Scaffolding small groups' mathematics learning

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Improving the quality of mathematical discussions: The impact of small-group scaffolding

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ABSTRACT

Guiding small groups working on mathematical tasks is challenging for teachers. In this study, we investigated whether using a tool that helps teachers scaffold small student groups during mathematical discussions (the SGS-Tool) leads to more and qualitatively better mathematical discussions. The participants were eight teachers and 272 seventh grade students drawn from two schools. Five teachers used the tool (SGS condition), while three did not (control condition). SGS teachers provided more support than control teachers. SGS teachers also implemented various steps of the SGS-Tool, whereas the control teachers mostly offered content support. Significantly more and qualitatively better mathematical discussions occurred in the SGS condition. We provide a qualitative illustration of two contrasting teacher–small-group interactions (one in the control condition and one in the SGS condition), followed by an analysis of the interaction processes associated with one student group in each class during one lesson.

Keywords: collaborative learning, small-group scaffolding tool, small-group scaffolding, quality of mathematical discussions, mathematical discussions

4.1 INTRODUCTION

Several studies have shown that mathematical discussions in small, heterogeneous groups, where students reflect on mathematical structures and activities, can raise the mathematical level of students (Dekker & Elshout-Mohr, 1998, 2004; Pijls & Dekker, 2011; Pijls et al., 2007b). Moreover, it has been found that the more students are involved in mathematical discussions, the more they learn (Webb et al., 2014; Ing et al., 2015). However, small-group work does not automatically benefit learning (Barron, 2003). Students might struggle to communicate their mathematical ideas to each other (Sfard & Kieran, 2001). Brodie (2001) recommended the development of teacher strategies to support mathematical discussions since teachers find it difficult to support small groups during collaborative learning (Van Leeuwen & Janssen, 2019; Webb, 2009). We mention here three specific aspects that make the task of supporting of small groups in collaborative learning challenging. First, teachers must decide which type of support to provide. A second challenge pertains to the task of exercising the correct degree of control when providing content support. Finally, it is challenging to determine when it is time to return control over the learning process to the group. In addition, Webb et al. (2019) emphasized the necessity for the development of tools that help teachers improve the quality of mathematical discussions to enhance mathematics learning. Freudenthal (1991) stated that high-quality mathematical discussions in which small, heterogeneous groups of students reflect on mathematical activities contribute significantly to increasing students' mathematical level. Mathematical discussions are therefore promoted in, for example, the Process Standards of the National Council of Teachers of Mathematics of the USA (NCTM, 2000). Webb et al. (2019) also emphasized the necessity of tools that help teachers decide what kind of support to provide.

To improve teachers' support for small, heterogeneous, mixed-ability groups in mathematics classes, and help them face the challenges mentioned above, we previously developed a Small-Group Scaffolding Tool (the SGS-Tool) in a study (Calor et al., 2022).

The SGS-Tool describes how scaffolding can be performed at a group level during mathematical discussions. Support at the group level is important since mathematical discussions are a group activity. Other research has focused on scaffolding individual students in group settings (Van de Pol et al., 2010) or scaffolding whole classes (Smit et al., 2012). Our focus is on the scaffolding of a small group of students. The SGS-Tool helps teachers decide, based on the group's needs, when it is best to provide process support (stimulate mathematical discussions) and when to provide content support. In both cases, the aim is for mathematical discussions to continue, thus enhancing the amount and quality of mathematical discussions in the group.

We found that teachers were able to apply the various steps of the SGS-Tool (Calor et al., 2022). However, we do not yet know the actual effect of applying the SGS-Tool on the quality of mathematical discussions. This study investigates whether working with the SGS-Tool improves the quality and amount of mathematical discussions in small student groups.

4.2 THEORETICAL FRAMEWORK

4.2.1 Quality of mathematical discussions

In mathematical discussions during small-group work, students reason about mathematical subjects and are challenged to reflect on mathematical structures and activities (Freudenthal, 1991; Van Hiele, 1986). Many practical guides of good teaching promote mathematical discussions, e.g., the Process Standards of the National Council of Teachers of Mathematics of the USA (NCTM, 2000) and the Australian Association of Mathematics Teachers Standards for Excellence in Teaching Mathematics in Australian Schools (2006). Mathematical discussions in which mathematical ideas are shared to promote learning have been extensively investigated (e.g., Dekker & Elshout-Mohr, 1998, 2004; Ing et al., 2015; Palha et al., 2014; Pijls et al., 2007a; Mercer & Littleton, 2007; Webb, 2009; Webb et al., 2014; Yackel et al., 1991). The effects of small-group work on learning depend on the quality of the discussion (Van Boxtel et al., 2000; Van der Linden et al., 2000; Ing et al., 2015). In particular, the extent to which students give and receive explanations, share ideas, co-construct new knowledge, critique each other's work, and justify and reconstruct their work have been shown to be important elements (Van Boxtel et al., 2000; Mercer & Littleton, 2007; Palha et al., 2014; Webb, 2009).

Dekker and Elshout-Mohr (1998) developed a process model (PM) that describes the elements that characterize a qualitatively good mathematical discussion. The model distinguishes two types of learning activities as crucial elements of such a discussion: key activities and regulating activities. Key activities are learning activities that invoke reflection on mathematical activities, which in turn contributes to mathematical learning (Freudenthal 1991). Four key activities are distinguished: tell/show work, explain work, justify work, and reconstruct work. Regulating activities are activities that regulate key activities. Three regulating activities are distinguished in the model: ask to tell/show work, ask to explain work and critique work. Research has shown that performing key and regulating activities has a positive effect on students' mathematical learning outcomes (Dekker & Elshout-Mohr, 2004; Pijls et al., 2007a). It is therefore important that students perform key and regulating activities during discussions. In addition, the PM stresses the importance of both types of sharing activities (tell/show work and ask to tell/show work) and elaborating activities (explain work, justify work, reconstruct work, ask to explain work, and critique work); see Table 4.1.

Table 4.1

Sharing and Elaborating Activities in the Process Model

	Regulating activities	Key activities
Sharing activities	Ask to show work	Tell/show work
Elaborating activities	Ask to explain work	Explain work
	Critique work	Justify work
		Reconstruct work

In a body of work exploring the practices used by teachers to promote productive dialogue and learning in mathematics classrooms, Webb et al. (2019) demonstrated that high quality discussions involving sharing and elaborating activities often have a positive influence on mathematical learning. Sharing activities make a student's mathematical ideas visible to his or her fellow students and thus open to elaboration. Sharing without elaborating activities may be less beneficial for learning, as elaborating, for example, encourages the explainer to fill in gaps in his or her own understanding and recognize his or her own misconceptions, while receiving explanations may help students correct their misconceptions and strengthen connections between new knowledge and previously learned knowledge (Webb, 2009). In particular, the extent to which students give and receive explanations, share ideas, co-construct new knowledge, critique each other's work, and justify and reconstruct their work have been shown to be important elements of high-quality discussions (Van Boxtel et al., 2000; Mercer & Littleton, 2007; Palha et al., 2014; Webb, 2009).

However, merely organizing students into small groups does not automatically inspire such discussions; students do not automatically know how to engage in high-quality discussions (Barron, 2003; Mercer & Sams, 2006). Previously (Calor et al., 2020), we found that merely placing students in a small group led to lower-quality discussions with many key activities and far fewer regulating activities. Additionally, more utterances were related to sharing activities, ask to tell/show work and tell/show work, than to elaborating activities, explain work, justify work, reconstruct work, ask to explain work, and critique work.

In the present study, we consider the occurrence of both key and regulating activities in mathematical discussions as indicators of good quality, particularly the occurrence of the elaborating activities explain work, justify work, reconstruct work, ask to explain work, and critique work. Teachers' guidance should be directed to supporting these activities to improve the quality of mathematical discussions and thus enhance mathematical learning.

4.2.2 Small-Group Scaffolding Tool

Teacher support is a crucial factor that can affect the quality of mathematical discussions, e.g., when students have questions or when discussions become stuck. Teacher support can take place on the level of the whole class (e.g., Stein et al., 2008 and Smit et al., 2012) or on the level of the group, which is our focus. It is also important to distinguish between different types of support. Dekker and Elshout-Mohr (1998) have thus distinguished between process support and content support in the context of mathematical group discussions. Process support is support that the teacher uses to stimulate small groups to perform elaborating activities during mathematical discussions, while content support involves mathematical content. Process support can be a more effective form of support than content support, as the former encourages students to elaborate on their mathematical ideas (Dekker & Elshout-Mohr, 2004). However, process support is often insufficient, and content support is necessary to further the discussion.

Studies have found that scaffolding individual students in a group setting can have positive effects on their learning (Van de Pol et al., 2010; Webb, 2009). A key feature of scaffolding is that content support is contingent, i.e., adapted to the level of understanding of the learner (Van de Pol et al., 2010); in other words, such support is provided in the zone of proximal development (ZPD) of a student. Vygotsky (1978) described the ZPD in terms of the distance between what a student can achieve independently (actual understanding) and what a student can achieve under the guidance of a more knowledgeable person (potential understanding). Van de Pol and colleagues (2010) focused on scaffolding individual students working in a group context. These authors defined three key characteristics of content-related scaffolding of individuals in a group setting: 1) contingency with regard to individuals in the context of groupwork (adapting content support to the level of the individual student), 2) fading (phasing out content support over time), and 3) transfer of responsibility (returning responsibility for learning to students). Since mathematical discussions are a group effort, the content support that teachers provide needs to be adapted to the level of understanding of the whole group.

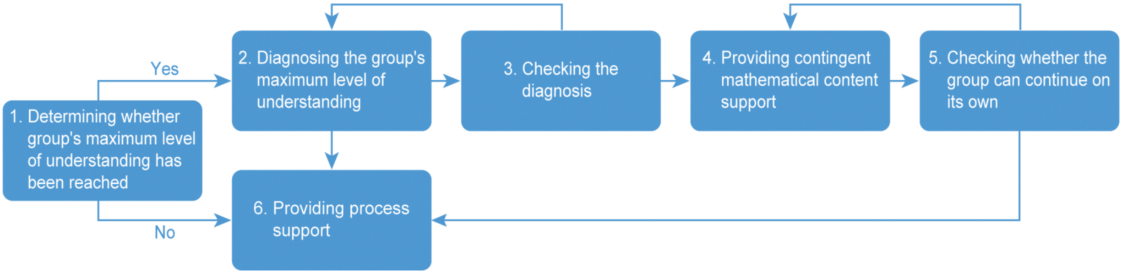
In a previous study (Calor et al., 2022), we described three key characteristics of *small-group scaffolding at the group level during mathematical discussions* in the following terms:

1) Contingency to the *group*
    a. Providing contingent mathematical content support at the maximum group level to the whole group when the group is stuck;
    b. Providing process support when the group has not yet reached its maximum level of understanding;
2) Phasing out mathematical content support when the *group* is able to continue on its own, i.e., providing contingent mathematical content support until a *group* is able to continue the discussion by itself; and
3) Returning responsibility for learning to the *group* by providing process support at the moment when a group is able to continue the discussion by itself.

The SGS-Tool that we developed previously was based on these characteristics and helped teachers scaffold small groups at the group level in the context of mathematical discussions (Calor et al., 2022). The SGS-Tool helps teachers decide which type of support to give and suggests concrete teacher moves and questions teachers can ask. The first step involved in using the tool is to determine whether or not the maximum level of understanding has been reached by the group. This task can be accomplished by determining whether the question that was asked is a question that is relevant to the whole group or a question that was asked by one or two individual students within the group, whereas others within the group might know the answer. When the question is not a question from all students (i.e., the maximum level of understanding of the group has not yet been reached), the group is able to advance on its own. In that case, the teacher is recommended to provide process support (step 6 of the tool) and then end his or her interaction with the group. When the question is a question from the whole group, the whole group has encountered an obstacle (i.e., the maximum level of understanding of the group has been reached), the group is not able to advance independently. In that case the teacher is recommended to provide mathematical content support to further the discussion. The next step is for the teacher to diagnose what the group's maximum level of understanding actually is (step 2 of the tool), followed by checking that this diagnosis is correct (step 3 of the tool) and offering mathematical content support until the group understands the issue at hand and can further the discussion on its own (step 4 of the tool). That support should be contingent, that is adapted to the maximum mathematical level that the group has achieved together, and should be continued until the group can continue the mathematical discussion independently. In other words, teachers provide content support in the group's zone of proximal development (ZPD) (Nyikos & Hashimoto, 1997). Teachers diagnose the maximum level of understanding by asking the group to share the mathematical ideas they have discussed thus far. Teachers decide when it is time to phase out content support (step 5 of the tool) by determining whether at least one student understood the content support. When at least one student understood the content support, teachers are recommended to return responsibility for learning to the group by offering process support (step 6 of the tool). Thus, by applying the SGS-Tool, teachers enhance the amount and quality of mathematical discussions in student groups to improve mathematical understanding. The SGS-Tool is distinctive in that it focuses on the support provided at the group level to ensure that mathematical discussions can progress. A brief description of the SGS-Tool is shown in Figure 4.1. For a more detailed description of the SGS-Tool, see Calor et al. (2022).

Figure 4.1

Small-Group Scaffolding Tool for Mathematical Discussions



In a previous quasi-experimental study of four teachers and their classes (Calor et al., 2022), we found that teachers were able to apply the various steps of the SGS-Tool. They were able to adapt their support to the needs of a small group by determining whether the group could continue on its own when it seemed stuck or a question was asked. The SGS teachers engaged in the following small-group scaffolding behavior more frequently than the control teachers: 1) determining whether the student group's maximum level of understanding had been reached, 2) diagnosing the student group's maximum level of understanding and providing process support when the group's maximum level of understanding had not been reached, 3) providing mathematical content support when the group's maximum level of understanding had been reached but failing to verify the diagnosis of the maximum level of understanding, 4) determining whether the group could continue the discussion independently, 5) returning responsibility for learning to the group and phasing out content support once the group was able to continue the discussion independently.

We also found that students who were guided with the SGS-Tool participated more in teacher–small-group interactions.

4.3 RESEARCH QUESTION AND HYPOTHESES

There is a need for tools that help teachers guide students during mathematical group work to improve the quality of mathematical discussions (Webb et al., 2019). The aim of the current study was to determine whether guiding seventh-grade student groups with the SGS-Tool in early algebra classes fosters mathematical discussions. Therefore, we conducted a quasi-experimental study in which we compared the mathematical discussions of student groups that were guided by the SGS-Tool to the discussions of student groups that were not.

Our research question is as follows: How does guiding seventh-grade student groups with the SGS-Tool affect the amount and quality of students' mathematical discussions?

We hypothesize that in the SGS condition, compared to the control condition,

1. More mathematical discussion occurs, and
2. More elaborating key and regulating activities occur than sharing key and regulating activities.

We answer our research question based on quantitative analysis of the interactions of teachers with students collaborating in small groups and illustrate these results with two contrasting examples excerpts from teacher–small-group interactions: one of a teacher who applied the SGS-Tool and the other of a teacher who did not.

4.4 METHOD

A quasi-experimental study was conducted at two schools in Amsterdam, The Netherlands. In the SGS condition, teachers were trained to apply the SGS-Tool, and in the control condition, teachers were asked to support small groups as they would normally do.

4.4.1 Participants

The participants were eight teachers (7 males and 1 female) and 272 seventh grade students (11 classes) approximately 12–13 years of age from two schools. All teachers volunteered to participate in the study and were not rewarded for participating. For the first school, six teachers were paired based on comparable age and teaching experience. For each pair, one teacher was randomly assigned to the SGS condition, while the other was assigned to the control condition. For the second school, the two teachers were both assigned to the SGS condition. The SGS condition consisted of five teachers with a mean experience of 7.6 years ($M = 7.6$, $SD = 3.3$), seven classes (four from school 1 and three from school 2), 46 groups (28 from school 1 and 18 from school 2), and 171 students (104 from school 1 and 67 from school 2). The control condition consisted of three teachers with a mean experience of 7.7 years ($M = 7.7$, $SD = 3.2$), four classes, 28 groups, and 101 students (all teachers and students were from school 1). The purpose of the study was known only to the teachers in the SGS condition. All students worked in small, heterogeneous groups of mixed ability that were formed by the class mentor. The groups consisted of four students with an occasional group of three due to class size restrictions. In both schools, collaborative learning was part of the educational concept of the school. Students were used to working collaboratively in small groups in all subjects.

4.4.2 Lessons

Students in both conditions worked on an early algebra lesson unit of 12 one-hour lessons over five weeks. Five of these lessons contained tasks that were explicitly designed to invoke mathematical discussions in a previous study (Calor et al., 2020). The tasks were adaptations of the regular assignments of the regular textbook. The lessons were developed according to the design principles of Palha et al. (2013). These design principles are based on Freudenthal's theory, which postulates that reflection on mathematical activities leads to a deeper understanding of mathematics (Freudenthal, 1991). Reflection, in turn, is induced by mathematical discussions (Dekker & Elshout-Mohr, 2004; Palha et al., 2013). In addition, mathematics started at a level that was experientially real to students. The learning goal is to gain knowledge of early algebra, in particular formulae. According to Janvier (1987), learning early algebra occurs best by switching back and forth between algebra representations, situations, graphs, tables, and formulae, one of the main elements of learning early algebra. Janvier called the ability to switch from one representation to another a translations skill. The tasks in the lessons contained all possible switches between the algebra representations. In the same study (Calor et al., 2020), we found that the five adapted lessons evoked more mathematical utterances than the lessons with regular tasks and that the quality of the mathematical utterances in the adapted lesson condition was better than that in the regular task condition. However, more sharing activities occurred in the adapted lesson condition, and far fewer elaborating activities occurred. Appendix C contains a task of the fifth lesson meant to evoke a mathematical discussion (lesson 12 of the lesson unit).

Each of the five lessons started with a ten-minute introduction by the teacher, followed by a 50-minute working phase in which the students worked collaboratively on the assignment. In the other lessons, students worked on regular assignments from the textbook and sat together in the same small, heterogeneous groups.

4.4.3 SGS condition

The teachers in the SGS condition were trained to apply the SGS-Tool by the first author in a three-hour training session. The SGS teachers were shown several videotapes of groups that needed support during mathematical discussions. They were asked to think of how they would give support according to the SGS-Tool and share their answers with each other. When the teachers asked questions, they were scaffolded according to the SGS-Tool so that the first author could model the desired scaffolding behavior. In addition, the SGS teachers were coached by the first author (who attended all the lessons) before and after the lessons to enable them to make connections between their learning and classroom practices (Borko et al., 2010) and to reflect on how they used the SGS-Tool in their lessons. The importance of not revealing the nature of the study to the control teachers was explained to the SGS teachers by the first author every time they met. The teachers gave assurance that they had not revealed it to the control teachers. The teachers kept an A3-sized poster with a schematic representation of the SGS-Tool on their desk during the lessons so that they could consult it to determine which step to take. As part of the coaching, after every lesson, the teachers were asked whether and how they had been able to use the SGS-Tool and whether they had experienced difficulties. In addition, the teachers' questions were answered.

4.4.4 Implementation check

The first author or a research assistant observed every lesson to check whether the lesson plans were carried out as intended and whether teachers used the SGS-Tool when guiding the small groups. In addition, teachers were interviewed about the use of the tool after each lesson. The first author or a research assistant also later reviewed a videotape of each lesson.

4.4.5 Data collection and analysis

We used a coding scheme to code the teachers' turns according to the steps of the SGS-Tool that we developed in a previous study (Calor et al., 2022). The unit of analysis was a teacher's turn in the context of an interaction fragment. The interrater reliability between two coders (first and second author) regarding teacher turns was determined (based on approximately 10% of the data) in a previous study (Calor et al., 2022). A high level of interrater reliability agreement was observed (91%), and based on the Fleiss kappa benchmark (El Emam, 1999), a good Cohen's kappa value was also obtained (.82).

To illustrate how applying the SGS-Tool might have influenced the quality of the mathematical discussions, we selected contrasting examples in which teachers reacted to a similar event and coded the students' mathematical discussions and teachers' scaffolding behavior. This provided insight into how teachers' support may or may not contribute to the quality of the mathematical discussions.

In addition, to analyze the quality of the mathematical discussions and test the hypotheses, we randomly chose one group from each class and videotaped those groups during the fifth mathematical discussion lesson. This was the last lesson of the lesson unit, and we assumed that the teachers would be more familiar with the SGS-Tool by then. Prior to this data collection, every group had been videotaped during one lesson, so the students were used to the camera. We report on the results regarding 10 groups since the quality of the audio of one of the recordings was poor. In total, we collected approximately 10 hours of video observation of student interactions for the purposes of analysis. A total of 3434 student utterances were obtained in the SGS condition, of which 825 utterances pertained to mathematical content (mathematical discussions), and a total of 2679 student utterances were obtained in the control condition, of which 361 utterances pertained to mathematical content (mathematical discussions). Group interaction processes were transcribed and analyzed using the software program Multiple Episode Protocol Analysis (Erkens, 2002). Utterances were coded according to a coding scheme by the Calor et al. (2020); see Table 4.2. Following Van Boxtel et al. (2000), an utterance was defined as "an individual message unit that is distinguished from another utterance through a 'perceptible' pause, comma or full stop." We selected utterances that involved students discussing task content with each other while working among themselves and coded these utterances as mathematical discussions. These utterances were divided further into seven subcategories according to the regulating and key activities of the PM (Dekker & Elshout-Mohr, 2004) (see Table 4.2).

Table 4.2

Coding Scheme: Seven Elements of a Mathematical Discussion

Regulating activities	Example
Ask to tell/show work	"What do you have?"
Ask to explain work	"How did you do that?"
Critique work	"That is in the wrong order."
Key activities	**Example**
Tell/show work	"Look what I have done."
Explain work	"I have 6, because you have to double it."
Justify work	"I am right, because 5 times 5 = 25."
Reconstruct work	"We'd better do it like this, times 5 plus 20."

The interrater reliability for the coding of the seven elements of a mathematical discussion was calculated between two coders, namely, the first author and a research assistant. The interrater reliability was satisfactory, with 84% agreement (good) and Cohen's kappa .67 (satisfactory). The first author coded all of the data in this study.

To test the hypotheses, we used relative frequency cross-table analyses. To test whether more mathematical discussion occurred in the SGS condition (hypothesis 1), we determined whether the relative frequencies of utterances in mathematical discussions and other utterances differed between the conditions (see Table D1 in Appendix D). To test whether more elaborating key and regulating activities occurred compared to sharing activities in the SGS condition (hypothesis 2), we determined whether the relative frequencies of utterances of sharing key and regulating activities and utterances of elaborating key and sharing activities differed between the conditions. With regard to sharing key and regulating activities, we examined tell/show and ask to tell/show work; see Table D2 and Table D3 in Appendix D. With respect to elaborating key and regulating activities, we examined explain work, justify work, reconstruct work, ask to explain work and critique work; see table D4 in Appendix D. We calculated the two-sided p values for Fisher's exact test (Agresti, 1992).

After each lesson, the five SGS teachers were interviewed for approximately 15 minutes each. After the intervention, they were interviewed for approximately 30 minutes each. The interviews (30 in total) were conducted by the first author or a research assistant.

4.5 RESULTS

First, we present the results of the implementation check that was carried out by the first author or a research assistant. Second, we illustrate differences in the teachers' moves in the teacher–small-group interactions between the two conditions. Subsequently, we illustrate the use of the SGS-Tool with two contrasting examples of teacher–small-group interaction.

Then, we describe the types of utterances that occurred in the (mathematical) discussions, followed by an analysis of whether more mathematical discussions occurred in the SGS condition. Finally, we analyze how guiding seventh grade students with the SGS-Tool affected the quality of the mathematical discussions.

4.5.1 Implementation check

In both conditions, the teachers implemented the lesson plans as intended. Students in both conditions worked collaboratively on the assignments and were able to finish the assignments on time. In some cases, the students finished a few minutes early and spent the extra time on pursuits other than mathematics. The review of a videotape of each lesson confirmed the findings mentioned above.

The observations showed that teachers in the experimental condition scaffolded the small groups as intended and that teachers consulted the A3-sized poster with a schematic representation of the SGS-Tool several times during a lesson. After each lesson, the teachers in the SGS condition indicated that they had indeed used the SGS-Tool when giving support to the small groups. One of the teachers said that she enjoyed seeing students discuss things in a different way because of how she scaffolded the groups. She also mentioned that because students knew that she gave less direction, discussion in some groups increased, and students tried to explain to each other. Finally, she and another teacher indicated that scaffolding the groups with the use of the SGS-Tool encouraged students to think more about the subject and that the SGS-Tool helped teachers allow students to figure out problems for themselves by providing them with small steps to ensure that they did not blindly copy what teachers said.

4.5.2 Steps of the SGS-Tool implemented by teachers

To illustrate differences in the teachers' moves in the teacher–small-group interactions between the two conditions, we present the frequencies of the steps taken by teachers in both conditions in Table 4.3.

Table 4.3

Frequencies (%) of Steps Taken by Teachers in Both Groups

	SGS condition	Control condition
Step 1	16 (13.79)	0 (0)
Step 2	45 (38.79)	3 (9.38)
Step 3	0 (0)	0 (0)
Step 4	38 (32.76)	24 (75.00)
Step 5	10 (8.62)	4 (12.50)
Step 6	7 (6.03)	1 (3.13)
Total	116 (100)	32 (100)

Table 4.3 shows that SGS teachers offered relatively more support than control teachers. SGS teachers also implemented various steps of the SGS-Tool, whereas control teachers mostly provided content support.

4.5.3 Contrasting examples excerpted from teacher–Small-group interactions

To illustrate what teacher–small-group interaction looked like, we discuss two contrasting examples of a similar situation. We selected a similar situation to which teachers in the SGS condition and control condition responded in different ways. In this situation, the teachers discover an error in the story or formula created by a small group of students. First, we present a typical teacher–small-group interaction in the control condition. Then, in contrast, we show a typical example from the SGS condition. We added coding of the students' mathematical discussions and teachers' scaffolding behavior to the excerpts.

4.5.3.1 Contrasting example 1, control condition

The following coded excerpt serves as a typical example of teacher support in the control condition, with the teacher providing support and giving the solution to individual students in the group and not engaging the other students in the group. The teacher's support involves only step 2, diagnosing the group's maximum level of understanding, and step 4, providing mathematical contingent content support (in a noncontingent way; i.e., the teacher does not build on the level of understanding of the students and immediately gives the solution to the students) of the SGS-Tool. The four students in the small group worked together on the first part of the assignment shown in Appendix C, in which they were asked to create a story that corresponds to a given graph. After working together for a few minutes, two of the students, Student 1 and Student 4, stopped contributing to the story and started talking about social issues that did not involve mathematics. Student 2 and Student 3 worked further on their own. They created a story about visiting a resort called Walibi for which a subscription and entry fee had to be paid. After finishing the first part of the assignment in Appendix C, they continued with the second part of the assignment, in which the students were asked to create a formula that corresponded with the given graph and the story they created in the first assignment. Student 2 and Student 3 created a formula but did not know how to write the formula down. They were not sure whether to write "number of visitors *times* 5" or "number of visitors x 5." Student 3 asked the teacher for help. The question was a straightforward question. Meanwhile, Student 1 and Student 4 were still talking about social matters and continued to do so even during the support the teacher provided to Student 3.

1	Student 3	(Raises her hand) Teacher, I don't know.	
2	Student 2	Doesn't matter, I do times 5, … plus [while writing on the A3 poster].	
3	Student 3	Ah, there he [the teacher] is.	
4	Teacher	[The teacher reads what Student 2 is writing on the A3 poster (and discovered an error in the story the group wrote down).]	Step 2
5	Teacher	The number of people is 20. [The teacher points to the starting point of the graph in the exercise on the table of Student 3.] Subscription, 20 euros.	Step 4
6	Student 2	[Student 2 corrects what she wrote down by writing the subscription is 20 euros.]	
7	Teacher	[The teacher walks away after answering Student 3]	

Before addressing the question of Student 3, the teacher read the A3 poster on which Student 2 was writing, i.e., applied step 2 of the SGS-Tool (diagnosing the maximum level of understanding of the group), and discovered a mathematical error. He immediately provided the correct answer to Student 2 in line 4, which is step 4 of the SGS-Tool (give mathematical content support in a noncontingent way); i.e., he did not build on the mathematical ideas of the whole group. Student 2 wrote down the correct answer without discussing it with the other students in the group. The teacher did not involve the other students in the conversation with Student 2. Instead, he started to answer the question of Student 3, who had called him to the table, missing the opportunity to start a discussion in the group about the mistake that had been made. Students misinterpreted starting point 20 of the graph when translating from an algebra representation graph to a representation situation. A discussion about why the translation of "the starting point 20" to "the number of people is 20" was not correct could have improved the students' algebra translating skills, which in turn could have enhanced their early algebra knowledge. Instead, the teacher provided the students with the correct answer (the translation of "starting point 20" to "subscription 20"). In addition, the teacher did not apply step 1 (determining whether the group's maximum level of understanding has been reached), step 3 (checking the diagnosis), step 5 (checking whether the group can continue on its own) or step 6 (providing process support) of the SGS-Tool. After answering the question of Student 3, the teacher walked away. After the teacher left, Student 2 and Student 3 started talking about social matters that did not involve mathematics. After a few minutes, they proceeded to work on the next question.

4.5.3.2 Contrasting example 2, SGS condition

The following coded excerpt serves as a typical example of teacher support in the SGS condition in which the teacher's support enhances the quality of the mathematical discussion. In this excerpt, the teacher applies step 1, step 2, step 4, step 5, and step 6 of the SGS-Tool. The four students were working on the same assignment, shown in Appendix C, as the students in the previous excerpt in the control condition. The students had created a story for part 1 of the assignment and finished part 2, in which they had to create a formula. The group agreed what the formula had to be, but it contained an error. Students made an error when switching from the algebra representation graph to the representation formula.

The formula did not contain the starting point 20 of the graph. The students were ready to move on to the next question when Student 1 critiqued the amount the group chose for the fee entrance in their story for visiting a waterpark called Tikibad. Student 1 found the fee of 5 euros per person too expensive. The teacher approached the group, listened for a while to the conversations of the students, and read what Student 2 had written on the A3 poster. The teacher discovered an error, and the formula did not correspond to the given graph since the starting point 20 of the graph was missing in the formula. She interrupted the ongoing mathematical discussion and asked Student 3 (pointing at the A3 poster) if what Student 2 had written down corresponded with the given graph.

1	Teacher	How is it going?	
2	Student 2	Alright.	
8	Teacher	[The teacher reads from the poster.]	Step 2
9	Teacher	I have a question.	
10	Student 3	Yes?	
11	Teacher	She wrote that the number of people times 5 euros is the amount [pointing to the A3 poster on which Student 2 was writing]. Does that really correspond, this formula and this graph [pointing to the graph on the assignment]?	Step 1
12	Student 3	Every time, it increases by 5.	Tell/show work
13	Teacher	Yes, but look if …	
14	Teacher	Student 1 [the teacher called Student 1's name to draw his attention back to what she and Student 4 were discussing. Student 1 was distracted by falling snow outside, which he could see through the classroom window. Thereafter, Student 1 returned his attention to what was being said].	
15	Teacher	Think of what else; it's not completely correct [addressing the whole group].	Step 4
16	Teacher	[The teacher started to walk away slowly but stopped after one step to listen to what the students were saying.]	Step 5
17	Student 2	Oh, it's wrong. Because if you do 2 times 5, then it is not, it is not 30 euros.	Explain work
18	Teacher	It's almost correct. One thing is still lacking.	Step 4
19	Teacher	Discuss among yourselves; what needs to be added? [Teacher walks away.]	Step 6

When the teacher in this excerpt reached the group, she asked how the work was going and listened for a while to the conversation among the four students. Student 2 claimed that the work was going alright. The teacher proceeded to read the answers of the group on the A3 poster, thus performing step 2 of the SGS-Tool: diagnose the maximum level of understanding of the group. When reading the A3 poster, the teacher discovered that the students made a mistake when switching from the algebra representation graph to the representation formula: the 20 of the starting point of the given graph (with coordinates (0, 20) had not been used in the formula. She asked whether Student 3 agreed with what Student

2 had written down (step 1 of the SGS-Tool: determining whether the group's maximum level of understanding had been reached). In the midst of the conversation between the teacher and Student 4, Student 1 was distracted by falling snow outside, which he could see through the classroom window. The teacher successfully reengaged Student 1 in the conversation that she was having with the group. She indicated to the group that the solution they had developed was not completely correct and that they should consider what could be missing (step 4 of the SGS-Tool). Student 2 discovered the mistake the group had made and explained it to the other members in the group (key activity Explain work). The teacher was slowly walking away but stopped to listen to the conversation among the students. When she heard that Student 2 was explaining the mistake to the other group members, she walked away, confident that Student 2 understood what had gone wrong (step 5 of the SGS-Tool: checking whether the group can continue on its own) and that mathematical discussions about how to fix the mistake, i.e., how to make the right translation from situation graph to situation formula, were continuing (return responsibility for learning to the group). After the teacher left, the mathematical discussion continued in the group. The students discussed what the formula should be. After a while they made the connection between the representation graph and representation formula by correctly translating "starting point 20" to "+ 20" (which was missing in their previous solution) and thus created the correct formula.

20	Student 1	The formula.	Tell/show work
21	Student 3	Let me see.	
22	Student 2	Times, number of people times 5 is the amount [shows the poster to the other students].	
23	Student 3	It's from 20 till 50 …	Tell/show work
24	Student 3	It's 30. Thirty divided by 6 equals 5.	Explain work
25	Student 2	Wait a minute.	Critique work
26	Student 3	Because here is a whole section with nothing in it [points up and down from 0 to 20 on the y-axis].	Explain work
27	Student 2	Yes.	Tell/show work
28	Student 2	However, 0 persons is at 20…, 1 person is 25 euros, so in between …	Explain work
29	Student 3	[Nods her head.]	
30	Student 2	1 times, 1 times 5 plus 20, because it is in any case already 20.	Explain work
31	Student 3	Yes! Times 5.	Tell/show work
32	Student 2	Number of people times 5 plus 20 [writes the correct formula on the poster].	Reconstructs work

During this mathematical discussion after receiving support, the students used the key activity Tell/show work four times (lines 20, 23, 27 and 31), the key activity Explain work three times (lines 24, 26, 28 and 30) and the regulating activity Critique work once (line 25).

4.5.3.3 Comparison of the examples

Both teachers were confronted with the same situation: the students were not able to correctly incorporate starting point 20 of the graph. The teacher in the control condition was called to the group to answer a formal question, but when he arrived at the group, he discovered an error in the story (translation error from algebra representation graph to representation situation). He immediately proceeded with providing mathematical content support (step 4) in a noncontingent way by giving the correct answer to the student who was writing on the poster. He also did not involve other students in the group in the interaction. Consequently, the opportunity to start a discussion about the translation of "starting point 20" in the group was missed, and no discussion among the students followed. We previously investigated the scaffolding behavior exhibited by teachers who worked with the SGS-Tool and compared it with the behavior of teachers who did not work with the SGS-Tool (Calor et al., 2022). We found that the teachers who did not work with the SGS-Tool showed similar scaffolding behavior, i.e., leading students to the answer, similar to the teacher in the control condition. The teacher in the SGS condition first diagnosed what the students had done thus far by reading the poster (step 2) and diagnosed that the mistake in the formula on the poster was a mistake of the whole group (step 1) and that the group had reached its maximum level of understanding and, thus, would not be able to create a correct formula on their own. Because she took these steps, she was able to adapt her mathematical content support to the needs of the group, i.e., provide contingent mathematical support (step 4) by pointing out that they made a translation mistake in the formula. When she was confident that one student understood the support (step 5: the discussion is able to continue on its own), she returned responsibility for learning to the group by encouraging the students to discuss what was wrong with the answer on the A3 poster, i.e., provided process support (step 6). Her support elicited a mathematical discussion in the group. Finally, the teacher listened to the discussion until she was confident that the mathematical discussion about how to fix the mistake was continuing and only then left the group. After a while students were able to create the correct formula by reflecting on their mathematical activities. They made the correct translation from the situation graph to the situation formula, which in turn enhances the learning of early algebra.

Ultimately, the support of the teacher in the control condition did not elicit further discussion about the translation of the starting point of the graph and thus did not enhance the quality of the mathematical discussion when he discovered an error on the poster, whereas the support of the teacher in the SGS condition elicited a discussion about the translation of the starting point of the graph and thus enhanced the quality of the mathematical discussion when she discovered an error on the poster.

4.5.4 Number of utterances in mathematical discussions

Table 4.4 shows the total number of utterances of the group interactions per condition, the absolute number of mathematical discussion utterances and the relative frequencies of regulating and key activities. The total number of utterances per group ranged from 167 to 954 in the SGS condition (M = 604.5, SD = 302.7) and from 349 to 1333 in the control condition (M = 734.0, SD = 421.0).

Table 4.4

Total Number of Utterances in Small-Group Interaction per Condition and Absolute Number of Mathematical Discussion Utterances and Absolute and Relative Frequencies of Regulating and Key Activities in Both Conditions

	SGS condition	%	Control condition	%
Total utterances	**3627**		**2936**	
Mathematical discussion utterances	824	100	361	100
Regulating activities	208	25.24	71	19.67
Ask to show work	82	9.95	46	12.74
Ask to explain work	13	1.58	0	0.00
Critique work	113	13.71	25	6.93
Key activities	616	74.76	290	80.33
Tell/show work	494	59.95	275	76.18
Explain work	46	5.58	6	1.66
Justify work	44	5.34	6	1.66
Reconstruct work	32	3.88	3	0.83

In both conditions, more key activities than regulating activities occurred. Additionally, many Ask to tell/show work and Tell/show work sharing activities occurred in both conditions. Finally, the relative frequency of the elaborating regulating activity Critique work was remarkably high in the SGS condition.

4.5.4.1 Hypothesis 1: More mathematical discussions occur in the SGS condition than in the control condition.

In Table 4.5, we show the total number of utterances made during the group interactions and the absolute number and relative frequencies of utterances made during mathematical discussions in both conditions.

Table 4.5

Total Number of Utterances in Small-Group Interactions and Absolute Number and Relative Frequencies of Utterances in Mathematical Discussions in Both Conditions

	SGS condition	%	Control condition	%
Total utterances	3627	100	2936	100
Utterances in mathematical discussions	824	22.7	361	12.3

Using a cross-table (see Table D1 in Appendix D), we determined whether the relative frequencies of utterances in mathematical discussions (361 in the control and 824 in the SGS condition) and other utterances (2575 in the control and 2803 in the SGS condition) differed between the conditions. Fisher's exact test (Agresti, 1992) showed that the relative frequency of utterances of mathematical discussions was significantly higher for the SGS condition ($p < .001$). Therefore, hypothesis 1 is confirmed: more mathematical discussions occur in the SGS condition.

4.5.5 Quality of mathematical discussions

4.5.5.1 Hypothesis 2: More elaborating key and regulating activities than sharing key and regulating activities occur in the SGS condition.

To determine whether the elaborating key activities Explain work, Justify work and Reconstruct work occurred more in the SGS condition than in the control condition, we present the absolute number and relative frequencies of utterances related to key activities in both conditions in Table 4.6.

Table 4.6

Absolute Number and Relative Frequencies of Key Activities in Both Conditions

	SGS condition	%	Control condition	%
Key activities	616	100	290	100
Tell/show work	494	80.19	275	94.83
Explain work	46	7.47	6	2.07
Justify work	44	7.14	6	2.07
Reconstruct work	32	5.19	3	1.03

Table 4.6 shows that 494 Tell/show work sharing key activities occurred in the SGS condition and 275 in the control condition, and 122 Explain work, Justify work and Reconstruct work elaborating key activities occurred in the SGS condition and 15 in the control condition. The cross-table shown in Table D2 in Appendix D demonstrates that the relative frequencies of utterances of the sharing key activity Tell/show work and utterances elaborating key activities (the sum of frequencies of utterances of the key activities Explain work, Justify work, and Reconstruct work) differ between the conditions. Fisher's exact test shows that the relative frequency of utterances of the sharing key activity Tell/show work is significantly higher for the control condition ($p < .001$). In addition, Table 4.6 shows that the elaborating key activities Explain work, Justify work and Reconstruct work occurred slightly more often in the SGS condition than in the control condition.

We show the absolute number and relative frequencies of the utterances of regulating activities made in both conditions in Table 4.7.

Table 4.7

Absolute Number and Relative Frequencies of Utterances of Regulating Activities in Both Conditions

	SGS condition	%	Control condition	%
Regulating Activities	208	100	71	100
Ask to show work	82	39.42	46	64.79
Ask to explain work	13	6.25	0	0.00
Critique work	113	54.33	25	35.21

Table 4.7 shows that 82 Ask to tell/show work sharing regulating activities occurred in the SGS condition and 46 in the control condition, and 126 Ask to explain work and Critique work elaborating regulating activities occurred in the SGS condition and 25 in the control condition. The cross-table shown in Table D3 in Appendix D demonstrates that the relative frequencies of utterances of the sharing regulating activity Ask to tell/show work and elaborating regulating activities (the sum of frequencies of the regulating activities Ask to explain work and Critique work) differ between the conditions. Fisher's exact test shows that the relative frequency of utterances of the sharing regulating activity Ask to tell/show work is significantly higher for the control condition ($p < .001$). In addition, Table 4.7 shows that the elaborating regulating activities Ask to explain work and Critique work occurred more in the SGS condition than in the control condition.

In Table 4.8, we show the absolute number and relative frequencies of the utterances of sharing and elaborating activities made in both conditions.

Table 4.8

Absolute Number and Relative Frequencies of Utterances of Sharing and Elaborating Activities in Both Conditions

	SGS condition	%	Control condition	%
Mathematical discussion utterances	824	100	361	100
Sharing activities	576	69.90	321	88.92
Elaborating activities	248	30.10	40	11.08

Note. Elaborating activities = sum of frequencies of key activities explain work, justify work, reconstruct work, and regulating activities ask to explain work and critique work. Sharing activities = sum of frequencies of key activity tell/show work and regulating activity ask to tell/show work.

Table 4.8 shows that 576 sharing key and regulating activities occurred in the SGS condition and 321 in the control condition. In addition, Table 4.8 shows that 248 elaborating key and regulating activities occurred in the SGS condition and 40 in the control condition. The cross table in Table D4 in Appendix D demonstrates that the relative frequencies of utterances of the elaborating key and regulating activities (the sum of frequencies of the key activities Explain work, Justify work, and Reconstruct work, and regulating activities Ask to explain work and Critique work) differ between the conditions. Fisher's exact test shows that the relative frequency of utterances of the elaborating key and regulating activities is significantly higher for the SGS condition ($p < .001$).

Therefore, hypothesis 2 is confirmed. More elaborating key and regulating activities than sharing key and regulating activities occur in the SGS condition.

4.6 CONCLUSIONS AND DISCUSSION

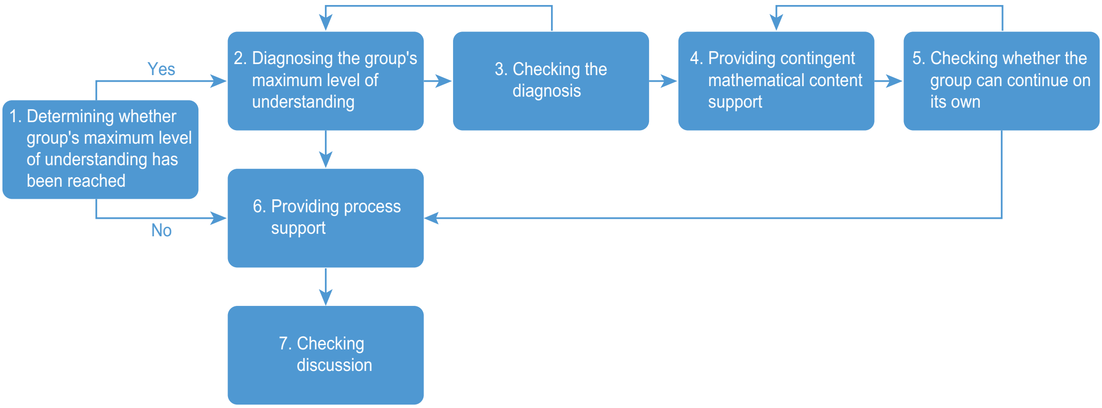
Mathematical discussions are considered an important means for the learning of mathematics. There is a need for tools that guide teachers in supporting small groups in order to improve the quality of the mathematical discussions. In this quasi-experimental study, we investigated the effect of guiding seventh grade students with the SGS-Tool, which aims to help teachers scaffold small groups at the group level during mathematical discussion by describing the steps a teacher can take (Calor et al., 2022). We investigated the effects of teachers' use of the SGS-Tool on the number and quality of mathematical discussions in ten seventh-grade classes. We considered the occurrence of key and regulating activities (in particular the elaborating key and regulating activities Explain work, Justify work, Reconstruct work, Ask to explain work, and Critique work) to be an indicator of a high-quality mathematical discussion. We hypothesized that in the SGS condition compared to the control condition, 1) more mathematical discussion would occur, and 2) more elaborating key and regulating activities than sharing key and regulating activities would occur. Both hypotheses were confirmed. Significantly more mathematical discussion utterances occurred in the SGS condition. Moreover, in the SGS condition, more elaborating key activities and elaborating regulating activities occurred. These outcomes suggest, with regard to our research question, that guiding seventh grade students with the SGS-Tool can elicit qualitatively better mathematical discussions.

Teachers in the SGS condition implemented more steps of the SGS-Tool than teachers in the control condition. The former also implemented various steps of the SGS-Tool, whereas the control teachers predominantly provided content support (step 4 of the tool). Two contrasting example excerpts illustrated our findings. The teacher in the SGS condition stimulated the students in the group to perform sharing and elaborating activities, while the teacher in the control condition provided content support to only one student in the group and guided this student to the correct answer and did not involve other students in the interaction. By doing so, he missed the opportunity to let the group discuss the algebra translation error they made. We found similar scaffolding behavior in teachers who did not work with the SGS-Tool in a previous study in which we compared the scaffolding behavior of teachers working with the SGS-Tool and teachers who did not work with the SGS-Tool. The teacher's support in the SGS condition elicited more and qualitatively good mathematical discussions in which students made the connection between the algebra representation graph and representation formula, thus enhancing their early algebra knowledge. In the control condition, no mathematical discussion followed after teacher support.

The excerpt in the SGS condition also illustrated that before the teacher walked away from the group, she stopped listening to the discussion to ensure that it continued. Although we did not analyze this in detail, we noticed similar helping behavior by other teachers in the SGS condition but not in the control condition. Van de Pol et al. (2019) found that the uptake of support applied correctly during scaffolding of individual students in a group work setting was related to providing correct answers during student-student interaction. They also found that teachers were likely to fade the support too soon, returning responsibility for learning to the students when they did not know how to proceed. Consequently, listening to whether the discussion continues after providing support is an important step in which the teacher can verify whether the group correctly takes up the support and thus whether responsibility for learning has been returned to the group at the appropriate time. Consequently, we suggest adding an extra step, "Checking discussion," to the SGS-Tool, i.e., listening to determine whether the group's uptake of support, applying support correctly, is sufficient and thus to determine whether responsibility for learning has been returned to the group at the appropriate time. We added this seventh step, "Checking discussion," after step 6, "Providing process support," to the SGS-Tool; see Figure 4.2.

Figure 4.2

Adapted Small-Group Scaffolding Tool for Mathematical Discussions with Seventh Step 'Checking discussion'



This study was conducted at two schools, which limits the generalizability of our findings. Further research could be expanded to more schools as well as to other grades and other mathematical subjects to acquire more robust findings. An additional limitation is that we investigated only one group in each class during the fifth lesson. Selection of other groups or all groups for analysis might have yielded different findings. It might thus be interesting to analyze the discussions of other groups and other lessons. However, selecting a limited number of small groups enabled us to perform detailed analyses.

Our coding scheme enabled us to analyze in detail students' interactions during group work and, most importantly, the quality of the mathematical discussions. Nevertheless, it is difficult to capture all processes that occur. For example, the elaborating key activity Reconstruct work can also occur in students' minds, where it cannot be observed. In addition, the coding scheme does not take into account whether the content of what a student has said is correct.

Although we matched teachers into pairs based on age and experience, followed by randomly assigning one member of each pair to a condition, there still might be differences between the teachers of the control condition and the teachers of the SGS condition. Nevertheless, we think that we took enough measures to avoid any influence of these differences by carefully matching the teachers into pairs.

We have indications that working with the SGS-Tool leads to qualitatively better mathematical discussions. However, we need to be careful when drawing these conclusions since other factors such as sociomathematical norms in small groups, e.g., expecting to listen to others' mathematical thinking (Sjöblom & Meaney, 2021), and language proficiency (Erath, 2021) may also play a role in the quality of discussions. Also, more research is needed to examine the relation between teachers' scaffolding behavior and the discussions in further detail. An interesting question in that respect is whether some scaffolding steps are more strongly related to some aspects of mathematical discussion than others. Another question that requires further research is whether scaffolding according to the SGS-Tool actually leads to an increase in the mathematical level.

Webb et al. (2019) stressed the need for the development of tools that support teachers in improving the quality of mathematical discussions. As our study suggests that guiding students using the SGS-Tool contributes to qualitatively better mathematical discussions, the SGS-Tool can be considered to meet this need. It supports teachers in deciding which type of support to provide to stimulate mathematical discussions and to improve the quality of such discussions. The SGS-Tool can also be used by teachers to stimulate discussions in their classrooms. Additionally, the SGS-Tool can be used in teachers' institutes to train in-service teachers to scaffold small groups and thus invoke qualitatively better mathematical discussions.