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Essays in pension economics and intergenerational risk sharing

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Chapter 5

Redesigning the Dutch occupational pension contract: Simulation of alternative contracts involving soft and hard entitlements

5.1 Introduction

Dutch pension arrangements are generally praised for their generosity, their wide coverage and their high degree of funding. The system consists of a public pay-as-you-go first pillar and a funded second pillar with employees participating in a company pension fund, an industry-wide fund or an occupation-related scheme for self-employed. However, the existing pension contract has come under pressure for a number of reasons. First, life expectancy continues to rise, implying that existing commitments to older workers and retirees are underfunded. Further, the economic and financial crisis of 2008-2009 produced a simultaneous slump in equity markets and a sharp decrease in the interest rate on high-quality public debt implying that existing liabilities had to be discounted at a lower rate. For many pension funds both factors have reduced funding ratios (the ratio of assets over liabilities) to levels that force them to undertake restoration measures. The main steering instruments in this regard are the pension
contribution rate and rate of indexation to nominal wage or price rises. The aforementioned developments have raised doubts about the robustness of the current collective pension contract for demographic and financial markets developments. Those doubts have been made explicit in the Goudswaard Report (Goudswaard, 2010) and have subsequently led to a headline agreement between the "social partners", i.e. the representatives of employees and employers, to revise the pension contract. This agreement (called the pension agreement or pension deal) specifies that social partners and pension funds can choose to stay in the existing contract with some minor changes, or move to the new contract in which all entitlements are 'soft' or conditional, so that no guarantees are given to participants with respect to their future pension benefits.

In this chapter we present some alternatives to the pension agreement, in which a participant can have both unconditional 'hard' entitlements - as in the old contract - and conditional 'soft' entitlements - as in the new contract of the pension agreement. We will quantify the consequences of redesigning the pension contract in such a way at the level of the pension fund and the consequences at the level of the individual fund participant. The main question to be answered for the pension fund concerns the impact of the new contract on its funding ratio under regular and extreme circumstances, while the main questions concerning the individual participants pertain to the distribution of replacement rates under the new contract and the evolution of the transition phase from the old to the new contract. We investigate various formats for the new pension contract allowing for financial market shocks. However, we mostly abstract from demographic shocks, because there is a consensus that the best way to deal with life expectancy shocks is to change the labor market participation of the elderly and the retirement age (see Bovenberg and Knaap (2005), Conesa and Garriga (2007) and Bohn (1999b)).

The various pension contracts considered below feature a number of common elements. Each year, active participants make a contribution to the fund. In return, they accumulate entitlements to their future pension. The way the accrual of entitlements takes place is currently under review and constitutes the focal point of this chapter. Under the current contract, there exists only one type of entitlement. This is a promise of a constant annual nominal payment as of retirement age. This payment is supposed
to be very safe in the sense that, once it has been promised to participants, it can only be reduced when the degree of underfunding is so large that it can no longer be undone through deployment of the regular instruments, a situation that is supposed to be very exceptional. The guaranteed nominal benefits are the main reason why pension liabilities are to be discounted against a risk-free interest rate.

The new pension contracts are intended to be robust against financial market shocks. In general terms, this is done by making the contract more complete in the sense that it will be made clearer ex ante how the consequences of unexpected shocks will be allocated across the various participants. Practically speaking, this takes place through the introduction of two types of entitlements. The first type is a very safe entitlement that we will refer to as a ”hard entitlement”. As described above, all entitlements accumulated until now were intended to be certain in nominal terms, although in reality they may be less safe than expected, as recent events have made clear. The second type of entitlement we will refer to as a ”soft entitlement”. Its role is to act as a buffer against financial shocks, thereby protecting the safety of the hard entitlements. Of course, it is conceivable that soft entitlements may evaporate completely due to a sequence of particularly unfortunate realizations in the financial markets. In that case also the hard entitlements have to be reduced, implying that even hard entitlements can never be completely safe.

The three new pension contracts differ in the way hard and soft entitlements are acquired. In the first contract, the ’rolling window’ contract, newly accrued entitlements are soft. After a fixed period, these entitlements are then converted into hard entitlements. In the second contract, the ’fraction’ contract, newly accrued entitlements consist of a fixed fraction of soft entitlements and hard entitlements. In the third contract, the ’split’ contract, all newly accrued entitlements are soft. If the funding ratio of the pension fund rises above a certain level, a fraction of the soft entitlements is transformed into hard entitlements.

Obviously, to guarantee a sufficiently high degree of safety of hard entitlements, it is necessary to have a sufficiently large buffer of soft entitlements. In fact, we are particularly interested in computing the likelihood that the stock of soft entitlements is insufficient to absorb all the financial shocks. The different proposals we consider
will lead to different distributions of the stocks of hard and soft entitlements across the various generations. Obviously, different allocations of the two types of entitlements will have important consequences for the incidence of the shocks across the various generations. In addition, they may have serious implications for the financial situation of the pension fund and the extent to which hard entitlements are truly safe.

Under the new proposals, indexation is higher and more readily provided than under the current system. Hence, pension buffers are lower, while current retirees benefit from the switch from the old contract to the new contract. Out of the three new proposals, only the Fraction contract succeeds in effectively guaranteeing hard entitlements and providing relatively stable indexation and buffers over time without inducing substantial intergenerational transfers. The results suggest that in a system with both hard and soft entitlements, the only way to prevent substantial intergenerational transfers is to have all generations accumulate both types of entitlements. Because financial shocks are absorbed through the soft entitlements, this way the costs and benefits of those shocks can be spread over all the generations.

The remainder of the chapter is structured as follows. In Section 5.2 we lay out our model. In Section 5.3, we describe the different designs of the new contract that we explore. In Section 5.4 we describe the calibration of our model and in Section 5.5 we present the results for the various contracts we consider. Section 5.6 concludes this chapter.

### 5.2 The Model

This section describes the set-up of the model.

#### 5.2.1 Demographics

Each period $t$, there are a total number of $N_t$ individuals, divided over $J$ generations and $I$ types. Further, $N_{i,j,t}$ is the number of type $i$ of age $j$ individuals alive at time $t$. Hence, $N_t = \sum_{i=1}^{I} \sum_{j=1}^{J} N_{i,j,t}$. Differences in types of individuals arise because of differences in skills or in gender. Those differences lead to differences in income within a cohort.
We denote by $\psi_j$ the age-dependent probability for a person that is currently of age $j$ to survive until age $j + 1$. We assume that it is constant over time and across the various types of agents $i$. Hence, the probability of an individual of age $j$ to survive until some age $l \geq j + 1$ is then given by $\prod_{k=0}^{l-j-1} \psi_{j+k}$.

Each year a new generation of size $(1 + n_t) \sum_{i=1}^{I} N_{i,1,t-1}$ is born, where $n_t$ is the growth rate in the size of the newborn generations. We assume that $n_t$ is constant. Hence, $n_t = n_{t+1} = \cdots \equiv n$. The maximum age an individual can reach is $J$. Hence, an individual of age $J$ dies for sure the coming year, implying that $\psi_J = 0$. We neglect the childhood of individuals and assume that they enter the labour force and become employed immediately after birth at age 1. Further, retirement happens at the mandatory age $R$. We denote by $N_{i,w,t} = \sum_{j=1}^{R-1} N_{i,j,t}$ the number of working individuals of type $i$ at time $t$. Hence, the total number of working individuals is given by $N_{t,w} = \sum_{i=1}^{I} N_{i,w,t} = \sum_{i=1}^{I} \sum_{j=1}^{R-1} N_{i,j,t}$.

### 5.2.2 Wage income and pension fund contributions

Depending on its type $i$, an individual in period $t$ earns a wage income $w_{i,t}$. Over time, the wage income of all types grows at the exogenous nationwide nominal growth rate $g_t$:

$$w_{i,t} = (1 + g_t) w_{i,t-1}, \quad \forall i \in I, \quad \forall t \geq 1, \quad (5.1)$$

$$g_t = g + \epsilon_t^g, \quad (5.2)$$

where $\epsilon_t^g$ is a shock to aggregate wages in period $t$ with properties detailed in Section 5.2.5.

Each working individual (i.e., individual of age $1 \leq j < R$) of type $i$ pays a contribution $p_{i,t} \geq 0$ to the pension fund. Collectively, pension funds constitute the ”second pillar” of the retirement benefit system. The contribution equals a fraction $\theta$ of income $w_{i,t}$ above the so-called franchise level $f r_t$ for those who earn more than $f r_t$ and is zero otherwise:

$$p_{i,t} = \max \left[ 0, \theta (w_{i,t} - f r_t) \right], \quad \forall i \in I, \quad \forall t \geq 1, \quad (5.3)$$
The franchise is the part of wage income over which no pension contributions are paid and over which no pension entitlements are accrued. The existence of the franchise originates in the fact that in the Netherlands the government provides a basic first-pillar PAYG pension (the ”AOW” in Dutch) for everyone. The second-pillar pension acquired through participation in the pension fund comes on top of the AOW. In other words, contributions to and accrual of entitlements in the second pillar apply only to the part of the wage above the franchise. Note that since all individuals of type $i$ earn the same wage, irrespective of their age $j$, the pension contribution does not depend on $j$. The franchise is indexed to inflation each year, since the AOW is indexed to inflation under our baseline:

$$fr_t = (1 + \pi_t) fr_{t-1},$$

where $\pi_t$ is the period $t$ inflation rate.

### 5.2.3 Second-pillar entitlements and liabilities under the old contract

**Entitlements**

The baseline of the old contract features only one type of pension entitlement. In each period $t$, the accrual of new pension entitlements $m_{i,t}$ of an individual of type $i$ equals the accrual rate $\mu$ times the difference between the nominal wage and the franchise:

$$m_{i,t} = \mu \max [0, w_{i,t} - fr_t]. \tag{5.4}$$

Note that, because the wage is independent of the age $j$ of an individual, the accrual of new entitlements is also independent of the individual’s age. The new entitlements built up in a given period $t$ are added to the individual’s beginning-of-period stock of entitlements $M_{i,j,t}$. Depending on the financial situation of the pension fund, the fund can apply either uniform indexation $\omega_t \geq 0$ or a uniform reduction $\omega_t < 0$ to the existing stock of entitlements accumulated in the pension fund.\footnote{The fund’s decision rules will be described in Section 5.3.} Hence, the law of
motion for the stock of entitlements of a working individual is given by:

\[ M_{i,j,t+1} = (1 + \omega_t) (M_{i,j,t} + \mu \max[0, w_{i,t} - fr_t]), \]

while the law of motion for the stock of entitlements of retirees is given by:

\[ M_{i,j,t+1} = (1 + \omega_t) M_{i,j,t}. \] (5.5)

The period \( t \) second-pillar pension benefit paid out to a type \( i \) age \( j \) retiree equals his stock of entitlements at the beginning of the period:

\[ b_{i,j,t} = M_{i,j,t}. \] (5.6)

**Liabilities**

The present value of the pension benefits of all participants together constitutes the liabilities of the pension fund. Because \( b_{i,j,t} = M_{i,j,t} \), the liability to each individual of age \( j \) and type \( i \) at the beginning of period \( t \) is given by:

\[ L_{i,j,t} = \sum_{l=R-j}^{J-j} \prod_{k=1}^{l} \frac{\psi_{j+k-1}}{(1 + r_{l,t})} M_{i,j,t}, \quad j < R, \] (5.7)

\[ L_{i,j,t} = \left(1 + \sum_{l=1}^{J-j} \prod_{k=1}^{l} \frac{\psi_{j+k-1}}{(1 + r_{l,t})} \right) M_{i,j,t}, \quad j \geq R, \] (5.8)

where \( r_{l,t} \) is the \( l \)-year interest rate on bonds in period \( t \). The total liabilities of the pension fund \( L_t \) equals the sum of the liabilities to all individuals in period \( t \):

\[ L_t = \sum_{i=1}^{I} \sum_{j=1}^{J} N_{i,j,t} L_{i,j,t}. \] (5.9)

Finally, the indicator of the financial health of the pension fund that is relevant for the fund’s supervisor is the funding ratio \( F_t \) given by:

\[ F_t = \frac{A_t}{L_t}, \] (5.10)
where \( A_t \) is the value of the fund’s assets at the start of period \( t \).

### 5.2.4 Assets

The total value of the pension fund’s assets at the start of period \( t \) is given by:

\[
A_t = \left( 1 + r_f^t \right) A_{t-1}^{\text{end}},
\]

where \( r_f^t \) is the return to the pension fund’s asset portfolio from period \( t - 1 \) to period \( t \) and \( A_{t-1}^{\text{end}} \) is the value of the fund’s assets at the end of period \( t - 1 \), i.e. after contributions for period \( t - 1 \) have been collected and benefits have been paid out. We assume that a share \( \alpha \) of the pension fund’s total asset portfolio is invested in equity and a share \( (1 - \alpha) \) is invested in bonds:

\[
E_{t-1} = \alpha A_{t-1}, \quad B_{t-1} = (1 - \alpha) A_{t-1},
\]

where \( E_{t-1} \) and \( B_{t-1} \) are the fund’s equity and bond investments, respectively. The bond portfolio is constructed to match the maturity structure of the liabilities. Specifically, if a share \( \zeta^k \) of total discounted benefit payments (i.e. liabilities of the fund) is expected to take place after \( k \) years time, then the fund invests a share \( \zeta^k \) of its bond portfolio in bonds maturing after \( k \) years:

\[
b_{k,t-1} = \zeta^k B_{t-1}, \quad \forall k \in [1, J],
\]

where \( b_{k,t-1} \) is the amount invested in bonds maturing after \( k \) years. Assuming that the fund invests only in zero-coupon bonds, the return on a maturity \( k \) bond at time \( t \) is given by:

\[
1 + Ret_{k,t}^b = \frac{1/(1 + r_{k,t})^k}{1/(1 + r_{k+1,t-1})^{k+1}} = \frac{(1 + r_{k+1,t-1})^{k+1}}{(1 + r_{k,t})^k}
\]

where \( r_{t,k} \) is the yield to maturity on a zero-coupon bond with \( k \) years to go until the face value is paid out. The return on equity is given by \( 1 + r_{t}^{eq} \), so that we can write

\[\text{footnote}{\text{Alternatively interest rate swaps could be used with the same pay-off structure as the bond portfolio.}}\]
the total gross portfolio return as:

\[ 1 + r^f_t = (1 + r^eq_t) \alpha + \sum_{k=1}^{K} \left( 1 + Ret^b_{k,t} \right) \zeta^k (1 - \alpha). \]  

(5.15)

Plugging this expression into the law of motion for the fund’s assets we obtain:

\[ A_t = (1 + r^eq_t) \alpha A^end_{t-1} + \sum_{k=1}^{K} \left( 1 + Ret^b_{k,t} \right) \zeta^k (1 - \alpha) A^end_{t-1} \]  

(5.16)

Then, in period \( t \), after the returns on the equity and bond holdings have materialised, contributions and benefits are paid. After this, the pension fund rebalances its portfolio so that the original investment shares \( \alpha \) in equity and \( 1 - \alpha \) in bonds are restored (and the bond portfolio again matches the maturity structure of its liabilities):

\[ E_t = \alpha \left( A_t + \sum_{i=1}^{I} N^w_{i,t} p_{i,t} - \sum_{i=1}^{I} \sum_{j=R}^{J} N_{i,j,t} b_{i,j,t} \right), \]  

(5.17)

\[ B_t = (1 - \alpha) \left( A_t + \sum_{i=1}^{I} N^w_{i,t} p_{i,t} - \sum_{i=1}^{I} \sum_{j=R}^{J} N_{i,j,t} b_{i,j,t} \right). \]  

(5.18)

End-of-period asset holdings of the pension fund are then given by:

\[ A^end_t = E_t + B_t. \]  

(5.19)

Note that because financial market risk is not the only source of risk, even with "matching investments" (\( \alpha = 0 \)) the retirees’ benefits still run risk. Moreover, if the total bond portfolio is smaller than the total value of the liabilities, the pension fund still faces some financial market risk, because it can not fully match its liabilities with bonds.

### 5.2.5 Economic shocks

The modelling of the economic shocks follows closely Beetsma and Bucciol (2011). There are four stochastic macro-economic variables: the inflation rate (\( \pi_t \)), the nominal wage growth rate (\( g_t \)), the yield on bonds with a maturity of one year (\( r_{1,t} \)) and the
equity return \((r^e_{t})\). These variables obey the following processes:

\[
\begin{bmatrix}
\pi_t \\
g_t \\
r_{1,t} \\
r^e_{t}\end{bmatrix} = \begin{bmatrix}
\pi \\
g \\
b \\
eq\end{bmatrix} + \begin{bmatrix}
\epsilon^\pi_t \\
\epsilon^g_t \\
\epsilon^b_t \\
\epsilon^e_t\end{bmatrix},
\]

where the first vector on the right-hand side contains the averages of the respective stochastic variables and the second vector on the right-hand side contains the (mean-zero) shocks to these variables. This vector of shocks follows a VAR(1) process:

\[
\begin{bmatrix}
\epsilon^\pi_t \\
\epsilon^g_t \\
\epsilon^b_t \\
\epsilon^e_t\end{bmatrix} = B \begin{bmatrix}
\epsilon^\pi_{t-1} \\
\epsilon^g_{t-1} \\
\epsilon^b_{t-1} \\
\epsilon^e_{t-1}\end{bmatrix} + \begin{bmatrix}
\eta^\pi_t \\
\eta^g_t \\
\eta^b_t \\
\eta^e_t\end{bmatrix},
\]

where \(B\) is a matrix filled with constants (including zeroes). Thus, each individual shock \(\epsilon^i_t, i \in \{\pi, g, b, eq\}\) is a combination of the vector of shocks in the previous year \([\epsilon^\pi_{t-1}, \epsilon^g_{t-1}, \epsilon^b_{t-1}, \epsilon^e_{t-1}]'\) plus an innovation \(\eta^i_t\). The vector of innovations \([\eta^\pi_{t-1}, \eta^g_{t-1}, \eta^b_{t-1}, \eta^e_{t-1}]'\) follows a normal process with mean zero and variance-covariance matrix \(\Sigma_f\).

The full term structure \(\{r_{k,t}\}_{k=1}^{K}\) is constructed as follows: each yield to maturity in the term structure \(r_{k,t}\) has a gross mark-up factor \(\nu_k\) on the one year interest rate \(r_{1,t}\). After realization of the one-year interest rate for a given year, we construct the full term structure by multiplying the mark-up factor of each maturity with the one-year yield:

\[
r_{k,t} = \nu_{k}r_{1,t}, \quad \forall k.
\]

### 5.2.6 The timing

Within any period \(t\), events take place in the following order:

1. Participants become one year older or die, a new generation of participants of age 1 enters the pension fund and economic shocks and returns materialise.
2. New pension entitlements are accrued, contributions are paid into the fund and benefits are paid out to the retirees.

3. The indexation policy is implemented.

4. The asset portfolio is rebalanced to ensure that a share $\alpha$ is invested in equity and to match the liability structure after indexation.

5.3 Pension fund policy

This section describes the various pension contracts we consider and discusses how these contracts differ. First, we describe the current contract. Then, we describe three proposals for the new contract. All three proposals for the new contract split entitlements in two types: hard and soft. The idea of this division of entitlements is that the hard entitlements provide the participants with some minimum benefit level during retirement. These entitlements should be very safe in the sense that individuals can only lose these entitlements in exceptional circumstances. The soft entitlements form the buffer around the hard entitlements: shocks to the asset position of the pension fund are absorbed first by the soft entitlements, so that these fluctuate with the value of the assets.

For each version of the new contract we consider, we will spell out a policy rule that addresses precisely how soft and hard entitlements are accumulated by participants in the pension fund. The total value of entitlements of an individual under the proposals for the new contract equals the sum of the stocks of hard and soft entitlements:

$$M_{i,j,t} = M^h_{i,j,t} + M^s_{i,j,t} \quad t \geq 0,$$

where $M^h_{i,j,t}$ is the stock of hard entitlements and $M^s_{i,j,t}$ is the stock of soft entitlements of individual $i$ of type $j$ at time $t$. 
5.3.1 The current contract

Under the current contract, there is only one type of entitlement, as described above. Whether one can view this entitlement as a hard or a soft entitlement depends on the interpretation of the pension contract (see also Bovenberg and Nijman (2011) and Bovenberg and Ewijk (2011)). The supervisor, the Dutch Central Bank (DNB), considers the entitlements as hard and allows a reduction in those entitlements only in extreme circumstances.

Contributions are such that they cover the expected future cost of the pension in terms of benefit payments. We now describe the rule that the pension fund uses to deal with underfunding and indexation.

Underfunding

Underfunding is defined as a situation in which the funding ratio is below some level $F_{lb}$. When the pension fund is underfunded, it has to set up a plan to restore the funding ratio to at least $F_{lb}$ within three years by using its policy instruments, which consist of a reduction in indexation, higher contributions and, as a last resort, a reduction in entitlements.

A reduction in indexation means that the pension fund does not provide sufficient indexation for the existing stock of entitlements to grow at the same rate as the price level or the nominal wage rate. This can go so far as to not providing any indexation at all, so that $\omega_t = 0$. The contribution rate $\theta$ can be raised up to a maximum of $\bar{\theta}$. If setting $\omega_t = 0$ and $\theta = \bar{\theta}$ for a period of three years is still not sufficient to bring the funding ratio back to $F_{lb}$ after these three years, the pension fund reduces existing entitlements ($\omega_t < 0$) by as much as is needed to achieve this goal.

Indexation

If the funding ratio exceeds $F_{lb}$, the pension fund uses the following indexation rule for the existing entitlements. If the funding ratio exceeds $F^{ub} > F_{lb}$, then the fund provides full indexation on all entitlements to either the price level or nominal wage growth (depending on the indexation goal of the pension fund). Moreover, in this case
the fund also provides catch-up indexation to compensate for missed indexation in
previous years to the extent that all missed indexation is compensated and the funding
ratio does not fall below $F^{ub}$. If the funding ratio is in between $F^{lb}$ and $F^{ub}$ then
indexation is a fraction $\kappa_t$ of full indexation according to:

$$\kappa_t = \frac{F_t - F^{lb}}{F^{ub} - F^{lb}}.$$  \hfill (5.24)

Hence, the rate of indexation provided is

$$\omega_t = \kappa_t g_t,$$  \hfill (5.25)

if the fund aims at keeping benefits in line with the nominal wage rate, or by

$$\omega_t = \kappa_t \pi_t,$$  \hfill (5.26)

if the fund aims at keeping the purchasing power of existing entitlements constant.

5.3.2 Proposals for the new contract

We investigate three different proposals for the new pension contract: the Rolling
Window contract, where participants accrue soft entitlements based on a rolling window
formula, the Fraction contract, where participants accrue a fixed fraction of hard and
soft entitlements each year and the Split proposal, where all existing entitlements at
the moment of transition are hard and all newly accrued entitlements are soft.

5.3.3 The Rolling Window contract

The first variant of the new pension contract was suggested by FNV Bondgenoten,
the largest labor union in the Netherlands. We will call it the ”Rolling Window”
proposal because soft entitlements are accrued based on a rolling window formula that
we describe below.

The existing stock of entitlements at the moment $t = 0$ of the transition from the
old to the new contract will be divided into a share $\xi$ of hard and a share $1 - \xi$ of soft
entitlements:

\begin{align*}
M_{i,j,0}^h &= \xi M_{i,j,0}^o \quad (5.27) \\
M_{i,j,0}^s &= (1 - \xi) M_{i,j,0}^o \quad (5.28)
\end{align*}

where \( M_{i,j,0}^o \) is the stock of ’old’ pension entitlements of individual type \( i \) and age \( j \) at the start of period 0 just before the transition to the new system is implemented.

All new entitlements accrued through working are soft entitlements. After \( Q \) years, these entitlements including their accumulated indexation (positive or negative) are transformed into hard entitlements. Hence, in period \( t \), the stock of soft entitlements evolves as follows:

\begin{align*}
M_{i,j,t+1}^s &= \begin{cases} 
(1 + \omega_t^s) \left( M_{i,j,t}^s + \mu \max[0, w_{i,t} - f r_t] \right), & j \leq Q, \\
(1 + \omega_t^s) \left( M_{i,j,t}^s - \tilde{m}_{i,j,t-Q}^s + \mu \max[0, w_{i,t} - f r_t] \right), & Q + 1 \leq j < R, \\
(1 + \omega_t^s) \left( M_{i,j,t}^s - \tilde{m}_{i,j,t-Q}^s \right), & R \leq j < R + Q, \\
0, & j \geq R + Q,
\end{cases}
\end{align*}

where \( \omega_t^s \) is the indexation rate of soft entitlements in year \( t \) and \( \tilde{m}_{i,j,t-Q}^s \) is the amount of soft entitlements that were earned by working exactly \( Q \) years ago, including the indexation that has been accumulated on those entitlements in the intervening years:

\begin{equation}
\tilde{m}_{i,j,t-Q}^s = \prod_{q=t-Q}^{t-1} (1 + \omega_q^s) m_{i,j,t-Q}^s, \quad (5.29)
\end{equation}

where \( m_{i,j,t-Q}^s \) is the amount of new soft entitlements that was earned by working in period \( t - Q \). The stock of hard entitlements evolves as follows:

\begin{align*}
M_{i,j,t+1}^h &= \begin{cases} 
0, & j \leq Q, \\
(1 + \omega_t^h) \left( M_{i,j,t}^h + \tilde{m}_{i,j,t-Q}^h \right), & Q < j < R + Q, \\
(1 + \omega_t^h) M_{i,j,t}^h, & j \geq R + Q,
\end{cases}
\end{align*}

where \( \omega_t^h \) is the indexation rate of hard entitlements in year \( t \).

Hence, at his retirement age \( R \) an individual has a stock of \( R - Q - 1 \) years of accumulated hard rights and \( Q \) years of accumulated soft rights. After \( Q \) years into
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At retirement a retiree has a stock of $R - 1$ years of accumulated hard rights and has no soft rights left. As before, the calculation of the liabilities and the funding ratio follows equations (5.7)-(5.10), given that the total stock of rights of an individual is given by equation (5.23).

We define full indexation of hard entitlements as $\omega_h^t = \pi_t$, if the fund tries to maintain the purchasing power of the hard entitlements, and as $\omega_h^t = g_t$ if it tries to track nominal wage growth. Full indexation of soft entitlements is defined as $\omega_s^t = \pi_t + \nu^\omega$, in the case that the fund tries to track price inflation, and as $\omega_s^t = g_t + \nu^\omega$, in the case that the fund tries to track nominal wage growth. Here, $\nu^\omega \geq 0$ is a mark-up to compensate for the higher riskiness of the soft entitlements.

**Underfunding**

The Rolling Window proposal is designed to adjust entitlements such that a situation of underfunding never occurs. This is accomplished as follows. If at the start of some period $t$, after asset returns have materialised and new pension entitlements have been earned, the pension fund has a funding ratio below $F^{lb}$, which serves as a lower bound on the funding ratio, it marks down the value of its soft entitlements (by setting $\omega_s^t < 0$) such that $F_t = F^{lb}$. If, after having completely marked down the stock of soft entitlements (i.e., $\omega_s^t = -1$), the funding ratio is still below $F^{lb}$, hard rights will be marked down (by setting $\omega_h^t < 0$) until $F_t = F^{lb}$.

This underfunding rule highlights the distinction between the soft and hard entitlements: the soft entitlements are the flexible part of the stock of entitlements that can be adjusted on an annual basis. They act as a buffer against the financial market risk that the pension fund faces in its asset portfolio. The hard entitlements are intended to provide a relatively safe guarantee for a minimal pension benefit at retirement and can only be adjusted downward once the entire stock of soft entitlements has been wiped out already.

How well the stock of soft entitlements performs its buffer role obviously depends on its size relative to the stock of hard entitlements. It also depends on the volatility of the financial market shocks hitting the pension fund and on the pension fund’s asset
mix. A fund with a relatively safe asset mix can likely guarantee its hard entitlements with a relatively small stock of soft entitlements.

Indexation

If the funding ratio exceeds $F^{lb}$, the pension fund provides indexation with the hard entitlements receiving priority over the soft entitlements. Specifically, the indexation rule obeys the following sequencing:

1. First, as long as $F_t$ does not fall below $F^{lb}$, the pension fund provides restoration of previous (unrestored) reductions in hard rights and missed indexation on hard rights. Missed indexation occurs when less than full indexation is given to the target (price or nominal wage inflation) pursued by the fund.

2. Next, if after applying the aforementioned restorations the funding ratio still exceeds $F^{lb}$, and as long as $F_t$ does not fall below $F^{lb}$, the pension fund provides the new indexation (to a maximum of full indexation) of hard entitlements. For this purpose it potentially marks down the value of the soft entitlements.

3. If the funding ratio still exceeds $F^{lb}$, the pension fund indexes the soft entitlements until either the funding ratio is $F^{lb}$ or soft entitlements have been fully indexed, including the mark-up $\nu^*$ of 0.5%. However, neither soft entitlements that were marked down earlier, nor missed indexation on these soft rights are restored. Soft entitlements function as a ‘memory-less’ buffer against shocks, absorbing both the good and the bad shocks while no track is kept of what happened in earlier years.

4. If after the full indexation including the mark-up of soft entitlements the funding ratio exceeds $F^{ub}$, the pension fund provides additional indexation to the soft rights, until the funding ratio is exactly $F^{ub}$ again. This is done to prevent the pension fund buffer from exploding and to award the holders of soft rights some of the upward potential of the pension fund investments as a compensation for the downward risk they are exposed to.
5.3.4 The Fraction contract

Under the second type of new contract, the "Fraction contract", the transition from old to new entitlements is handled in the same way as in the Rolling Window contract: a fraction $\xi$ of the old entitlements is transformed into hard entitlements, while a fraction $1 - \xi$ is transformed into soft entitlements:

$$M_{i,j,0}^h = \xi M_{i,j,0}^o,$$  \hspace{1cm} (5.30)  
$$M_{i,j,0}^s = (1 - \xi) M_{i,j,0}^o.$$  \hspace{1cm} (5.31)

However, the accrual of hard and soft entitlements as a result of working takes place in the same proportion as the transformation of the old entitlements: each year a fraction $\xi$ of the new entitlements is accumulated as hard entitlements, while the remaining fraction $1 - \xi$ is accumulated as soft entitlements. Hence, for working generations ($j < R$) we have:

$$M_{i,j,t+1}^h = (1 + \omega^h_t) \left( M_{i,j,t}^h + \mu \max[0, \xi (w_{i,t} - fr_t)] \right),$$  \hspace{1cm} (5.32)  
$$M_{i,j,t+1}^s = (1 + \omega^s_t) \left( M_{i,j,t}^s + \mu \max[0, (1 - \xi) (w_{i,t} - fr_t)] \right),$$  \hspace{1cm} (5.33)

while for retired generations ($j \geq R$) we have:

$$M_{i,j,t+1}^h = (1 + \omega^h_t) M_{i,j,t}^h,$$  \hspace{1cm} (5.34)  
$$M_{i,j,t+1}^s = (1 + \omega^s_t) M_{i,j,t}^s.$$  \hspace{1cm} (5.35)

The total entitlements of each individual are again the sum of the stocks of the soft and hard entitlements as in equation (5.23), while as before the values of the liabilities and the funding ratio are calculated according to (5.7)-(5.10). Notice that, in contrast to the Rolling Window proposal, retirees continue to hold soft entitlements and so they continue to share in the risks affecting the pension fund.

Underfunding
The rule for dealing with underfunding is exactly the same as under the Rolling Window contract.

**Indexation**

The rule for indexation is exactly the same as under the Rolling Window contract.

### 5.3.5 The Split contract

The Split contract focuses on a separation between hard and soft entitlements in the transition from the old to the new system. The stock of old entitlements becomes the stock of new hard entitlements in a one-to-one conversion:

\[
M_{i,j,0}^h = M_{i,j,0}^o, \quad (5.36)
\]
\[
M_{i,j,0}^s = 0. \quad (5.37)
\]

Hence, at the moment of the transition to the new system, there will be only hard entitlements. Subsequently, all newly accrued entitlements are soft entitlements. As soon as the funding ratio exceeds $F_{ub}$, some of the soft entitlements are transformed into hard entitlements. For working generations ($j < R$) entitlements evolve as:

\[
M_{i,j,t+1}^h = (1 + \omega^h_t) M_{i,j,t}^h + T_t, \quad (5.38)
\]
\[
M_{i,j,t+1}^s = (1 + \omega^s_t) \left( M_{i,j,t}^s + \mu \max[0,(w_i - f r_t)] \right) - T_t, \quad (5.39)
\]

where $T_t$ is the potential transformation of soft entitlements into hard entitlements in period $t$, as will be described below.

For retired generations ($j \geq R$) we obtain:

\[
M_{i,j,t+1}^h = (1 + \omega^h_t) M_{i,j,t}^h + T_t, \quad (5.40)
\]
\[
M_{i,j,t+1}^s = (1 + \omega^s_t) M_{i,j,t}^s - T_t. \quad (5.41)
\]
Underfunding

The policy in the case of underfunding is identical to that under the Rolling Window and Fraction arrangements.

Indexation

As long as the funding ratio is below $F_{ub}$, indexation policy is identical to that under the Rolling Window and Fraction proposals. However, when the funding ratio exceeds $F_{ub}$, step 4 of the indexation rule becomes as follows. First, soft entitlements are uniformly increased until the funding ratio has fallen to $F_{ub}$. After that some of the soft entitlements may be transformed into hard entitlements. If soft entitlements as a share of the total (hard plus soft) entitlements are smaller than $\phi$, nothing happens. If soft entitlements as a share of the total exceed $\phi$, soft entitlements are transformed into hard entitlements on a one-for-one basis, until the soft entitlements as a share of the total entitlements are exactly equal to $\phi$. The transformation of entitlements is thus given by:

$$T_t = \max \left\{ 0, \left[ \frac{(1 + \omega_t^s) M_{i,j,t}^s}{M_{i,j,t}} - \phi \right] \tilde{M}_{i,j,t} \right\}.$$

(5.42)

where $\tilde{M}_{i,j,t} = (1 + \omega_t^h) M_{i,j,t}^h + (1 + \omega_t^s) M_{i,j,t}^s$. Notice that indexation is awarded first, before the transformation takes place.

5.3.6 Contribution policy

The contribution rate is fixed and constant at its cost-covering level. Hence, the contribution rate is not used as a policy instrument by the pension fund. The cost-covering contribution rate $\theta$ to the pension fund is defined as the contribution rate such that in expectation the discounted sum of all contributions equals the discounted sum of all benefits payed by the pension fund:

$$E_1 \sum_{t=1}^{R-1} \frac{\theta (\bar{w}_t - f r_t)}{(1 + r_t^f)^t} = E_1 \sum_{t=R}^{J} \frac{\bar{b}_t}{(1 + r_t^f)^t-1}.$$

(5.43)
Chapter 5

where \( \bar{w}_t \) and \( \bar{b}_t \) are the population averages of wages and benefits of the pension fund participants.

5.4 Calibration and simulation

5.4.1 Calibration

Individuals enter the labour market at age 25 and work for 42 years, so that they retire at age 67. We normalise the entry age to 1, hence \( R = 43 \) in the model. The maximum age that an individual can reach and live is 99 years (as soon as the individual turns 100, he dies for sure), which corresponds to a maximum of 75 years of life of the model \( (J = 75) \). Survival probabilities from year to year are taken from the latest projections by the Dutch Actuarial Society (AG, 2010). Data for the distribution of wages over age groups and gender specific income classes are provided by MN for the PMT pension fund for the year 2010. These figures pin down the initial situation for our simulations.

We set the pension fund parameters so as to mimick the situation for PMT. The pension fund starts with a funding ratio of exactly 100%. The contribution rate is \( \theta = 18.6\% \), which equals the cost-covering contribution rate in 2010 plus a solvency mark-up. The accrual rate of pension entitlements is \( \mu = 2.236\% \), which is very close to the legally allowed maximum of 2.25%. We set \( \alpha = 0.5 \), hence the pension fund invests half of its portfolio in equity and the other half in bonds. The lower and upper bounds of the indexation schedules are set at \( F^{ib} = 100\% \) and \( F^{ub} = 140\% \). The choice for the lower bound is obvious, as a funding ratio below 100% means that the pension fund is technically insolvent. The choice for the upper bound is loosely based on observed indexation policies in the past and the idea that we do not want the buffers to become unrealistically large. For instance, participants might pressure the fund into spending some of the buffer in the form of lower contributions or higher benefits or the government might start taxing away very high pension buffers. Finally, we need to pin down some parameters for whose values we do not have any a priori guidelines. We set the mark-up for the indexation of soft entitlements at \( \nu^s = 0.5\% \), the length of the rolling window of soft entitlements \( Q \) in the Rolling Window proposal to 10 years, the fraction of hard entitlements in the Fraction proposal at \( \xi = 0.5 \) and the fraction of
Table 5.1: Base case parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>43</td>
</tr>
<tr>
<td>$J$</td>
<td>75</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.186</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.02236</td>
</tr>
<tr>
<td>$F_0$</td>
<td>1</td>
</tr>
<tr>
<td>$F^{lb}$</td>
<td>1</td>
</tr>
<tr>
<td>$F^{ub}$</td>
<td>1.4</td>
</tr>
<tr>
<td>$\nu^\omega$</td>
<td>0.005</td>
</tr>
<tr>
<td>$Q$</td>
<td>10</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.02</td>
</tr>
<tr>
<td>$g$</td>
<td>0.03</td>
</tr>
<tr>
<td>$r^b$</td>
<td>0.03</td>
</tr>
<tr>
<td>$r^{eq}$</td>
<td>0.068</td>
</tr>
</tbody>
</table>

soft entitlements after transformation of entitlements in the Split contract at $\phi = 0.2$.

Further, we set the average value of inflation at 2%, average nominal wage growth at 3%, the average one-year bond return at 3% and the average equity return at 6.8%. By taking these values we take the parameter values prescribed by the Parameters (2009) for Dutch pension funds. The values for the covariance matrix $\Sigma_f$ of the innovations and the VAR coefficient matrix $B$ are taken from Beetsma and Bucciol (2011). Their estimates are based on times series over the period 1976-2005 for the Dutch consumer price index, Dutch hourly wages - both taken from OECD (2009) - U.S. one-year bond yields, taken from Reserve (2009) and the MSCI U.S. equity index, taken from Datastream (2009). The mark-ups for bond maturities longer than one year are based on the average mark-ups in the Dutch term structure over the period 2001-2010 as published by Bank (2011).

5.4.2 The simulation setup

We start the simulation by generating an initial distribution of individuals and entitlements based on the PMT data. This initial situation is identical in all versions of
the model, as it reflects the situation as it currently is, which does not depend on the form the new contract takes. Next, we draw 1000 sets of 50 (or 250 to check if the pension scheme is viable in the very long run) realisations for the economic shocks. We then use these realisations to compute a scenario for each set of shock realisations. Thus we obtain 1000 scenario’s for all variables in the model. We use these scenarios to compute the relevant statistics for each version of the model, such as medians and standard deviations of the funding ratio, the share of soft entitlements, indexation of hard and soft entitlements and replacement rates of the retiring cohorts.

5.5 Results

For each scenario we present a table with numerical results and a graph. We start with the baseline parameter combination as reported in Table 5.1, first considering the case of indexation to nominal wage growth, followed by the case in which the pension fund indexes entitlements to price inflation.
5.5.1 Indexation to wages

Table 5.2 and Figures 5.1, 5.2 and 5.3 show the consequences for the financial position of a pension fund that targets wage indexation under both the current contract and the three different versions of the new pension contract. Notice that in the current contract there is only one kind of entitlement (hard), so there are no table entries for soft entitlements under the current contract.

The median values of the funding ratios differ across the contracts. The current contract features the highest median funding ratio of 123.7%, while in the rolling window contract the funding ratio is lowest at 114.8%. This corresponds to the fact that in the current contract the median indexation of hard and, hence, total entitlements is 1.9%. This is substantially below the median indexation of hard entitlements in the proposals for the new contract, which are 3.1% under the Rolling Window contract and 3.2% under the Fraction and Split contracts. It is also substantially below the median indexation of soft entitlements of 3.4% under the Rolling Window proposal and 3.8% under the Fraction and Split proposals. Hence, the overall median indexation rate under the new proposals is substantially higher than the median indexation rate under the current contract.

We see that the standard deviation of the funding ratio is virtually identical across contracts. The reason is that it is almost entirely driven by the financial shocks, which are the same for each contract.

The standard deviation of the indexation of hard entitlements is highest under the current contract. This is exactly as expected, since there is no buffer of soft entitlements under the current contract. Of the proposals for the new contract, the standard deviation under the Fraction proposal is notably below the standard deviation under the other two proposals, indicating that hard entitlements are more stable under this scenario. The percentage of time that the hard rights have to be reduced differs materially across the scenarios: the Fraction scenario does best in this regard. Only in 0.3% of the observed outcomes are hard entitlements reduced. The Split proposal is intermediate at roughly 3%, while under the Rolling Window proposal in more than 6% of the cases hard entitlements are reduced. The main reason for these differences is the size of the buffer of soft entitlements. Under the Fraction proposal the median share of
soft entitlements as a share of total entitlements is 57.5%, under the Rolling Window scenario it is only 25.8%, while under the Split scenario it is even lower at 22.1%. The substantially larger buffer of soft rights under the Fraction proposal implies that hard entitlements are safer than under the other two proposals.4

Finally, the median replacement rates under the Rolling Window and Fraction scenarios are quite close (1.037 versus 0.995). However, under the Rolling Window proposal the replacement rate is much more volatile. The reason is that under the Rolling Window proposal the share of the soft entitlements held by the retired is much lower than under the Fraction proposal; the median shares of soft entitlements are 10.3% and 34.9%, respectively. Hence, under the Rolling Window proposal the elderly share less in the risks affecting the pension fund and, hence, the accumulation of entitlements by the workers is more uncertain.

At 0.838, the median replacement rate under the Split proposal is substantially lower than under the other two proposals. This is caused by the fact that the share of hard entitlements is relatively large, while the young possess all or most of the soft entitlements, especially in earlier periods. The small share of soft entitlements serves as the buffer for the hard entitlements and the chance that a large share of the soft entitlements is wiped out through the shocks that hit the pension fund is relatively high. Because the soft entitlements process is memoryless, these losses will not be restored. The burden of these losses thus falls primarily on the young who will on average be confronted with a lower funding ratio at retirement than under the other two alternatives.

If we look at Figures 5.1, 5.2 and 5.3, we see the same picture emerging in a slightly more detailed way. The figures depict the funding ratio, indexation of hard entitlements, the share of soft entitlements in total entitlements, and the replacement rate of the retiring cohort for each of the three proposals for the new contract, respectively. In Figure 5.1 we see that in the Rolling Window version of the contract, the median funding ratio goes up in the first few years of the simulation period and then steadily

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4Note that the share of soft entitlements is higher than $\xi = 0.50$, because of the higher indexation awarded to the soft entitlements when the funding ratio is between 100 and 140%, and the additional indexation on top of that when the funding ratio is higher than 140%.
declines. We see that indexation of hard entitlements is pretty stable for the first 20 years of the simulation period, but after 20 years the 5% and 95% intervals strongly diverge: the standard deviation of the indexation of hard entitlements goes up sharply. In the lower left panel we see why this happens: in the first 20 years the pension fund ‘burns’ its buffer of soft entitlements, so that after 20 years the stock of soft entitlements is not large enough anymore to absorb most financial shocks.

In Figure 5.2 we see that for the Fraction contract, both the volatility and the levels of all four variables depicted are relatively stable. In Figure 5.3 we see that for the Split proposal, indexation of hard entitlements in the first 5 years is highly uncertain (because the stock of soft entitlements is then very small). After the first 5 years, the situation is more or less stable. In the lower right panel, which depicts the replacement rate of the retiring cohort, we see that after about 30 years, the median replacement rates of the retiring cohorts start declining. As described above, this is caused by the fact that these are the generations that possessed few or no hard entitlements at the moment of transition and thus absorbed most of the financial shocks the pension fund faced.

### 5.5.2 Indexation to prices

We now repeat the simulations of the previous subsection for indexation to price inflation. The results are presented in Table 5.3. We see that in all three scenarios the median funding ratio is higher while its volatility is unchanged, a direct consequence of the fact that price inflation tends to be lower than nominal wage growth. That means that the pension fund provides on average less indexation to its participants, as can be seen when we compare median rates of indexation of the hard and soft entitlements for the two cases. The counterpart of lower indexation is a higher buffer. Since buffers are slightly higher under price indexation, the percentages of times that hard and soft rights have to be reduced are lower than under wage indexation. Since hard entitlements are on average indexed by less under price indexation, soft entitlements can be indexed more often and, hence, the median share of soft entitlements is higher in this case.

The median replacement rates of the retiring cohorts are virtually identical under
Table 5.3: Outcomes under price indexation

<table>
<thead>
<tr>
<th></th>
<th>Current</th>
<th>RW</th>
<th>Fraction</th>
<th>Split</th>
</tr>
</thead>
<tbody>
<tr>
<td>median funding ratio</td>
<td>1.252</td>
<td>1.189</td>
<td>1.229</td>
<td>1.236</td>
</tr>
<tr>
<td>standard deviation funding ratio</td>
<td>0.161</td>
<td>0.162</td>
<td>0.159</td>
<td>0.160</td>
</tr>
<tr>
<td>median indexation rate hard entitlements</td>
<td>0.014</td>
<td>0.022</td>
<td>0.021</td>
<td>0.022</td>
</tr>
<tr>
<td>stand. dev. indexation rate hard entitl.</td>
<td>0.120</td>
<td>0.044</td>
<td>0.014</td>
<td>0.038</td>
</tr>
<tr>
<td>median indexation rate soft entitlements</td>
<td>0.027</td>
<td>0.029</td>
<td>0.031</td>
<td></td>
</tr>
<tr>
<td>stand. dev. indexation rate soft entitlements</td>
<td>0.783</td>
<td>0.300</td>
<td>0.706</td>
<td></td>
</tr>
<tr>
<td>% of time reduction of hard entitlements</td>
<td>0.204</td>
<td>0.055</td>
<td>0.001</td>
<td>0.025</td>
</tr>
<tr>
<td>% of time reduction of soft entitlements</td>
<td>0.241</td>
<td>0.176</td>
<td>0.184</td>
<td></td>
</tr>
<tr>
<td>median share soft rights</td>
<td>0.285</td>
<td>0.626</td>
<td>0.228</td>
<td></td>
</tr>
<tr>
<td>standard deviation share soft entitlements</td>
<td>0.191</td>
<td>0.159</td>
<td>0.106</td>
<td></td>
</tr>
<tr>
<td>median replacement rate</td>
<td>1.000</td>
<td>1.058</td>
<td>0.996</td>
<td>0.836</td>
</tr>
<tr>
<td>standard deviation replacement rate</td>
<td>0.367</td>
<td>0.588</td>
<td>0.351</td>
<td>0.279</td>
</tr>
<tr>
<td>median share soft retired</td>
<td>0.108</td>
<td>0.364</td>
<td>0.169</td>
<td></td>
</tr>
<tr>
<td>standard deviation share soft retired</td>
<td>0.042</td>
<td>0.157</td>
<td>0.092</td>
<td></td>
</tr>
</tbody>
</table>

wage growth and price inflation indexation for the Fraction and Split proposals, while it is slightly higher under price indexation than under wage indexation for the Rolling Window proposal. This might seem counterintuitive, because higher indexation should imply a higher replacement rate. However, we obtain this result, because under wage indexation the retiring generations are relatively well off in the first years, leading to a lower pension buffer and, hence, increasing the chances of low or negative indexation for later generations. On balance, over the 50-years horizon we consider here, the median replacement rate of the retiring cohort is lower under wage indexation.

5.5.3 Robustness checks

To save space, we report the robustness checks only for wage indexation. The robustness checks for price indexation are qualitatively very similar.

Varying the volatility on asset returns

In this subsection we vary the standard deviations of the returns on bonds and equity. In the "low volatility" scenario, the standard deviation of the return on bonds is 0.85% and that of the return on equity is 10%. In the "high volatility" scenario, the standard
Table 5.4: Varying the volatility of the asset returns

<table>
<thead>
<tr>
<th></th>
<th>$\sigma^q = 0.1$</th>
<th>$\sigma^b = 0.0085$</th>
<th>$\sigma^q = 0.175$</th>
<th>$\sigma^b = 0.02$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RW</td>
<td>Fraction</td>
<td>Split</td>
<td>RW</td>
</tr>
<tr>
<td>median funding ratio</td>
<td>1.177</td>
<td>1.196</td>
<td>1.266</td>
<td>1.121</td>
</tr>
<tr>
<td>st. dev. funding ratio</td>
<td>0.151</td>
<td>0.149</td>
<td>0.146</td>
<td>0.167</td>
</tr>
<tr>
<td>median ind. rate hard</td>
<td>0.031</td>
<td>0.031</td>
<td>0.031</td>
<td>0.031</td>
</tr>
<tr>
<td>st. dev. ind. rate hard</td>
<td>0.022</td>
<td>0.014</td>
<td>0.020</td>
<td>0.073</td>
</tr>
<tr>
<td>median ind. rate soft</td>
<td>0.035</td>
<td>0.036</td>
<td>0.041</td>
<td>0.031</td>
</tr>
<tr>
<td>st. dev. ind. rate soft</td>
<td>0.325</td>
<td>0.159</td>
<td>0.357</td>
<td>1.879</td>
</tr>
<tr>
<td>% time reduction hard</td>
<td>0.020</td>
<td>0.001</td>
<td>0.008</td>
<td>0.101</td>
</tr>
<tr>
<td>% time reduction soft</td>
<td>0.189</td>
<td>0.143</td>
<td>0.101</td>
<td>0.347</td>
</tr>
<tr>
<td>median share soft ent.</td>
<td>0.282</td>
<td>0.516</td>
<td>0.242</td>
<td>0.244</td>
</tr>
<tr>
<td>st. dev. share soft ent.</td>
<td>0.143</td>
<td>0.135</td>
<td>0.122</td>
<td>0.217</td>
</tr>
<tr>
<td>median replacement r.</td>
<td>0.899</td>
<td>0.914</td>
<td>0.755</td>
<td>1.165</td>
</tr>
<tr>
<td>st.dev. replacement r.</td>
<td>0.296</td>
<td>0.244</td>
<td>0.164</td>
<td>0.943</td>
</tr>
<tr>
<td>median share soft retired</td>
<td>0.108</td>
<td>0.322</td>
<td>0.187</td>
<td>0.102</td>
</tr>
<tr>
<td>st. dev. share soft retired</td>
<td>0.033</td>
<td>0.138</td>
<td>0.094</td>
<td>0.044</td>
</tr>
</tbody>
</table>

deviations become 2% for the bond returns and 17.5% for the equity returns.

Not surprisingly, increasing the volatility of the assets returns raises the standard deviations of all variables and the percentage of the time that soft and hard entitlements are reduced increases. Further, under both the Rolling Window and the Split contracts, raising the volatility of asset returns leads to a fall in the median funding ratio and a rise in the median replacement rate. When volatility is low, the funding ratio is almost always between 100% and 140%. Hence, the pension fund almost never provides additional indexation on soft rights, so that the median replacement rate of the retiring cohort is lower, but at the same time this ensures that the buffer is on average a bit higher.

The asset portfolio

We now consider more defensive mixes of the fund’s asset portfolio. In particular, we consider the cases of 10% and 25% equity shares.

With more defensive asset mixes, the impact of the equity returns diminishes and the impact of the bond returns increases. Because the fund invests its bond portfolio so as to match its liability structure, a larger share of total assets invested in bonds implies
Table 5.5: Varying the equity share in the fund’s portfolio

<table>
<thead>
<tr>
<th></th>
<th>$\alpha = 0.1$</th>
<th></th>
<th>$\alpha = 0.25$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RW</td>
<td>Fraction</td>
<td>Split</td>
<td>RW</td>
</tr>
<tr>
<td>median funding ratio</td>
<td>1.215</td>
<td>1.239</td>
<td>1.340</td>
<td>1.212</td>
</tr>
<tr>
<td>st. dev. funding ratio</td>
<td>0.150</td>
<td>0.151</td>
<td>0.136</td>
<td>0.151</td>
</tr>
<tr>
<td>median ind. rate hard</td>
<td>0.031</td>
<td>0.032</td>
<td>0.032</td>
<td>0.031</td>
</tr>
<tr>
<td>st. dev. ind. rate hard</td>
<td>0.014</td>
<td>0.014</td>
<td>0.014</td>
<td>0.017</td>
</tr>
<tr>
<td>median ind. rate soft</td>
<td>0.037</td>
<td>0.039</td>
<td>0.046</td>
<td>0.037</td>
</tr>
<tr>
<td>st. dev. ind. rate soft</td>
<td>0.139</td>
<td>0.081</td>
<td>0.260</td>
<td>0.291</td>
</tr>
<tr>
<td>% reduction hard ent.</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.010</td>
</tr>
<tr>
<td>% reduction soft ent.</td>
<td>0.128</td>
<td>0.107</td>
<td>0.043</td>
<td>0.155</td>
</tr>
<tr>
<td>median share soft ent.</td>
<td>0.269</td>
<td>0.534</td>
<td>0.240</td>
<td>0.263</td>
</tr>
<tr>
<td>st. dev. share soft ent.</td>
<td>0.111</td>
<td>0.120</td>
<td>0.166</td>
<td>0.138</td>
</tr>
<tr>
<td>median replacement r.</td>
<td>0.952</td>
<td>0.951</td>
<td>0.770</td>
<td>1.015</td>
</tr>
<tr>
<td>st. dev. replacement r.</td>
<td>0.323</td>
<td>0.242</td>
<td>0.174</td>
<td>0.362</td>
</tr>
<tr>
<td>median share soft retired</td>
<td>0.113</td>
<td>0.346</td>
<td>0.227</td>
<td>0.111</td>
</tr>
<tr>
<td>st. dev. share soft retired</td>
<td>0.028</td>
<td>0.142</td>
<td>0.114</td>
<td>0.031</td>
</tr>
</tbody>
</table>

that the fund is able to better match its assets to the liabilities. This decreases
the risk of underfunding. However, it also decreases the potential for indexation, because
the average portfolio return is lower.

The standard deviations of most variables under consideration become lower as
the equity share is reduced. Hard entitlements can be guaranteed more effectively.
The fraction of time hard entitlements are reduced decreases substantially under all
three schemes, to essentially never if equity is only 10% of total assets. However, soft
entitlements are still reduced regularly under the three schemes, implying that the
benefits are still uncertain.

The disadvantage of reducing the equity share is that the median replacement rates
fall. Under the Rolling Window proposal, a reduction from 50% to 25% equity share
reduces the median replacement rate of the retiring cohort from 1.04 to 1.02, while
with a 10% equity share the median replacement rate falls to 0.95. Under the Fraction
scenario, the median replacement rate goes down in a comparable way. For the Split
contract the median replacement rate goes down in a comparable way. For the Split
contract the median replacement rate was lower to start with. However, the reduction
is similar as the equity share goes down.

Summarising, we find that reducing the equity share in the fund’s portfolio is a
Table 5.6: Wage indexation, different retirement ages

<table>
<thead>
<tr>
<th></th>
<th>$R = 38$</th>
<th></th>
<th>$R = 41$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RW Fraction</td>
<td>Split</td>
<td>RW Fraction</td>
</tr>
<tr>
<td>median funding ratio</td>
<td>1.133 1.168</td>
<td>1.203</td>
<td>1.142 1.185</td>
</tr>
<tr>
<td>st. dev. funding ratio</td>
<td>0.160 0.160</td>
<td>0.161</td>
<td>0.162 0.160</td>
</tr>
<tr>
<td>median ind. rate hard</td>
<td>0.031 0.031</td>
<td>0.032</td>
<td>0.031 0.031</td>
</tr>
<tr>
<td>st. dev. ind. rate hard</td>
<td>0.042 0.025</td>
<td>0.046</td>
<td>0.049 0.021</td>
</tr>
<tr>
<td>median ind. rate soft</td>
<td>0.032 0.035</td>
<td>0.038</td>
<td>0.033 0.036</td>
</tr>
<tr>
<td>st. dev. ind. rate soft</td>
<td>0.528 0.459</td>
<td>0.720</td>
<td>0.667 0.436</td>
</tr>
<tr>
<td>% reduction hard ent.</td>
<td>0.049 0.012</td>
<td>0.035</td>
<td>0.064 0.008</td>
</tr>
<tr>
<td>% reduction soft ent.</td>
<td>0.297 0.240</td>
<td>0.216</td>
<td>0.296 0.217</td>
</tr>
<tr>
<td>median share soft ent.</td>
<td>0.299 0.489</td>
<td>0.212</td>
<td>0.270 0.537</td>
</tr>
<tr>
<td>st. dev. share soft ent.</td>
<td>0.179 0.192</td>
<td>0.109</td>
<td>0.188 0.185</td>
</tr>
<tr>
<td>median replacement r.</td>
<td>0.743 0.768</td>
<td>0.623</td>
<td>0.931 0.927</td>
</tr>
<tr>
<td>st. dev. replacement r.</td>
<td>0.326 0.258</td>
<td>0.163</td>
<td>0.479 0.324</td>
</tr>
<tr>
<td>median share soft retired</td>
<td>0.225 0.406</td>
<td>0.257</td>
<td>0.147 0.374</td>
</tr>
<tr>
<td>st. dev. share soft retired</td>
<td>0.052 0.152</td>
<td>0.097</td>
<td>0.046 0.157</td>
</tr>
</tbody>
</table>

good way of guaranteeing hard entitlements. However, it comes at the cost of lower or zero median indexation and thus lower median replacement rates for retiring cohorts.

**Varying the retirement age**

This subsection varies the retirement age. It considers retirement ages of 62 years, which corresponds to a retirement age in the model of $R = 38$, and 65 years, which corresponds to $R = 41$.

If workers retire earlier, they pay contributions for a fewer number of years and, hence they accumulate fewer entitlements. Hence, median replacement rates are lower under all contract types. The effects on other variables are not very pronounced, because the lower lifetime contributions and the lower benefits during retirement have offsetting effects on the funding ratio.

**Policy parameters**

This subsection varies a number of policy parameters. Under the Rolling Window contract, we vary the rolling window phase of soft entitlements to $Q = 5$ and $Q = 15$ years. Under the Fraction contract, we vary the fraction of hard entitlements to
### Table 5.7: Varying the policy parameters

<table>
<thead>
<tr>
<th></th>
<th>RW</th>
<th>Fraction</th>
<th>Split</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q = 5$</td>
<td>$Q = 15$</td>
<td>$\xi = 0.75$</td>
</tr>
<tr>
<td>median funding ratio</td>
<td>1.097</td>
<td>1.174</td>
<td>1.181</td>
</tr>
<tr>
<td>st. dev. funding ratio</td>
<td>0.162</td>
<td>0.161</td>
<td>0.162</td>
</tr>
<tr>
<td>median ind. rate hard</td>
<td>0.030</td>
<td>0.031</td>
<td>0.031</td>
</tr>
<tr>
<td>st. dev. ind. rate hard</td>
<td>0.079</td>
<td>0.033</td>
<td>0.047</td>
</tr>
<tr>
<td>median ind. rate soft</td>
<td>0.028</td>
<td>0.036</td>
<td>0.036</td>
</tr>
<tr>
<td>st. dev. ind. rate soft</td>
<td>1.739</td>
<td>0.546</td>
<td>2.193</td>
</tr>
<tr>
<td>% reduction hard ent.</td>
<td>0.141</td>
<td>0.029</td>
<td>0.049</td>
</tr>
<tr>
<td>% reduction soft ent.</td>
<td>0.379</td>
<td>0.243</td>
<td>0.243</td>
</tr>
<tr>
<td>median share soft ent.</td>
<td>0.082</td>
<td>0.399</td>
<td>0.387</td>
</tr>
<tr>
<td>st. dev. share soft ent.</td>
<td>0.141</td>
<td>0.206</td>
<td>0.222</td>
</tr>
<tr>
<td>median replacement r.</td>
<td>1.056</td>
<td>1.024</td>
<td>1.028</td>
</tr>
<tr>
<td>st. dev. replacement r.</td>
<td>0.679</td>
<td>0.510</td>
<td>0.412</td>
</tr>
<tr>
<td>median share soft retired</td>
<td>0.057</td>
<td>0.153</td>
<td>0.302</td>
</tr>
<tr>
<td>st. dev. share retired</td>
<td>0.023</td>
<td>0.060</td>
<td>0.194</td>
</tr>
</tbody>
</table>

$\xi = 0.25$ and $\xi = 0.75$, while under the Split contract, we vary the threshold for the transformation of soft into hard contracts to $\phi = 0.1$ and $\phi = 0.3$.

For the Rolling Window model, we see that if the build up phase of soft entitlements is only 5 years, the buffer of soft entitlements becomes much smaller and is much less effective at insulating the hard entitlements to shocks: the standard deviation of the indexation rate to hard entitlements rises to 0.08, while hard entitlements are more often reduced, namely in 14% of all scenarios.

The median share of soft rights is only slightly more than 8%. The median replacement rate of the retiring cohorts is actually slightly higher (1.06 versus 1.04) than with the 10 year rolling window, because (older) workers have a larger share of hard entitlements for which the soft entitlements act as a buffer. This is a benefit for the older workers in the earlier years: the first 35 generations that retire under the 5 year rolling window are better off, while all generations retiring after that are worse off, since they absorbed most shocks in their soft entitlements when young. The standard deviation of the replacement rate rises to 0.68. In case the build up phase is lengthened to 15 years, the effects are the opposite of a shortening of the rolling window phase.

Under the Fraction proposal, if the share of hard entitlements is raised to $\xi = 0.75$
the hard entitlements will be harder to guarantee, because the buffer of soft entitlements is lower than in the base case. The standard deviation of hard entitlements goes up from 0.017 to 0.047, while the fraction of time the hard entitlements are reduced goes up from 0.003 to 0.049. Not surprisingly, when the fraction of hard entitlements is reduced, the effects go into the opposite direction. Finally, increasing the threshold for the transformation of soft into hard entitlements to $\phi = 0.3$ under the Split proposal, the result will be a larger median fraction of soft entitlements and the consequences are qualitatively the same as those of raising the share of soft entitlements under the Fraction proposal. Similarly, decreasing the threshold to $\phi = 0.1$ has qualitatively similar consequences as reducing the share of soft entitlements under the Fraction proposal.

**Simulation length**

Running the simulations for 250 periods instead of 50 periods, so that all transition dynamics in both the build up phase and demographics disappear produces the results in Table 5.8. We see that for the Rolling Window contract, the median funding ratio is slightly lower, but not that much. The main difference for the Rolling Window contract is that in the long run the stock of soft entitlements is not nearly large enough

<table>
<thead>
<tr>
<th></th>
<th>RW</th>
<th>Fraction</th>
<th>Split</th>
</tr>
</thead>
<tbody>
<tr>
<td>median FR</td>
<td>1.106</td>
<td>1.193</td>
<td>1.000</td>
</tr>
<tr>
<td>s.d. FR</td>
<td>0.161</td>
<td>0.159</td>
<td>0.156</td>
</tr>
<tr>
<td>median hard indexation</td>
<td>0.031</td>
<td>0.032</td>
<td>0.030</td>
</tr>
<tr>
<td>s.d. hard indexation</td>
<td>0.065</td>
<td>0.019</td>
<td>0.151</td>
</tr>
<tr>
<td>median soft indexation</td>
<td>0.029</td>
<td>0.037</td>
<td>-0.104</td>
</tr>
<tr>
<td>s.d. soft indexation</td>
<td>1.001</td>
<td>0.434</td>
<td>6.083</td>
</tr>
<tr>
<td>% of time reduction of hard rights</td>
<td>0.104</td>
<td>0.005</td>
<td>0.220</td>
</tr>
<tr>
<td>% of time reduction of soft rights</td>
<td>0.355</td>
<td>0.193</td>
<td>0.515</td>
</tr>
<tr>
<td>median Share soft</td>
<td>0.170</td>
<td>0.592</td>
<td>0.029</td>
</tr>
<tr>
<td>s.d. share soft</td>
<td>0.195</td>
<td>0.188</td>
<td>0.119</td>
</tr>
<tr>
<td>median replacement rate</td>
<td>1.145</td>
<td>1.186</td>
<td>1.471</td>
</tr>
<tr>
<td>s.d. replacement rate</td>
<td>1.112</td>
<td>0.587</td>
<td>3.985</td>
</tr>
<tr>
<td>median share soft retired</td>
<td>0.114</td>
<td>0.423</td>
<td>0.146</td>
</tr>
<tr>
<td>st. dev. share soft retired</td>
<td>0.041</td>
<td>0.136</td>
<td>0.103</td>
</tr>
</tbody>
</table>
to provide a meaningful buffer for the hard entitlements, which essentially become soft themselves. In over 10% of the scenarios, hard entitlements have to be reduced. Also, under the Rolling Window contract there are particularly lucky generations and particularly unlucky generations: the standard deviation of the replacement rate of the retiring cohort is quite high (1.112).

For the Fraction contract, the differences between the 50 periods and the 250 periods simulation are very small, indicating that there were hardly any transitional dynamics in the first place.

For the Split contract, we see large differences between the 50 and 250 periods simulations. The median funding ratio is 100% in the 250 periods simulation, whereas it was over 120% in the 50 periods simulation. Median indexation of soft entitlements is -10.4%, because the buffer of soft entitlements gradually vanishes. The median share of soft entitlements over the entire time horizon is 2.9%. Closer inspection reveals initially this is higher (over 22% for the first 50 years), but that after 100 periods the median share of soft entitlements falls to 0. The median replacement rate is quite high, but also the standard deviation is high, implying some generations are well off, while others suffer. A generation’s luck in this regard depends to a large extent on the size of the buffer of soft entitlements at the moment of entering the pension fund.

5.6 Conclusion

In this chapter we have simulated the consequences for benefits and pension funding ratios of several proposals to replace the current Dutch occupational contract. The common feature of all proposals was the replacement of the current system of only hard entitlements by a system with both hard and soft entitlements, where the latter acted as a buffer for the former. That is, negative shocks were first absorbed by the soft entitlements and only once the buffer of soft entitlements was depleted hard entitlements would be affected. Our simulations were carried out both for the case of indexation of entitlements to price inflation and indexation to wage inflation.

Under the proposals considered here, indexation is higher and more readily provided than under the current system. This implies that the currently retired generations
benefit more under the new proposals than under the current contract. Out of the three new proposals, only the Fraction contract succeeds in effectively guaranteeing hard entitlements and providing relatively stable indexation and buffers over time. The results suggest that in a system with both hard and soft entitlements, the only way to prevent substantial intergenerational transfers is to have all generations accumulate both types of entitlements. Because financial shocks are absorbed through the soft entitlements, this way the costs and benefits of those shocks can be spread over all the generations.

A possible extension of the analysis in this chapter would be to include additional sources of shocks. Obvious candidates are unexpected changes in survival probabilities and fertility. Another extension would to vary the parameters of the pension contract. For example, it would be interesting to vary the recovery period. Under the current proposals, a fall of funding ratio below 100% is immediately offset through a cut in entitlements. The resulting volatility in the entitlements could be dampened by considering longer recovery periods. We would expect disposable income to become more stable, which would benefit the fund’s retirees to the extent that they have difficulties in smoothing consumption.
APPENDICES

5.A Derivation of (5.14)

We assume zero-coupon bonds for simplicity. (Using coupon bonds would be conceptually equivalent, but would complicate the notation needlessly). The return on a zero-coupon bond in year $t$ with $k$ years to maturity at the end of the year equals the capital gain in year $t$:

$$1 + Ret_{k,t}^b = \frac{P_{k,t}}{P_{k+1,t-1}}$$

where $P_{k,t}$ is the price of the bond at the end of the year and $P_{k+1,t-1}$ is the price at the beginning of the year, when the time to maturity is $k + 1$. For a zero coupon bond, we can write these prices as:

$$P_{k,t} = \frac{FV}{(1 + r_{k,t})^k},$$

$$P_{k+1,t-1} = \frac{FV}{(1 + r_{k+1,t-1})^{k+1}},$$

where $FV$ is the face or principal value of the bond. Plugging these expressions into (5.44) yields (5.14):

$$1 + Ret_{k,t}^b = \frac{FV/(1 + r_{k,t})^k}{FV/(1 + r_{k+1,t-1})^{k+1}} = \frac{(1 + r_{k+1,t-1})^{k+1}}{(1 + r_{k,t})^k}. \quad (5.47)$$

5.B Indexation Policy Rolling Window proposal

In this appendix we provide the exact formulas for the indexation policy under the Rolling Window proposal as laid out in Section 5.3.3. Throughout this appendix we assume price-indexation, so that full indexation of hard entitlements is defined as $\omega^h_t = \pi_t$ and full indexation of soft entitlements is defined as $\omega^s_t = \pi_t + \nu\omega$. For a pension fund instead following a policy of wage indexation, we would simply need to replace $\pi_t$ by $g_t$ in all instances.
5.B.1 Reduction of hard entitlements

We keep track of the stock of reductions and missed indexation of hard entitlements as follows:

\[ M^m_{i,j,t} = (1 + \omega^h_t) (M^m_{i,j,t-1} - \tilde{m}^h_{i,j,t}) + m^m_{i,j,t}, \]  

(5.48)

where \( \tilde{m}^h_{i,j,t} \) is the amount of previously missed entitlements that is now awarded to the participants of the pension fund through a catch-up indexation and \( m^m_{i,j,t} \), the addition to the stock of missed entitlements, is given by:

\[ m^m_{i,j,t} = (\pi_t - \omega^h_t) M^h_{i,j,t} \text{ if } \omega^h_t < \pi_t, \]

and zero otherwise. The amount \( \tilde{m}^h_{i,j,t} \) is determined as:

\[ \tilde{m}^h_{i,j,t} = \min[1, \max(0, \frac{A_t}{F^{ub}_t} - 1)] * M^m_{i,j,t-1}. \]  

(5.49)

This expression says that the amount of previously missed entitlements that are restored through catch-up indexation \( \tilde{m}^h_{i,j,t} \) is a fraction of previously missed entitlements. This fraction depends on the actual funding ratio \( F_t \) relative to the funding ratio at which full indexation is awarded, \( F^{ub} \). It is capped at 1 because it is at most equal to the total stock of missed entitlements. Its minimum value is 0 because catch-up indexation is not allowed to be negative.

5.B.2 Indexation of hard entitlements

If, after catch-up indexation, \( A_t > L^h_t \), the pension fund provides indexation of hard entitlements until either \( A_t = L^h_t \) or \( \omega^h_t = \pi_t \):

\[ \omega^h_t = \min[\pi_t, \max(0, \frac{A_t - L^h_t}{L^h_t})]. \]  

(5.50)

Note that the fact that the pension fund only considers its asset position and the amount of hard liabilities means that it can potentially reduce soft entitlements to provide indexation of hard entitlements.
5.B.3 Indexation of soft entitlements

If, after indexation of hard entitlements, \( A_t > L_t \), the pension fund provides indexation of soft entitlements until either \( A_t = L_t \) or \( \omega^s_t = \pi_t + \nu^s \):

\[
\omega^s_t = \min[\pi_t + \nu^s, \max(0, \frac{A_t - L_t}{L^s_t})]
\]  
(5.51)

In the case that after awarding indexation to the soft entitlements the funding ratio is above its upper bound, \( F_t > F^{ub} \), then the pension fund awards extra indexation to the soft entitlements until \( F_t = F^{ub} \). In that case the expression for the total indexation of the soft entitlements becomes:

\[
\omega^s_t = L^{ub} - L_t
\]  
(5.52)

where \( L_t \) and \( L^s_t \) denote the values of total entitlements and total soft entitlements, respectively, after indexation to hard entitlements has been awarded, but before any indexation to soft entitlements has been awarded. Further, \( L^{ub} \) is the value of liabilities such that \( F_t = F^{ub} \). Hence, \( L^{ub} = A_t / F^{ub} \).
5.B.4 Figures for base scenario

Figure 5.1: Rolling Window wage indexation
Figure 5.2: Fraction wage indexation
Figure 5.3: Split wage indexation
5.B.5 Figures for replacement rates base scenario

Figure 5.4: Replacement Rates Rolling Window version
Figure 5.5: Replacement Rates Fraction version
Figure 5.6: Replacement Rates Split version