varsigma^*, a_1, a_2, a_3\}$, where Z^* includes all asterisked outcomes (cf. fn 84).

A strategy of player i is a mapping from the set of unresolved histories to the set of admissible proposals, $\sigma_i : H \to \hat{Z}$. It meets the following two conditions:

(S1) For all $h_\tau \in H$, $\sigma_i(h_\tau) \in Z_i(h_\tau)$

(S2) For all $h_\tau \in H$, a time $\varepsilon > 0$ exists such that $\sigma_i(h_{\tau'}) = \sigma_i(h_\tau) \forall h_{\tau'}$ with $h_\tau \subseteq h_{\tau'} \subseteq h_{\tau+\varepsilon}(h_\tau)$.

(S1) ensures moves are in the set of admissible proposals. (S2) ensures that the strategies result in well-defined histories with piecewise constant, right-continuous paths. By Σ we denote the set of all profiles that meet (S1) and (S2).

Proposition 5.2 shows that $\sigma \in \Sigma$ induces from any well-defined unresolved history $h_\tau \in H$ a unique well-defined resolved history $\bar{h} \in \bar{H}$:

Proposition 5.2 *The game Γ_T^L is a well-defined mapping $\Gamma_T^I : H \times \Sigma \to \bar{H}_{T+\rho}$.*

Proof: See Appendix 5.7.3.

The intuition underlying the proof is that for a given strategy profile and unresolved history h_τ, either no player would do anything after h_τ until the game ends or one can find a well-defined first action at or after τ. This action yields a new history, which is either a resolved history or an unresolved history. Hence, one can repeat this procedure of searching for the first action until it yields a resolved history.

We can now define subgames and equilibria of Γ_T^I. A subgame $\Gamma_{T\backslash h_\tau}^I : \{h \in H : h \supseteq h_\tau\} \times \Sigma \to \bar{H}_T$, represents the game that starts at h_τ. Let $U_i(\sigma_i;\sigma_{-i} \mid h_\tau) \equiv u_i(z(\Gamma_{T\backslash h_\tau}^I(h_\tau,\sigma)))$ be the payoff player i receives from the outcome that σ induces on h_τ and let $U_i(\sigma_i;\sigma_{-i}) \equiv U_i(\sigma_i;\sigma_{-i} \mid h_0)$. A Nash Equilibrium of $\Gamma_{T\backslash h_\tau}^L$ is a strategy profile σ such that $U_i(\sigma_i;\sigma_{-i} \mid h_\tau) \geq U_i(\sigma_i';\sigma_{-i} \mid h_\tau)$ for each i and all $\sigma_i' \in \Sigma_i$. σ is a Subgame Perfect Equilibrium (SPE) of Γ_T^I if it is a Nash Equilibrium in all of its subgames: $U_i(\sigma_i;\sigma_{-i} \mid h_\tau) \geq U_i(\sigma_i';\sigma_{-i} \mid h_\tau)$ for each i and all $\sigma_i' \in \Sigma_i$ and in all $h_\tau \in H$.

Finally, we are able to show by an explicit construction that every point of the interval $[-a, b] \subset R$ and the disagreement point δ itself can be an outcome of a subgame-perfect equilibrium of Γ_T^I. This yields Proposition 5.3.

Proposition 5.3 *The set of SPE outcomes contains* $[-a, b] \cup \{\delta\}$ *for every continuous game* Γ_T^I *with* $T > \rho$.

Proof: See Appendix 5.7.3.

Many of the SPEs in Proposition 5.3 may seem unintuitive. For instance, the at first sight unlikely outcome b (which is the ideal point of the wing player furthest from the median player) can be supported by an equilibrium in which players 1 and 2 always propose b. Player 3 will accept b as soon as she can, while players 1 and 2 cannot individually profitably deviate, as the other player will anyhow propose b, which will be readily accepted by player 3.

This large set of SPE cannot be refined in any standard way. A simple set of standard tie-breaking rules, such as those used for Γ_T^F, would be much too weak to have any effect. Stationarity only has a very small bite (b and δ can, for instance, be sustained by stationary strategies for any a and b). A procedure of iterated elimination of weakly dominated strategies, as proposed by Moulin (1979), and used by Baron & Ferejohn (1989), is of little avail in our case, due to the fact that typically in many subgames of Γ_T^I multiple actions per player will survive, so that hardly any strategy will eventually be eliminated in the complete game. In addition, a refinement based on trembling-hand perfection (Selten, 1975), if it can be adapted to continuous time and space, will not eliminate these unintuitive equilibrium-outcomes either. For instance, the reason that player 1 does not propose 0 instead of b in the equilibrium discussed above, could be that she is afraid that player 2 might tremble and play $b + \varepsilon$ instead of b. Hence, there are few strategic restrictions on the equilibrium strategies in the informal game.

Nonetheless, there are some points in the outcome set that strike us as more 'likely' than others (for instance points in the uncovered set). Their plausibility might be the result of their *focal* nature due to the constellation of preferences and winning coalitions (which is captured by the cooperative solution concepts of the bargaining problem).

5.3.3 Overview of Theoretical Results

In Table 5.2 we summarize the main results obtained for the outcomes of the equilibria of the two strategic games analyzed in this section, Γ_T^F and Γ_T^I, together with the solutions of the cooperative game Γ.

TABLE 5.2
THEORETICAL RESULTS MAIN CASES

		Polarization		
		Weak: $a<1$	Moderate: $1<a<2$	Strong: $a>2$
Cooperative Game Γ (N, X, u_i, W)	Core	0	$\varnothing$	δ
	Uncovered Set	0	$\{1-a, 0, a-1^{*}, \delta\}$	δ
Formal (convergence), Γ_T^F		0	No convergence**	δ
Informal (for $T \geq \rho$), Γ_T^I			$[-a\ ,\ b] \cup \{\delta\}^{***}$	

Notes: Cells give the solution concepts for the two games as derived above for the three generic cases of polarization ($a<1$, $1<a<2$, $a>2$). Solution concepts used are described in the previous subsections.

*$a-1$ is only included if $a = b$; ** There are some exceptions, in which case the outcome may converge but never to a single outcome in Z. ***Outcomes can also lie in the interval $[\max\{b-1, -b\}, a]$ if $\max\{b-1, -b\} < -a$.

5.4 Experimental Procedures and Design

The experiment was run at the Center for Research in Experimental Economics and political Decision making (CREED) of the University of Amsterdam. It was computerized using z-Tree (Fischbacher, 2007). An English translation of the Dutch experimental instructions is provided in Appendix 5.7.4. Subjects had to correctly answer a quiz before proceeding to the experiment. In total, 102 subjects were recruited from CREED's subject pool.[89] They earned a €5 show-

[89] The subject pool consists of around 2000 individuals who have voluntarily registered. Almost all of these are undergraduate students at the University of Amsterdam, of which approxi-

up fee plus on average €11.65 in 90-120 minutes. In the experiment, payoffs are in 'francs'. The cumulative earnings in francs are exchanged for euros at the end of the session at a rate of 1 euro per 10 francs.

We ran six sessions. Each session consists of 24 periods. In each period subjects are rematched in groups of three. We use matching groups of 6 or 9 subjects.[90] After groups have been formed, subjects are randomly appointed the role of player 'A', 'B', or 'C'. To avoid focality, players do not play the normalized game described above (e.g. B's position is not set equal to 0 and it is not necessarily the case that A's ideal value is closer to B's value than C's is). For analysis, the bargaining problem subjects face can easily be normalized to correspond to the model of section 5.2.

Each player is appointed an 'ideal value', which is an integer between 0 and 100 (inclusive).[91] Player A's ideal value is always the smallest and player C's the largest. Players know all ideal values. Each group has to choose an integer between 0 and 100 (inclusive). If the group chooses a player's ideal value, this player receives 20 francs. For every unit further from her ideal point, one franc is subtracted from her earnings. Hence, earnings are negative for a player if the group chooses a number that is more than 20 larger or smaller than her ideal value. To avoid negative total earnings at the end of the experiment, each subject starts with a positive balance of 100 francs.[92]

The procedure was varied in a between subjects design, which consisted of a *formal* and an *informal* treatment. We ran three sessions per treatment, so that we have six matching groups per treatment. In both treatments, proposals are

mately 40% major in Economics or Business. When the experiment was announced, all received an invitation to sign up and participation was on a first-come, first-serve basis.

[90] Subjects are told that they are in a session with 15 or 18 participants and will be rematched in every round.

[91] The restriction to natural numbers is done for practical purposes. It is sufficiently finemazed to avoid affecting the equilibria described in the previous section in any relevant way. One difference is that if the outcome converges in T to 0, it converges in finite time and, for our parameter values, in fact it already converges for $T < 10$. In Table 5.3, we report the equilibria of the high structure game with a discretized line and 10 rounds (which we calculated numerically).

[92] Still, five subjects ended their session with negative earnings. They were sent off with no pay other than the €5 show-up fee. Data which involved these individuals were deleted from the sample due to possible incentive problems. Including these individuals makes little difference, except that statistical results become somewhat less conclusive due to the extreme behavior of one subject who would have earned −14.70 euros and showed erratic behavior after his earnings became negative.

made consisting of any integer between 0 and 100 (inclusive) or δ (called "end"). If the disagreement point is the outcome, each player receives a payoff of zero.[93]

In the *formal* treatment, subjects play the game Γ_T^F of section 5.3.1 with $T = 10$; i.e., negotiations were held for a maximum of 10 rounds per period. We use the strategy method (for proposals) where in every round every player is asked to make a proposal. One proposal is randomly selected and put to the other two group members to vote on. If at least one of the two accepts this proposal, it becomes the group choice and the game ends. If the proposal is rejected by both players, a new round begins, unless 10 rounds have been finished. In the latter case, the outcome is the disagreement point.

In the *informal* treatment, subjects are given two-and-a-half minutes to reach an agreement. At any time, any group member can make a proposal, change a previous own proposal or accept one made by another member. They do so by typing a number (proposal) and clicking on an ok-button, respectively selecting another member's proposal and clicking on an accept-button. As soon as a proposal has been accepted, this becomes the group choice for the period and negotiations are finished. If no proposal is accepted within the time-span, the disagreement point is the outcome. This treatment follows our informal model closely. We do not impose a reaction or waiting times. In the model, these times do not represent procedural restrictions but rather cognitive and physical restrictions, which are allowed to be arbitrarily small. Nor do we require that players need to retract their own proposal before they can accept a proposal.[94] Otherwise, the rules are identical as in Γ_T^L.

To help a subject in determining her choices during the negotiations for a group decision, her screen always shows a history of previous rounds,[95] current

[93] In both treatments, a round of bilateral messages precedes group negotiations: each player may send a private message (consisting of a number between 0 and 100 or δ) to either or both other player(s). This is meant to reflect pre-negotiation lobbying. This cheap-talk does not affect the theoretical analysis presented in section 5.3.

[94] In the model, we need this requirement to guarantee a well-defined outcome. In the experiment, we do not require this, as the probability that a player accepts a proposal at the exact same time her own proposal is accepted is zero. Implementing the additional restriction would not make a big behavioral difference (it would take two mouse clicks instead of one to accept a proposal), but would make the interface unnecessarily more cumbersome.

[95] The history showed for each previous round what happened in the group the player participated in. In particular, it specified (*i*) the ideal point for each role,(*ii*) the role the player herself had, (*iii*) the outcome and (*iv*) the earnings for all three roles.

earnings, a scrollable help-box with instructions, a history of offers in the current round and a device to calculate payoffs for any hypothetical proposal.

Polarization is varied in a within-subjects design by using 12 sets of ideal values. Each set was used once in the *first half* (first 12 periods) and once in the *second half* (last 12 periods) of a session. The sets were chosen such that for the normalized parameters there were six with $a<1$ and six with $1<a<2$ (*cf.* Table 5.3, below). We chose not to use parameters with $a\geq 2$ in the experiment because it seems obvious that participants will always agree on the disagreement point of no earnings if there is no outcome where at least two players have positive earnings.

Table 5.3 gives the (normalized) parameters used, the periods in which they were used and the theoretical predictions for each set. We can conclude a few things from this table about the predictions of the cooperative solutions and the equilibrium of the formal game. First, as long as $a<1$ (weak polarization) the median's payoff does not depend on the level of polarization. Second, when $a>1$ the median's payoff can be expected to decrease with polarization. Third, when the core is empty, there are many instances where the SPE-outcomes of the formal game are not in the uncovered set.

TABLE 5.3
PARAMETERS AND PREDICTIONS

Parameters		Periods	Cooperative[1]	Formal game[2]			Informal game
a	b			1 starts	2 starts	3 starts	
0.2	1.4	5, 23	0	0	0	0	$[-0.2, 1.4]\cup\{\delta\}$
0.5	1.1	3, 21	0	0	0	0	$[-0.5, 1.1]\cup\{\delta\}$
0.5	1.7	11, 13	0	0	0	0	$[-0.5, 1.7]\cup\{\delta\}$
0.8	0.8	1, 19	0	0	0	0	$[-0.8, 0.8]\cup\{\delta\}$
0.8	1.4	9, 15	0	0	0	0	$[-0.8, 1.4]\cup\{\delta\}$
0.8	2	8, 24	0	0	0	0	$[-0.8, 2.0]\cup\{\delta\}$
1.1	1.1	7, 17	$\{-0.1, 0, 0.1,\delta\}$	–0.3	0	0.3	$[-1.1, 1.1]\cup\{\delta\}$
1.1	1.7	10, 22	$\{-0.1, 0, \delta\}$	–0.4	–0.15	0.4	$[-1.1, 1.7]\cup\{\delta\}$
1.1	2.3	2, 14	$\{-0.1, 0, \delta\}$	–0.45	–0.15	0.45	$[-1.1, 2.3]\cup\{\delta\}$
1.4	1.4	12, 20	$\{-0.4, 0, 0.4,\delta\}$	δ	0	δ	$[-1.4, 1.4]\cup\{\delta\}$
1.4	2	4, 16	$\{-0.4, 0, \delta\}$	–0.55	0.2	δ	$[-1.4, 2.0]\cup\{\delta\}$
1.7	1.7	6, 18	$\{-0.7, 0, 0.7,\delta\}$	δ	$\{-0.5,0.5\}$	δ	$[-1.7, 1.7]\cup\{\delta\}$

Notes: Cells give the theoretical prediction (*cf.* Table 5.1) applied to the experimental parameter set. The predictions for the formal procedure are for the game with a discretized outcome set and 10 rounds, as played in the experiment.

[1]For $a<1$ the prediction is given by the core (=uncovered set); for $a>1$ it is given by the uncovered set.

[2]The column gives the (refined) SPE conditional on the player chosen to make the first offer. Player 2 is defined as the median position, 1 is the other player closest to the median.

5.5 Experimental Results

We focus on how formality (and its interaction with polarization) affects the ability of the median player to reach agreements close to her ideal point. All tests used below are two-sided and non-parametric, using each matching group (of six or nine participants) as one independent data point. We use the Wilcoxon signed rank tests for within comparisons and the Mann-Whitney test for between comparisons. Whenever we report statistically significant results for pooled *Formal/Informal* data only, the results are also significant at the 0.05 level for the disaggregated data where *Formal* and *Informal* are tested separately. *P*-values that are (unrounded) smaller than 0.05 are marked by an asterisk.

5.5.1 Earnings

We start with players' earnings from negotiations. Figure 5.2 shows the payoffs for different levels of polarization (captured by a) and the two treatments. Most relevant are the payoffs of the median player. First consider the effect of polarization. Theory predicts that for weak polarization ($a < 1$) the median player will be able to secure her maximum payoff (of 1), whereas moderate levels ($1<a<2$) of polarization would hurt her (*cf.* Table 5.3).

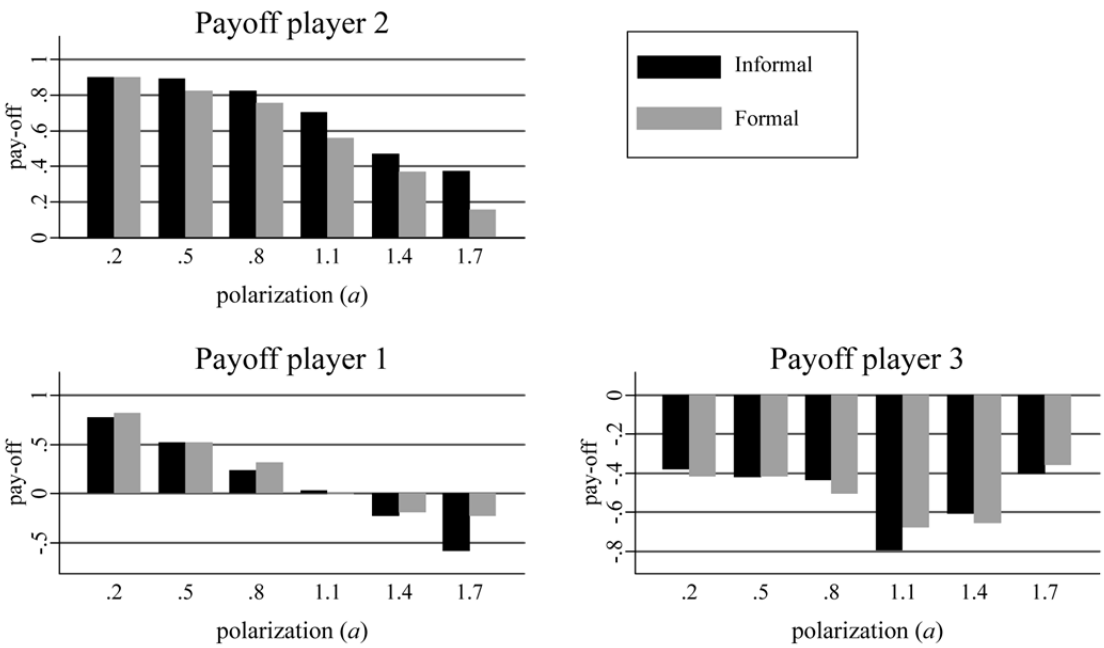


Figure 5.2
This figure shows payoffs.The bars show the average payoffs of players per period. Player 2 is the median player and player 1 is the other player closest to her.

The experimental results show no obvious change at $a = 1$. Increasing polarization clearly affects the median player (player 2) negatively, even when $a < 1$. For example, Player 2 earns approximately 0.9 (close to the maximum of 1) when a=0.2 (for both *Formal* and *Informal*) but only just over 0.79 for a=0.8 in the informal setting. Her earnings are significantly lower for a=0.8 than for a=0.2 (p=0.01*). As predicted by theory, the median's payoff is significantly lower for moderate (a>1) than for weak (a<1) polarization (p<0.01*).

Second, formality also has a clear effect. The median player is (significantly) better off in the *Informal* treatment than in the *Formal* treatment (p=0.03*). The difference between treatments seems to increase with the extent of polarization. When polarization is very weak (a=0.2) the procedure does not affect player 2's earnings from negotiations. When it is relatively strong (a=1.7) the median earns more than twice as much in *Informal* than in *Formal*. Next, we further explore what drives these results.

5.5.2 How Polarization & Formality Affect The Median Player

We start by looking at whether participants manage to reach an agreement before the deadline. Figure 5.3 shows the number of proposals needed to reach agreement.

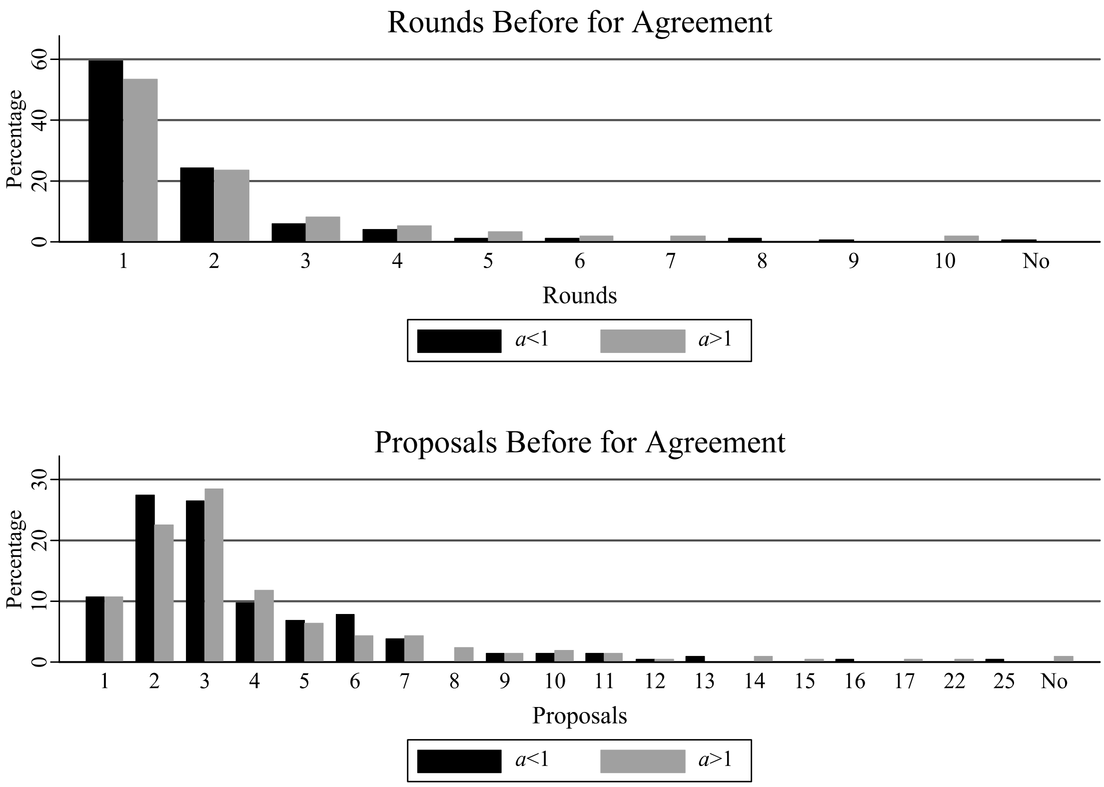


Figure 5.3
This figure shows the rounds or proposals before agreement. Bars show the fraction of agreements using the number of proposals depicted on the horizontal axis for Formal (top panel) and Informal (bottom panel). In Formal, a proposal in any round could only be made by the player selected to do so and there was a maximum of 10 rounds. In Informal, any player could make a proposal at any time during a period of at most 150 seconds.

In both treatments, agreement was reached within the limit (150 seconds or 10 rounds, respectively) in 99% of all cases. Hence, it almost never occurred that the disagreement point was forced upon the negotiators for missing their limit. Moreover, agreement was generally reached very quickly. In *Formal,* agreement was reached in at most 3 rounds in 88% of the cases and in *Informal* agreement was reached in at most 30 seconds in 82% of the cases. Consequently, binding (time) limits do not appear to be of any influence (for treatment effects). Players make significantly more proposals in *Informal* (4) than in *Formal* (2) (p<0.01*), however.

The outcome of the game can be characterized by three dimensions. First, whether it is a *real number* (as opposed to disagreement). If so, second, its value ('*location*') and, third, its distance to the median position, i.e., its absolute value ('*distance*'). We will look at each of these in turn. Figure 5.4 shows the percentages of outcomes that were a real number ('frequency'). As long as polarization is weak (a<1), virtually all outcomes are real numbers and polarization is immaterial.

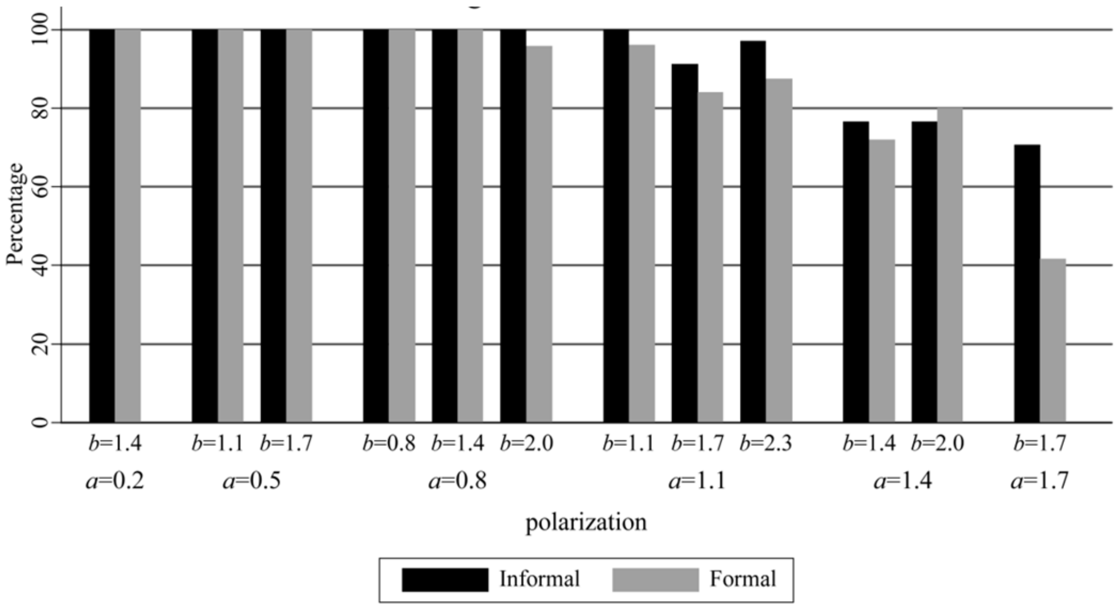


Figure 5.4

This figure shows the frequency (of outcomes were a real number). The bars show the percentage of outcomes that were a real number.

The frequency (of real number agreements) is, however, clearly and statistically significantly lower for moderate than for weak polarization (p<0.01*). Hence, a decrease in real number agreements may partly explain why moderate polarization is worse for the median player than weak polarization. However, it cannot explain why she cannot obtain her optimal payoff even when polarization is weak. Furthermore, there is no clear treatment effect. Real number outcomes are somewhat less likely in *Formal*, but the effect is small and insignificant (p=0.33 for a<1 and p=0.22 for a>1). Hence, the percentage of real number outcomes cannot explain why the median player is better off in *Informal*.

In search of such an explanation, we take a closer look at the real number outcomes players agreed upon. For completeness sake, we first depict the location of real number agreements in Figure 5.5 (although the location *per se* is not relevant for player 2's payoff). This clearly shows that the average agreement is typically between the ideal points of the median player and player 1 (the other player closest to the median). In fact, there are only two cases with $a \neq b$ where the average agreement lies between the ideal points of the median player and player 3. In both cases, player 1 still earns more that player 3. We will discuss the coalitions observed in more detail further on.

Given that an outcome is a real number, the median player's payoff is fully determined by its distance to the median ideal (0). This is shown in Figure 5.6. This figure clearly shows that the distance increases with polarization. Distance is significantly higher for moderate than for weak polarization (p<0.01*). Distance matters even within weak levels of polarization: it is significantly higher for a=0.8 than for a=0.2 (p=0.01* (pooled), p=0.12(*Low*), p=0.05* (*High*)).

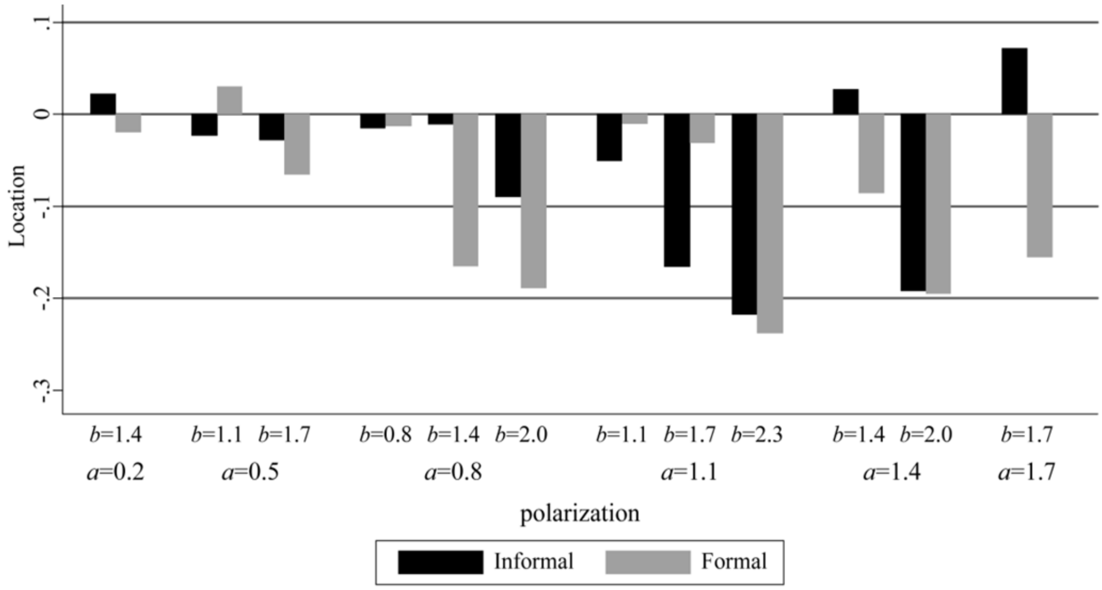


Figure 5.5
This figure shows the location of real number outcomes. Bars show the average normalized location of agreements, when groups agreed on a real number. Negative (positive) numbers indicate an agreement in between the ideal points of players 1 (3) and 2. The median position is an agreement at 0. Whenever $a{=}b$, the non-median players are randomly located as players 1 and 2, so any agreement is equally likely to be normalized to a positive or negative number.

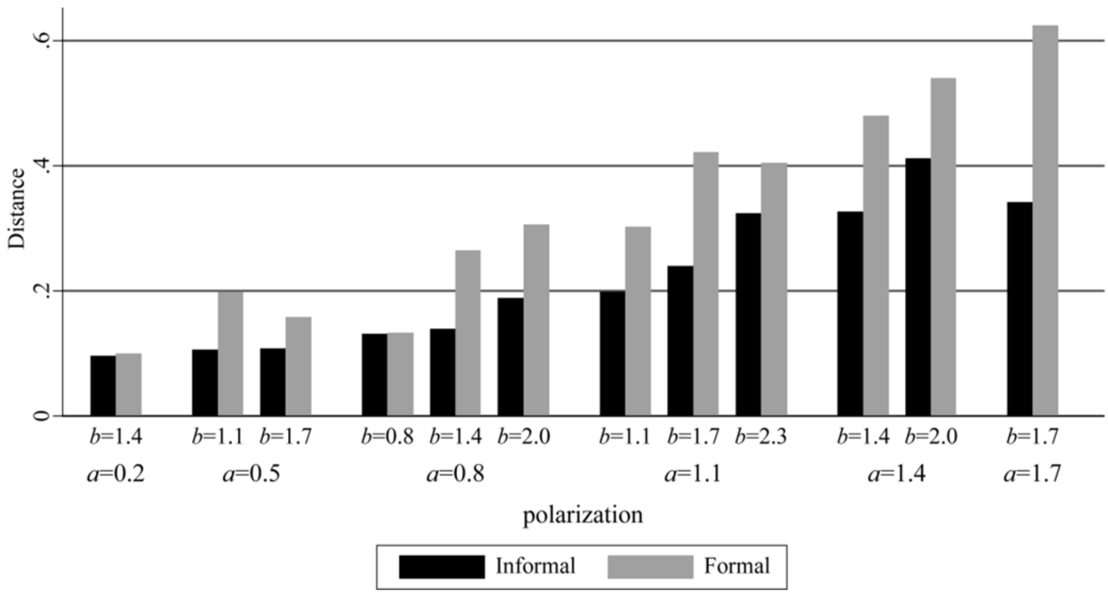


Figure 5.6
This figure shows the distance of real number outcomes. Bars show the average absolute distance between agreements and the median point, when groups agree on a real number.

Figure 5.6 also shows a clear treatment-effect. Distance is significantly lower for *Informal* than for *Formal*. Hence, player 2 seems to exploit her superior bargaining position better in *Informal* than in *Formal*. A possible explanation is that players are freer to make proposals in the *Informal* negotiations, so that they can negotiate better. Recall that players make significantly more proposals in *Informal* (4.0) than in *Formal* (2.0) (p<0.01*), In addition, we find that in *Formal* players use slightly less proposals in the last 12 periods (1.9) than in the first 12 periods (2.1). In contrast, in *Informal,* players use significantly more proposals in the last 12 periods (4.5) than in the first 12 periods (3.5) (p=0.03*).

We conclude that the main driving force underlying the higher profits for the median player in the *Informal* treatment is that the more flexible bargaining procedure allows her to secure real number agreements closer to her preference.

5.5.3 Intra-coalitional Fairness *vs.* Inter-coalitional Competition

One intriguing question that remains is why even weak polarization hurts the median player, while her ideal is the unique core element. To address this question, we consider coalitions and the way in which outcomes distribute payoffs within them. Figure 5.7 shows the distribution of real number agreements divided by a. Hence, -1 represents an agreement at $-a$ (i.e., player 1's ideal point), 0 represents the median ideal 0 and 1 represents a.

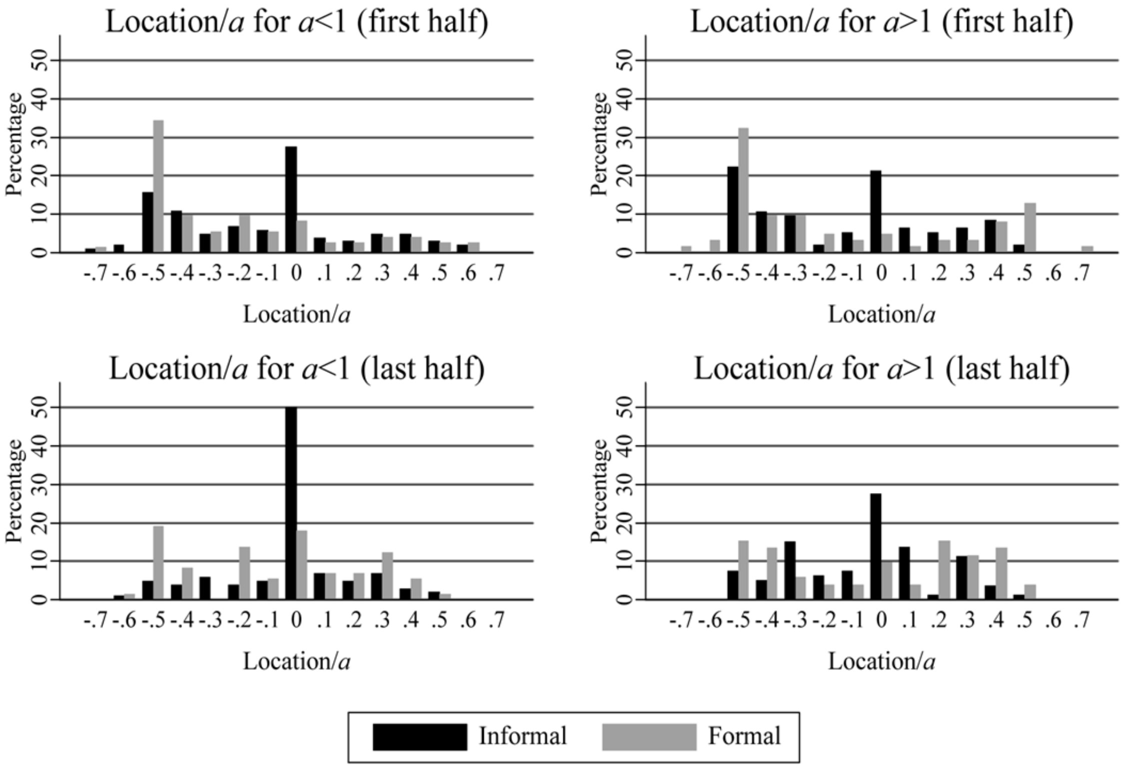


Figure 5.7

This figure shows the distribution of real number outcomes and learning effects. Bars show the fraction of real number outcomes that are within 0.05 of the outcome depicted on the horizontal axis. The horizontal axis gives the normalized outcome divided by a. The left panels shows the distribution for $a<1$, the right for $a>1$. The top panels show the distribution for the first half (first 12 periods), the bottom for the last half (last 12 periods).

Strikingly, almost all real number outcomes lie between $-a/2$ and $a/2$, with $-a/2$ being one of the most frequently chosen outcomes. Note that $-a/2$ equalizes payoffs between players 1 and 2, but is a rather unfair outcome for player 3; in fact worse than the median preference. From a fairness perspective it might seem remarkable that the players in the coalition do not seem to care much about the player outside of the coalition. Nonetheless, this is in line with the findings in the three-person ultimatum games (Güth & Van Damme (1998), Bolton & Ockenfels (1998)), where the third powerless person (who can neither propose nor reject) is given little consideration.

It seems that players 1 and 3 in many cases demand some part of the 'surplus' in a coalition with player 2. However, player 2 does not give more than the fair split to player 1. Furthermore, player 3 does not obtain a better outcome than $a/2$, since player 2 probably feels that she can certainly obtain $-a/2$ in a coalition with player 1. Such considerations of intra-coalitional fairness yield real number agreements increasing in a, even for weak levels of polarization, as we observe. Note, however, that as a increases it becomes more costly to the median player to give her coalition partner a 'fair share.'

Figure 5.7 also shows that there is a strong learning effect: the distribution of outcomes in the first half (first 12 periods) is very different from that in the last half (last 12 periods). In the first half, intra-coalitional fairness considerations seem to play an important role, certainly within coalitions of players 1 and 2. Furthermore, coalitions tend to consist of players 1 and 2, in particular in *Formal* (see Figure 5.8).

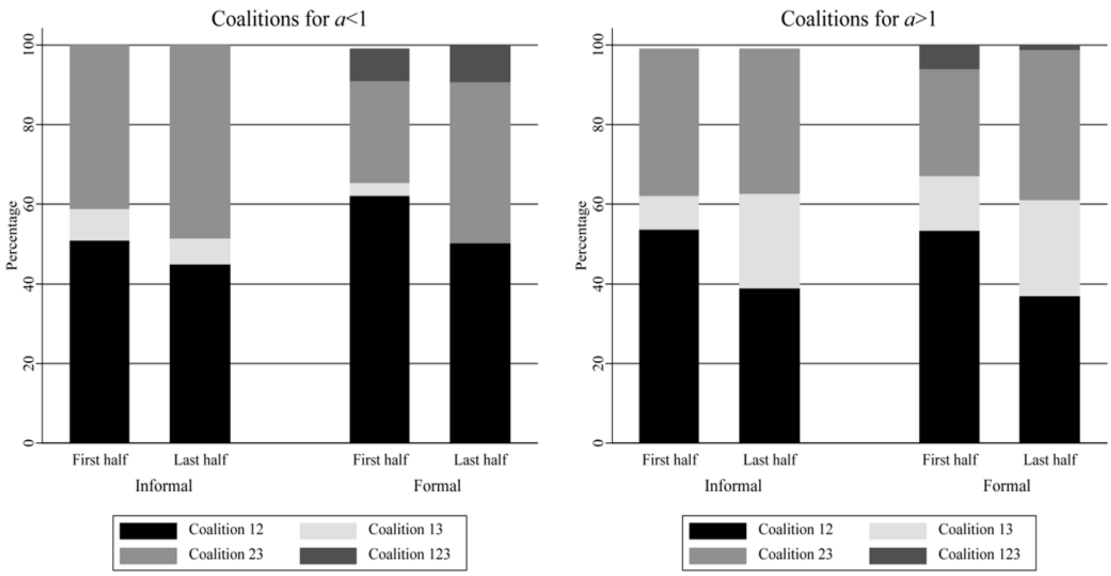


Figure 5.8
This figure shows coalitions and learning effects. Stacked bars show the distribution of distinct coalitions. A coalition ij is defined as an outcome proposed by i and accepted by j or vice versa. A coalition ijk is an outcome with 2 yes votes (only possible in formal).

In the course of the experiment, inter-coalitional competition becomes more important. In the second half, more coalitions arise of players 2 and 3 than in the first half (p=0.03* (pooled), p=0.46 (*Informal*), p=0.05* (*Formal*)), resulting in a more even spread of positive and negative agreements. Furthermore, for a>1 more coalitions are formed in the second half than in the first half between players 1 and 3 (p=0.03* (pooled), p=0.03* (*Informal*), p=0.21 (*Formal*)).

Note that having the viable 'outside option' of a coalition with player 3 means that the median player can offer less to player 1. Median players appear to realize this remarkably well in the second half of the experiment. Agreements tend to be closer to 0 in the last half (see Figure 5.7). In particular, the number of fair 1-2 compromises drops considerably ($p<0.01^*$ (pooled), $p=0.04^*$ (*Informal*), $p=0.05$ (*Formal*)) with an accompanying increase in the number of outcomes at the median ideal ($p=0.01^*$ (pooled), $p=0.03^*$ (*Informal*), $p=0.14$ (*Formal*)).

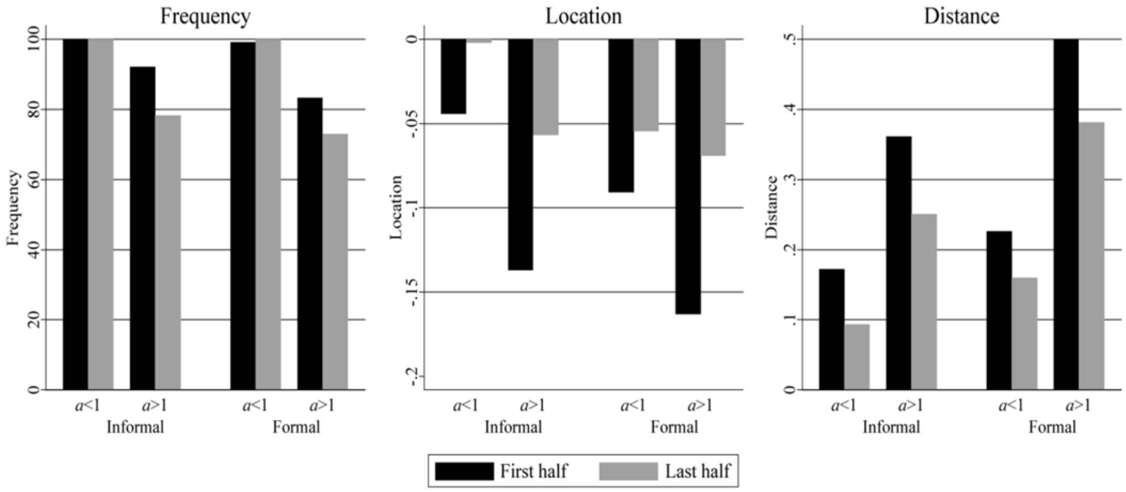


Figure 5.9
This figure shows the learning effects in frequency, location and distance. The left chart shows the percentage of outcomes that were real numbers. The middle chart shows the average location of real number agreements and the right chart the average distance between real number outcomes and the median preference.

Figure 5.9 splits the data depicted in Figure 5.4-Figure 5.6 and shows the frequency of real number agreements, their location and distance separately for the first and last half. From Figure 5.9 we learn firstly that the greater number of coalitions between players 1 and 3 results in a significantly lower frequency of real number agreements in the second half when $a>1$ ($p=0.04^*$ (pooled), $p=0.03^*$ (*Informal*), $p=0.25$ (*Formal*)). Secondly. the more equal spread of 1-2 and 2-3 coalitions also results in an average location closer to zero ($p=0.04^*$, (pooled), $p=0.12$ (*Low*), $p=0.18$ (*High*). Finally, we see that the strongest learning effect is that the distance of real number outcomes to the median ideal becomes significantly lower ($p<0.01^*$). Hence, it is the median player who benefits most from the increase in inter-coalitional competition.

5.6 Conclusion

In this chapter, we have addressed the question whether the outcome of the legislative process is affected by the formality of the bargaining procedure. We compared an informal bargaining procedure where players can freely make and accept proposals to a formal bargaining procedure where agenda-setting and voting is regulated by a Baron-Ferejohn alternating offers scheme. We studied the effect of formality in a legislative bargaining problem that consisted of a three-player median voter setting modified to have an external disagreement point. This allowed us to study formality both when the core exists and when it is empty, and to study whether an external disagreement point can explain why the polarization of a legislature can affect the legislative outcome. We derived cooperative solutions for the bargaining problem, studied the equilibrium properties of the formal and informal bargaining games, and tested the two procedures in the laboratory.

Our first result pertains to polarization. Theoretically, we find that polarization should matter when there is an external disagreement point and in our experiment, we find that this is indeed the case.[96] In particular, polarization hurts the median player. As predicted by theory, in our experiments the median player is significantly worse off at moderate than at weak levels of polarization. In contrast to what theory predicts, however, more polarization hurts the median player even when her ideal is the unique core element. This seems to be the result of intra-coalitional fairness considerations. Over time, inter-coalitional competition appears to attenuate these fairness considerations and polarization hurts the median player less.

Our second and main result is that formality matters. Theoretically, it is difficult to analyze the effects of formality, as a key characteristic of informal bargaining is that it imposes very few strategic restrictions on the negotiators. We find that in the informal game, all plausible outcomes are supported as subgame perfect equilibrium points. This is an important motivation to run experiments. The data show a clear treatment effect of formality. The median

[96] Recall that polarization does not matter in the classic median voter setting (Black, 1948; 1958).

player in our experiment is significantly better off under an informal bargaining procedure without rules about the timing of proposal and acceptance decisions. Outcomes in the *Informal* treatment are significantly more often the median ideal and significantly less often the fair compromise between players 1 and 2. It appears that the informal procedure gives the median player more flexibility to exploit its superior bargaining position. This result supports the armchair observation that players in a better bargaining position prefer less regulation of negotiations. To put this result on a stronger footing, more research is needed as we compare two representative but still specific procedures. Recently, this result has received support from Drouvelis, Montero & Sefton (2010).

Our results are relevant for the application of game theoretic models to the legislative process. The fact that formality influences the payoffs of certain players and the performance of specific predictions means that 'neutral' simplifying assumptions (i.e., assumptions that do not favor any player *prima facie*) made to obtain tractable results need not be as innocuous as is often assumed. For instance, a highly stylized alternating offers game may not be a suitable model of the legislative process if a significant part of the bargaining is informal.

Finally, understanding the influence of formality is relevant for studying institutional choice and parliamentary procedure. In particular, legislatures have to decide on a bargaining *procedure* –either from scratch or from a set of previously established procedures– before they can decide on the outcome itself. Even if the extent of formality may seem like a neutral parameter, it can significantly influence the bargaining outcome. Consequently, parties may have preferences for a formal or informal bargaining procedure. For instance, parties in the center of a political spectrum may prefer to prolong backroom discussions until agreement has been reached. Our results point to the more general idea that parties in a superior bargaining position will prefer less formal bargaining institutions, as these give them more room to exploit their bargaining position.

5.7 Appendix

5.7.1 Cooperative Solution Concepts

This appendix gives an overview of the dominance relations and the cooperative solutions concepts in our bargaining problem. The dominance relations (as shown in Table 5.4) depend mainly on the polarization parameter a and to some degree on b.

TABLE 5.4

DOMINANCE RELATIONS Γ

	Position	Polarization
$z' \succ z$	$\lvert z'\rvert < \lvert z\rvert$ for $z, z' \in \mathbb{R}$	
$z \succ \delta$	$z \in (-1, 1)$	$a + b < 2$
	$z \in (-1, -a+1) \cup (b-1, 1)$	$b < 2$
	$z \in (-1, -a+1)$	$a < 2$
$\delta \succ z$	$z \in (-a+1, b-1)$	$a, b \leq 2,\ a + b > 2$
	$z \in (-a+1, 1)$	$a \leq 2,\ b > 2$
	$z \in [-1, 1]$	$a > 2$
	$z \in (-\infty, -a-1) \cup (b+1, \infty)$	all a, b

Notes: The table summarize dominance relations between alternatives in the cooperative game.

For example, the second row states that when two alternatives are real numbers, the one closest to the median ideal of 0 dominates the alternative further away. The third and fourth rows compare real numbers to the disagreement point. For example, row 3 shows for a<2 that any real number between –1 and –a+1 dominates δ (because it gives players 1 and 2 strictly positive utility). On the other hand, if a>2, any real number between –1 and 1 gives both wing players negative utility, so they both prefer the disagreement point, which then dominates the real number (row 4).

With these dominance relations, we can analyze the set of cooperative solution concepts for our bargaining problem. Aside from the core ($\mathcal{C}(\Gamma)$) and uncovered set ($\mathcal{U}(\Gamma)$), this includes two refinements of the uncovered set, the *von*

Neumann Morgenstern set ($\mathcal{L}(\Gamma)$)[97] and the *bargaining set* ($\mathcal{B}(\Gamma)$)[98]. Both are unique.

All four solutions are finite sets for all values of a and b. The size of each depends on the polarization parameter a. The four solutions coincide whenever the core is non-empty. This is the case in both extremes, i.e., for weakly and for strongly polarized preferences. In the remaining case of moderately polarized preferences ($1 < a < 2$), the other three solution sets are non-empty and satisfy the following inclusions.

$$\varnothing = \mathcal{C}(\Gamma) \subset \mathcal{B}(\Gamma) \subset \mathcal{L}(\Gamma) \subset \mathcal{U}(\Gamma)$$

It turns out that all inclusions are strict. Under the general additional assumption that $a < b$, the bargaining set $\mathcal{B}(\Gamma)$ consists of a single point, the von Neumann Morgenstern set $\mathcal{L}(\Gamma)$ consists of two points, and the uncovered set $\mathcal{U}(\Gamma)$ consists of three points. In the special case when $1 < a = b < 2$ all three sets contain an additional solution point, due to symmetry considerations. The solution sets are listed in Table 5.5.

TABLE 5.5

COOPERATIVE SOLUTIONS (COMPLETE)

Polarization		$\mathcal{C}(\Gamma)$	$\mathcal{B}(\Gamma)$	$\mathcal{L}(\Gamma)$	$\mathcal{U}(\Gamma)$
weak: $a \leq 1$		0	0	0	0
moderate $1<a<2$	$a < b$	$\varnothing$	$\{-a + 1\}$	$\{-a + 1, \delta\}$	$\{-a + 1, 0, \delta\}$
	$a = b$		$\{-a + 1, b- 1\}$	$\{-a + 1, 0, \delta\}$	$\{-a + 1, 0, b-1, \delta\}$
strong: $b \geq 2$		δ	δ	δ	δ

Notes: For $\Gamma = \Gamma(N, Z, u_i, W)$, the table gives the elements in the core ($\mathcal{C}(\Gamma)$), bargaining set $\mathcal{B}(\Gamma)$, vNM set ($\mathcal{L}(\Gamma)$) and uncovered set ($\mathcal{U}(\Gamma)$) for the levels of polarization distinguished in the first column.

[97] Formally, a subset L of Z is a von Neumann Morgenstern set if elements of L do not dominate each other and every element of $Z \backslash L$ is dominated by at least one element of L.

[98] The bargaining set is the set of efficient points z in Z such that for any z' which dominates z and player $k \in N$ who prefers z over z' there exists z'' which weakly dominates z' and is for player k at least as good as z. (We use Maschler's (1992) definition of the bargaining set.)

The results for weak and strong polarization are straightforward. In the former case, the median ideal (0) dominates all other alternatives and the same holds for the disagreement point in the latter case. Here, we briefly explain the arguments underlying the results for moderate polarization, specifically for a and b such that $1 < a < 2 < b$.[99] The results for parameters where equalities hold in this relationship are straightforward.

($\mathcal{C}$) All $z \in \mathbb{R}$ unequal 0 are dominated by 0, 0 itself is dominated by δ, which in turn is dominated by any z between –1 and $(-a+1)$. Hence, the core is empty.

($\mathcal{B}$) The proposal $-a+1$ is dominated by any alternative z' closer to 0 than itself. Player 1 who prefers $-a+1$ above z' has a counter-objection $z'' = \delta$ which dominates z' and which gives him the same utility of 0 as the original proposal z. For any other proposal $z \in Z$ an objection exists for which there is no counter-objection justifying the original proposal. Hence, $(-a+1)$ is the unique element of the bargaining set $\mathcal{B}(\Gamma)$.

($\mathcal{L}$) Player 1 is indifferent between $(-a+1)$ and δ while players 2 and 3 have opposite preferences for these alternatives, hence, they do not dominate each other. Points on $\mathbb{R}$ between $(-a+1)$ and $(b-1)$ are dominated by δ, those beyond these limits are dominated by $(-a+1)$ itself. Hence, $\{-a+1, \delta\}$ is a vNM set $\mathcal{L}(\Gamma)$. One can easily verify that in Γ there is no other vNM set.

($\mathcal{U}$) (*i*) $z = -a+1$ is only dominated by $z' \in (-a+1, 1-a)$, which in turn are dominated by $z'' = \delta$ which does not dominate z,

(*ii*) $z = 0$ is dominated only by $z' = \delta$, which in turn is dominated by, for example, $z'' = -a/2$ which does not dominate z,

[99] The properties of the core and the uncovered set known from the literature on cooperative games are invariably obtained under the assumption of convexity of the values of all coalitions, which does not hold here. The bargaining sets and the von Neumann Morgenstern sets have been studied mostly in the context of TU games. Hence, all results in Table 5.5 must be verified case by case. We do not claim validity for any of these relations beyond the scope of bargaining problems as described here, with a one-dimensional set of alternatives augmented with a disagreement point and single-peaked preferences for the trio of players.

(iii) $z = \delta$ is dominated by any $z' \in (-1, -a + 1)$, all of which are dominated by $z'' = 0$ which does not dominate z,

(iv) all z with $|z| > -a + 1$ are covered by $-a + 1$, while all $z \in (-a + 1, 0) \cup (0, a - 1]$ are covered by 0.

Hence, the uncovered set $\mathcal{U}(\Gamma)$ consists of the triple $\{-a + 1, 0, \delta\}$.

5.7.2 Proof of Proposition 5.1 (Formal)

In this appendix, we provide the proof of Proposition 5.1, which characterizes the equilibrium outcome of Γ_T^F (when it converges). Since Γ_T^F is (highly) non-convex due to the exterior disagreement point, we cannot use standard results and techniques to derive equilibria; rather it involves a *tour de force* in backward induction. We also ran simulations, which illustrate (and corroborate) the results of the proposition. In particular, they shed some light on what happens if the outcome does not converge. At the end, we provide a figure that illustrates the cyclic dependence of the outcome on a and b for $1 < a < 2$ and $a < b < 3$ (as obtained by simulations).

Proof

Before we can determine the equilibrium, we need to introduce some notation. Due to backward induction and players having a unique best response at each information set, the equilibrium proposal and voting strategies only depend on how many rounds are ahead. Hence, we will count the rounds by the remaining number of rounds $r \equiv T - t + 1$. Hence, the first round has $r = T$ and the last round $r = 1$. Furthermore, this implies that the equilibrium strategy for round r is the same for each game Γ_T^F with $T \geq r$. Hence, it is meaningful to talk in general about the (sub)game Γ_r^F. The equilibrium (behavioral) strategy for player i, σ_i, specifies for each round $r \leq T$ (i) for the proposal stage, a probability distribution over possible proposals $\pi_i^r : Z \to [0,1], p_i^r \mapsto \pi_i^r(p_i^r)$,[100] and (*ii*) for the voting stage, an acceptance function $\nu_i^r : Z \to \{0,1\}, p^r \mapsto \nu_i^r(p^r)$. The equilibrium outcome of Γ_r^F can be characterized by the probability distribution of the equilibrium outcomes $\mu^r : Z \to [0,1]$.[101] The continuation value $EU_i^r = E_{\mu^r}[u_i(z)] = \sum_{\operatorname{supp}\mu^r} \mu^r(z)u_i(z)$ is the expected utility of player i of the (sub)game Γ_r^F. We can conveniently express EU_i^r in terms of $f^r \equiv \mu^r(\delta)$, $L^r \equiv E_{\mu^r}[z \mid z \in R]$ and $D^r \equiv E_{\mu^r}[|z| \mid z \in R]$. (Note that $D^r \geq |L^r|$). Define the indicator function $I_R(x) \equiv [1 \text{ if } x \in R, 0 \text{ if } x \notin R]$, the acceptance probability $\xi_i^r(x) \equiv 1-(1-\nu_j^r(p))(1-\nu_k^r(p))$ and the probability of delay $P_{\mu^r}^r[delay] \equiv 1-\frac{1}{3}\sum_N \sum_{\operatorname{supp}\mu_i^r} \pi_i^r(x)\xi_i^r(x)$. Then, we get:

$$f^r = \tfrac{1}{3}\sum_N \sum_{\operatorname{supp}\pi_i^r} \left(\pi_i^r(x)\xi_i^r(x)I_R(x)\right) + P^r[delay]f^{r-1}, \quad f^0 \equiv 0$$

If $f^r = 0$, then $L^r \equiv 0, D^r \equiv 0$. Otherwise:

$$L^r \equiv \frac{1}{f^r}\left(\tfrac{1}{3}\sum_N \sum_{\operatorname{supp}\pi_i^r\setminus\{\delta\}} \left(\pi_i^r(x)\xi_i^r(x)x\right) + P^r[delay]f^{r-1}L^{r-1}\right) \tag{5.1}$$

$$D^r \equiv \frac{1}{f^r}\left(\tfrac{1}{3}\sum_N \sum_{\operatorname{supp}\pi_i^r\setminus\{\delta\}} \left(\pi_i^r(x)\xi_i^r(x)|x|\right) + P^r[delay]f^{r-1}D^{r-1}\right)$$

Since $E_{\mu^r}[u_i(z)] = \Pr_{\mu^r}[z=\delta]\ E_{\mu^r}[u_i(z)|\ z=\delta] + \Pr_{\mu^r}[z\neq\delta]\ E_{\mu^r}[u_i(z)|\ z\neq\delta]$ and $u_i(z)$ are linear in z for $z \in [-a,b] \cup \delta$, from $u_i(z) = 1-|z-z_i|$ we get that (for $\operatorname{supp}\mu^r \subseteq [-a,b]\cup\delta$):

[100] $|\operatorname{supp}\pi_i^r| \leq 2$

[101] μ^r is a probability mass function and has countable support: $|\operatorname{supp}\mu_i^r| \leq 3r \max_{r\leq T, i\in N} |\operatorname{supp}\pi_i^r| = 6r$.

$$
\begin{aligned}
EU_1^r &= f^r(1-a-L^r) \\
EU_2^r &= f^r(1-D^r) \\
EU_3^r &= f^r(1-b+L^r)
\end{aligned}
\tag{5.2}
$$

A player i will accept a proposal p in round $r + 1$ if and only if $u_i(p) \geq EU_i^r$. This allows us to characterize for round $r + 1$ (i) $\overline{L}_1^{r+1}$, the largest proposal player 1 accepts, (ii) $\underline{L}_3^{r+1}$, the smallest proposal player 3 accepts, and (iii) $\overline{D}_2^{r+1}$, the largest absolute value a proposal can have for player 2 to accept it:

$$
\begin{aligned}
\overline{L}_1^{r+1} &= \left(1-f^r\right)\left(1-a\right)+f^r L^r \\
\underline{L}_3^{r+1} &= \left(1-f^r\right)\left(b-1\right)+\ f^r L^r \\
\overline{D}_2^{r+1} &= \left(1-f^r\right)+f^r D^r
\end{aligned}
\tag{5.3}
$$

Players will only delay if they cannot make a proposal that will be accepted and gives them at least their continuation value. Player 1 or 2 will only delay in round $r + 1$ if $\overline{L}_1^{r+1} \leq -\overline{D}_2^r$, which is equivalent to $(1 - f^r)$ $(2 - a)$ + f^r $(L^r$ + $D^r)$ $<$ 0. This can only hold if $a > 2$ and $f^r < 1$. Hence, players 1 and 2 will never delay if $a \leq 2$ and, by the same reasoning, player 3 will never delay if $b \leq$ 2.

Note that if $p_i^{r+1} \in R, i \neq 2$ is accepted in equilibrium, it must be accepted by player 2 in round $r + 1$ and, hence, $u_2(p_i^{r+1}) \geq EU_2^r$. If $a < 2$, then player 2 will propose $1 - a$ (or $a - 1$) in round T. This means that $EU_2^r > 0$ for all r and that she will never accept nor propose δ. Furthermore, if $a < 1$, player 1 can always propose 0 so that she will never propose δ. Finally, $EU_2^r > 0$ implies $L^r < 1$, and hence $EU_3^r < 0$ if $b \geq 2$. In the following Lemma, we summarize these facts and some conditions that are easily derived.

Lemma 5.1 *For $a < 2$, the equilibrium $\{\sigma_1, \sigma_2, \sigma_3\}$ is determined by:*

1. *$\nu_1^r(p) = 1$ for $p \in [-a, b]$ iff $p \leq \overline{L}_1^r$ and $\nu_1^r(\delta) = 1$ iff $EU_1^{r-1} \leq 0$.*
2. *$\nu_2^r(p) = 1$ for $p \in [-a, b]$ iff $|p| \leq \overline{D}_2^r$ and $\nu_2^r(\delta) = 1$ iff $EU_2^r \leq 0$.*
3. *$\nu_3^r(p) = 1$ for $p \in [-a, b]$ iff $p \geq \underline{L}_3^r(\sigma)$ and $\nu_3^r(\delta) = 1$ iff $EU_3^{r-1} \leq 0$.*

4. $\pi_1^r(\delta)=1$ *iff* $-\overline{D}_2^r \geq 1-a$ *and* $EU_3^{r-1} \leq 0$; $\pi_1^r(-a)=1$ *iff* $-\overline{D}_2^r < -a$;

$\pi_1^r(-\overline{D}_2^r)=1$ *iff* $-a \leq -\overline{D}_2^r < 1-a$ *or* $-\overline{D}_2^r \geq -a$ *and* $EU_3^{r-1} > 0$;

5. $\pi_2^r(0)=1$ *iff* $\overline{L}_1^r > 0$ *or* $\underline{L}_3^r < 0$; $\pi_2^r(\underline{L}_1^r)=1$ *iff* $\overline{L}_1^r \leq 0$, $\underline{L}_3^r > 0$ *and* $|\overline{L}_1^r| < \underline{L}_3^r$;

$\pi_2^r(\underline{L}_3^r)=1$ *iff* $\overline{L}_1^r < 0$, $\underline{L}_3^r \geq 0$ *and* $-\overline{L}_1^r > \underline{L}_3^r$;

$\pi_2^r(\overline{L}_1^r)=\pi_2^r(\underline{L}_3^r)=\frac{1}{2}$ *iff* $\overline{L}_1^r \leq 0$, $\underline{L}_3^r \geq 0$ *and* $|\overline{L}_1^r| = \underline{L}_3^r$;

6. $\pi_3^r(\text{delay})=1$ *iff* $\underline{L}_3^r > \overline{D}_2^r$ *and* $EU_1^r > 0$ *(only if* f^{r-1} *<1);*

$\pi_3^r(\delta)=1$ *iff* $\overline{D}_2^r \leq b-1$ *and* $EU_1^{r-1} \leq 0$; $\pi_3^r(b)=1$ *iff* $\overline{D}_2^r > b-1$;

$\pi_3^r(\overline{D}_2^r)=1$ *iff* $b-1 < \overline{D}_2^r \leq b$ *or* $EU_1^r > 0$ *and* $\overline{D}_2^r \leq b$.

Now we are ready to look at whether the equilibrium outcome converges. Let $x^* \equiv \lim_{r\to\infty} x^r$. The probability distribution μ^* is the limit of μ^r if it holds that $\lim_{r\to\infty} \mu^r(z) = \mu^*(z)$ for all z in the support of μ^*. As defined in section 5.3.1, , we say that the equilibrium outcome μ^r converges to μ^* if $\mu^* \equiv \lim_{r\to\infty} \mu^r$; if this limit does not exist, we say that μ^r does not converge.

The equilibrium outcome converges to 0 if $\mu^*(0)=1$, which is equivalent to $f^*=1$ and $D^*=1$. The equilibrium outcome converges to δ if $\mu^*(\delta)=1$, which is equivalent to $f^*=0$ and $\lim_{r^*\to\infty} \sup_{r\leq r^*} D^r \in \mathbb{R}$. Finally, it is straightforward that μ^r does not converge if (*i*) f^* does not exist or (*ii*) $f^*>0$ and L^* or D^* do not exist.

Proposition 5.1

(i) *If* $0 \leq a < 1$ *or* $a=b=1$, *then the equilibrium outcome converges to 0*

(ii) *If* $a \leq b < 2$ *and* $b > 1$, *the equilibrium outcome does not converge, unless* $\frac{13}{7} \leq a = b < 2$ or $b=\frac{3}{2}, \frac{5}{4} \leq a < \frac{3}{2}$ or $\frac{8}{3} \leq b < \frac{13}{14}, a=\frac{5}{7}b-\frac{4}{7}$ or

$\frac{3}{2} \leq b < \frac{7}{3}$, $\max\{\frac{7}{5} - \frac{1}{10}b, \frac{7}{5}b - \frac{8}{5}\} < a < \frac{6}{5} + \frac{1}{5}b$. *In these latter cases the outcome may converge, but never to a single outcome in Z.*

(iii) *If $a \geq 2$, the equilibrium outcome is δ.*

Proof:

(i.a) We show that if $0 \leq a < 1$, then $f^* = 1$ *and* $D^* = 0$.

Throughout the proof, we will use the following sufficient condition for convergence: For $0 \leq a < 1$, $f^* = 1$ and $D^* = 0$ if

(SC) there exists a round $r^\dagger$ such that $f^{r^\dagger} = 1$ and $EU_1^{r^\dagger} > 0$

Let (SC) hold for $r^\dagger$. $EU_1^{r^\dagger} > 0$ ($\Leftrightarrow L^{r^\dagger} < 1 - a$) implies that player 1 will not accept nor propose δ in round $r^\dagger + 1$. $f^{r^\dagger} = 1$ implies that player 3 will not delay and that either player 1 or 3 accept 0 in round $r^\dagger + 1$. Consequently, $p_2^{r^\dagger+1} = 0$, $p_1^{r^\dagger+1} = -\overline{D}_2^{r^\dagger+1} = -D^{r^*}$ and $p_3^{r^\dagger+1} = \overline{D}_2^{r^\dagger+1} = D^{r^\dagger}$. Hence, $f^{r^\dagger+1} = 1$, $L^{r^\dagger+1} = \frac{1}{3}(-D^{r^\dagger} + 0 + D^{r^\dagger}) = 0 < 1 - a$ and $D^{r^\dagger+1} = \frac{1}{3}(D^{r^\dagger} + 0 + D^{r^\dagger}) = \frac{2}{3}D^{r^\dagger}$. Thus, (SC) holds for $r^\dagger + 1$ and by induction for all $r \geq r^\dagger$. As a result, $f^* = 1$ and $D^* = \lim_{m\to\infty} D^{r^*+m} = \lim_{m\to\infty} \left(\frac{2}{3}\right)^m D^{r^*} = 0$. A sufficient condition for (SC) to hold is that:

(SC') there is a round r' such that
(i) $\underline{L}_3^{r'} \leq \overline{D}_2^{r'} \leq a$ and $\overline{L}_1^{r'} < 1 - a$ and $(ii)\underline{L}_3^{r'} \geq -\overline{L}_1^{r'}$ or $\underline{L}_3^{r'} < 3(1-a)$

$\underline{L}_3^{r'} \leq \overline{D}_2^{r'} \leq a$ and $\overline{L}_1^{r'} < 1 - a$ imply that $p_1^{r'} = -\overline{D}_2^{r'}$ and $p_3^{r'} = \overline{D}_2^{r'}$. Hence, $f^{r'} = 1$ and $L^{r'} = \frac{1}{3}E[p_2^{r'}]$. If $\underline{L}_3^{r'} \geq -\overline{L}_1^{r'}$ or $\underline{L}_3^{r'} < 3(1-a)$, then $E[p_2^{r'}] < 3(1-a)$ and $L^{r'} < (1-a)$. Hence (SC) holds for r'. In the remainder of the proof we divide the $(a,\ b)$ parameter-set into regions and show that (SC) holds for each region.

We start by looking at the last four rounds. In the final round, $p_1^1 = -a$ and $p_2^1 = 0$. If $b < 2$, then $p_3^1 = \min\{b,1\}$, $f^1 = 1$ and $L^1 < \frac{1}{3}(1-a)$, such that round 1 satisfies (SC). So, let $b \geq 2$. Then $p_3^1 = \delta$ and $f^1 = \frac{2}{3}$, $D^1 = -L^1 = \frac{1}{2}a$. Hence, $\overline{L}_1^2 = \frac{1}{3}(1-2a) < 1 - a$, $\overline{D}_2^2 = \frac{1}{3}(1+a)$ and $\underline{L}_3^2 = \frac{1}{3}(b-a-1)$. Since $EU_1^1 > 0$, p_3^2 $=$ *delay* iff $\overline{D}_2^2 < \underline{L}_3^2$ iff $b > 2\ (a + 1)$. Let us first consider $b \leq 2\ (a + 1)$ and $a \leq ½$. Then $p_1^2 = -a$, $p_2^2 = 0$ and $p_3^2 = \overline{D}_2^2$, so that $f^2 = 1$ and $L^2 = \frac{1}{9}(1-2a) < 1 - a$. Hence, round 2 meets (SC).

Let us now consider $b \leq 2\ (a + 1)$ and $a > ½$. In this case, $\underline{L}_3^2 \leq \overline{D}_2^2 \leq a$, so that round 2 satisfies (SC') if $-\overline{L}_1^2 \leq \underline{L}_3^2$ or $\underline{L}_3^2 < 3(1-a)$. So let $3(1 - a) \leq \underline{L}_3^2 < -\overline{L}_1^2$. This means that $10 - 8a \leq b < 3a$ and $a > \frac{10}{11}$. Furthermore, $p_1^2 = -\overline{D}_2^2$, $p_2^2 = \underline{L}_3^2$ and $p_3^2 = \overline{D}_2^2$, so that $f^2 = 1, L^2 = \frac{1}{3}p_2^2 > 1-a$ and $D^2 = \frac{1}{9}(1 + a + b) \leq a$. Hence, $p_1^3 = -\overline{D}_2^3 = -D^2$, $p_2^3 = 0$ and $p_3^3 = \delta$, so that $f^3 = \frac{2}{3}$, $D^3 = -L^3 = \frac{1}{2}D^2$. From this, $\overline{L}_1^4 = \frac{1}{27}(8-10a-b)$, $\overline{D}_2^4 = \frac{1}{27}(10+a+b)$ and $\underline{L}_3^4 = \frac{1}{27}(8b-10-a)$). $2 < b \leq 2\ (a + 1)$ and $a > ½$ imply $\overline{D}_2^4 \leq a$ and $\overline{L}_1^4 < (1-a)$. $10 - 8a \leq b < 3a$ implies $-\overline{L}_1^4 < \underline{L}_3^4 < \overline{D}_2^4$. Hence, round 4 satisfies (SC). Thus (SC) holds if

$$\text{(A)} \qquad b \leq 2(a+1)$$

Let $b > 2\ (a + 1)$ from now on. $EU_3^r < 0$ for all r and player 3 can now delay consecutive rounds and alternatingly delay and not delay. This requires a careful characterization of the dynamic before we proceed. We will call a set of consecutive rounds in which player 3 delays a delaying sequence. We index these sequences by s (again backwards), with $s = 1$ the final delaying sequence, $s = 2$ the pre-final delaying sequence etc. Let $R(s)$ be the set of rounds in the s-th delaying sequence and define $\overline{r}(s) \equiv \max R(s)$ and $\underline{r}(s) \equiv \min R(s)$. Finally, let $m(s) = \overline{r}(s) - \underline{r}(s) + 1$ be the number of delaying rounds in *R(s).*

Let us look at $\underline{r}(s)$-1. $p_3^{\underline{r}(s)} = delay$ implies $EU_1^{\underline{r}(s)-1} > 0$ and $f^{\underline{r}(s)-1} = \frac{2}{3}$. As player 1 accepts δ in $\underline{r}(s)$ - 1, she also accepts 0 and $p_2^{\underline{r}(s)-1} = 0$. Since $p_1^{\underline{r}(s)-1} = -\overline{D}_2^{\underline{r}(s)-1}$, this means that $D^{\underline{r}(s)-1} = -L^{\underline{r}(s)-1} = -\frac{1}{2}\overline{D}_2^{\underline{r}(s)-1}$. We proceed to rounds $r \in R(s)$. Since player 3 delays, $\underline{L}_3^{\underline{r}(s)} > \overline{D}_2^{\underline{r}(s)}$ and $p_2^{\underline{r}(s)} = \min\{0, \overline{L}_1^{\underline{r}(s)}\} \leq 0$. As $D^{\underline{r}(s)} = -L^{\underline{r}(s)}$ and $f^{\underline{r}(s)-1} = \frac{2}{3}$, by (5.1) and Lemma 5.1 it must be that $f^1 < 1$, $L^r = -D^r < 0$ and $EU_1^r > 0$ for all $r \in R(s)$. Furthermore:

$$\begin{aligned}
&\text{If } D^r = -L^r, \text{ then:}\\
&\alpha_+^{r+1} \equiv \overline{D}_2^{r+1} + \overline{L}_1^{r+1} = \left(1 - f^r\right)(2 - a)\\
(5.4)\qquad &\alpha_-^{r+1} \equiv \overline{D}_2^{r+1} - \overline{L}_1^{r+1} = \left(1 - f^r\right)a + 2f^r D^r\\
&\gamma_+^{r+1} \equiv \overline{D}_2^{r+1} + \underline{L}_3^{r+1} = \left(1 - f^r\right)b\\
&\gamma_-^{r+1} \equiv \underline{L}_3^{r+1} - \overline{D}_2^{r+1} = \left(1 - f^r\right)(b - 2) - 2f^r D^r
\end{aligned}$$

In particular, $D^r = -L^r$ holds for $r = \underline{r}(s) - 1,..,\underline{r}(s)$.

Moreover, as player 3 delays in rounds $r \in R(s)$ and $f^{\underline{r}(s)-1} = \frac{2}{3}$:

$$f^r = \tfrac{2}{3} + \tfrac{1}{3} f^{r-1} \forall r \in R(s) \tag{5.5}$$

$$f^{\underline{r}(s)+m} = \frac{3^{m+2} - 1}{3^{m+2}} \text{ for } m = -1, 0, 1, .., m(s) - 1 \tag{5.6}$$

From (5.4) we get $\alpha_-^{\underline{r}(s)} = \frac{1}{3}a + \frac{4}{3}D^{\underline{r}(s)-1}$ and it turns out that $\alpha_-^r = \alpha_-^{\underline{r}(s)} \equiv \alpha_-(s)$ for $\underline{r}(s) \leq r \leq \overline{r}(s) + 1$. For $s = 1$ it is simple. Suppose $\alpha_-^r = a$ and D^r = -L^r = $\frac{1}{2}a$. Then immediately $\alpha_-^{r+1} = a$. Furthermore, due to the symmetry D^{r+1} = -L^{r+1} = $\frac{1}{2}a$. Since $D^{\underline{r}(1)-1} = D^1 = -L^1 = \frac{1}{2}a$ and $\alpha_-^{\underline{r}(1)-1} = \alpha_-^1 = a$, by induction it follows that $\alpha_-^r = a$ for $\underline{r}(1) \leq r \leq \overline{r}(1) + 1$. For $s > 1$, we need to assume that $\overline{L}_1^{\underline{r}(s)} \leq 0$ and $\overline{D}_2^{\underline{r}(s)} \leq a$ and justify it later. Suppose $\overline{L}_1^r \leq 0$, $\overline{D}_2^r \leq a$ and $p_3^r = delay$. Hence, $p_1^r + p_2^r = -\alpha_-^r$ and, using (5.1), we get D^r =

$\frac{\frac{2}{3}\cdot\frac{1}{2}(-p_1^r - p_2^r) + \frac{1}{3}f^{r-1}D^{r-1}}{f^r} = \frac{(1-f^{r-1})a + 3f^{r-1}D^{r-1}}{3f^r}$. Substituting this term and using (5.5), we get that $\alpha_-^r - \alpha_-^{r+1} = a\,(f^r - f^{r-1}) + 2\,f^{r-1}D^{r-1} - 2\,f^r D^r = 0$. Hence, $\alpha_-^{r+1} = \alpha_-^r$. Furthermore, using the same substitutions, we get $\overline{D}_2^{r+1} - \overline{D}_2^r = -\frac{1}{3}(2-a)(1-f^{r-1}) < 0$ and $\overline{D}_2^{r+1} < \overline{D}_2^r \le a$. Finally, $\overline{L}_1^{r+1} = \overline{D}_2^{r+1} - \alpha_-^{r+1} = \overline{D}_2^{r+1} - \alpha_-^r < \overline{D}_2^r - \alpha_-^r = \overline{L}_1^r \le 0$. Hence, as $p_3^r = delay$ for all $r \in R(s)$, $\alpha_-^r = \alpha_-^{\underline{r}(s)}$ for $\underline{r}(s) \le r \le \overline{r}(s)+1$.

Using (5.4) and (5.6), we get the following results for $m = 0, 1, ..., m(s)$:

$$\text{(5.7)}\qquad \begin{aligned}
\overline{L}_1^{\underline{r}(s)+m} &= \tfrac{1}{2}(\alpha_+^{\underline{r}(s)+m} - \alpha_-^{\underline{r}(s)+m}) = \frac{1-\frac{1}{2}a}{3^{m+1}} - \tfrac{1}{2}\alpha_(s)\\
\overline{D}_2^{\underline{r}(s)+m} &= \tfrac{1}{2}(\alpha_+^{\underline{r}(s)+m} + \alpha_-^{\underline{r}(s)+m}) = \frac{1-\frac{1}{2}a}{3^{m+1}} + \tfrac{1}{2}\alpha_(s)\\
\underline{L}_3^{\underline{r}(s)+m} &= \gamma_+^{\underline{r}(s)+m} - \overline{D}_2^{\underline{r}(s)+m} = \frac{\frac{1}{2}a+b-1}{3^{m+1}} - \tfrac{1}{2}\alpha_(s)\\
\gamma_-^{\underline{r}(s)+m} &= \gamma_+^{\underline{r}(s)+m} - 2\overline{D}_2^{\underline{r}(s)+m} = \frac{a+b-2}{3^{m+1}} - \alpha_(s)
\end{aligned}$$

Since player 3 only delays in round r iff $EU_1^{r-1} > 0$ and $\gamma_-^r > 0$, from (5.7) we get that $m(s) = \min\{m \in R : \gamma_-^{\underline{r}(s)+m} \le 0\} = \text{ceiling}\left[\frac{\ln(a+b-2) - \ln(3\alpha\_(s))}{\ln(3)}\right]$. Equivalently, since $\gamma_-^{\underline{r}(s)+m(s)-1} > 0$ and $\gamma_-^{\underline{r}(s)+m(s)} \le 0$, we get:

$$\text{(5.8)}\qquad 2 - a + 3^{m(s)}\alpha\_(s) < b \le 2 - a + 3^{m(s)+1}\alpha\_(s)\forall s$$

In round $\overline{r}(s)+1 = \underline{r}(s)+m(s)$ player 3 will not delay. Since $EU_1^{\overline{r}(s)} > 0$ and $\underline{L}_3^{\overline{r}(s)+1} \le \overline{D}_2^{\overline{r}(1)+1} < \overline{D}_2^{\overline{r}(s)} \le a$,

$$\text{(5.9)}\qquad \overline{r}(s)+1 \text{ satisfies (SC') if } \underline{L}_3^{\overline{r}(s)+1} < 3(1-a)$$

$\underline{L}_3^{\overline{r}(1)+1} \le \overline{D}_2^{\overline{r}(1)+1} \le \overline{D}_2^{\underline{r}(1)+1} = \frac{1}{9}(1+4a)$. Hence, by (5.9) $\overline{r}(1)+1$ satisfies (SC') if

(B) $a < \frac{26}{31}$

Let $a > \frac{26}{31}$ .It turns out that if $\overline{r}(s)+1$ does not satisfy (SC), then $\overline{r}(s)+3 = \underline{r}(s+1)$. Let $\overline{r}(s)$ + 1 not satisfy (SC). In this case $EU_1^{\overline{r}(s)+1} \leq 0$ and $p_2^{\overline{r}(s)+1} = \underline{L}_3^{\overline{r}(s)+1}$. Consequently, $p_2^{\overline{r}(s)+2} = 0$, $p_3^{\overline{r}(s)+2} = \delta$ and $p_1^{\overline{r}(s)+2} = -\overline{D}_2^{\overline{r}(s)+2} = -D^{\overline{r}(s)+1} = -\frac{1}{3}(2\overline{D}_2^{\overline{r}(s)+1} + \underline{L}_3^{\overline{r}(s)+1}) = -\frac{1}{6}\left(\frac{2-a+2b}{3^{m(s)+1}} + \alpha^-(s)\right)$ (using (5.7). Thus, $f^{\overline{r}(s)+2} = \frac{2}{3}$, $EU_1^{\overline{r}(s)+2} > 0$ and $D^{\overline{r}(s)+2} = -L^{\overline{r}(s)+2} = -\frac{1}{2}p_1^{\overline{r}(s)+2}$. Using (5.4) , we get $\gamma_-^{\overline{r}(s)+3} = \frac{1}{9}\left(3b-6-\alpha\_(s) - \frac{2-a+2b}{3^{m(s)+1}}\right)$. Furthermore, since $D^{\underline{r}(s)-1} = -\frac{1}{2}p_1^{\underline{r}(s)-1} \leq \frac{1}{2}a$, $\alpha\_(s) = \frac{1}{3}a + \frac{4}{3}D^{\underline{r}(s)-1} \leq a$ and, by (5.8), $\frac{1}{3^{m(s)+1}} \leq \frac{\alpha\_(s)}{a+b-2}$. Hence, $\gamma_-^{\overline{r}(s)+3} \geq \frac{1}{9}\left(3b-6-a\left(1+\frac{2-a+2b}{a+b-2}\right)\right)$. Since $b > 2$ $(a + 1)$ and $a \geq \frac{26}{31}$, $\gamma_-^{\overline{r}(s)+3} > \frac{2}{9}(2a-1) > 0$. Thus, $p_3^{\overline{r}(s)+3} = \mathit{delay}$ and $\overline{r}(s)+3 = \underline{r}(s+1)$. As a consequence, $\alpha\_(s+1) = \frac{1}{3}a + \frac{4}{3}D^{\underline{r}(s+1)-1} = \frac{1}{3}a + \frac{4}{3}D^{\overline{r}(s)+2}$:

$$\alpha\_(s+1) = \frac{1}{3}a + \frac{1}{9}\alpha\_(s) + \frac{2-a+2b}{3^{m(s)+3}} \tag{5.10}$$

We conclude our characterization of the delaying sequences by showing we can indeed assume $\overline{L}_1^{\underline{r}(s)} \leq 0$ and $\overline{D}_2^{\underline{r}(s)} \leq a$ for $s > 1$. Since $\alpha\_(s) \leq a$ and $a \geq \frac{26}{31}$, $\overline{D}_2^{\underline{r}(s)} = \frac{1}{3}(1-\frac{1}{2}a) + \frac{1}{2}\alpha\_(s) < a$. Showing $\overline{L}_1^{\underline{r}(s)} \leq 0$ requires some work. Let $\overline{L}_1^{\underline{r}(s)} \leq 0$ or $s = 1$. Using (5.3), $\overline{L}_1^{\underline{r}(s+1)} = \overline{L}_1^{\overline{r}(s)+3} = -\frac{1}{18}\left(\frac{2-a+2b}{3^{m(s)+1}} + \alpha\_(s) + 6(a-1)\right)$. Since $\overline{r}(s)+1$ does not satisfy (SC'), $\underline{L}_3^{\overline{r}(s)+1} \geq 3(1-a)$ and, using (5.7), this implies $\frac{1}{3^{m(s)+1}} \geq \frac{6-6a+\alpha\_(s)}{-2+a+2b}$. Furthermore, (5.10) implies that $\alpha\_(s) \geq \frac{1}{3}a$ for $s > 1$. Hence, $\overline{L}_1^{\underline{r}(s+1)} \leq -\dfrac{2(18+9a^2+a(b-27))}{27(2b+a-2)} < 0$ (as $b > 2(a+1)$). Since in particular $\overline{L}_1^{\underline{r}(2)} \leq 0$, by induction it follows that $\overline{L}_1^{\underline{r}(s)} \leq 0$ for all $s > 1$.

We proceed by dividing the parameter-plane not covered by (A) and (B) according to $m(1) \geq 1$, the number of rounds player 3 delays in the first delaying cycle, and proof that (SC) holds for some $\overline{r}(s)+1$. By $\alpha\_(1) = a$ and (5.8)

$$2-a+3^{m(1)}a < b \leq 2-a+3^{m(1)+1}a \tag{5.11}$$

By (5.9), $\overline{r}(1)+1$satisfies (SC') if $\underline{L}_3^{\overline{r}(1)+1} = \dfrac{\frac{1}{2}a+b-1}{3^{m(1)+1}} - \frac{1}{2}a < 3$ (1 - a) if

$$b < 1+3^{m(1)+2} - \tfrac{1}{2}(1+5\cdot 3^{m(1)+1})a \tag{C}$$

Now, let $\overline{r}(1)+1$ not satisfy (SC) and (A) - (C) not hold. Using $\alpha\_(1) = a$ and (5.10), we get $\alpha\_(2) = \dfrac{2-a+2b}{3^{m(1)+3}} + \frac{4}{9}a$. Hence, by (5.7), $\gamma_-^{\underline{r}(2)+m(1)} = \dfrac{10a+7b-20}{3^{m(1)+3}} - \frac{4}{9}a$. This is positive iff:

$$b > \tfrac{2}{7}(10-5a+2\cdot 3^{m(1)+1}a) \tag{D}$$

This means that if (D) holds $m(2) > m(1)$ and $\underline{L}_3^{\overline{r}(2)+1} \leq \underline{L}_3^{\underline{r}(2)+m(1)+1}$. Using (5.7) and the upper bound for b in (5.11), we get $\underline{L}_3^{\underline{r}(2)+m(1)+1} = \frac{2}{9}\left(\dfrac{a+b-2}{3^{m(1)+1}} - a\right) \leq 0 < 3(1-a)$. Hence, if (D) is met $\overline{r}(2)+1$ satisfies (SC').

Finally, let $\overline{r}(2)+1$ not satisfy (SC) and (A) - (D) not hold. As long as $m(s) = m(1)$, from (5.10) we get

$$\alpha\_(s+1) = \frac{1}{3}a + \frac{1}{9}\alpha\_(s) + \frac{2-a+2b}{3^{m(1)+3}} \tag{5.12}$$

The unique steady state of this difference equation is $\alpha_{\_} = \frac{1}{8}\left(3a + \frac{2-a+2b}{3^{m(1)+1}}\right)$, which is a global attractor with a monotonic dynamic since $0 < \frac{d\alpha_{\_}(s+1)}{d\alpha_{\_}(s)} < 1$. Using the upper bound for b in (5.11), $m(1) \geq 1$ and $a \geq \frac{26}{31}$, we get that $\alpha_{\_} \leq \frac{1}{12}(1+7a) \leq a = \alpha_{\_}(0)$. Hence, $\alpha_{\_}(s)$ decreases monotonically to $\alpha_{\_}$. Suppose $m(s) = m(1)$ for all delaying sequences. Using the right-hand side of (C) as lower bound for b and $0 \leq a < 1$, we get $\frac{b}{3^{m(1)+1}} - \alpha_{\_} \geq \frac{1}{4}\left(9(1-a) + \frac{2-a}{3^{m(1)+1}}\right) > 0$. Hence, if $m(s) = m(1)$ for all s, there exists an $\hat{s}$ such that $\underline{L}_3^{\overline{r}(\hat{s})+1} + \overline{L}_1^{\overline{r}(\hat{s})+1} = \frac{b}{3^{m(1)+1}} - \alpha_{\_}(\hat{s}) > 0$ and $\hat{s}$ meets (SC').

$m(s)$ is increasing in s, because $\alpha_{\_}(s)$ is decreasing in s and, by (5.8), $m(s)$ is decreasing in $\alpha_{\_}(s)$. This means that if $m(s)$ is not equal to $m(1)$ for all s, there exists an s' such that $m(s') > m(s'-1) = m(1)$. Furthermore, $\alpha_{\_}(s) > \frac{1}{3}a + \frac{2-a+2b}{3^{m(1)+3}}$ (for $s > 1$). Using (5.7), this implies that $\underline{L}_3^{\overline{r}(s')+1} \leq \left(\frac{2(a+b)-4}{3^{m(1)+3}} - \frac{1}{6}a\right)$. As a consequence, $\underline{L}_3^{\overline{r}(s')+1} \leq 0$ and $\overline{r}(s')+1$ satisfies (SC) if

(E) $$b \leq \frac{1}{4}(8 - 4a + 3^{m(1)+2}a)$$

Hence, if (E) holds, either $m(s) = m(1)$ for all s or not, both of which imply that (SC) holds for some $\overline{r}(s)+1$. Furthermore, the right-hand side of (E) minus the right-hand side of (D) is $\frac{3}{28}\left(\left(4 + 5 \cdot 3^{m1}\right)a - 8\right)$ and this is positive if $a \geq \frac{26}{31}$ and $m(1) \geq 1$. Hence, since (A) – (D) do not hold, (E) must hold.

In conclusion, for each $(a,b) \in [0,1) \times [a,\infty)$ there exists some round $r \in \mathbb{N}$ that satisfies (SC) and, hence, the outcome converges to 0 as r increases.

(i.b) We show that if $a = b = 1$, then $\lim_{r\to\infty} f^r = 1$ and $\lim_{r\to\infty} D^r = 0$.

In round 1, $p_1^1 = -1$, $p_2^1 = 0$ and $p_3^1 = 1$. In round $r > 1$, $p_1^r = -\overline{D}_2^r$, $p_2^r = 0$ and $p_3^r = \overline{D}_2^r$, with $f^r = 1$ and $D^r = \frac{2}{3}\overline{D}_2^r = \frac{2}{3}D^{r-1}$. Consequently, $\lim_{r\to\infty} f^r = 1$ *and* $\lim_{r\to\infty} D^r = 0$.

(ii) We show that if $1 \le a < 2$ and $b > 1$, then $f^* \neq 0$ and f^*, L^* or D^* does not exist, except if $\frac{13}{7} \le a = b < 2$ or $b = \frac{3}{2}, \frac{5}{4} \le a < \frac{3}{2}$ or $\frac{8}{3} \le b < \frac{13}{14}, a = \frac{5}{7}b - \frac{4}{7}$ or $\frac{3}{2} \le b < \frac{7}{3}$,$\max\{\frac{7}{5} - \frac{1}{10}b, \frac{7}{8}b - \frac{8}{5}\} < a < \frac{6}{5} + \frac{1}{5}b$. In these latter case, the outcome may converge but never to a single outcome in Z.

Let $1 \le a < 2$ and $b > 1$. First, we show that if the outcome converges there exists an $r^\dagger$ such that $p_3^r \neq delay$ for all $r > r^\dagger$. Since $EU_2^r > 0$ and $\overline{L}_1^r > -\overline{D}_2^r$, $\overline{L}_1^r > -1$ and $p_2^r \in \mathbb{R}$ for all $r \ge r^\dagger$. Suppose there exists an *r'* such that $p_3^r = delay$ for all $r \ge r'$. This implies that $EU_1^r > 0$, $p_2^r = \overline{L}_1^r < 0$ and $p_1^r = -\overline{D}_2^r > -a$ for all $r \ge$ *r'* and hence by (5.1) that $f^* = 1$. Furthermore, using (5.3), (5.4), and the logic behind (5.7), we get that $\alpha_-^r = \alpha_-^{r'} > 0$ and, hence, $D^r = \frac{2}{3}\frac{1}{2}(-p_1^r - p_2^r) + \frac{1}{3}D^{r-1} \ge \frac{1}{3}(-p_1^r - p_2^r) = \frac{1}{3}\alpha_-^{r'}$ for all $r \ge$ *r'*. Now, $p_3^r = delay$ only if $\overline{D}_2^r < \underline{L}_3^r$, which implies by (5.4) that $(1 - f^r)(b - 2) - 2f^r D^r > 0$ for all $r \ge$ *r'*. However, this is not possible, since $f^* = 1$ and $D^r \ge \frac{1}{3}\alpha_-^{r'} > 0 \forall r \ge r'$. Hence, there does not exist an *r'* such that $p_3^r = delay$ for all $r \ge$ *r'*. Convergence (of $\underline{L}_3^r - \overline{D}_2^r$) implies the opposite holds:

(5.13) $$\text{there exists an } r^\dagger \text{ such that } p_3^r \neq delay \text{ for all } r \ge r^\dagger$$

Second, there can be no convergence to δ as $p_2^r \in \mathbb{R}$ for all r.

Third, there can be no convergence to 0. $\lim_{r\to\infty} f^r = 1$ and (5.13) would imply that there exists an r' such that $f^r = 1$ for $r \geq r'$. This means that $p_1^r = -\overline{D}_2^r$, $p_2^r = 0$ and $p_3^r = \overline{D}_2^r$ for all $r \geq r' + 1$. Consequently, $L^r = 0$ and, hence, $EU_1^r \leq 0$ and $EU_3^r < 0$ for $r \geq r' + 1$. However, if this is the case $p_3^{r+1} = \delta$, contradicting $f^r = 1$ for $r \geq r' + 1$.

Finally, we show there can be no convergence to anything else then 0 or δ, save for four exceptions. Suppose that f^*, L^* and D^* exist, but $0 < f^* \leq 1$ and $D^* > 0$. Now, $f^* = \frac{1}{3}$, $\frac{2}{3}$ or 1. We have seen above that $f^* = 1$ is not possible. Let $f^* = \frac{1}{3}$. This means that there exists a round r' such that $f^r = \frac{1}{3}$, $\overline{D}_2^r \leq a - 1$ and, thus, $D^r \leq 3a - 5$ for $r \geq r'$. Suppose $p_2^r = \min\{0, \overline{L}_1^r\}$ for $r \geq r'$. This implies that $L^{r-1} = -D^{r-1}$ and $L^{r-1} < 0$, such that $\overline{L}_1^r < 0$. Furthermore, $D^r = -\overline{L}_1^r = \frac{2}{3}(a-1) + \frac{1}{3}D^{r-1}$. Convergence implies that $\lim_{r\to\infty} D^r - D^{r-1} = 0$ and solving for $D^r = D^{r-1}$ yields $D^* = -L^* = a - 1$. However, since $a < 2$, $D^* = a - 1 > 3a - 5$ and $D^r \leq 3a - 5$ cannot hold for all $r \geq r'$. Hence, $p_2^r \neq \min\{\overline{L}_1^r, 0\}$ and a similar reasoning (with $a - 1 \leq b - 1 < 3a - 5$) shows that $p_2^r \neq \max\{0, \underline{L}_3^r\}$. Thus, let $\pi_2^r(\overline{L}_1^r) = \pi_2^r(\underline{L}_3^r) = \frac{1}{2}$ for $r \geq r'$. This means that $L^r = 0$, $-\overline{L}_1^r = \underline{L}_3^r$ and, hence, $a = b$ for $r \geq r'$. Furthermore, $D^r = \frac{1}{2}(\underline{L}_3^r - \overline{L}_1^r)$ $= \frac{2}{3}(a-1)$. Hence, $L^* = 0$ and $L^* = \frac{2}{3}(a-1)$. $D^r \leq 3a - 5$ requires $\frac{13}{7} \leq a = b < 2$.

Let, ultimately, $f^* = \frac{2}{3}$. This means that there exists a round r' such that $f^r = \frac{2}{3}$, $1 - a \leq \overline{L}_1^r$, $a - 1 < \overline{D}_2^r \leq b - 1$, $p_1^r = -\overline{D}_2^r$, and $p_3^r = \delta$ for $r \geq r'$. In particular, this implies

$$(5.14) \qquad L^r \geq 1 - a \text{ and } \tfrac{3}{2}a - 2 < D^r \leq \tfrac{3}{2}b - 2 \; \forall r \geq r'$$

To begin, suppose $p_2^r = 0$. Now, $L^r = -D^r$ and $D^r = \frac{1}{2}\overline{D}_2^r = \frac{1}{6} + \frac{1}{3}D^{r-1}$ for $r > r'$. Solving for $D^r = D^{r-1}$ yields $D^* = -L^* = \frac{1}{4}$. As $L^* = -\frac{1}{4}$, $\overline{L}_1^{r^*} < 0$ and $p_2^r = 0$ requires $\underline{L}_3^{r^*} \leq 0$. Together with (5.14), this implies that $b = \frac{3}{2}$ and $\frac{5}{4} \leq a < \frac{3}{2}$. Suppose now that $p_2^r = -\overline{L}_1^r < 0$ for $r \geq r'$. Hence, $L^r = -D^r$ and $D^r = \frac{1}{2}(\overline{D}_2^r - \overline{L}_1^r) = \frac{1}{6}a + \frac{2}{3}D^{r-1}$. Solving for $D^r = D^{r-1}$ gives $D^* = \frac{1}{2}a = -L^*$. However, this is not possible due to (5.14), since $L^r \geq 1 - a > -\frac{1}{2}a$. To continue, suppose $\pi_2^r(\overline{L}_1^r) = \pi_2^r(\underline{L}_3^r) = \frac{1}{2}$ for $r \geq r'$. $\underline{L}_3^{r+1} + \overline{L}_1^{r+1} = \frac{1}{3}(4L^r + b - a) = 0$ implies $L^r = \frac{1}{4}(a-b)$ for $r \geq r'$. Furthermore, $D^r = \frac{1}{2}\overline{D}_2^r + \frac{1}{4}\underline{L}_3^r - \frac{1}{4}\overline{L}_1^r = \frac{1}{12}\left(a + b + 4D^{r-1}\right)$ and $L^r = -\frac{1}{2}\overline{D}_2^r + \frac{1}{4}\underline{L}_3^r + \frac{1}{4}\overline{L}_1^r = \frac{1}{12}\left(b - a - 2 - 4D^{r-1} - 4L^{r-1}\right)$. Solving for $L^r = L^{r-1}$ and $D^r = D^{r-1}$ and using $L^r = \frac{1}{4}(a-b)$, we get that $L^* = \frac{1}{4}(a-b)$ and $D^* = \frac{1}{8}(a+b)$. (5.14) implies that $\frac{8}{3} \leq b < \frac{13}{14}$ and $a = \frac{5}{7}b - \frac{4}{7}$.

The last possibility is $p_2^r = \underline{L}_3^r > 0$. Thus, $\underline{L}_3^r + \overline{L}_1^r < 0$ and $L^r < \frac{1}{4}(a-b)$ for $r \geq r'$. Furthermore, $D_r = \frac{1}{2}\left(\underline{L}_3^r + \overline{D}_2^r\right) = \frac{1}{6}\left(b + 2\left(D^{r-1} + L^{r-1}\right)\right)$, $L^r = \frac{1}{2}\left(\underline{L}_3^r - \overline{D}_2^r\right) = \frac{1}{6}\left(b + 2\left(L^{r-1} - D^{r-1} - 1\right)\right)$. Solving for $L^r = L^{r-1}$ and $D^r = D^{r-1}$, we get $L^* = \frac{1}{10}b - \frac{2}{5}$ and $D^* = \frac{3}{10}b - \frac{1}{5}$. (5.14) and $L^r < \frac{1}{4}(a-b)$ imply that $\frac{3}{2} \leq b < \frac{7}{3}$ and $\max\{\frac{7}{5} - \frac{1}{10}b, \frac{7}{5}b - \frac{8}{5}\} < a < \frac{6}{5} + \frac{1}{5}b$.

In conclusion, if $1 \leq a < 2$ and $b > 1$, then a necessary condition for convergence is that either of the following holds:

$(i)\ \frac{13}{7} \leq a = b < 2$ with $f^* = \frac{1}{3}, L^* = 0, D^* = \frac{2}{3}(a-1)$
$(ii)\ b = \frac{3}{2}, \frac{5}{4} \leq a < \frac{3}{2}$ with $f^* = \frac{2}{3}, D^* = -L^* = \frac{1}{4}$
$(iii)\ \frac{8}{3} \leq b < \frac{13}{14}, a = \frac{5}{7}b - \frac{4}{7}$ with $f^* = \frac{2}{3}, L^* = \frac{1}{4}(a-b)$, $D^* = \frac{1}{8}(a+b)$;
$(iv)\ \frac{3}{2} \leq b < \frac{7}{3}, \max\{\frac{7}{5} - \frac{1}{10}b, \frac{7}{5}b - \frac{8}{5}\} < a < \frac{6}{5} + \frac{1}{5}b$
with $f^* = \frac{2}{3}, L^* = \frac{1}{10}b - \frac{2}{5}$, $D^* = \frac{3}{10}b - \frac{1}{5}$.

(Note that these four regions covers a very small part of the parameter plane.)

(iii) If $a \geq 2$*, then* $f^{*} = 0$ and $\lim_{r^* \to \infty} \sup_{r \leq r^*} D^r \in \mathbb{R}$

It is immediate that $p_i^r = \delta$ for all r and i. Hence, $f^r = 0$ and $D^r = 0$ for all r.

Q.E.D.

Cycles

To illustrate the cyclic dependence of the equilibrium outcome on T when the core is empty, we provide below the simulation results for $1 < a < 2$ and $a < b < 3$. The color of the area indicates the period of the cycle. White regions indicate that there is a steady state. The darker the color of the area, the higher the period of the cycle. The darkest color indicates the period is equal or higher than 10.

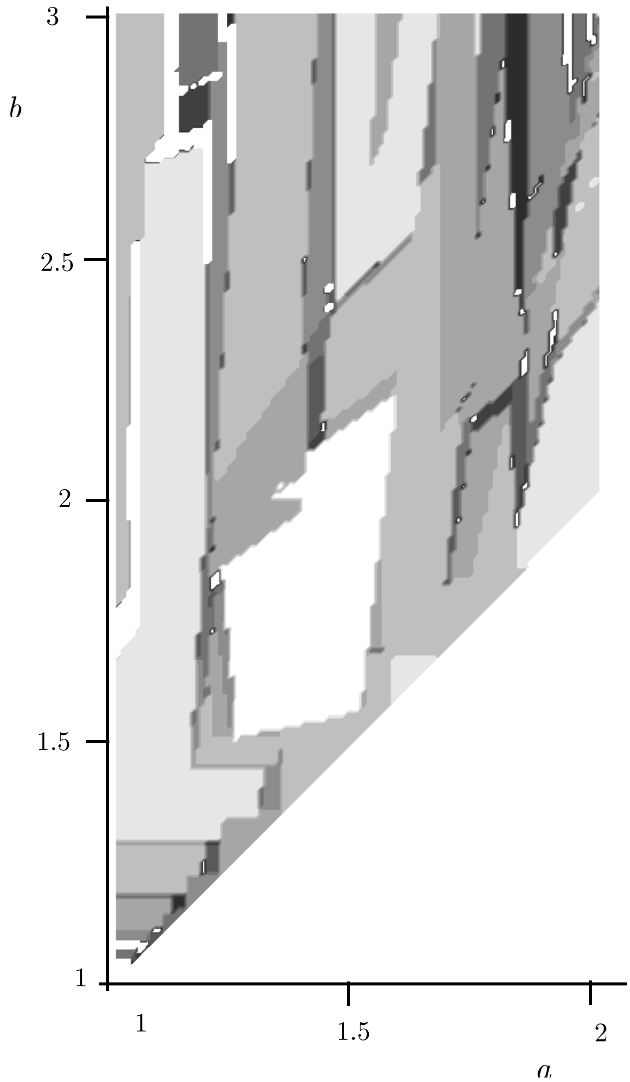


Figure 5.10
Cycles

5.7.3 Proofs of Proposition 5.2 and Proposition 5.3 (Informal)

Proof of Proposition 5.2

To prove that Γ_T^I is well-defined, we need to show how the game proceeds given some profile $\sigma \in \Sigma$ and some history h_τ. We first define the first moment of movement:

Definition 5.5 *Given* $\sigma \in \Sigma$ *and* $h_\tau \in H$ *and let* $R_i(\sigma_i \mid h_\tau) \equiv \{\tau \leq t \leq T$: $\sigma_i(h_t(h_\tau)) \neq \sigma_i(h_{t-}(h_\tau))\}$. (i) $r_i(\sigma_i \mid h_\tau)$ is *the first moment of movement of player i. If* $R_i(\sigma_i \mid h_\tau) = \varnothing$, then $r_i(\sigma_i \mid h_\tau) \equiv T + \rho$ *and player i would not move at any* $h_t(h_\tau) \subseteq h_T(h_\tau)$. *If* $R_i(\sigma_i \mid h_\tau) \neq \varnothing$, *then* $r_i(\sigma_i \mid h_\tau) = \min R_i(\sigma_i \mid h_\tau)$. (ii) *We define the first moment of movement* $r(\sigma \mid h_\tau) \equiv \min_{i \in N}\{r_i(\sigma_i \mid h_\tau)\}$.

If $R_i \neq \varnothing$, $\min R_i$ must exist, because otherwise (S2) would not hold for $h_{\inf R_i}(h_\tau)$.

Now we can define a function γ that returns a history h' as a function of any unresolved history h_s. The function determines whether the (absence of a) first moment of action directly leads to a resolved history. If this is not the case, it returns another unresolved history with $\tau(h') \geq \tau(h) + \rho$.

Definition 5.6 *Define* $\gamma : H \times \Sigma \to \tilde{H} \cup \bar{H}$, $(h_\tau, \sigma) \to h'$, *as follows:*

- $h_r(h_\tau) \subset h'$, with $r = r(\sigma \mid h_\tau)$.
- If $r > T$, then $h' = h_{T+\rho}(h_\tau) \in \bar{H}$.
- If $r \leq T$, then $p_i^t(h') = \sigma_i(h_r(h_\tau))\ \forall i, j\ \forall t \in [r, \tau(h'))$
- $h' \in \bar{H}$ if $T - \rho < r \leq T$ or $\sigma_i(h_\tau) = a_j$ for some *i,j*
- $h' \in H$ and $\tau(h') = r + \rho$ if $r \leq T - \rho$ and $\sigma_i(h_\tau) \neq a_j \forall i, j$

Now, it is straightforward to show that that Γ_T^I is well-defined.

Proposition 5.2 *The game* Γ_T^I *is a well-defined mapping* $\Gamma_T^L(G): H \times \Sigma \to \overline{H}_{T+\rho}, (h_\tau, \sigma) \to \overline{h}$

Proof: Consider $\sigma \in \Sigma$ and $h_\tau \in H$. Γ_T^I applies γ iteratively. It starts with $\gamma^0(h_\tau) = h_\tau$. If $\gamma^k(h_\tau) \in H$, then $\gamma^{k+1}(h_\tau) = \gamma(\gamma^k(h_\tau))$. If $\gamma^k(h_\tau) \in \overline{H}$, then the procedure stops and $\overline{h} = \gamma^k(h_\tau)$. Because $\tau(\gamma(h_\tau)) \geq \tau(h_\tau) + \rho$ and T / ρ is finite, this procedure will always return a resolved history. *Q.E.D.*

Proof of Proposition 5.3

By p we denote the vector $(p_1, p_2, p_3)$and by $p_{\text{-}i}$ we denote the vector (p_j, p_k). For convenience, we set $u_i(\varsigma) \equiv -\infty$.

Definition 5.7 *By* $\hat{p} / \hat{z}$ *we denote the strategy profile such that the following 1-3 hold.*

1. *For each active history* $\tilde{h}_\tau \in \tilde{H}_T$ with $\tau > T - \rho$

   (i) $\sigma_i(\tilde{h}_\tau) = a_j$ iff (a) $u_i(p_j^{\tau-}(\tilde{h}_\tau)) \geq u_i(\delta)$ and $u_i(p_j^{\tau-}(\tilde{h}_\tau)) > u_i(p_k^{\tau-}(\tilde{h}_\tau))$ or (b) $p_j^{\tau-}(\tilde{h}_\tau) = \hat{z}$, $u_i(\hat{z}) \geq u_i(\delta)$ and $u_i(p_k^{\tau-}(\tilde{h}_\tau)) \leq u_i(\hat{z})$, or (c) $p_j^{\tau-}(\tilde{h}_\tau) \neq \hat{z}$ and $u_i(p_j^{\tau-}(\tilde{h}_\tau)) = u_i(p_k^{\tau-}(\tilde{h}_\tau)) \geq u_i(\delta)$ and either $p_j^{\tau-}(\tilde{h}_\tau) > p_k^{\tau-}(\tilde{h}_\tau)$ or $p_j^{\tau-}(\tilde{h}_\tau) = \delta$.

   (ii) $\sigma_i(\tilde{h}_\tau) = \delta$ iff $\sigma_i(\tilde{h}_\tau) \notin \{a_1, a_2, a_3\}$

2. *For each active history* $\tilde{h}_\tau \in \tilde{H}_T$ with $T - \rho < \tau \leq T - \rho$

   (i) $\sigma_i(\tilde{h}_\tau) = a_j$ iff (a) $u_i(p_j^{\tau-}(\tilde{h}_\tau)) \geq u_i(\hat{z})$ and $u_i(p_j^{\tau-}(\tilde{h}_\tau)) > u_i(p_k^{\tau-}(\tilde{h}_\tau))$ or (b) $p_j^{\tau-}(\tilde{h}_\tau) = \hat{z}$ and $u_i(p_k^{\tau-}(\tilde{h}_\tau)) \leq u_i(\hat{z})$ or (c) $p_j^{\tau-}(\tilde{h}_\tau) \neq \hat{z}$ and $u_i(p_j^{\tau-}(\tilde{h}_\tau)) = u_i(p_k^{\tau-}(\tilde{h}_\tau)) \geq u_i(\hat{z})$ and either $p_j^{\tau-}(\tilde{h}_\tau) > p_k^{\tau-}(\tilde{h}_\tau)$ or $p_j^{\tau-}(\tilde{h}_\tau) = \delta$.

(*iii*) $\sigma_i(\tilde{h}_\tau) = \hat{p}_i$ iff $\sigma_i(\tilde{h}_\tau) \notin \{a_1, a_2, a_3\}$

3. $z\left(\Gamma^I_{\tilde{h}_\tau}\left(\hat{p} / \hat{z}\right)\right) = \hat{z}$ if $p^{\tau -}(h_\tau) = \hat{p}$ and $\tau \leq T - \rho$.

Such profiles have a special property:

Lemma 5.2 *If $\sigma = \hat{p} / \hat{z}$ is a SPE of some subgame $\Gamma^I_{T \backslash h^*}$ with $h^* \in \tilde{H}_{T-\rho}$, then it is a SPE for any subgame $\Gamma^I_{T \backslash \hat{h}}$.*

Proof: Let $\sigma = \hat{p} / \hat{z}$ be a SPE of $\Gamma^I_{T \backslash h^*}$ and let $\Gamma^I_{T \backslash \hat{h}_\tau}$ be a subgame of Γ^I_T. For all $\hat{h}$ with $\tau(\hat{h}) > T - \rho$, it is immediate from the definition of $\hat{p} / \hat{z}$ that σ is a SPE of $\Gamma^L_{T \backslash \hat{h}}$.

Hence, consider some $\hat{h}$ with $\tau(\hat{h}) \leq T - \rho$ and let us look at whether there exists some $\hat{\sigma}_i$ such that $U_i(\hat{\sigma}_i; \sigma_{-i} \mid \hat{h}) > U_i(\sigma_i; \sigma_{-i} \mid \hat{h})$. If players j and k adhere to σ_{-i}, then $p^{\tau' -}_{-i}(\tilde{h}) = \hat{p}_{-i}$ for all $\tilde{h} \supset \hat{h}_\tau$. At $\hat{h}_\tau$ each player i accepts according to σ any proposal yielding a higher payoff than $u_i(\hat{z})$. Hence, a necessary condition for a profitable deviation is that one of the following holds:

(*i*) a subhistory $h'_\tau \supseteq \hat{h}$ exists such that player i does not accept under σ_i the most attractive proposal yielding her a higher payoff than $u_i(\hat{z})$ (i.e. $\exists j \neq k : \sigma_i(h'_\tau) \neq a_j,\ p^{\tau -}_i(h'_\tau) = \varsigma,\ \ u_i(p^{\tau -}_j(h'_\tau)) > u_i(\hat{z})$ and $u_i(p^{\tau -}_j(h'_\tau)) \geq u_i(p^{\tau -}_k(h'_\tau))$.)

(*ii*) a subhistory $h'_\tau \supseteq \hat{h}$ exists where she can make a deviating proposal with a higher payoff than $\hat{z}$ that will be accepted in the next active history $h_{\tau+\rho} \supset h'_\tau$. (i.e. for some active history h'_τ, $\exists z :$ $\sigma_i(h'_\tau) \neq z$, $u_i(z) > u_i(\hat{z})$ and for some j, $\sigma_j(h_{\tau+\rho}) = a_i$ for $h_{\tau+\rho} \supset h'_\tau$, $p^{\tau+\rho -}_i(h'_{\tau+\rho}) = z$ and $p^{\tau+\rho -}_{-i}(h'_{\tau+\rho}) = \hat{p}_{-i}$.)

(*iii*) $u_i(\delta) > u_i(\hat{z})$ and a subhistory $h'_\tau \supseteq \hat{h}$ exists where she can deviate by moving to be silent such that at the next active history no proposal is accepted. (i.e. $\sigma_j(h_{\tau+\rho}), \sigma_k(h_{\tau+\rho}) \notin (a_1, a_2, a_3)$ for $h_{\tau+\rho} \supset h'_\tau$ with $p_i^{\tau+\rho-}(h'_{\tau+\rho}) = q$ and $p_{-i}^{\tau+\rho-}(h'_{\tau+\rho}) = \hat{p}_{-i}$.)

From the definition of $\hat{p} / \hat{z}$ it follows that $\sigma_i(\tilde{h}_\tau) = a_j$ iff $\sigma_i(\tilde{h}_T^*) = a_j$ for $\tilde{h}_T^* \supset h^*$ with $p^{T-}(\tilde{h}_T^*) = p^{\tau-}(\tilde{h}_\tau)$. Hence, if either of aforementioned (*i*)-(*iii*) would hold, then player i could also profitably deviate at either $\tilde{h}_T^*$ or h^* and σ would not be a SPE of $\Gamma^I_{T \backslash h^*}$.

Hence, no player i can profitably deviate from σ_i, and σ is a SPE of $\Gamma^I_{T\backslash \hat{h}}$. *Q.E.D.*

We are now ready to characterize the equilibrium outcomes of Γ_T^I.

Proposition 5.3 *The set of SPE outcomes is equal to* $[c,b] \cup \delta$ *for any* Γ_T^I *with* $T \geq \rho$, *where* $c = \min\{-a, \max\{-b, b-1\}\}$.

Proof: We first show by construction that $[c,b] \cup \delta$ are SPE outcomes of Γ_T^I if $T \geq \rho$. For $z = 0$, we simply need to observe that $(0, \varsigma, 0) / 0$ is an SPE of any $\Gamma^I_{T\backslash \tilde{h}_{T-\rho}}$ and hence Γ_T^I. For $z \in [-a, 0)$ consider the following profile: σ is equal to $\hat{p} / \hat{z} = (0, \varsigma, 0) / 0$, except that $\sigma(h_0) = (\varsigma, z, z)$ and $\sigma(\tilde{h}'_\rho) = \sigma(\tilde{h}''_\rho) = (\varsigma, -a, -a)$ with $p^{\rho-}(\tilde{h}'_\rho) = (\varsigma, z, \delta)$ and $p^{\rho-}(\tilde{h}''_\rho) = (\varsigma, \delta, z)$. Now, $(0, \varsigma, 0) / 0$ is an SPE of any subgame and h_0 is the only active subhistory of h'_ρ and h''_ρ. Hence, it only remains to be shown that no player can profitably change strategies at $h_0, \tilde{h}'_\rho$ and $\tilde{h}''_\rho$. At $\tilde{h}'_\rho$, player 1 will obtain her maximal payoff. Furthermore, at $\tilde{h}'_\rho$ player 2 nor 3 can profitably deviate: neither of them can accept the other's proposal and, whatever they propose at $\tilde{h}'_\rho$, player 1 will accept $-a$ at $h_{2\rho}(\tilde{h}'_\rho)$ given that the other proposes $-a$. By the same reasoning, at $\tilde{h}''_\rho$ no player can profitably deviate. Finally, no player can profitably deviate at h_0. If player 1 moves away from ς, the outcome will be 0, which is worse for her than z. Players 2 and 3 cannot do better by proposing anything else; in particular, even if *1* - a < z < 1 proposing δ at *t=0* is not attractive for them, because that will trigger the subgame in which –a is the outcome (rather than player 1 accepting δ). Hence, $\hat{p}/\hat{z}$ is an SPE of Γ_T^I.

In a similar way, SPE of Γ_T^I can be constructed that support z∈(0, b] as an outcome. An SPE of Γ_T^I that supports δ is $\hat{p}/\hat{z}=(\delta,\delta,\delta)/\delta$, which is obviously an SPE of any $\Gamma^I_{T\backslash\tilde{h}_{T-\rho}}$. Finally, an SPE that supports $z\in[\max\{-b,b-1\},-a)$ (if –b < b – 1 < -a) is the following profile: σ is equal to $\hat{p}/\hat{z}=(\delta,\delta,\delta)/\delta$, except that $\sigma(h_0)=(z,z,\varsigma)$ and $\sigma(\tilde{h}'_\rho) = (b,b,\varsigma)$ for all $\tilde{h}'_\rho$ with $p_1^{\rho-}(\tilde{h}'_\rho)\neq z$ or $p_2^{\rho-}(\tilde{h}'_\rho)\neq z$. It is easily verified that no player can profitably deviate from σ at any $\tilde{h}'_\rho$ or h_0.

Second, we show that all points in ℝ outside of $[c,b]$ cannot be equilibrium outcomes. Suppose σ' is an SPE with outcome $\hat{z}\in\mathbb{R}\setminus[c,b]$. In this case, player 2 can in equilibrium never accept x ≠ 0 at a history h_τ, because then either player 1 or 3 could profitably deviate by proposing 0 at $\tilde{t}(h_\tau)$. If 0 is proposed, namely, then in equilibrium either player 2 will accept this, or it will trigger a subgame in which 0 is the outcome under σ'.

Player 1 will in equilibrium never accept x with |x| > a, because then player 2 could profitably deviate by proposing –a at t = $\tilde{t}(h_\tau)$ by the same reasoning. Similarly, player 3 will never accept an x with |x| > b in equilibrium. This immediately rules out the possibility that $z\in(-\infty,-b)\cup(b,\infty)$. If $\hat{z}\in$ [–b, min{–a, max{–b, b–1}), then it must be accepted by player 3. However, player 3 could then profitably deviate by at no history accepting $\hat{z}$. Since players 1 and 2 will never accept a proposal outside [-*a,* a] in equilibrium, the outcome would always be better than $\hat{z}$ for player 3. *Q.E.D.*

5.7.4 Experimental Instructions

We present the English translation of the original instructions in Dutch for both treatments.

Instructions Informal Treatment

INSTRUCTIONS

You will initially have fifteen minutes to go through these instructions. When time is up, we will ask whether there is anyone who would like some more time. In case you need more time, please raise your hand and we will simply give you the extra time you need.

Introduction

In a moment you will participate in a decision making experiment. The instructions are simple. If you follow them carefully, you can earn a considerable amount of money. Your earnings will be paid to you individually at the end of the session and separately from the other participants.

You have already received five euros for showing up. In addition, you can earn more money during the experiment. In the experiment the currency is 'francs.' At the end of the session, francs will be changed into euros. The exchange rate is **1 euro for each 10 francs.**

In this experiment you can also lose money. To prevent that your earnings become negative, you will receive at the beginning of the experiment 75 francs extra. In the unlikely situation that your final earnings will be negative, your earnings will be zero (but you keep the five euros for showing up.)

Your decisions will remain **anonymous**. They will not be linked in any way to your name. Other participants cannot possibly figure out which decisions you have made.
You are not allowed to talk to other participants or communicate with them in any other way. If you have a question, please raise your hand.

Periods and Groups

The experiment consists of **24 periods**, each of which will be carried out in **groups of three players**.

At the beginning of **each period**, participants will again be randomly divided into groups of three. The chances that you will be with any other participant in the same group for two consecutive periods are therefore very small.

Choices and Earnings

In each period, your group of three participants negotiates about **choosing a number**. The chosen number **determines the earnings** of each of the three participants for that period. The group can choose **any integer between 0 and 100**. The group can also choose not to determine any number (the **"no number"** option).

Hence, the number chosen by the group determines the earnings for each member of the group. These earnings are different per member nevertheless. How much a player earns depends, in addition of the chosen number, also on her '**ideal value**.' Each player in a group receives an ideal and unique value **between 0 and 100**. The earnings for a player increase as the outcome lies closer to this ideal value.

If the outcome is exactly equal to the ideal value of a player, then this player receives the **maximum earnings of 20 francs**. The difference between the ideal value and outcome (if any) decreases the earnings by the **same amount in francs**. For instance, suppose your ideal value in a certain round is 54. Then you receive 20 francs if the outcome of the period is 54, 19 if the outcome is 53 or 55, 18 if the outcome is 52 or 56 etc. Your earnings may also be negative. If the group, for instance, chooses the number 20, then with an ideal value of 54, your earnings will be equal to -14.

The outcome of a period can also be that the group reaches **no agreement**. Hence, one chooses "**no number**." In this case each member of the group receives **0 francs**.

During a round, **players are identified by a letter: A, B and C**. These are based on their ideal value: the player with the lowest ideal value is A and the player with the highest ideal value is C. For instance, suppose the ideal values of the three players are 16, 54 and 86. Then the player with ideal value 16 is player A, the player with ideal value 54 is player B and, finally, the player with ideal value 56 is player C.

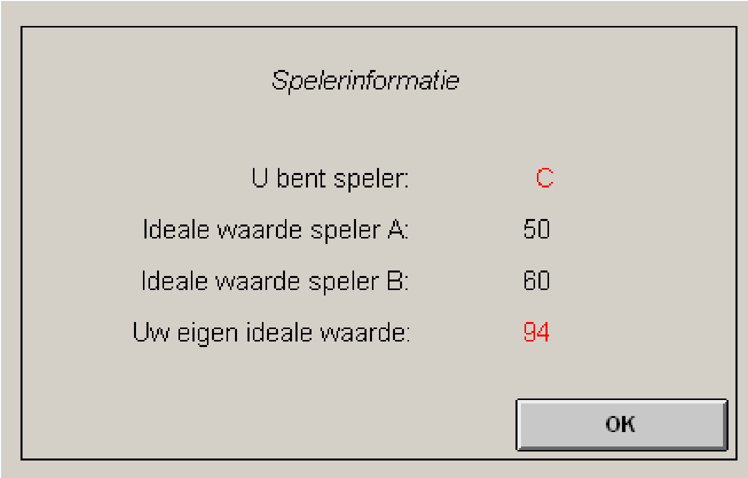
The negotiations

The group negotiations on how to choose a number consist of several steps. First, we give an overview. Afterwards, we discuss the separate steps one at a time.

1. **Before** the negotiations start, each player can send a **private message** to **each other member** of the group. A message is a suggestion for the number to choose. Each message from one player to another remains **secret for the third player**.
2. Then, there will be 2.5 minutes during which participants can make and accept proposals. A proposal is a **number between 0 and 100** or a proposal to **end the negotiations**.
3. As soon as a proposal is **accepted** by a player other than the proposer, **the negotiations end**. The accepted proposal is the **group's choice** for that period.
4. **The period also ends** if after **two and half minutes** no proposal has been accepted. The outcome is then "**no number**" and all players earn 0 francs.

Information screen

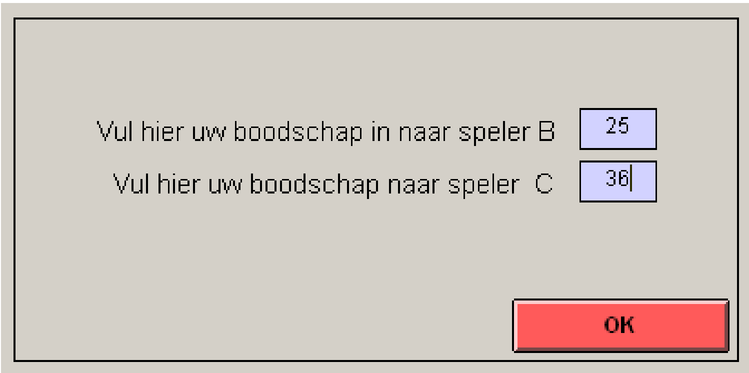
The first screen that you will see in a period, will show which player you are (A, B or C) and the ideal values of you and your group members. Your own letter is marked in red.



If you are ready to proceed, before the time has elapsed, you can press the OK-button.

Sending and receiving messages

Subsequently, you will be able to send a message to each of your two group members and they will be able to send a message to you.



A message is either an integer between 0 and 100 or the word "end." A number is a **suggestion for the group choice**. With "end" you tell the two players that you do not want to negotiate (and therefore have earnings 0). You can also choose to send **no message** by **not filling out anything or typing the space bar**. To send a message, you fill out a number or "end" in one or both cells and you press OK.

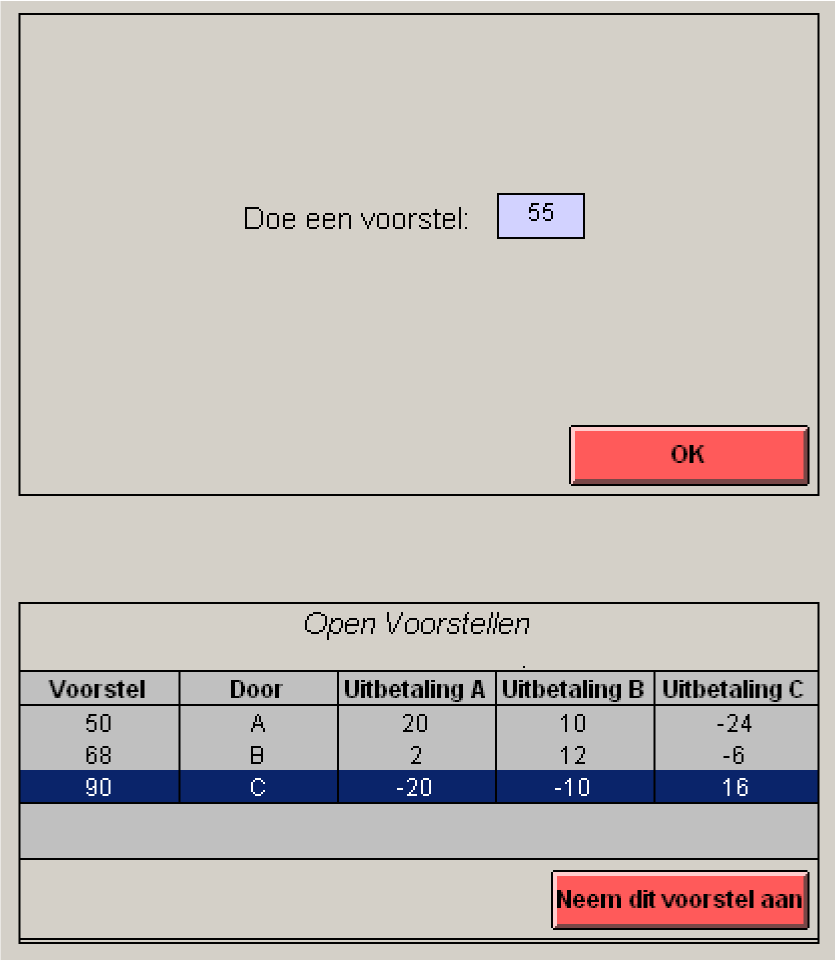
Attention: suggestions you send as a message are not put to a vote and will only be seen by the player who receives the message.

You receive **30 seconds** to send messages. If you do not fill out anything and **press OK** within this time, then no message will be sent. The other players will only see a space at their cell in this case.

After the 30 seconds have elapsed, you will see the messages that the other players sent to you. **You will NOT see what the other players sent to each other**.

Making and accepting proposals

You are then ready to make and accept proposals. In this phase you will see in the top-left corner of your screen all the necessary information (your identity, the messages, the ideal values). At the end of these instructions, we will show you the entire screen lay out.



As a group you will have **two and a half minutes** (150 seconds) to accept a proposal (or not accept one). A proposal can once again be **any integer between 0 and 100 or the word "end."**

During this phase, **you can do three things**: make your own proposal, revise your own proposal or accept a proposal by another player.

To make a proposal, you fill out the number or word you want to propose and press on the "OK" button. This proposal will become immediately visible to the other players in the list "outstanding proposals." Each of the other two players can accept a proposal you make.

To revise your proposal, you simply make another proposal. This must be different from the previous proposal. The old proposal disappears from the list "Outstanding Proposals" (but, as we shall see later, it will remain in the list "Made proposals" on the left of your screen). The new proposal replaces the old one in the list "Outstanding Proposals"

If one of the other layers has made a proposal, then you can **accept a proposal**. You do this by clicking on the proposal you want to accept in the list "Open Proposals" and press the button "Accept this Proposal."

As soon as a **proposal has been accepted** by a player, the **period ends**. The choice of the group for this period is then the accepted proposal. If no proposal is accepted within the two and a half minutes, then the group chooses "no number" and all players receive 0 points.

Results

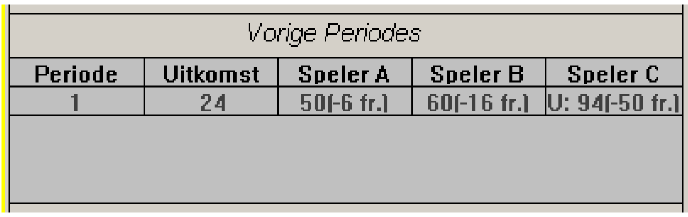
At the **end of each period**, you will get to see the outcome and the corresponding earnings.

Screens

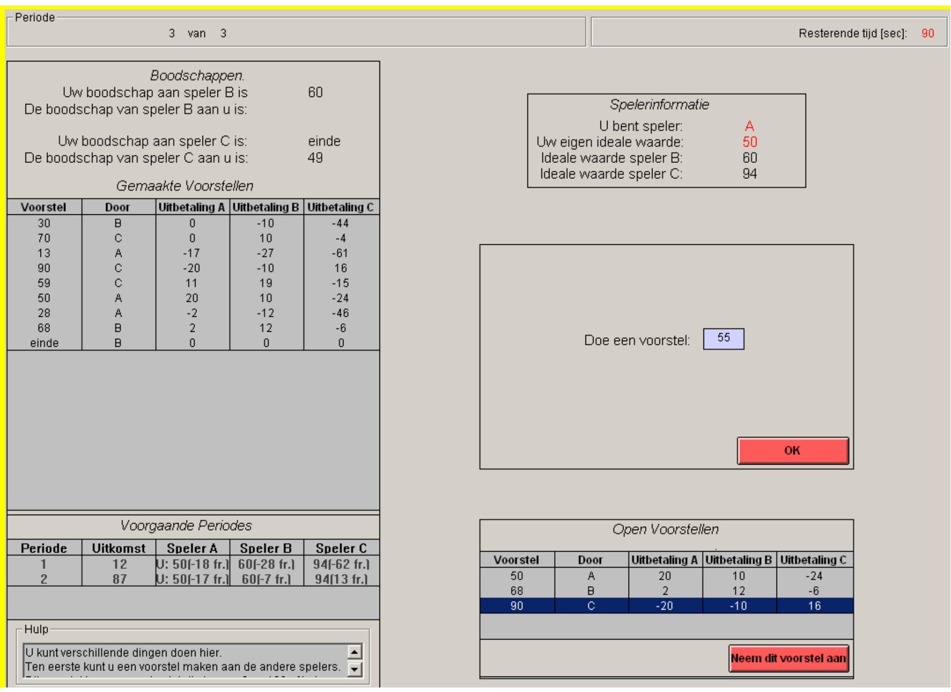
There is a lot of information you can use while you are making your choices. You can find:

- the player you are
- the ideal values of each player
- the messages you sent and received
- the proposals that have been rejected
- the outcomes of previous periods

At "Previous Periods," you can find the outcomes of previous periods, together with the ideal values of the player and, between brackets their earnings. The word "You" before the value and payment indicates which player you were.



At the far-left corner below you see in red the total amount of points (Earnings) that you have made across rounds. Because you received 75 francs at beginning, the counter starts at 75. Divide the final score by 10 to determine your earnings in euros. All information about previous periods is shown together on the left side of the screen. On the right side of the screen you will find new information and/or what action you have to take. On top, the ideal values of all players are displayed. Finally, you can find in the far-left corner below a help box with short description of what you have to do.



Instructions Formal Treatment

INSTRUCTIONS

You will initially have fifteen minutes to go through these instructions. When time is up, we will ask whether there is anyone who would like some more time. In case you need more time, please raise your hand and we will simply give you the extra time you need.

Introduction

In a moment you will participate in a decision making experiment. The instructions are simple. If you follow them carefully, you can earn a considerable amount of money. Your earnings will be paid to you individually at the end of the session and separately from the other participants.

You have already received five euros for showing up. In addition, you can earn more money during the experiment. In the experiment the currency is ‘francs.’ At the end of the session, francs will be changed into euros. The exchange rate is **1 euro for each 10 francs.**

In this experiment you can also lose money. To prevent that your earnings become negative, you will receive at the beginning of the experiment 75 francs extra. In the unlikely situation that your final earnings will be negative, your earnings will be zero (but you keep the five euros for showing up.)

Your decisions will remain **anonymous**. They will not be linked in any way to your name. Other participants cannot possibly figure out which decisions you have made.

You are not allowed to talk to other participants or communicate with them in any other way. If you have a question, please raise your hand.

Periods and Groups

The experiment consists of **24 periods**, each of which will be carried out in **groups of three players**.

At the beginning of **each period**, participants will again be randomly divided into groups of three. The chances that you will be with any other participant in the same group for two consecutive periods are therefore very small.

Choices and Earnings

In each period, your group of three participants negotiates about **choosing a number**. The chosen number **determines the earnings** of each of the three participants for that period. The group can choose **any integer between 0 and 100**. The group can also choose not to determine any number (the **“no number”** option).

Hence, the number chosen by the group determines the earnings for each member of the group. These earnings are different per member nevertheless. How much a player earns depends, in addition of the chosen number, also on her '**ideal value**.' Each player in a group receives an ideal and unique value **between 0 and 100**. The earnings for a player increase as the outcome lies closer to this ideal value.

If the outcome is exactly equal to the ideal value of a player, then this player receives the **maximum earnings of 20 francs**. The difference between the ideal value and outcome (if any) decreases the earnings by the **same amount in francs**. For instance, suppose your ideal value in a certain round is 54. Then you receive 20 francs if the outcome of the period is 54, 19 if the outcome is 53 or 55, 18 if the outcome is 52 or 56 etc. Your earnings may also be negative. If the group, for instance, chooses the number 20, then with an ideal value of 54, your earnings will be equal to -14.

The outcome of a period can also be that the group reaches **no agreement**. Hence, one chooses "**no number**." In this case each member of the group receives **0 francs**.

During a round, **players are identified by a letter: A, B and C**. These are based on their ideal value: the player with the lowest ideal value is A and the player with the highest ideal value is C. For instance, suppose the ideal values of the three players are 16, 54 and 86. Then the player with ideal value 16 is player A, the player with ideal value 54 is player B and, finally, the player with ideal value 56 is player C.

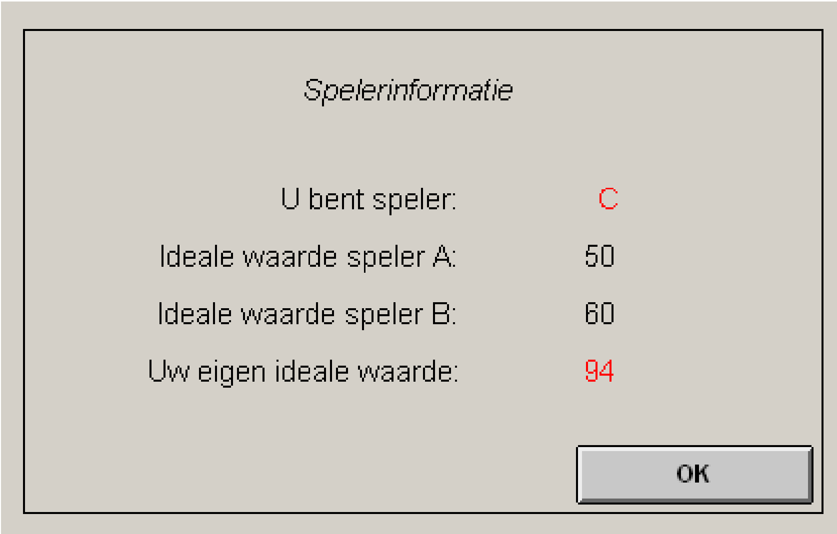
The negotiations

The group negotiations to choose a number consist of several steps. First, we give an overview. Afterwards, we will discuss the separate steps one at a time.

1. **Before** the negotiations start, each player can send a **separate message** to **each other member** of the group. A message is a suggestion for the number the group can choose. Each message from one player to another remains **secret for the third player**.
2. Next, **at most 10 rounds** follow with making proposals and voting.
3. During each round, **each of the three participants makes a proposals**. This proposal can be any number between 0 and 100 or a proposal to end the negotiations. Subsequently, **one of the three proposals is randomly chosen to be put to a vote**. The other two participants can then vote "**For**" or "**Against**" the chosen proposal (the player who made the chosen proposal automatically votes in favor).
4. If one of these two participants votes "For," then the **proposal is accepted** and the period ends. If both participants vote "Against," then the proposal is rejected and there will be a next round of making proposals and voting. This can continue until nine proposals have been rejected; if also the **tenth proposal is rejected**, then the period **ends** and the outcome is 'no number.'
5. If a proposal to **end the negotiations** is accepted, then the outcome is "**no number**" and, consequently, all players receive 0 francs. If a proposed number is accepted, then this number is the choice of the group for that period.

Information screen

The first screen that you will see in a period, will show which player you are (A, B or C) and the ideal values of you and your group members. Your own letter is marked in red.



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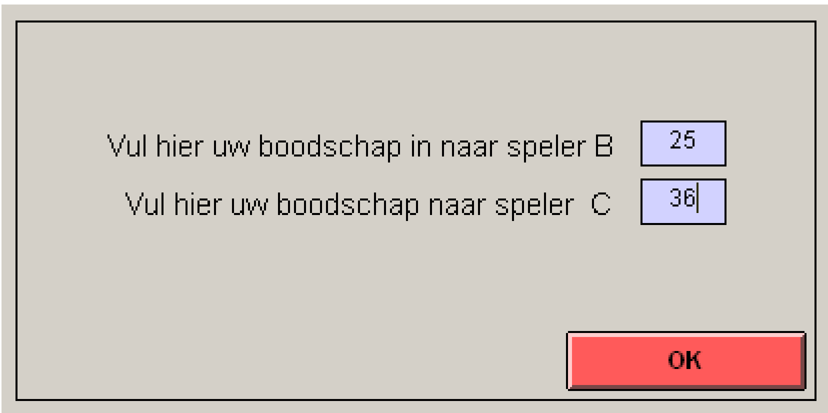
Sending and receiving messages

Subsequently, you will be able to send a message to each of your two group members and they will be able to send a message to you.



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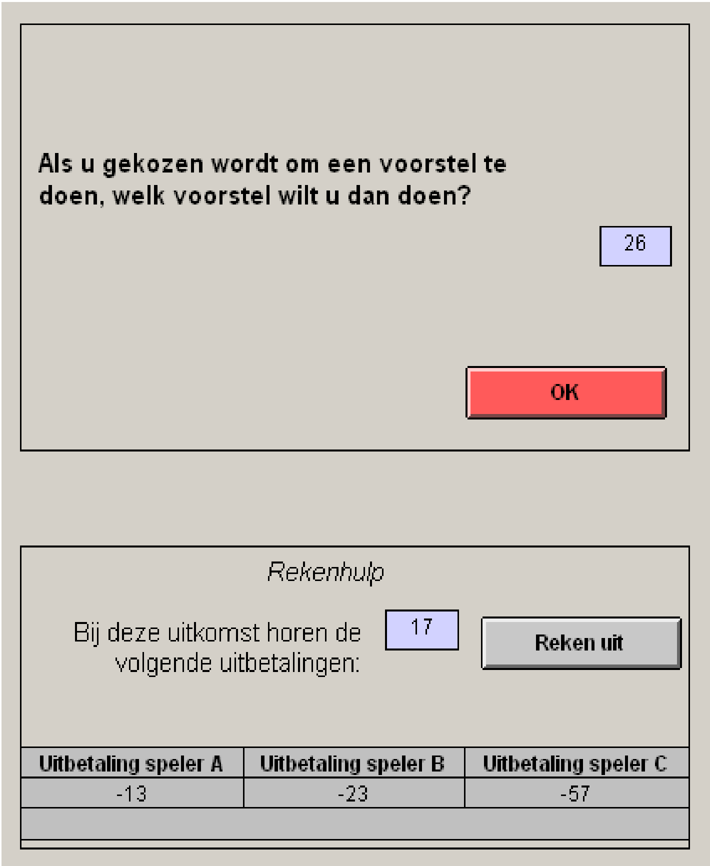


Making a proposal

You are then ready to make and accept proposals. In this phase you will see in the top-left corner of your screen all the necessary information (your identity, the messages, the ideal values). At the end of these instructions, we will show you the entire screen lay out.

A proposal can once again be **any integer between 0 or 100 or the word "end."**To make a proposal, you fill out this number or word and press "OK."

To help you calculate quickly which payments belong to which proposal, you also have a **earnings-calculator** at your disposal. If you fill out any number and press "Calculate" then the earnings will appear that all members would receive should that proposal be accepted. This device is only meant to help you. Nothing that you type there, will be seen by the other players.

ATTENTION: In each rounds, everybody fills out a proposal. However, only one of these proposals is (randomly) chosen. This proposal will be revealed to the others and be put to a vote.

You will receive 40 seconds to make your proposal. If you do not type in anything within this time, then **'0'** will be your proposal.

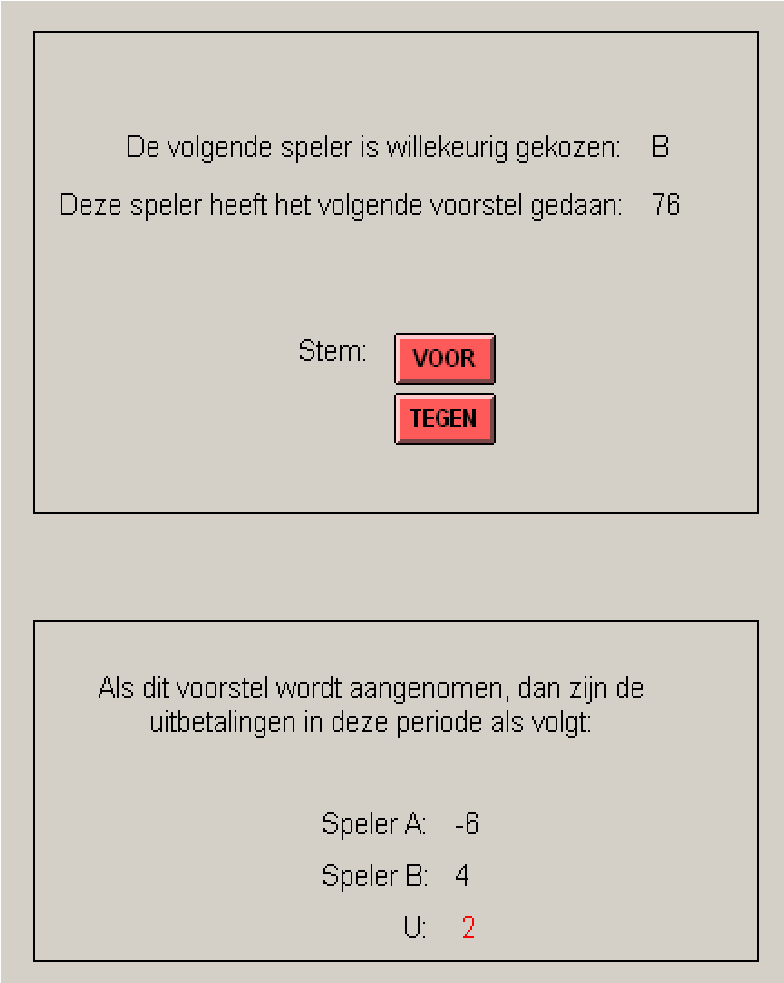
Voting

After everybody has made a proposal, it will be revealed whose proposal has been chosen. Moreover, the payments everyone would receive if this proposal would be accepted are also shown.

Next, the proposal will be put to a vote. The player who made the proposal, automatically votes "For" and **does not press any button**.

The other members can vote by simply pressing “For” or “Against.”

If at least **one of the two votes is “For”**, then the proposal is accepted and it will be the outcome of that period. The group has then made a decision and the **period ends**.



If **both vote “Against,”** then the proposal is **rejected** and you will proceed to a next round of proposing and voting. This can continue until nine proposals have been rejected. If the **tenth proposal is also rejected**, then the group was not able to reach a decision and the period **ends**. In this case, the outcome is **“no number.”**

Results

At the end of each round, you will see how each player voted, whether the proposal has been accepted and whether or not you will go to a next round.

At the **end of each period**, you will see the outcome and your corresponding earnings.

Screens

There is a lot of information you can use while you are making your choices, You can find:

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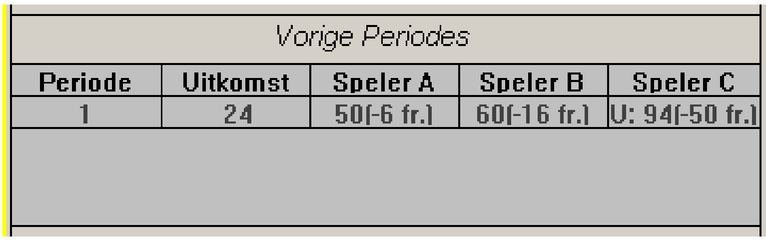

Vorige Periodes

Periode	Uitkomst	Speler A	Speler B	Speler C
1	24	50(-6 fr.)	60(-16 fr.)	U: 94(-50 fr.)

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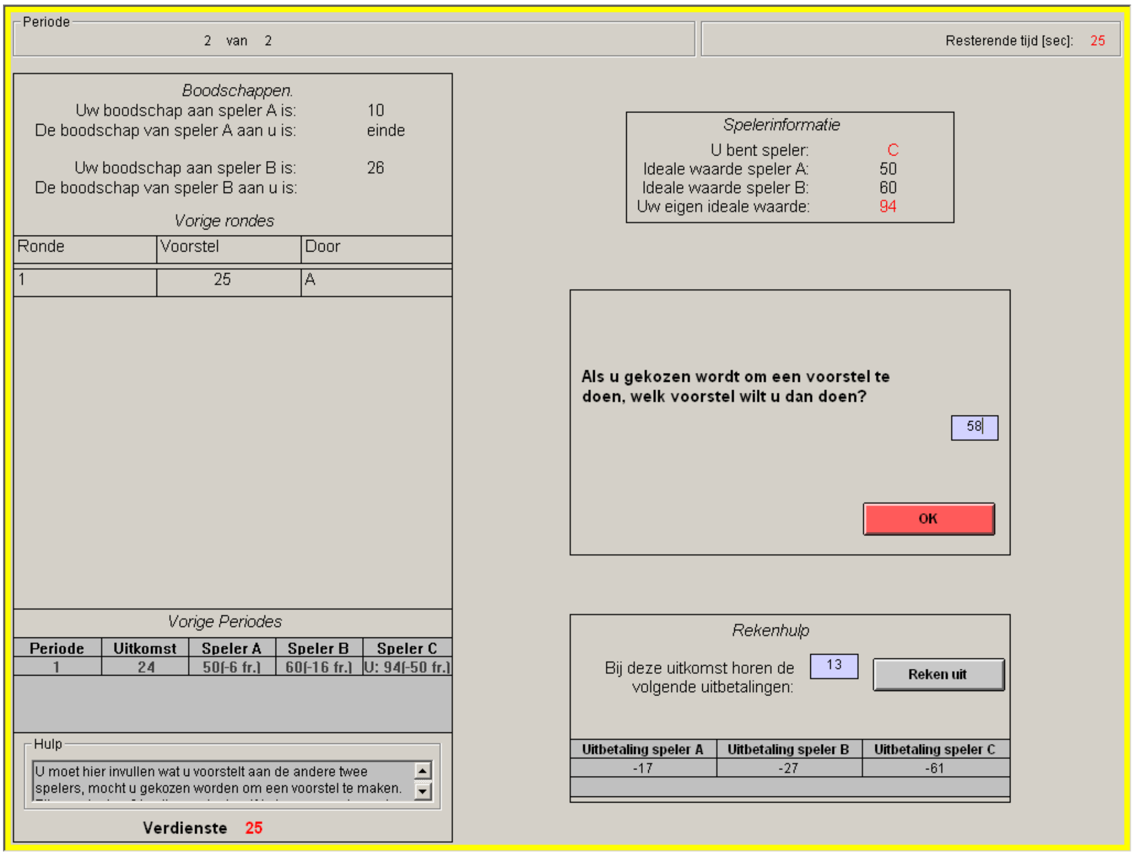

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