Chapter 1

Introduction

The start of the Large Hadron Collider (LHC) in 2009 located at CERN, Geneva, has opened a new window for the search of elementary particles and will provide stringent tests for proposed theoretical frameworks of these particles and their forces. Our current understanding is described by the Standard Model (SM). This theory describes the interaction of six quarks and six leptons by the strong, weak and electromagnetic force, mediated by bosons. The model is completed with the Higgs mechanism that provides mass to the particles, and is largely confirmed by the recent discovery of the what seems to be SM Higgs boson. The model has been tested to impressive accuracy. Successful though it is, it fails to provide an explanation for astronomical observations as dark matter and energy. Moreover, the model only takes three fundamental forces into account, neglecting the force of gravity. One therefore often views the SM as an effective theory of a larger model.

Usage of the SM as an effective theory is sufficient to describe previous experiments. These unanswered questions however, have motivated proposals of more encompassing theories such as supersymmetry, strong dynamics models, etc.. While we expect the validity of the SM to be limited, it is a central question of modern particle physics where and how such new physics will show itself. The collision energy of the LHC is up to seven times larger than its predecessor the Tevatron at Fermi National Accelerator Laboratory, USA, and will allow for the testing of such new physics. The question posed translates to where and how to look for the deviations of the SM as predicted by these new theories, and this demands an increasingly higher accuracy of SM predictions. Standard Model predictions for collider observables are made using perturbative expansions in the coupling of the theory. The calculations can be depicted by Feynman diagrams, named after their inventor, representing a graphical interface for complicated mathematical expressions. The most interesting part of the collision takes place in the heart of the beam pipe at the primary interaction point, creating both stable and short-lived particles detected through their decay products. The prediction of a scattering process therefore takes all possible particles into account that may have propagated the intermediate state, given a certain initial and final state. The leading order (LO) contribution is defined by the sim-
plest diagrams, corresponding to those with the fewest vertices, possible in accordance with the required external particles. The next-to-leading-order (NLO) correction is given by diagrams that are two orders higher in the coupling. In most cases only the Quantum Chromodynamics (QCD) corrections, that dominate quark and gluon interactions, need to be taken into account. An example is given in fig. [1]. The figure on the left depicts the leading order contribution to the scattering of a quark-antiquark pair, and the two rightmost figures examples of next-to-leading-order corrections. Being a quantum theory the predictions require the absolute value squared of the sum of diagrams, which explains that two orders higher in the coupling yields a diagram with a loop which gets contracted with a LO diagram, or diagrams with an additional final state particle contracted with itself.

Figure 1.1: Examples of Feynman diagrams for the scattering of a quark and antiquark pair producing an intermediate gluon. The figure on the left denotes a LO diagram, and the two rightmost figures NLO diagrams.

For a proton-proton collider as the LHC the description is complicated by the initial state. Perturbative QCD cannot describe the strong binding effects that cause bound states as in the case of a proton. This makes it impossible to describe proton-proton collisions exactly. The factorization theorem allows one to separate the calculation in two parts, a hard interaction accounting for the small distance scattering of two partons (quarks or gluons), and the prediction of finding a certain parton in the proton [1, 2]. The parton model presents a basic picture of why we are allowed to view partons as individual non-interacting proton constituents. The relativistic proton is Lorentz contracted, while the interaction time between partons within the proton is time dilated, and therefore the proton can essentially be regarded as individual partons with a momentum fraction $x$ while crossing the colliding proton. It is possible to prove [3] that such a picture holds under the influence of QCD corrections, with the derivation leading to the factorization theorem. Parton Distribution Functions (PDF) are fitted to sets of well calibrated data such as from Deep Inelastic Scattering (DIS). The cross-section $\sigma$ for a scattering $pp \to j\ell$ can now be written as

$$\sigma_{pp \to j\ell} = \sum_{ik} \int \frac{d x_1}{x_1} \frac{d x_2}{x_2} f_i(p)(x_1, \mu_F^2) f_k(p)(x_2, \mu_F^2) \sigma_{ik \to j\ell}(Q^2, \mu_F^2),$$

with $x_1$ and $x_2$ the energy fractions of the partons $i$ and $k$, the PDFs are indicated with the function $f$, $Q^2$ indicates the scale of the hard interaction and $\mu^2$ the factorization scale at which the hard scattering is separated from long distance interactions.
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The hard scattering process, the perturbatively calculated piece, will yield an expression dependent on the momenta of the external particles, and needs to be integrated over unobserved phase space in order to obtain the total cross-section. Generally these expressions get increasingly more complicated as more massive particles and/or more than two final state particles are involved. This drives one to the use of numerical methods in order to solve the integrations. A naive approach uses an estimate of the average value to calculate the integrand \( I \) over its whole domain. For a one-dimensional function \( g(x) \), with \( x \in [a, b] \) this corresponds to

\[
I = \int_{a}^{b} g(x) \, dx \simeq \frac{(b-a)}{N} \sum_{i=1}^{N} g(x_i) + \mathcal{O}\left( \frac{1}{\sqrt{N}} \right),
\]

with \( i \) denoting random points chosen from a uniform distribution between \( a \) and \( b \). The central limit theorem states that the error to this estimate of the integrand may be approximated by \( 1/\sqrt{N} \) for large values of \( N \). An obvious improvement of the approach is to focus on regions that yield the largest contribution to the integral. An interesting by-product of the calculation is the generation of an ‘event’, associating Feynman diagrams with explicit values for the momentum vectors of the particles. The complete cross-section is approximated by

\[
\sigma = \int \frac{d\sigma(x_1, \ldots, x_n)}{dx_1 \ldots dx_n} \, dx_1 \ldots dx_n.
\]

The phase space of a cross-section containing \( N \) outgoing particles requires integration over \( 3N - 4 \) variables, accounted for by the variables \( x_i \). We may naively interpret eq. (1.3) as the generation of an event with a weight \( d\sigma/dx_1 \ldots dx_n \). The weight indicates the importance of that phase-space point to the integrand. Therefore these distributions need to be unweighted in order for event generators to create events with the same weight. Such a procedure can be implemented with a ‘hit-and-miss’ approach. The algorithm requires the sacrifice of some events, to allow for the remainder to share equal weights. This approach consists of the following steps

- Select an event
- Generate a random number between zero and the maximum weight present.
- If the random number is smaller or equal to the weight accept the event, and reject otherwise.

We have assumed all weights to be positive definite. An intuitive way of understanding this, is by comparing the weights of two events, \( w_1 \) and \( w_2 \). Assuming \( w_2 > w_1 \), the probability for accepting \( w_2 \) is \( w_2/w_1 \) times larger than the probability of accepting \( w_1 \). As the random numbers are generated uniformly, we expect that when considering enough events, there will eventually be \( w_2/w_1 \) times more events accepted that started with a weight \( w_2 \) than events that started with a weight \( w_1 \). Because it can be time consuming to sacrifice events, the choice is often made to use all events with the allocated weights.
The previous procedure automatically generates distributions in $x_i$. The accuracy can, in principle, be made as high as desired given sufficiently large $N$.

With this description of events it is possible to simulate signals and background, and thereby explore the influence of new physics in collider events. One approach that is gaining much interest is to focus on the spin of the particle. The spin of a particle influences the angular orientation of its decay products. While, for instance, a top quark always carries spin-\(\frac{1}{2}\), the preference for spin up or spin down (polarization) can be different depending on the model. Event simulation is used to investigate if spin information present in the hard scattering can be measured at the detector level. The top quark decays before the strong force causes it to hadronize, and thereby retains possibly interesting spin information in the top decay products. We start with exploring the spin dynamics in top production in chapter 2 together with the definition of various laboratory frame observables allowing for a connection between a theory’s parameters and its decay products through the use of polarization. In chapter 3 the effect of NLO corrections to the observables is examined for the SM in comparison to Two Higgs Doublet Models. The focus on top polarization is completed with a survey of its sensitivity to the supersymmetric parameter space in the context of the MSSM.

For now, we have been generating events based on the Feynman diagrammatic approach. This treated the hard-scattering as a whole, creating intertwined products of momenta. An alternative way of building up a scattering event is by treating the particle radiations and interactions iteratively. Instead of requiring a certain initial and final state, the approach is to start with a specific LO process and radiate particles iteratively off the remainders of the LO process, a procedure commonly referred to as a parton shower. One can employ monopole radiation (radiation off one parent particle) or dipole radiation (radiation of a third particle between two parent particles). This transforms the LO process to a final state with additional partons. An interesting question that has to be addressed is how to incorporate the knowledge of NLO precision in such an approach. In chapter 5 a review of the VINCIA antenna shower is discussed for a $Z$ boson decaying to two partons, together with a demonstration of the trivial matching to NLO while chapter 6 focusses on the generalization of consecutive NLO matching for an additional final state particle. The final chapter contains our conclusions.