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Beta-decay implications for the W-boson mass anomaly

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We point out the necessity to consider β-decay observables in resolutions of the W-boson anomaly in the Standard Model Effective Field Theory that go beyond pure oblique corrections. We demonstrate that present global analyses that explain the W-boson mass anomaly predict a large, percent-level, violation of first-row Cabibbo Kobayashi Maskawa unitarity. We investigate what solutions to the W-boson mass anomaly survive after including β-decay constraints.

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I. INTRODUCTION

The recent announcement by the CDF Collaboration of a new measurement of the W-boson mass $m_W = 80433.5 \pm 9.4$ MeV [1] is very exciting. Taken at face value, it implies significant tension with the Standard Model (SM) prediction [2] as well as with earlier, less precise, determinations [3,4]. While the impact of potential beyond-the-SM (BSM) physics on m_W has previously been considered in the literature [5–10], the announcement has sparked a lot of new interest to understand the BSM implications of this measurement, if correct. In particular, several groups studied the W-boson mass anomaly (as we will call this from now on) in terms of the Standard Model Effective Field Theory (SMEFT) under various assumptions to limit the number of independent operators and associated Wilson coefficients.

Most effort has been focused on explanations through oblique parameters [11–14] probing universal theories [15–17]. References [18–23] went beyond this approach by using a more general set of SMEFT operators. For example, Ref. [18] fitted electroweak precision observables (EWPO) under the assumption of flavor universality, finding that the W-boson mass anomaly requires nonzero values of various dimension-6 SMEFT Wilson coefficients. An important ingredient in these fits is the decay of the muon that enters in the determination of the Fermi constant. The hadronic counterparts of muon decay provide complementary probes of the Fermi constant in the form of β-decay processes of the neutron and atomic nuclei [24] and semileptonic meson decays [25]. Although the combination of EWPO and β-decay processes has been discussed in the literature before [10,26], very few of the recent analyses of m_W [20,27] included the low-energy data. In this work, we argue that not including constraints from these observables in global fits generally leads to too large deviations in first-row Cabibbo Kobayashi Maskawa (CKM) unitarity, much larger than the mild tension shown in state-of-the-art determinations. We investigate what solutions to the W-boson mass anomaly survive after including β-decay constraints.

II. MASS OF THE W BOSON IN SMEFT

We adopt the parametrization of SMEFT at dimension 6 in the *Warsaw* basis [28–30].

$$\mathcal{L}_{\rm SMEFT}^{\rm dim-6} = \mathcal{L}_{\rm SM} + \sum_i C_i \mathcal{O}_i^{\rm dim-6}, \tag{2.1}$$

where C_i are Wilson coefficients of mass dimension -2.

Calculated at linear order in SMEFT, the shift to W mass from the SM prediction due to dimension-6 operators is given by [8,31]

$$\frac{\delta m_W^2}{m_W^2} = v^2 \frac{s_w c_w}{s_w^2 - c_w^2}\left[2C_{HWB} + \frac{c_w}{2s_w} C_{HD} + \frac{s_w}{c_w}(2C_{Hl}^{(3)} - C_{ll})\right], \tag{2.2}$$

where $v \simeq 246$ GeV is the vacuum expectation value of the Higgs field, $s_w = \sin\theta_w$, and $c_w = \cos\theta_w$. The Weinberg

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TABLE I. List of the most relevant SMEFT dimension-6 operators that are involved in this analysis.

$\mathcal{O}_{HWB}$	$H^\dagger \tau^I H W^I_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{HD}$	$\lvert H^\dagger D_\mu H\rvert^2$
$\mathcal{O}^{(3)}_{Hl}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
$\mathcal{O}^{(3)}_{Hq}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
$\mathcal{O}_{ll}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$
$\mathcal{O}^{(3)}_{lq}$	$(\bar{l}_p \tau^I \gamma_\mu l_r)(\bar{q}_s \tau^I \gamma^\mu q_t)$

angle θ_w is fixed by the electroweak input parameters $\{G_F, m_Z, \alpha_{\rm EW}\}$ [32]. Here, we define $\delta m_W^2 = m_W^2(\text{SMEFT}) - m_W^2(\text{SM})$. For a detailed derivation of δm_W^2, see Appendix.

The mass of the W boson receives corrections from four Wilson coefficients, namely, C_{HWB}, C_{HD}, $C^{(3)}_{Hl}$, and C_{ll}. For the corresponding operators, see Table I. C_{HWB} and C_{HD} are related to the oblique parameters S and T [16]. They have been thoroughly studied for constraining "universal" theories [17,33] with electroweak precision observables as well as in light of the W-boson mass anomaly [11–14]. The linear combination of Wilson coefficients shown in Eq. (A11) $(2C^{(3)}_{Hl} - C_{ll})$ is related to the shift to the Fermi constant.

III. EWPO FITS AND CKM UNITARITY

Under the assumption of flavor universality, ten operators affect the EWPO at tree level, but only eight linear combinations can be determined by data [18]. Following Ref. [18], these linear combination are written with $\hat{C}_i$ notation and given by $\hat{C}^{(1)}_{Hf} = C^{(1)}_{Hf} - (Y_f/2)C_{HD}$, where f runs over left-handed lepton and quark doublets and right-handed quark and lepton singlets, and $\hat{C}^{(3)}_{Hf} = C^{(3)}_{Hf} + (c_w/s_w)C_{HWB} + (c_w^2/4s_w^2)C_{HD}$, where f denotes left-handed lepton and quark doublets, and $\hat{C}_{ll} = (C_{ll})_{1221}$. Here, Y_f is the hypercharge of the fermion f.

Reference [18] reports the results of a fit to EWPO data including the correlation matrix from which we can reconstruct the χ^2. We obtain very similar results by constructing a χ^2 using the SM and SMEFT contributions from Refs. [34,31], respectively, and the experimental results of Refs. [35–38]. For concreteness, we use the "standard average" results of Ref. [18], but our point would hold for the "conservative average" as well. To investigate the consequences of CKM unitarity on the fit, we will assume the flavor structures of the operators follow minimal flavor violation (MFV) [39,40]. That is, we assume the operators are invariant under a $U(3)_q \times U(3)_u \times U(3)_d \times U(3)_l \times U(3)_e$ flavor symmetry. In addition, we slightly change the operator basis and trade the Wilson coefficient $\hat{C}_{ll}$ for the linear combination

TABLE II. Results from the dimension-6 SMEFT fit of Ref. [18], before and after the inclusion of $\Delta_{\rm CKM}$. All Wilson coefficients are given in units of TeV^{-2}.

Coefficient	Result	Result with CKM
$\hat{C}^{(1)}_{\varphi l}$	-0.007 ± 0.011	-0.013 ± 0.009
$\hat{C}^{(3)}_{\varphi l}$	-0.042 ± 0.015	-0.034 ± 0.014
$\hat{C}_{\varphi e}$	-0.017 ± 0.009	-0.021 ± 0.009
$\hat{C}^{(1)}_{\varphi q}$	-0.0181 ± 0.044	-0.048 ± 0.04
$\hat{C}^{(3)}_{\varphi q}$	-0.114 ± 0.043	-0.041 ± 0.015
$\hat{C}_{\varphi u}$	0.086 ± 0.154	-0.12 ± 0.11
$\hat{C}_{\varphi d}$	-0.626 ± 0.248	-0.38 ± 0.22
C_Δ	-0.19 ± 0.09	-0.027 ± 0.011

$$C_\Delta = 2[C^{(3)}_{Hq} - C^{(3)}_{Hl} + \hat{C}_{ll}], \tag{3.1}$$

which allows for a more direct comparison to low-energy measurements. In principle, one could trade C_Δ for one of the $\hat{C}^{(3)}_{Hf}$ coefficients instead of $\hat{C}_{ll}$, but this would not change the determination of C_Δ.

We then refit the Wilson coefficients to the EWPO and obtain the results in the second column of Table II. In particular, we obtain

$$C_\Delta = -(0.19 \pm 0.09)\ \text{TeV}^{-2}. \tag{3.2}$$

This combination of Wilson coefficients contributes to the violation of unitarity in the first row of the CKM matrix tracked by $\Delta_{\rm CKM} \equiv |V_{ud}|^2 + |V_{us}|^2 - 1$, where we neglected the tiny $|V_{ub}|^2$ corrections. Within the MFV assumption, we can write [41]

$$\Delta_{\rm CKM} = v^2[C_\Delta - 2C^{(3)}_{lq}]. \tag{3.3}$$

The $C^{(3)}_{lq}$ operator that appears here does not affect EWPO and does not play a role in the fit of Ref. [18]. If one assumes this coefficient to be zero, Eq. (3.2) causes a shift,

$$\Delta^{\rm EWfit}_{\rm CKM} = -(0.012 \pm 0.005), \tag{3.4}$$

implying large, percent-level, deviations from CKM unitarity. We note that, to a lesser extent, large values of C_Δ and $\Delta_{\rm CKM}$ were already preferred by fits to EWPO before the recent CDF determination of m_W. Using an older average of determinations of $m_W = 80.379 \pm 0.012$ GeV [34], we find $C_\Delta = -0.15 \pm 0.09$ TeV^{-2} and $\Delta_{\rm CKM} = -(0.9 \pm 0.5) \times 10^{-2}$ (assuming $C^{(3)}_{lq} = 0$).

Based on up-to-date theoretical predictions for $0^+ \to 0^+$ transitions and kaon decays [42–49], the particle data group average indicates that unitarity is indeed violated by a bit more than two standard deviations [38],

$$\Delta_{\rm CKM} = -0.0015(7), \tag{3.5}$$

but in much smaller amounts than predicted by Eq. (3.4). This exercise shows that global fits to EWPO and the W mass anomaly which assume MFV and $C_{lq}^{(3)} = 0$, but include BSM physics beyond the oblique parameters S and T, such as the one of Ref. [18], are disfavored by β-decay data. While we did not repeat the fits of Refs. [20,23], the central values of their Wilson coefficients also indicate a negative percent-level shift to $\Delta_{\rm CKM}$, consistent with Eq. (3.4).

Indeed, combining the EWPO with $\Delta_{\rm CKM}$, we find that the minimum χ^2 increases by 3.3 and Wilson coefficients are shifted, as shown in Table II. Again, this shows that the Cabibbo universality test has a significant impact and should be included in EWPO analyses of the W-boson mass anomaly. These statements are illustrated in Fig. 1, which shows the values of $\Delta m_W = m_W - m_W^{\rm SM}$ obtained by fitting EWPO alone or EWPO *and* $\Delta_{\rm CKM}$ for two single-operator scenarios and the global analysis involving all operators.

Another way to proceed is to effectively decouple the CKM unitarity constraint from EWPO by letting $C_{lq}^{(3)} \neq 0$, which is consistent with the MFV approach. The $\Delta_{\rm CKM}$ observable is then accounted for by a nonzero value,

$$C_{lq}^{(3)} = -(0.082 \pm 0.045)\ {\rm TeV}^{-2}, \tag{3.6}$$

while the values of the other Wilson coefficients return to their original value given in the second column of Table II. However, care must be taken that such values of $C_{lq}^{(3)}$ are not excluded by LHC constraints [50–56]. In particular, Ref. [57] analyzed 8 TeV $pp \to ll$ data from Ref. [58] in the SMEFT at dimension 8. Limiting the analysis to MFV dimension-6 operators, we find

$$\begin{aligned} C_{lq}^{(3)} &= -(0.028 \pm 0.028)\ {\rm TeV}^{-2} \\ &\quad ({\rm Single\ coupling}, 95\%\ {\rm C.L.}), \\ C_{lq}^{(3)} &= -(0.05 \pm 0.1)\ {\rm TeV}^{-2} \quad ({\rm Global\ fit}, 95\%\ {\rm C.L.}), \end{aligned} \tag{3.7}$$

when in the first line only $C_{lq}^{(3)}$ is turned on, while in second line seven operators were turned on: $C_{lq}^{(1)}$, $C_{lq}^{(3)}$, C_{qe}, C_{lu}, C_{ld}, C_{eu}, and C_{ed}.

The resulting constraints from EWPO, $\Delta_{\rm CKM}$, and the LHC are shown in Fig. 2. As mentioned above, a simultaneous explanation of m_W and $\Delta_{\rm CKM}$ requires a nonzero value of $C_{lq}^{(3)}$, which implies effects in collider processes. The single-coupling bound from $pp \to ll$ in Eq. (3.7) is already close to excluding the overlap of the EWPO and $\Delta_{\rm CKM}$ regions, while a global fit allows for somewhat more room. Nevertheless, should the current discrepancy in the EWPO fit hold, the preference for a nonzero $C_{lq}^{(3)}$ could be tested by existing 13 TeV $pp \to ll$ [59] and $pp \to l\nu$ data [60] and, in the future, at the High-Luminosity upgrade of the Large Hadron Collider (HL-LHC).

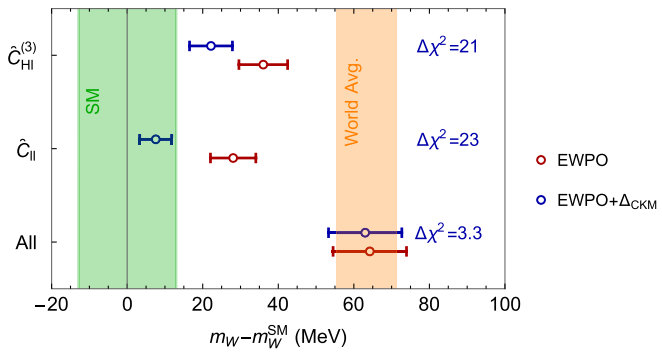

FIG. 1. The resulting values of $\Delta m_W = m_W - m_W^{\rm SM}$ when turning on $\hat{C}_{Hl}^{(3)}$, $\hat{C}_{ll}$, and all Wilson coefficients that are probed by EWPO. The red bars indicated the predicted Δm_W from the EWPO fit, while the blue bars show the resulting Δm_W after inclusion of $\Delta_{\rm CKM}$. The shown values of $\Delta\chi^2$ denote the differences in the minimum χ^2 between the blue and red points. The SM prediction and world average, taken from Ref. [18], are depicted by the green and orange bands, respectively.

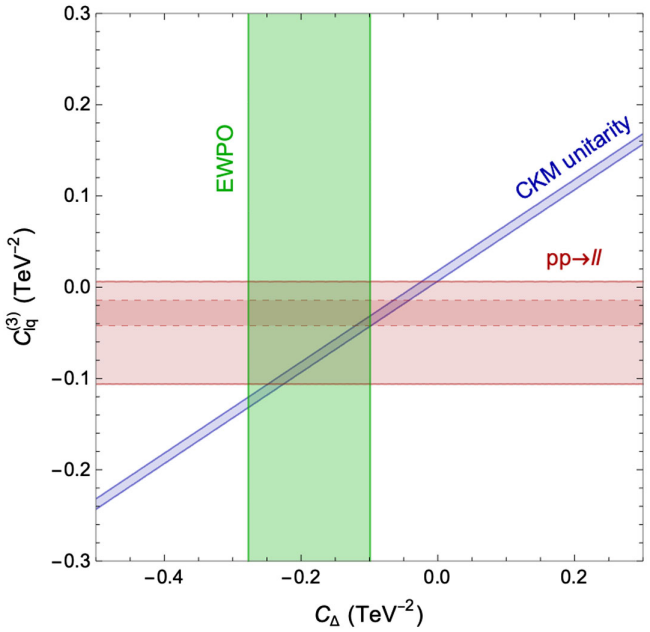

FIG. 2. The 1σ constraints from EWPO in green, a global (single-coupling) analysis of LHC measurements in (dashed) red, and low-energy beta decays in blue.

IV. CONCLUSION

In this paper, we have pointed out that global analyses of EWPO (beyond oblique parameters) in the general SMEFT

framework, while explaining the W-boson mass anomaly, tend to predict a large, percent-level, violation of Cabibbo universality, parametrized by $\Delta_{\rm CKM}$. This result is not consistent with precision beta decay and meson decay phenomenology and calls for the inclusion of a first-row CKM unitarity test in the set of EWPO, which is not commonly done. The inclusion of $\Delta_{\rm CKM}$ also requires adding $O_{lq}^{(3)}$ to the set of SMEFT operators usually adopted in EWPO analyses. We have illustrated this and shown that in this case Cabibbo universality can be recovered at the 0.1% level while still explaining the W mass anomaly. This extended scenario is currently consistent with constraints on $C_{lq}^{(3)}$ from $pp \to ll$ at the LHC and will be tested/challenged by future LHC data.

In this work, we have considered the case of $U(3)^5$ flavor invariance for the SMEFT operators. Several global fits in the literature adopted the same flavor assumption [18,20], so we can meaningfully compare our results. These flavor assumptions are ideal for describing flavor-blind BSM physics and, in practice, drastically reduce the number of Wilson coefficients entering the fit. Relaxing this hypothesis has several implications: first, one should consider flavor nonuniversal Wilson coefficients for the usual set of operators in EWPO fits (see, e.g., Ref. [23]); second, when including semileptonic low-energy processes, one should extend the operator set to include operators with more general Lorentz structures than $O_{lq}^{(3)}$, such as right-handed currents [61], that provide additional ways to decouple the $\Delta_{\rm CKM}$ constraint from the W mass. The inclusion of flavor nonuniversal Wilson coefficients [23] in the EWPO fit has been shown to provide a good solution to the W mass anomaly. In this context, addressing the role of CKM unitarity constraints and lepton-flavor universality tests in meson decays is an interesting future direction.

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Note added.—Recently, a discussion on the impact of $\Delta_{\rm CKM}$ on the fit to EWPO, Higgs, and diboson data has been added to Ref. [20]. Although this analysis includes more observables and operators than were considered here, Ref. [20] obtains qualitatively similar conclusions, finding that CKM unitarity can have a significant impact on the fit.

APPENDIX: DERIVATION OF δm_W^2

We now derive the correction to the W-boson mass at linear order in the SMEFT power counting; see also Refs. [8,31]. The W mass is not modified by the dimension-6 operators directly. The correction eventually comes from the shift to electroweak input parameters $\{\alpha_{\rm ew}, m_Z^2, G_F\}$, which is the input scheme most suitable for this study. These corrections propagate to the W mass through the Lagrangian parameters $\{g_1, g_2, v\}$, where g_1 and g_2 are the gauge couplings of hypercharge and weak interaction, and $v \simeq 246$ GeV is the vacuum expectation value of Higgs field.

1. Shift to Fermi constant G_F

We define the local effective interaction for muon decay as

$$\mathcal{L}_{G_F} = -\frac{4G_F}{\sqrt{2}}(\bar{\nu}_\mu \gamma^\mu P_L \mu)(\bar{e}\gamma_\mu P_L \nu_e). \tag{A1}$$

The effective Fermi constant gets corrections from several SMEFT operators,

$$-\frac{4G_F}{\sqrt{2}} = -\frac{2}{v^2} + (C_{\substack{ll \\ \mu e e \mu}} + C_{\substack{ll \\ e\mu\mu e}}) - 2(C_{\substack{Hl^{(3)} \\ ee}} + C_{\substack{Hl^{(3)} \\ \mu\mu}}). \tag{A2}$$

In the limit of MFV, the shift to G_F simplifies to

$$\delta G_F = [G_F]_{\rm SMEFT} - [G_F]_{\rm SM} = \sqrt{2}\left(C_{Hl}^{(3)} - \frac{1}{2}C_{ll}\right). \tag{A3}$$

In our notation, a δ in front of some quantity always indicates the SMEFT-corrected form of that quantity minus the original SM one.

2. Shift to the Z mass and $\alpha_{\rm ew}$

In the broken phase, the relevant kinetic and mass terms of the electroweak gauge bosons in SMEFT are[1]

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}W^+_{\mu\nu}W^{\mu\nu}_- - \frac{1}{4}W^3_{\mu\nu}W_3^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &-\frac{1}{2}v^2 C_{HWB} W^3_{\mu\nu}B^{\mu\nu} + \frac{1}{4}g_2^2 v^2 W^+_\mu W^{-\mu} \\ &+\frac{1}{8}v^2(g_2 W^3_\mu - g_1 B_\mu)^2 + \frac{1}{16}v^4 C_{HD}(g_2 W^3_\mu - g_1 B_\mu)^2.\end{aligned} \tag{A4}$$

The mass basis is given by [62]

[1]Several other Wilson coefficients enter this Lagrangian, such as C_{HW} and C_{HB}. However, their effects can be absorbed by a redefinition of g_1, g_2, and v and are irrelevant to the W mass.

$$\begin{bmatrix} W_\mu^3 \\ B_\mu \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2}v^2 C_{HWB} \\ -\frac{1}{2}v^2 C_{HWB} & 1 \end{bmatrix} \times \begin{bmatrix} \cos\bar{\theta} & \sin\bar{\theta} \\ -\sin\bar{\theta} & \cos\bar{\theta} \end{bmatrix} \begin{bmatrix} Z_\mu \\ A_\mu \end{bmatrix}, \tag{A5}$$

where the rotation angle is defined as

$$\tan\bar{\theta} = \frac{g_1}{g_2} + \frac{1}{2}v^2\left(1 - \frac{g_1^2}{g_2^2}\right)C_{HWB}. \tag{A6}$$

After the rotation, the Z mass is shifted:

$$\delta m_Z^2 = \frac{1}{8}v^4(g_1^2 + g_2^2)C_{HD} + \frac{1}{2}v^4 g_1 g_2 C_{HWB}. \tag{A7}$$

Substituting Eq. (A5) into the Standard Model covariant derivative gives

$$\begin{aligned} D_\mu &= \partial_\mu + ig_2 W_\mu^a t^a + ig_1 YB \\ &= \partial_\mu + i\frac{g_2}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) + ieQA_\mu \\ &\quad + ig_Z(T_3 - \sin^2\bar{\theta}Q)Z_\mu, \end{aligned} \tag{A8}$$

where $Q = T_3 + Y$, with Y and T_i the hypercharge and $SU(2)_L$ generators. Extracting the coefficient of A_μ, we get the SMEFT correction to the electric coupling e and also to the fine-structure constant $\alpha_{\rm ew}$,

$$\delta\alpha_{\rm ew} = 2\left(\frac{\delta e}{e}\right)\alpha_{\rm ew} = -\frac{v^2 g_1^3 g_2^3}{2\pi(g_1^2 + g_2^2)^2}C_{HWB}. \tag{A9}$$

3. Shift to the W mass

We may express the W mass as

$$m_W^2 = \frac{[m_Z^2]_{\rm SM}}{2}\left(1 + \sqrt{1 - \frac{2\sqrt{2}\pi[\alpha_{\rm ew}]_{\rm SM}}{[m_Z^2]_{\rm SM}[G_F]_{\rm SM}}}\right). \tag{A10}$$

This can be expressed in terms of the input scheme observables, $\mathcal{O}_i = \{\alpha_{\rm ew}, m_Z^2, G_F\}_i$, by substituting $\mathcal{O}_i = [\mathcal{O}_i]_{\rm SM} + \delta\mathcal{O}_i$ into Eq. (A10),

$$\begin{aligned} \frac{\delta m_W^2}{m_W^2} &= \frac{v^2}{g_1^2 - g_2^2}\left[2g_1 g_2 C_{HWB} + \frac{1}{2}g_2^2 C_{HD} + g_1^2(2C_{Hl}^{(3)} - C_{ll})\right] \\ &= v^2\frac{s_w c_w}{s_w^2 - c_w^2}\left[2C_{HWB} + \frac{c_w}{2s_w}C_{HD} + \frac{s_w}{c_w}(2C_{Hl}^{(3)} - C_{ll})\right], \end{aligned} \tag{A11}$$

where $s_w = \sin\theta_w$ and $c_w = \cos\theta_w$, with θ_w being the Weinberg angle in the SM.

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