Measurement of the polarisation of $W$ bosons produced with large transverse momentum in $pp$ collisions at $\sqrt{s} = 7$ TeV with the ATLAS experiment


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Measurement of the polarisation of $W$ bosons produced with large transverse momentum in $pp$ collisions at $\sqrt{s} = 7$ TeV with the ATLAS experiment

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Abstract This paper describes an analysis of the angular distribution of $W \rightarrow e\nu$ and $W \rightarrow \mu\nu$ decays, using data from $pp$ collisions at $\sqrt{s} = 7$ TeV recorded with the ATLAS detector at the LHC in 2010, corresponding to an integrated luminosity of about 35 pb$^{-1}$. Using the decay lepton transverse momentum and the missing transverse momentum, the $W$ decay angular distribution projected onto the transverse plane is obtained and analysed in terms of helicity fractions $f_0$, $f_L$ and $f_R$ over two ranges of $W$ transverse momentum ($p^W_T$): $35 < p^W_T < 50$ GeV and $p^W_T > 50$ GeV. Good agreement is found with theoretical predictions. For $p^W_T > 50$ GeV, the values of $f_0$ and $f_L - f_R$, averaged over charge and lepton flavour, are measured to be: $f_0 = 0.127\pm0.030\pm0.108$ and $f_L - f_R = 0.252\pm0.017\pm0.030$, where the first uncertainties are statistical, and the second include all systematic effects.

1 Introduction

This paper describes a measurement with the ATLAS detector of the polarisation of $W$ bosons with transverse momenta greater than 35 GeV, using the electron and muon decay modes, in data recorded at 7 TeV centre-of-mass energy, with a total integrated luminosity of about 35 pb$^{-1}$. The results are compared with theoretical predictions from MC@NLO [1] and POWHEG [2–5].

The paper is organised as follows. Section 2 describes the theoretical framework of this analysis. Section 3 reviews the relevant components of the ATLAS detector, the data, the corresponding Monte Carlo simulated data sets, and the event selection. The estimation of backgrounds after this selection is explained in Sect. 4, and the comparison of data and Monte Carlo simulations for the most relevant variable ($\cos(\theta_{2D})$) is given in Sect. 5. The construction of helicity templates and its validation using Monte Carlo samples is described in Sect. 6, while the uncorrected results are given in Sect. 7. The systematic uncertainties associated with the fitting procedure are discussed in Sect. 8 and the final results, corrected for reconstruction effects, are given in Sect. 9. Section 10 is devoted to the conclusions.

2 Theoretical framework and analysis procedure

Measuring the polarisation of particles is crucial for understanding their production mechanisms.

At hadron colliders, $W$ bosons with small transverse momentum are mainly produced through the leading order electroweak processes

$$ud \rightarrow W^+ \quad \text{and} \quad d\bar{u} \rightarrow W^-$$

At the LHC the quarks generally carry a larger fraction of the momentum of the initial-state protons than the antiquarks. This causes the $W$ bosons to be boosted in the direction of the initial quark. In the massless quark approximation, the quark must be left-handed and the antiquark right-handed. As a result the $W$ bosons with large rapidity ($y_W$) are purely left-handed.

For more centrally produced $W$ bosons, there is an increasing probability that the antiquark carries a larger momentum fraction than the quark, so the helicity state of the $W$ bosons becomes a mixture of left- and right-handed states whose proportions are respectively described with fractions $f_L$ and $f_R$.

For $W$ bosons with large transverse momentum, three main processes contribute (taking the $W^+$ as example):

$$ug \rightarrow W^+d, \quad ud \rightarrow W^+g \quad \text{and} \quad g\bar{d} \rightarrow W^+\bar{u}$$
Given the vector nature of the gluon, present in all three reactions, the simple argument used at low $p_T^W$ no longer applies. Predictions require detailed helicity state calculations. Leading-order (LO) and next-to-leading-order (NLO) QCD predictions have been available for $p p$ interactions for some time [6] and more recently for proton-proton interactions [7]. At high transverse momenta more complex production mechanisms contribute, and polarisation in longitudinal states is also possible (the proportion of longitudinal $W$ bosons is hereafter described by $f_0$). This state is particularly interesting as it is directly connected to the massive character of the gauge bosons.

2.1 Theoretical framework

The general form for inclusive $W$ production followed by its leptonic decay can be written as [6]:

$$\frac{d\sigma}{d\left(p_T^W\right)^2 dy_W d\cos\theta d\phi} = \frac{3}{16\pi} \frac{d\sigma^u}{d\left(p_T^W\right)^2 dy_W} \times \left[ (1 + \cos^2\theta) 
+ \frac{1}{2} A_0 (1 - 3 \cos^2\theta) + A_1 \sin 2\theta \cos \phi 
+ \frac{1}{2} A_2 \sin^2\theta \cos 2\phi + A_3 \sin \theta \cos \phi 
+ A_4 \cos \theta + A_5 \sin^2\theta \sin 2\phi 
+ A_6 \sin 2\theta \sin \phi + A_7 \sin \theta \sin \phi \right] \quad (1)$$

where $\sigma^u$ is the unpolarised cross-section and $\phi$ and $\theta$ are the azimuthal and polar angles of the charged lepton in a given $W$ rest frame. The $A_i$ coefficients are functions of $p_T^W$ and $y_W$ and depend on the parton distribution functions (PDFs). For $p_T^W \rightarrow 0$ all reference frames used in [6–12] become identical, with the $z$-axis directed along the beam axis. In these conditions the dependence on $\phi$ disappears and only the term with $(1 + \cos^2\theta)$ and the terms proportional to $A_0$ and $A_4$ remain.

The $A_0$ to $A_4$ coefficients in Eq. (1) receive contributions from QCD at leading and higher orders, while $A_5$ to $A_7$ appear only at next-to-leading order. Their expression as a function of $p_T^W$ and $y_W$ depends on the reference frame used for the calculation.

Several papers have been published to discuss and predict these coefficients, first for $p p$ colliders [6, 8–12] and more recently for the LHC [7]. While at $p \bar{p}$ colliders, because of CP invariance, the $A_i$ coefficients are either equal ($A_0, A_2, A_3, A_5, A_7$) or opposite ($A_1, A_4, A_6$) for $W^+$ and $W^-$ production, there is no such simple relationship at $p p$ colliders. However it has been observed [7] that $A_3$ and $A_4$ change sign between $W^+$ and $W^-$, while the other coefficients ($A_0, A_1, A_2, A_5, A_7$) do not and are similar in magnitude between $W^+$ and $W^-$. In all cases, the pure NLO coefficients ($A_5$ to $A_7$) are small. They are neglected in this analysis.

Experimental measurements have been reported from the Tevatron by CDF [13], from HERA by H1 [14] and recently from the LHC by CMS [15].

2.2 Helicity fractions

Helicity is normally measured by analysing the distribution of the cosine of the helicity angle ($\theta_{3D}$ in the following), defined as the angle between the direction of the $W$ in the laboratory frame and the direction of the decay charged lepton in the $W$ rest frame. The distribution of this angle as generated by mc@NLO is shown in Fig. 1 without phase space restriction, as well as with the acceptance ($p_T^\ell, \eta^\ell$ and $p_T^{\nu}$) and $W$ transverse mass $m_T^W$ cuts (where $m_T^W = \sqrt{2(p_T^\ell p_T^{\nu} - \vec{p}_T^{\ell} \cdot \vec{p}_T^{\nu})}$), described in Sect. 3.4.

The differential cross-section in the helicity frame is expressed by using $\theta_{3D}$ and $\phi_{3D}$ in Eq. (1). Integrated over $y_W$ and $\phi_{3D}$, Eq. (1) then takes the form:

![Fig. 1 Cosine of the helicity angle of the lepton from W decay at generator-level for positive charge (left) and negative charge (right). Solid lines are without selection, dashed lines are after all acceptance plus $m_T^W$ cuts except the $\eta^\ell$ cuts and dotted lines are after all acceptance plus $m_T^W$ cuts. “All events” distributions are normalised to unity.

1ATLAS uses a right-handed coordinate system with its origin at the nominal interaction point (IP) in the centre of the detector and the z-axis along the beam pipe. The x-axis points from the IP to the centre of the LHC ring, and the y-axis points upward. Cylindrical coordinates $(r, \phi)$ are used in the transverse plane, $\phi$ being the azimuthal angle around the beam pipe. The pseudorapidity is defined in terms of the polar angle $\theta$ as $\eta = -\ln \tan(\theta/2)$.

2The helicity frame is the $W$ rest frame with the $z$-axis along the $W$ laboratory direction of flight and the $x$-axis in the event plane, in the hemisphere opposite to the recoil system.
\[ \frac{1}{\sigma} \frac{d\sigma}{d\cos\theta_{3D}} = \frac{3}{8} \left[ (1 + \cos^2\theta_{3D}) + A_0 \frac{1}{2} (1 - 3 \cos^2\theta_{3D}) + A_4 \cos\theta_{3D} \right] \] (2)

Comparing Eq. (2) to the standard form \[16\] using helicity fractions:

\[ \frac{1}{\sigma} \frac{d\sigma}{d\cos\theta_{3D}} = \frac{3}{8} f_L (1 \mp \cos\theta_{3D})^2 + \frac{3}{8} f_R (1 \pm \cos\theta_{3D})^2 + \frac{3}{4} f_0 \sin^2\theta_{3D} \] (3)

yields the relations between the \(A_i\) coefficients and the helicity fractions:

\[ f_L (y_W, p_T^W) = \frac{1}{4} (2 - A_0 (y_W, p_T^W) \mp A_4 (y_W, p_T^W)) \]

\[ f_R (y_W, p_T^W) = \frac{1}{4} (2 - A_0 (y_W, p_T^W) \pm A_4 (y_W, p_T^W)) \] (4)

\[ f_0 (y_W, p_T^W) = \frac{1}{2} A_0 (y_W, p_T^W) \]

where the upper (lower) sign corresponds to \(W^+ (W^-)\) boson production respectively. It is interesting to notice that the difference between the left- and right-handed fraction is proportional to \(A_4\) only, as:

\[ f_L - f_R = \mp \frac{A_4}{2} \] (5)

From general considerations, the longitudinal helicity fraction \(f_0\) is expected to vanish for \(p_T^W \rightarrow 0\) as well as for \(p_T^W \rightarrow \infty\), with a maximum expected around 45 GeV \[7\].

2.3 Analysis principle and variable definitions

When analysing data, a major difficulty arises from the incomplete knowledge of the neutrino momentum. The large angular coverage of the ATLAS detector enables measurement of the missing transverse momentum, which can be identified with the transverse momentum of the neutrino. The longitudinal momentum can be obtained through the \(W\) mass constraint. However, solving the corresponding equation leads to two solutions, between which it is not possible to choose in an efficient way. The approach taken in this analysis is to work in the transverse plane only, using the “transverse helicity” angle \(\theta_{3D}\) defined by:

\[ \cos\theta_{2D} = \frac{\vec{p}_{T}^\ell \cdot \vec{p}_{T}^W}{|\vec{p}_{T}^\ell| |\vec{p}_{T}^W|} \] (6)

where \(\vec{p}_{T}^\ell\) is the transverse momentum of the lepton in the transverse \(W\) rest frame and \(\vec{p}_{T}^W\) is the transverse momentum of the \(W\) boson in the laboratory frame. The angle \(\theta_{2D}\) is a two dimensional projection of the helicity angle \(\theta_{3D}\). Its determination uses only fully measurable quantities, defined in the transverse plane. Its use is limited to sizeable values of \(p_T^W\), which corresponds to the physics addressed in this work.

The correlations between \(\cos\theta_{2D}\) and \(\cos\theta_{3D}\) for events where \(p_T^W > 50\) GeV are represented in Figs. 2(a) and 2(b) for positive and negative leptons respectively. This information is obtained using a sample of events simulated with MC@NLO after applying acceptance and \(m_W^\ell\) cuts, as defined in Sect. 3.4.

![Fig. 2](image)

### Fig. 2

Representation of \(\cos\theta_{2D}\) as a function of \(\cos\theta_{3D}\) in events where the \(W\) transverse momentum is greater than 50 GeV, for (a) positive and (b) negative leptons. Events are simulated with MC@NLO after applying the acceptance and \(m_W^\ell\) cuts, as defined in Sect. 3.4

The enhancement near \(-1\) for positive leptons reflects that the maximum of the left-handed part of the decay distribution (first term in Eq. (3)) falls within detector acceptance, as opposed to the case of negative leptons where the maximum (near +1) falls largely beyond the \(\eta_L\) acceptance, resulting in a more “symmetric” distribution between forward and backward hemispheres. This effect is also seen in Fig. 1 when comparing \(\cos\theta_{3D}\) distributions at generator-level, before and after the lepton pseudorapidity cut.

The measurement of helicity fractions is made by fitting \(\cos\theta_{2D}\) distributions with a weighted sum of templates obtained from Monte Carlo simulations, which correspond to longitudinal, left- and right-handed states. This is described in detail in Sect. 6.

3 Detector, data and simulation

3.1 The ATLAS detector

The ATLAS detector \[17\] at the LHC covers nearly the entire solid angle around the collision region. It consists of an inner tracking system surrounded by a thin superconducting solenoid, electromagnetic and hadronic calorimeters, and a muon spectrometer incorporating three large superconducting toroid magnets.

The inner detector (ID) is immersed in a 2 T axial magnetic field and allows charged particle tracking in the range
The high-granularity silicon pixel detector covers the vertex region and typically provides three measurements per track. It is followed by the silicon microstrip tracker which usually provides four two-dimensional measurement points per track. These silicon detectors are complemented by the transition radiation tracker, which enables radially extended track reconstruction up to $|\eta| = 2.0$. The transition radiation tracker also provides electron identification information based on the fraction of hits (typically 30 per track) above an energy threshold corresponding to transition radiation.

The calorimeter system covers the pseudorapidity range $|\eta| < 4.9$. Within the region $|\eta| < 3.2$, electromagnetic calorimetry is based on barrel and end-cap high-granularity lead liquid-argon (LAr) electromagnetic calorimeters, with an additional thin LAr presampler covering $|\eta| < 1.8$ to correct for energy loss in material upstream of the calorimeters. Hadronic calorimetry is provided by a steel/scintillating-tile detector, segmented into three structures within $|\eta| < 1.7$, and two copper/LAr hadronic endcap calorimeters. The solid angle coverage is completed with forward copper/LAr and tungsten/LAr calorimeter modules optimised for electromagnetic and hadronic measurements respectively.

The muon spectrometer (MS) comprises separate trigger and high-precision tracking chambers measuring the deflection of muons in a magnetic field generated by superconducting air-core toroids. The precision chamber system covers the region $|\eta| < 2.7$, with three layers of monitored drift tubes complemented by cathode strip chambers in the region beyond $|\eta| = 2.0$ where the background is highest. The muon trigger system covers the range $|\eta| < 2.4$ with resistive plate chambers in the barrel, and thin gap chambers in the endcap regions.

A three-level trigger system is used to select interesting events [18]. The Level-1 trigger is implemented in hardware and uses a subset of detector information to reduce the event rate to a design value of at most 75 kHz. This is followed by two software-based trigger levels which together reduce the event rate to about 200 Hz.

### 3.2 Data sample

The data used in this analysis were collected from August to October 2010. Requirements on beam, detector and trigger conditions, as well as on data quality, were used in the event selection, resulting in integrated luminosities of 37.3 pb$^{-1}$ for the electron channel and 31.4 pb$^{-1}$ for the muon channel (data where the muon trigger conditions varied too rapidly were not included).

The integrated luminosity measurement has an uncertainty of 3.4% [19, 20].

### 3.3 Simulation

Signal and background samples were processed through a GEANT4 [21] simulation of the ATLAS detector [22] and reconstructed using the same analysis chain as the data.

The signal samples were generated using MC@NLO 3.4.2 with HERWIG [23] parton showering, and with POWHEG 1.0 and PYTHIA parton showering. Both used the CTEQ 6.6 [24] PDF set. All background samples were generated with PYTHIA 6.4.21 [25] except $t\bar{t}$ for which MC@NLO was used. In order to study the sensitivity of the angular distributions to different NLO PDF sets, the MC@NLO sample was reweighted [26] according to MSTW 2008 [27] and HERAPDF 1.0 [28] PDF sets.

The radiation of photons from charged leptons was simulated using PHOTOS [29], and TAUOLA [30] was used for $\tau$ decays. The underlying event [31] was simulated according to the ATLAS tune [32]. The Monte Carlo samples were generated with, in average, two soft inelastic collisions overlaid on the hard-scattering event. Events were subsequently reweighted so that the distribution of the number of reconstructed vertices matched that in data, which was 2.2 on average.

### 3.4 Event selection

Events in this analysis are first selected using either a single-muon trigger with a requirement on the transverse momentum $p_T^\mu$ of at least 13 GeV, or a single-electron trigger, with a $p_T^e$ requirement of at least 15 GeV [18]. Subsequent selection criteria closely follow those used for the $W$ boson inclusive cross-section measurement reported in [33].

Events from $pp$ collisions are selected by requiring a reconstructed vertex compatible with the beam-spot position and with at least three associated tracks each with transverse momentum greater than 0.5 GeV.

Electron candidates are required to satisfy $p_T^e > 20$ GeV, $|\eta| < 2.47$ (but removing the region where barrel and end-cap calorimeters overlap, i.e. $1.37 < |\eta| < 1.52$) and to pass the “tight” identification criteria described in [34]. This selection rejects charged hadrons and secondary electrons from conversions by fully exploiting the electron identification potential of the detector. It makes requirements on shower shapes in the electromagnetic calorimeter, on the angular matching between the calorimeter energy cluster and the ID track, on the ratio of cluster energy to track momentum, and on the number of hits in the pixels (in particular a hit in the innermost layer is required), in the silicon microstrip tracker and in the transition radiation tracker.

Muon candidates are required to be reconstructed in both the ID and the MS, with transverse momenta satisfying the
conditions $|p_T^{\text{MS}} - p_T^{\text{ID}}|/p_T^{\text{ID}} < 0.5$ and $p_T^{\text{MS}} > 10$ GeV. The two measurements are then combined, weighted by their respective uncertainties, to form a combined muon. The W candidate events are required to have at least one combined muon track with $p_T^\mu > 20$ GeV, within the range $|\eta| < 2.4$. This muon candidate must also satisfy the isolation condition $(\Sigma p_T^{\text{ID}})/p_T^\mu < 0.2$, where the sum is over all charged particle tracks around the muon direction within a cone of size $\Delta R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} = 0.4$. Finally, to reduce the contribution of cosmic-ray events, and beam-halo induced by proton losses from the beam, the analysis requires the reconstructed vertex position along the beam axis to be within 20 cm of the nominal interaction point.

The missing transverse momentum ($E_T^{\text{miss}}$) is reconstructed as the negative vector sum of calibrated “objects” (jets, electrons or photons, muons) to which the energies of calorimeter cells not associated to any of the objects are added. $E_T^{\text{miss}}$ is required to be larger than 25 GeV. A cut $m_W > 40$ GeV is finally applied.

In addition to these cuts, called in the following standard cuts, additional selections are used for this analysis. A low $m_W$ cut at 50 GeV is applied to minimise backgrounds, and a high $m_W$ cut at 110 GeV is applied to remove tails of badly reconstructed events. Finally a $p_T^W$ selection in two bins ($35 < p_T^W < 50$ GeV, and $p_T^W > 50$ GeV) is made. The numbers of events passing these cuts are shown in Table 1.

The data are compared to expectations based on Monte Carlo simulations. In addition to the signal ($W$ production followed by leptonic decay to an electron or a muon), the following electroweak backgrounds are considered: $W \rightarrow \tau\nu$, $Z \rightarrow e\nu$, $Z \rightarrow \mu\mu$ and $Z \rightarrow \tau\tau$, as well as $t\bar{t}$ events with at least one semi-leptonic decay. Jet production via QCD was also simulated, but the final estimate of this background is obtained from data, as explained in Sect. 4.2.

### Table 1 Numbers of events in data and signal Monte Carlo samples, after standard and analysis cuts (see text), classified according to lepton flavour and charge. The remaining numbers of events after standard plus analysis cuts are also represented as a percentage of the numbers of events passing the standard selection.

<table>
<thead>
<tr>
<th></th>
<th>$\mu^+$</th>
<th>$\mu^-$</th>
<th>$e^+$</th>
<th>$e^-$</th>
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<td><strong>Data</strong></td>
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</tr>
<tr>
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<td>67130</td>
<td>45690</td>
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<td>3778 (5.6 %)</td>
<td>2656 (5.8 %)</td>
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<td>2640 (5.1 %)</td>
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<td>2572 (5.6 %)</td>
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<td>39528 (5.1 %)</td>
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<td><strong>POWHEG</strong></td>
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<td>47115 (4.6 %)</td>
<td>50705 (4.8 %)</td>
<td>37792 (4.9 %)</td>
</tr>
</tbody>
</table>

4 Signal normalisation and background estimate

4.1 Signal normalisation

The $W^\pm \rightarrow \ell\nu$ production cross-sections and the decay branching ratios used in this study are normalised to the NNLO predictions of the FEWZ program [35] with the MSTW 2008 PDF set:

$$\sigma_{W^\pm \rightarrow \ell\nu}^{\text{NNLO}} = 6.16 \text{ nb}$$

$$\sigma_{W^\pm \rightarrow \ell\nu}^{\text{NNLO}} = 4.30 \text{ nb}$$

The estimated uncertainties on each cross-section coming from the factorisation and renormalisation scales as well as from the parton distribution functions are expected to be approximately 5 % [33].

4.2 Background estimates

$W$ events decaying into $\tau$-leptons with subsequent leptonic $\tau$ decays contribute as background to both electron and muon channels. Contributions from $Z \rightarrow \mu\mu$ decays are significant in the muon channel, where the limited $\eta$ coverage of the tracking and muon systems can result in fake $E_T^{\text{miss}}$ when one of the muons is missed. On the contrary, the $Z \rightarrow ee$ background is almost negligible in the electron channel due to the nearly hermetic calorimeter coverage over $|\eta| < 4.9$. For both the electron and the muon channels, contributions from $Z \rightarrow \tau\tau$ decays and from $t\bar{t}$ events involving at least one leptonic $W$ decay are also taken into account. The latter is particularly relevant for the large transverse momentum $W$ bosons studied here.

The normalisation of electroweak and $t\bar{t}$ backgrounds is based on their total theoretical cross-sections. These cross-sections are calculated at NLO (plus next-to-next-to-leading-log corrections) for $t\bar{t}$ [36, 37], and at NNLO for the
others. The contributions of these backgrounds to the final data sample have been estimated using simulation to model acceptance effects.

One of the major background contributions, especially in the electron channel, is from dijet production via QCD processes. The selected leptons from these processes have components from semi-leptonic decays of heavy quarks, hadrons misidentified as leptons, and, in the case of the electron channel, electrons from conversions. The missing transverse momentum is due mainly to jet mismeasurement. For the muon channel, jet event fractions are small and measured with larger uncertainties than for electrons, a value of 2% with an uncertainty of ±2% is used for both $W^+$ and $W^-$. Table 2 shows the statistical uncertainties from the jet template method. Uncertainties on the measurement due to background modelling are described in Sect. 8.1.

5 Data to Monte Carlo comparison of transverse helicity

As shown in [33], MC@NLO and POWHEG give a rather good description of inclusive $W$ production. However both generators were shown [38] to underestimate the fraction of events at large $p_T^W$ (see also Table 1). While this affects the relative fraction of data versus Monte Carlo events retained in the two $p_T^W$ bins of the analysis, it should not significantly impact the angular distributions used to measure the $W$ polarisation. This is discussed in more detail in Sect. 8.3.

Figures 3 and 4 show the cos$\theta_{2D}$ distributions for electrons and muons and both charges, compared to the predictions from MC@NLO and POWHEG and to the expected behaviour of unpolarised $W$ bosons (the unpolarised distributions are obtained by averaging the longitudinal, left- and right-handed MC@NLO templates with equal weights, see Sect. 6.1). The good agreement of

### Table 2

<table>
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<th>Fractions (%)</th>
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<th>$e^-$</th>
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<td>3.1 ± 0.2</td>
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<td>3.6 ± 0.1</td>
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<td>0.4</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>$W \rightarrow \tau\nu$</td>
<td>2.6</td>
<td>2.8</td>
<td>2.3</td>
<td>2.5</td>
</tr>
<tr>
<td>$Z \rightarrow \tau\tau$</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$Z \rightarrow ℓℓ$</td>
<td>2.9</td>
<td>3.9</td>
<td>0.1</td>
<td>0.2</td>
</tr>
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<td>Analysis cuts</td>
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<tr>
<td>jet</td>
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<td>2 ± 2</td>
<td>2.4 ± 0.4</td>
<td>2.5 ± 0.5</td>
</tr>
<tr>
<td>$t\bar{t}$</td>
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<td>0.7</td>
<td>0.6</td>
<td>0.9</td>
</tr>
<tr>
<td>$W \rightarrow \tau\nu$</td>
<td>2.1</td>
<td>2.4</td>
<td>1.8</td>
<td>1.9</td>
</tr>
<tr>
<td>$Z \rightarrow \tau\tau$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$Z \rightarrow ℓℓ$</td>
<td>2.9</td>
<td>3.9</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>Analysis cuts</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>($p_T^W$ &gt; 50 GeV)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>jet</td>
<td>2 ± 2</td>
<td>2 ± 2</td>
<td>2.0 ± 0.3</td>
<td>2.5 ± 0.4</td>
</tr>
<tr>
<td>$t\bar{t}$</td>
<td>2.8</td>
<td>4.1</td>
<td>3.5</td>
<td>5.0</td>
</tr>
<tr>
<td>$W \rightarrow \tau\nu$</td>
<td>2.1</td>
<td>2.0</td>
<td>1.9</td>
<td>2.0</td>
</tr>
<tr>
<td>$Z \rightarrow \tau\tau$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$Z \rightarrow ℓℓ$</td>
<td>2.6</td>
<td>3.5</td>
<td>0.3</td>
<td>0.4</td>
</tr>
</tbody>
</table>
both the MC@NLO and POWHEG distributions with data is demonstrated also by the $\chi^2$ values reported in Table 3. It is also clear from Table 3 and Figs. 3 and 4 that the production of unpolarised $W$ bosons does not match the data.

For the electron channel, the jet background clusters around $\cos \theta_{2D} = 1$, which supports the assumption that these were two-jet events, where one of the jets was misidentified as an electron. On the other hand, in the muon channel, the jet background clusters around $\cos \theta_{2D} = -1$, in agreement with the assumption that the background originates mainly from semi-leptonic decay of heavy-flavour in jets.

6 Helicity templates and Monte Carlo closure test

6.1 Construction of helicity templates

In order to measure the helicity fractions, it is necessary to construct $\cos \theta_{2D}$ distributions corresponding to samples of longitudinal, left- and right-handed $W$ bosons that decay...
Fig. 4 The $\cos \theta_{2D}$ distributions for $p_T^W > 50$ GeV. The data (dots) are compared to the distributions from POWHEG (dashed line), MC@NLO (solid line), and for unpolarised $W$ bosons (dotted line) in the muon (top) and electron (bottom) channel, split by charge. The bottom parts of each plot represent the ratio of data, POWHEG and unpolarised distributions to MC@NLO.

The $\cos \theta_{3D}$ distributions corresponding to the three polarisation states were also made. All these distributions are called helicity templates in the following. The templates were built independently from MC@NLO and from POWHEG using the following reweighting technique.

It was first verified that, at the generator-level, and in bins of limited size in $p_T^W$ and $y_W$, $W$ decays generated with the Monte Carlo simulations are well described by Eq. (3). The generator-level $\cos \theta_{3D}$ distributions were then fitted with the distribution corresponding to this equation, which gave the values of $f_L$, $f_0$ and $f_R$ in $y_W$ and $p_T^W$ bins. The results, in terms of $f_0$ and $f_L - f_R$, are shown in Fig. 5 for MC@NLO. The size of the bins results from a compromise between the rate of variation of the coefficients and the size of the available samples.

Several conclusions may be drawn from Fig. 5. The longitudinal fraction, which is very small for low $p_T^W$, grows with $p_T^W$ (especially at low $|y_W|$), before flattening out and then starting to decrease. The difference between the fractions of left- and right-handed $W$ bosons is small for low $|y_W|$ and grows quickly with $|y_W|$, reaching up to 70% for $|y_W| = 3$. As already explained in Sect. 1, a smaller left-right difference is expected for negative than for positive $W$ bosons; however in the $p_T^W$ range analysed here, these differences differ by at most a few percent. The analysis of systematic uncertainties described in Sect. 8.5, shows that it is experimentally advantageous to average the measured values of $f_L - f_R$ between the two charges. As an anticipation of this observation, it can be seen in Fig. 5 that this averaging is physically meaningful.

An equivalent analysis for POWHEG shows a similar trend for $f_L - f_R$ as observed for MC@NLO. For $f_0$, in the $p_T^W$ range analysed here, POWHEG exhibits a much flatter dependence on $y_W$ than MC@NLO, the average values being, however, very close to each other. Analytical calculations at NNLO reported in [7] by the BlackHat collaboration are very close to POWHEG. This is illustrated in Fig. 6.

Samples representing longitudinal, left- and right-handed states are obtained by reweighting the MC@NLO or POWHEG
Fig. 5  Computed values of $f_0$ (top) and $f_L - f_R$ (bottom) using fits with Eq. (3) to MC@NLO samples in $|(y_W|, p_T^W)$ bins, split by charge. These values are used to calculate the weights needed to create helicity templates.

Fig. 6  Evolution of the longitudinal polarisation fraction as a function of $|y_W|$, in MC@NLO, POWHEG and a calculation based on BlackHat, for $W^+$ (top) and $W^-$ (bottom) for two $p_T^W$ bins.
simulated events according to:

\[
\frac{1}{f_\ell (1 \pm \cos \theta_{3D})^2} + \frac{3}{8} f_R (1 \pm \cos \theta_{3D})^2 \pm \frac{3}{4} f_0 \sin^2 \theta_{3D}
\]

where

\[
\frac{1}{\sigma^\pm} \frac{d\sigma^\pm}{d\cos \theta_{3D}} \bigg|_{L/R} = \frac{3}{8} \begin{cases} (1 \pm \cos \theta_{3D})^2 \\ 2 \sin^2 \theta_{3D} \end{cases} \frac{(1 \pm \cos \theta_{3D})^2}{(1 \pm \cos \theta_{3D})^2}
\]

and where the denominator corresponds to the general form of the differential cross-section in which the coefficients are taken from Fig. 5 (or its equivalent from POWHEG), for the corresponding value of \( p_T^W \) and \( |y_W| \). In these equations, the upper (lower) sign corresponds to \( W^+ (W^-) \) boson.

6.2 Fit procedure applied to Monte Carlo samples

The fitting procedure with templates was first applied to the simulated samples, at three different levels:

– all events using generator information for \( \cos \theta_{3D} \) distributions;
– events remaining after applying acceptance and \( m_W^T \) cuts using generator information for \( \cos \theta_{3D} \) distributions;
– events after the complete event selection (standard plus analysis cuts), using fully simulated information followed by reconstruction for \( \cos \theta_{2D} \) distributions.

The fits of \( \cos \theta_{3D} \) and \( \cos \theta_{2D} \) distributions were performed using a binned maximum-likelihood fit [39, 40]. Since the parameters of the fit, \( f_0 \), \( f_\ell \) and \( f_R \), must sum to 1, only two independent parameters, chosen to be \( f_0 \) and \( f_\ell - f_R \), are reported. The parameters were not individually constrained to be between 0 and 1.

For the second and third steps, numerical results for \( f_0 \) and \( f_\ell - f_R \) fits are summarised in Table 4 for \( 35 < p_T^W < 50 \) GeV and \( p_T^W > 50 \) GeV. In Table 4 and in the following, the coefficients \( f_0 \) and \( f_\ell - f_R \) represent helicity fractions, averaged over \( y_W \), within a given \( p_T^W \) bin.

Template fit results using the \( \cos \theta_{3D} \) distributions at the generator-level, without any cut, reproduce the average value of the numbers quoted in the relevant \( p_T^W \) bin of Fig. 5. With respect to these fit results, the numbers shown in the first lines of Table 4 for the two \( p_T^W \) bins reflect the effect of the acceptance and \( m_W^T \) cuts, which is small on \( f_0 \) but is sizeable on \( f_\ell - f_R \), typically reducing it by 25 % (relative). Indeed, the detector has a small acceptance for the events produced at high \( |y_W| \), for which \( f_\ell - f_R \) is largest.

Comparisons of the first row of each part of Table 4 (\( \cos \theta_{3D} \) at generator-level, within acceptance) to the second row (\( \cos \theta_{2D} \) after full simulation) indicates that the values of \( f_0 \) are rather stable for \( W^- \) while for \( W^+ \) there is in several cases a significant increase. Similar effects are observed with POWHEG. Corrections applied at the analysis level (see Sect. 8.6) are intended to remove these effects to obtain the final, corrected results.

7 Fit results

The raw helicity fractions for each of the four analysed channels were obtained by fitting the experimental \( \cos \theta_{2D} \) distributions, after background subtraction, with a sum of templates (see Eq. (3)) corresponding to longitudinal, left- and right-handed states.

In order to correct for systematic effects associated with the choice of the variable used in the fit (\( \cos \theta_{2D} \)), and for

| Table 4 | Results (as percentages) of fitting \( \cos \theta_{3D} \) and \( \cos \theta_{2D} \) distributions from MC@NLO simulated samples using helicity templates. The fits are performed at generator-level, after applying acceptance and \( m_W^T \) cuts, and on fully simulated events, after applying standard plus analysis selections using \( \cos \theta_{2D} \). |

<table>
<thead>
<tr>
<th>( \mu^+ )</th>
<th>( \mu^- )</th>
<th>( e^+ )</th>
<th>( e^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td>35 &lt; ( p_T^W &lt; 50 ) GeV</td>
<td>( \cos \theta_{3D} ) generator-level</td>
<td>( f_0 ) (%)</td>
<td>14.6 ± 0.8</td>
</tr>
<tr>
<td>( \cos \theta_{3D} ) generator-level</td>
<td>( f_\ell - f_R ) (%)</td>
<td>27.9 ± 0.7</td>
<td>26.5 ± 0.8</td>
</tr>
<tr>
<td>( \cos \theta_{2D} ) fully simulated</td>
<td>( f_0 ) (%)</td>
<td>30.1 ± 2.4</td>
<td>19.5 ± 2.2</td>
</tr>
<tr>
<td>( \cos \theta_{2D} ) fully simulated</td>
<td>( f_\ell - f_R ) (%)</td>
<td>31.8 ± 1.4</td>
<td>26.5 ± 1.2</td>
</tr>
<tr>
<td>( p_T^W \geq 50 ) GeV</td>
<td>( \cos \theta_{3D} ) generator-level</td>
<td>( f_0 ) (%)</td>
<td>18.3 ± 1.0</td>
</tr>
<tr>
<td>( \cos \theta_{3D} ) generator-level</td>
<td>( f_\ell - f_R ) (%)</td>
<td>26.9 ± 0.8</td>
<td>25.8 ± 0.9</td>
</tr>
<tr>
<td>( \cos \theta_{2D} ) fully simulated</td>
<td>( f_0 ) (%)</td>
<td>25.1 ± 1.9</td>
<td>20.7 ± 2.2</td>
</tr>
<tr>
<td>( \cos \theta_{2D} ) fully simulated</td>
<td>( f_\ell - f_R ) (%)</td>
<td>29.7 ± 1.1</td>
<td>26.2 ± 1.2</td>
</tr>
</tbody>
</table>
resolution effects, the raw results have been corrected in a second step by the differences observed in Monte Carlo events between the fits at the generator level with the $\cos\theta_{3D}$ distribution after acceptance plus $m_W$ cuts and the fit on $\cos\theta_{3D}$ distributions after full simulation. The two sets of templates obtained from MC@NLO or from POWHEG were used, and their bias corrected for accordingly. Differences between the results obtained with the two Monte Carlo generators were used to estimate a systematic uncertainty associated with the choice of templates (see Sect. 8.6).

The minimisation [39] gives the uncertainties and correlations between the parameters. The $\chi^2$ values, in Table 5, obtained using MC@NLO and POWHEG templates, are similar. They are significantly lower, in most cases, than in Table 3, especially for muons, even taking into account that the number of degrees of freedom is reduced from 19 to 17.

The values of the fitted parameters, using MC@NLO and POWHEG templates, are reported in Table 6. The contributions of the individual fitted helicity states, and their sum, are also shown, for the MC@NLO case, in Fig. 7 for $35 < p_T^W < 50$ GeV, and in Fig. 8 for $p_T^W > 50$ GeV. These histograms show the contributions of each polarisation state (separately and summed together), with a normalisation which, in addition to the value of $f_0$, $f_L$ and $f_R$, also takes into account the relative average acceptance for each of the three polarisation states. The data show a dominance of the left-handed over the right-handed fraction in about the same proportion as in the Monte Carlo simulations.

The $f_0$ values obtained with the POWHEG templates are in general larger (see Table 6). For the negative charges, the increase of $f_0$ is correlated with a decrease of $f_L - f_R$, while for positive charges the reverse is observed, though with a smaller increase, especially in the higher $p_T^W$ bin.

8 Systematic effects

In addition to the choice of templates, which is treated separately, the measurement suffers from systematic effects due to limited knowledge of backgrounds, charge misidentification, choice of PDF sets, uncertainties on the lepton energy scale and resolution, and uncertainties on the recoil system energy scale and resolution. The uncertainties on helicity fractions have been estimated using MC@NLO and are reported in Table 7, in absolute terms.

The effect of reweighting simulated events to restore a $p_T^W$ distribution closer to that observed [38] was also assessed.

8.1 Backgrounds

The electroweak and $t\bar{t}$ backgrounds have been studied previously and found to be well modelled by Monte Carlo simulations [33, 41–43]. As these backgrounds are subtracted

<p>| Table 5 | Values of the $\chi^2$ from the fit of data with MC@NLO and POWHEG helicity templates (see Figs. 7 and 8 for MC@NLO). The number of degrees of freedom in the fits is 17 |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|</p>
<table>
<thead>
<tr>
<th>$\chi^2$ between data and $35 &lt; p_T^W &lt; 50$ GeV</th>
<th>$50 &lt; p_T^W$</th>
<th>$\mu^+$</th>
<th>$\mu^-$</th>
<th>$e^+$</th>
<th>$e^-$</th>
<th>$\mu^+$</th>
<th>$\mu^-$</th>
<th>$e^+$</th>
<th>$e^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC@NLO templates</td>
<td></td>
<td>13.5</td>
<td>23.1</td>
<td>7.6</td>
<td>25.3</td>
<td>29.3</td>
<td>21.1</td>
<td>24.8</td>
<td>16.9</td>
</tr>
<tr>
<td>POWHEG templates</td>
<td></td>
<td>11.1</td>
<td>20.7</td>
<td>8.2</td>
<td>20.8</td>
<td>30.1</td>
<td>26.6</td>
<td>20.9</td>
<td>13.1</td>
</tr>
</tbody>
</table>

| Table 6 | Summary of raw data results for helicity fractions (as percentages) for $35 < p_T^W < 50$ GeV and $p_T^W > 50$ GeV obtained with MC@NLO or with POWHEG template fits (see Figs. 7 and 8 for MC@NLO). The errors represent the statistical uncertainties only |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\mu^+$ | $\mu^-$ | $e^+$ | $e^-$ | $50 < p_T^W$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Data with MC@NLO | $f_0$ (%) | 26.6 ± 5.1 | 10.9 ± 5.6 | 23.2 ± 5.7 | 9.9 ± 10.2 |
| $f_L - f_R$ (%) | 20.6 ± 3.9 | 27.1 ± 4.3 | 17.9 ± 4.2 | 33.0 ± 4.0 |
| Data with POWHEG | $f_0$ (%) | 42.8 ± 5.1 | 35.1 ± 5.7 | 36.9 ± 9.1 | 26.5 ± 6.1 |
| $f_L - f_R$ (%) | 25.6 ± 3.9 | 21.8 ± 4.3 | 21.3 ± 5.3 | 25.1 ± 4.3 |
| Data with MC@NLO | $p_T^W > 50$ GeV |
| $f_0$ (%) | 8.3 ± 5.0 | −0.0 ± 7.3 | 9.7 ± 5.7 | 20.0 ± 5.6 |
| $f_L - f_R$ (%) | 27.5 ± 3.3 | 29.9 ± 3.4 | 29.3 ± 3.5 | 19.7 ± 3.9 |
| Data with POWHEG | $f_0$ (%) | 15.3 ± 4.4 | 13.0 ± 5.0 | 19.6 ± 5.7 | 26.6 ± 6.9 |
| $f_L - f_R$ (%) | 27.7 ± 3.2 | 19.9 ± 3.6 | 29.5 ± 3.6 | 13.3 ± 4.2 |
from data for the final fit, an associated systematic uncertainty has been estimated by changing the global normalisation of the subtracted distributions by $\pm 6.8\%$ ($\pm 3.4\%$ to take into account the uncertainty on the integrated luminosity, $\pm 5\%$ for the uncertainty on background cross-sections relative to signal, and $\pm 3\%$ for the influence of PDFs on the acceptance [44]).

Furthermore, the amount of jet background was varied inside the uncertainty estimated by the dedicated fit (see Table 2).

8.2 Charge misidentification

Since charge misidentification is well reproduced by simulations [34], the possible associated effect on the results presented here has been measured by comparing helicity fractions extracted from fully simulated events where the charge assignment was taken either from generator-level information or after full reconstruction. The effect on $f_0$ and $f_L - f_R$ is estimated to be about 0.4% in the electron case, and is negligible for muons.

8.3 Reweighting of $p_T^W$ distribution

MC@NLO and, to a lesser extent POWHEG, underestimate the fraction of $W$ events at high $p_T^W$. In order to investigate the possible consequences of such a bias on this measurement, the MC@NLO Monte Carlo signal sample, weighted event-by-event so as to restore a $p_T^W$ spectrum compatible with data, was fitted using unchanged helicity templates (both POWHEG and MC@NLO templates were used for this test). The effect of the reweighting was found to have a small impact on the fitted values of $f_0$ (less than 2%). For $f_L - f_R$ sizeable effects were observed (up to 5% in the low $p_T^W$ bin). However, they are of opposite sign for the positive and negative lepton charges, and almost perfectly cancel when analysing charge-averaged values (see Table 7).

8.4 PDF sets

Using the PDF reweighting method, the uncertainty associated with PDFs was estimated by keeping the templates unchanged and using MSTW 2008 and HERAPDF 1.0 instead of the CTEQ 6.6 PDFs for the simulation of the signal...
Fig. 8 Results of the fits to \( \cos \theta_{2D} \) distributions using helicity templates (built from MC@NLO), for \( W \to \mu \nu \) (top) and \( W \to e \nu \) (bottom) events in data with \( p_T^W > 50 \) GeV, after background subtraction. Each template distribution is represented: left-handed contribution (dashed line), longitudinal contribution (dotted-dashed line) and right-handed contribution (dotted line).

The impact on \( f_0 \) and \( f_L - f_R \) is in the range of 1 % to 2 %.

8.5 Energy scales

While a coherent change of the lepton and recoil energy scales would leave the angles in the transverse plane unchanged, both in the laboratory and in the transverse \( W \) rest frame, an effect on \( \cos \theta_{2D} \) arises when only one of the two measured objects (lepton, recoil) changes, or if they change by different amounts.

Using simulated events, it has been observed that an increase of the lepton transverse momentum alone gives a positive slope to the \( \cos \theta_{2D} \) distribution, which in turn induces an increase of the left-handed fraction in the negative lepton sample, and a decrease of the left-handed fraction in the positive lepton sample. As expected, the reverse happens for an increase of the recoil transverse energy.

The value of \( f_L - f_R \) when averaged over the two charges is largely independent of the lepton and recoil energy scales, as can be seen in Table 7.

The same compensation mechanism is however not present for \( f_0 \), for which an increase in the recoil energy scale induces an increase of \( f_0 \) for both charges.

The lepton energy scale is precisely determined from \( Z \to \ell \ell \) decays: using the precisely-known value of the \( Z \) boson mass, scale factors have been extracted by \( \eta_\ell \) regions, which in the muon case depend also on the muon charge \([34, 45]\). The reconstructed \( Z \) boson mass spectrum has also been used to derive smearing corrections to be applied to Monte Carlo electrons and muons in order to reproduce the observed \( Z \) mass peak resolution. The resulting uncertainties are about 3 % to 5 % on \( f_0 \) and around 2 % on \( f_L - f_R \).

The recoil system in general contains one or several jets with \( p_T > 20 \) GeV, and may also include additional “soft jets” (\( 7 < p_T < 20 \) GeV), and clusters of calorimeter cells not included in the above objects. The uncertainty on the energy scale of these objects (typically 3 % for jets, 10.5 % for soft jets and 13.5 % for isolated clusters) was propagated as described in \([46]\). This is the largest systematic uncertainty on the helicity fractions measured in this study. In the worst case (muons in the low \( p_T^W \) bin), the resulting uncertainty
on $f_0$ is 16 %. This uncertainty is largely correlated between the muon and electron channels.

Given the anti-correlation observed between the impacts on positive and negative leptons, the uncertainties from energy scale variations enter with ± or ∓ in Table 7, depending on whether the effect goes in the same direction as an energy increase or in the opposite direction. As already pointed out, in the case of $f_L - f_R$ the effects largely cancel when considering the average between negative and positive charges.

### 8.6 Choice of the Monte Carlo generator

The results of the template fits to real and fully simulated data are affected by the imperfect correlation between $\cos \theta_{2D}$ and $\cos \theta_{3D}$ and by resolution effects.

In order to compare results directly to theoretical models, the raw results from Sect. 7 are corrected by adding the difference, found using simulations, between the “true” values which would be given by fits to $\cos \theta_{3D}$ distributions obtained at the generator level within acceptance and $m_W$ cuts as used here, and the results obtained using fully-simulated $\cos \theta_{2D}$ distributions. In order to be able to average results from muons and electrons, the electron results are corrected to the same $\eta_\ell$ acceptance as for muons (i.e. without the barrel-endcap calorimeters overlap region around 1.5, and with a maximum $|\eta_\ell|$ value of 2.4).

The corrections for results obtained using MC@NLO templates were determined from the difference between:

- results of a fit of MC@NLO (3D) templates to $\cos \theta_{3D}$ distributions of the POWHEG Monte Carlo samples at the generator-level with acceptance and $m_W$ cuts;
- results of a fit of MC@NLO (2D) templates to $\cos \theta_{2D}$ distributions of the same POWHEG Monte Carlo samples, after full simulation and with standard plus analysis cuts.

The corrections for results obtained using POWHEG templates were derived in the same way as above, interchanging the roles of MC@NLO and POWHEG.

In a further step, after averaging over the charges for each lepton flavour:

- the corrected data result, for $f_L - f_R$ and $f_0$, was obtained by averaging the numbers obtained with MC@NLO and with POWHEG templates;

### Table 7 Summary of systematic uncertainties on helicity fractions for $35 < p_T^W < 50$ GeV and $p_T^W > 50$ GeV. The effect of lepton and recoil energy scales, and of $p_T^W$ reweighting, on $f_L - f_R$ is also estimated on the mean between the two charges. The larger errors appear with the ± (±) sign if they vary in the same (opposite) direction as the parameter studied, in order to highlight the correlations used in calculating the errors on the means.

<table>
<thead>
<tr>
<th>Source</th>
<th>$35 &lt; p_T^W &lt; 50$ GeV</th>
<th>$p_T^W &gt; 50$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>EW background</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta f_0$ (%)</td>
<td>0.5</td>
<td>0.6</td>
</tr>
<tr>
<td>$\delta (f_L - f_R)$ (%)</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>Jet background</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta f_0$ (%)</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>$\delta (f_L - f_R)$ (%)</td>
<td>0.3</td>
<td>0.7</td>
</tr>
<tr>
<td>$p_T^W$ scale</td>
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<td></td>
</tr>
<tr>
<td>$\delta f_0$ (%)</td>
<td>±4.5</td>
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<tr>
<td>$\delta (f_L - f_R)$ (%)</td>
<td>±2.5</td>
<td>±2.0</td>
</tr>
<tr>
<td>Recoil scale</td>
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</tr>
<tr>
<td>$\delta f_0$ (%)</td>
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<td>±16.8</td>
</tr>
<tr>
<td>$\delta (f_L - f_R)$ (%)</td>
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<td>±10.9</td>
</tr>
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<td>PDF set</td>
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<td></td>
</tr>
<tr>
<td>$\delta f_0$ (%)</td>
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<td>2.0</td>
</tr>
<tr>
<td>$\delta (f_L - f_R)$ (%)</td>
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<td>1.5</td>
</tr>
<tr>
<td>Charge misID</td>
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<td></td>
</tr>
<tr>
<td>$\delta f_0$ (%)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\delta (f_L - f_R)$ (%)</td>
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<td>0.4</td>
</tr>
<tr>
<td>$p_T^W$ resolution</td>
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</tr>
<tr>
<td>$\delta f_0$ (%)</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$\delta (f_L - f_R)$ (%)</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$p_T^W$ reweighting</td>
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<td></td>
</tr>
<tr>
<td>$\delta f_0$ (%)</td>
<td>2.5</td>
<td>1.1</td>
</tr>
<tr>
<td>$\delta (f_L - f_R)$ (%)</td>
<td>±4.9</td>
<td>±5.2</td>
</tr>
</tbody>
</table>
The corrected results and the associated systematic uncertainties are shown in Table 8 for \( f_L - f_R \) and \( f_0 \).

The systematic uncertainty associated with the differences between the two sets of templates is large for \( f_0 \), for which other systematic effects are also large.

Another correction procedure was tried, using the same Monte Carlo generator for producing the templates and calculating the corrections. The resulting central values of the helicity fractions are very close to those shown in Table 8 (within less than 2%), but the systematic uncertainties of the corrections are slightly larger (by about 10% in relative terms).

Finally, a full simulation based on SHERPA 1.2.2 [47], made only for the electron channel, was also used to obtain, similarly as above, first raw results, and then correction terms found by applying SHERPA templates to simulated data produced with both MC@NLO and POWHEG. The corrected measurement obtained in this way are shown in Table 9, together with the “electron average” results from Table 8. In the case of SHERPA, only the uncertainty associated with the choice of template is reported. A very good agreement is observed.

### 9 Results

The corrected final measurements of \( f_L - f_R \), already shown in Table 8, are compared in Table 10 to the values obtained from the MC@NLO and POWHEG samples, at the generator-level with the acceptance and \( m_T^W \) cuts, using a template fit to the \( \cos \theta_{3D} \) distributions.

In the low \( p_T^W \) bin the data lie in between the MC@NLO and POWHEG predictions, slightly closer to the former. For \( p_T^W > 50 \text{ GeV} \), the data are close to the MC@NLO values, while POWHEG predicts a somewhat smaller difference between left- and right-handed states than observed in the data.

The same good agreement between data and MC@NLO remains after averaging results over lepton flavours (Table 11). While the complete NNLO cross-section calculation of [7] has not been implemented in a Monte Carlo generator, it can be seen in Fig. 5 and its equivalent (not shown) for BlackHat, that at the particle level, without any cuts, the \( f_L - f_R \) values from [7] are on average about 5% lower (in absolute terms) than the MC@NLO predictions. They are thus quite close to POWHEG and somewhat lower than the data.

The measurements shown in Table 11, where all systematic uncertainties have been combined, are the main result of this study concerning \( f_L - f_R \), and the directly related coefficient \( A_4 \) (Eq. (5)).

For \( f_0 \), and the directly related coefficient \( A_0 \) (Eq. (4)), the systematic uncertainties associated with the recoil and lepton energy scales do not cancel between negative and positive charges. In order to reduce the statistical uncertainties, which are also large, and the uncorrelated instrumental and analysis systematic uncertainties, the measurements in each \( p_T^W \) bin were averaged over charges and lepton flavours. The uncertainties from the recoil energy scale were taken to be fully correlated among all four measurements. The uncertainty associated with the template model (Table 8) was combined quadratically with the other systematic uncertainties.

A comparison between the corrected experimental results and the predicted values, within the acceptance and \( m_T^W \) cuts (Table 11), indicates that:

- in the low \( p_T^W \) bin the data are compatible with both MC@NLO and POWHEG predictions, which are mutually consistent;
- in the high \( p_T^W \) bin, the data favour \( f_0 \) values smaller than the predictions of MC@NLO and POWHEG, which are close to each other.

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**Table 8** Percentage values of \( f_L - f_R \) and \( f_0 \) averaged over charges, separately for electrons and muons, obtained by averaging results with templates from MC@NLO (see Figs. 7 and 8) and from POWHEG. The first uncertainty is statistical, the second covers the systematic uncertainties from instrumental and analysis effects, and the last one the differences between templates constructed with the two generators.

<table>
<thead>
<tr>
<th>( 35 &lt; p_T^W &lt; 50 \text{ GeV} )</th>
<th>( p_T^W &gt; 50 \text{ GeV} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_L - f_R ) (%)</td>
<td>( f_L - f_R ) (%)</td>
</tr>
<tr>
<td>Muon average</td>
<td>21.7 ± 3.0 ± 3.6 ± 2.0</td>
</tr>
<tr>
<td>Electron average</td>
<td>26.0 ± 2.8 ± 3.4 ± 2.0</td>
</tr>
<tr>
<td>( f_0 ) (%)</td>
<td>( f_0 ) (%)</td>
</tr>
<tr>
<td>Muon average</td>
<td>23.6 ± 3.8 ± 12.0 ± 7.2</td>
</tr>
<tr>
<td>Electron average</td>
<td>20.1 ± 6.9 ± 12.0 ± 5.0</td>
</tr>
</tbody>
</table>

---

**Table 9** Corrected values of \( f_L - f_R \) and \( f_0 \) (as percentages) obtained using SHERPA templates, compared to the standard result (Table 8), for the electron channels averaged over charges. In the SHERPA case the only uncertainty quoted is associated with the two ways of calculating the correction term: applying SHERPA templates either to MC@NLO or to POWHEG simulated data.

<table>
<thead>
<tr>
<th>( 35 &lt; p_T^W &lt; 50 \text{ GeV} )</th>
<th>( p_T^W &gt; 50 \text{ GeV} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_L - f_R ) (%)</td>
<td>( f_L - f_R ) (%)</td>
</tr>
<tr>
<td>Data (SHERPA)</td>
<td>25.5 ± 2.2</td>
</tr>
<tr>
<td>Data (standard)</td>
<td>26.0 ± 2.8 ± 3.4 ± 2.0</td>
</tr>
<tr>
<td>( f_0 ) (%)</td>
<td>( f_0 ) (%)</td>
</tr>
<tr>
<td>Data (SHERPA)</td>
<td>21.0 ± 9.1</td>
</tr>
<tr>
<td>Data (standard)</td>
<td>20.1 ± 6.9 ± 12.0 ± 5.0</td>
</tr>
</tbody>
</table>
Due to the large uncertainties on the measurements, however, no stringent constraints nor clear inconsistencies can be deduced. The measured values of $f_L - f_R$ and $f_0$ are plotted in Fig. 9 within the triangular region allowed by the constraint $f_L + f_0 + f_R = 1$, together with the predictions from MC@NLO and POWHEG.

### 10 Summary and conclusions

The results presented in this paper show that MC@NLO and POWHEG reproduce well the shape of the angular distributions in the transverse plane of charged leptons from high-$p_T$ W boson decays ($p_T^W > 35$ GeV), a regime where the leading-quark effect in quark-antiquark annihilation is subordinate to the dynamics of quark-gluon interactions producing W bosons.

The variable used for the analysis in terms of helicity fractions (respectively $f_0$, $f_L$ and $f_R$) is the cosine of the "transverse helicity" angle $\cos \theta_{2D}$. Given that the three helicity fractions are constrained to sum to unity, the independent variables chosen in this study are $f_0$ and $f_L - f_R$. Their values have been derived by fitting $\cos \theta_{2D}$ distributions with templates representing longitudinal, left- and right-handed W bosons. Two sets of templates were used, obtained from MC@NLO and POWHEG.

The experimental results have been corrected for the difference between the distribution of the measured quantity, the "transverse helicity" angle $\cos \theta_{2D}$, and the distribution of the true helicity angle, $\cos \theta_{3D}$. The correction includes resolution effects, as well as systematic differences between the two sets of templates. Corrected results correspond to

---

**Table 10** Corrected values of $f_L - f_R$ (as percentages), averaged over charge, separately for electrons and muons, for the data, MC@NLO and POWHEG, and for $35 < p_T^W < 50$ GeV and $p_T^W > 50$ GeV. For data the first uncertainty is statistical, the second covers the systematic uncertainties from instrumental and analysis effects, and the last one the differences between templates constructed with the two generators. For MC@NLO and POWHEG the uncertainties are only statistical.

<table>
<thead>
<tr>
<th>$35 &lt; p_T^W &lt; 50$ GeV</th>
<th>$p_T^W &gt; 50$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Muon average</td>
</tr>
<tr>
<td>Data</td>
<td>$21.7 \pm 3.0 \pm 3.6 \pm 2.0$</td>
</tr>
<tr>
<td>MC@NLO</td>
<td>$27.2 \pm 0.8$</td>
</tr>
<tr>
<td>POWHEG</td>
<td>$19.9 \pm 0.8$</td>
</tr>
</tbody>
</table>

**Table 11** Corrected values of $f_L - f_R$ and $f_0$ (as percentages), averaged over charges and lepton flavours, for the data, MC@NLO and POWHEG, and for $35 < p_T^W < 50$ GeV and $p_T^W > 50$ GeV (Fig. 9). For data the first uncertainty is statistical, the second covers all systematic uncertainties. For MC@NLO and POWHEG the uncertainties are only statistical.

<table>
<thead>
<tr>
<th>$35 &lt; p_T^W &lt; 50$ GeV</th>
<th>$p_T^W &gt; 50$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_L - f_R$ (%)</td>
</tr>
<tr>
<td>Data</td>
<td>$23.8 \pm 2.0 \pm 3.4$</td>
</tr>
<tr>
<td>MC@NLO</td>
<td>$27.1 \pm 0.7$</td>
</tr>
<tr>
<td>POWHEG</td>
<td>$19.9 \pm 1.0$</td>
</tr>
</tbody>
</table>
the following acceptance region: $|\eta_\ell| < 2.4$, $p_T^{\ell} > 25$ GeV, $p_T^T > 20$ GeV and $50 < m_W^T < 110$ GeV.

The longitudinal fraction is the most difficult to extract and has rather large systematic uncertainties, especially in the low $p_T^W$ bin, mostly associated with the recoil energy scale and with the choice of Monte Carlo generator. In the low $p_T^W$ bin the data are compatible with both MC@NLO and POWHEG predictions while in the high $p_T^W$ bin, they favour lower values than predicted by either of the simulations, which agree well with each other.

When averaging over charges, $f_L - f_R$ is measured with a small statistical uncertainty and a relatively small systematic uncertainty. The agreement between data and MC@NLO, separately for the four measurements (two lepton flavours and two $p_T^W$ bins) is good. Predictions by POWHEG are somewhat smaller than data, especially in the high $p_T^W$ bin.

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