Scaffolding small groups' mathematics learning

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Conclusions and discussion

The Small-Group Scaffolding Tool helped me get students to solve problems on their own by giving them small steps so they don't blindly copy what I say.
Participating teacher

6.1 INTRODUCTION

Early algebra is challenging for students in secondary education (Mullis et al., 2016; OECD, 2016, 2019). Students find it difficult, for example, to understand how patterns and formulae are related (Sfard & Linchevski, 1994). One way to help students is to have them collaborate with other students. Collaborative learning is known to improve students' mathematics learning (Dekker & Elshout-Mohr, 1998, 2004; Pijls et al., 2007b; Webb, 2009, Yackel et al., 1991). Freudenthal (1991) argued that mathematical discussions, i.e., discussions in which students reflect on mathematical activities, are important for learning mathematics. Often, however, small groups arrive at a point where they need assistance from a teacher to further their discussions. Teachers themselves find it challenging to support small groups during collaborative learning (Van Leeuwen et al., 2015; Webb, 2009), as providing such support is complex (Van Leeuwen & Janssen, 2019). The general aim of this study is therefore to gain insight into how teachers can guide small groups during collaborative learning to enhance the quality of mathematical discussions and students' mathematical understanding.

This research is underpinned by the concept of scaffolding (Van de Pol et al., 2010) and by the conceptual distinction between process and content support (Dekker & Elshout-Mohr, 2004). Teachers often tend to provide content support. Dekker and Elshout-Mohr (2004) found that when a teacher provided elaborate content support to a student who asked a question, other students in the group stopped participating in the mathematical discussion. They therefore emphasized the importance of providing process support. Process support aims at stimulating small groups to elaborate on their mathematical ideas, while content support focuses on mathematical content. Process support has been shown to be effective during mathematical discussions (Dekker & Elshout-Mohr, 2004). However, process support is not always sufficient to further mathematical discussions, and small groups may become stuck. In those cases, some kind of content support is needed to further the discussion.

Scaffolding individual students, i.e., adapting content support to the level of the individual student (contingency to individual students) during group work, is known to enhance learning (Van de Pol et al., 2010). However, scaffolding is often directed to one student of the group. As a result, help may not be contingent for other students in the group. As mathematical discussions are a group effort, the support that is offered by the teacher should be contingent at the level of the group; that is, support should be adapted to the needs of the group. This may *either* be content support that is adapted to the level of understanding of the group *or* process support. What matters when guiding small groups is giving the right kind of support at the right time.

To help teachers support small student groups, we developed a Small-Group Scaffolding Tool (SGS-Tool) that might help teachers adapt their support to the needs of the group, i.e., to be contingent to the group. When the group has reached their maximum level of understanding, i.e., the point at which the group cannot move further on its own, teachers are recommended to provide mathematical content support that is contingent at the maximum level of the whole group. When the group has not yet reached their maximum level of understanding, teachers are recommended to provide process support. In other words, *contingency to the group* involves choosing when and how to support small groups according to the needs of the group. The desired outcome of scaffolding small groups at the group level is to further the discussion the group was already having before intervention of the teacher so that reflection on mathematical activities during the mathematical discussion can continue and learning can occur.

The main research question of this dissertation was as follows: *What is the effect of small-group scaffolding on the quality of mathematical discussions, and does it raise the mathematical knowledge level of students in seventh grade early algebra classes?*

To answer this question, we conducted four quasi-experimental studies in three phases. The first phase (study 1, chapter 2) was a preparatory phase in which we investigated the effect of shift-problem lessons (lessons that invite student groups to discuss a mathematical problem) for seventh-grade early algebra. The aim was to develop qualitatively good shift-problem lessons that could be used in the subsequent phases of this research. In the second phase (study 2, chapter 3), we developed a tool for small-group scaffolding (SGS-Tool) and investigated whether teachers were able to apply this tool and whether teachers' small-group scaffolding behavior improved. In the final phase, we investigated the effect of small-group scaffolding, i.e., teachers applying the SGS-Tool, on the quality of mathematical discussions (study 3, chapter 4) and the mathematical knowledge of students (study 4, chapter 5) and determined whether the quality of mathematical discussions mediated the effect of small-group scaffolding on the mathematical knowledge of students (study 4, chapter 5).

In this final chapter, I present a summary of the main findings, after which I discuss my conclusions about small-group scaffolding, reflect on the methodology used, identify limitations of the study, and formulate suggestions for future research and implications for practice.

6.2 MAIN FINDINGS

6.2.1 Main findings chapter 2

In study 1, the aim was to develop high-quality collaborative lessons (shift-problem lessons) on the topic of early algebra for seventh grade and to investigate whether these lessons resulted in more and qualitatively better mathematical discussions and more mathematical level raising than working with conventional lessons in a small-group setting.

A quasi-experimental study (pre- and posttest, control group) was conducted in 6 seventh-grade classes (N=160). In the shift-problem condition, five out of twelve lessons from a conventional textbook were replaced with shift-problem lessons. Analysis of the mathematical discussions of one randomly chosen small group in each class during the third shift-problem lesson showed that significantly more mathematical discussions (30% of all utterances) occurred in the shift-problem condition, and barely any mathematical discussions (3% of all utterances) occurred in the conventional textbook condition. Moreover, the discussions in the shift-problem condition were of better quality, as more key activities (tell/show work, explain work, justify work, and reconstruct work) and regulating activities (ask to tell/show work, ask to explain work, and critique work) occurred. Nevertheless, there was room for improvement, as justifying work and reconstructing work rarely occurred, whereas these activities allow students to reflect on their mathematical activities.

The students' mathematical knowledge levels in early algebra were measured by means of a mathematical knowledge test. The mathematical levels of the students were measured based on the translation skills from Janvier (1987). The highest level implies being able to switch from the representation 'formula' to the other representations and vice versa, and the lowest level implies not being able to make these switches at all. Although students' mathematical knowledge level was increased by a fair amount in both conditions, no significant differences between the conditions were found.

We concluded that shift-problem lessons elicited more and qualitatively better mathematical discussions in seventh grade but that applying shift-problem lessons may be a necessary but not sufficient condition to improve mathematical level raising. In the subsequent phases of the research, all students worked on this lesson series, containing the five shift-problem lessons for early algebra.

6.2.2 Main findings chapter 3

The next step was to investigate how teachers can guide small groups of students in such a way that the group discussions continue and students improve their mathematical knowledge. When guiding groups, teachers face various challenges, such as deciding which type of support to provide, exercising the correct degree of control, i.e., deciding how much mathematical content support to provide and determining when to return control over the learning process to the group. The aim of study 2 was to investigate whether working with the SGS-Tool offers solutions to some of these challenges, particularly whether working with the tool supports teachers' scaffolding behavior.

We identified three key characteristics of scaffolding small groups at the group level. The first is contingency *to the group*, i.e., adapting support to the needs of the group (providing either process support or contingent content support at the *group level*). The second key characteristic is phasing out mathematical content support when the group is able to continue on its own, i.e., providing contingent mathematical content support until a group is able to continue the discussion by itself. The third key characteristic of scaffolding small groups *at the group level* is returning responsibility for learning to the group by providing process support at the moment that a group is able to continue the discussion by itself. We developed the SGS-Tool (Appendix E) based on these characteristics. The tool consists of six steps: 1) determining whether the group's maximum level of understanding has been reached, 2) diagnosing the group's maximum level of understanding, 3) checking the diagnosis, 4) providing contingent mathematical content support, 5) checking whether the group can continue on its own, and 6) providing process support.

We conducted a quasi-experimental study in which four teachers and students in their four seventh grade classes (N=109) participated. Analysis of the scaffolding behavior of each teacher during interaction with one randomly chosen group during five lessons showed that teachers using the SGS-Tool provided more overall support than teachers who did not use the SGS-Tool. They implemented the various steps distinguished in the SGS-Tool, except the "checking the diagnosis" step. The control teachers mainly implemented the "provide contingent content support" step. Teachers in both conditions provided contingent content support. Crosstable analysis showed no difference between the contingency of the content support given by the teachers in the two conditions. An important observation was that teachers in the SGS condition mainly gave content support until the group could continue their discussion on their own, while the teachers in the control condition mainly gave content support until the problem was solved.

We analyzed three excerpts of two teachers (two from the SGS condition and one from the control condition) of typical support in a qualitative way. How the teacher in the control condition gave mathematical content support and continued to do so until the assignment was completed was illustrated. On the other hand, the teacher in the SGS condition stopped providing mathematical content support (phased out) when it was clear that mathematical discussions could continue, and after determining that the mathematical discussion continued, the teacher returned responsibility for learning to the group by walking away. Moreover, the SGS teacher provided process support and contingent content support at the appropriate time, as recommended by the SGS-Tool.

The four teachers indicated in interviews that they were able to work with the SGS-Tool, that the tool was clear, that they thought that they had not skipped any steps (they had performed the "checking the diagnosis" in their minds) and that all of the steps of the tool were necessary. Furthermore, the teachers indicated that it was challenging to refrain from providing too much mathematical content support and that it was especially challenging to determine what amount of support was enough and what amount was too much.

We concluded that the SGS-Tool helped teachers support small groups that discussed mathematics, offered teachers support regarding whether content or process support was appropriate and helped them return responsibility for learning to the group when a group could continue their discussion independently.

6.2.3 Main findings chapter 4

After it was determined that the SGS-Tool could help teachers scaffold small groups at the group level, the next question was how guiding seventh-grade student groups with the SGS-Tool affected the amount and quality of students' mathematical discussions. Study 3 involved a quasi-experimental study with eight teachers and their 272 seventh-grade students from two schools. Analysis of the teacher moves in teacher–small-group interaction with one randomly chosen group per class in the last lesson of the unit in both conditions showed that SGS teachers gave relatively more support than control teachers. SGS teachers also implemented the various steps of the SGS-Tool, whereas control teachers mostly provided content support. Analysis of the discussions of one randomly chosen group per class in the last lesson of the unit showed that significantly more mathematical discussions occurred in the SGS condition than in the control condition. These discussions were of higher quality; significantly more elaborating key (explain work, justify work and reconstruct work) and regulating activities (ask to explain work and critique work) occurred in the SGS condition, whereas in the control condition, significantly more sharing key activities (tell/show work) and sharing regulating activities (ask to tell/show work) occurred.

Qualitative analysis of two contrasting teacher–small-group interactions in response to a similar event illustrated how the SGS teacher was able to further the discussion, whereas the control teacher did not. The control teacher, similar to the control teacher in chapter 3, immediately gave the correct answer to one student, whereas the SGS teacher provided contingent mathematical support until the group was able to continue the discussion on their own. The SGS teacher only left the group after listening to the mathematical discussions.

We concluded that guiding seventh-grade students with the SGS-Tool elicited qualitatively better mathematical discussions.

6.2.4 Main findings chapter 5

The next question addressed in study 4 was the effect of small-group scaffolding on the mathematical knowledge level raising of seventh-grade students in early algebra classes. We performed a quasi-experimental study (pre- and posttest, control group design) with the same dataset from study 3 to investigate the effect of small-group scaffolding on the mathematical knowledge level raising of these students. In addition, we were interested in whether the quality of mathematical discussions mediated the relation between small-group scaffolding and mathematical knowledge level raising.

An adapted version of the mathematical knowledge test of study 1 was used to measure students' mathematical knowledge levels in early algebra. We found that the growth in mathematical knowledge in the SGS condition was significantly larger than that in the control condition. To analyze the quality of the mathematical discussions, we used the data from study 3. In study 3, the utterances of individual students were the unit of analysis, whereas in study 4, we aggregated these utterances to the group level. We used the quality of the discussions that was observed at the group level to test for mediation (Baron & Kenny, 1986). We found no evidence, however, that the quality of the mathematical discussions mediated the effect of small-group scaffolding on mathematical level raising.

The conclusion was that small-group scaffolding positively affected students' mathematical knowledge level and the quality of discussions. However, the quality of mathematical discussions did not play a mediating role.

To summarize, the general aim of this study was to gain insight into how teachers can guide small groups during collaborative learning to enhance the quality of mathematical discussions and students' mathematical understanding. Based on the key characteristics of small-group scaffolding, we developed the SGS-Tool.

We found that teachers were able to use the SGS-Tool and that their scaffolding behavior at the group level improved. They did not immediately provide content support but determined first whether the maximum level of understanding of the group had been reached. If the maximum level of understanding had been reached, teachers diagnosed this level of understanding and then provided contingent content support until the group could continue independently, determined whether the group could continue on its own and provided process support when this was the case. Furthermore, when the maximum level of understanding of the group had not been reached, teachers provided process support. Mathematical discussions of small groups who were guided with the SGS-Tool were qualitatively better than the mathematical discussions of small groups that were not. In addition, students who were guided with the SGS-Tool developed more mathematical knowledge than students who were not.

Our main research question is answered as follows: This research showed that small-group scaffolding improves the quality of mathematical discussions and enhances the mathematical knowledge levels of students.

6.3 DISCUSSION

In this section, I will discuss my conclusions about small-group scaffolding, the SGS-Tool and the effect of small-group scaffolding on mathematical discussions and mathematical knowledge.

6.3.1 Small-Group Scaffolding

Recently, there is quite some interest in the concept of scaffolding in mathematics education (Bakker et al., 2015). Originally, the notion of scaffolding was associated with support adapted to the level of understanding of an individual (Cazden, 1979; Wood et al., 1976). In subsequent studies, it was also used in the context of supporting an individual student during group work (Van de Pol et al., 2010) or supporting whole classes (Smit et al., 2013).

An important theoretical contribution of this dissertation is the translation of the key characteristics of individual scaffolding in the context of group work (Van de Pol, 2012) into key characteristics of scaffolding small groups *at the group level*. When teachers scaffold individuals in a group, not all students may benefit from the support because the support is not contingent to every student in the group, i.e., not all students may benefit from the support. Content support that is given at the maximum level of understanding of the group is important, as it helps the group continue when it cannot move further on its own. This research shows that small-group scaffolding, i.e., applying the SGS-Tool improves teachers' support for small groups.

When teachers scaffold small groups at the group level, support is supposed to be contingent on the whole group. I argued that contingency to the group may entail either content support or process support. Process support is recommended when the group has not reached its maximum level of understanding, and contingent content support is recommended when the group has reached its maximum level of understanding. This adds to the literature, where contingency related to an individual is only associated with content support: to be contingent upon, i.e., to adapt to, the level of understanding of an individual student (Van de Pol, Volman, Elbers, et al., 2012).

Contingency at the group level entails content support given in what might be considered the group's ZPD (Nyikos & Hashimoto, 1997). I considered the group's ZPD as the distance between the actual developmental level of the group as a whole (that was reached through mathematical discussions) and the potential development under the guidance of a teacher. To further the group discussion, it is important to provide content support to the whole group within the ZPD of the group when a group is stuck or provide process support when a group is able to further the discussion on their own. Teachers are advised to determine whether the maximum level of understanding has been reached by determining whether the question that is asked is a question from the whole group or from one individual student.

From the results of study 2 (chapter 3), we can conclude that small-group scaffolding offers a solution for the challenges teachers face when guiding groups, first, by suggesting whether content or process support is appropriate, whereupon the teacher can provide that type of help; second, by helping teachers exercise the correct degree of support, i.e., providing contingent mathematical content support when appropriate and phasing out mathematical content support; and third, by suggesting when to return responsibility for learning to the group.

6.3.2 Small-Group Scaffolding Tool

Based on the key characteristics of small-group scaffolding, we developed the SGS-Tool to help teachers enhance the mathematical discussions of small collaborating student groups. This aligns well with the observation by Webb et al. (2019) that teachers need concrete tools to help them guide small groups during collaborative learning.

This research showed that teachers were able to work with the SGS-Tool and scaffold small groups at the group level. The outcomes of studies 2 (chapter 3) and 3 (chapter 4) showed that teachers who worked with the tool implemented various steps of the tool, gave more support and gave process support and content support at the appropriate time according to the needs of the groups. In contrast, teachers who did not use the tool mainly gave content support to one or two individual students in the group until the problem was solved.

This research showed that using the SGS-Tool contributed to the amount and quality of mathematical discussions. Students who were guided with the tool were more engaged in mathematical discussions than students who were not. Moreover, these discussions contained more elaborating key activities and elaborating regulating activities, indicating mathematical discussions of better quality. Finally, this research showed that the mathematical knowledge levels of students who were guided with the tool increased more than those of students who were not. The SGS-Tool can therefore be considered a powerful tool to help teachers scaffold small groups at the group level.

The SGS-Tool is useful for practice. Teachers stated that the tool helped them get students to solve problems on their own by giving them help in small steps. Teachers checked more often whether one or more students understood the support, and when this was the case, teachers stimulated students to continue the mathematical discussion. Teachers also stated that scaffolding the group's learning made students think more. They indicated that they enjoyed seeing students discuss things in a different way because of the way they provided support to the group.

Applying the SGS-Tool can also confirm certain classroom norms (sociomathematical norms) (Yackel & Cobb, 1996) that imply that students should first share their knowledge with each other during mathematical discussions before asking the teacher for help.

Although we found positive effects of using the SGS-Tool, there are still some questions to be answered. Three issues in particular remain to be resolved.

First, teachers did not seem to perform the step "checking diagnosis" in study 2 (chapter 3) and study 3 (chapter 4). In the interviews, some teachers indicated that they had performed the diagnosis in their minds. In their study on scaffolding individual students in a group context, Van de Pol and colleagues found that the step "checking diagnosis" rarely occurred (Van de Pol, Volman, Elbers, et al., 2012). However, checking the diagnosis is important to come to a mutual understanding between teacher and student about the problem at hand (Van de Pol, Volman, Elbers, et al., 2012). When teachers carry out this step out loudly, students have the opportunity to confirm whether what the teacher thinks the problem is, is indeed the problem the group is facing. The maximum level of understanding of the group can thus be determined together with the students. Based on these considerations, we decided to keep the step "checking the diagnosis" in the SGS-Tool. A question that remains to be answered is: is it indeed necessary for teachers to perform the "checking the diagnosis" step (after performing steps 1 and 2) out loud?

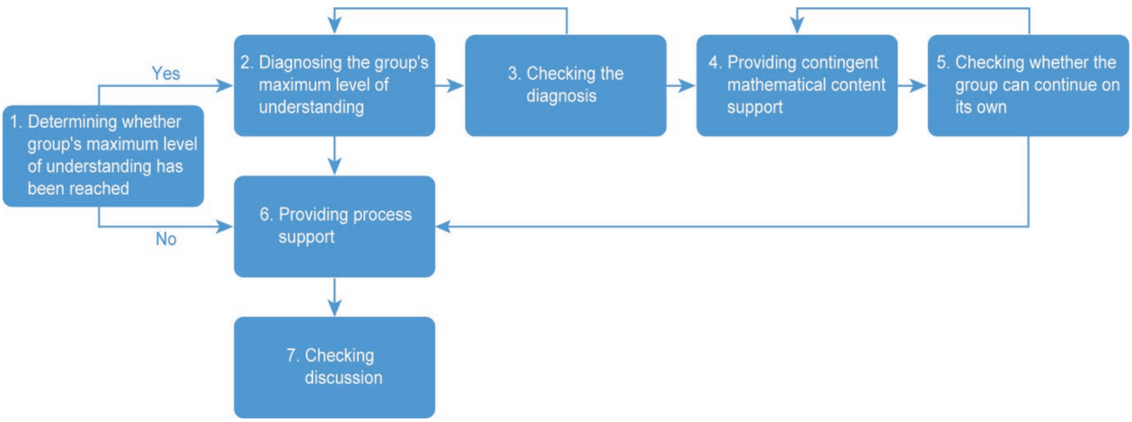
Second, in study 2, we found that content support in the SGS condition was more contingent than content support in the control condition, but this difference was not significant. We observed but did not analyze in detail that teachers in the SGS condition mostly gave contingent content support to the groups when the groups had reached their maximum level of understanding, while teachers in the control condition gave contingent content support to individuals in the group without determining whether the groups' maximum level of understanding had been reached. This would imply that content support given in the SGS condition was mostly given in the groups' ZPD, whereas the content support given in the control condition was mostly given to one or two students in the group and not given in the group's ZPD.

Third, we did not analyze whether teachers made the right decision to give process support or content support. Therefore, we do not know whether their diagnosis of whether the group's maximum level of understanding has been reached was correct.

During our studies, the original SGS-Tool, as presented in chapter 3 and Appendix E, was adapted twice. In study 2, we found that situations needed teachers to take steps that occasionally differed from the sequences outlined by the SGS-Tool. We found that other combinations of steps and repetitions of combinations, e.g., a combination of steps providing mathematical content support and checking whether the group can continue on its own and a combination of providing mathematical content support and providing process support, are indications of teachers determining when to return control over the learning process to the group. As a result of these findings, the tool was adapted by adding loops between steps 3 (Checking the diagnosis) and 2 (Determining the group's maximum level of understanding) and between steps 5 (Checking whether the group can continue on its own) and 4 (Providing contingent mathematical content support) (Appendix F for the

Figure 6.1

Final Version of the Small-Group Scaffolding Tool for Mathematical Discussions adapted with seventh step "Checking discussion"



first adapted version of the SGS-Tool). In study 3, we observed that before SGS teachers walked away from the group, they listened to the discussion to ensure that it continued in a productive manner. This observed behavior of teachers in the SGS condition gave reason to expand the tool with an additional step: the teacher checks how the discussion continues (Checking discussion), i.e., listening to determine whether the group's uptake of the support was correct and thus to determine whether responsibility for learning had been returned to the group at the appropriate time. Adding this step also relates to recent literature on the role of listening during mathematical discussions (Hintz & Tyson, 2015; Sjöblom & Meaney, 2021). As a result, the SGS-Tool is now as follows (Figure 6.1 and Appendix G).

6.3.3 Mathematical Discussions

Mathematical discussions during collaborative learning are important for learning mathematics (Freudenthal, 1991). Freudenthal (1991) stated that high-quality mathematical discussions in which small, heterogeneous groups of students reflect on mathematical activities contribute to increasing students' mathematical level. In this research, I considered the occurrence of both key and regulating activities in mathematical discussions as indicators of high quality, particularly the occurrence of elaborating key activities; explain work, justify work, reconstruct work, and elaborating regulating activities; ask to explain work and critique work.

In the first study (chapter 2), I found that shift-problem lessons, especially designed to evoke mathematical discussions, did indeed evoke more mathematical discussions than conventional lessons. However, there was still room for improvement, as the mathematical discussions consisted predominantly of the sharing key activity tell/show work. This might also explain why we did not find significant differences in mathematical level raising. Although there were more discussions in the experimental group, discussions in both groups were of the same quality as the key activity of telling and showing work, i.e., activities in which students share their mathematical ideas (rather than elaborate on them), was dominant in both conditions. I concluded that applying shift-problem lessons may be a necessary but not sufficient condition to achieve more mathematical level raising.

The desired outcome of scaffolding small groups at the group level was to further the discussions of the group was already having before intervention of the teacher. In this way, the quality of the discussions could be enhanced so that reflection on mathematical activities during the mathematical discussion could continue and learning could occur. For the discussion to continue, process support and content support need to be carefully combined. Process support needs to be given to stimulate the group discussion and help students share and elaborate on their ideas (Dekker & Elshout-Mohr, 2004). Content support needs to be given only when the group is stuck, and such support needs to be adapted to the maximum level of understanding of the entire group. The SGS-Tool offers teachers concrete steps regarding when and how to provide process support and content support. It also helps teachers not provide too much content support, which can have a negative influence on collaborative learning processes (Van Leeuwen & Janssen, 2019) and stop discussions from continuing (Dekker & Elshout-Mohr, 2004).

In the third study (chapter 4), I found that significantly more mathematical discussion utterances occurred in the SGS condition. Similar to the shift-problem condition of study 1, mathematical discussions of students in the control condition predominantly consisted of sharing key activity tell/show work. In the SGS condition, more elaborating key activities and elaborating regulating activities occurred compared to the control group. In other words, guiding seventh-grade students with the SGS-Tool can elicit qualitatively better mathematical discussions.

Furthermore, based on our third and fourth studies (chapters 4 and 5), I can conclude that applying shift-problem lessons *together with* scaffolding small groups at the group level contributes to mathematical level raising. Students' mathematical knowledge improved more in the SGS condition, where teachers combined process support and contingent content support according to the suggestions of the SGS-Tool, than in the control condition. This suggests that the combination of process support and content support was better for level raising than the support given by the teachers in the control condition.

Study 4 showed that the quality of mathematical discussions, however, did not mediate the effect of small-group scaffolding on mathematical level raising. A possible explanation might be that students in the small-group scaffolding condition learned from the contingent content support of the teacher rather than from the group discussion. In that case, the mechanism underlying the effect of the SGS-Tool, i.e., that qualitatively better mathematical discussions lead to more mathematical level raising, would be slightly different than originally hypothesized. However, even if the improved quality of the mathematical discussions would not explain the learning of the students, the group discussion would still be part of the explaining mechanism, since discussion is necessary for groups to reach the point at which a teacher can give content support in the ZPD of the whole group. Another explanation for not finding a mediating effect of the quality of mathematical discussion could be that not all students in a group participated in the discussion. Students who did not engage in a discussion might have learned less than those who did not, since student participation in mathematical discussions plays a mediating role in the relationship between teacher support of student participation and students' mathematical achievement (Ing et al., 2015). A mediating effect might have been found if we had been able to include more than ten groups.

6.4 METHODOLOGICAL CONSIDERATIONS

In this section, I will reflect on some of the methodological decisions made in this research.

A first issue is that I not only conducted the research but also participated as a teacher in the first study of this research project. Part of the lessons that were analyzed were my own lessons. I did not consider it a problem to participate in the first study as a teacher, as the object of study was the mathematical discussions of the small groups and not teacher support. I therefore did not participate in subsequent studies, as the support that teachers provided was the object of study, and my participation might have influenced the outcomes.

Being part teacher, part researcher did have several advantages. The school management was enthusiastic about my research project and supported me in every respect. Furthermore, it was easy for me to conduct pilot studies and test the shift-problem lessons and the pre- and posttests. I could instantly observe whether the shift-problem lessons worked and how they affected the students' mathematical knowledge.

For the analysis of the mathematical discussions, we used the Process Model of Dekker and Elshout-Mohr (1998) as a coding scheme. This coding scheme enabled us to analyze in detail students' conversations during group work and, most importantly, the quality of their mathematical discussions. We adapted this coding scheme in study 3 to distinguish between elaborating and sharing activities to connect more to the literature (e.g., Van Boxtel et al., 2000; Webb et al., 2019). We rearranged the key and regulating activities into elaborating and sharing activities. A limitation of the coding scheme worth mentioning here is that it did not account for the correctness of the content of the student utterances. An incorrect utterance might be accepted as true by the group, thus negatively affecting the quality of the discussion. On the other hand, incorrect utterances may initiate mathematical discussions, thus positively affecting the quality of the discussion.

For this research, I developed several measurement instruments that could be used in future research in mathematics education. First, an instrument was used to measure the mathematical knowledge levels of students. I developed two versions of this mathematical knowledge test. The first version of the mathematical knowledge test (adapted after a pilot test in one of my classes) was used in study 1. Hereafter, an adapted version of the study 1 test was used in study 4. When designing the test, I first defined what level raising in early algebra entails by connecting Freudenthal's level raising theory (1991) with Janviers' theory (1987) about how early algebra can be learned best. I defined level raising in early algebra as the ability to switch to and from the algebra representation 'formulae' (Janvier, 1987) and then operationalized these switches. The highest mathematical knowledge level implies being able to switch from the algebra representation 'formulae' to the other representations 'graphs', 'tables' and 'situation', and vice versa, and the lowest level implies not being able to make these switches at all. We carefully designed the test to include all switches from and to the algebra representation 'formulae'. I pilot tested and retested the test in my seventh-grade class. A similar approach could be taken when developing tests for other topics by connecting Freudenthal's level theory with the theory of how a specific topic can be learned by students. It was possible to score mathematical knowledge levels, and the interrater reliability for the pre- and posttest between the two coders was excellent.

Second, I developed an instrument for analyzing teachers' scaffolding behavior at the group level. I operationalized the scaffolding behavior of the teachers at the group level according to the steps described in the SGS-Tool. This coding scheme enabled us to analyze in detail the scaffolding behavior of teachers at the group level. One limitation is that we coded only verbal utterances. It is possible that a teacher would utilize a particular step but not explicate it verbally, which might have been the case with the step "checking the diagnosis". For the added step "checking the discussion" in the final version of the SGS-Tool, it might be interesting to code the nonverbal behavior of teachers to determine whether teachers listen if the discussions are continuing. For the analysis of contingency, we could analyze only what occurred during the interactions. Previous lessons or a teacher's knowledge of his or her students may affect a teacher's actions (Mercer, 2008), but they were not accounted for when determining contingency of the content support.

Coding contingency turned out to be an elaborate endeavor. Utterances that are coded as mathematical content support need to be coded for contingency in a second round. Rules for coding contingency involve dealing with many different possibilities. For example, there are exception coding rules when no student understanding can be determined or when students do not demonstrate their understanding. In their study on the scaffolding behavior of teachers scaffolding individual students in a group context, Van de Pol (2012) acknowledged that coding contingency is an elaborate endeavor. Nevertheless, the SGS-Tool, used as a coding scheme, enabled us to analyze the scaffolding behavior of teachers at the group level. This is important for research on small-group scaffolding, which can be followed up in future research.

In this research project, I used a mixed method approach in which qualitative and quantitative research designs were combined in quasi-experimental studies. This turned out to be a rich combination; the qualitative analyses helped us to illustrate how the mechanisms underlying certain quantitative findings might work. They showed, for example, how teachers in the control condition tended to give students the answer to a problem, whereas teachers in the SGS condition provided contingent content support at the group level until the group could continue on their own and find the answer themselves. Additionally, I was able to code the student–student interactions and scaffolding behavior of teachers in detail in a qualitative way and quantify them afterward. This gave us an in-depth insight into the discussions of the groups and support that teachers provided and enabled us to compare outcomes between the conditions.

6.5 LIMITATIONS AND FUTURE RESEARCH

In this section, I discuss four important limitations of this research project and formulate suggestions for future research.

A first limitation of this research is that the studies were conducted at only two schools. Collaborative learning was part of the educational system of these schools, which implies that teachers were used to guiding students who were working together in small heterogeneous groups. Although many teachers and students participated in this research, caution should be taken when generalizing our findings to other school settings.

A second limitation concerns the analysis of the interaction processes. As the analysis of interaction is a time-consuming process, only a limited number of group discussions and lessons were selected for this analysis (five student groups and the third lesson in study 1 and ten groups and the last lesson in study 3 and 4). One group per class was randomly chosen. Weaker or stronger groups may have been selected, which might have influenced our findings. It is also important here to realize that all groups were mixed ability groups. Nevertheless, differences between mixed ability groups might occur, but the differences between groups are not expected to be as large as when groups were composed of the strongest or weakest students of the class. Furthermore, a large amount of data was analyzed in detail. For study 1, data from 5 groups from the third lesson were collected, which resulted in approximately 5 hours of video, for a total of 2973 student utterances. For studies 3 and 4, data from 10 groups during the last lesson were collected, resulting in approximately 10 hours of video, for a total of 6113 student utterances. I decided to analyze one lesson in studies 1, 3 and 4. Analyses of other lessons might have generated other results. I selected the third lesson for analysis of mathematical discussions in study 1 because this was a high-qualitatively shift-problem lesson. For the analysis of mathematical discussions in studies 3 and 4, I focused on the last lesson. I chose the last lesson because I assumed that teachers might be familiar with the SGS-Tool by then and would be able to apply it better than in the beginning. For the analysis of teachers' scaffolding behavior in study 2, we analyzed teachers' behavior in teacher–small-group interactions with one random group in all five lessons.

A third limitation of this study is that our coding schemes did not include correctness of content. The coding scheme for measuring teachers' scaffolding behavior did not account for the correctness of the content or quality of the support teachers gave to small groups. In addition, the coding scheme for measuring mathematical discussions did not account for the content. An incorrect utterance might be accepted as true by the group. On the other hand, incorrect utterances might initiate more mathematical discussions.

In this research, the mediating role of the quality of mathematical discussions was investigated with ten groups. Future research could investigate the mediating role of the quality of mathematical discussions regarding the effect of small-group scaffolding on mathematical level raising on a larger scale. It could be also interesting to investigate the mediationg role of student participation in the relationship between small-group scaffolding and students' mathematical achievement. Ing et al. (2015) found that student participation mediated the relationship between teacher support for student participation and students' mathematical achievement.

Being guided with the SGS-tool was new to the students in this research. It would be interesting to investigate how students experienced this guidance and whether they thought it was helpful. Related to this, it might be interesting to investigate what happens when students become more experienced with scaffolding at the group level; will they ask fewer questions as they first try to solve the problem as a group, will they ask different kind of questions, and what will the implication of that be for the quality of the mathematical discussions?

Future research might also focus on how the scaffolding behavior of teachers develops over time. Will teachers, for example, find it easier to refrain from providing too much content support over time?

I observed that SGS teachers checked whether groups correctly took up the support that was given in a previous encounter with the group. When this was not the case, teachers corrected mistakes. Research into scaffolding individuals during group work showed that the correct uptake of teacher support was linked to the correctness of utterances of student-student interactions (Van de Pol et al., 2019). How teachers check whether small groups take up their support correctly might also be a topic of future research.

In research into scaffolding on the whole class level, teachers adapted learning materials between lessons (Smit et al., 2013). Future research might focus on how teachers, for example, adapt their scaffolding strategies between lessons.

We investigated small-group scaffolding in the context of mathematics learning. It might be interesting to use it for other school subjects as well. Applying process support and content support during collaborative learning is also important for subjects other than mathematics. The SGS-Tool could suggest concrete steps for teachers of other subjects to take since supporting small-group work is also a challenging activity for these teachers (Van de Grift et al., 2011). Future research could focus on applying the tool for other subjects.

6.6 IMPLICATIONS FOR PRACTICE

This research project shows that when mathematics teachers scaffold small groups *at the group level* with the use of the SGS-Tool, both the quality of mathematical discussions and the mathematical knowledge levels of students can be improved. This is a good reason for teachers to make use of the SGS-Tool when they support students working on mathematical assignments in small groups. However, teachers (and student teachers) need knowledge and skills to be able to apply the SGS-Tool during collaborative learning. Training might be necessary.

When mathematics teachers want to apply the SGS-Tool during collaborative learning in their classrooms, it is important to carefully pick a topic and adapt an assignment into a group task using the design principles for developing shift-problem lessons of Palha et al. (2013). When designing shift-problem lessons for mathematics, I recommend using literature on how students learn the chosen mathematical topic. In this research project, Janvier's theory (1987) was used, in which he states that algebra is best learned when switching back and forth between algebra representations. Similar to the intervention of this research, teachers who teach other topics might substitute a conventional lesson with a shift-problem lesson in which students work collaboratively on group tasks addressing that topic.

In schools where collaborative learning is less common, the use of the SGS-Tool can help teachers set sociomathematical norms that support meaningful mathematical discussions, as it suggests that students should first discuss their mathematical ideas with each other before asking the teacher for help.

This research project was conducted with heterogeneous small groups (groups of mixed ability), but we think that small-group scaffolding might also work with homogeneous groups. Freudenthal (1991) has suggested that level raising can be achieved by collaborative learning. He also argued that this is due to reflection on mathematical activities, which often occur during collaboration in heterogeneous small groups. Often, one of these students will understand the subject matter (jumping to a higher level) and then start explaining to the other members of the group by reflecting on what he or she just learned. This process might also occur in homogeneous groups, and if this does not happen, teachers can provide contingent content support (as suggested by small-group scaffolding) until one or more of the students understand the support and discussions can continue without the support of the teacher.

Teacher training institutes might incorporate the use of the SGS-Tool in their training program and in professionalization programs. This training might focus on helping teachers decide whether to provide process support or content support and refrain from providing too much content support when appropriate, i.e., exercise the correct degree of control. As a teacher educator, I have taught student teachers how to scaffold small groups at the group level who worked on shift-problem lessons on a mathematical topic of their choice. I taught them how to find literature on how students learn that particular topic and how to design shift-problem lessons for that topic. Furthermore, student teachers learned to code mathematical discussions and their own scaffolding behavior. In my experience, student teachers were able to design shift-problem lessons, scaffold small groups at the group level, and code discussions and scaffolding behavior. In my opinion, applying small-group scaffolding during collaborative learning helped student teachers reflect on teaching and learning mathematics in the sense that learning is not merely an individual activity but that students can learn from each other and that guiding small student groups entails more than just giving the answer to a problem.

In summary, the SGS-Tool can help teachers guide small groups during collaborative learning by offering them concrete steps on how and when to offer process support or content support and when to phase out support and return responsibility for learning to small groups. This study showed that it is a powerful tool that can enhance the quality of mathematical discussions and students' mathematical understanding.