Top quark spin and QCD corrections in event generation

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Chapter 3

Top Polarization in $Ht$ and $Wt$ Production

The polarization-dependent observables discussed in section 2.3 were so far studied at leading order (LO) accuracy. For a given polarization-dependent observable, such a calculation represents a best case scenario in which polarization effects in the production of the top quark are least diluted by kinematic effects. Beyond this order in QCD perturbation theory, additional radiation may carry away energy and/or angular momentum. In this section we extend the study to next-to-leading order (NLO) accuracy, including also the effects of a parton shower. Studying the observables at NLO + shower level and comparing them to the LO result provides a handle on their robustness. We start by introducing a rather generic physics scenario.

3.1 Two Higgs Doublet Model

When a top quark is produced together with a $W$ boson in the Standard Model, the $V-A$ nature of the weak interaction eq. (2.13) implies that the top quark is always left-handed at the moment of production. If the production however is followed by a massive top propagator mixing terms arise. However, top quarks produced by BSM processes can have a different polarization than is predicted in the SM. Hence, the polarization of the produced top can help distinguish between BSM and the SM.

An example is charged Higgs plus top production, a process analogous to $Wt$ production in the SM. The leading order contributions to this process are similar to fig. 2.3 with the $W$ replaced by a charged Higgs boson. In contrast to the SM, this theory consists of two Higgs doublets $\phi_1, \phi_2$, predicting therefore not one, but five Higgs bosons. Motivations for introducing another doublet can for instance be found in the extension of the SM with supersymmetry. We will elaborate further on the impact of supersymmetry in the next chapter, but for now it is sufficient to know that supersymmetry postulates a symmetry between all known SM particles and their supersymmetric partners that differ
3.1. Two Higgs Doublet Model

by spin-$\frac{1}{2}$. A second Higgs doublet is required in this model to preserve the cancellation of gauge anomalies, see for instance [16]. The Higgs potential is constructed from hermitian combinations of these two doublets subject to the conditions of renormalizability and gauge invariance. Focussing on the terms quadratic in the doublet fields we find

$$V(\phi_1, \phi_2) = \lambda_1 \left( \phi_1^\dagger \phi_1 - v_1^2 \right)^2 + \lambda_2 \left( \phi_2^\dagger \phi_2 - v_2^2 \right)^2 + \lambda_3 \left[ \left( \phi_1^\dagger \phi_1 - v_1^2 \right) + \left( \phi_2^\dagger \phi_2 - v_2^2 \right) \right]^2 + \ldots$$  (3.1)

The occurrence of $v_i = \langle \phi_i \rangle$ with $i = 1, 2$ denotes that we are now dealing with two vacuum expectation values. These parameters are commonly varied together according to their ratio $\tan(\beta) = v_2/v_1$. Observing the production of a charged Higgs particle would provide very valuable information on the structure of the Higgs sector. Studies have shown [17, 18] that top polarization can be used to extract information on the model parameters of a Two Higgs Doublet Model (2HDM). However, these analyses were carried out only at LO in perturbation theory. While the higher order corrections coming from the chirality and parity conserving QCD interactions will not affect the top polarisation, they can change the kinematics of the produced top quark and hence it is important to verify that the conclusions of the LO analysis are robust against NLO corrections.

In our investigation of the effects of NLO corrections we will assume that the charged Higgs boson has a larger mass than the top quark, so that the top quark cannot decay to it. A general form for a charged Higgs-top quark interaction is

$$G_{H - \bar{t}b} = iV_{tb}(a - b\gamma_5).$$

This reduces to its SM equivalent of $Wt$ production by setting $a = b = \frac{1}{2\sqrt{2}}g$, with $g$ the SU(2) gauge coupling. For later event simulation we must make explicit choices for these parameters. We have followed the approach of [19], that derives a specific form of $a$ and $b$ based on the counterterms resulting from the charged Higgs Yukawa couplings. We will consider a type II two Higgs doublet model, where the up type quarks couple to one of the Higgs doublets and down type quarks couple to the other Higgs doublet. The coupling of the charged Higgs to the top and bottom quarks is given by

$$G_{H - \bar{t}b} = -\frac{i}{v\sqrt{2}}V_{tb}\left[ m_b \tan \beta(1 - \gamma_5) + m_t \cot \beta(1 + \gamma_5) \right].$$  (3.2)

Here the vacuum expectation values of the two Higgs doublets are $v \cos \beta = v_1$ and $v \sin \beta = v_2$, with $v^2 = v_1^2 + v_2^2$. Now that we have established the fundamental interactions for a BSM single top production mode let us discuss the effects of NLO corrections to the polarization.
3.2 Effects of NLO Corrections to Polarization Dependent Observables

The NLO correction to the $Ht$ production includes QCD interactions, which conserve parity and chirality. Kinematic effects on the other hand do change when going to NLO + shower accuracy. In particular, as will be shown explicitly in fig. [3.1] the boost of the top quark, as measured by the $B$ parameter of eq. (2.10), increases a few percent in the highest bin with respect to the LO boost due to the higher order corrections.

For the LO computation of the $H^- t$ production process, we use MadGraph 5 [13,14], where we extended the Standard Model to include the charged Higgs coupling. Contributing diagrams at LO are comparable to those in fig. 2.3 where the $W$-boson must be exchanged with a charged Higgs boson. The NLO calculation matched to a parton shower was performed using the MC@NLO software package described in [19–24], with spin correlations between the partons in the hard scattering and the Higgs and/or top decay products, implemented according to the algorithm of [25].

The $Wt$ production process poses a conceptual problem at NLO, due to the fact that some of the real emission diagrams beyond LO involve an intermediate top quark pair. The contribution from such diagrams is large when the $\bar{t}$ becomes resonant, reflecting an interference between the $Wt$ and top-pair production processes. How to most accurately model the sum of $Wt$ and top-pair production then becomes a topic of debate, and there are two main points of view. The first is that all singly and doubly resonant diagrams must be combined, thus including all interference (and off-shell) effects (see, for example, [31, 32]). A major deficiency of such calculations, however, is that they typically do not include NLO corrections, which for top pair production are known to be large. NLO corrections for the $WWb\bar{b}$ final state have been presented [33], also including decay of the $W$ bosons [34], in the so-called four flavour scheme in which all initial state $b$ quarks are explicitly generated via gluon splitting, although these results have yet to be interfaced with a parton shower.

The second point of view is that singly and doubly resonant contributions may be safely regarded as separate production processes, which may be meaningfully combined subject to suitable analysis cuts, an approach followed by e.g. [20, 29, 35, 36]. This amounts to defining a subtraction term, which removes doubly resonant contributions from the $Wt$ cross-section. A potential deficiency of such an approach is that gauge invariance is violated by terms $\sim O(\Gamma_t/m_t)$, where $\Gamma_t$ is the top quark width, although it is usually (convincingly) argued that this is more a problem of principle than one of practice. Another way to think about this procedure is that the subtraction term avoids the double counting that would result upon naively adding the $Wt$ and top pair cross-sections at NLO. Such on-shell subtraction schemes are in fact a common feature in many NLO calculations involving extensions to the Standard Model, in which intermediate heavy particles abound (see e.g. [37–40]). Indeed, in this context, the interference problem is usually referred to in terms of being a double counting issue.

1 Alternative methods for matching NLO computations with a parton shower have been presented in [26,27] and chapter 5 and 6. See also [28–30] for implementations of the processes discussed in this chapter.
3.3 Results for $H^-t$ Production

It is not our intention to revisit the issue on the validity of on-shell subtraction schemes. But, in order to discuss $Wt$ production at all, we must necessarily take the view that it makes sense to separate singly and doubly resonant production modes. For a detailed recent discussion of this viewpoint, see [41]. In that paper, it was argued that $Wt$ is unambiguous for suitable analysis cuts, and we will assume the validity of this approach in what follows.

The MC@NLO code for $Wt$ production includes two definitions of $Wt$ production, labelled Diagram Removal (DR) and Diagram Subtraction (DS), where the difference between these is intended to represent the systematic uncertainty due to interference with top pair production. Roughly speaking, DS subtracts doubly resonant (i.e. top pair) contributions at the cross-section level (thus is gauge invariant up to terms $\sim O(\Gamma_t/m_t)$), and DR subtracts such contributions at the amplitude level. The difference between these then mostly measures the interference between $Wt$ and $tt\bar{t}$ production, up to ambiguities in the subtraction term. However, one only trusts each calculation if the DR and DS results agree closely, which relies upon the imposition of suitable analysis cuts for reducing the interference. We will not implement such cuts in the calculation of the observables for $H^-t$ production as this study aims to present a first investigation of the effect of NLO corrections + parton shower without the complication of a full experimental analysis. Despite this, we will show the results obtained from both the DR and DS calculations.

3.3 Results for $H^-t$ Production

In the previous chapter, we reviewed the observables which are designed to be sensitive to the polarization state of produced top quarks. In this section, we study these observables for single top production in association with a charged Higgs boson. The latter does not occur in the Standard Model of particle physics, but exhibits a somewhat generic presence in possible extensions, including supersymmetry.

The top quark polarization in the $H^-t$ production process does not follow directly from eq. (3.2). As explained in detail in Ref. [17], the polarization vanishes if $m_H = 6m_t$ and if $\tan \beta = \sqrt{m_t/m_b}$. The latter point in parameter space cancels the axial contribution in eq. (3.2). In addition, it was shown in figure 4 of that paper that the $\tan \beta$ dependence of the polarization is different for different Higgs masses. For Higgs masses below $6m_t$ it is negative if $\tan \beta < \sqrt{m_t/m_b}$ and positive for higher values of $\tan \beta$. The polarization for higher Higgs masses has the opposite behaviour. In the rest of this section, we will often show distributions for $m_H = 200$ GeV and $m_H = 1500$ GeV as representative examples. For a given value of $\tan \beta$, the former is more strongly polarized than the latter.

One may study how the observables of section 2.3 vary throughout the two dimensional parameter space $(m_H, \tan \beta)$. In what follows, we will do this at LO and NLO, as specified in section 3.2. Note that the aim of this section is not to undertake a fully comprehensive phenomenological analysis, including all relevant backgrounds together with realistic experimental cuts. Rather, we wish to study the efficiency of the different observables that reflect the polarization of the parent top, and in particular their robustness.
when one includes higher order effects.

In order to present results, we consider the LHC with a centre of mass energy of 14 TeV, and define parameters as follows: the top mass and width are \( m_t = 172.5 \text{ GeV} \) and \( \Gamma_t = 1.4 \text{ GeV} \) respectively. The \( W \) mass and width are respectively \( m_W = 80.42 \text{ GeV} \) and \( \Gamma_W = 2.124 \text{ GeV} \). Factorization and renormalization scales are set to \( \mu_r = \mu_f = m_t \). We calculate LO and MC@NLO results using MSTW 2008 LO and NLO parton sets [42–44]. Note that the \( b \) mass entering the Yukawa coupling is run as in [45], from a pole mass of \( m_b = 4.95 \text{ GeV} ^2 \).

As explained in section 2.3, the polarization-dependent observables are affected considerably by the kinematics of the top. Therefore we first briefly discuss the boost parameter \( B \) and the top transverse momentum \( p_T^t \). On the left-hand side of fig. 3.1 the distribution of the boost parameter is shown for two different values of the charged Higgs mass. On the right-hand side, the LO and NLO + parton shower distributions are compared. The distribution is much more strongly peaked for the high Higgs mass, as expected from the fact that the top quark must recoil against the heavy particle. In addition we see that the NLO+parton shower effects increase the boost parameter slightly. This can be traced back to the definition of eq. (2.10), coupled with the fact that the energy of the top quark softens more on average than its momentum when higher order effects are included.

### 3.3.1 Azimuthal Angle \( \phi_l \)

Fig. 3.2 shows the \( \phi_l \) distribution defined in section (2.9) for two different values of \( \tan \beta \), and two different charged Higgs masses at NLO + parton shower. For \( \tan \beta = 5 \), there is a pronounced difference between the two \( \phi_l \) distributions at different mass values, with the higher mass value showing more asymmetry. At high \( \tan \beta \), there is very little difference between the two Higgs mass values. The reason for this behaviour can be traced back to the polarization of the top. At low \( \tan \beta \) a light Higgs yields a negatively polarized top, so in the rest frame the lepton tends to be emitted in the backward direction (cf. eq. (2.8)). For a heavy Higgs the top is positively polarized for low values of \( \tan \beta \), so the lepton is emitted in the forward direction. Since the top is boosted more for higher Higgs masses, the kinematics enhance this polarization effect. For large \( \tan \beta \), the top polarization has the opposite sign, so in that case the kinematics cancel the effect of the polarization.

In fig. 3.3 the \( \phi_l \) distribution is shown at LO and MC@NLO level for \( \tan(\beta) = 5 \) and two different charged Higgs masses. The results can be compared to figure 6 of [17], and indeed the qualitative trend of the curves is the same as in [17]. In the case of a high Higgs mass the distribution becomes slightly flatter due to the NLO corrections and parton shower. This is caused by competing kinematic effects. As shown in fig. 3.1 the top boost increases slightly due to the higher order corrections, but the \( p_T^t \) distribution is typically softer compared to LO, and progressively more so for higher Higgs masses as the top then showers more on average. The higher top boost leads to a sharper \( \phi_l \)

\(^2\)Strictly speaking, one should run the \( b \) mass at one-loop order for the LO results, and two-loop order for the NLO results. To facilitate a more direct comparison between the LO and MC@NLO results we adopt the LO prediction, given that the relative proportion of right- and left-handed \( H^- t \) couplings is governed by the value of \( m_b(\mu_r)/m_t(\mu_r) \). We have checked that the difference in running is a small effect.
3.3. Results for $H^- t$ Production

Figure 3.1: The distribution of the boost parameter of in $H^- t$ production for $\tan \beta = 5$ and two different Higgs masses is shown on the left-hand side. On the right-hand side the boost parameter is shown at LO and NLO plus parton shower level.

Figure 3.2: Azimuthal angle ($\phi_l$) of the decay lepton from the top quark, as defined in the text, at NLO plus parton shower level.

distribution, but for high Higgs masses the effect of the softer $p_T^t$ distribution is stronger, resulting in a flatter distribution in the end. We can quantify this further by calculating the asymmetry parameter of eq. (2.9). We show this in fig. 3.4 for the two Higgs mass values used above and a range of $\tan \beta$ values. Both LO and MC@NLO results are shown for comparison, where for the MC@NLO results we include an error band stemming from statistical uncertainty. The shape of fig. 3.4 is very similar to the results of [17]: for the large charged Higgs mass value, a high asymmetry is observed for low $\tan \beta$, which decreases at large $\tan \beta$. For the low charged Higgs mass value, the opposite trend is seen.

The MC@NLO results show less of a difference between the two Higgs mass values than the LO results. This is caused by the competing kinematic effects we already saw in fig. 3.3. The higher top boost leads to a larger value of the asymmetry $A_{\phi}$, but for high Higgs masses the effect of the softer $p_T^t$ distribution is stronger, yielding a net reduction
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Figure 3.3: Azimuthal angle ($\phi_l$) of the decay lepton from the top quark, as defined in the text, comparing LO and NLO + parton shower.

Figure 3.4: Azimuthal asymmetry parameter for $H^-t$ production, as defined in eq. (2.9). LO (MC@NLO) results are shown in blue (black), for $m_H = 200$ GeV (lower curves) and $m_H = 1500$ GeV (upper curves). The error band is statistical. Results for $Wt$ production, using both the DR and DS approaches in [20], are shown in red.

We see that the difference between the DR and DS results is much less than the difference between $Wt$ and $H^-t$ production, which gives us confidence that the interference of $A_\phi$. At NLO, the difference between the two Higgs mass values is smaller than at LO, even at low $\tan \beta$. However, a pronounced asymmetry is still visible, with a strong dependence on the charged Higgs parameters, so the azimuthal asymmetry appears to be quite robust with respect to higher order corrections.
3.3. Results for $H^-t$ Production

issue does not get in the way of getting an estimate of the asymmetry parameter for $Wt$. Thus, the fact that $Wt$ and $H^-t$ production lead to rather different $A_\phi$ values (for essentially any choice of $m_H$ or $\tan \beta$), as has already been observed at LO [17], remains true at NLO and after a parton shower has been applied.

3.3.2 Polar Angle $\theta_l$

One may also consider the polar angle between the decay lepton and the top quark direction. Fig. 3.5 shows the NLO+parton shower results for the same extremal values of $\tan \beta$ and $m_H$ as in fig. 3.2. We see that the distribution is more sensitive to the Higgs mass at small $\tan \beta$ than at large $\tan \beta$, which is again due to the enhancement (cancellation) of the polarization effects by the kinematics at low (high) $\tan \beta$.

The distribution of $\theta_l$ at LO and MC@NLO level is shown in fig. 3.6. As with the $\phi_l$ distribution, the NLO distribution strongly resembles the LO results. The NLO distribution is peaked towards $\theta_l = 0$ somewhat more due to the slight increase in the top boost parameter.

![Figure 3.5: Polar angle ($\theta_l$) of the decay lepton from the top quark, measured with respect to the top quark direction, at NLO plus parton shower level.](image)

In all cases, the distribution shows a strong peak at low values of $\theta_l$, with a fall-off at higher values. For normalized distributions it follows that a distribution which has a slower fall-off must correspondingly have a lesser peak, and vice versa. This motivates the definition of the following asymmetry parameter:

$$A_\theta = \frac{\sigma(\theta_l < \pi/4) - \sigma(\theta_l > \pi/4)}{\sigma(\theta_l > \pi/4) + \sigma(\theta_l < \pi/4)}.$$  \hspace{1cm} (3.3)

We have here used $\pi/4$ as representative of the point at which distributions corresponding to different points in parameter space cross each other. However, we have found no obvious analytic justification for this result, so this number can in principle be varied in order to enhance the asymmetry.
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Results for the polar asymmetry parameter are shown in fig. 3.7. Again we show both LO and MC@NLO results, where a statistical uncertainty band is included for the latter. One sees that the MC@NLO values of \( A_\theta \) are higher than the LO results, as expected from the higher value of the top boost at MC@NLO level compared to LO. In contrast to the azimuthal asymmetry, there is a significant difference between the extremal charged Higgs mass values at large \( \tan \beta \). This makes the polar angle very useful as a complementary

Figure 3.6: Polar angle \((\theta_l)\) of the decay lepton from the top quark, measured with respect to the top quark direction, at LO and NLO plus parton shower level.

Figure 3.7: Polar asymmetry parameter for \( H^- t \) production, as defined in eq. (3.3). LO (MC@NLO) results are shown in blue (black), for \( m_H = 200 \) GeV (lower curves) and \( m_H = 1500 \) GeV (upper curves). The error band is statistical. Results for \( Wt \) production, using both the DR and DS approaches in [20], are shown in red.
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observable to the azimuthal angle, as the latter is relatively insensitive to the charged Higgs mass at large $\tan\beta$.

Similarly to the azimuthal case, one sees from fig. 3.7 that typical values for the polar asymmetry are markedly different to the result obtained for $Wt$ production, as estimated by the DR and DS results. Again this is presumably a reliable conclusion, given that the difference between the two $Wt$ results is much less than the difference between the $H^-t$ and $Wt$ results. This information is a potentially valuable tool in being able to distinguish charged Higgs boson production from the $Wt$ background.

3.3.3 Energy Ratio Observables

In the previous sections, we presented results for angular distributions of the decay lepton in $H^-t$ and $Wt$ production, finding these to be robust discriminators of the charged Higgs parameter space, as well as of use in distinguishing a charged Higgs signal from the Standard Model background. In this section, we consider the energy ratios $z$ and $u$ of eq. (2.11), which were first defined in [12].

Note that both the $z$ and $u$ observables depend on the energy of the $b$ quark emanating from the top quark decay. In a leading order calculation, this can be straightforwardly identified. In an experimental environment, one must use event selection cuts which require the presence of a tagged $b$ jet, and use the energy of this jet in constructing eq. (2.11). A full phenomenological analysis is beyond the scope of this chapter: we here wish to present a first analysis of the $z$ and $u$ parameters in the context of $H^-t$ production, unshrouded by the full complications of an experimental analysis. There is then a choice to be made regarding which energy to use in presenting results from MC@NLO. One option is to use the energy of the $b$-flavoured hadron that contains the $b$ quark from the top decay, requiring this to be stable. However, to facilitate a more direct comparison with the LO results, we instead define $E_b$ via the energy conservation relation

$$E_b = E_t - E_l - E_\nu,$$

(3.4)

where $E_t$, $E_l$ and $E_\nu$ are the energies of the top quark, decay lepton and decay neutrino respectively. The latter is, of course, unmeasurable in a real experiment but can be identified in a Monte Carlo event generator. Our definition of $E_b$ then means that our comparisons between LO and MC@NLO results measure the collective effect of a single hard additional emission (from the NLO matrix element), together with the parton shower, but with no non-perturbative contributions from e.g. hadronization or the underlying event. We deem such an approach to be valid in assessing the robustness of energy ratio observables against perturbative higher order corrections, which is our present aim. For a realistic situation an alternative definition implementing analysis cuts in terms of jets would allow for $E_b$ to be the energy of the $b$-jet which enters the cuts.

The energy ratios of eq. (2.11) are more sensitive to the top quark polarization in the kinematic region in which the decaying top quark is highly boosted. It is important to check which values of a cut on the boost parameter are sufficient in order to isolate the desired sensitivity to the top quark polarization.
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The energy ratios of eq. (2.11) can provide additional information when the top is boosted and the resolution of the top decay products is insufficient. First we will investigate the dependence of the influence of a boost cut on the energy ratios $z$ and $u$ of eq. (2.11) for different values of this cut in fig. 3.8. One sees that the results with a cut are markedly different from those with no cut (as expected). However, the difference between results with $B > 0.9$ and $B > 0.8$ is much less, suggesting that a cut of $B > 0.8$ is sufficient.

The distribution of $u$ at MC@NLO level after the cut $B > 0.8$ has been applied is shown in fig. 3.9 for two values of $m_H$.

Figure 3.8: Distribution of $u$ (left-hand plot) and $z$ (right-hand plot) for $\tan\beta = 1$ and $m_H = 200$ GeV, at NLO plus parton shower level. Results are shown for different cut values on the boost parameter $B$ of eq. (2.10).

Figure 3.9: Distribution of $u$, as defined in eq. (2.11), where a cut on the boost parameter $B > 0.8$ has been applied, at NLO plus parton shower level. Results are shown for $m_H = 200$ GeV (left-hand plot) and $m_H = 1500$ GeV (right-hand plot).

The shape of the plots can be compared to the corresponding figures in [12], which are presented for the ideal case in which the top quark is completely polarized and infinitely
### 3.3. Results for $H^- t$ Production

**Figure 3.10:** Distribution of $u$ with a cut on the boost parameter $B > 0.8$.

**Figure 3.11:** Distribution of $z$, as defined in eq. (2.11), where a cut on the boost parameter $B > 0.8$ has been applied, at NLO plus parton shower level. Results are shown for $\tan\beta = 1$ (left-hand plot) and $\tan\beta = 40$ (right-hand plot).

**Figure 3.12:** Distribution of $z$ at LO and MC@NLO level, with a boostcut of $B > 0.8$. 
boosted, i.e. $P_t = \pm 1$ and $B \to 1$. The latter seem to show a much more pronounced difference between the curves for positive and negative helicity top quarks. This is mostly due to the fact that in our case the top quarks are not completely polarized. The high Higgs mass in particular does not yield a strong top quark polarization. For the lower Higgs mass, the shapes are broadly consistent with the results of [12]: for the negatively polarized top quarks ($\tan \beta = 1$), the distribution falls off more sharply for higher values of $u$. Also, the curvature of the distributions is different for lower values of $u$ for the two different $\tan \beta$ values.

The $u$ variable at LO and MC@NLO level with a boostcut of $B > 0.8$ is shown in fig. 3.10. We see that the general shape does not change when including NLO+parton shower corrections.

We may also consider the $z$ distribution, which is shown for our two extremal $\tan \beta$ values in fig. 3.11. The plots have three distinct regimes. Firstly, there is a sharp fall-off as $z \to 0$, due to the finite mass of the $b$ quark. Then, there is an intermediate regime $0.1 \lesssim z \lesssim 0.7$, over which the $z$ distribution is approximately linear, with the sign of the slope correlated with the polarization of the top quark (i.e. positive and negative for negatively and positively polarized top quarks respectively). Finally, there is another fall-off as $z \to 1$, due to the finite $W$ boson mass. Again one sees very little correlation for the charged Higgs mass of 1500 GeV due to the small value of the polarization.

In fig. 3.12 we see that this is not due to the NLO and parton shower effects. The distribution is changed by these effects, but the correlation with the charged Higgs mass is not very strong even at LO. For the lower Higgs mass we also see that the NLO+parton shower corrections change the distribution more than for the angular distributions.

For the angular observables of the previous section, we used asymmetry parameters which efficiently distill the difference between different regions of the charged Higgs parameter space into single numbers. It is perhaps useful to also adopt this strategy for the energy ratios $u$ and $z$. Regarding the former, one may first note that the normalisation of the distribution means that a slower fall-off above the peak region entails less events below the peak region. One may exacerbate this effect by defining the corresponding asymmetry parameter

$$A_u = \frac{\sigma(u > 0.215) - \sigma(u < 0.215)}{\sigma(u > 0.215) + \sigma(u < 0.215)},$$  \hspace{1cm} (3.5)

Here $u \approx 0.215$ is chosen as the approximate position of the peak, motivated by the analysis of [12]. As in the case of the polar angle asymmetry of eq. (3.3), however, this choice can in principle be varied in order to enhance the result.

The behaviour of $A_u$ is shown in fig. 3.13 for a cut on the boost parameter of $B > 0.8$. For comparison purposes, we also show the result one would obtain with no cut on the boost parameter, where the $u$ observable suffers significant contamination from contributions which are insensitive to the top quark polarization. As expected, the $A_u$ variable has more discriminating power for the lower Higgs mass, since the top is more strongly polarized in that case. In addition one sees that the cut on the boost parameter has a larger effect for the lower Higgs mass than for the higher one, although this effect is somewhat weaker at MC@NLO level, where the top is more boosted on average. Generally, there is
more of a pronounced difference between the LO and MC@NLO values than in the case of the angular asymmetries considered in the previous section. Furthermore, decorrelation with LO is more pronounced for heavier Higgs masses, due presumably to the fact that the top quark showers more on average.

![Figure 3.13: The asymmetry parameter $A_u$ for $H^{-}t$ production, as defined in eq. (3.5). LO (MC@NLO) results are shown in blue (black), for $m_H = 200$ GeV (upper curves at large $\tan \beta$) and $m_H = 1500$ GeV (lower curves at large $\tan \beta$). The error band is statistical. Results for $Wt$ production, using both the DR and DS approaches in [20], are shown in red (in the left-hand plot the DS and DR results are on top of each other).](image1)

![Figure 3.14: The asymmetry parameter $A_z$ for $H^{-}t$ production, as defined in eq. (3.5). LO (MC@NLO) results are shown in blue (black), for $m_H = 200$ GeV (upper curves at large $\tan \beta$) and $m_H = 1500$ GeV (lower curves at large $\tan \beta$). The error band is statistical. Results for $Wt$ production, using both the DR and DS approaches in [20], are shown in red (in the right-hand plot the DR and DS results are on top of each other).](image2)

As for the angular asymmetry, we also show results for $Wt$ production in fig. 3.13. Before a cut on the boost parameter is applied, the $Wt$ result sits more or less in the middle of the $H^{-}t$ results over most of the range in $\tan \beta$. This is not the case once a cut is applied, and indeed a significant difference is observed between the $Wt$ and $H^{-}t$
results. Admittedly, this difference appears larger (and thus more useful) for smaller charged Higgs masses, and is only 3% or so for the largest Higgs mass we consider.

We may also define an asymmetry parameter for the energy ratio $z$ of eq. (2.11). This is perhaps most conveniently done by considering only the linear regime in fig. 3.11 occuring at intermediate values of $z$, as it is the sign of the slope in this kinematic region that distinguishes the cases of positive and negatively polarized tops. We therefore define

$$A_z = \frac{\sigma(0.1 \leq z \leq 0.4) - \sigma(0.4 < z \leq 0.7)}{\sigma(0.1 \leq z \leq 0.4) + \sigma(0.4 < z \leq 0.7)}.$$  (3.6)

We have chosen the values at which to define the intermediate region by eye from fig. 3.11. Again, these could be varied in order to maximise the resulting asymmetry.

The behaviour of $A_z$ is shown in fig. 3.14. A first notable feature is the lack of smoothness, even in the LO results. This is due to the fact that the boundaries of the intermediate regime will themselves depend on the value of $\tan \beta$, leading to fluctuations such as those observed in the figure. It may be that such fluctuations can be ameliorated by tuning of these boundaries, with a corresponding trade-off in the size of the asymmetry observed. The sign of the asymmetry flips for each charged Higgs mass as the full range in $\tan \beta$ is scanned, which is expected since the sign of the polarization changes. Note that there is again a marked difference between the LO and NLO results, particularly for the higher Higgs mass, and that the boost cut has a larger effect for the lower Higgs mass.

As before, one may compare the $H^-t$ and $Wt$ results. Here, though, a note of caution is necessary, because the difference between the DR and DS results for $Wt$ appears more pronounced for this parameter. In particular, it varies considerably before and after the boost cut is applied. This greater variation is perhaps exacerbated by the smallness of the asymmetry (which is at best only a few percent), but also suggests that interference with top pair production may be an issue in interpreting the $Wt$ results. It is nevertheless the case that the difference with $Wt$ is most pronounced at either low Higgs mass and high $\tan \beta$, or high Higgs mass and low $\tan \beta$. In both these cases, the sign of the top polarization in $H^-t$ production is opposite to the one in $Wt$ production. This results in a small asymmetry of opposite sign to the $Wt$ case, but roughly comparable in size.

To summarise, we have here presented results for a number of angular and energy-related distributions and, building upon the analysis of [9, 17], defined a corresponding asymmetry parameter for each that efficiently encodes the difference in these distributions for different regions in the charged Higgs parameter space, as well as the differences between $Wt$ and $H^-t$ production. All of these asymmetries seem to be fairly robust against NLO and parton shower corrections. In addition, they complement each other, since different observables are sensitive to different parts of the parameter space. This suggests that they may indeed be very useful in isolating a charged Higgs boson, with subsequent identification of its properties. In the following section, we consider a second context in which such observables may be useful, namely that of isolating $Wt$ production itself as a signal.
3.4 Results for Wt Production

In the previous section, we examined the angular and energy distributions introduced in section 2.3 in H−t production, and defined asymmetry parameters which are potentially highly useful in elucidating the properties of a charged Higgs boson. In this section, we investigate whether these same observables have anything useful to say about Standard Model Wt production.

There are three production modes for a single top quark in the Standard Model. Two of these, the so-called s− and t− channel modes, have been observed in combination at both the Tevatron [46–49] and LHC [50–52]. The theoretical state of the art is also highly advanced, and includes fixed order computations [53–57], NLO plus parton shower implementations [58, 59], resummed results [60], and finite top width corrections [61, 62]. For related phenomenological studies, see [63–66]. As already stated in the introduction, Wt production offers a complementary window through which to look at top quark interactions, being sensitive to corrections to the Wtb vertex, but not to four fermion operators which may affect the s− and t− channel modes. The investigation of Wt production as a signal in its own right was first explored in [67]. Since then, computations have been carried out at NLO [35,36], and also matched to a parton shower at this accuracy [20,29].

The aim of this section is to examine angular observables and energy ratios for both Wt and top pair production, for semi-realistic analysis cuts, and to reflect upon whether these results may be useful in enhancing the signal to background ratio of the former process. To this end, we adopt the following Wt signal cuts, similar to those used in [41]:

**Wt signal cuts**

1. The presence of exactly 1 b jet with \( p_T > 50 \) GeV and \(|\eta| < 2.5\). No other b jets with \( p_T > 25 \) GeV and \(|\eta| < 2.5\).

2. The presence of exactly 2 light flavour jets with \( p_T > 25 \) GeV and \(|\eta| < 2.5\). In addition, their invariant mass should satisfy \( 55 \) GeV < \( m_{j_1j_2} < 85 \) GeV.

3. Events are vetoed if the invariant mass of the b jet and light jet pair satisfies

\[
150 \text{ GeV} < \sqrt{(p_{j_1} + p_{j_2} + p_b)^2} < 190 \text{ GeV}
\]

4. The presence of exactly 1 isolated lepton with \( p_T > 25 \) GeV and \(|\eta| < 2.5\). The lepton should satisfy \( \Delta R > 0.4\) with respect to the two light jets and the b jet, where \( R \) is the distance in the \((\eta, \phi)\) plane.

5. The missing transverse energy should satisfy \( E_T^{miss} > 25 \) GeV.

Here the first cut is the most useful one in getting rid of top pair production, as one expects two b jets on average in \( tt \) production, but only one b jet in \( Wt \). The other cuts pick out semi-leptonic decays. That is, one W boson decays to leptons (we would want

\[\text{Note that to increase the statistics in our analysis, we will explicitly generate semi-leptonic decays using MC@NLO. The above analysis cuts, however, will still affect the shapes of distributions.}\]
Chapter 3. Top Polarization in $Ht$ and $Wt$ Production

This to be the $W$ boson from the top quark decay), and the other decays to quarks. We thus expect two light jets whose invariant mass reconstructs the $W$ mass, as well as a lepton and missing energy from the neutrino. The only difference with respect to the cuts used in [41] is the presence of an additional cut involving the invariant mass of the $b$ jet and light jet pair, restricting this to be different than the top mass. This ensures that the selected semi-leptonic events are such that the top quark in $Wt$ decays leptonically, and the $W$ hadronically, as is required in order to use the decay lepton as a marker of top quark polarization effects.

It was shown in [41] that, for these signal cuts (minus the invariant mass requirement for the three jets, which was unnecessary in that analysis), $Wt$ is a well-defined scattering process in that interference with pair production can be neglected. This was found by comparing the DR and DS results from MC@NLO. The results in this section were obtained using the DR subtraction method. Furthermore, the $Wt$ cross-section was found to be larger than the scale-variation uncertainty associated with the top pair cross-section. If this had not been true, then $Wt$ production would be swallowed up in the uncertainty of the top pair prediction, and much more care would be needed in order to be able to claim that it can be observed independently. We thus use the above cuts as an example of a fairly minimal analysis which guarantees that $Wt$ is a well-defined signal. We will see that even for this analysis, the angular and energy-related observables defined in section 2.3 display pronounced differences between $Wt$ and top pair production.

Note that in this section, in order to be more realistic, we consider distributions constructed from the isolated lepton entering the cuts. This is not guaranteed to be the decay lepton from the top quark, although the likelihood of this is increased by the event selection cuts. Also, we assume that the top quark direction is reconstructed with perfect resolution. In practice this would be done by considering the four-momenta of the $b$ jet and isolated lepton passing the cuts, together with missing energy. A full determination of the uncertainty induced in the reconstruction of the top quark (also including detector effects) is beyond the scope of the present study. Note that in $Wt$ and $W\bar{t}$ production, we assume that the top and antitop quark is reconstructed respectively. In top pair production, one constructs either the top or antitop quark which decays to give the isolated lepton passing the selection cuts. In contrast to the $H^-t$ results of the previous section, we present results for a centre of mass energy of 7 TeV. Jets are clustered using the $k_T$ algorithm [68] with D=0.7.

We first consider the azimuthal angle $\phi_l$, whose distribution is shown in fig. 3.15 for both $Wt$ and top pair production.

The first thing to notice is that there is a distinct shape difference between the $Wt$ and top pair curves. The $Wt$ results include a slight peak structure at $\theta = \pi$, due to the contribution from events in which the $W$ boson decays leptonically, rather than the top quark. This structure is missing in the case of top pair production, due to the symmetrical nature of the final state. For the choice of analysis cuts given above, one may evaluate the asymmetry parameter $A_{\phi}$, which is shown in tab. 3.1. The values for $Wt$ and top pair production are significantly different. This is potentially a useful distinguishing feature between the two production processes.

Next, we consider the polar angle $\theta_l$, again defined in terms of the isolated lepton
3.4. Results for $Wt$ Production

Figure 3.15: Azimuthal angle distribution of the isolated lepton which enters the $Wt$ signal cuts, for both $Wt$ and top pair production, at NLO plus parton shower level.

Figure 3.16: Polar angle distribution of the isolated lepton which enters the $Wt$ signal cuts, for both $Wt$ and top pair production, at NLO plus parton shower level.
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<table>
<thead>
<tr>
<th>$B_{\text{cut}}$</th>
<th>$Wt$</th>
<th>Top pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.33 ± 0.01</td>
<td>0.63 ± 0.02</td>
</tr>
<tr>
<td>0.8</td>
<td>0.41 ± 0.02</td>
<td>0.70 ± 0.05</td>
</tr>
<tr>
<td>0.9</td>
<td>0.42 ± 0.03</td>
<td>0.70 ± 0.07</td>
</tr>
<tr>
<td>0.95</td>
<td>0.44 ± 0.04</td>
<td>0.68 ± 0.08</td>
</tr>
</tbody>
</table>

Table 3.1: Results for the azimuthal asymmetry parameter $A_\phi$ of eq. (2.9), evaluated using the isolated lepton entering the $Wt$ selection cuts, and for different values of a cut $B > B_{\text{cut}}$ on the boost parameter of the top quark.

<table>
<thead>
<tr>
<th>$B_{\text{cut}}$</th>
<th>$Wt$</th>
<th>Top pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.02 ± 0.01</td>
<td>0.26 ± 0.02</td>
</tr>
<tr>
<td>0.8</td>
<td>0.18 ± 0.02</td>
<td>0.38 ± 0.04</td>
</tr>
<tr>
<td>0.9</td>
<td>0.49 ± 0.03</td>
<td>0.75 ± 0.07</td>
</tr>
<tr>
<td>0.95</td>
<td>0.70 ± 0.05</td>
<td>0.97 ± 0.10</td>
</tr>
</tbody>
</table>

Table 3.2: Results for the polar asymmetry parameter $A_\theta$ of eq. (3.3), evaluated using the isolated lepton entering the $Wt$ selection cuts, and for different values of a cut $B > B_{\text{cut}}$ on the boost parameter of the top quark.

eventing the $Wt$ signal cuts. The distribution of this angle is shown in fig. 3.16. There is a notable difference between the $Wt$ and top pair production, due to the negative polarization of the top in the former case. The corresponding asymmetry parameters $A_\theta$ are shown in tab. 3.2. Again the results are different between the two production processes which, as in the azimuthal case, is a potentially useful discriminator between the two processes.

In the case of $H^-t$ production considered in section 3.3, we also considered various observables which depended upon the boost of the top quark. This is clearly of practical importance for heavy charged Higgs masses, which do indeed lead to heavily boosted top quarks in a sizeable fraction of events, as is clear from fig. 3.1. One expects boosted top observables to be less useful in $Wt$ production, due to the fact that the $W$ boson is much lighter. Nevertheless, it is perhaps worth examining the dependence of various observables on the boost parameter of the top quark. If sizeable differences between $Wt$ and top pair production were to be observed, the impact on the signal to background ratio would then outweigh the loss in signal cross-section.

The distribution of the boost parameter $B$ of eq. (2.10) is shown for both $Wt$ and top pair production in fig. 3.17 and one sees that there is a reasonable fraction of events in both cases which have $B > 0.8$, albeit not as large a fraction as in the $H^-t$ case of the previous section. This is not surprising, given that charged Higgs masses of at least 200 GeV were considered there, so that the top recoiled against a much more massive particle than a $W$ boson. Here we also have a lower centre of mass energy. The $\phi_l$ distributions for the two processes are shown in fig. 3.18 for different values of a cut $B > B_{\text{cut}}$. One
3.4. Results for $Wt$ Production

Figure 3.17: Distribution of the boost parameter $B$ of eq. (2.10), at NLO plus parton shower level.

Figure 3.18: Azimuthal angle distribution of the isolated lepton which enters the $Wt$ signal cuts, for $Wt$ and top pair production, for different values of a cut $B > B_{cut}$ on the boost parameter of eq. (2.10), at NLO plus parton shower level.
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Figure 3.19: Polar angle distribution of the isolated lepton which enters the $W_t$ signal cuts, for $W_t$ and top pair production, for different values of a cut $B > B_{\text{cut}}$ on the boost parameter of eq. (2.10), at NLO plus parton shower level.

Figure 3.20: Distributions of $u$ and $z$, as defined in eq. (2.11), where a cut on the boost parameter $B > 0.8$ has been applied, at NLO plus parton shower level.

sees that, whilst there is some dependency on the boost parameter, the qualitative features remain identical. The corresponding asymmetries $A_\phi$ are given in tab. 3.1. One sees that the absolute value of the difference between the asymmetries for the two processes is roughly independent of the boost cut. However, the relative difference decreases.

One expects a much greater effect from the boost on the polar angle distribution, as the requirement of a boosted top will concentrate the decay products in polar angle. The $\theta_l$ distributions as a function of $B_{\text{cut}}$ are shown in fig. 3.19.

The effect of the higher boost cut is to increase the peak region of the distribution at the expense of the tail, as expected. The corresponding $A_\theta$ values are collected in tab. 3.2. Unsurprisingly, both sets of results display an increase in $A_\theta$ as the boost cut is increased. This implies that a boost cut is actually detrimental in this case, as the relative difference between the asymmetry parameters in the two processes decreases.

Finally, we present results for the energy ratios of eqs. (2.11), which were shown to
be useful for $H^−t$ production in section 3.3. In that case, we defined the energy of the $b$ quark via eq. (3.4), which is possible in a Monte Carlo study but not in a real experiment. Here, given that we have explicitly implemented analysis cuts in terms of jets, we define $E_b$ to be the energy of the $b$ jet which enters the cuts. Then the distributions of $z$ and $u$, with a cut on the boost parameter of $B > 0.8$, are shown in fig. 3.20. The first thing to note is that the results for the $u$ distribution do not show a significant difference between $Wt$ and top pair production. This is perhaps not so surprising given that we have already seen in section 3.3 that oppositely polarized top quarks tend to exhibit smaller differences in energy-related distributions than in angular distributions. Here we are essentially probing the difference between a polarized top quark and one which is unpolarized on average, and thus one expects an even smaller difference in behaviour.

The $z$ distribution in fig. 3.20 shows some difference between the $Wt$ and top pair distributions. However, the top pair result does not closely resemble the flat profile one would expect for unpolarized top quarks, due presumably to that fact that the shape has been sculpted somewhat by the event selection cuts, in particular those which implement restrictions on jet invariant masses.

Given the above results, it does not seem particularly useful to examine the asymmetry parameters of eqs. (3.5, 3.6) in the present context. Nevertheless, the fact that a shape difference persists in the $z$ distribution between $Wt$ and top pair production still makes this a potentially useful observable in discriminating the two processes, perhaps as an ingredient in a neural net analysis. One must also bear in mind the result for the polar asymmetry from above, namely that a boost cut will decrease the relative difference between the angular asymmetries in $Wt$ and top pair production. Thus, and perhaps unsurprisingly, the utility of boost cuts in $Wt$ production is somewhat limited.

### 3.5 Conclusion

In this chapter, we have examined the role that observables that are sensitive to top quark polarization can play in exploring the parameter space of charged Higgs models, and also in distinguishing $H^−t$ production from (Standard Model) $Wt$ production. In particular, we examined the azimuthal and polar angles $\phi_l$ and $\theta_l$ of [9, 17], and the energy ratios $z$ and $u$ of [12], defining corresponding asymmetry parameters analogous to that already defined for the azimuthal angle in [17]. Importantly, we found that polarization effects are robust up to NLO and including parton shower corrections\footnote{A similar robustness has already been observed in (Standard Model) $s-$ and $t-$ channel single top production [66].}. At this level, each of the asymmetry parameters showed significant difference between different regions in the charged Higgs parameter space ($m_H, \tan \beta$), and also between $H^−t$ and $Wt$ production. The full set of asymmetries taken together thus provides a potentially highly useful probe of charged Higgs properties. Angular observables are sensitive only to corrections to the production of a top quark, and the polar angle is able to discriminate between charged Higgs masses at high $\tan \beta$ values, where the azimuthal angle cannot. Energy observables are sensitive to corrections to both the production and decay of top quarks. Although more
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difficult to construct (owing to the need for a cut on the boost parameter of the top quark), they give useful complementary information, particularly on the value of the charged Higgs mass at intermediate and high $\tan\beta$ values.

As a second application of these observables, we considered the problem of distinguishing Standard Model $Wt$ production from top pair production, which is a significant background. Under the assumption that it is meaningful to separate $Wt$ and top pair production, we observed significant differences, for semi-realistic $Wt$ analysis cuts, between angular distributions relating to the isolated lepton entering the cuts. It is worth pointing out that the cuts we used are fairly minimal in terms of signal to background ratio [41]. Nevertheless, large differences are obtained between the two production processes, which suggests that our findings would persist in a more realistic study, including detector effects etc.

One may also consider boosted top quark observables in Standard Model $Wt$ production, and we gave a couple of examples in section 3.4. These seem less useful than in $H^-t$ production, however. In the angular observables, a cut on the boost parameter does not increase the absolute difference between the asymmetry parameters for $Wt$ and top pair production, and decreases the relative difference. For energy observables, one sees only a small difference between the $u$ distributions even when a boost cut is applied. This is due mainly to the fact that one is comparing a polarized top quark in $Wt$ with an (on average) unpolarized top quark in top pair production, rather than an oppositely polarized top quark. A larger difference is observed in the $z$ distribution, which may yet be a useful observable in distinguishing $Wt$ and top pair production.

To summarise, the observables studied in this paper are useful probes of both $H^-t$ and $Wt$ production, and seem to be robust against higher order perturbative corrections. They therefore deserve further investigation.