Essays in pension economics and intergenerational risk sharing

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Publication date
2012

Chapter 2

Intergenerational risk sharing, pensions and endogenous labour supply in general equilibrium

There is a trend towards a greater degree of funding in pension systems in OECD countries – see OECD (2011). In a number of countries, such as Chile, Denmark, the Netherlands, the U.K. and the U.S., pension funds already play a prominent role in the social security system. Anticipating the rising future costs of pension provision caused by population aging, more countries are setting up or expanding their funded pension pillars, often with mandatory participation. Examples of countries that have recently moved towards more funding are Israel and Norway. This trend will have important implications for the distribution of economic risks in society.

In this chapter the optimal design of two-tier pension systems in an overlapping generations general equilibrium model with endogenous labor supply is explored. While the first tier allows for both systematic redistribution and risk sharing between the young and old generations, the second pillar only allows for potential intergenerational risk sharing, as it is fully funded. Funded pension benefits can be of the defined contribution (DC) type or of the defined benefit (DB) type. Under DC, the contribution rate is fixed and the pension benefit is uncertain, while under DB the contribution rate is stochastic and adjusts to guarantee a fixed benefit. Of the latter type, I shall explore a defined real benefit (DRB) system, where the pension benefit is ex ante determined

This chapter is joint work with Roel Beetsma and Ward Romp and is forthcoming in the Scandinavian Journal of Economics.

in real terms, and a defined wage-indexed benefit (DWB) system, where the benefit is linked to the realized wage rate. With a DC fund, no risk sharing is possible through the second pension pillar, because the entire value of the fund is paid out to the retired. Hence, the social optimum cannot generally be replicated. With a DRB fund, optimal risk sharing requires wage risks to be shared via the first pillar. However, this requires a distortionary pension premium to be levied on wages, which, in turn, distorts the labor supply. Hence, also with a DRB fund, the social optimum cannot generally be achieved. The only system that enables the market economy to replicate the social optimum is a properly designed DWB system. Such a system allows a complete separation between systematic redistribution, which is the task of the first pillar, and optimal risk sharing, which is the domain of the second pillar. This way, the labour supply is undistorted and the first best can be mimicked.

Finding funded arrangements that minimise distortions in the labour market is of particular relevance nowadays for countries that face substantial pension deficits, while at the same time their labour forces are shrinking. In these circumstances, the mentioned trend of moving from solely pay-as-you-go systems towards more funding is to be welcomed. However, these new funded arrangements are usually of the DC type, and in existing funded schemes there is a tendency to replace DB arrangements with DC arrangements. This happened on a large scale in the U.K. and is starting to happen in the Netherlands as well. Since our results suggest that this development is not optimal as far as the scope for intergenerational risk sharing is concerned, policymakers would do well to carefully consider the design of funding arrangements.

Related to this chapter is Beetsma and Bovenberg (2009). However, there the labour supply is exogenous and DRB and DWB both achieve the first best. Hence, relaxing the assumption that the labour supply is exogenous has substantial implications for the optimal design of the funded pension pillar. There is a growing literature studying intergenerational risk sharing via pension systems. For example, Wagener (2004) and Gottardi and Kubler (2011) focus on risk-sharing within PAYG systems. Matsen and Thøgersen (2004) explore the optimal division between PAYG and funding from a risk-sharing perspective. However, they do not consider funded systems of the DB type. Neither do Teulings and De Vries (2006), who study a funded system in which each generation builds up its own pension account. Moreover, in contrast to this chapter, they adopt a partial-equilibrium setting and assume that capital markets are complete such that individuals can already invest in equity before they are born. Cui et al. (2011) do study funded systems of the DB type, but, as all the other papers mentioned so far, they do not consider endogenous labour supply. Bonenkamp and Westerhout (2010) is one of the few papers that combines both funded DB pension systems and endogenous labour supply. They compare the welfare gains from intergenerational risk sharing with the losses due to the labour market distortions caused by the income-dependent contributions in such systems. Their model lacks the analytical tractability of the model employed here. More importantly, these authors are not concerned with the design of optimal pension arrangements like I am in this chapter.

A second paper that combines funded DB schemes and endogenous labour supply is Mehlkopf (2011). The author finds that in a sixty generations OLG model, the presence of labour supply distortions forces the pension fund to deviate from optimal consumption smoothing and absorb a relatively large fraction of shocks when they occur, in order to avoid an accumulation of shocks and thus very high distortionary welfare costs in the future. The focus of Mehlkopf's paper is different from this chapter. He employs a partial equilibrium model of a funded pension fund, which is modelled in detail and focuses on the question how, quantitatively, for different parameter settings of the model the pension fund should distribute shocks to pension fund assets over different participating generations. In contrast, this chapter features a general equilibrium model, where a PAYG first pillar is included besides the funded second pillar pension fund, and where shocks do not only affect pension fund savings, but also wages and the capital stock in the economy. The two-period set-up of the chapter allows for analytical results for the social planner's objective of optimal risk sharing of the shocks occurring in the economy.

The remainder of this chapter is structured as follows. Section 2.1 lays out the model and presents the social planner's (first-best) solution. Section 2.2 discusses the market economy with the different pension systems. Section 2.3 shows that only DWB achieves the first best. Finally, Section 2.4 discusses the main results.

2.1 The command economy

In this section I derive the conditions that characterise the social planner's solution in an economy with overlapping generations that cannot share their risks through direct trade in the financial markets. The planner's solution is presented as the benchmark that the market economy combined with a pension arrangement would ideally be able to achieve.

2.1.1 Individuals and preferences

I assume a closed economy that runs for two periods (0 and 1). Periods are denoted by subscripts. In period 0, a continuum of identical individuals of total mass 1 is born. This generation lives through periods 0 and 1 and is termed the "old generation". It is denoted by the supercript "o". Utility of each agent from this generation is

$$u\left(c_0^o\right) - z\left(n_0^o\right) + \beta \mathrm{E}_0\left[u\left(c_1^o\right)\right], \tag{2.1}$$

where c_0^o denotes its consumption in period 0 and c_1^o its consumption period 1. Further, n_0^o is the endogenous labour supply in period 0, $-z(.)$ is the disutility of work effort, β is the discount factor and $\mathrm{E}_0[.]$ denotes expectations conditional on information in period 0. I assume that $u' > 0$, $u'' < 0$, $z' > 0$ and $z'' > 0$.

In period 1, a new generation (the "young generation") is born that also consists of a continuum of identical individuals of total mass 1. It lives just for this period and during this period it overlaps with the other generation. Utility of each agent from this generation is

$$u\left(c_1^y\right) - z\left(n_1^y\right), \tag{2.2}$$

which is defined over consumption c_1^y and endogenous work effort n_1^y in period 1.

2.1.2 Investment and production

In period 0 each member of the old generation receives an initial non-stochastic endowment of capital k_0. With a mass 1 of old generation members, this implies an initial aggregate capital stock equal to $K_0 = k_0$. The aggregate capital stock in period 1 is

$$K_1 = (1 - \delta_0) K_0 + I_0, \tag{2.3}$$

where I_0 is aggregate investment in period 0 and δ_0 is the (non-stochastic) depreciation rate in period 0.

Production is endogenous in both periods. Given the aggregate labour supply N_0^o and N_1^y in periods 0 and 1, respectively, production in these periods is given by

$$Y_0 = A_0 F(K_0, N_0^o), \quad Y_1 = A_1 F(K_1, N_1^y). \tag{2.4}$$

Further, A_0 is the (non-stochastic) total factor productivity in period 0, while A_1 is the total factor productivity in period 1, which I assume to be stochastic. Finally, I assume that function F exhibits constant returns to scale.

Following Bohn (1999a) and Smetters (2006), depreciation risk is introduced to reduce the correlation between labour and capital income. A growing number of recent articles argue that depreciation shocks are an important source of economic fluctuations – see, for example, Barro (2006, 2009) and Liu et al. (2010). Such shocks can occur for a variety of reasons, such as natural disasters, armed conflicts and other violence causing harm to the capital stock. Barro (2006) documents evidence of these types of shocks and finds that they occur with a probability of roughly 2% per year and an impact ranging from a decrease of 15% to 64% of real GDP per capita. Other sources of depreciation risk are unexpected technological advances and the associated creative destruction that renders capital obsolete. Further, changes in environmental regulation and other regulatory standards (such as town planning) may affect the value of the existing capital stock.

2.1.3 The resource constraints

The period 0 and 1 resource constraints are, respectively,

$$C_0^o = A_0 F(K_0, N_0^o) + (1-\delta_0) K_0 - K_1, \tag{2.5}$$

$$C_1^y + C_1^o = A_1 F(K_1, N_1^y) + (1-\delta_1) K_1, \tag{2.6}$$

where the left-hand sides denote aggregate consumption. Further, $0 \leq \delta_1 \leq 1$ is the stochastic depreciation rate of the capital stock between periods 0 and 1. The right-hand side of (2.5) represents total production minus investment. Because the world ends after period 1, whatever capital is left after production in this period is used for consumption. Hence, the right-hand side of (2.6) is total production plus capital left after depreciation.

2.1.4 The social planner's solution

The vector $\xi_0 \equiv \{A_0, \delta_0\}$ is known at the start of period 0, while the vector of shocks for period 1, $\xi_1 \equiv \{A_1, \delta_1\}$, is unknown in period 0 and only becomes known before period 1 variables are determined. As a benchmark, I consider a utilitarian social planner who chooses an optimal state-contingent plan in period 0 to maximize the sum of the expected utilities of all individuals. The consumption levels and the labour supply in period 1 are functions of the shocks, which implies $c_1^o = c_1^o(\xi_1)$, $c_1^y = c_1^y(\xi_1)$ and $n_1^y = n_1^y(\xi_1)$. Since the masses of the old and the young generations are both unity, the planner realizes that $N_0^o = n_0^o$, $C_0^o = c_0^o$, $N_1^y = n_1^y$, $C_1^o = c_1^o$ and $C_1^y = c_1^y$. Using this the Lagrangian of the planner's problem can be written as:

$$\begin{aligned}
\pounds &= \int \left[\begin{array}{c} \left[u(c_0^o) - z(n_0^o) + \beta u(c_1^o(\xi_1))\right] + \beta\left[u(c_1^y(\xi_1)) - z(n_1^y(\xi_1))\right] \\ +\beta\lambda_1(\xi_1)\left[A_1 F(K_1, n_1^y(\xi_1)) + (1-\delta_1) K_1 - c_1^y(\xi_1) - c_1^o(\xi_1)\right] \end{array} \right] f(\xi_1)\, d\xi_1 \\
&+ \lambda_0 \left[A_0 F(K_0, n_0^o) + (1-\delta_0) K_0 - K_1 - c_0^o\right].
\end{aligned}$$

Here, $f(\xi_1)$ stands for the probability density function of the vector of stochastic shocks ξ_1. The Lagrange multipliers on the resource constraints in periods 0 and 1 are denoted by λ_0 and $\lambda_1(\xi_1)$, respectively.

The optimality conditions are

$$c_1^y = c_1^o, \forall\ \xi_1, \tag{2.7}$$

$$z'(n_0^o)/u'(c_0^o) = A_0 F_{N_0}, \tag{2.8}$$

$$z'(n_1^y)/u'(c_1^y) = A_1 F_{N_1}, \forall \xi_1, \tag{2.9}$$

$$u'(c_0^o) = \beta \mathrm{E}_0\left[\left(1 + r_1^{kn}\right) u'(c_1^o)\right]. \tag{2.10}$$

where $r_1^{kn} \equiv A_1 F_{K_1} - \delta_1$ is the "net-of-depreciation return on capital" in period 1 and F_{K_t} (F_{N_t}) is the marginal product of capital (labour) in period t. (I drop function arguments whenever this does not create ambiguities.) Condition (2.7) equalizes the marginal utilities of the two generations, (2.8) and (2.9) provide the optimal consumption - leisure trade-offs for the old, respectively young generation, while (2.10) determines the optimal intertemporal trade-off.

2.2 The decentralized economy

This section describes the decentralized market economy in which individuals and firms maximize their objective functions under the relevant constraints. A key question will be which pension system can replicate the planner's solution. (2.7) can be interpreted as the condition for ex-ante trade in risks between the young and old generations in complete financial markets. However, in a decentralized economy, the two generations cannot trade risk in financial markets, because the young generation is born only after the shock vector ξ_1 has materialized. Other institutions thus have to replace this missing market and this chapter shall explore to what extent the pension system can perform that role.

In the decentralized market economy events unfold as follows. In period 0, given their initial capital holdings k_0 and the known vector ξ_0, the members of the old generation take their investment, consumption and labour supply decisions, while firms take hiring and production decisions. At the beginning of period 1 the shock vector ξ_1 materializes. After this, firms take hiring and production decisions for period 1, while the members of the young generation choose their consumption - labour trade-off.

2.2.1 The pension systems

The market economy features a two-tier pension system, with a pay-as-you-go (PAYG) first tier and a fully-funded second tier. The former consists of each young in period 1 paying to the old generation a lump-sum transfer θ^l and a fraction θ^w of their wage income. Hence, given that the two generations are both of size 1, the PAYG transfer per old person is $\theta^l + \theta^w w_1 N_1^y$.

The second tier consists of a pension fund that in period 0 per old-generation member collects a fraction θ^s of their labour income as a mandatory contribution, so total payments to the fund equal $\theta^s w_0 N_0^o$. The fund invests aggregate amounts B_1^s and K_1^s in real bonds and capital, respectively, such that

$$\theta^s w_0 N_0^o = B_1^s + K_1^s. \tag{2.11}$$

The corresponding investments per individual contributor will be denoted by b_1^s and k_1^s. The total value of the fund in period 1 is

$$(1 + r_1^a)\theta^s w_0 N_0^o = \left(1 + r_1^f\right) B_1^s + \left(1 + r_1^{kn}\right) K_1^s, \tag{2.12}$$

where r_1^a is the average fund return and r_1^f is the non-stochastic real-debt return. r_1^s denotes the net return in period 1 to the old generation members on their pension fund investment. Depending on the type of benefit scheme, the value of the fund may differ from the value of the total pay-out $(1+r_1^s)\theta^s w_0 N_0^o$ to the old. The young are the fund's residual claimants and receive the difference $(r_1^a - r_1^s)\theta^s w_0 N_0^o$. The difference (positive or negative) is spread out over the young generation in a lump-sum fashion.

The possibility that r_1^s differs from r_1^a allows for potential intergenerational risk sharing. It is assumed that the second pillar is fully funded in utility terms in an *ex ante* sense, which means that an old individual is indifferent between paying an additional unit into the fund and consuming it now (or investing it privately). Hence,

$$u'(c_0^o) = \beta E_0[(1 + r_1^s)u'(c_1^o)]. \tag{2.13}$$

The members of the old generation pay their mandatory contribution $\theta^s w_0 n_0^o$ into the fund and this contribution generates a pay off $(1 + r_1^s)\theta^s w_0 n_0^o$. Hence, the total payout to each member of this generation depends on the individual contribution. The full funding condition is necessary to ensure that the old generation makes an optimal consumption-savings decision in the first period of their life and to prevent a distortion on the labour market via the wage dependent pension contribution. Hence, it is a necessary condition for a market equilibrium with a pension arrangement to replicate the first best.

The net flows between the generations can be summarized by the generational accounts:

$$\begin{aligned} g^o &= \theta^l + \theta^w w_1 N_1^y + (r_1^s - r_1^a)\theta^s w_0 n_0^o, \\ -g^y &= \theta^l + \theta^w w_1 n_1^y + (r_1^s - r_1^a)\theta^s w_0 N_0^o, \end{aligned} \tag{2.14}$$

where g^o and g^y are the accounts for each old, respectively young, generation member and where the assumption that each generation's size is 1 has been used. Terms involving aggregate labour supply variables are the result of lump-sum transfers and will be taken as given at the individual level, because each individual is too small to affect aggregate variables by changing its own labour supply. Of course, in equilibrium $n_1^y = N_1^y$ and $n_0^o = N_0^o$ and, hence, $G^o + G^y = 0$, where G^y and G^o are defined as the aggregate accounts of the young and old generations, respectively.

Defined contribution (DC)

If the second-pillar pension is of the DC type, the total payout is simply equal to the value of the fund, i.e. assets and liabilities are always equal. Hence, $r_1^s = r_1^a$ and the second pillar provides no additional intergenerational risk-sharing opportunities. Since

$r_1^s = r_1^a$, the generational account of a young generation member reduces to

$$-g^y = \theta^l + \theta^w w_1 n_1^y. \tag{2.15}$$

Defined real benefit (DRB)

With a DRB system, each old receives a safe real return on its contribution. Full funding excludes *ex ante* intergenerational redistribution and, hence, requires

$$r_1^s = r_1^f. \tag{2.16}$$

The young receive what is left over of the pension fund after the old have been paid their safe real benefit. Hence, the young absorb the mismatch risk of the pension fund and they receive $(r_1^a - r_1^f)\theta^s w_0 N_0^o = \left(r_1^{kn} - r_1^f\right) K_1^s$, where (2.11), (2.12) and (2.16) have been used. In this case, the generational account of a young generation member becomes:

$$-g^y = \theta^l + \theta^w w_1 n_1^y + \left(r_1^f - r_1^{kn}\right) K_1^s. \tag{2.17}$$

Defined wage-indexed benefit (DWB)

Finally, with a DWB system, each old receives

$$\left(1 + r_1^s\right) \theta^s w_0 n_0^o = \theta^{dwb} N_1^y w_1, \tag{2.18}$$

where θ^{dwb} is the (non-stochastic) fraction of the aggregate wage sum in period 1. The pension benefit received by the old generation depends on the wage rate w_1 per unit of labour and is stochastic since w_1 is determined by market forces only after the shocks have materialised. The pension contribution is like an investment in a wage-linked bond, with a payout that depends on aggregate wage developments.

Combining the full-funding condition (2.13) with (2.18) yields[1]

$$\theta^s = \theta^{dwb} \frac{\beta \mathrm{E}_0 \left[N_1^y w_1 u'\left(c_1^o\right)\right]}{n_0^o w_0 u'\left(c_0^o\right)}. \tag{2.19}$$

[1] Multiply both sides of (2.18) by $u'\left(c_1^o\right)$ and take expectations $\mathrm{E}_0\left[.\right]$ on each side of the resulting equation. Combine the result with (2.13).

In the absence of a risk-free bond, $\theta^s w_0 n_0^o$ would simply be fully invested in capital. When risk-free bonds exist, as in our economy, the pension fund issues or accepts risk-free bonds to create a wedge between the collected contributions and its investment in physical capital. The generational account of a young generation member becomes

$$-g^y = \theta^l + w_1\left(n_1^y\theta^w + N_1^y\theta^{dwb}\right) - \left(1 + r_1^f\right)B_1^s - \left(1 + r_1^{kn}\right)K_1^s. \tag{2.20}$$

Because the payment $\theta^{dwb}N_1^y w_1$ is distributed in a lump-sum fashion over young generation individuals and a change in the labour supply of such an individual has a negligible effect on the aggregate wage sum, the presence of θ^{dwb} does not distort this individual's labour supply decision. Hence, the factor in front of θ^{dwb} in (2.20) should be the aggregate labour supply N_1^y of the young generation.

2.2.2 Individual budget constraints

With voluntary private investments b_1^p and k_1^p in real bonds and capital, period 0 consumption of each member of the old generation is

$$c_0^o = (1 - \theta^s)\, n_0^o w_0 + \left(1 + r_0^{kn}\right)k_0 - \left(b_1^p + k_1^p\right), \tag{2.21}$$

while period 1 consumption of, respectively, each young and old generation member is:

$$c_1^y = w_1 n_1^y + g^y, \tag{2.22}$$

$$c_1^o = \left(1 + r_1^{kn}\right)k_1^p + \left(1 + r_1^f\right)b_1^p + \left(1 + r_1^a\right)\theta^s n_0^o w_0 + g^o. \tag{2.23}$$

2.2.3 Individual and firm optimization

The model is solved through backwards induction. Under all three pension schemes, the optimal consumption - leisure trade-off to the period 1 young is

$$z'\left(n_1^y\right)/u'\left(c_1^y\right) = (1 - \theta^w)\, w_1. \tag{2.24}$$

In period 0, the old generation decides about its labour supply and the allocation of its savings over risk-free bonds b_1^p and risky capital k_1^p according to:

$$z'\left(n_0^o\right)/u'\left(c_0^o\right) = w_0 \tag{2.25}$$

$$\beta\left(1+r_1^f\right)\mathrm{E}_0\left[u'\left(c_1^o\right)\right] = u'\left(c_0^o\right), \tag{2.26}$$

$$\beta\mathrm{E}_0\left[\left(1+r_1^{kn}\right)u'\left(c_1^o\right)\right] = u'\left(c_0^o\right). \tag{2.27}$$

Appendix 2.B contains the details on the derivation of the first-order conditions.

Assuming perfectly competitive representative firms, the profit-maximization conditions in periods 0 and 1 for firms are:

$$A_tF_{N_t} = w_t, \quad A_tF_{K_t} - \delta_t = r_t^{kn}, \quad t = \{0, 1\}. \tag{2.28}$$

2.2.4 Market equilibrium conditions

The model is completed with the labour, capital and bond market equilibrium conditions. The labour market equilibrium conditions are $N_1^y = n_1^y$ and $N_0^o = n_0^o$. The capital market equilibrium condition for period 0 is $K_0 = k_0$. In period 1, the total capital stock must be equal to total privately held capital plus total capital held by the pension fund, $K_1 = K_1^p + K_1^s$. Further, since the mass of the old generation is 1, $K_1 = k_1$, $K_1^p = k_1^p$ and $K_1^s = k_1^s$. The aggregate net supply of bonds must be zero, so that total private bond holdings and total pension fund bond holdings cancel out: $B_1^p + B_1^s = 0$. Finally, $B_1^p = b_1^p$ and $B_1^s = b_1^s$.

2.3 Optimality of pension systems

2.3.1 Pension fund optimality conditions

It is now explored whether and how the market economy can replicate the social optimum with an appropriate choice of the first and second pension pillars. It is easy to see that when a pension arrangement produces (2.7) - (2.9) for all possible realizations of the shock vector ξ_1, then the market equilibrium reproduces the socially-optimal allocation under this arrangement. From equations (2.25) and (2.28) note that the pension system always satisfies (2.8). It is assumed that the pension system parameters θ^l, θ^w, θ^s, θ^{dwb}, K_1^s and B_1^s are not shock-contingent, as would be required for a realistic arrangement.

The optimality condition (2.7) requires the right-hand sides of (2.22) and (2.23) to be equal for all possible shock vectors ξ_1. Because both generations are of unity mass and populated by representative agents, the aggregate versions of these expressions can be used. Hence, condition (2.7) requires the generational accounts to vary such that:

$$\tfrac{1}{2}A_1 F_{N_1} N_1^y - \tfrac{1}{2}\left(1 + r_1^{kn}\right) K_1 = -G^y, \tag{2.29}$$

where (2.12), (2.28), $B_1^p + B_1^s = 0$, $K_1 = K_1^p + K_1^s$ and $G^y + G^o = 0$ have been used. Hence, if profit income plus the scrap value of capital of the old generation in period 1, $\left(1 + r_1^{kn}\right) K_1$, exceeds the wage income of the young generation, $A_1 F_{N_1} N_1^y$, the old would have more per-capita resources for consumption in period 1 than the young. Intergenerational equality of period 1 consumption requires the generational accounts to offset these income differences.

Reproduction for all shocks of (2.9) by (2.24) is possible if and only if

$$\theta^w = 0. \tag{2.30}$$

In other words, replication of the social optimum requires (at least) the elimination of the wage-linked part of the first pillar.

2.3.2 Optimality of different pension systems

The main result of this chapter can now be stated:

Proposition 2.1. *(a) With a DC second pillar it is generally not possible to replicate the social optimum. (b) With a DRB second pillar it is generally not possible to replicate the social optimum. (c) With a DWB pillar it is possible to replicate the socially-optimal allocation for all possible shock combinations. The appropriate parameters of the pension arrangement are* $B_1^s = \theta^s N_0^o w_0 - K_1^s$, $\theta^l = \left(1 + r_1^f\right) B_1^s$, $\theta^w = 0$, $\theta^{dwb} = \frac{1}{2}$

and $K_1^s = K_1^p$, *where* θ^s *follows from (2.19) with* N_0^o *substituting for* n_0^o.

Proof. Part (a): Under a DC fund, $-G^y = \theta^l + \theta^w A_1 F_{N_1} N_1^y$. Substitution into (2.29) yields as a necessary condition for reproducing the social optimum that $\theta^l + \theta^w A_1 F_{N_1} N_1^y = \frac{1}{2} A_1 F_{N_1} N_1^y - \frac{1}{2}\left[A_1 F_{K_1} + (1-\delta_1)\right] K_1$. It is immediately obvious that this expression cannot hold for all possible shock realizations for a constant parameter combination (θ^l, θ^w).

Part (b): Under a DRB scheme, $-G^y = \theta^l + \theta^w A_1 F_{N_1} N_1^y + \left(r_1^f - r_1^{kn}\right) K_1^s$. Substitution into (2.29) yields as a necessary condition for reproduction of the social optimum:

$$\theta^l + \theta^w A_1 F_{N_1} N_1^y + \left(r_1^f - r_1^{kn}\right) K_1^s = \frac{1}{2} A_1 F_{N_1} N_1^y - \frac{1}{2}\left[A_1 F_{K_1} + (1-\delta_1)\right] K_1.$$

There are three instruments $\left(\theta^l, \theta^w, K_1^s\right)$ to produce equality of the constant terms and the shock coefficients on both sides of this expression. The solution is $K_1^s = K_1^p$, $\theta^l = -\left(1 + r_1^f\right) K_1^s$ and $\theta^w = \frac{1}{2}$. The solution for θ^w contradicts (2.30).

Part (c): Under a DWB fund, $-G^y = \theta^l + A_1 F_{N_1} N_1^y \left(\theta^w + \theta^{dwb}\right) - \left(1 + r_1^f\right) B_1^s - \left(1 + r_1^{kn}\right) K_1^s$. Substitute this into (2.29). It is easy to check that the proposed solution ensures that the resulting expression holds for all possible shock combinations. Because $\theta^w = 0$ is part of the proposed solution, also (2.9) is fulfilled for all possible shock combinations. □

Intuitively, the first pillar can only be used to offset possible systematic transfers between generations via its lump-sum part. This pillar should not contain a wage-linked part since this would distort the young generation's labour supply decision. Hence, this pillar cannot be employed to share wage risks. Notice that the lump-sum component of the first pillar is a necessary part of the optimal arrangement, because making a lump-sum transfer out of the second-pillar fund would break the full-funding condition. Hence, inter-temporal optimisation would be distorted and the first-best would not be reached. Since a DC scheme does not allow for any risk sharing, it is obvious that a pension system consisting of a DC scheme and a lump-sum PAYG transfer cannot mimic the social planner's allocation. A lump-sum PAYG plus a DRB second pillar cannot achieve the first best either, contrary to the results in Beetsma and

Bovenberg (2009). In their paper, labour supply is exogenous, so a wage-linked PAYG part does not distort the labour supply decision. They use the first pillar to share wage risks optimally and the second pillar to share financial risks. Here the PAYG pillar does distort the labour supply, so the second pillar should share both risks. This is only possible under a DWB scheme.

2.4 Discussion

This chapter has shown that a two-tier pension arrangement with a DWB second tier is able to combine optimal intergenerational redistribution with optimal intergenerational risk sharing, without distorting the labour market. The appropriate DWB arrangement completely separates the roles of both pension system pillars, where that of the first pillar is to provide the right amount of systematic redistribution and that of the second pillar is to provide for optimal risk sharing. This contrasts with the DRB system where a distortionary pension premium is needed to share wage risks between the two generations via the first pension pillar.

Our results have clear implications for the design of pension arrangements. From the perspective of the sustainability of adequate future pension provision the trend towards more funding is to be welcomed. However, the design of new funding arrangements tends to be of the defined-contribution type, which implies that risk sharing through the second pillar of the pension system will be very limited or non-existent. Shifting the task of providing risk sharing to the PAYG first pillar creates distortions in the labour market. Hence, policymakers would do well to carefully consider the design of funded arrangements, since our results indicate that a properly designed funded DB arrangement improves welfare of participants.

An obvious extension of the present analysis would be to cast the analysis into an infinite horizon framework with endogenous labour supply and production in every period. In this infinite horizon model, the young and the pension fund must save for the new capital stock, whereas in the model in this chapter the world ends after period 1 and there is no need for this capital stock. Due to this additional complication it is no longer possible to exactly replicate the social planner's allocation in the infinite horizon model, even with a DWB system. The pension system must ensure that the two generations living at the same time have the same exposure to productivity and depreciation shocks and have the same level of consumption in a base scenario. The parameter constellation proposed in this chapter equalises exposure to economic shocks, but the old generation does not contribute to the new capital stock. Moreover, the optimal new capital stock will depend on previous technological and depreciation shocks. In the infinite horizon model, pension planners must make a trade-off between perfect risk sharing and equalisation of consumption. The optimal constrained allocation has features of both. However, I again find that to avoid labour market distortions, it is necessary that the first pillar is only used for redistribution and not for risk sharing. Although it is impossible to replicate the social planner's solution exactly, our main result still holds. A DWB system outperforms a DRB system with respect to risk sharing since any risk sharing allocation that is possible with a DRB system, is also possible with a DWB system, but without the distorting effect via the wage-linked part of the second pillar.

APPENDICES

2.A Derivation of the planner's solution

Maximization of the planner's program with respect to c_0^o, n_0^o, K_1, $c_1^y(\xi_1)$, $c_1^o(\xi_1)$ and $n_1^y(\xi_1)$ for all ξ_1 yields the following first-order conditions:

$$
\begin{aligned}
u'(c_0^o) &= \lambda_0, \\
z'(n_0^o) &= \lambda_0 A_0 F_{N_0}, \\
\lambda_0 &= \int \beta\lambda_1(\xi_1)\left(1+r_1^{kn}\right) f(\xi_1)\, d\xi_1, \\
u'(c_1^y(\xi_1)) &= \lambda_1(\xi_1), \forall\xi_1, \\
u'(c_1^o(\xi_1)) &= \lambda_1(\xi_1), \forall\xi_1, \\
z'(n_1^y(\xi_1)) &= \lambda_1(\xi_1) A_1 F_{N_1}, \forall\xi_1.
\end{aligned}
$$

By eliminating the Lagrange multipliers from these first-order conditons, we obtain

$$u'(c_1^y) = u'(c_1^o), \forall\xi_1,$$

and (2.8)-(2.10). This reduces to (2.7)-(2.10).

2.B Individual first-order conditions

2.B.1 Period 1 individual first-order conditions

The young generation solves:

$$\max_{c_1^y, n_1^y}\left\{u(c_1^y) - z(n_1^y)\right\},$$

subject to the following budget constraint, which differs according to the pension scheme that is in place:

$$\begin{aligned}
\text{DC: } c_1^y &= (1-\theta^w) w_1 n_1^y - \theta^l, \\
\text{DRB: } c_1^y &= (1-\theta^w) w_1 n_1^y - \theta^l - \left(r_1^f - r_1^{kn}\right) K_1^s, \\
\text{DWB: } c_1^y &= (1-\theta^w) w_1 n_1^y - \theta^{dwb} w_1 N_1^y - \theta^l + \left(1 + r_1^f\right) B_1^s + \left(1 + r_1^{kn}\right) K_1^s.
\end{aligned}$$

In all three cases, the first-order conditions for c_1^y and n_1^y are given by, respectively,

$$\begin{aligned}
u'(c_1^y) &= \mu, \\
z'(n_1^y) &= \mu (1-\theta^w) w_1,
\end{aligned}$$

where μ is the Lagrange multiplier on the budget constraint. The first-order conditions combine to (2.24).

2.B.2 Period 0 individual first-order conditions

We can write consumption per old individual in period 1 as:

$$c_1^o = \left(1 + r_1^{kn}\right) k_1^p + \left(1 + r_1^f\right) b_1^p + (1 + r_1^s) \theta^s w_0 n_0^o + \theta^l + \theta^w w_1 N_1^y. \tag{2.31}$$

In period 0 a member of the old generation solves:

$$\max_{c_o^o, c_1^o, n_0^o, k_1^p, b_1^p} \left\{ u(c_0^o) - z(n_0^o) + \beta \mathrm{E}_0 \left[u(c_1^o) \right] \right\},$$

subject to (2.21) and (2.31). The first-order conditions are (2.26), (2.27) and:

$$\begin{aligned}
u'(c_0^o)(1-\theta^s) w_0 - z'(n_0^o) + \beta \theta^s w_0 \mathrm{E}_0 \left[(1 + r_1^s) u(c_1^o) \right] &= 0 \Leftrightarrow \\
u'(c_0^o) w_0 - z'(n_0^o) - \theta^s w_0 u'(c_0^o) + \theta^s w_0 \beta \mathrm{E}_0 \left[(1 + r_1^s) u(c_1^o) \right] &= 0 \Leftrightarrow \\
u'(c_0^o) w_0 - z'(n_0^o) &= 0,
\end{aligned}$$

where we have used (2.13).

2.C Infinite horizon model

2.C.1 Notation

In the paper we use o and y superscripts to identify generations (so c_0^o is consumption in period 0 of the generation born in period 0 and c_1^o consumption of the same generation in period 1). This is not possible with an infinite number of generations, so in this appendix we use a subscript to identify the timing of the variable and a superscript to indicate whether this generation was born in the previous period or in this period (c_0^y is consumption in period 0 by the generation born in period 0, c_1^o is this generation's consumption in period 1). People only work when young, so individual labour supply does not need an age indicator (n_t is labour supply by someone born in period t).

2.C.2 Social Planner

Full Diamond-Samuelson OLG model for the central planner

$$\max_{c^o, c^y, n} \sum_{t=0}^{\infty} \beta^t E\Big[u(c_t^y) - z(n_t) + \beta u(c_{t+1}^o)\Big] + u(c_t^o) \tag{2.32}$$

$$\text{s.t.} \quad c_t^o + c_t^y + K_{t+1} = A_t F(K_t, n_t) + (1 - \delta_t) K_t \quad \text{for each } t \geq 0 \tag{2.33}$$

A_0 and δ_0 are known (non-stochastic), future A_t and δ_t for $t \geq 1$ are stochastic.

The FOC's are

$$c_t^o = c_t^y \quad \forall\, t \geq 0 \tag{2.34}$$

$$\frac{z'(n_t)}{u'(c_t^y)} = A_t F_N(K_t, n_t) \quad \forall\, t \geq 0 \tag{2.35}$$

$$u'(c_t^y) = \beta E_t\Big[\Big(A_{t+1} F_K(K_{t+1}, n_{t+1}) + (1 - \delta_{t+1})\Big) u'\left(c_{t+1}^o\right)\Big] \quad \forall\, t \geq 0 \tag{2.36}$$

These conditions are comparable to the conditions in the two-period model in the paper. The first equalises consumption of everybody living at the same time, the second describes the optimal trade-off between leisure and consumption, the third is the Euler equation, describing the optimal intertemporal trade-off.

2.C.3 Decentralised economy

The pension system is as in the paper. The relevant incentives are

- The member of the young generation pays a fraction θ^w per received euro wage income plus a fixed contribution θ^l. Paying to the first pillar has an incentive effect!

- The member of the old generation receives $\theta^l + \theta^w w_{t+1} N_{t+1}$, regardless of the individual work history. The first pillar's pension payouts have no incentive effect.

- Each member of the young generation pays θ^s to the pension fund. This contribution has an incentive effect. Per paid euro this participant receives a stochastic pension when old, so this pension also has an incentive effect. The full funding condition ensures that these two effects cancel out.

- The residual value of the pension fund (positive or negative) is equally spread over the young generation. This has no incentive effect.

The pension fund's budget constraint is

$$B^s_{t+1} + K^s_{t+1} = (1 + r^f_t)B^s_t + (1 + r^k_t)K^s_t + \theta^s_t w_t N_t - (1 + r^s_t)\theta^s_{t-1} w_{t-1} N_{t-1} \tag{2.37}$$

The fully funded condition still holds, so the expected return on paid contributions must be equal to the expected return on other assets.

The individual budget constraint for each young generation is

$$c^y_t = w_t n_t - b^p_{t+1} - k^p_{t+1} - (\theta^l + \theta^w w_t n_t) - \theta^s_t w_t n_t \tag{2.38}$$

And when old

$$c^o_{t+1} = (1 + r^f_{t+1})b^p_{t+1} + (1 + r^k_{t+1})k^p_{t+1} + (\theta^l + \theta^w w_{t+1} N_{t+1}) + (1 + r^s_{t+1})\theta^s_t w_t N_t \tag{2.39}$$

Optimisation of individual utility, taking the factor payments as exogenous and using the pension fund's full funding condition gives (besides the two budget constraints)

$$\frac{z'(n_t)}{u'\left(c_t^y\right)} = (1-\theta^w)w_t \tag{2.40}$$

$$u'\left(c_t^y\right) = \beta E_t\Big[\Big(1+r_{t+1}^k\Big)u'\left(c_{t+1}^o\right)\Big] \tag{2.41}$$

$$u'\left(c_t^y\right) = \beta E_t\Big[\Big(1+r_{t+1}^f\Big)u'\left(c_{t+1}^o\right)\Big] \tag{2.42}$$

The possible distortionary effect of the second pillar is neutralised by the fully funded condition. These individual first order conditions are comparable to the ones in the paper. For equilibrium factor prices (so $w_t = A_t F_N(K_t, n_t)$ and $r_t = A_t F_K(K_t, n_t) - \delta_t$), markets ensure that the intertemporal trade-off is optimal. The first-pillar's wage component distorts the intratemporal trade-off and a necessary requirement to mimic the social planner's allocation is that this wage component is zero ($\theta^w = 0$).

As in the paper, the task of the pension system is to equalise consumption for the young and the old living at the same time. Substitution of the pension fund's budget identity into that of the young and using the equilibrium conditions on the market for real bonds ($B_{t+1}^p + B_{t+1}^s = 0$) and the capital market ($K_{t+1}^p + K_{t+1}^s = K_{t+1}$) gives for the consumption of the young

$$\begin{aligned} c_t^y = w_t n_t - K_{t+1} - (\theta^l + \theta^w w_t n_t) + (1+r_t^f)B_t^s \\ + (1+r_t^k)K_t^s - (1+r_t^s)\theta_{t-1}^s w_{t-1}N_{t-1} \end{aligned} \tag{2.43}$$

This equation differs from the paper in one crucial aspect: it includes K_{t+1}. In the paper, the economy ends after period 1 and there is no reason to save so $K_{t+1} = 0$. In this infinite horizon model, the young have to save for the new capital stock. Using the pension parameters proposed in the paper gives for the consumption of the old and young living at time t

$$c_t^y = \tfrac{1}{2}w_t n_t + \tfrac{1}{2}(1+r_t^k)K_t - K_{t+1} \tag{2.44}$$

$$c_t^o = \tfrac{1}{2}w_t n_t + \tfrac{1}{2}(1+r_t^k)K_t \tag{2.45}$$

The proposed parameters do give the old and the young the same exposure to economic shocks, but it will not equalise the level of consumption because the young also have to (and want to) save for the next period. They save k^p_{t+1} themselves and their pension fund saves K^s_{t+1}, but for the proposed parameters, it all comes from their consumption. Since the optimal new capital stock is non-linear in wages and interest rate shocks, and all pension parameters must be shock-independent, it is impossible to exactly mimic the social planner's allocation.

The pension system must ensure that the two generations living at the same time have the same exposure to productivity and depreciation shocks and have the same level of consumption in a base scenario. In the infinite horizon model, pension planners must make a trade-off between perfect risk sharing and equalisation of consumption. The optimal constrained allocation has features of both. However, we again find that to avoid labour market distortions, it is necessary that the first pillar is only used for redistribution and not for risk sharing. Although it is impossible to replicate the social planner's solution exactly, our main result still holds. A DWB system outperforms a DRB system with respect to risk sharing since any risk sharing allocation that is possible with a DRB system, is also possible with a DWB system, but without the distorting effect via the wage linked part of the second pillar.