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Development and simulation of microfluidic Wheatstone bridge for high-precision sensor

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Abstract. In this work we present the results of analytical modeling and 3D computer simulation of microfluidic Wheatstone bridge, which is used for high-accuracy measurements and precision instruments. We propose and simulate a new method of a bridge balancing process by changing the microchannel geometry. This process is based on the "etching in microchannel" technology we developed earlier (doi:10.1088/1742-6596/681/1/012035). Our method ensures a precise control of the flow rate and flow direction in the bridge microchannel. The advantage of our approach is the ability to work without any control valves and other active electronic systems, which are usually used for bridge balancing. The geometrical configuration of microchannels was selected based on the analytical estimations. A detailed 3D numerical model was based on Navier-Stokes equations for a laminar fluid flow at low Reynolds numbers. We investigated the behavior of the Wheatstone bridge under different process conditions; found a relation between the channel resistance and flow rate through the bridge; and calculated the pressure drop across the system under different total flow rates and viscosities. Finally, we describe a high-precision microfluidic pressure sensor that employs the Wheatstone bridge and discuss other applications in complex precision microfluidic systems.

1. Introduction

Lab-on-a-chip microfluidic systems are modern instruments for advanced material processing in biology, analytical chemistry and synthesis [1-3]. There are many simple or complex microfluidic systems for controlled gas or liquid flow through microchannel [4, 5]. Some non-trivial problems of high-precision measurements require new approaches and microfluidic devices.

In our work we investigate a microfluidic Wheatstone bridge (MWB). The system is a functional copy of a traditional electric Wheatstone bridge, which has a wide application in precision measurements. A microfluidic Wheatstone bridge was used for measuring viscosity and elasticity of blood [6]. It is a promising area of application, because MWB accurately assessed blood pulsation and flow in blood vessels, including arteries and capillaries, without any damage to the red blood cells. In [7] MWB was used for rapid sampling and confinement of suspended particles in aqueous solutions. MWB was also applied for measuring liquid electrokinetic properties and flow control based on biomolecular separation processes [8].

Usually microfluidic valves and other complex electronic systems with feedback loop are used to balance the microfluidic Wheatstone bridge [6-8]. This approach has disadvantages, because it involves additional parts in microfluidic system and reduces durability and measurement accuracy. To solve this problem, we suggest to develop a "perfectly balanced" MWB. A common microfabrication

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technology does not allow to produce a precisely balanced MWB, so we need a new technological approach. In this work we describe a novel method to make a "permanently balanced" microfluidic Wheatstone bridge, and present the results of the MWB investigation under different conditions.

2. Microfluidic Wheatstone bridge

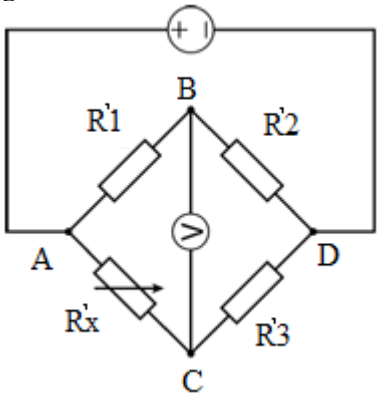


Figure 1. Electric circuit of the Wheatstone bridge.

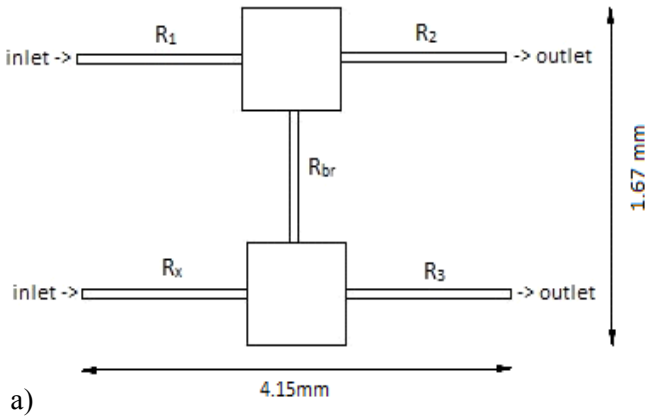
A classical Wheatstone bridge [9] is an electrical circuit shown in Figure 1. This scheme is used for measuring resistance R_2' by balancing the bridge with a varying resistance R_x'. If the bridge is balanced then the voltage between points C and B is zero. In this case equation (1) describes the branch resistance ratios:

$$\frac{R_1'}{R_x'} = \frac{R_2'}{R_3'} \tag{1}$$

where R' is an electrical resistance.

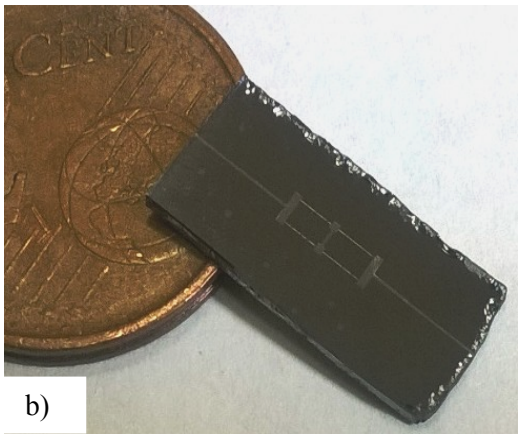
Based on the analogy of electric circuit to incompressible Newtonian fluid flow, fluid flow is analogous to electric current, pressure drop to voltage, and hydraulic resistance to electrical resistance. This way we can design a microfluidic Wheatstone bridge shown in Figure 2a.

A prototype of the MWB have been produced in our lab (see Figure 2b). The microchannels shown in Figure 2a are etched in the silicon wafer. The microfluidic structure was covered by a glass wafer, and then interconnectors for liquid supply were mounted (not shown in Figure 2b).



a)

Figure 2a. Scheme of the microfluidic Wheatstone bridge.



b)

Figure 2b. Silicon-glass prototype of the microfluidic Wheatstone bridge.

For analytical estimations we used a definition of the microchannel hydraulic resistance [10]:

$$R = \frac{\Delta P}{Q} \tag{2}$$

where R is hydraulic resistance, ΔP is pressure difference between the inlet and outlet, Q is flow rate.

Hydraulic resistance for microchannel with rectangular cross-section was calculated from approximation [11]:

$$R = \frac{12 \cdot \eta \cdot L}{1 - 0.63 \cdot \left(\frac{h}{w}\right)} \cdot \frac{1}{h^3 \cdot w} \tag{3}$$

where η is dynamic viscosity, L is length of the microchannel, h is height, w is width of the channel.

Based on equations (1-3) it is possible to design a microfluidic Wheatstone bridge. Experimental production of the MWB prototype has shown that it is difficult to make a balanced MWB because of the technology defects that slightly modify the microchannel resistance.

To solve this problem, we propose a new method to produce a balanced MWB by fine tuning the microchannel geometry. Our method is based on etching in microchannel technology, described in our previous work for silicon on glass wafer [12]. The main idea of the process has three steps: The first step is etching a microfluidic structure in a glass wafer. The second step is bonding glass with silicon wafer. And the third step is supplying silicon etchant into the channel. The solution etches the silicon and changes the geometry of the channel; the hydraulic resistance changes, too. This technology can be applied for many materials and devices. This idea of fine-tuning the microchannel resistivity can be applied to make a permanently balanced MWB without any additional valves and control systems.

To investigate the behavior of the MWB and the process of its balancing, we consider the MWB configuration shown in Figure 2a. Geometrical parameters are listed in Table 1. The bridge was calculated for Newtonian fluids with density range from 600 to 1200 kg/m^3, dynamic viscosity range 1.2-1.9·10^{-5} Pa·s and pressure difference between the inlet and outlet ΔP_{MWB} varied from 1000 to 15000 Pa. Under these conditions the highest Reynolds number is 1.9, which means a laminar flow in the microchannel.

Table 1. Parameters of the microfluidic Wheatstone bridge.

Channel (see Figure 2a)	Length, L (mm)	Height, h (μm)	Width, w (μm)	Resistance, R×10^{12} (Pa/m^3·s)
Channel R1	1.75	30	40	32.80
Channel R2	2.00	30	50	25.44
Channel R3	2.00	30	45	30.31
Channel RX	1.75	30	30-42, increment 1	29.97-62.26
Bridge Channel R_{Br}	0.87	30	150	2.62

3. Computational Fluid Dynamics model

To investigate the microfluidic Wheatstone bridge behavior, we used a computational fluid dynamics approach in 3D geometrical model. We solve the mass conservation law (4) and Navier-Stokes equations (5) for a steady-state laminar flow of incompressible fluid:

$$div(\boldsymbol{V}) = 0, \tag{4}$$

$$\frac{\mathbf{1}}{\boldsymbol{\rho}}\mathbf{grad}\,(\mathrm{p}) = \nu\,\Delta(\boldsymbol{V}) \tag{5}$$

where ρ is density, p is pressure, $\boldsymbol{V}$ is velocity vector, ν is kinematic viscosity.

Boundary conditions for flow velocity on all surfaces of microchannels are:

$$\boldsymbol{V}|_{wall} = 0 \tag{6}$$

Pressure at inlet is fixed at 100000 Pa. Pressure at outlet was varied in the range from 90000 Pa to 99000 Pa in a series of numerical experiments.

Equations (4, 5) were solved numerically by the finite volume method with conjugate gradient algorithm with a preconditioner.

We considered two series of numerical experiments: In the first series, we simulated a balancing process of MWB. For this purpose width of Channel "X" (R_X) was varied from 30 to 42 μm under the fixed pressure difference in MWB. The condition where velocity and pressure drop in the bridge channel (R_{Br}) are zero we will call a balance point (and the MWB is perfectly balanced). In the second series, we change the fluid viscosity for balanced and imbalanced MWB.

4. Results and discussion

Figure 3 presents velocity distribution in the MWB. To measure velocity in the bridge channel (R_{Br}), we take point A in the middle (see Figure 4). Between points B and C we measure the pressure drop (ΔP_{Br}) in the bridge channel.

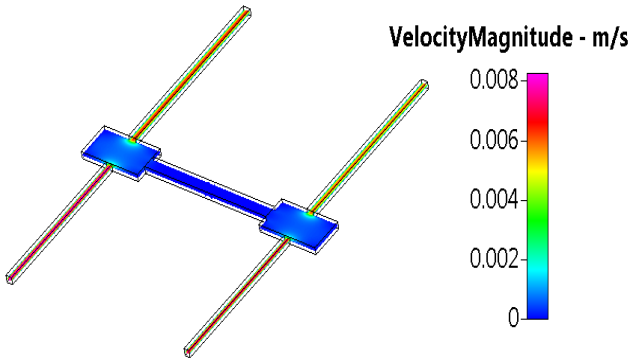


Figure 3. Velocity distribution in the MWB.

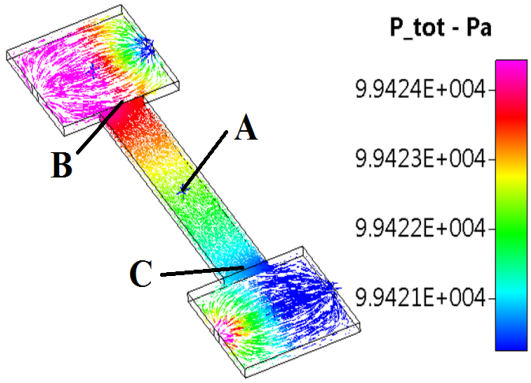


Figure 4. Velocity in the bridge channel

Influence of the width of Channel R_X on pressure drop and velocity in the bridge microchannel is shown in Figure 5. From these data we can find the balance point of MWB. In our case it is 36.39 micron of channel R_X width. It is important to note that the point does not change its position when we change the pressure difference in MWB (or total flow rate in it). Velocity magnitude for imbalanced MWB and P_{MWB} = 5000 Pa has the values consistent with the data given in [7].

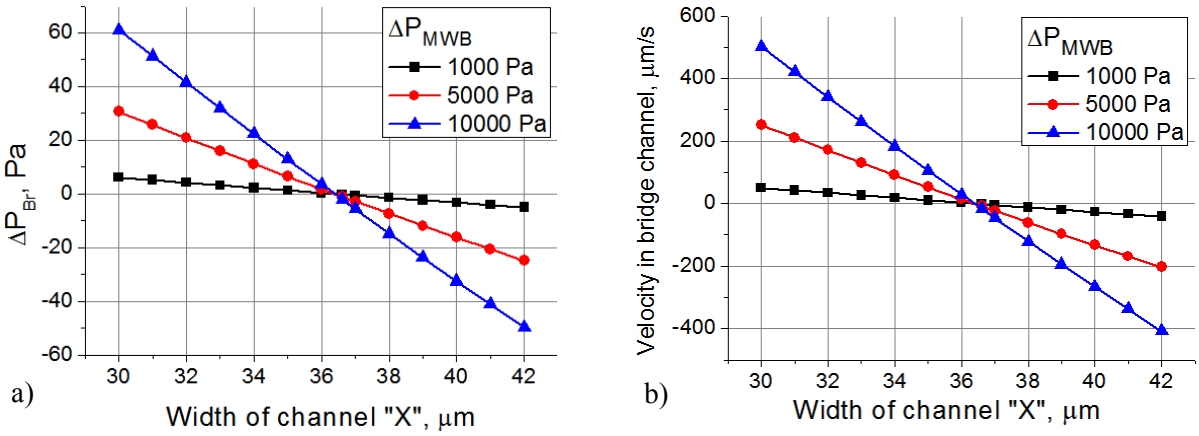


Figure 5. Pressure drop (a) and velocity (b) in the bridge microchannel for different width of the channel R_X and pressure in the MWB. Viscosity is 1.58910^{-5}Pa·s

In the second series of numerical experiments we investigated velocity in the bridge microchannel for different fluid viscosities for two cases of the MWB geometry: near the balance configuration and far from it. The results are presented in Figure 6. The dependences show a similar trend, but have a significant difference in values for balanced and imbalanced bridge: velocity magnitude in the imbalanced bridge (Figure 6b) is 200 times higher than that in the balanced bridge (Figure 6a). Based on this data we conclude that for the perfectly balanced bridge, velocity in the bridge microchannel should be zero and independent from viscosity and temperature. Bridge near the balance point has a high stability from viscosity variation. In addition, it means a temperature stability because liquids usually have a strong dependence of its viscosity from temperature

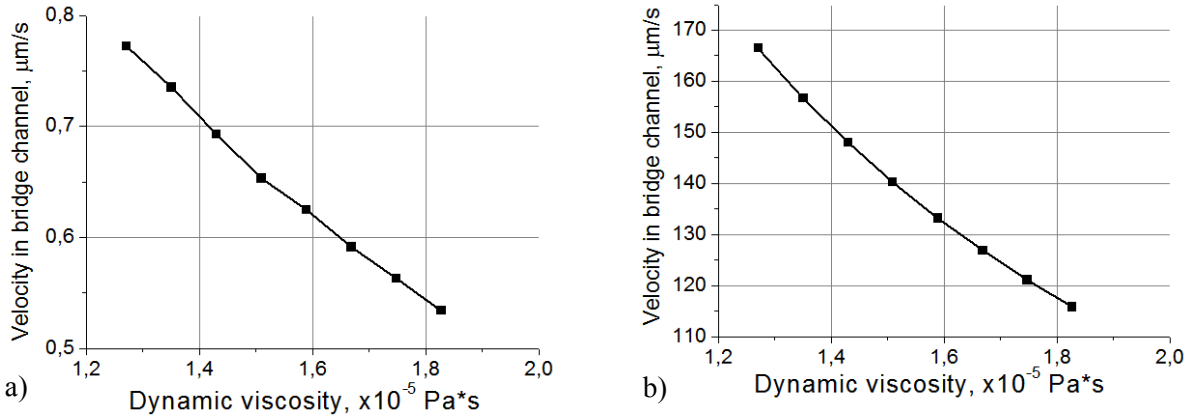


Figure 6. Velocity in the bridge microchannel for different viscosity of fluid and ΔPMWB =5000 Pa. a)Width of "X" channel is 36.4 μm (near balance configuration); b) Width of "X" channel is 40 μm.

One of the most interesting properties of the MWB is the ability to reduce pressure drop in the bridge (ΔP_{Br}) compared to the pressure drop in the system (ΔP_{MWB}) by several orders of magnitude. Let us define the pressure reduction coefficient K as:

$$K=\Delta P_{MWB}/\Delta P_{Br} \quad (7)$$

where ΔP_{MWB} is pressure difference between inlet and outlet of the microfluidic Wheatstone bridge, ΔP_{Br} is pressure difference between two ends of the bridge microchannel.

Figure 7 illustrates the dependence of ΔP_{Br} on ΔP_{MWB} for different width of channel "X". We note its linear trend and conclude that the angle of the slope strongly depends on balancing the MWB. Figure 8 shows a significant growth of pressure reduction coefficient (K) as the width of microchannel "X" is reducing and approaching the balance configuration (width of channel "X" is near 36.39μm). This property of pressure reduction can be applied in high-precision microfluidic devices.

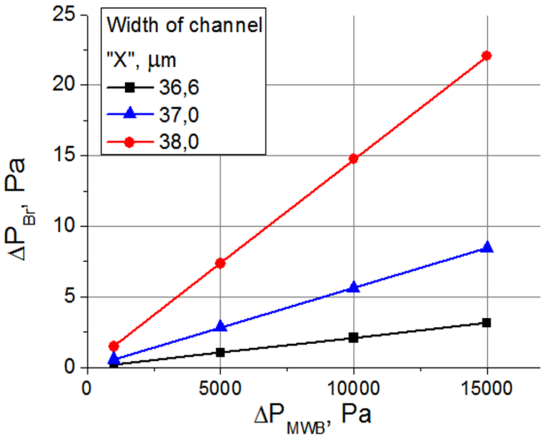


Figure 7 ΔP_{Br} as a function of ΔP_{MWB} for different width of channel "X".

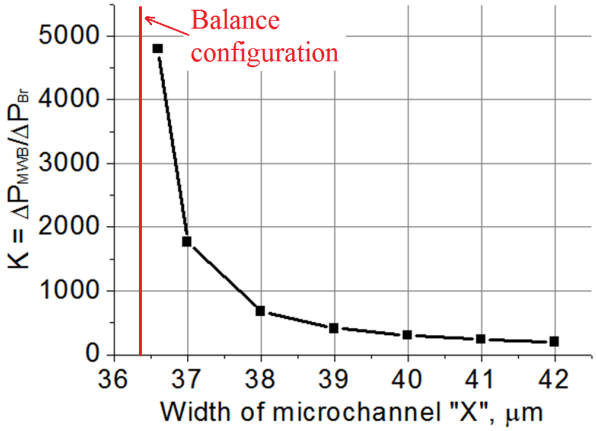


Figure 8 Pressure reduction coefficients for different width of the microchannel "X".

5. Application of microfluidic Wheatstone bridge to high-precision sensors

Based on the studied properties of the microfluidic Wheatstone bridge, we propose a design of high–precision microfluidic pressure sensor. The architecture of the system is shown in Figure 9.

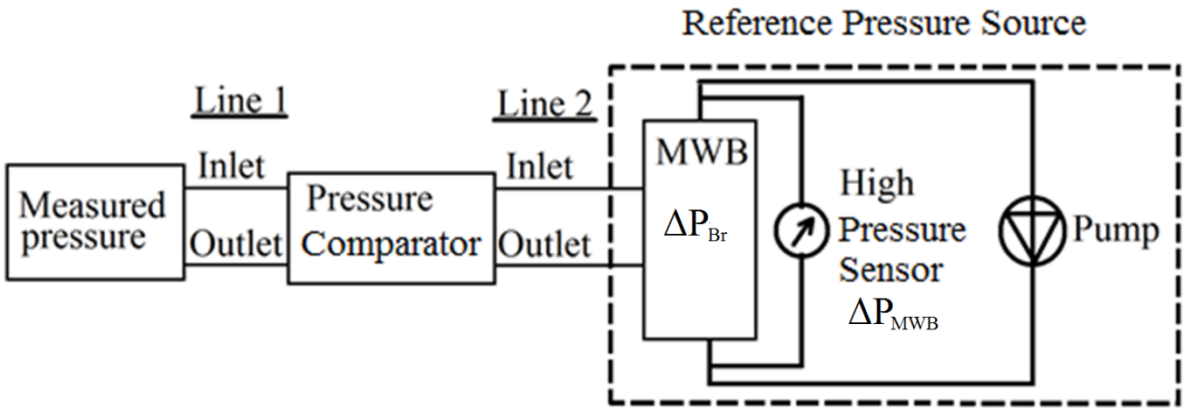


Figure 9. Architecture of the high accuracy microfluidic pressure sensor based on the MWB.

The device has two main parts: the Reference Pressure Source and the pressure Comparator. The Comparator has two measurement lines. Line1 connects to the place where pressure difference should be measured. Line2 connects to the Reference Pressure Source. Pressure difference in Line2 can be varied in some range. The Comparator alternately measures pressure difference in Line 1 and Line 2 and signals when the parameters are equal.

The design of the Reference Pressure Source includes circulation circuit, microfluidic pump microfluidic Wheatstone bridge and High Pressure Sensor. The pump drives a fluid flow in the circuit by providing a pressure drop in the MWB (ΔP_{MWB}).Value of the pressure drop is measured by the High Pressure Sensor. ΔP_{Br} is "K" times lower than ΔPMWB (see equation 7). This pressure drop (ΔP_{Br}) is supplied to Line 2 of Pressure Comparator.

Varying the flow in the circulation circuit, we change a pressure difference in Line 2. When the Pressure Comparator detects an equality of the pressure drops in L1 and L2, we note a value ΔP_{MWB} from High Pressure Sensor. Measured pressure difference in Line 1 (ΔP_{L1}) is calculated by equation 8

$$\Delta P_{L1} = \Delta P_{L2} = \Delta P_{Br} = \Delta P_{MWB}/K \quad (8)$$

where ΔP_{L1} is measured pressure drop (Line 1), ΔP_{L2} is pressure in Line 1 (equal to pressure difference in bridge channel ΔP_{Br}), ΔP_{MWB} is pressure drop in MWB, which is measured by high pressure sensor, K is pressure reduction coefficient.

Based on equation (8) we conclude that error of measurements by High-Pressure Sensor is reduced in K times. This properties of our microfluidic pressure sensor is very important, because it allows to measure a low pressure difference with high accuracy. For example, if ΔP_{MWB} =100 000 Pa, pressure reduction coefficient K = 5000 and error of high pressure sensor is 0.5% (typical value for High-Pressure Sensor) or ε=500 Pa, then the error of low pressure measurement is ε=0.1 Pa. This value corresponds to the best devices for high-precision measurements. It is possible to improve the metrological characteristics of described systems by increasing the pressure reduction coefficient. Such a microfluidic pressure sensor can be applied for measuring a pressure drop in ventilation systems, clean rooms and filters. Using additional devices, it is possible to measure also velocity, flow rate, or fluid viscosity with very high accuracy.

6. Conclusions and future work

In this work we investigated the microfluidic Wheatstone bridge. Starting from the classical electrical Wheatstone bridge, we design a microfluidic structure with similar operation principles. In addition, we propose to use a technology of etching in microchannel for balancing the MWB, improving the stability of the device and simplifying it. Analytical models of hydraulic resistance were used to design the geometry of the MWB. Applying Navier-Stokes equations for a laminar flow of viscous incompressible liquid, we created a 3D computational model of the microfluidic Wheatstone bridge and carried out two series of numerical experiments. We considered a pressure reduction coefficient in MWB as a function of the bridge balance. It was shown that the microfluidic structure near the balance point has a very high pressure reduction coefficient (*K*=5000) and its value can be changed during the production process by etching in microchannel technology. Based on the properties of the MWB, we developed the principles of high-precision microfluidic pressure sensor, which has perfect metrological characteristics and can be applied in many areas.

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References

[1] Sackmann E K, Fulton A L, Beebe D J 2014 *Nature*, *507*(7491), 181-189.
[2] Liu D, et al. *Advanced Materials* 2015, 27.14: 2269-2269.
[3] Nightingale A M, deMello J C, 2013 *Advanced Materials*, *25*(13), 1806-1806.
[4] Mark D, Haeberle S, Roth G, von Stetten F, Zengerle R,.2010 *Chemical Society Reviews*, *39*(3), 1153-1182.
[5] Castillo-Leon J, Svendsen W E 2014 *Lab-on-a-Chip Devices and Micro-Total Analysis Systems: A Practical Guide*. (Springer).
[6] Kang Y J, Lee S-J 2013 *Biomicrofluidics*, *7*(5), 054122.
[7] Tanyeri M, Ranka M, Sittipolkul N and Schroeder C M 2011 *Lab on a Chip*, *11*(24), 4181-4186.
[8] Plecis A and Chen Y 2008*Analytical chemistry*, *80*(10), 3736-3742.
[9] Morris A S, Langari R 2012 *Measurement and instrumentation: theory and application*. (Academic Press).
[10] Tabeling P 2005 *Introduction to Microfluidics* (Great Britain: Oxford University Press)
[11] Bruus H 2007 *Theoretical Microfluidics* (Denmark: Oxford University Press)
[12] Konakov S A, Krzhizhanovskaya V V, 2016 *Journal of Physics: Conference Series,* **681**(1), 012035