Scaffolding small groups’ mathematics learning

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General introduction

> *In principle, you make students think more if you scaffold the group's learning.*
> Participating teacher

The idea for this research stems from my studies to obtain my master's degree in teaching mathematics at the University of Amsterdam. As a preservice teacher, I taught a seventh grade mathematics class consisting of students who had received five different track advice levels, ranging from vocational to preuniversity advice levels, since primary school. After receiving instructions, the students sat together in groups of mixed ability and worked on assignments during the working phase. This scenario is characteristic of the educational system of the school where I worked. Consequently, I began experimenting with collaborative learning in my classes but struggled with what kind of support I should give to small groups working together on mathematical assignments. Students seemed to learn a great deal from each other when they were discussing the mathematical topics at hand and explained matters to each other, but sometimes small groups became stuck and needed my support. I distinctly remember walking with lead in my shoes toward a group of students that needed my assistance. All kinds of possible support that I could give ran through my mind. As I arrived at the group, I decided to just give them the answer to the problem. The students were happy that I provided the answer. However, since I provided the answer to the problem, the discussions the group was having stopped altogether.

My experiences were in line with what is found in the literature : Teachers in general tend to find supporting small groups that work together difficult (Van Leeuwen & Janssen, 2015). They are confronted with complex choices of how, when and what kind of support to give (Van de Pol et al., 2014). Consequently, the idea behind this research was born. How can mathematics teachers support small groups so that they continue with their discussions? I chose to focus my research on seventh grade early algebra since my students experienced difficulty learning this subject. The results of my endeavor are described in the rest of this book.

Early algebra is difficult for students in secondary education. High-quality discussions about mathematics during collaborative learning in which students reflect on mathematical activities are important for learning mathematics (Freudenthal, 1991). However, merely placing students together in small groups does not automatically lead to such discussions; students do not automatically know how to engage in high-quality discussions (Barron, 2003; Mercer & Sams, 2006). The guidance of a teacher is important during collaborative learning (Webb et al., 2019). Teachers, however, face challenging decisions of how and when to give support during such learning (Van de Pol et al., 2014).

The general aim of this study is therefore to gain insight into how teachers can guide small groups during collaborative learning to enhance the quality of mathematical discussions and students' mathematical understanding.

First, I discuss the theoretical framework of this research project. Second, I list the research questions and elaborate on the general aim of this research project. Finally, I present the research design and structure of this project.

1.1 THEORETICAL FRAMEWORK

1.1.1 Mathematical level raising in early algebra

Algebra is an important part of the secondary mathematics curriculum in the Netherlands as well as in other countries. In the Netherlands, where early algebra is taught in seventh grade, research has shown that students experience difficulty learning this subject (Van Stiphout et al., 2011). The finding that learning early algebra in secondary education is not an easy task for students has been repeatedly shown in comparative studies such as the Trends in International Mathematics and Science Study (TIMSS) (Mullis et al., 2016) and the Programme for International Student Assessment (PISA) (OECD, 2016; OECD, 2019). Seventh grade early algebra is concerned with the relation between two variables in a formula. Janvier (1978) defined four algebra representations, namely, situation, graphs, tables, and formulae, as four ways to describe this relation. In early algebra, students are typically asked to make a switch from one algebra representation to the other, e.g., to create a formula for a situation. Sfard and Linchevski (1994) found that students find it particularly difficult to make a switch from studying patterns, at a lower level, to creating a formula, at a higher level.

Learning mathematics was described in terms of discrete levels by Van Hiele (1986) and Freudenthal (1991). Van Hiele distinguished four levels of understanding mathematics: 1) the visual level, in which the forms of objects are being studied; 2) the descriptive level, in which the properties of objects are being studied; 3) the theoretical level, in which the relations between the properties of objects are being studied; and 4) the formal logical level, in which the relations between theorems are being studied. Freudenthal (1991) built on Van Hiele's level theory and stated that mathematical level raising occurs every time an activity that is the object of study at a lower level consciously becomes the object of study at a higher level. In the case of early algebra, switching from Janvier's situation, graphs, and tables representations to a formulae representation represents a jump to a higher mathematical level. For example, learning to switch from studying patterns, at a lower level, to formulating formulae for the patterns of two variables constitutes level raising.

1.1.2 Collaborative learning and mathematical discussions

Freudenthal (1991) has suggested that level raising can be achieved by collaborative learning. He also argued that this is due to reflection on mathematical activities, which often occurs during collaboration in small student groups of mixed ability. Often, one of these students will understand the subject matter (jumping to a higher level) and then start explaining to the other members of the group by reflecting on what he or she just learned.

Collaborative learning is known to enhance students' achievement (e.g., Kyndt et al., 2014; Mercer & Littleton, 2007; Mercer & Sams, 2006; Van Leeuwen & Janssen, 2019). Van der Linden et al. (2000, p.39) define collaborative learning as "learning in cooperation with others offers the opportunities for an active learning process; it entails that one has to come to a mutual agreement as to the interpretation of what is to be learned and how to go about it." Collaborative learning has been proven to be successful in the mathematics classroom (Dekker & Elshout-Mohr, 1998, 2004; Pijls et al., 2007a; Webb, 2009; Yackel et al., 1991), and the impact of collaborative learning on students' mathematics understanding has also been extensively researched over time (Campbell & Yeo, 2021; Elbers & Streefland, 2000; Francisco, 2013; Ing et al, 2015; Mercer & Sams, 2006; Webb, 1982; Webb et al., 2021). To explain how discussions impact students' mathematical learning (ages 9-10), researchers such as Mercer and Sams (2006) have used the notion of exploratory talk. In exploratory talk, students share all relevant information and elaborate on each other's ideas; they contribute to discussions by sharing their reasons and challenging each other. Mercer and Sams (2006) found that students are able to use talk for mathematical reasoning and that such discussions contribute to students' mathematical knowledge level. Furthermore, they argued that students do not automatically know how to engage in high-quality discussions and that students need guidance from a mathematics teacher to do so.

Sharing ideas and elaborating on other students' ideas are at the core of many of these studies and lie at the heart of explanations of how collaborating with other students might foster learning. In a body of work exploring the teacher practices that promote productive dialogue and learning in mathematics classrooms, Webb et al. (2019) have shown that sharing and elaborating activities often have a positive influence on mathematical learning. Sharing activities make a student's mathematical ideas visible to his or her fellow students and thus makes them open to elaboration. Sharing without elaborating activities may be less beneficial for learning, as elaborating, for example, encourages the explainer to fill in gaps in his or her own understanding and recognize his or her own misconceptions (Webb, 2009). Moreover, elaboration activities such as explaining one's mathematical ideas might help students reflect on their mathematical activities, which in turn might lead to mathematical level raising (Freudenthal, 1991). Van Boxtel et al. (2000) found that elaborative student interaction is particularly valuable for concept learning. Following Dekker and Elshout-Mohr (2004), we identify the following activities as elaborating activities: explain work, justify work, reconstruct work, ask to explain work, and critique work.

For collaborative learning to lead to learning, powerful group tasks that invoke mathematical discussions where students elaborate on their mathematical ideas are necessary (Cohen, 1994). These group tasks should involve ill-structured problems, problems that are not clear-cut and cannot be solved by individuals (Cohen, 1994). In conventional mathematics, textbook tasks are meant to be solved by individual students. Additionally, students are often led through predefined problem-solving processes in a step-by-step manner (Lithner, 2008; Mayer, 2002). It has been shown that this kind of step-by-step guidance can result in fragmented mathematical knowledge and low mathematical reasoning skills in students (Bergqvist et al., 2008). Inspired by the level theory of Freudenthal (1991), Palha et al. (2013) developed design principles for group tasks, so-called shift-problem lessons, specifically for mathematics. In addition to aiming to help students cultivate a deeper knowledge of mathematics, shift-problem lessons aim to enhance students' reasoning skills and thus prevent fragmented mathematical knowledge. The design principles suggest that mathematics in group tasks has to start at a level that is experientially real to students, that the goal of group tasks is the development of a deeper understanding of mathematics and that reflection on mathematical activities can be induced through mathematical discussions (Palha et al., 2013). These design principles were used to design group tasks in early algebra in this research project.

This approach, however, does not yet solve the problem of teachers' difficulties in guiding students during collaborative learning. There is still a need for tools that help mathematics teachers guide students to engage in high-quality mathematical discussions (Webb et al., 2019).

1.1.3 Process support

Teachers can provide support in many situations, for example, during whole-class instruction, one on one or in groups. In this dissertation, I focus on the latter. It is also important to distinguish among the several types of support. Dekker and Elshout-Mohr (1998) have thus distinguished between process support and content support during mathematical group discussions. Process support is support in which the teacher prompts small groups to perform elaborating activities during student–student interactions, while content support involves mathematical content. Process support is typically a more effective kind of support than content support (Dekker & Elshout-Mohr, 2004). However, in some cases, process support is not sufficient and content support is necessary to further the discussions. The amount of content support plays an important role in getting discussions back on track. When a teacher gives extensive content support to an individual student in a small group, other members of the group stop contributing to the discussion, and discussion comes to a halt (Dekker & Elshout-Mohr, 2004). Effectively getting this discussion back on track probably involves a combination of process support and content support. However, it is still unclear how to effectively combine process support and content support; and when to provide process support or content support.

1.1.4 Scaffolding

There are indications that adapting content support to the needs of individuals in the context of group work, i.e., scaffolding, enhances students' learning (Van de Pol et al., 2010). In the field of construction, a scaffold is a temporary structure that is used to help construct or renovate a building. In the field of education, scaffolding is related to the sociocultural theory of Vygotsky (1978). It is used as a metaphor to describe the temporary adaptive content support a teacher can provide to a student in the student's zone of proximal development (ZPD) (Mercer & Littleton, 2007). Vygotsky (1978) described the ZPD as the distance between what a student can achieve independently (actual understanding) and what a student can achieve under the guidance of a more knowledgeable person (potential understanding).

The term “scaffolding” was originally used to describe how teachers could support individual students (Wood, et al., 1976). Van de Pol and colleagues (2010) focused on scaffolding individual students working in a group context, a topic of interest in this study, as I focus on how teachers can guide small groups effectively. Van de Pol and colleagues (2010) have defined three key characteristics of content-related scaffolding of individuals in a group setting: 1) contingency to individuals in the context of groupwork (adapting content support to the level of the individual student), 2) fading (phasing out content support over time), and 3) transfer of responsibility (returning responsibility for learning to students). Smit et al. (2013) applied this notion to a whole-class situation in their research on whole-class language scaffolding in a multilingual setting.

The focus in this dissertation is not on scaffolding individuals in a group or in the whole-class setting but rather how to scaffold a whole group at the group level, i.e., how to contingently adapt support to the needs of a group. Among other objectives, we aim to define small-group scaffolding by extending the notion of scaffolding individuals within a group (Van de Pol et al., 2010) to scaffolding a group as a whole at the group level.

1.2 AIMS AND RESEARCH QUESTIONS

The general aim of this study was to gain insight into how teachers can guide small groups during collaborative learning to enhance the quality of mathematical discussions and students’ mathematical understanding.

In this research project, we explore what small-group scaffolding entails, develop and test a tool (SGS-Tool) that helps teachers scaffold small groups at the group level during mathematical discussions, and we investigate whether small-group scaffolding enhances the quality of mathematical discussions and raises the mathematical knowledge level of students in seventh grade early algebra classes. To create a context in which this topic can be studied, we developed specially designed and tested lessons, so-called shift-problem lessons.

The main research question I address in this dissertation is thus as follows: What is the effect of small-group scaffolding on the quality of mathematical discussions, and does it raise the mathematical knowledge level of students in seventh grade early algebra classes?

The main research question is addressed through five sub questions:
1. Do shift-problem lessons for early algebra in seventh grade result in more and qualitatively better mathematical discussions and more mathematical level raising than working with conventional lessons in a small-group setting?
2. Does working with the Small-Group Scaffolding tool (SGS-Tool) elicit teachers’ scaffolding behavior at the group level during mathematical discussions?
3. How does guiding seventh-grade student groups with the SGS-Tool affect the amount and quality of students’ mathematical discussions?
4. What is the effect of small-group scaffolding on the mathematical knowledge level raising of seventh-grade students in early algebra classes?

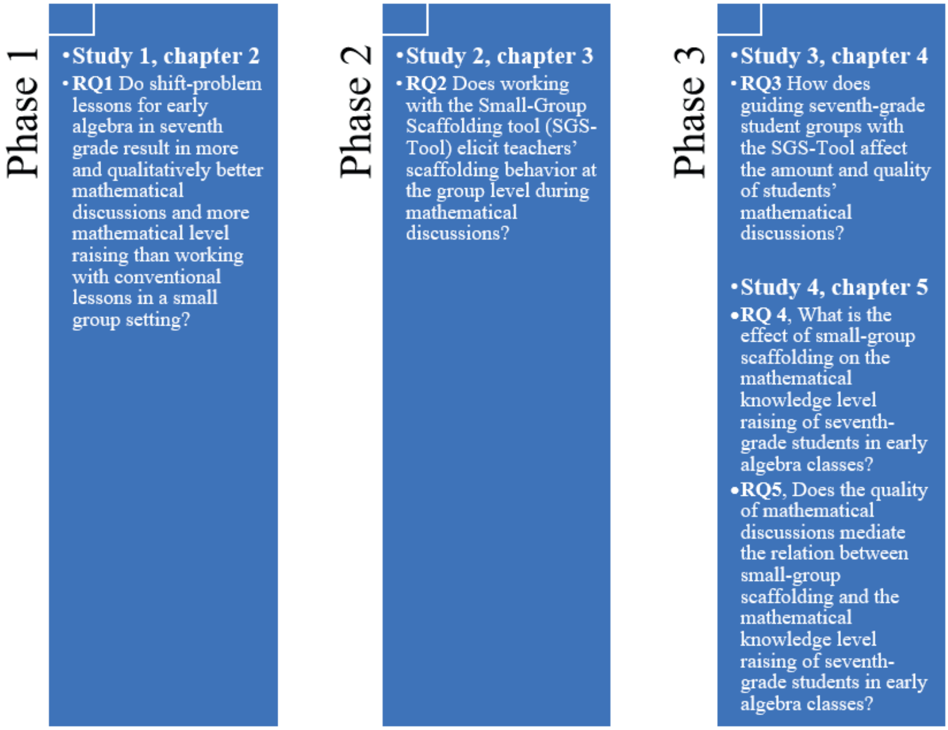
5. Does the quality of mathematical discussions mediate the relation between small-group scaffolding and the mathematical knowledge level raising of seventh-grade students in early algebra classes?

1.3 RESEARCH DESIGN

This research was conducted in seventh-grade early algebra classes. This context of heterogeneous (mixed-ability) groups was selected following Freudenthal (1991), who stated that discussions in such settings often invoke reflections on mathematical activities, which in turn can lead to mathematical level raising. We purposely chose to conduct the first two phases (study 1 and study 2) of this research project at one school because of the educational design of this school in which students sit together during the working phase in small heterogeneous groups of mixed ability. I was one of the teachers at this school and participated in the first study. In the third phase (study 3 and 4), we added another school with a similar educational design to the experimental condition to increase the number of participating teachers.

Figure 1.1

Phases, Studies and Research Questions of this Research Project



Phase 1

- **Study 1, chapter 2**
- **RQ1** Do shift-problem lessons for early algebra in seventh grade result in more and qualitatively better mathematical discussions and more mathematical level raising than working with conventional lessons in a small group setting?

Phase 2

- **Study 2, chapter 3**
- **RQ2** Does working with the Small-Group Scaffolding tool (SGS-Tool) elicit teachers' scaffolding behavior at the group level during mathematical discussions?

Phase 3

- **Study 3, chapter 4**
- **RQ3** How does guiding seventh-grade student groups with the SGS-Tool affect the amount and quality of students' mathematical discussions?

- **Study 4, chapter 5**
- **RQ 4**, What is the effect of small-group scaffolding on the mathematical knowledge level raising of seventh-grade students in early algebra classes?
- **RQ5**, Does the quality of mathematical discussions mediate the relation between small-group scaffolding and the mathematical knowledge level raising of seventh-grade students in early algebra classes?

This research project consisted of three phases in which we conducted four studies in total (see Figure 1.1). In phase 1 (study 1), the aim was to develop high-quality collaborative lessons on the topic of early algebra. To ensure that the group tasks evoked mathematical discussion, we investigated whether shift-problem lessons result in more and better quality mathematical discussions and more mathematical level raising for seventh-grade early algebra. We thus tried to answer research question 1: Do shift-problem lessons in early algebra in seventh grade result in more and qualitatively better mathematical discussions and more mathematical level raising than working with conventional lessons in a small-group setting? Therefore, we designed and tested shift-problem lessons in which students collaborate with each other, based on the design principles of Palha et al. (2013). A quasi-experimental study with a pre- and posttest control group design was conducted. Students in the experimental condition worked on a lesson unit of 12 60-minute lessons on early algebra, of which five lessons were replaced with shift-problem lessons. Students in the control condition worked with the conventional textbook. The interactions among students working in small groups were taped to measure the quality of the mathematical discussions. In addition, we designed a test that measures students' mathematical level for early algebra. The participants were 160 students, aged between 12 and 15 years old, from 6 seventh-grade classes, and 6 teachers (5 males, 1 female) of one school in an ethnically diverse suburban neighborhood in Amsterdam, the Netherlands. I participated as a teacher in this first study. I did not participate in the subsequent studies, since the support that teachers gave was the object of those studies and my participation might have influenced the outcomes. I report on the findings of phase 1 (study 1) in chapter 2.

In phase 2 (study 2), the aim was to investigate whether we can assist teachers in scaffolding small groups at the group level. We were particularly interested in developing a tool that helps teachers scaffold small groups at the group level, and answering research question 2: Does working with the Small-Group Scaffolding Tool (SGS-Tool) elicit teachers' scaffolding behavior at the group level during mathematical discussions? To do so, we first defined what small-group scaffolding entails, then we developed an appropriate tool. A quasi-experimental study was conducted in which teachers in the experimental condition used the SGS-Tool and teachers in the control condition did not. Students in both conditions worked on the same lesson unit of phase 1, study 1. The participants were four teachers (3 males, 1 female) and their 109 seventh-grade students at the same school of phase 1, study 1. To assign the teachers to one of the groups, they were matched by age and teaching experience. In each pair, one teacher was randomly assigned to the SGS condition and the other to the control condition. I trained the teachers in the SGS condition to use the SGS-Tool. We analyzed and compared the scaffolding behavior of the teachers who worked with the SGS-Tool with that of teachers who did not work with the tool. To do so, I videotaped the interactions between teachers and student groups to measure the scaffolding behavior of the teachers. I report on the findings of phase 2, study 2, in chapter 3.

Phase 3 consisted of two studies (study 3 and study 4). In phase 3, the aim was to determine whether small-group scaffolding (i.e., guiding student groups with the SGS-Tool) in seventh-grade early algebra classes fosters mathematical discussions (study 3) and whether this has a positive effect on students' mathematical knowledge level (study 4).

In phase 3, study 3, we answered research question 3: "How does guiding seventh-grade student groups with the SGS-Tool affect the amount and quality of students' mathematical discussions?". A quasi-experimental study at two schools in the Netherlands was conducted. The participants were eight teachers (7 males and 1 female) and 272 seventh grade students (11 classes) approximately 12–13 years of age. At the first school, six teachers with comparable age and teaching experience were paired. In each pair, one teacher was randomly assigned to the SGS condition, while the other was assigned to the control condition. For the second school, the two teachers were both assigned to the SGS condition. In the experimental condition, the teachers were trained to use the SGS-Tool, and in the control condition, the teachers gave support as they normally would. To determine whether small-group scaffolding fosters mathematical discussions, a quantitative and qualitative analysis of the interactions of students in small groups was conducted. I report on the findings of phase 3, study 3, in chapter 4.

In phase 3, study 4, the aim was to gain insight into whether small-group scaffolding has a positive effect on students' mathematical knowledge level in seventh-grade early algebra classes. We aimed to answer two research questions (4 and 5): What is the effect of small-group scaffolding on the mathematical knowledge level raising of seventh-grade students in early algebra classes? Does the quality of mathematical discussions mediate the relation between small-group scaffolding and the mathematical knowledge level raising of seventh-grade students in early algebra classes?

A quasi-experimental study was conducted in which the mathematical knowledge levels of students who were scaffolded at the group level with help of the SGS-Tool were compared to those of students who were not. In addition, we investigated whether the effect of scaffolding at the group level with help of the SGS-Tool on mathematical knowledge level was mediated by the quality of the mathematical discussions. The same dataset on mathematical discussions was used in study 3 and 4. However, in study 3 the utterances of individual student were the unit of analysis, whereas in study 4, we aggregated these utterances to the group level.

To determine whether small-group scaffolding has a positive effect on students' mathematical knowledge level raising (research question 4), we measured the difference in the growth of students' mathematical level with an adapted version of the pre- and posttests that were designed in phase 1. Additionally, we determined whether the quality of the mathematical discussions in the experimental condition mediates the effect of small-group scaffolding on mathematical level raising (sub research question 5). I report on the findings of phase 3, study 4, in chapter 5.

In the closing chapter, I discuss the main findings regarding our research questions and present my conclusions. Additionally, I consider the methodology used in this research project and discuss limitations, future research and implications for practice.