Development of a modelling learning path

van Buuren, O.P.M.

Link to publication

Citation for published version (APA):

General rights
It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations
If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: https://uba.uva.nl/en/contact, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.
First version of the modelling learning path*

Abstract

Until recently most research on computational modelling in education has been focussed on the higher educational levels, often with courses on a project base only. During the past two years we developed a learning path on system dynamical graphical modelling, integrated into the Dutch physics curriculum, starting from the initial phases (age 13-15 years). A first version of this path has been tested in classroom.

In this paper, after giving an overview of this path, we will focus on the part in which students had to build a complete model themselves for the first time. Student's understanding of the graphical relation structure, of the distinction between difference equations and direct relations, and of the relation between difference equations and stock-flow diagrams is investigated. It appears that students mix up different aspects of dependences of variables. We identified a number of misconceptions which students showed when discriminating between direct and difference equations. Finally, we detected a number of problems students may have when building graphical models.

3.1 Introduction

In this study on modelling in physics education, the emphasis will be on quantitative computational modelling with graphical modelling tools, such as Stella, PowerSim, and Coach 6, based on the ‘system dynamical’ stock and flow approach developed by J.W. Forrester in the early 1960s (Forrester, 1968). From a mathematical point of view, this mainly concerns the numerical integration of ordinary difference and differential equations.

It appears that understanding the conceptual network of system dynamics can be difficult for students (Booth Sweeney & Sterman, 2000; Westra, * This chapter has been presented in 2010 as a paper at the GIREP-ICPE-MPTL Conference and has been published as Van Buuren, O., Uylings, P., & Ellermeijer, T. (2011). A modelling learning path, integrated in the secondary school curriculum, starting from the initial phases of physics education. In Proceedings GIREP-ICPE-MPTL Conference 2010, Teaching and Learning Physics Today: Challenges. Reims.
A new way of thinking is required which takes considerable time to learn. Therefore modelling should not be limited to only one theme or subject, but should be integrated into the curriculum (Schecker, 1998).

There are several reasons to start such a curriculum from the initial phases of science education (Van Buuren, Uylings, & Ellermeijer, 2010). Research has indicated that it is possible to involve students in quantitative modelling at the age of 14-15 years (Mulder, Slooten, Uylings, & Wieberdink, 2008).

In the past two years we have been working on the design of a learning path on modelling, integrated into the first two years of physics education (age 13-15), with the intention of expanding it to the subsequent years in future. A pilot version of this path has now been tested in classroom. Our approach can be classified as educational design research (Van den Akker, Gravemeijer, Mckenney, & Nieveen, 2006). In educational design research, materials are designed, tested and improved in subsequent cycles. In this paper, our focus is on establishing the required orientation elements for building system dynamical graphical models.

3.2 Main design principles

3.2.1 The modelling process as a starting point

Modelling involves the modelling process as a whole. This process is described schematically in Figure 3.1, which is derived from the proposal of the commissions for renewal of the Dutch science curricula. (see for example: Commissie Vernieuwing Natuurkundeonderwijs havo/vwo, 2006). This proposal states:

“Student must be able to analyse a situation in a realistic context and reduce it to a manageable problem, translate this into a model, generate outcomes, interpret these outcomes, and test and evaluate the model.”

This process is often called a cycle, because for the testing, validation and evaluation of the model it is necessary to revisit the realistic context situation.
Similar modelling cycles can be found elsewhere (Blum & Leiß, 2005; Galbraith & Stillman, 2006; Maaß, 2006).

We used this process as a framework on which the determination of the elements for our learning path can be based. A first description of this framework and its implications has been given elsewhere (van Buuren, Uylings, & Ellermeijer, 2009). In this paper, we confine ourselves to the translation of the manageable problem into a system dynamical graphical model.

### 3.2.2 Analysis of the main elements of system dynamical graphical models

Following Gal’perin (Van Parreren & Carpay, 1972), we want to establish a complete orienting base for working with the main elements of system dynamical modelling. Therefore we will give a brief analysis, using examples from Coach 6 (Heck, Kedzierska, & Ellermeijer, 2009).

Five types of variables can be distinguished: stock- (or state-) variables, flow variables, the independent variable, auxiliary variables, and constants (Figure 3.2). The type of a variable is not an intrinsic property, but must be derived from its role in the process, which is a process of numerical integration of a system of equations. There are two types of relations, difference equations and direct relations.

![Figure 3.2: Representation of variables in Coach 6.](image)

The graphical equivalent of a difference equation is a combination of a stock variable and one or more flow variables (Figure 3.3). The flow variables are to be integrated; the stock variable is the integral and needs an initial value. Often, stock-flow combinations are introduced in a qualitative, intuitive way: “the flow tells the stock how to change”.

All other relations are direct relations. All five types of variables can be part of direct relations, but only flow variables and auxiliary variables can be defined by them. They are indicated by connectors (see Figure 3.2; in Dutch: ‘relation arrows’).
In this paper we will make a distinction between variables which are connected directly and variables with one or more other variables in between. We will call the former “direct linked” and the latter “secondary linked”. Direct linked variables are used explicitly in the definition (formula) of the variable they are linked to, secondary linked variables are not (Figure 3.4).

Figure 3.3: Difference equation and its graphical representation. The independent variable $t$ is not visualised.

Figure 3.4: Variables a and b are directly linked to $y$, c and d are secondary linked to $y$.

We consider the following, not necessarily exhaustive list of aspects to be important for the formation of a complete orienting base:

1. Difference equations, as describing differences or changes (of the stock variable).
2. The role and use of stock variables.
3. The need of an initial value for the stock variable.
5. The role of the independent variable and its step size.
6. The role and use of flow variables.
7. The role and use of auxiliary variables and constants.
8. The graphical representation of difference equations.
9. The distinction between difference equations and direct relations.
10. The graphical representation of direct relations.
11. The role of connectors in showing the relational structure of the model.
These aspects must be addressed on our learning path.
3.3 Design and implementation of the learning path

3.3.1 Overview of the learning path

We want our students to be able to build simple models before the end of the second year of physics education (age 14-15 years). Before we can establish the required orienting base, an orientation on graphical models must be given (for example, by using models for simulation), students must become acquainted with the software tool, an understanding of graphs must be developed, and a few of the above aspects, such as difference equations and initial values, must already be introduced. For the development of the orienting base it is important that modelling activities are not restricted to one domain. We choose six domains. For each new domain, before the start of a modelling activity, some conceptual domain knowledge must be developed first.

Table 3.1 gives an overview of our design, starting with the module on kinematics, in which students use simple models for simulation as a first orientation, and ending with the module on heat, in which a model must be built without the help of the teacher. In the preceding module on dynamics all elements required for model building are integrated for the first time, so at this point the required orienting base should be established.

We need a curriculum to which not only interventions must be added, but which must be adaptable as a whole. The self-written curriculum of the HML (the Hague Montessori Lyceum) fulfils this purpose.

3.3.2 First version of the learning path

The first version of the learning path we tried out in classroom was a shorter version, to be used as a pilot. It consisted of only four modules: kinematics part 1, vibrations, and both modules on dynamics. We faced two disadvantages of this approach. Not all aspects could be addressed as they should be, and not everything worked out as intended. These disadvantages were counteracted by additional activities in the modules on dynamics. An overview of this version is shown in Table 3.2. Tests with a second, more complete version of the path are still in progress and will be reported about in future.
Table 3.1
Overview of the learning path.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Kinematics, part 1</th>
<th>Kinematics, part 2</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical domain content</td>
<td>x,t-graphs, $\Delta x = v_{av} \cdot \Delta t$</td>
<td>$v,t$-graphs</td>
<td>$\Delta E = P_{in},\Delta t, E_{t}$ &amp; $P_{t}$-graphs</td>
</tr>
<tr>
<td>1. Difference equations</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>2. Stock variable</td>
<td>X</td>
<td></td>
<td>Explicit</td>
</tr>
<tr>
<td>3. Initial value</td>
<td>Explicit</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>4. Numerical integration</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>5. Independent variable</td>
<td>X</td>
<td>Step size</td>
<td>X</td>
</tr>
<tr>
<td>6. Flow variable</td>
<td>Number</td>
<td>Sketched function</td>
<td>Explicit</td>
</tr>
<tr>
<td>7. Auxiliary variables</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>8. Graphical representation of difference equations</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>9. Difference equations $\leftrightarrow$ direct relations</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>10. Graphical representation of direct relations</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>11. Connectors &amp; structure</td>
<td></td>
<td></td>
<td>(qualitatively)</td>
</tr>
</tbody>
</table>

“X” means that an aspect is used more or less tacit, or is repeated.
Second year of the learning path. Age: 14-15 years.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Molecules</th>
<th>Vibrations</th>
<th>Dynamics: Newton</th>
<th>Dynamics: Falling</th>
<th>Heat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical domain content</td>
<td>Vacuum pump</td>
<td>( u_t = u_1 + u_2 )</td>
<td>( \Delta v = \frac{F}{m} \cdot \Delta t )</td>
<td>( F_{air} = k \cdot v^2 )</td>
<td>( \Delta Q = C \cdot \Delta T; \ P = K \cdot (T_0 - T_f) )</td>
</tr>
<tr>
<td>1. Difference equations</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Stock variable</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Initial value</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Numerical integration</td>
<td></td>
<td>Iteration</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Independent variable</td>
<td>Pump beat</td>
<td>Part of relation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Flow variable</td>
<td>Simple direct relation</td>
<td></td>
<td>Explicit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Auxiliary variables</td>
<td>Explicit</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Graphical representation of difference equations</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. Difference equations ↔ direct relations</td>
<td></td>
<td></td>
<td></td>
<td>Explicit</td>
<td></td>
</tr>
<tr>
<td>10. Graphical representation of direct relations</td>
<td>X</td>
<td>Explicit</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>11. Connectors &amp; structure</td>
<td>Explicit</td>
<td>Explicit</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

"X" means that an aspect is used more or less tacit, or is repeated.

Table 3.1 (continued) Overview of the learning path.

---

Dynamics: Falling

\[ F_{air} = k \cdot v^2 \]

Dynamics: Newton

\[ \Delta v = \frac{F}{m} \cdot \Delta t \]

Vibrations

\[ u_t = u_1 + u_2 \]

Molecules

\[ P = K \cdot (T_0 - T_f) \]

Heat

\[ \Delta Q = C \cdot \Delta T \]

---

First version of the modelling learning path
### Overview of the first, pilot version of the learning path.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Kinematics, part 1</th>
<th>Vibrations</th>
<th>Dynamics: Newton</th>
<th>Dynamics: Falling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date try-out</td>
<td>Feb-Mar 2009</td>
<td>Dec 2009 - Feb 2010</td>
<td>Apr-May 2010</td>
<td>May-June 2010</td>
</tr>
<tr>
<td>Typical domain content</td>
<td>$x,t$-graphs, $\Delta x = v \Delta t$</td>
<td>$u_i = u_1 + u_2$</td>
<td>$\Delta v = (F/m) \Delta t$</td>
<td>$F_{tot} = k \cdot v^2$</td>
</tr>
<tr>
<td>1. Difference equations</td>
<td>X</td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>2. Stock variable</td>
<td>X</td>
<td></td>
<td>Explicit*</td>
<td></td>
</tr>
<tr>
<td>3. Initial value</td>
<td>Explicit</td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>4. Numerical integration</td>
<td></td>
<td>Explicit*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Independent variable</td>
<td>X</td>
<td>Part of relation</td>
<td>Step size*</td>
<td></td>
</tr>
<tr>
<td>6. Flow variable</td>
<td>Number</td>
<td>Sketched function*</td>
<td>Explicit*</td>
<td></td>
</tr>
<tr>
<td>7. Auxiliary variables</td>
<td></td>
<td>(Explicit)</td>
<td>Explicit*</td>
<td></td>
</tr>
<tr>
<td>8. Graphical representation of difference equations</td>
<td>X</td>
<td></td>
<td>Explicit</td>
<td></td>
</tr>
<tr>
<td>9. Difference equations ↔ direct relations</td>
<td></td>
<td></td>
<td>Explicit</td>
<td></td>
</tr>
<tr>
<td>10. Graphical representation of direct relations</td>
<td>(Explicit)</td>
<td></td>
<td>Explicit*</td>
<td></td>
</tr>
<tr>
<td>11. Connectors &amp; structure</td>
<td></td>
<td>(Explicit)</td>
<td>Explicit*</td>
<td></td>
</tr>
</tbody>
</table>

Typical model

"X" means that an aspect is used more or less tacit, or is repeated. Some aspects should be introduced in modules that are missing in this version and some other aspects did not work out as intended (in brackets). We solved these problems temporarily with additional activities in the modules on dynamics (with *). Age of the students: kinematics part 1: 13-14 years; all other modules: 14-15 years.
3.3.3 Design of the module on falling

In the module on falling, the integration of the orienting base for model building is to be established. A second goal of the module is the presentation of modelling as a process starting with simple models that are improved in subsequent cycles. In the module, two of these cycles are completed. In the first, students predict characteristics of free fall, by making calculations based on Newton’s second law, introduced in the preceding chapter as the difference equation

$$\Delta v = \frac{F_{\text{net}}}{m} \Delta t$$

with gravity as the sole force, without a computer. These predictions are tested experimentally. In the second cycle, after analysis of video measurements, the model is revisited and expanded with air resistance. The formula $F_{\text{air}} = k \cdot v^2$ as a candidate for air resistance creates the need for a computer model. At this point, the orienting base is introduced. See Section 3.3.4. Finally, the model is built, validated and explored. This module takes 4 lessons of 80 minutes.

3.3.4 Design of the introduction of the complete orienting base for model building

The purpose of this part of the module on falling is the integration of all main elements of graphical model building. All aspects of Section 3.2.2 must be addressed, especially the new ones. In addition, students must learn to use the interface for creating models.

For the introduction of the elements a variety of topics is used, both from within physics as from other fields, in order to force our students to orientate on the essential features. In order to avoid the risk of the module becoming too complicated for our students, we decided:

1. to confine ourselves, implicitly, to time as the independent variable;
2. not to make the distinction between constants and auxiliary variables yet, but to use the latter for constants as well;
3. to confine ourselves to models containing only one stock variable.

For difference equations the term “change formulas” is introduced, because we expected this to be a more meaningful term to our students.

The lessons start with an introduction of the main elements, followed by a number of learning tasks. The six most important tasks are three pen-and-paper exercises and three model building tasks (Table 3.3).
Design considerations of the learning tasks are the following:
1. Exercise 1 aims at the development of an understanding of the main structure of graphical models, especially with respect to direct relations. Students are asked about dependencies in a given graphical model.
2. Exercise 2 is on the recognition of the difference between direct relations and difference equations. Eleven realistic ‘situations’, from physics, econ-
Economy and biology are presented. In each situation a direct relation or a difference equation is involved. In some situations a formula is given, other situations are described in words only. Students must decide whether a situation corresponds to a “change formula” or to a “direct relation”. Situations involving feedback mechanisms, e.g. about interest, are not included, because in a small pilot study with upper level students, we noticed that students got mixed-up by these situations, whereas at this stage these situations do not help in clarifying the difference between both types of equations.

3. Exercise 3 is on the graphical representation of difference equations. Students must be able to translate a difference equation into a stock-flow combination and also to interpret a stock-flow combination as a difference equation. In this exercise students are asked to do the latter.

4. In building task 1 the required equipment knowledge is developed. Students must build the simple stock-flow model of Figure 3.3, supported by an instructional video, integrated in Coach.

5. The purpose of building task 2 is the development of a more general view on stock and flow variables. Students must learn that variables can have different roles, depending on the equations they are part of. Therefore, we let them build a model in which one of the variables from building task 1 was involved, but in a different role. See Figure 3.5.

6. In building task 3 an extended model must be build. This is the final goal of the module. In order to save time and, possibly, some frustration, a semi-built model is provided, in which all variables are already present, but to which all relations and values must be added (Figure 3.6). This model must be finished, fitted to experimental data, and explored.

**Figure 3.5:** Model from building task 2. In building task 1, $v$ was a flow variable, in this task it is a stock variable.

**Figure 3.6:** Initial modelling screen of building task 3.
3.4 Set up of the classroom experiment on model building

In this experiment we focus on the understanding by students of aspects of the elements of graphical models.

3.4.1 Research questions

Partial research questions are:
1. How do students understand relation structures?
2. How do students understand the difference between difference equations and direct relations?
3. How do students understand the relation between stock-flow patterns and difference equations?
4. Which problems do students have when building an extended model?

3.4.2 Setting

The lessons were tested in four third-year classes (14-15 years), numbered 3A to 3D, consisting of 124 students, with two different teachers. The researcher was one of the teachers. Students worked individually or in small groups. Because they were allowed to collaborate, results of individual students may not be independent. The lessons were planned near the end of the year, and for several reasons we ran out of time. Therefore, in 3A and 3B, we decided to concentrate on experimenting and modelling and to skip other parts of the module. In 3C and 3D, we let students work at their own pace. As a consequence, not all students completed all modelling activities, but those who did followed the learning trajectory as intended. This may have lead to a bias in our results for faster students.

3.4.3 Data collection

Data from three of the exercises and two of the modelling tasks were analysed. During classroom activities, notes were made of the questions and problems of our students. We analysed delivered results, consisting of produced models and written or typed answers. In addition, we made screen recordings and a few detailed observations of individual students. See Table 3.4.
3.5 Results

Results of this first, incomplete version on the learning path, with a possible bias towards faster students, must be viewed only as indicatory. The results will be used to make changes in future versions, however. In presenting the results, we follow the order of the research questions.

### 3.5.1 Understanding of the relational structure

In exercise 1 students’ understanding of the structures of graphical models as shown by connectors is investigated. Students were given a picture of a model (Figure 3.7) and were asked on which other variables some specific variables depended, according to the model. Both direct and secondary linked variables were present. In the subsequent questions they were asked explicitly about dependencies on the secondary linked variables. The terms “direct” and “secondary” were not mentioned to our students.

From analysis of 24 students’ answers on the first question it appeared that, in 20 of these, all direct linked variables were mentioned correctly, but in only three of these the secondary linked variables were mentioned also. This is in accordance with our classroom observations. Many students wondered whether secondary linked variables should be regarded as “dependent” on each other. To the subsequent explicit questions about dependencies on secondary linked variables, 21 answers were correct, but many students added comments, like “Partially”, or “Yes, in a very indirect way”. Apparently, these questions

<table>
<thead>
<tr>
<th>Research question</th>
<th>Task</th>
<th>Classroom observation</th>
<th>Delivered results</th>
<th>Screen recordings</th>
<th>Indiv. Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Relation structure?</td>
<td>Exercise 1</td>
<td>3A+3B (55)</td>
<td>24 (42)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Difference equation or direct relation?</td>
<td>Exercise 2</td>
<td>(pilot 4HV) 3A+3B (55)</td>
<td>20 (36)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3. Stock-flow pattern and difference equation?</td>
<td>Exercise 3</td>
<td>3A+3B (55)</td>
<td>21 (38)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Building problems?</td>
<td>Building task 2</td>
<td>3A+3B (55)</td>
<td>28 (48)</td>
<td>3 (6)</td>
<td></td>
</tr>
<tr>
<td>1. Relation structure?</td>
<td>Building task 3</td>
<td>3B (28)</td>
<td>24 (44)</td>
<td>2* (3)</td>
<td>1</td>
</tr>
</tbody>
</table>

Students were working in small groups or individually. Numbers between brackets are the numbers of individuals involved. *: 1 including sound.
draw attention to a relational aspect of secondary linked variables that is not automatically recognized properly. For two reasons this aspect is important.

First, students may misinterpret models when they do not realize that the influence of secondary linked variables is direct and instantaneous. Second, when students start building models themselves, they may follow a qualitative, dependency-based approach, by drawing the relational structure first and adding the formulas later, as is advised by some authors (Westra, 2008). PowerSim even forces its users to do so. Our results indicate that this may lead to a less adequate relational structure. An example of this can be seen in a screen recording of building task 3 (Figure 3.8). Some of our students started building their model by drawing a connector from the friction constant $k$ to the acceleration $a$. Although $a$ is influenced by $k$, $k$ does not appear directly in the formula $a = \frac{F_{\text{net}}}{m}$, so it should have been secondary linked, through the

![Figure 3.7: Model used for showing how connectors reveal dependencies.](image)

![Figure 3.8: Screenshot of the initial phase of the model of two students. The flow variable is the acceleration $a$.](image)
friction force $F_w (= F_{\text{air}})$, especially since our students were not familiar with substitution. With a more quantitative, equation based approach, this problem may be avoided.

3.5.2 Understanding of the distinction between direct relations and difference equations

We used classroom observations and 20 sets of delivered student answers of the 11 questions of exercise 2 in order to investigate how students understood the difference between direct relations and difference equations. On average, in each set of student answers, 8 questions were answered correctly. Therefore, we conclude that it is possible for students to determine the type of relation in most situations. We analysed these data further, looking for criteria that students may have been using, consciously or not, when answering these questions. Results are summarised in Table 3.5.

<table>
<thead>
<tr>
<th>Criterion: presence of:</th>
<th>Valid</th>
<th>Students’ conclusion</th>
<th>Sets of student answers*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. formula with $\Delta$-notation</td>
<td>+</td>
<td>Change formula</td>
<td>20</td>
</tr>
<tr>
<td>2. variable “time”</td>
<td>-</td>
<td>Change formula</td>
<td>$\leq 6$</td>
</tr>
<tr>
<td>3. variable with a name suggesting change</td>
<td>-</td>
<td>Change formula</td>
<td>10</td>
</tr>
<tr>
<td>4. written physics formula vs. a relation in words from outside physics.</td>
<td>-</td>
<td>Change formula / direct relation</td>
<td>3</td>
</tr>
</tbody>
</table>

*Sets of student answers in which this criterion may have been used (max. 20).

Only the question in which $\Delta$-notation was used explicitly was answered correctly by all students. $\Delta$-notation apparently is a good trigger. However, the idea that “a change formula always involves time”, as one student explicitly stated, may have interfered. We found six sets of answers that might be based on this invalid idea, including the 2 sets that were totally correct. This idea may be the consequence of our simplification only to use time as independent variable. Orientation on this aspect will be improved in the next version. This requires both situations with other independent variables and situations with time as part of a direct relation.

The highest number of wrong answers occurred on the only question concerning direct relations to a variable suggesting change. See Figure 3.9.
Figure 3.9: “The expenses of a restaurant depend on the number of employees, the rent of the building, the mean salary and the purchases” may be modelled as in this picture. If “Expenses” is seen as “change of capital”, its role is twofold.

Between “Expenses” and four auxiliary variables exists a direct relation, but it is tempting to consider “Expenses” as a flow variable to a stock-variable “capital”. The possibly twofold roles of flow variables can be confusing.

Analysing a number of errors that appeared to be correlated, we arrived at the possibly underlying misconception that “a written physics formula means a difference equation, a relation in words from outside physics indicates a direct relation.” This must be investigated more closely in future.

Finally, students may interpret situations not as intended, but slightly different, with correct alternative models as a possible consequence. An example is shown in Figure 3.10.

Figure 3.10: Intended and alternative solution for ‘The population of a country increases through birth and immigration’.

3.5.3 Understanding the relation between stock-flow patterns and difference equations

In order to investigate students’ understanding of the relation between stock-flow patterns and difference equations, we asked students to give the equation corresponding to a stock flow model (Figure 3.11, exercise 3). We received 21 students’ answers, from which 6 answers were clearly wrong.
In modelling tasks 1 and 2 students had to do the opposite, building models corresponding to the equations \( \Delta x = v \Delta t \) and \( \Delta v = a \Delta t \). We observed many students having problems with task 2. These could be solved by stressing the similarity of the formulas and highlighting the twofold role of \( v \). Eventually, 23 out of 28 models that were sent in appeared to be correct.

Initially, we expected a system of coupled difference equations to be too complicated, but confronting students with such a system may actually be an effective way to clarify the possibly twofold role of variables. Results of a pilot with such a system (Figure 3.12) with five student couples were promising, but further testing is necessary.

3.5.4 Building problems and errors

In order to detect building problems, we did classroom observations and analysed screen- and audiorecordings of building task 3, in which students had to build a more extended model. The variables for this model were already on the modelling screen, but relations still had to be defined (Figure 3.6). All required physics formulas could be known to our students. We detected the following building problems and errors of students:

1. Not understanding if a variable must be defined by a number or a formula;
2. Use of cyclic definitions;
3. Defining a stock variable by a direct relation;
4. Finding a building order.

Some students initially thought that only numbers are used in the definitions of variables. Even if students knew that formulas could be used, some of them found it hard to decide whether a number or a formula was required. An example is the value for the friction constant \( k \), which eventually must be
determined by curve fitting. It was not clear to some students why $k$ should not be defined by the formula $k = \frac{F_{\text{air}}}{v^2}$. In order to solve this problem, the special icon for constants could be introduced, but then the concept of a constant must still be clarified.

Some students tried to use relations in a cyclic way, defining the acceleration $a$ as $\frac{F_{\text{net}}}{m}$, while simultaneously defining $F_{\text{net}}$ as $m \cdot a$. Another example is the cyclic definition of friction force and friction constant.

A number of students, said not to know where to start building. This may be an underlying problem for the problems mentioned above. For students it is not a priori clear which variable should be defined by which other variable. The question for students is how to determine an appropriate building order. Some authors advise to start building from a variable 'in which we are interested'. The question is, if this approach is adequate in all cases. For example, in building task 3 we are 'interested' in determining the constant $k$, but $k$ is not an appropriate starting point. A more fundamental approach follows from the fact that our type of modelling is developed for integration only. The stock (the integral), is determined by the flow, the opposite is not possible. This largely determines the order in which the model can be built. After putting a stock and one or more flows on the screen, we can advance by defining the flows. For our students, to whom the difference between $\Delta x = v \cdot \Delta t$ and $v = \frac{\Delta x}{\Delta t}$ is just the difference between multiplying and dividing, this one way dependence must be clarified.

Some students tried typing “$a \cdot t$” in the window for the initial value of the stock variable $v$. Apparently, they tried to define $v$ in this way. This might be caused by an asymmetry in graphical models between direct relations, which must be defined explicitly, and difference equations, that are defined automatically by stocks and flows, and by a lack of understanding of the integrating process.

### 3.6. Conclusions

We conclude that students can understand simple relation structures as shown by connectors, but attention must be paid to the difference between direct linked and secondary linked variables. Students are able to distinguish between difference equations and direct relations in realistic situations, but a better base of orientation must be provided to prevent misconceptions, and more attention must be paid to the possibly twofold role of variables in one model. The latter may also facilitate the translation from a difference equation to a stock-flow model, and vice versa, for students.
When building graphical models, problems can arise concerning the definitions of variables and the building order. These problems will be investigated more closely in a future version.

References


