Development of a modelling learning path
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Lower secondary students’ use of formulas in computer models*

Abstract

By means of a classroom experiment and a questionnaire, the understanding by lower level secondary school students of the calculation process and the use of formulas (direct relations) in a graphical computer model was investigated. It appeared that students could understand the calculation process on a numerical level, but had problems with the creation and use of formulas. Students did not yet have a clear understanding of the concept of a formula. The term ‘formula’ must be defined more clearly. Suggestions are given for improvement of the learning sequence leading to the use of direct relations in a computer model.

4.1 Introduction

Until recently, most research on quantitative computational modelling in education has been focussed on the higher educational levels, often with modelling courses on a project base only. There are good reasons for starting with modelling at an earlier age (Van Buuren, Uylings, & Ellermeijer, 2010), and in integrating it into the curriculum (Schecker, 1998). Therefore, in the past three years we have been working on the design of a learning path on modelling, integrated into the Dutch physics curriculum, starting from the initial phases (age: 13-14 years) (Van Buuren et al., 2010; Van Buuren, Uylings, & Ellermeijer, 2011). This learning path is tested in school practice. The modelling approach used is the ‘system dynamical’ stock and flow approach developed by

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* This chapter is a paper that is presented in 2011 at the GIREP-EPEC Conference and has been published as Van Buuren, O., Uylings, P., & Ellermeijer, T. (2012). The use of formulas by lower level secondary school students when building computer models. In A. Lindell, A.-L. Kähkönen, & J. Viiri (Eds.), Proceedings GIREP-EPEC Conference 2011, Physics Alive (pp. 140–149). Jyvaskyla: University of Jyväskylä.
Forrester (1968). Examples of software tools built for this approach are Stella (Steed, 1992) and Coach 6 (Heck, Kedzierska, & Ellermeijer, 2009).

With computer models, students can solve difference equations numerically that otherwise would be beyond their mathematical capabilities. In graphical computer models these difference equations are not entered in the model as formulas. Instead, combinations of graphical ‘stock and flow’ symbols are used. Although in this way the difference equations are not used explicitly, a certain level of understanding of the different roles of the variables is still required. In addition, students still need the ability to use formulas for variables that are defined by direct relations. Even more advanced students don’t possess the required levels of understanding automatically (Doerr, 1996; Hogan & Thomas, 2001; Schecker, 2005; Westra, 2008; Van Buuren et al., 2011). Therefore, the development of understanding of formulas and variables, and their connection to graphical models, must be tuned carefully.

The terms ‘formula’ and ‘variable’ both have many different meanings in mathematics and science (Heck, 2001; Malisani & Spagnolo, 2008). Here, we consider the term variable, which is of great importance to the concept of a formula. We confine ourselves to three of its meanings, stemming from mathematics and increasing in level of abstraction:

1. Placeholder: a variable that stands for one number, known or unknown.
2. Generalised number: an indeterminate number that appears in generalisations and in general methods.
3. Variable object, a symbol for an object with varying value (Heck, 2001), often in functional relationship to another variable, as “a thing that varies” (Malisani & Spagnolo, 2008), with a “changing nature” (Graham & Thomas, 2000).

In school practice in physics, in exercises students often replace symbols in formulas by numbers as soon as possible. What they are actually doing then is turning generalised numbers into placeholders. In this way, a more formal use of formulas can be avoided for quite a long time. In modelling however, direct relations must be used explicitly. Initially, they just can be entered into the model. Eventually, students must learn to build simple formulas themselves.

Research questions are:

1. **Which level of understanding of formulas and variables is required for students if they are to build models themselves?**
2. **How must a learning path be shaped to achieve the required level of understanding?**

The calculation process of computer models is a process of iteration. In order to understand the use of formulas in this process, students must understand the process itself. Therefore, an additional research question is:

3. **Do our students understand the calculation process as performed by the computer model?**
Our approach can be classified as educational design research (Van den Akker, Gravemeijer, McKenney, & Nieveen, 2006): educational materials are designed, tested in classroom, and redesigned in several cycles. In this paper, we describe the design and test results of the module in which a variable was to be defined by means of a direct relation for the first time by our students. Because it appeared that a number of students had problems using a formula in a model, a questionnaire was developed in addition to investigate the understanding of our students of the term “formula” itself.

4.2 Method

4.2.1 Graphical modelling

A more detailed description of the graphical stock and flow approach has been given in Van Buuren et al. (2011). Here, we restrict ourselves to the relation between graphical models and formulas, using examples from Coach 6. Fundamentally, graphical models consist of two types of formulas: difference equations and direct relations. Graphical modelling boils down to the numerical integration of the difference equations. Their graphical equivalent is a combination of a stock variable, represented by a square, and one or more flow variables, represented as ‘thick’ arrows (Figure 4.1). The flow variables are to be integrated; the stock variable is the integral and needs an initial value. If a variable is required that is not a stock or a flow variable, it is defined as an ‘auxiliary variable’ and is represented by a circle (Figure 4.2). Connectors (thin arrows) are used to indicate direct relations (Figures 4.2 and 4.3). These relations must be entered into the model after double clicking on the symbol that is to be defined.

\[ \Delta E = (P_{in} - P_{out}) \cdot \Delta t \]

**Figure 4.1:** Difference equation of one stock and two flow variables, and its graphical representation. The independent variable \( t \) is not visualised.

**Figure 4.2:** Auxiliary variables are used for variables that are not stock or flow variables.
4.2.2 Design of the module

**Preceding part of the learning path.** Before students can use direct relations in a computer model, they must learn to use formulas for calculations, they must get acquainted with both direct relations and difference equations, and they must learn to handle the software. Such aspects are addressed in the preceding modules of our learning path (Van Buuren et al., 2010, 2011). In these modules, there is an increase in complexity in models and model input. The first model consists of only one stock and one, constant, flow variable. The first varying flow is not defined by means of a formula, but by means of a graph, to be sketched by the students.

**Design considerations.** We expected our students to be able to enter formulas into the model. Therefore, our design mainly aimed at the following targets:
- the creation of an understanding of the iterative calculation process;
- the creation of an understanding of the direct relation to be used;
- the creation of a need for a computer model.

To facilitate thinking in iterations, the process to be modeled must be discrete and cyclic by nature. The required physics must not be too abstract, students must be able to visualise the process. The model must consist of only one stock and only one flow variable, and one direct relation. Finally, the subject should fit into the curriculum. This led us to the following principle set-up of the learning trajectory:
1. to make the process concrete, students perform real experiments first;
2. in order to develop an understanding of the calculation process on a concrete level, students calculate several cycles of the process by hand, without using formulas, filling in a table;
3. the required direct relation emerges from the repetitive character of the calculations;
4. the huge amount of calculations creates a need for a computer model;
5. students complete a partially built model by entering the required direct relation and initial value;
6. the model is adapted to fit the real measurements by changing parameters; the resemblance between model results and experimental results enhances students’ confidence in the model and in their own modelling skills.
**Implementation of the design.** A manually driven vacuum pump met our requirements. It could simply be added to the existing module on molecules. For an ideal gas with constant temperature and volume, pressure is proportional to the number of molecules. When air is pumped out of a vessel, each pump beat the same fraction of the molecules is removed. This enables a comparison of the real process, in which pressure can be measured, with the computer model, in which the number of molecules can be calculated, both as functions of the number of pump beats. A complication is the determination of the pump factor, the fraction of molecules pumped out each beat. This factor depends on the volumes of the pump and the vessel. We expected our students to be able to calculate it after some practice.

For step 2 of the trajectory a simplified imaginary pump was used, in which the dimensions were chosen in such a way that the arithmetic would not block students’ progress (Figure 4.4). The teacher assisted in making step 3, the emergence of the formula. This led to both a formula for the pump factor

$$p = \frac{V_{\text{pump}}}{V_{\text{vessel\-pump}}}$$

and the required direct relation for the number of molecules in the pump

$$N_{\text{p}} = p \times N,$$

with \(N\) the total number of molecules in vessel and pump.

![Figure 4.4: Imaginary pump with simplified dimensions.](image)

For steps 5 and 6, students had to calculate the pump factor for the real vessel and pump from estimated dimensions. After this, the formula for \(N_{\text{p}}\) had to be entered. Finally, the pump factor had to be adapted to get a better fit with the results of the measurements. After the module, a test was given to the students.

### 4.2.3 Set up of the classroom experiment

**Setting and instruments.** The module was tested a few weeks after summer holiday in five third-year classes (14-15 years), numbered 3A to 3E, consisting of 146 students, doing senior general secondary education or pre-university education (in Dutch: HAVO or VWO). The researcher was one of the two teachers. Up to a point students were allowed to work at their own pace,
individually or in small groups. The measurements (step 1) had to be done in special hours, for which students had to subscribe. Research data consist of:

- observation notes, and audio recordings of lessons and of individual students,
- worksheets for step 2 (39 sheets, involving 83 students),
- screen recordings of the Coach task (steps 5 and 6; 3 recordings of 7 students),
- uploaded Coach files (36 files of 80 students), including answers to questions to the Coach task,
- students answers on the questions on modelling in the final test of the module.

Test. In the test, students were asked to do a similar task as they had done in step 2. For an imaginary pump and vessel for which the dimensions and the initial total number $N$ of molecules was given, students were asked to calculate the initial value of $N_p$, and the values of $N$ and $N_p$ after one and two pump beats. Thereto, they had to calculate the pump factor. After this, they were asked to give the formula for $N_p$ that should be entered into the computer model.

4.2.4 Additional questionnaire

As will be described below, a number of students appeared to have problems using the formula in the model. Especially, some of them used numbers where variables are required. Therefore a questionnaire was developed, consisting of five questions, to investigate students’ understanding of the term “formula” itself (see Appendix 1). Research questions are:

a. to what extent do our students consider expressions in which a variable is equated to an expression consisting only of numbers as formulas?

b. which criteria do our students use, consciously or unconsciously, to determine whether an expression is a formula?

Research question ‘a’ is addressed in the questions 1, 3 and 4. In question 1 and 4, several expressions from mathematics and physics were offered. Students were asked which they considered as formulas. In question 3, students were asked to compare a formula with a ‘filled-in’ version of it. Students may have problems using formulas because they don’t feel themselves familiar with symbols. Therefore, in question 3, word formulas were offered as an extra alternative.

Research question ‘b’ is addressed in questions 1 and 4, but also in questions 2 and 5. In these questions, students were asked to explain why they considered certain expressions as formulas.

This questionnaire was given to the students of 3A, 3B and 3C. Accidentially, 3B received an older version, in which question 4 was missing.
4.3 Results

4.3.1 Learning process

Most students needed two lessons of 80 minutes for the learning trajectory.

**Determination of the pump factor and understanding of the calculation process.** From observation and from the worksheets, it appeared that only a few students had problems determining the pump factor for the simplified pump. The determination of the factor for the real pump in Coach task appeared to be more difficult, but finally only 5 out of 83 students did not succeed. Understanding the calculation process appeared not to be problematic. We detected only a few student errors, and most of these were corrected during the learning process. Errors were:

- keeping the number of molecules in the pump \( N_p \) constant over all pump cycles (5 out of 39 worksheets, 11 out of 83 students);
- mixing up the pump factor and \( N_p \) (2 out of 39 worksheets, 6 out of 83 students);

**Determination of the formula for \( N_p \) and entering it into the graphical model.** Determining the correct formula for \( N_p \) and entering this formula into the model appeared to be more difficult. Many students needed assistance of the teacher or of each other. In 7 of the 36 delivered final Coach results (17 out of 80 students), the formula still was not correct. The most frequently occurring errors were:

- assigning a constant value to \( N_p \) (5 Coach results, 11 students); in a number of cases, this value was optimized to fit the measurements as much as possible.
- mixing up variables, such as volume, pump factor, and number of molecules (3 Coach results, 6 students);
- assigning an expression to \( N_p \) consisting of numbers (constant values) only; some students called such an expression literally a “formula” (1 Coach result, 2 students; 1 screen recording, 4 students);
- some students did use symbols when calculating manually, but not did not use symbols in the graphical model.

4.3.2 Results of the test

We received 130 student tests. However, because of a failure of the computer network, most 3C students did not do the Coach task. In the other classes, some students did not do this task either, or did not deliver their results. Because the Coach task is crucial to our experiment, we decided to analyse in detail only the 80 tests of the students who had delivered their Coach task. Results are summarised in Table 4.1.
Student answers on question 1 of the questionnaire are summarised in Table 4.2. Answers in group 3B on this question deviate from answers in 3A and 3C, probably due to the explanation in 3B a few days before of the difference between ‘formula-notation’ and ‘function-notation’ by the mathematics teacher.

Table 4.2

<table>
<thead>
<tr>
<th>Correct of consequent</th>
<th>Determination of pump factor</th>
<th>Manual calculated values for pumping process</th>
<th>Formula for (N_p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct</td>
<td></td>
<td>49%</td>
<td>59%</td>
</tr>
<tr>
<td>‘Reasonable’ calculation error</td>
<td></td>
<td>23%</td>
<td></td>
</tr>
<tr>
<td>Not correct</td>
<td></td>
<td>19%*</td>
<td>29%</td>
</tr>
<tr>
<td>No answer</td>
<td></td>
<td>10%</td>
<td>13%</td>
</tr>
</tbody>
</table>

*13% used the pump factor from the example of the worksheet.

Table 4.1

Results of the test.

<table>
<thead>
<tr>
<th>Determination of pump factor</th>
<th>Manual calculated values for pumping process</th>
<th>Formula for (N_p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct</td>
<td></td>
<td></td>
</tr>
<tr>
<td>‘Reasonable’ calculation error</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Not correct</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No answer</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.1**

In Table 4.3, student answers to question 3 are summarised. Of all respondents, 83% chooses a formula or a word-formula, only 17% prefers a ‘filled-in’ version, an expression consisting of numbers. Symbols (options B and D in Table 4.3) are preferred above word variables (options A and C) by 53% of all respondents. Some students spontaneously commented on the options: “they are all the same”.

Table 4.3

<table>
<thead>
<tr>
<th>Is (y = 7 \times 8 + 27) a formula, according to you?</th>
<th>‘Yes’ &amp; ‘I think so, but I’m not sure’</th>
<th>‘No’ &amp; ‘I don’t think so, but I’m not sure’</th>
</tr>
</thead>
<tbody>
<tr>
<td>3A (28)</td>
<td>68%</td>
<td>21%</td>
</tr>
<tr>
<td>3B (25)</td>
<td>28%</td>
<td>64%</td>
</tr>
<tr>
<td>3C (24)</td>
<td>67%</td>
<td>13%</td>
</tr>
</tbody>
</table>

The number in brackets is the number of respondents.

Table 4.2
Table 4.3

Results of question 3.

<table>
<thead>
<tr>
<th></th>
<th>3A (28)</th>
<th>3B (25)</th>
<th>3C (24)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Word formula</td>
<td>43%</td>
<td>28%</td>
<td>38%</td>
</tr>
<tr>
<td>B. Formula (consisting of symbols)</td>
<td>39%</td>
<td>48%</td>
<td>54%</td>
</tr>
<tr>
<td>C. Word formula, ‘filled-in’</td>
<td>4%</td>
<td>12%</td>
<td>8%</td>
</tr>
<tr>
<td>D. Formula, ‘filled-in’</td>
<td>14%</td>
<td>4%</td>
<td></td>
</tr>
<tr>
<td>E. Other: ‘They are all the same’</td>
<td></td>
<td></td>
<td>8%</td>
</tr>
</tbody>
</table>

(Exercise: velocity and time are given). “Your teacher asks you to write down the formula that can be used to calculate the distance. What do you write down?”

Table 4.3

![Figure 4.5](image-url)

*Figure 4.5: Percentage of students considering the given expression to be a formula.*

In Figure 4.5, results of 3A and 3C on question 4 are summarised. As can be seen, a significant number of students considers an expression as a formula as soon as there is a symbol in it. For many students, an '='-sign is not required. When students were asked to explain when they considered expressions as formulas (questions 2 and 5), there appeared to be much doubt. Answers were very diverse and not always consistent. Analyzing all answers, we arrived at the following 'student criteria':
• you can calculate something with it / get an answer from it (~50%)
• it contains letters or symbols (~50%), but only a small part (~13%) mentions that there must be more than one letter or symbol.
• A number of students refers to mathematics (~16%)
• Very few students referred to something like a varying quantity.

4.4 Conclusions, and recommendations for a next round

We can conclude that a majority of students can understand the calculation process (Research question 3). However, the understanding is on a numerical level only. Students have more difficulty understanding the formula and using the formula in the model. Some use numbers where variables are required. From the questionnaire it appears that our students do not have a clear understanding of the concept of a formula. Expressions consisting of numbers and only one symbol are considered to be formulas by approximately half our students (Table 4.2 and Figure 4.5). This may indicate that their understanding of the concept of a variable is on the level of a placeholder. ‘Varying object’ would suit the variables in this module better (Research question 1). However, there are more simple explanations for the difficulties of our students, leading to a number suggestions for answers to research question 2:

1. A clear definition of the term ‘formula’ was not given to our students. The deviating student answers in class 3B on question 1 of the questionnaire (Table 4.2) indicate that some sort of definition does make a difference. The term ‘formula’ needs a more precise introduction.

2. In the preceding modules, model input mainly consisted of constant numbers. The difference equations needed not to be entered. This may give rise to the impression that there is no difference between a variable and a constant number. A recent remark of one of our students points in that direction. The relation between a difference equation and the graphical symbols should be made more explicit.

3. Students used the formula explicitly in the model only. They probably did not get acquainted with it enough.

4. Students may have had not enough training in the use of more than one formula in a module. It may be advantageous to add a module in which more than one formula must be used, preceding the module on the vacuum pump.

5. In our module, students not only had to use a formula, they also had to reconstruct it. Construction of formulas is difficult, even for higher level students (Schaap, Vos, Ellermeijer, & Goedhart, 2011). More training is required.

6. Students do not always rehearse experiments and modelling tasks when preparing for a test. They should be invited to do so, or the textbook must be adjusted, to refer more to the experiments and models.

These suggestions will be tried out in the next version of our learning path.
References


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Appendix 1: Questionnaire on student perceptions of the concept ‘formula’.

Dear student,

We are doing research in order to improve the teaching of physics. We want to know how students understand some concepts often used in physics. We ask you to fill in this short questionnaire according to your own ideas. It is not a test, it is not about being right or wrong, it is about your perception of these concepts.

1. Is \( y = 7 \times 8 + 27 \) a formula, according to you? Choose the answer that suits your opinion best:
   A. Yes
   B. I think so, but I’m not sure
   C. I am in doubt
   D. I don’t think so, but I’m not sure
   E. No

2. Explain your answer.

3. In science class, you get an exercise about a car that drives for 3.7 hours with a velocity of 97 km/h. Your teacher asks you to write down the formula that can be used to calculate the distance travelled. What do you write down?
   A. distance = velocity \( \times \) time span
   B. \( \Delta x = v_{av} \Delta t \)
   C. distance = 97\( \times \)3.7
   D. \( \Delta x = 97 \times 3.7 \)
   E. something else, namely: ...........................................

4. Which of the “expressions” below are formulas, according to you?
   A. \( 3x + 2 = 5x - 7 \)  
   B. \( ax = by \)
   C. \( f = \frac{1}{T} \)
   D. \( \frac{m}{V} \)
   E. \( \frac{325}{25} = 13 \)
   F. 30:8
   G. Gravity is proportional to mass
   H. \( y = \sqrt{x} \)
   I. \( a = 17^2 \)

5. Try to explain what a formula is, according to you.