



UvA-DARE (Digital Academic Repository)

Inducing good behavior

van der Veen, A.

[Link to publication](#)

Citation for published version (APA):
van der Veen, A. (2012). *Inducing good behavior.*

General rights

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations

If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: <https://uba.uva.nl/en/contact>, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

F. Proofs of Propositions “Keeping out Trojan Horses”

Proof of Proposition 5.1. Let $\tilde{u}(\theta, \tilde{\theta})$ be the utility of bidder 1 with type θ who bids as if having type $\tilde{\theta}$ “close” to θ while the other two bidders bid according to the same strictly increasing bidding function B with $B(\theta) < 2\theta$. Then,

$$\begin{aligned}\tilde{u}(\theta, \tilde{\theta}) &= \int_0^{\tilde{\theta}} \int_0^{\tilde{\theta}} \max \left\{ \theta + \theta_2 + \theta_3 - B(\tilde{\theta}), 0 \right\} d\frac{\theta_2}{100} d\frac{\theta_3}{100} - \frac{1}{20,000} c \left[B(\tilde{\theta}) - \theta \right]^2 \\ &= \frac{1}{60,000} \left[\theta + 2\tilde{\theta} - B(\tilde{\theta}) \right]^3 - \frac{1}{30,000} \left[\theta + \tilde{\theta} - B(\tilde{\theta}) \right]^3 - \frac{1}{20,000} c \left[B(\tilde{\theta}) - \theta \right]^2.\end{aligned}$$

The first [second] term on the right-hand side in the first line refers to situations in which bidder 1 does not go [goes] bankrupt. The first-order condition of the equilibrium is given by

$$\left. \frac{\partial \tilde{u}(\theta, \tilde{\theta})}{\partial \tilde{\theta}} \right|_{\tilde{\theta}=\theta} = \frac{\frac{1}{2} [3\theta - B(\theta)]^2 [2 - B'(\theta)] - [2\theta - B(\theta)]^2 [1 - B'(\theta)] - cB'(\theta) [B(\theta) - \theta]}{10,000} = 0$$

from which differential equation (5.10) follows. □

Proof of Proposition 5.2. Let B be the equilibrium bid function. According to the ranking lemma (see e.g., Milgrom, 2004), the proposition holds true if $B(0) = 0$ and if $B(\theta) = \frac{5}{3}\theta$ implies that $B'(\theta) < \frac{5}{3}$. It is standard that $B(0) = 0$ must hold in a symmetric equilibrium. Moreover, suppose that bidders 2 and 3 bid according to B and that bidder 1 with signal θ bids as if having signal $\tilde{\theta}$. Bidder 1’s utility equals

$$\tilde{u}(\theta, \tilde{\theta}) = \int_0^{\tilde{\theta}} \int_0^{\tilde{\theta}} u(\theta + \theta_2 + \theta_3 - B(\tilde{\theta})) d\frac{\theta_2}{100} d\frac{\theta_3}{100}.$$

The first-order condition of the equilibrium implies that if $B(\theta) = \frac{5}{3}\theta$,

$$\begin{aligned}
0 &= 10,000 * \tilde{u}_2(\theta, \theta) \\
&= 2 \int_0^\theta u(2\theta + \theta_2 - B(\theta)) d\theta_2 - B'(\theta) \int_0^\theta \int_0^\theta u'(\theta + \theta_2 + \theta_3 - B(\theta)) d\theta_2 d\theta_3 \\
&= 2 \int_0^\theta u\left(\frac{1}{3}\theta + \theta_2\right) d\theta_2 - B'(\theta) \int_0^\theta \left[u\left(\frac{1}{3}\theta + \theta_2\right) - u\left(\theta_2 - \frac{2}{3}\theta\right) \right] d\theta_2 \Rightarrow \\
B'(\theta) &= \frac{2 \int_0^\theta u\left(\frac{1}{3}\theta + \theta_2\right) d\theta_2}{\int_0^\theta \left[u\left(\frac{1}{3}\theta + \theta_2\right) - u\left(\theta_2 - \frac{2}{3}\theta\right) \right] d\theta_2} < \frac{5}{3}.
\end{aligned}$$

The third equality follows by direct integration and by substituting $B(\theta) = \frac{5}{3}\theta$. The inequality follows because the strict concavity of implies that

$$\int_0^\theta \left[u\left(\frac{1}{3}\theta + \theta_2\right) + 5u\left(\theta_2 - \frac{2}{3}\theta\right) \right] d\theta_2 < u'(0) \int_0^\theta \left[\left(\frac{1}{3}\theta + \theta_2\right) + 5\left(\theta_2 - \frac{2}{3}\theta\right) \right] d\theta_2 = 0.$$

□

Proof of Corollary 5.1. The expected winning bid equals

$$\mathbb{E} \left\{ \min \left(\frac{\delta_n \theta_m}{1 - \delta_n}, \delta_n \theta_n \right) + \theta_k \right\} \leq \mathbb{E} \{ \delta_n \theta_n + \theta_k \} \leq \mathbb{E} \{ \theta_n + \theta_k \} \leq \mathbb{E} \{ \theta^{(1)} + \theta^{(2)} \} = 125 = R_E^\infty,$$

from which the result immediately follows.

□

Proof of Proposition 5.4. Suppose both opponents of bidder 1 bid according to (5.19). Bidder 1 wishes to step out of the auction at a price equal to her (perceived) expected value. If both of her opponents step out at the same price p , bidder 1 knows that both have signal

$$\theta = \frac{p - 100\chi}{3 - 2\chi}.$$

She steps out at price p equal to her perceived expected value, i.e.,

$$v = \theta_1 + 2(1 - \chi)\theta + 100\chi = \theta_1 + 2(1 - \chi)\frac{p - 100\chi}{3 - 2\chi} + 100\chi = p.$$

It is readily verified that $B_E^{1,\chi}$ in (5.19) is a solution. Similarly, $B_E^{2,\chi}$ follows by taking into account that bidder 1 updates her beliefs about the signal of the lowest bidder with probability $1 - \chi$.

□

Proof of Proposition 5.5. Let $\tilde{u}(\theta, \hat{\theta})$ be the perceived utility of bidder 1 with type θ who bids

as if having type $\tilde{\theta}$ while the other two bidders bid according to the same strictly increasing bidding function B . Then,

$$\tilde{u}(\theta, \tilde{\theta}) = \tilde{\theta}^2 \left[(1 - \chi) (\theta + \tilde{\theta}) + \chi (\theta + 100) - B(\tilde{\theta}) \right].$$

The first-order condition of the equilibrium is given by

$$\left. \frac{\partial \tilde{u}(\theta, \tilde{\theta})}{\partial \tilde{\theta}} \right|_{\tilde{\theta}=\theta} = 2\theta [2\theta (1 - \chi) + \chi (\theta + 100) - B(\theta)] + \theta^2 [(1 - \chi) - B'(\theta)] = 0.$$

It is readily verified that (5.20) is a solution. □

Proof of Proposition 5.6. Bidder 1 steps out at price p equal to her perceived expected value of winning given that her two opponents bid according to equilibrium. Because bidder 1 is fully cursed, she assumes that the other two bidders' signals are uniformly distributed on $[0, 100]$ regardless of her winning the auction and regardless of the price at which an opponent steps out. Therefore, she indeed steps out at a price p which solves $\tilde{U}(p, \theta) = 0$. □

Proof of Proposition 5.7. Let $\tilde{u}(\theta, \tilde{\theta})$ be the utility of bidder 1 with type θ who bids as if having type $\tilde{\theta}$ while the other two bidders bid according to the same strictly increasing bidding function B . Then

$$\tilde{u}(\theta, \tilde{\theta}) = G(\tilde{\theta}) \tilde{U}(B(\tilde{\theta}), \theta)$$

where

$$G(\theta) \equiv \frac{\theta^2}{10,000}$$

is the distribution function of the higher of two draws from $U[0, 100]$. Equation (5.27) follows immediately from the first-order condition of the equilibrium:

$$\left. \frac{\partial \tilde{u}(\theta, \tilde{\theta})}{\partial \tilde{\theta}} \right|_{\tilde{\theta}=\theta} = G'(\theta) \tilde{U}(B(\theta), \theta) + G(\theta) \tilde{U}_1(B(\theta), \theta) B'(\theta) = 0.$$

□

Proof of Corollary 5.3. (The proof proceeds along the same lines as Maskin and Riley's (1984) proof of their Theorem 4.) Conditional on a bidder with type θ winning, the expected winning

bid in EN is given by

$$R_E(\theta) = \int_0^\theta \frac{b_E^{\chi=1}(t)}{G(\theta)} dG(t)$$

where G is the distribution function of the higher of two draws from $U[0, 100]$. Consequently,

$$R'_E(\theta) = \left[b_E^{\chi=1}(\theta) - R_E(\theta) \right] \frac{G'(\theta)}{G(\theta)}.$$

The winning bid in FP equals $R_F(\theta) = b_F^{\chi=1}(\theta)$. Therefore,

$$R'_F(\theta) = b_F^{\chi=1'}(\theta) = -\frac{\tilde{U}(b_F^{\chi=1}(\theta), \theta) G'(\theta)}{\tilde{U}_1(b_F^{\chi=1}(\theta), \theta) G(\theta)}.$$

Because $b_E(0) = b_F(0)$, it follows that $R_E(0) = R_F(0)$. According to the ranking lemma (see e.g., Milgrom (2004)), the proposition follows if $R_E(\theta) = R_F(\theta) \Rightarrow R'_E(\theta) > R'_F(\theta)$, which is equivalent to

$$b_E^{\chi=1}(\theta) - b_F^{\chi=1}(\theta) > -\frac{\tilde{U}(b_F^{\chi=1}(\theta), \theta)}{\tilde{U}_1(b_F^{\chi=1}(\theta), \theta)}.$$

Consider the left- and right-hand sides as functions of b_F . For $b_F = b_E$, both sides vanish. The derivative of the right-hand side is equal to $-1 + \frac{\tilde{U}\tilde{U}_{11}}{(\tilde{U}_1)^2} < -1$ whereas the derivative of the left-hand side equals -1. Therefore, because $b_F^{\chi=1}(\theta) < b_E^{\chi=1}(\theta)$, we conclude that the inequality is satisfied. □