The nature of nurture: the role of gene-environment interplay in the development of intelligence

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CHAPTER 5

ON THE INTERPRETATION OF THE CHC FACTOR GC

Abstract

On the one hand, the factors in the Cattell-Horn-Carroll (CHC) model of intelligence (e.g., g, Gf and Gc) are hypothesized to represent individual differences in unique psychological or biological capacities. On the other hand, the CHC factors Gf and Gc are interpreted as representing the theoretical variables fluid and crystallized intelligence in investment theory. This leads to a theoretical conflict because crystallized intelligence is purely a statistical entity. We conclude that if CHC factor Gc represents a capacity it cannot represent crystallized intelligence, and if Gc represents crystallized intelligence, it does not represent a capacity. We also conclude that in investment theory, the CHC factors Gf and g *represent one and the same capacity. In support of these conclusions, we present a reanalysis (confirmatory factor analysis) of a HCA (Human Cognitive Abilities project) data set. As expected, Gc was effectively absent, and Gf and* g *were statistically equivalent. Factors Gc and* g *could be removed from the CHC model without any reduction in model fit. We contend that these factors are redundant as explanatory variables.*

5.1 Is Gc an elusive construct?

In a recent editorial in *Intelligence*, McGrew (2009, p.1) made three recommendations. First, he recommended “to begin using the [Cattell-Horn-Carroll] taxonomy as a common nomenclature for describing research findings and a theoretical framework from which to test hypotheses regarding various aspects of human cognitive abilities”. Second, he recommended “to reinvigorate the investigation of the structure of human intelligence”. Third, he recommended to "access the Carroll [Human Cognitive Abilities Data Set Archive] datasets to test and evaluate structural models of human intelligence with contemporary methods (confirmatory factor analysis)”. Encouraged by this editorial, we hope to contribute to the theoretical framework underlying the Cattell-Horn-Carroll (CHC) model, and to reinvigorate the investigation of the structure of human intelligence. We aim to do so by discussing the interpretation of CHC factor Gc. The discussion sheds light on the relation between the factors Gf, and *g*. Additionally, we test hypotheses stemming from this discussion by applying structural equation modeling to a dataset in the Human Cognitive Abilities Data Set Archive (HCA).

The CHC taxonomy (McGrew, 1997, 2009) is considered to be a well-validated model of human intelligence (e.g. Evans, Floyd, McGrew, & Leforgee, 2002). It is a synthesis of Cattell and Horn’s extended ‘Gf-Gc model’ (Cattell, 1987; Horn, 1968, 1991; Horn & Stankov, 1982) and Carroll’s (1993) ‘three-stratum model’. From the statistical point of view, the common factors in these models represent the common variance among test scores. However, substantively, they are often interpreted in terms of substantive underlying variables. Carroll, for instance, stated:

> “[U]nderlying each factor of the three-stratum theory there is a specific state or substrate that exists in the individual and that accounts for his or her ability or inability to perform tasks in which that ability is called for” (Carroll, 1996, p.15).

Although factor analysis may support such substantive, or realist, interpretation of common factors, it cannot prove that this interpretation is correct (see Borsboom & Mellenbergh. 2002; Bartholomew, 2004; van der Maas et al., 2006; Borsboom & Dolan, 2006). Hence, to attach substantive interpretation to the results of factor analysis, i.e., to interpret a given common factor as a substantive, realistic underlying variable, theory is required.

Concerning the theoretical status of the factors in CHC model, we note certain disagreements among Cattell, Horn, and Carroll, despite large agreement about the statistical structure. Consider the following two examples. First, in the CHC model the second order factors are positively intercorrelated, which opens the possibility of positing a general intelligence factor at the apex of the hierarchy. The three-stratum model includes such a factor (*g*), but the Gf-Gc model does not. This salient difference between the three-stratum model and the Gf-Gc model has a theoretical background: Whereas Carroll (1998) took a realist position concerning the general factor by interpreting it as a unique cognitive ability, Horn (e.g. Horn & Noll, 1997) rejected this realist interpretation, and viewed general intelligence as merely a statistical entity; he considered it to be as nothing more than an aggregate of various cognitive abilities.

Second, Carroll (1993) argued that it is a matter of preference whether Gc is interpreted as verbal ability or as crystallized intelligence, whereas Cattell (1987) maintained that verbal ability and crystallized intelligence are distinct. In view of these disagreements, we conclude that the theoretical status of the CHC factors is open to debate. We also subscribe to Keith and Reynolds (2010)'s conclusion, after discussing "what we have learned over the past 20 years of research on the nature and measurement of intelligence from a CHC perspective." (p. 647):

> "Gc remains an elusive construct, and researchers often talk past each other when discussing Gc, with it being referred to as crystallized intelligence, academic achievement, verbal ability, or comprehension/knowledge, to name a few [...] Clarification about the nature of Gc versus verbal ability and achievement would be useful" (Keith & Reynolds, 2010, p. 643).

Our first aim is therefore to discuss the status of CHC factor Gc as representing an underlying variable. Our second aim is to address the question whether or not Gf and *g* represent distinct underlying variables.

In accepting the CHC model, and referring to factors Gf and Gc as fluid and crystallized intelligence, researchers seem to subscribe, implicitly or explicitly, to Cattell's (1963; 1971; 1987) investment theory of intelligence. This theory includes two central hypotheses. One hypothesis states that the variable fluid intelligence (represented by CHC factor Gf) is a real, substantive cause of individual differences in intelligence test performance, e.g. a relation perceiving or reasoning capacity. The second hypothesis, the investment hypothesis, states that during the course of development people 'invest' their fluid intelligence to acquire domain specific knowledge and skills (e.g. vocabulary and arithmetic skills), which are called crystallized abilities. The conglomerate of crystallized abilities is crystallized intelligence. Individual differences in the variable crystallized intelligence are usually denoted as Gc. In some interpretations of the CHC model, this theoretical variable crystallized intelligence is supposed to be represented by the factor Gc.

We discuss, in light of Cattell's (1987) investment theory, the theoretical status of CHC factor Gc as representing (1) crystallized intelligence, and (2) a substantive underlying variable. In so doing, we hope to advance our understanding of the relations between Gf, *g*, and Gc in the CHC model, hence the theory behind the CHC model. The chapter is organized as follows. We first discuss the concept of a latent (underlying) variable, and the scientific philosophical positions that one can take with regard to this concept (i.e., realist versus nonrealist), because the explication of one's position with respect to latent variables is important in general, but indispensible in a discussion of the nature of Gc. Next, we present a brief review of investment theory, the development of the CHC taxonomy, and various interpretations of the CHC factor Gc as given in the literature.

We conclude first that crystallized intelligence is purely a statistical entity, which does not represent an underlying cause of individual differences in intelligence, and we conclude that within investment theory, fluid and general intelligence represent one and the same variable (see also Kvist & Gustafsson, 2008). Second, we conclude that within investment theory two interpretations of CHC factor Gc as representing a realistic underlying variable remain viable. Gc may represent individual differences in exposure to information via education, which is consistent with Cattell (1971, 1987). Alternatively, Gc may represent individual differences in verbal comprehension, which is consistent with Johnson and Bouchard (2005). Which interpretation is preferable depends on the exact constitution of the (CHC) test battery, and thus on what tests are found to load on the factor designated as Gc.

Next, in support of our conclusions, we present a confirmatory factor analysis of a subset of HCA data set ALLI00 (Allison, 1960; Carroll, 1993). In this dataset, CHC factor Gc was found to be indistinguishable from a verbal comprehension factor. In addition, CHC factor Gf, the variable that represented individual differences in fluid intelligence, was indeed indistinguishable from the CHC factor *g*. Our final conclusions are that in the CHC model and investment theory (a) crystallized intelligence does not represent a true psychological or biological capacity; and (b) CHC factors Gf and *g* represent one and the same capacity. The implications for the CHC taxonomy are that Gc and Gf or *g* can be removed from the CHC factor model. We end with a brief discussion concerning the theoretical status of other CHC factors, and, following McGrew (2009), we offer some recommendations concerning the CHC taxonomy as a common nomenclature.

5.2 The Interpretation of a Latent Variable

In the literature, the term latent variable is used in more ways than one (Borsboom, Mellenbergh, & van Heerden, 2003). Firstly, the term latent variable can refer to a formal, mathematical concept. This concept is used in mathematical treatments of measurement and structural relation models, and is usually symbolized by a letter, e.g. θ in Item Response Theory, or η in Structural Equation Modeling. Beyond the mathematical treatment the formal concept has no meaning, i.e., it does not reveal anything about reality. Secondly, the term latent variable can refer to an operational concept, which is the result of an algebraic function of observed scores, e.g. a weighted sum score, like a full-scale IQ score. The factors extracted from a test battery (e.g. the CHC factors) represent instances of the operational concept. Since there is nothing latent about an algebraic function, there is nothing latent about the operational concept. The link between the formal concept (label) and the operational concept (algebraic result) is not self-evident, and requires theoretical interpretation, such as '*g* represents individual differences in general intelligence, and a part of the common variance in intelligence tests scores corresponds to individual differences in general intelligence'.

With respect to the theoretical interpretation of the link between formal concept (label) and operational concept (algebraic result), there are essentially three philosophical positions (Borsboom et al., 2003). First, one can take an operationalist position. This position holds that there is nothing beyond the operational variable (algebraic result). This implies, *inter alia*, that different sets of items necessarily measure different variables (e.g. different IQ test batteries or subsets of batteries measure different variables, or 'intelligences', so to speak; see Borsboom & Mellenbergh, 2002). We therefore maintain that "operationalism and latent variable theory are fundamentally incompatible." (Borsboom et al., p. 207). Second, one can take a constructivist position.[8] This position holds that the latent variable is a construction of the scientist. It implies that different IQ batteries may measure or estimate the same variable (or set of variables), but that this variable need not be attributed existence independent of the scientist (e.g. although a construct called general intelligence is not a real-world entity, different batteries estimate this construct; the variable *g* represents individual differences in this construct). Third, one can take a realist position. A realist position holds that the unobserved (i.e. latent) variable does exist independent of the scientist (e.g. *g* represents individual differences in a real-world entity, and the tests on IQ batteries estimate one's relative position on this variable).

With respect to causation, the constructivist and realist positions are associated with different interpretations of measurement (e.g. Borsboom et al., 2003) and structural relations (e.g. Diamantopoulos et al., 2008). The constructivist position implies that a formative model is appropriate, while the realist position implies a reflective model is appropriate (Bollen & Lennox, 1991; Edwards & Bagozzi, 2000; Edwards, 2011, Diamantopoulos et al., 2008). The latent variable in a reflective model (as depicted in Figure 5.1a) is conceptualized as *determining* the measurements,

[8] Strictly, operationalism is a kind of constructivism, but we mean the term constructivism to denote a broader class of views (for example, the view of van Fraassen, 1980).

whereas the latent variable in a formative model (as depicted in Figure 5.1b) is conceptualized as *constructed from* measured (or lower level) variables.

Reflective modeling is customary in psychometric measurement models (e.g. IRT models) and in many structural relation models (e.g. factor models of intelligence), and appears to be implicit in much psychological theory. For instance, the measurement of cognitive abilities, say, working memory capacity, usually involves reflective modeling. Here, one assumes that individual differences in the unobserved variable give rise to individual differences in the indicators. So, if persons differ in working memory capacity (the unobserved variable), these differences will give rise to individual differences, hence variance, in observed indicators of working memory capacity, e.g. digit span.

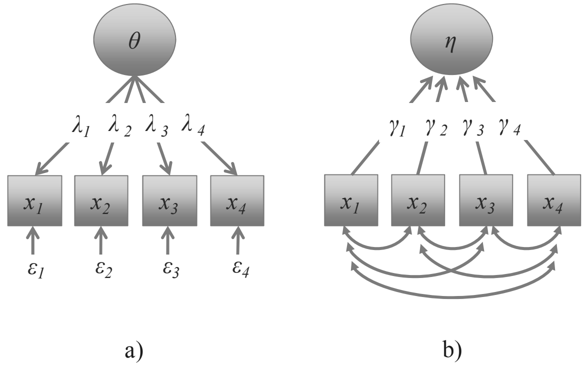

Figure 5.1 Two measurement models. The reflective model (a), in which individual differences in the latent variable (e.g. cognitive ability) cause individual differences in the indicators. Positive correlations between the indicators are the result of the common dependence on the latent variable. The formative model (b), in which individual differences in the indicators determine the unobserved variable. It summarizes the positive correlations among the indicators.

Although formative modeling (Edwards & Bagozzi, 2000) is less customary in psychology (in contrast to economic modeling and marketing research), it is increasingly the subject of debate (Diamantopoulos et al., 2008; Kievit et al., 2011). Recently, several special issues were devoted to reflective versus formative modeling (see, e.g. Diamantopoulos, 2008). As an example of formative modeling consider the measurement model for SES (socioeconomic status). The construct SES is a linear combination of relevant variables, such as income and occupation. Here, differences in the indicators give rise to differences in the unobserved variable, rather than vice versa. For instance, individual differences in salary determine individual differences in SES, and a raise in one's salary results in an increase in one's level of SES. Constructivist variables as 'general health', 'fitness', and 'mental health' may also be conceptualized as formative variables; one does not have all kinds of health problems because of poor general health, rather one has a poor general health, because of all kinds of health problems.

We note that the interpretation of a latent variable as a realistic, underlying variable is compatible with a reflective model, but cannot be justified merely by reference to the results of factor analysis, or the application of any other measurement or structural relation model (Borsboom et al., 2003). The common factor model may be compatible with a reflective model, but the results of a factor analysis cannot prove the existence of a causal underlying variable. To elucidate this, consider the following. In intelligence research, the positive correlations amongst cognitive ability test scores and the presence of a statistical general factor are often explained by positing a common dependence on a realistic underlying variable, *g* (Spearman, 1927; Jensen, 1998; Carroll, 1993). However, several alternative explanations have been proposed that can also account for the positive correlations, hence for the statistical general factor (Thorndike, 1927; Thomson, 1951; Cattell, 1987, pp. 365-433; Anderson, 2001; Penke, Denissen, & Miller, 2007; van der Maas et al., 2006; Bartholomew et al., 2009; Dickens, 2008). In the mutualism theory of van der Maas et al., for instance, the positive correlations are the result of mutually beneficial interactions among cognitive processes that are assumed to take place throughout development. In this case, the general factor obtained in a factor analysis does not correspond to a realistic, underlying causal variable; it simply summarizes the positive correlations among cognitive abilities. In the mutualism theory, the general factor is conceptualized as a constructivist variable; from the realist standpoint this general factor does not represent a causal underlying variable. However, a formative model of general intelligence (e.g. Diamantopoulos et al., 2008, Figure 3) would be appropriate given mutualism.

In light of the above, we maintain that the variables and relations between the variables revealed in factor analyses require theoretical interpretation, and that, in this interpretation, the scientist's philosophical position (realist versus non-realist) concerning these variables is important.

Throughout this chapter, we approach the factors in the Cattell-Horn-Carroll taxonomy from the realist position, because these factors are often interpreted as such (e.g. Carroll, 1993). We note that, within the CHC setup, Gc is considered to represent individual differences in crystallized intelligence. The present aim is to demonstrate theoretically and empirically that the interpretation of Gc as representing a substantive, causal underlying variable (e.g. a biological or psychological capacity), and the interpretation of Gc as crystallized intelligence are mutually incompatible. That is, if Gc represents a capacity it cannot represent crystallized intelligence, and if Gc represents crystallized intelligence, it does not represent a capacity. To demonstrate this theoretically, we review the investment theory of fluid and crystallized intelligence, individual differences models of intelligence that include a factor denoted as Gc (e.g. the CHC model), and interpretations of Gc as an substantive underlying variable. Subsequently, to support our main conclusions empirically, we fit the CHC model to a HCA data set.

5.3 Cattell's Investment Theory of Fluid and Crystallized Intelligence

According to Cattell (1941; 1943; 1963), intelligence tests measure two kinds of abilities, which differ greatly with respect to the role knowledge. One kind is measured by tests in which individual differences in prior knowledge, rather than individual differences in cognitive processing, determine individual differences in the subtest scores. Cattell called these abilities crystallized abilities. The other kind, which Cattell called fluid abilities, is measured by tests in which prior knowledge does not discriminate. That is, although knowledge is required to solve the items on a fluid ability test, this knowledge is considered to be equally novel or equally known to the testees. Hence, individual differences in the subtest scores are not primarily determined by differences in prior knowledge, but by differences in cognitive processing.

Furthermore, in his early work, Cattell argued that Spearman's general factor of intelligence, *g*, can be split into two distinct factors, although these factors, "[b]eing cooperative, are very difficult to separate" (Cattell, 1963, p.2). In addition, fluid and crystallized abilities were found to load on both factors, but fluid abilities loaded primarily on the one factor and crystallized abilities primarily on the other. Therefore Cattell postulated that Spearman's *g* comprised two general factors of intelligence: fluid intelligence and crystallized intelligence.[9]

Cattell (1963, 1987) entertained specific ideas about the nature and ontology of fluid and crystallized intelligence. He viewed fluid intelligence as a 'source trait' (Cattell, 1987, p.15), i.e., an underlying causal variable, or a latent variable of the reflective kind (Bollen & Lennox, 1991). He considered fluid intelligence to be a 'single relation perceiving capacity', which is related to the maturation of the cortex (Cattell, 1987, p. 138). In addition, in his investment theory, Cattell attributed a causal role to fluid intelligence in the development of other cognitive abilities. He hypothesized that people 'invest' their fluid intelligence to acquire specific skills, strategies, and knowledge in all kinds of fields (Cattell, 1963; Cattell, 1987, pp. 138-147), for example vocabulary

[9] Other observations have been cited in support of the distinction between fluid and crystallized abilities (Cattell, 1963, 1987; Hunt, 2000). For example, fluid and crystallized abilities display different developmental curves (see also Horn & Cattell, 1967, McArdle et al, 2002; cf. Schaie & Strother, 1968). In addition, brain injuries have distinct effects on crystallized abilities and fluid abilities (Cattell, 1963; Lee, Choi, & Gray, 2007).

and arithmetic skills. These acquired abilities are the crystallized abilities. The terms fluid and crystallized have their origin in investment theory: “The term crystallized is meant to imply this freezing in a specific shape of what was once fluid ability” (Cattell, 1987, p. 140). Crystallized abilities are thus domain specific; their expressions are ‘tied to particular areas’ (Cattell, 1987, p. 139), whereas fluid intelligence, in contrast, is hypothesized to be domain transcending, i.e., “has the fluid quality of being directable to almost any problem” (Cattell, 1987, p. 97). The investment hypothesis boils down to the hypothesis that knowledge acquisition, hence learning, requires relation perceiving, with fluid intelligence representing the capacity to do this.

It follows from the investment hypothesis that, *ceteris paribus*, people with high levels of fluid intelligence tend to acquire more and better-developed crystallized abilities than people with low levels of fluid intelligence. Or, stated more mundanely, they learn faster. As a result, provided investment occurs in several areas, crystallized abilities across these areas become positively intercorrelated, and a factor analysis of scores on tests that measure the relevant crystallized abilities will thus yield a common factor. Accordingly, crystallized intelligence is defined as a conglomerate of crystallized abilities. Individual differences in crystallized intelligence are denoted Gc or gc, and individual differences in fluid intelligence Gf or gf. Because fluid intelligence is invested to acquire crystallized abilities across a variety of domains, crystallized intelligence, and hence the theoretical variable Gc, is also associated with abilities across domains (see also Hunt, 2000).

In the literature, ‘investment’ is taken quite literally. Consider the following examples:

> “As the term ‘investment theory’ indicates, gf is liable, in its generation of gc, to all risks of an investment. Laziness may cause it scarcely to be invested at all; differences of individual interest may cause it to be invested in directions different from that in which ‘traditional’ intelligence tests measure it.” (Cattell, 1987, p. 334).

> “Persons high in Gf tend to acquire more Gc (i.e., they reap greater returns on their initial investment) from their opportunities for learning than persons of lower Gf.” (Jensen, 1998, p.123)

In psychometric modeling fluid and crystallized intelligence have become popular constructs: In the model of Cattell and Horn (Cattell, 1987; Horn; 1968), Carroll’s three stratum model (Carroll, 1993), as well as in combined models (e.g. Gustafsson, 1984, Alfonso et al., 2005), the higher order factor Gf is considered to represent individual differences in fluid intelligence, whereas Gc is considered to represent individual differences in crystallized intelligence. Since crystallized intelligence is considered to be the result of investment of fluid intelligence, researchers consider *factor* Gc, in its interpretation as representing individual differences in crystallized intelligence, to arise as result of investment of Gf: “Gc […] develops out of the investment of Gf.” (Ackerman, 2003).

Just as we take the reflective model seriously (as a causal model of inter-individual differences), we take investment theory seriously. That is, we take Cattell’s theory as a theory of investment of peoples’ fluid intelligence in a variety of areas or domains. We note, however, that investment theory concerns an intra-individual process, i.e., one at the level of the individual, whereas the CHC model of intelligence concerns the structure of interindividual differences in intelligence. The link between investment theory and the factors in the CHC model is not self-evident and has yet to be established. One difficulty in establishing this link is that investment theory has not undergone any significant changes, whereas the psychometric models of intelligence have. The relation between investment theory and the CHC model has become more complicated and less clear than the relation between investment theory and early models.

To establish the link between investment theory and the factors in the CHC model, a historical overview is useful. Therefore, below we consider the development of the CHC model in more detail. We also discuss the relation between Gf and *g*. We consider this discussion to be relevant for two reasons. First, some researchers consider crystallized intelligence to be the result of investment of general intelligence (e.g. Carroll, 1993; Jensen, 1998). Second, factors denoted as *g* and Gf are sometimes found to be indistinguishable in factor analysis (e.g. Gustafsson, 1994), and are therefore considered to represent the same underlying variable (e.g. Jensen, 1998). This finding that Gf and *g* are indistinguishable has been associated with investment theory (Kvist & Gustafsson 2008).

5.4 The (Development of the) CHC factor model

As mentioned above, in his early work Cattell (Cattell, 1943, 1957, 1963) argued that Spearman's general factor of intelligence (*g*) can be split into two distinct factors. Because fluid abilities tended to load primarily on the one factor, and crystallized abilities on the other, he called the one factor fluid intelligence (Gf) and the other one crystallized intelligence (Gc). Originally the theory of fluid and crystallized intelligence involved a bipartite taxonomy of individual differences in cognitive abilities, and investment theory was an attempt to account for the ontology of these two factors.

This bipartite Gf-Gc model was expanded following the inclusion of a broader range of subtests measuring cognitive abilities and intellectual achievement (Horn, 1968, 1991; Horn & Stankov, 1982). Additional higher order factors were extracted from diverse batteries. These higher order factors are instances of the operational concept of a latent variable (see above). The factors were labeled Gv, Gsm, Glr, Gs, Ga, Gt, Gq, Grw, and were interpreted as visual perception, short-term memory, long-term storage and retrieval, processing speed, auditory processing ability, reaction time, quantitative ability, and broad reading and writing ability, respectively. These interpretations provide the theoretical link between the formal and operational concepts (see above). As argued above, in order to interpret a factor as representing a causal underlying variable or as merely a statistical summary, one requires an explicit scientific philosophical position, that is, realist or nonrealist.

Because the theory of fluid and crystallized intelligence provided its point of departure, the extended model is still referred to as the Gf-Gc model (Horn & Noll, 1997, Alfonso et al. 2005). This extended Gf-Gc model closely resembles Carroll's three-stratum model (Carroll, 1993; 1996; 2003; Bickley, Keith, Wolfle, 1995). The latter may be regarded as a synthesis of the Gf-Gc model and Spearman's *g* model (Alfonso et al. 2005; McGrew, 2009). Like the Gf-Gc model, the three-stratum model incorporates higher order factors, denoted Gf and Gc, as well as many other higher order factors, defined by Horn and Cattell. However in some aspects the models differ. A relatively minor difference is that several narrow second order factors in the Gf-Gc model are subsumed under broader second order factors in the three-stratum model (e.g. Grw in the Gf-Gc model is subsumed under Gc in the three-stratum model). The major difference between the models is that Carroll posited a general factor (*g*) at the highest level in order to account for the positive intercorrelations between the broad factors at the second stratum, whereas Horn and Cattell did not.

Correlated factors Gf and Gc are invariably included in the CHC model, but the inclusion of *g* remains an issue of debate (McGrew, 1997, Alfonso et al. 2005). In some representations of the CHC model *g* is included at the apex of the hierarchical factor model (e.g. Vanderwood, McGrew, Flanagan, Keith, 2001), whereas in others it is omitted (e.g. Alfonso et al., 2005). Note in this connection that

> "The exclusion of *g* does not mean that the integrated model does not subscribe to a separate general human ability or that *g* does not exist. Rather, it was omitted […] since it has little practical relevance to cross-battery assessment and interpretation." (Flanagan, Ortiz, & Alfonso, 2006, p. 6; McGrew, 1997).

Whether or not to include a general factor is related to its actual theoretical conceptualization. Horn in particular objected to the extraction of a general factor, because in his opinion such a general factor does not represent a unique cognitive ability: "[G]eneral factors represent different mixture measures, not one general intelligence." (Horn & Noll, 1997, p. 68, but see Johnson et al., 2004; Johnson et al., 2008, for a different view). We conclude that in Horn's view, *g*, as an aggregate rather than a unique cognitive ability, is better conceptualized as a constructivist variable, and that a formative model of *g* is more appropriate than a reflective (i.e., underlying variable) model (e.g. Diamantopoulos et al., 2008, Figure 2-II, p. 1207).

Related to the discussion of whether *g* represents a substantive underlying variable is the discussion whether fluid and general intelligence are distinct cognitive abilities or capacities. Some researchers (e.g. Jensen, 1998) have argued that *g* and Gf represent one and the same variable in view of the following. In another attempt to synthesize existing models of cognitive abilities, Gustafsson (1984) extracted a third-order general factor (*g*) as well as a number of second order factors, including factors referred to as fluid intelligence (Gf) and crystallized intelligence (Gf). In this model, which he called the HILI model (HIerarchical LIsrel model) of intelligence, the correlation between *g* and Gf approached one, which renders them indistinct as common factors, i.e., as statistical entities.

This finding was replicated in some (Gustafsson, 1988, Keith, 2005; Reynolds & Keith, 2007), but not all other studies (Carroll, 1993, 2003). We conclude therefore that the relation between *g* and Gf remains puzzling, and that the theoretical status of *g* remains open to debate. With our discussion of Gc, we hope to shed light on this relation.

Cattell's (1987, p 141) proposal that *g* represents "the fluid ability of yesteryear, which fathered the present fluid ability" can be regarded as a first step towards explaining the mixed results concerning the distinctiveness of Gf and *g*, and as providing an account of the relation between these variables. However, this proposal raises questions about investment theory, and thus about the ontology of Gf and Gc, because originally Cattell argued that highest order factor *g* can be split into fluid intelligence (a reasoning factor) and crystallized intelligence (a knowledge factor). But with this proposal Cattell is reintroducing *g* as single variable at the highest order.

A second step towards explaining these mixed results is provided by Kvist and Gustafsson (2008), who argued that the relation between Gf and *g* is a function of cultural background:

> " [T]he Gf-factor would be equal to the *g*-factor in populations which are homogeneous with respect to opportunity to having learned the knowledge and skills measured but […] this relationship would not hold in heterogeneous populations where subgroups differ with respect to opportunity to learn" (Kvist & Gustafsson, 2008, p. 433).

They consider this to be consistent with Cattell's investment theory of fluid and crystallized intelligence. But this also raises questions about investment theory. As has been suggested by Jensen (1998, see below), the splitting of *g* into separate factors Gf and Gc might be due to heterogeneity of the subject sample.

To advance our understanding of the relations among the factors *g*, Gf, and Gc, it is important to establish whether the interpretation of Gc as crystallized intelligence is compatible with the interpretation of the CHC factor Gc as a substantive underlying variable.

5.5 The Interpretation of Factor Gc as a Crystallized Intelligence

In the literature, the common factor denoted Gc has been interpreted in many ways (see Table 5.1). These interpretations are associated with different scenarios concerning the ontogeny of crystallized intelligence in the investment hypothesis. In the following two sections, we discuss the interpretations of CHC factor Gc in the light of the investment hypothesis.

The first scenario is based on a literal interpretation of the investment hypothesis (see Figure 5.2). People invest their fluid intelligence, and so acquire crystallized abilities in a variety of domains. Gf represents individual differences on the psychological dimension fluid intelligence (Cattell, 1963, 1987). The investment hypothesis predicts that people who have a high level of fluid intelligence (have a high position on Gf) will acquire more and better developed crystallized abilities than people with low levels of fluid intelligence. Given that people generally spread their investments, and given they do so in similar ways within a given society, crystallized abilities become positively correlated. The conglomerate or total of a person's crystallized abilities is a person's crystallized intelligence.

We note that in this literal interpretation of the investment hypothesis, factor Gf is the ultimate source of the correlations between the crystallized abilities. In a factor analysis of a battery that includes both fluid and crystallized abilities, a distinct Gc factor should not appear. Given this scenario, we should conclude that (1) crystallized intelligence is not a latent variable of the reflective kind (i.e., an underlying variable), but of the formative kind, and (2) this scenario does not support factor models that include two separate factors Gf (fluid intelligence) and Gc (crystallized intelligence), e.g. the CHC model.

We also note that crystallized intelligence, as measured in psychometric intelligence tests, emerges as a particular composite of crystallized abilities (namely the total of those crystallized abilities that are measured by the subtests). In the ASVAB, for example, the factor designated crystallized intelligence summarizes mathematical and verbal skills (Roberts et al. 2000); in the Woodcock-Johnson (WJ)-III achievement battery this factor summarizes general and academic knowledge (McGrew & Woodcock, 2001). Furthermore, in the view of Horn and Blankson (2005), the best estimate of crystallized intelligence is the total of all achievements in the WJ-R achievement battery. In this sense, crystallized intelligence is merely an operational variable (see above), because different intelligence tests measure different conglomerates of crystallized abilities, hence measure different 'crystallized intelligences'.

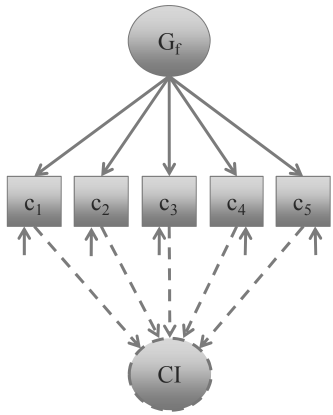

Figure 5.2 A literal interpretation of the investment hypothesis. Gf represents individual differences on the psychological dimension fluid intelligence. Fluid intelligence is invested to acquire crystallized abilities in a variety of domains. As a result, crystallized abilities are positively correlated. In a factor analysis including Gf, a distinct Gc factor does not appear; Gf accounts for the correlations between the crystallized abilities. Crystallized intelligence (CI) is not a latent variable of the reflective kind, but of the formative kind.

In the second scenario, we reverse the reasoning. We suppose crystallized intelligence is a distinct causal variable, i.e., a substantive reflective latent variable, and ask whether this ability can be reasonably considered to be the result of investment. As mentioned, in investment theory crystallized intelligence is associated with crystallized abilities across domains, so assuming Gc is a substantive reflective latent variable, it has to exert its influence across domains. This interpretation of Gc is consistent with, for example, Schweizer and Koch (2002)'s revision of investment theory, in which crystallized intelligence is defined as 'the potential to perform on the basis of' crystallized abilities (p. 58), or with Marks' (2010, p.646) contention that crystallized intelligence should not be equated with knowledge and skills itself, but that it is represents the ability to use them.

We maintain that in this second scenario Gc cannot both be the result of investment of fluid intelligence and a unique domain transcending cognitive ability (e.g. as proposed by Schweizer & Koch, 2002, and Marks, 2010), because this would imply that transfer of training occurs across domains. To illustrate this, imagine that one invests one's fluid intelligence in a particular area, say, by practicing a mathematical skill. Then, according to the investment hypothesis, this increases one's level of crystallized intelligence, i.e., Gc (the ability or potential to perform on the basis of knowledge). An increase in Gc (ability or potential), in turn, should have a direct positive effect on all the indicators of Gc, such as vocabulary, general information, spelling, etc.[10] So, in this scenario, training the mathematical skill should result in an increase in all other crystallized abilities. This poses a problem, because although there is empirical evidence for transfer of training *within* domains, evidence for transfer of training *across* domains is absent (e.g. Cattell, 1987; Ceci, 1990, p.185). In this light, we conclude that this scenario is not tenable.

Given the investment hypothesis, we can draw the following conclusions. First, crystallized intelligence is not a latent variable of the reflective kind, but of the formative kind; it is not a substantive latent variable. Second, crystallized intelligence should not appear as a common factor (Gc) that is distinct from Gf. Hence, the question remains what meaning to attach to the finding in factor analysis of the distinct factor commonly designated Gc. To answer this, we have to identify a second domain transcending cause of individual differences, which we denote *X* (see Figure 5.3). In fact, Cattell's *investment theory* is broader than solely *the investment hypothesis* of fluid intelligence, because it stipulates that such second cause of individual differences is necessary to explain the patterns of covariance between fluid and crystallized abilities (Cattell, 1987). Regardless of what this second cause, *X*, may represent, it must represent a variable different from crystallized intelligence. In the next section, we discuss candidates for *X*.

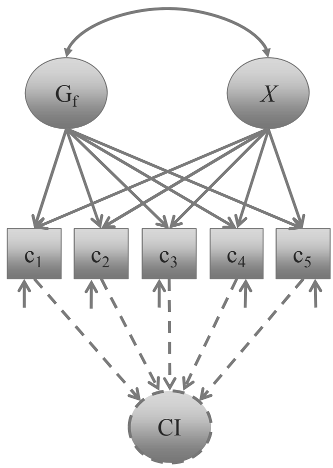

Figure 5.3 If a second factor Gc is found in factor analysis and if this factor is interpreted as a variable of the reflective kind (and as a variable different from Gf), it must represent variable *X*. Crystallized intelligence (CI) is a latent variable of the formative kind. Plausible candidates for *X* are exposure to information via education and verbal comprehension.

5.5 The Interpretation of Gc as a Causal Variable

If we find Gc in a factor analysis and want to interpret this factor as an underlying cause of individual differences (and different from Gf; i.e., the *X* in Figure 5.3), then what can it be? As Guilford (1980) pointed out, the factor Gc may well represent an environmental cause of individual differences, rather

[10] On the level of individual differences it implies the following. Given that 2 persons (person A and person B) have same level of fluid intelligence and the same history of investment, if person A improves a mathematical skill, thereby increasing the level of Gc (hence taking a higher position on the dimension Gc), person A must tend to perform better on all crystallized tasks (vocabulary, general information, spelling, etc.) as compared to person B.

than a psychological (or biological) capacity. Furthermore, assuming that it represents a psychological capacity, it can represent a cognitive capacity or some other psychological capacity.

If we interpret *X* as the potential to perform on the basis of acquired knowledge (Schweizer & Koch, 2002) - but not as the result of investment (see above) - we can regard *X* as the capacity to store, retain or retrieve information, thus as an capacity to memorize. However, the CHC model (Horn, 1991; Carroll, 1993; McGrew, 1997) already includes a short term memory (Gsm) and a broad retrieval and storage capacity (Glr) factor, and both are distinct from this substantiation of *X*. Furthermore, if *X* represents such a capacity, it is difficult to explain why *X* has additional loadings on tests in which information is new to the participants (Cattell, 1963; Carroll, 1993), i.e., tests on which better memory, or storage or retrieval capacity confers no clear advantage. We conclude that the capacity to store, retain or retrieve information is not a plausible candidate for *X*.

As discussed by Cattell (1987), other candidates for *X* include motivation, broad interest, and persistence, provided that these can feature as unitary, causal variables. In view of empirical results, variable *X* must be quite highly correlated with fluid intelligence (because Gc correlates substantially with Gf). However, fluid intelligence is not particularly highly correlated with any of these variables (e.g. Ackerman, 1996). In addition, although crystallized abilities are correlated with them, the correlations are low to moderate, which suggests that motivation, broad interest, or persistence cannot be equated with Gc. We conclude that these variables are not plausible candidates for variable *X* either.

Are there any other candidates? From the literature, we conclude that two interpretations of *X* (Gc) as representing an underlying variable are tenable. Consistent with Cattell (1971, 1987), it may represent individual differences in exposure to information via education. Consistent with Johnson and Bouchard (2005), it may represent individual differences in verbal comprehension. Below, we discuss these options in more detail.

5.5.1 Candidate 1: *X* is Exposure to Information Via Education

Exposure to information via education represents the effect of individual differences in educational attainment (especially those due to enrollment in formal educational systems), and the quality of that education. It is affected by opportunities, by social influences, such as cultural values, and by psychological influences, such as motivation, interests, and persistence.[11]

We consider the interpretation of Gc as exposure to information via education to be plausible, because it is consistent with salient findings in intelligence research. First, the high correlation between Gf (or *g*) and exposure can be explained as follows: People who have a higher level of fluid or general intelligence generally receive more (longer and better) education. Second, the educational curriculum affords many opportunities to acquire specific abilities in all kinds of domains. Hence, exposure to information via education can explain additional common variance across domains after Gf (or *g*) has been taken into account. Third, in many IQ batteries, subtests that load on the variable designated Gc typically measure knowledge and skills acquired during the formal education. Fourth, exposure to information via education is consistent with the empirical finding that delinquents often have disproportionately low levels of crystallized abilities (e.g. Blecker, 1983; Law & Faison, 1996). They are likely to have invested their fluid intelligence to acquire knowledge, skills, and solving strategies that are not measured by intelligence tests (Cattell, 1987, p. 137).[5]

The interpretation of Gc as exposure to information via education is also consistent with Cattell's investment theory. First, the distinction between fluid and crystallized abilities remains valid. Second, it is consistent with the observation that IQ subtests display different age curves as a function of knowledge (McArdle et al., 2002). The more differences in prior knowledge (rather than differences in reasoning) determine differences in the testees' test scores, the less the age curve

[11] In a society in which social economic opportunities influence access to educational institutions, SES will predict exposure to education. In a society in which such opportunities do not influence access, SES will not predict well exposure to education. Group differences in exposure to education between people from different cultures, however, may well reflect group differences in (social economic) circumstances or cultural values, even if these variables do not predict well exposure to education within a group (see also Kvist & Gustaffson, 2008).

declines. Also, the more time one invests to acquire or maintain this knowledge, the less one's age curve declines. We therefore expect the smallest decline in one's area(s) of expertise. Third, crystallized intelligence can still be defined in the original way, i.e., as a conglomeration of knowledge, skills, and solving strategies across domains (Cattell, 1963, 1987). Finally, Cattell himself proposed that the dislocation of the variable crystallized intelligence from Gf is due to the influence of education. Education itself, in turn, can be affected by influences described above, such as personality, motivation, persistence, and interests.

5.5.2 Candidate 2: *X* is Verbal Comprehension

Although exposure to information through education is a plausible candidate theoretically, we question whether in practice the factor Gc in the CHC taxonomy truly represents a domain transcending factor. Many of the tests that load on the factor denoted Gc rely, directly or indirectly, on language (Carroll, 1993; Hunt, 2000). For this reason, Carroll (1993) suggested that it is a matter of preference whether one views Gc as crystallized intelligence or verbal ability. Theoretically, however, it is important to make a distinction between crystallized intelligence and verbal ability for three reasons (Cattell, 1987; Hunt, 2000; Johnson & Bouchard, 2005; Johnson & Gottesman, 2006). First, as explained above, the investment hypothesis in itself does not explain why Gc ('crystallized intelligence') should emerge as a common factor, whereas equating Gc with verbal comprehension does (provided that verbal comprehension is the manifestation of some underlying system that is different from the system that underlies fluid intelligence). Second, with respect to causality, crystallized intelligence must be considered to be a formative variable, whereas verbal comprehension can reasonably be assumed to be a reflective variable (differences in verbal comprehension give rise to differences in its indicators). Third, investment theory aims to give a developmental account of cognitive abilities, and aims to explain differences in age trends of tests that depend mainly on reasoning and tests that depend mainly on knowledge (Cattell, 1963). In itself, the interpretation of Gc as verbal comprehension does not provide such explanation (Hunt, 2000).

In this light, Beauducell, Brocke, & Liepmann (2001), Beauducell & Kersting (2002), and Amthauer, Brocke, Liepmann, & Beauducell (2001) are relevant. In these studies, participants were tested using a battery comprising verbal, numerical, and figural *reasoning* tasks, as well as verbal, numerical, and figural *knowledge* tests. In factor analyses all tests loaded on factors denoted Gf and Gc. However, if tasks required mainly reasoning they loaded primarily on Gf, and if they required mainly prior knowledge they loaded primarily on Gc. These results are consistent with Ackerman's distinction between intelligence as a process and intelligence as knowledge (Ackerman, 1996, 2000), and with Cattell's (1987, p.114) remark that "verbal (synonym, analogies) tests can be made to load fluid intelligence very substantially (and crystallized relatively little) if the words are chosen to be easily within the vocabulary of the group tested" (for examples, see Horn, 1970, p. 59, and Jensen, 1998, p. 123). Thus, at first sight, the distinction between a domain transcending (fluid) reasoning factor and a domain transcending knowledge factor seems better supported empirically than a distinction between a nonverbal and verbal factor. However, a close inspection of item content suggests a finer distinction is necessary.

In the Beauducell et al. studies, the figural and numerical knowledge items are stated in such a way that they appeal to verbally stored information (e.g. "what's the name of this figure?"). Furthermore, loadings on Gf and Gc generally depend on the proportion of verbal and nonverbal content in the battery (Carroll, 1993). Finally, since transmission of knowledge commonly takes place verbally (Hunt, 2000), differences in verbal comprehension may have resulted in differences in acquired knowledge. We suggest that in investment theory the acquisition of knowledge requires investment of fluid intelligence and exposure to education, and mediation by verbal comprehension. This would give an account, *inter alia*, of why numerical and figural knowledge tests often have loadings on a verbal factor. We conclude that verbal comprehension is a plausible alternative interpretation of factor Gc. However, one final phenomenon, namely the fact that under certain circumstances factors can merge together, is important, because here the interpretations of Gc as exposure to education and Gc as verbal comprehension intersect.

Table 5.1 Interpretations of Gc (see main text)

Interpretation of Gc	Appropriate Measurement model	Broadness of domain	Source	Kind	Plausibility
Crystallized intelligence (knowledge)	Formative	Transcending	-	-	-
Crystallized intelligence (Potential to perform)	Reflective	Transcending	Psychological	Cognitive	Implausible
Motivation	Reflective	Transcending	Psychological	Noncognitive	Implausible
Persistence	Reflective	Transcending	Psychological	Noncognitive	Implausible
Broad interest	Reflective	Transcending	Psychological	Noncognitive	Implausible
Exposure to education	Reflective	Transcending	Environmental	Noncognitive	Plausible
Verbal cognitive ability	Reflective	Specific	Psychological	Cognitive	Plausible

5.5.3 Interpretations intersect

As mentioned above, in culturally homogeneous samples, Gf and *g* may correlate perfectly and thus appear as one, while in heterogeneous samples, Gf and *g* correlate less than one, and thus may appear distinct (Kvist & Gustafsson, 2008). The fact that Gf and *g* can merge is thus important in understanding the mixed results of studies concerning the distinctiveness of *g* and Gf (see above). In addition, such merging is also important in the discussion and understanding of Gc as representing an underlying variable. We contend that in culturally and educationally homogeneous samples Gc and verbal comprehension merge into one factor (note that this would explain why Carroll (1993) did not always find a separate crystallized intelligence factor). In other words: Given investment theory, once differences in language, cultural influences, and education have been taken into account, individual differences in fluid intelligence (the same ability as general intelligence) and verbal comprehension can account for individual differences in crystallized intelligence.

Obviously, Gc cannot represent both exposure to information via education and verbal comprehension. To clinch the meaning of Gc, we have to scrutinize the common factors in the factor structure of a given test battery. We note that researchers do not always label CHC factors consistently. For example, consider the Woodcock-Johnson batteries, in which Gc can refer to (1) a variable that summarizes covariance among tests of verbal knowledge in the cognitive battery (McGrew & Woodcock, 2001); (2) a variable that summarizes covariance among general and academic knowledge in the achievement battery (McGrew & Woodcock, 2001); (3) a variable that summarizes covariance among all tests in the achievement battery (Horn & Blankson, 2005). In the last case, Gc is a general achievement factor, and, as a general factor, resembles the *g* factor in the cognitive battery. If this general achievement factor (Gc) is not found to be identical to the general factor in the cognitive battery (*g*), a third level *g*-Gc model seems appropriate, which can be regarded as a reintroduction of Cattell's original bipartition of *g*. If the verbal ability factor in the cognitive battery and the general achievement factor (Gc) are indistinguishable, Gc can be interpreted as verbal comprehension.

5.5.4 Conclusion

The realist interpretation of the CHC factor that has been designated Gc depends on the exact constitution of the battery. Theoretically, Gc can either represent verbal comprehension or exposure to information through education. We suspect that in practice in most cognitive batteries, the CHC factor Gc will represent verbal comprehension (Carroll, 1993).

5.6 Reanalysis of the HCA ALLI00 Dataset

To corroborate our conclusions we reanalyzed a (HCA) data set (Carroll, 1993, McGrew, 2009) based on a sample that was homogeneous with respect to age and cultural background. We chose the HCA ALLI00 data set, because educational attainment was measured, which opened the possibility to control for educational differences. The data set was originally analyzed by Allison (1960), and reanalyzed by Snow, Kyllonen and Marshalek (1984), and by Carroll (1993). According to Carroll (1993, p. 281), the study was well designed and well conducted. A point of concern might be that the sample consists entirely of male subjects, which, in principle, may limit our conclusions to males. However, there is no clear theoretical reason to assume that investment theory only applies to men and not to women. Moreover, since measurement invariance with respect to sex may not be tenable (e.g. factor loadings may differ in male and female samples), mixed samples may introduce additional variance not ascribable to cognitive factors. We do not consider the sex limited sample as a real limitation with respect to testing our hypotheses. The sample was representative for the population otherwise.

Because the battery was designed to measure cognitive abilities (rather than educational achievement), we hypothesized that in the HCA ALLI00 data set, once educational differences have been taken into account (1) Gc and verbal comprehension are indistinct as common factors, and 2) Gf and *g* are indistinct as common factors. If so, Gc and one of the factors *g* and Gf can be removed from the model without a significant decrease in model fit.

We stress that in this chapter we limit ourselves to the test of the hypotheses that Gc and Gf (or *g*) are redundant as explanatory variables. We do not test, for example, whether the CHC model is the preferred statistical model (cf, Johnson et al, 2007), or via what pathways education influences intelligence exactly (cf, Dolan, et. al, 2001). These questions are important, but beyond the scope of the current chapter. Our present analyses are meant to serve as an example of how to test Gc is not a substantive latent variable, and as a starting point for future research into investment theory. We content that to investigate investment theory properly, one should use a longitudinal design, using same-aged, same-sex, cultural and educational homogeneous subject samples.

Table 5.2 Indicator variables (subtests) and the factors on which they are taken to load across analyses

Subtest	Carroll (1993) Factor	Allison (1960) Factor	Current analysis
Vocabulary	Verbal	Verbal Knowledge	Verbal Comprehension
Sentence Completion	Verbal	Verbal Knowledge	Verbal Comprehension
General Classification Test	Verbal	Verbal Knowledge	Verbal Comprehension
Arithmetic	Verbal	General Reasoning	Quantitative Reasoning
Division	Verbal	Number Facility	Quantitative Reasoning
Math Aptitude	Verbal	General Reasoning	Quantitative Reasoning
Number Series	Verbal	Induction	Reasoning
Letter Sets	Verbal	Induction	Reasoning
Ship Destination	Verbal	General Reasoning	Reasoning
Reasoning	Verbal	Deduction	Reasoning
False Premises	Verbal	Deduction	Verbal Comprehension
Words Association	Perceptual Speed	Speed of Association	(Verbal) Fluency
Word Checking	Perceptual Speed	Speed of Association	(Verbal) Fluency
Recognition	Perceptual Speed	Associative Memory	(Verbal) Fluency
Mechanical	Mechanical/Space	Mechanical Knowledge	Spatial ability
Mechanical Knowledge	Mechanical/Space	Mechanical Knowledge	Spatial ability
Paper Form Board	Mechanical/Space	Visualization	Spatial ability
Cards	Mechanical/Space	Spatial Relations & Orientation	Spatial ability
Paper Folding	Mechanical/Space	Visualization	Spatial ability
Cubes	Mechanical/Space	Spatial Relations & Orientation	Reasoning
Picture Discrimination	Mechanical/Space	Perceptual Speed	Spatial ability
Clerical Aptitude	Writing	Perceptual Speed	Speed
Picture Number	Association Memory	Associative Memory	Speed
Word Number	Association Memory	Associative Memory	Memory
First Names	Association Memory	Associative Memory	Memory
Addition	Numerical	Number Facility	Quantitative Reasoning

5.6.1 Method

Participants The sample consisted of 483 17-to-22-year-old (male) U.S. Naval recruits. Educational attainment ranged from 0 years in high school to 2 years in college. Mean IQ was 'slightly below average' (Allison, 1960, p. 48). The data analysis is based on the correlation matrix of cognitive ability test scores of 315 participants. The correlation matrix is available at www.iapsych.com/wmfhcaarchive/ALLI00.html.

Measures In his (re)analysis of data set ALLI00, Carroll (1993) interpreted the extracted first order factors as verbal reasoning, perceptual speed, mechanical/spatial ability, writing speed, associative memory, and numerical facilitation. In addition, he interpreted these variables as being dependent on fluid intelligence, crystallized intelligence, and broad memory. Because the CHC factor model is based largely on Carroll's analyses, we followed his analyses as closely as possible in our confirmatory factor analysis. However, we omitted the Armed Forces Qualification Test, the Otis Self-Administering Achievement Test, and the Oral Directions Test, because these are full scale IQ tests (Allison, 1960), hence they measure aggregated effects. Measures of writing speed were also omitted because we consider these to represent psychomotor abilities, which are not relevant to the present discussion. Table 5.2 includes the indicator variables.

5.6.2 Statistical analysis.

Because we omitted several subtests, we could not implement Carroll's model directly into a confirmatory factor model. Since, in general, the factor structure depends to a certain extent on the tests in the test battery, we considered it possible that the factor structure differed somewhat from Carroll's results (especially at the first order level). We therefore carried the statistical analyses out in the following five steps.

Step 1: Following Carroll, we carried out a Schmid-Leiman hierarchical factor analysis (Schmid & Leiman, 1957).

Step 2: Next, to implement the higher order factor model, we used maximum likelihood confirmatory factor analysis (CFA). Initially the factor structure was thus based on the Schmid-Leiman results. Loadings that were found to be small (< 0.15) in step 1 were omitted, but otherwise we included cross-loadings (as in Carroll, 1993).

Step 3: To evaluate the fit (Schermelleh-Engel, Moosbrugger, & Müller, 2003), we considered the ratio of the χ^2 to the degrees of freedom (χ^2/df), the Root Mean Square Error of Approximation (RMSEA), and the Non-normed Fit Index (NNFI). We considered a value of the RMSEA of 0.05 or lower and a value of χ^2/df of less than 2 as indicative of good fit. We considered an NNFI between .90 and .95 as acceptable, and above .95, as good. To obtain an acceptable model, we allowed for correlated errors in the model obtained in step 2 (but only if these correlations were open to reasonable theoretical interpretable).

Step 4: Since we hypothesized that once education differences are taken into account, Gc and verbal ability, on the one hand, and Gf and *g,* on the other, would be indistinct, we partialled out the educational variable. We did so by regressing all indicator variables on this variable in the model obtained in step 3. To evaluate our hypotheses, we examined correlations among the factors and the amount of explained variance in given factors (squared multiple correlations). If the correlation between two factors did not differ from 1, we took this to mean that we could remove one factor as being redundant in accounting for the observed covariance structure. If the variance of a given factor was fully explained by other factors (i.e., not significantly different from 100% explained variance), we interpreted the first as being nonexistent.

Step 5: Finally, we implemented the model leaving out any redundant factors.

5.6.3 Results

Step 1: On the basis of the results of the Schmid-Leiman procedure and theoretical considerations, we arrived at a model including 7 first order factors and, following Carroll's interpretation of the factors at the second stratum, 3 second-order factors. We interpreted the first order factors as inductive reasoning (Re), verbal comprehension (Ve), spatial reasoning (SA), quantitative reasoning (QR), (verbal) fluency (Fl), associative memory (AM), and speed (Sp). We denoted the second order factors fluid intelligence (Gf), a Gc factor (its interpretation yet to be evaluated), and BM, which Carroll interpreted as broad memory. Following Carroll (1993), speed (Sp) and associative memory (AM) were taken to load on the broad memory factor in the CFA. Following investment theory and Cattell's original observation that in his bipartite model fluid and crystallized tests loaded on both general factors, the other first order factors were taken to load both on Gf and on Gc. In the CFA, this leads to identification problems, which we solved by fixing the factor loadings of Re on Gc, and of Ve on Gf to zero. We consider this acceptable as these loadings are expected to be low: The tests that loaded on Reasoning can be assumed to require a minimum of prior knowledge, and the tests that loaded on Verbal comprehension were clearly knowledge tests (hence crystallized tests), so they can be assumed to require only a minimum of relation perceiving.

Step 2: We fitted the model and removed small loadings on Gc and Gf. Ultimately, Ve and Sp loaded on Gc, Re and Qu on Gf, and Fl on both Gc and Gf.

Step 3: We allowed for correlated residuals of the tests *vocabulary* and *sentence completion* (because *sentence completion* can be considered to rely on vocabulary), and correlated residuals of the tests *words association* and *word checking* (because *words association* can be assumed to involve mentally checking of words in order to prevent giving answers that have already been given).

For this model, we obtained a χ^2 of 492.05 (df = 275 , p <0.001), hence, $\chi^2/df < 2$, with RMSEA = 0.049 and NNFI = 0.94. That is, the fit was acceptable. Factor loadings are given in Table 5.3. Squared multiple correlations for this model are given in Table 5.4.

Step 4: Partialling out the educational variable resulted in a model (see Figure 5.4) in which the correlation between Gf and *g*, and the correlation between verbal comprehension and Gc were found to equal one. So statistically, Gf and *g* were indistinguishable. In addition, the correlation between the first order reasoning factor and Gf was also one. Hence, statistically, all variance in reasoning was explained by Gf (or *g*). Finally, statistically, Gc explained all variance in verbal comprehension.

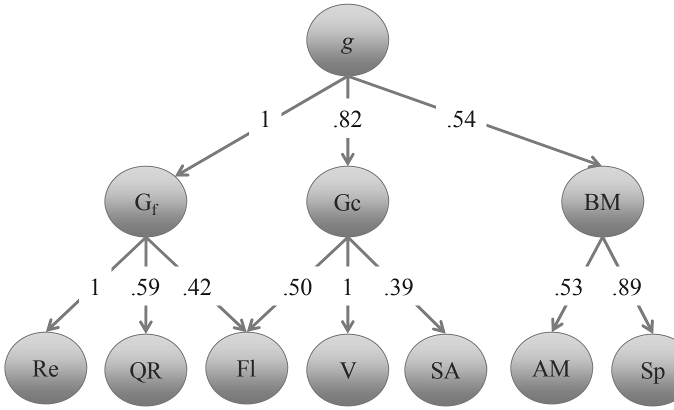

Figure 5.4 Hierarchical factor model derived from the Schmid-Leiman procedure and Confirmatory Factor Analysis (without Education regressed out).

Step 5: The redundant factors were omitted. This model (see Figure 5.5) yielded the exact same fit statistics as the hierarchical factor model.

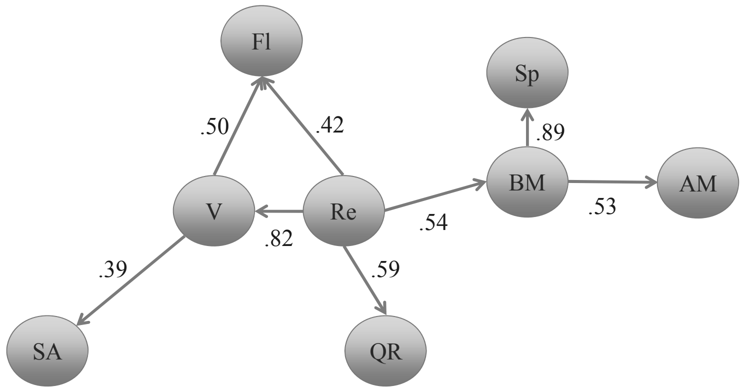

Figure 5.5 Causal model (interpretation of the factor model in Figure 5.4)

5.6.4 Conclusions

We hypothesized that in the ALLI00 data set, once educational differences were taken into account, Gc and verbal comprehension on the one hand, and Gf and *g*, on the other, would be indistinct. These hypotheses were confirmed. Because Gc was found to be redundant, we conclude that crystallized intelligence is not a unique capacity. Furthermore, Gf accounted for all the variance in the first order factor Re (reasoning). So, if the three factors *g,* Gf and Reasoning are interpreted as realistic underlying variables (e.g. as representing individual differences in psychological or biological capacities), there is no need to assume that they constitute different variables (hence capacities). We note that in the model of Step 5, verbal comprehension mediates not only (directly) the relationship between reasoning and verbal knowledge, but also (indirectly) the relationship between reasoning and spatial (mechanical) knowledge.

5.7 Discussion

The CHC factor Gc has been interpreted in different ways. On the one hand, Gc has been interpreted as a realistic, underlying cause of individual differences in intelligence subtest scores. On the other hand, Gc has been interpreted as crystallized intelligence, consistent with Cattell's investment theory of fluid and crystallized intelligence. We demonstrated that these interpretations are mutually incompatible, because in investment theory crystallized intelligence is not an underlying cause, but a statistical entity that (under certain circumstances) may emerge in factor analysis. We argued that, if one subscribes to investment theory, and to the interpretation of Gc as underlying cause of individual differences in intelligence subtest scores, two interpretations are tenable. We maintained that, depending on the exact constitution of the test battery, Gc can be interpreted as (1) exposure to information through education, or (2) verbal comprehension. We corroborated our main conclusions by presenting a reanalysis of a HCA (Human Cognitive Abilities project) data set. Our results supported the interpretation of Gc as verbal comprehension. We conclude that crystallized intelligence does not represent an underlying variable, i.e., a real, causal cognitive capacity, or any other psychological or biological capacity. Since crystallized intelligence is a statistical entity, it is better conceptualized as a formative variable (see Diamantopoulos, 2008, for example).

The objective of this chapter was to contribute to the theoretical framework of the CHC model, and to elucidate the relations among *g*, Gc and Gf in the light of investment theory. Therefore, we took the CHC model and investment theory to be correct. However, we do not mean to say that we consider the CHC taxonomy to be the best taxonomy or that investment theory is correct in reality. We do not rule out that other taxonomies (e.g. based on Johnson's VPR model) may provide more satisfactory results. In addition, in view of the mixed support for investment theory (Ferrer & McArdle, 2004; Kvist & Gustafsson, 2008; Schweizer & Koch, 2002), we also content that investment theory is itself is open to debate.

However, despite the mixed results concerning investment theory, we note that Gf is still found to be a predictor of Gc (Ferrer & McArdle, 2004). We argue that the essence of investment theory - the hypothesis that cognitive processing (including reasoning) is involved in the acquisition of knowledge, skills, and solving strategies - remains plausible. We also note that in Cattell's investment theory, which is broader than the investment hypothesis of fluid intelligence, Gf is not the only predictor of crystallized intelligence; exposure to information through education is another. From our results, we conclude that the acquisition of knowledge as a result of reasoning may be mediated by verbal comprehension.

We believe that Cattell's investment theory still provides a viable framework to test hypotheses in intelligence research.[12] For example, Kvist & Gustafsson (2008) hypothesized that the relation between fluid intelligence and the general factor is a function of cultural background, which they consider to be evidence for investment theory. They predicted a perfect correlation between Gf and *g* in samples homogeneous with respect to culture, age, and education. In our re-analysis of the HCA ALLI00, we confirmed this prediction. To study investment theory properly, it is important to be aware of the role of sample heterogeneity. Ideally, it should be investigated in a longitudinal design using same-aged, same-sex, culturally and educationally homogeneous samples.

We contend that the explication of one's scientific/philosophical position regarding the CHC factors will facilitate the integration of empirical results, and the investigation of old (e.g. the hypotheses in investment theory) and new hypotheses. Like Gc, Grw (Reading and writing), Gq (Quantitative knowledge), and Gkn (General specialized knowledge) (McGrew, 2009) are regarded alternately as achievement clusters (McGrew & Woodcock, 2001), the results of investment of cognitive abilities (McGrew, 2009), subsets of crystallized intelligence (Snow, 1996; Woodcock, 1994), and as non-unitary variables (McGrew & Evans, 2004). We conjecture that the appropriate measurement models for these achievement variables are formative, rather than reflective.

In the spirit of Cattell (1987, pp. 365-433), we argue for the usage of different notations for domain specific, domain transcending, and domain general variables, and for different notations for constructivist variables and realist variables. The adoption of such explicit notation will benefit the CHC theoretical framework. We expect that this will further increase our understanding of empirical findings, help to generate novel hypotheses, and, ultimately, reinvigorate the study of human intelligence.

[12] Although our hypotheses, derived from investment theory, were supported by our analyses, we leave open the possibility that the theory may be incorrect. The development of intelligence may be more dynamic and complex (e.g. reciprocally interactive) than suggested by investment theory. As dynamic modeling has shown that factors may arise from such interactions (van der Maas et al., 2006), each variable in the CHC model deserves critical examination.

Table 5.3 Subtests' loadings on the first order factors in the current Confirmatory Factor Analysis.

		First order factor loadings						
	Subtest	Ve	SA	Qu	Fl	AM	Sp	Re
1	Vocabulary	**.82**						
2	Sentence Completion	**.74**						
3	General Classification Test	**.99**						
4	Arithmetic			.45				**.54**
5	Division			**.85**			.24	
6	Math Aptitude			.26				**.53**
7	Number Series							**.75**
8	Letter Sets	.29					.18	**.31**
9	Ship Destination		.21					**.76**
10	Reasoning							**.68**
11	False Premises	**.44**						
12	Words Association				**.69**			
13	Word Checking				**.65**			
14	Recognition				**.56**			
15	Mechanical	.16	**.83**					
16	Mechanical Knowledge		**.77**					
17	Paper Form Board		**.54**					.23
18	Cards		**.50**					.22
19	Paper Folding		**.39**					.39
20	Cubes							.16
21	Picture Discrimination		.34				**.53**	.19
22	Clerical Aptitude						**.77**	
23	Picture Number					**.78**		
24	Word Number					**.55**		
25	First Names				**.43**	.41		
26	Addition			**.56**			.44	

Note: Ve = Verbal comprehension, SA = Spatial ability, Qu = Quantitative Reasoning, Fl= (Verbal) Fluency, Me = Memory, Sp = Speed, Re = Reasoning. Bold faced loadings denote most salient loadings

Table 5.4 Squared multiple correlations of the higher order factors in the Confirmatory Factor Analyses.

Order	Squared multiple correlations without education partialled out						
Third	*g*						
	100.0%						
Second	Gc	Gf	BM				
	21.9%	0.0%	12.0%				
First	Verbal	Spatial	Quantitative	Fluency	Memory	Speed	Reasoning
	0.0%	50.3%	12.9%	10.9%	43.4%	0.7%	0.0%
Order	Squared multiple correlations with education partialled out						
Third	*g*						
	100,0%						
Second	Gc	Gf	BM				
	20.0%	0.0%	13.9%				
First	Verbal	Spatial	Quantitative	Fluency	Memory	Speed	Reasoning
	0.0%	46.9%	13.3%	11.1%	43.3%	0.8%	0.0%

Note: *g* = general intelligence, Gf = fluid intelligence, Gc = to be interpreted