Development of a modelling learning path

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Understanding of relation structures of graphical models

Abstract

A learning path has been developed on system dynamical graphical modelling, integrated into the Dutch secondary physics curriculum, starting from the initial phases (age: 13-15 years). In this paper, the focus is on students’ understanding of the relation structures shown in the diagrams of graphical system dynamics based models. Only part of the students understand these structures correctly. Reality-based interpretation of the diagrams can conceal an incorrect understanding of diagram structures. As a result, students seemingly have no problems interpreting the diagrams until they are asked to construct a graphical model. Misconceptions have been identified that are the consequence of the fact that the equations are not clearly communicated by the diagrams or because the icons used in the diagrams mislead novice modellers. Suggestions are made for improvements.

7.1 Introduction

Because in the Netherlands a concrete implementation of a complete, well-designed and thoroughly tested learning path on modelling into the physics curriculum does not yet exist, in 2008 we started to develop a learning path on quantitative computational modelling. For several reasons, we decided to start this learning path from the initial phases of physics education (age: 13-14 years) and to integrate it into the physics curriculum (Van Buuren, Uylings, & Ellermeijer, 2010). As a modelling approach, we chose the graphical system dynamics approach developed by Forrester (1961, 1968). Henceforth, we refer to this approach as ‘graphical modelling’. As computer modelling environment, we used Coach 6, in which this graphical approach is implemented (Heck,

* This chapter has been submitted for publication as Van Buuren, O., Heck, A., & Ellermeijer, T. (2013). Understanding of relation structures of graphical models by lower secondary students.
Graphical modelling is considered to be an appropriate candidate for a modelling approach in secondary science education (Savelsbergh et al., 2008), but in practice, it is not without problems. Several authors report on difficulties that students have, especially when designing or adapting graphical models (Bliss, 1994; Kurtz dos Santos & Ogborn, 1994; Sins, Savelsbergh, & Van Joolingen, 2005; Lane, 2008; Westra, 2008; Van Borkulo, 2009; Ormel, 2010). Doerr (1996), Lane (2008), and Savelsbergh et al. (2008) state that more research is needed for successful integration of graphical modelling in education.

The first two years of this modelling learning path have now been designed and tested in school practice, using an iterative research and development approach (Van Buuren, Heck, & Ellermeijer, 2013a). As part of this research project, difficulties that students have when designing or adapting graphical models have been studied. Difficulties related to students’ conceptions of formulas and variables are described elsewhere (Van Buuren, Uylings, & Ellermeijer, 2012; Van Buuren, Heck, & Ellermeijer, 2013b). In this paper, the focus is on students’ understanding of the diagrams of the graphical models in relation to the model structures. The learning path on these diagrams is part of the entire learning path on modelling and is therefore called a partial learning path.

This paper describes two different but related stages of this partial learning path. In the first stage, students study the way in which direct relations are used in the diagrams. In the second stage, all elements for graphical modelling are integrated. These two stages are connected to two different modules in the physics curriculum, namely Sound and Force and Movement. There were approximately three months in between the try-outs of these two modules. The research for each stage can be considered as a separate round of research.

This paper is organized as follows. In Section 7.2, graphical modelling and approaches to graphical modelling are described. Section 7.3 contains the research methodology and main research question. The general design of the entire partial learning path on graphical models and graphical modelling and an overview of the implementation of the instructional design is presented in Sections 7.4 and 7.5. Outcomes of the partial learning path preceding the modules Sound and Force and Movement are summarised in Section 7.6. A general description of the research instruments and the classroom setting is given in Section 7.7. The implementation of the learning path, research instruments, and outcomes for the stage connected to the module Sound are presented in Sections 7.8, 7.9, and 7.10, and for the stage connected to the module Force and Movement in Sections 7.11, 7.12, and 7.13. Overall outcomes are discussed in Section 7.13. Unless stated otherwise, figures of graphical models in this paper stem from Coach 6.
7.2 Graphical models and graphical modelling

7.2.1 General description

From a mathematical perspective, graphical modelling mainly concerns the numerical solving of (systems of) one-dimensional difference (or differential) equations. From this perspective, there are two fundamental building blocks: difference equations and direct relations. Difference equations are not entered in a computer environment for graphical modelling as formulas. Instead, they are represented by means of stock-flow diagrams: combinations of ‘stock’ icons, represented by rectangles, and ‘flow’ icons, represented by ‘thick’ arrows (Figures 7.1 and 7.2). For brevity, we refer to variables represented by flow icons as flow variables or flows, and variables represented by stock icons as stock variables or stocks. The stock-flow diagrams can be understood metaphorically as flows out or into the stock (‘outflows’ and ‘inflows’). Each stock needs an initial value. Variables that are not explicitly part of a difference equation are referred to as ‘auxiliary variables’ and are represented by circular icons (Figure 7.2).  

\[ \Delta E = (P_{\text{in}} - P_{\text{out}}) \cdot \Delta t \]

Figure 7.1: Difference equation consisting of one stock and two flow variables, and its graphical representation. The independent variable \( t \) is not visualised.

Figure 7.2: Auxiliary variables are used for variables that are not stock or flow variables. A connector indicates that a variable is defined by a direct relation.

Contrary to difference equations, direct relations must be entered explicitly, as formulas, in the computer environment. This is done by double clicking in the diagram on the icon of the variable that must be defined and by subse-

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1 Unless stated otherwise, in this paper the term variable is used for a quantity that can either be a varying object or a constant.
2 By a direct relation, we mean a mathematical relationship between symbolized quantities in which at least one quantity can be isolated and written as a closed form expression of the other quantities.
quently entering the required direct relation into the input field of a dialog window. The direct relations are not directly visible in the diagrams, but connectors (thin arrows) indicate their presence. If a connector points from a first icon to a second, this means that the variable corresponding to the first icon is part of the formula that is used to define the variable corresponding to the second icon (Figure 7.2). For shortness, in this paper we refer to such a first icon as a defining variable and to such a second variable as the defined variable. Both auxiliary variables and flow variables must either be defined by means of direct relations or by means of numbers.

7.2.2 Approaches to graphical modelling

Goals of the diagrams. Forrester (1961, p. 81) considers the diagrams as “an intermediate transition between a verbal description and a set of equations.” The main goal of the diagrams is to communicate the causal assumptions and the main features of the mathematical model. The diagrams are used in two different ways, namely:

1. as a graphical expression of a mental model. This graphical expression (the diagram) can subsequently be turned into a quantitative computer model by entering the required direct relations. In the original approach of Forrester, model conceptualisation starts with identifying the stocks (or levels, as they are also called) and flows (or rates). In this way, an understanding of difference equations is not required. Often more qualitative intermediate representations are also used, such as causal loop diagrams (cf., Kurtz dos Santos & Ogborn, 1994; Ossimitz, 2000, 2002; Lane, 2000, 2008). Forrester (1994) only occasionally uses causal loops, and only for model explanation afterwards;

2. as a way of expressing the meaning of the equations and structures of equations that is also clear and concrete to people with less mathematical education. In this way, equations can form the starting point for model construction and their meanings may be clarified by the diagrams.

The diagrams serve the purpose of communication at their best if, eventually, students are able to consider the diagrams from both ‘mental’ and mathematical viewpoint. Students must be able to use at least one approach for model construction.

Stock-and-flows approaches to graphical modelling. In more traditional approaches to graphical modelling, stock variables, flow variables, and auxiliary variables are considered as the fundamental building blocks. In these approaches, first of all, students must be able to determine the type of a

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3 In this paper, the term stock-flow diagram is used exclusively for combinations of stock and flow icons, whereas a graphical representation of a system dynamics model in general, including auxiliary icons and connectors, is referred to as a diagram.
variable. The stock-flow metaphor, or similar metaphors, and/or causal reasoning are supposed to help students designing and understanding models. This is expected to be easier than establishing and understanding the equations. In educational practice, however, such an approach does not go without problems. The following problems have been identified:

- The distinctions between flows (or rates) and stocks (or levels) are not easily made by students (Tinker, 1993; Bliss, 1994; Schecker, 2005; Westra, 2008).
- Considering the type of a variable as a fundamental property is questionable. As it is shown in Figure 7.3, one model variable can be part of several equations, in each of which its ‘type’ can be different. Therefore these types are not basic, inherent properties of variables.
- According to Bliss (1994), the representation of the rate of change of a variable as itself another variable is problematic for students of age 12–14 years. When not confident of this idea, students cannot express themselves with stock-flow diagrams.
- Students of age 16–18 years tend to use objects, events, and processes instead of true variables (in the sense of measurable properties) when trying to conceptualize their models, although this seems to depend very much on the problem that is modelled (Kurtz dos Santos & Ogborn, 1994).
- Not all students have sufficient prior knowledge of the flow of an incompressible fluid through a valve and its accumulation in a tank to understand this as a metaphor for stock-flow diagrams (Tinker, 1993).
- The stock-flow metaphor may work for many people, but does not work for all (Lane, 2008). After an instruction based on this metaphor, the stock-flow diagrams are not yet easily grasped by many a student (Tinker, 1993; Sins et al., 2005).
- The stock-flow metaphor is not very useful for modelling dynamics, as it is hard to imagine ‘position’ as a stock where ‘velocity’ accumulates (Tinker, 1993).
- Even if the use of differential equations can be circumvented, the use of direct relations cannot be avoided. Lower secondary students can have difficulty using these relations in their models if they do not have the required notions of variable and formula (Van Buuren et al., 2012, 2013b).
- Students may think that variables only depend on variables to which they are directly linked (Van Buuren, Uylings, & Ellermeijer, 2011).

\[
\begin{align*}
  A. \quad \Delta x &= v \cdot \Delta t \\
  B. \quad F_{\text{air}} &= k \cdot v^2 \\
  C. \quad \Delta v &= a \cdot \Delta t
\end{align*}
\]

Figure 7.3: Possibly threefold role of one variable. In the difference equations A and C, the velocity \( v \) is a flow variable and a stock variable, respectively. In equation B, \( v \) is part of the direct relation used to define \( F_{\text{air}} \). In this equation, \( v \) can be any type of variable.
• Constructing models using a qualitative, dependency based approach, as is done by some authors (Westra, 2008), may sometimes lead to less adequate relational structures, as is demonstrated in Figure 7.4 (Van Buuren et al., 2011).

Figure 7.4: Example of an incorrect model structure as a result of a dependency-based approach. The acceleration of a body moving through air can be said to depend on the friction force $F_{\text{air}}$ and on its velocity $v$. This may lead to this diagram. However, the acceleration depends on the velocity via the friction force.

A relation approach to graphical modelling. The question is whether these problems also will arise if the equations form a more dominant first part in the creation of graphical models. Such a more equation-based approach possibly fits better to the existing physics curriculum, in which formulas are introduced and used almost from the beginning. As a consequence, physics students soon have a collection of well-known formulas at their disposal. The problems with determining the types of variables may be diminished by considering the relations between the variables, i.e., the difference equations and direct relations, as the dominant building blocks of models, instead of the individual variables. Finally, the underlying reason why the stock-flow metaphor or other metaphors may be useful as an aid for constructing models lies in the uniformity of the underlying difference equations. Therefore, in our opinion, these equations should not be concealed, but highlighted.

For these reasons we have chosen such a more mathematical approach to graphical modelling. We call it a relation approach. A similar approach, in which graphical models were consistently presented to students as representations of computer programs evaluating recurrence relations, was possible and successful in upper secondary mathematics education (Heck, 2012, p. 82, 245). Central to the relation approach are the link between a difference equation and a stock-flow diagram, and the interpretation of connectors as indicating structures of direct relations.

Even with such a more mathematics-oriented relation approach, the diagrams and the stock-flow metaphor still may be useful, initially as an aid to comprehending models, and at a later stage both for the construction of models and for the construction of difference equations. But, as it is mentioned above, the understanding of the diagrams and the stock-flow metaphor does not go without problems. Although these problems are well known, little research has
been done on the question exactly how the diagrams and the icons are understood by students (Doerr, 1996; Lane, 2008), or which icons best convey the nature of the variables (Lane, 2008, p.17). In order to answer these questions, first must be established what exactly can be derived from a diagram.

7.2.3 Interpretation of diagrams

In order to find out to what extent students are able to interpret diagrams, first we must reflect on what actually can be derived from the diagrams. In the diagrams, two different subsystems are used: a subsystem representing direct relations and a subsystem representing difference equations.

Subsystem 1: relation structures based on connectors.
1a. As is mentioned above, a connector provides information about a direct relation. It shows which model variables are defined by which other variables. This is the key feature of connectors. Connectors do not give information about the exact forms of direct relations.
1b. Especially, if no connector is pointing to a specific flow icon or auxiliary icon, this means that the corresponding model variable is defined by a number only.
1c. Because connectors provide information about direct relations, the structure of icons and connectors also provides information about dependencies in a model. If in such a structure it is possible to go from one icon to another by following a chain of connectors and icons, this means that the other icon depends on the first. Van Buuren et al. (2011) introduce the terms ‘directly linked’ for variables that are directly connected to each other by means of one connector, and ‘secondarily linked’ in case there are one or more variables in between. See Figure 7.5. By means of substitution, auxiliary variables often can be removed from a model, although this is not customary. In such a case, many secondary links disappear.

Figure 7.5: Variables \(a\) and \(b\) are directly linked to \(y\), and variables \(c\) and \(d\) are secondarily linked to \(y\).
Subsystem 2: stock-flow diagrams.

2a. The relationship between a stock-flow diagram and a difference equation is unambiguous. A difference equation can be translated into a stock-flow diagram, and vice versa. A condition for this is that the modeller knows which variable is the so-called independent variable (the variable of integration). Usually, this is time.

2b. Because graphical models are essentially about numerical integration and not differentiation, the relation structure in stock-flow diagrams is always the same: the stock depends on the flows that are connected to it, but a flow does not need to depend on the stock. The value of a stock also depends on previous values of the stock. The flows determine the change of the stock. Usually, this is a change in time, but time can optionally be replaced by another independent variable.

There are reasons not yet mentioned in Section 7.2.2 to expect that students may also have difficulties interpreting the diagrams if a more relation-based approach is used.

- In physics and mathematics, formulas can be inverted at will, to calculate each of its components, depending on which components are given in advance. Components in a formula depend on each other, and this dependence is mutually. There is a relationship, but this relationship is not a priori causal. Students are used to this idea that formulas can be inverted at will. But in graphical models, a formula is used to calculate only one of its components. Which component is calculated is determined by the model, as a consequence of the fact that the graphical model is about integration. In the model, the formula cannot be inverted at will. This is not straightforward for students (Van Buuren et al., 2011). Therefore it can be expected that students may have difficulty understanding points 1a, 2a and 2b. We refer to this as the relation inversion conception.

- An error that is related to this relation inversion conception is the use of cyclic definitions: within one model, one direct relation is used for defining two of the variables that occur in the relation. For example, students may define the acceleration \( a \) as \( \frac{F_{\text{net}}}{m} \) while simultaneously defining \( F_{\text{net}} \) as \( m \cdot a \) (Van Buuren et al., 2011).

- The relation between stock-flow diagrams and difference equations is not self-evident for many students (Van Buuren et al., 2013b). Therefore students may have problems with points 2a and 2b.

These problems may not only be present when students interpret the diagrams, but also when students construct models based on known equations. Because of these anticipated problems, we started a design research project in which students’ understanding of the diagrams is investigated and a partial learning path is developed, as part of our research project on modelling.
7.3 Research methodology and main research question

In this paper we describe a research project in which we try to develop the partial learning path on the use and understanding of graphical models, dealing with students’ difficulties identified in earlier work (Van Buuren et al., 2011, 2012). Our research approach can be classified as educational design research (Van den Akker, Gravemeijer, McKenney, & Nieveen, 2006): instructional materials are designed, tested in classroom, and redesigned in several cycles. The main research question for this part of the research project is:

How do students understand the relation structures used in graphical system dynamics based modelling after a relation-based instruction?

Here, the term understanding is operationalized as:
1. being able to derive the information provided by the diagrams as described in Section 7.2.3, and
2. being able to construct graphical models or parts of models based on given formulas.

In order to answer the research question, the features of the diagrams described in Section 7.2.3 have been addressed in the design of the learning path. This design is described in Section 7.4. This design is implemented in the curriculum and tested in classroom. Test questions have been designed to probe students’ understanding of the features of the diagrams. The general setting and the research instruments are described in Section 7.7, followed by outcomes of classroom testing in subsequent sections.

7.4 Design of the partial learning path on graphical models and graphical modelling

7.4.1 Design considerations

The design of the current version of the learning path is based on results from field testing of earlier versions. Here, all important adaptations compared to the earliest version, described by Van Buuren et al. (2011), are discussed:

• Instead of a modelling approach that starts with establishing the types (i.e., stock, flow, or auxiliary) of the involved variables, the relation approach described in Section 7.2.2 has been adopted. An illustration of this approach is the use of a system of coupled difference equations as shown in Figure 7.6. Such a system was avoided by Van Buuren et al. (2011) because the twofold role of one of the variables, as a stock in one equation and as a flow in the
other, was considered to be too complicated for young students. In the current version, such a system is explicitly used to clarify the relation between difference equations and stock-flow diagrams, and to show that the type of a variable is not an intrinsic property.

![Diagram](image)

Figure 7.6: Diagram for the system of coupled difference equations $\Delta x = v \cdot \Delta t$ and $\Delta v = a \cdot \Delta t$. This requires the introduction of a dummy variable ($v_-$) in the diagram.

- Lower secondary students’ notions of formula and variable turned out to be insufficient for modelling (Van Buuren et al., 2012). Therefore, students are provided with clear operational definitions of formula and variable. This is discussed in detail by Van Buuren et al. (2013b).
- As part of the relation approach, connectors are not introduced as primarily showing dependencies, but as providing information about the defining variables of a defined model variable. From this, dependencies are derived.
- Van Buuren et al. (2011) suggest that some of students’ problems with the early, incomplete version of the learning path may have been caused by a lack of understanding of the process of numerical integration. In the current version of the learning path, more explicit and more regular attention is paid to this process, starting from earlier on in the learning path (Van Buuren et al., 2013b).
- Because of students’ difficulties in finding a construction order, a general construction plan for graphical models is offered suiting the relation approach. It consists of three steps. Students are advised to:
  1. collect or establish all formulas (including difference equations) that are related to the situation to be modelled. This is an opportune first step, because in physics education many important formulas usually are already known to the students.
  2. create the stock-flow diagrams corresponding to the difference equations.
  3. proceed with construction of the model by defining the variables in the diagram that have not yet been defined.

For step 3, it is important that students understand that the stock-flow diagram automatically determines the difference equation by which the stock is defined (apart from the initial value), but that all flows need to be defined explicitly, by entering a direct relation. Also, students must understand that
direct relations cannot be used in a cyclic way in a computer model (Section 7.2.3): each formula can be used as definition for only one variable.

- Students must learn to understand each subsystem but they must also be able to combine both subsystems. From a pilot project with a group of 23 students, this turned out to be difficult for students. This follows most clearly from the students’ answers to a multiple choice question in which students are asked to choose the two options that correspond to two diagrams, a stock-flow diagram and a structure of auxiliary variables and connectors (Figure 7.7). None of these students selected both correct options. Apparently, the distinction between these two subsystems must be consciously addressed. Thereto, the question in Figure 7.7 is prominently incorporated in the learning path, to be studied by the students.

The design principles described by Van Buuren, Heck, and Ellermeijer (2013a) are used as general principles for the design of the learning path. One of them is the creation of collections of examples of new concepts that students can use for orientation and that subsequently can be used to form bases of orientation when the concepts are introduced formally. Others are the design criteria for learning sequences around modelling activities that are analogous to design criteria for practical work. This leads us to the principle setup of the partial learning path on graphical models described below.

7.4.2 Design of the partial learning path on graphical models

In this subsection, the design of the partial learning path is described from a more general point of view. Six phases are distinguished:

1. In the preparatory phase of the partial path on graphical models, students are provided with several direct relations and difference equations; Δ-notation is used from the beginning. They get acquainted with basic features of the computer learning environment and orient on numerical integration.

2. In this phase, all elements required for an understanding of diagrams are introduced one by one on an introductory level. The learning goal is orientation, and learning to use, rather than understanding at a more structural level.
<table>
<thead>
<tr>
<th>Module number and title</th>
<th>Phase</th>
<th>Content regarding the understanding of graphical models</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Second year of secondary education (age: 13-14 years)</strong></td>
<td></td>
<td></td>
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<tr>
<td>3 Density</td>
<td></td>
<td>- first direct relation;</td>
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<td>5 Velocity</td>
<td>1.</td>
<td>- first Δ-formula;</td>
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<td></td>
<td></td>
<td>- main features of the computer learning environment;</td>
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<td></td>
<td></td>
<td>- first use of a graphical model;</td>
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<td></td>
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<td>- initial value for a stock;</td>
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<td></td>
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<td>- numerical integration, role of the time step;</td>
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<td>7 Energy and Power</td>
<td></td>
<td>- second Δ-formula;</td>
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<td></td>
<td></td>
<td>- introduction of stock-flow diagrams for storage and flows of energy;</td>
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<td>- inflows and outflows;</td>
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<td></td>
<td></td>
<td>- first orientation on the relation between a stock-flow diagram and a Δ-formula;</td>
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<tr>
<td><strong>Third year of secondary education (age: 14-15 years)</strong></td>
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<tr>
<td>2 Resistance and Conductance</td>
<td>2.</td>
<td>- operational definition of formula;</td>
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<td></td>
<td></td>
<td>- first use of a direct relation in a computer learning environment;</td>
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<tr>
<td>3 The Vacuum Pump</td>
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<td>- operational introduction of constant and of variable as varying quantity;</td>
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<td></td>
<td>- second use of concepts of storage and flow;</td>
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<td>- relation between Δ-formula and stock-flow diagram;</td>
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<td>- process of numerical integration in case of a feedback mechanism;</td>
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<td>- first use of direct relations in a computer model;</td>
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<td>- first connector;</td>
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<tr>
<td>4 Sound</td>
<td>3.</td>
<td>- second use of a direct relation in a computer model;</td>
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<td></td>
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<td>- auxiliary variables;</td>
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<td></td>
<td>- connectors and their relation to the definitions of model variables (the key feature of connectors);</td>
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<td>- secondarily linked variables and dependencies;</td>
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<td>- flows both as defining variables and as defined variables;</td>
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<td>- stocks as defining variables;</td>
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<td>- introduction of the independent variable (time);</td>
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<td></td>
<td></td>
<td>- explicit use of the independent variable in a model formula;</td>
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<tr>
<td>5 Force and Movement</td>
<td>4.</td>
<td>- formal introduction of the distinction between difference equations and direct relations;</td>
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<td>- summary of preceding elements of the partial learning path;</td>
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<td>- dependency in stock-flow diagrams;</td>
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<td>- mathematical meaning of outflow;</td>
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<td>- differences between stock-flow diagrams and structures formed by connectors;</td>
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<td>- possibly twofold (or threefold) roles of a variable; coupled difference equations; dummy variables;</td>
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<td>- adaptation of difference equations to graphical modelling by splitting up the difference equation into a new difference equation and a direct relation;</td>
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<td></td>
<td>5.</td>
<td>- general construction plan;</td>
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<td></td>
<td></td>
<td>- construction of a more extended model based on given equations;</td>
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</table>

A shading pattern is used to indicate the six phases.
3. In the third phase, the focus is on subsystem 1, the structures formed by connectors. This phase starts with the introduction of the key feature of connectors: showing which variables are used in the direct relation that defines another variable. For reasons described in Section 7.8, the independent variable (time) is also introduced and explicitly used in a direct relation.

4. For an understanding of graphical models, an understanding of the distinction between difference equations and direct relations (or ‘Δ-formulas’ and ‘direct formulas’, as they are called in the instructional materials) is required. Therefore, in this phase both types of formulas are introduced formally, making use of the examples taken from preceding modules in the learning path.

5. In the fifth phase, the two subsystems used in the diagrams of the graphical models are coupled. The complete set of elements is summarized. Students practise with all aspects mentioned in Section 7.2.3. For this, pen-and-paper exercises are used. The relation between the structures of stock-flow diagrams and difference equations is clarified. For this, a computer task in which a system of coupled difference equations is translated into a system of coupled stock-flow diagrams is used (see Figure 7.6). Students must learn to adapt a difference equation to graphical modelling in case an integrand (a ‘flow’) in the equation actually is a compound quantity. Hereto, the difference equations must be split up into a new difference equation containing a new dummy flow variable and a direct relation to define this dummy flow variable.

6. Finally, the general construction plan is introduced and used by the students to construct a more extended model in the computer learning environment, based on a given set of equations.

A more detailed overview of the content of these phases is given in Table 7.1.

7.5 Overview of the implementation of the instructional design

The physics curriculum into which the modelling learning path is integrated consists of twelve modules. In the second year of secondary education (age: 13-14 years), which is the first year of physics education, these modules are the following: 1. Doing Research; 2. Light; 3. Density; 4. Electricity; 5. Velocity; 6. Forces and Bridges; 7. Energy and Power. In the third year of secondary education (age: 14-15 years), the modules are: 1. Solid, Liquid, and Gas; 2. Resistance and Conductance; 3. The Vacuum Pump; 4. Sound; 5. Force and Movement. A detailed description of this modelling learning path is given by Van Buuren et al. (2013a). An overview of the distribution of the design of the partial learning path on graphical models over the modules from the learning path is shown in Table 7.1. Outcomes of field testing of the modules preceding Sound
are summarised in Section 7.6. The implementation of the partial learning path in the modules *Sound* and *Force and Movement* is described in Sections 7.8 and 7.11, respectively.

### 7.6 Summary of outcomes from modules preceding the module *Sound*

The preparatory phase of the partial path on graphical models largely coincides with the first year of physics education, and mainly with the module *Velocity*. This module has been used by approximately ten teachers at three different schools. Research data on the effects of this module need deeper analysis yet, but, according to the teachers, the early introduction of Δ-notation was not problematic and helped their students to distinguish between position and distance. The early introduction of numerical integration, with help of tables, turned out to be useful: students recognised the principles at later stages of the learning path. It may have been a disadvantage that no formulas have been entered by the students into the earliest models, because this may have contributed to the misconception found with some of the students that in computer programs numbers need to be entered only (Van Buuren et al., 2011, 2012).

Elements of the second phase of Section 7.4.2 are positioned at the end of the first year in the module *Energy and Power* and at the beginning of the second year in the modules *Resistance and Conductivity* and *The Vacuum Pump*.

Preliminary results from the final test of *Energy and Power* show that many students are able to draw stock-flow diagrams for devices in which energy can flow in, be stored and flow out. A majority of the students have also given correct names to the flows in these diagrams, but most students have trouble determining the type of energy stored. Some students have not given the name of the type of energy at all, some have given the name of the device instead of the type of energy. The reason appears to be related to the diagrams and not to the students' understanding of energy: in other test questions, in which no diagrams played part, most students have correctly determined the type of energy stored in the devices. A reason may be, that in some of the examples of stock-flow diagrams in the instructional materials, the type of stored energy has not explicitly been indicated, but another reason may be that students do not clearly distinguish between storage place and stock. In the final test of the module, only a minority of the students was able to give the correct difference equation to a given stock-flow diagram. Reasons may be that there was no clear need for students to understand the relation between a stock-flow diagram and a difference equation yet, and that students practised connecting stock-flow diagrams and difference equations only a few times in the last section of this last module of the year.
The two modules in which students’ conception of formulas and variables is developed, *Resistance and Conductivity* and *The Vacuum Pump*, are described in detail by Van Buuren et al. (2013a, 2013b).

### 7.7 Overview of research instruments and setting of the classroom experiments

**Overview of the research instruments**

The main research instruments are a subset of the questions in the final tests assessing students’ understanding of the modules *Sound* and *Force and Movement*. This subset probes students’ understanding of the diagrams. An overview of the research purposes of the test questions is shown in Table 7.2. Answers of a student on the test questions within one test can be expected to cohere. This qualitative coherence of answers may provide useful insights in students’ understanding and is therefore also studied. These test questions are described more specifically in Section 7.9 for the module *Sound* and in Section 7.12 for the module *Force and Movement*. Outcomes of these test questions are presented in Sections 7.10 and 7.13, respectively. A small part of the data presented in this paper (part of the data from questions *F2* and *F3*) have also been analysed by Van Buuren et al. (2013b), from a different perspective.

<table>
<thead>
<tr>
<th>Aspect of diagram</th>
<th>Type of relation between variables</th>
<th>Interpretation abilities</th>
<th>Construction abilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structures of connectors and direct relations</td>
<td>Occurrence in definitions</td>
<td><em>Sb, F1a, F1b</em></td>
<td><em>S, F2, F3b, F3c</em></td>
</tr>
<tr>
<td></td>
<td>Dependency</td>
<td><em>Sc</em></td>
<td></td>
</tr>
<tr>
<td>Stock-flow diagrams and difference equations</td>
<td>Occurrence in definitions</td>
<td><em>F1c</em></td>
<td><em>F2, F3a, F3c</em></td>
</tr>
<tr>
<td></td>
<td>Dependency</td>
<td><em>F1d</em></td>
<td></td>
</tr>
<tr>
<td>Effects of mixing</td>
<td>Occurrence in definitions</td>
<td><em>Sa, F1a, F1c</em></td>
<td><em>F2, F3</em></td>
</tr>
<tr>
<td></td>
<td>Dependency</td>
<td><em>Sd, F1d</em></td>
<td></td>
</tr>
</tbody>
</table>

Test questions from the module *Sound* are labelled with an *S*, questions from *Force and Movement* are labelled with an *F*, followed by a number indicating the question. Lower case letters indicate sub questions.
Setting of the classroom experiments

The part of the research project that focuses on the module *Sound* was carried out at HML with three third year classes (age: 14-15 years), together consisting of 82 students. In two of these classes, the first author of this paper was also the teacher. A third class was taught by an experienced teacher who had also participated as a teacher at an earlier stage of the project. Because of cancelled lessons this class did not finish the module *Force and Movement*. Therefore, the research related to this module was only carried out in the other two classes (57 students).

In general, all students worked individually or in small groups with the instructional materials. At HML, to some extent, students are allowed to work at their own pace and to postpone a test until they are ready for it. Also, students are allowed to redo a test if they fail the first time. In some cases, we must decide which test attempt of a student to use for analysis. Generally, we have used test results of the first attempt, for pragmatic reasons and to avoid a bias towards good results.

All classes consisted of mixed groups of students, in principle preparing for senior general secondary education or pre-university education. Near the end of the third year of secondary education, all students in the Netherlands must have chosen a set of courses for the next year. Motivation of students not applying for sciences often declines. At traditional schools, this may lead to bad test results for these less motivated students at the end of the third year. At HML, it leads to a diminishing number of students participating in the tests towards the end of the year. As a consequence, in the test results of *Force and Movement* there is probably a bias towards hard working and motivated students. Presumably, the aptitude for sciences of this group of students is more comparable with upper secondary science groups than with average third year lower secondary groups.

We received 67 final tests of *Sound* and 27 final tests of *Force and Movement*. There were more than three months between these final tests. This time was also spent on a chemistry module, on the first part of *Force and Movement*, and on holidays.

### 7.8 Design of the module *Sound*

The main goal of this module with respect to the diagrams used for graphical modelling is the introduction of structures of direct relations visualised by connectors. We need an intriguing physical phenomenon to provoke motivation. An additional requirement is that it is clear to students that the problem surpasses their mathematical capabilities. To them, this justifies the use of a computer model. For clarity, a model is preferred that only consists of direct relations, in which no stock-flow diagram appears. The ‘mysterious’ phenomen-
non of sound beats caused by two tuning forks with slightly different frequencies, as an example of momentary ‘anti-sound’, serves these purposes. The corresponding model concerns the addition of two sinusoids. This model is not a genuine dynamic model, because there is no difference equation to be integrated, but addition of functions certainly goes beyond the mathematical capabilities of third year secondary students, while the principle of addition can be understood by them for the individual points of these functions. In this context of functions, the independent variable \( t \) (time) appears explicitly. To make the use of the variable \( t \) visible, the special icon for this independent variable is used.

First, the required basic concepts (period, frequency, displacement, sinusoidal oscillation, and amplitude) are introduced. Because sine functions have not yet been introduced to the students in mathematics class at this stage of the curriculum, the sine function is introduced as ‘some mathematical function’ of time, frequency, and amplitude, and it is mainly represented as a graph. First, students study this function graphically, using a simulation driven by the model shown in Figure 7.8. This figure is also used to introduce auxiliary variables and to explain the role of connectors. Next, students search for a mathematical combination of two of these sine functions that leads to beats, using the almost complete model shown in Figure 7.9. Students practise with the role of connectors in exercises, in which also a few stock- and flow variables appear. In one of these exercises, a structure must be built for a given set of direct relations. In a subsequent exercise, Figure 7.9 is used to explain dependencies: although \( f_1, f_2, A_1, A_2, \) and \( t \) are not explicitly part of the formula for \( u \), they do have a direct influence on \( u \). This means that \( u \) depends on these five variables. Finally, students must enter a suitable formula for \( u \) in terms of \( u_1 \) and \( u_2 \).

**Figure 7.8:** Graphical model for a sine function \( u \), in which the connectors show that \( t, f, \) and \( A \) occur in the formula defining \( u \). The rectangle is the icon for the independent variable.

**Figure 7.9:** \( u \) must be defined as the addition of \( u_1 \) and \( u_2 \).
Chapter 7

7.9 Test questions of the module *Sound*

The main research instrument to the module *Sound* is one question in the final test of the module. In this paper, this test question is labelled with an S. Is has been designed for probing students’ understanding of structures formed by connectors. It consists of five sub questions, labelled Sa to Se. There are two versions of the test of the module *Sound*. In version 1 of the test, the context situation is on the increase of the number of customers of a shop as a result of investments in advertising (Figure 7.10). In version 2, a model is used with a similar structure: a farmer uses part of the revenue of the sale of apples for investments in fertiliser, which leads to better growth of apples. Because we have not found clear differences in the characteristics of students’ answers to the two versions, we combine the results of the two versions in our analysis and use version 1 for exemplifying outcomes.

![Diagram from the graphical model of test version 1, on the growth of the number of customers of a shop as a result of investments in advertising.](image)

*Figure 7.10: Diagram from the graphical model of test version 1, on the growth of the number of customers of a shop as a result of investments in advertising.*

**Sub questions Sa and Sb, on the key feature of connectors**

Sub questions Sa and Sb address students’ abilities to determine the defining variables for two of the variables from the connectors in the diagram. Sa and Sb have been split into Sa1 and Sa2, and Sb1 and Sb2, respectively. In Sa1 and Sb1, students determine the type of definition of a variable in the diagram. This type can be ‘formula’ or ‘only a number’. The subsequent sub questions Sa2 and Sb2 are conditional: in case students have chosen for ‘formula’, the defining variables must be determined. In this paper, a student is considered to understand the key feature of connectors (i.e., indicating defining variables) if this student correctly answers both Sa2 and Sb2.

In the text of the question, we used the term quantity instead of variable, because students have been taught to interpret variable as varying quantity, as opposed to constant, and therefore for a wrong reason may choose for formula. The correct answer is that one of the variables is defined by a number (and therefore is a constant) and the other by a formula. In order to investigate if students regard auxiliary variables as different from stocks with respect to connectors, both an auxiliary variable and a stock variable are part of this formula.
Sub questions Sc and Sd, on dependencies of secondarily linked variables

Sub questions Sc and Sd probe the students’ understanding of dependencies as indicated by the structures of connectors and icons. In Sc, students’ understanding of dependencies between secondarily linked variables is investigated for a chain of connectors and auxiliary variables. The flow variable (Customers_growth in Figure 7.10) from this diagram is the defined variable at the end of this chain, the starting point is the defining (constant) auxiliary icon from Sa (Revenue_per_customer in Figure 7.10). Students are asked whether the model variable at the end of the chain depends on the variable at the starting point. Here, the correct answer is “yes”. Students are asked to give an explanation of their answers based on the diagram.

Sd differs from Sc only with respect to the starting point of the structure of connectors and icons: in Sc, this is an auxiliary variable, whereas in Sd, it is a stock (Customers in Figure 7.10). By comparing students’ answers to these two sub questions, we want to study effects of mixing the subsystem for direct relations with the subsystem for difference equations. We expected this to be more difficult for students, because the stock and flow variable are not only secondarily linked through a chain of connectors and auxiliary variables, but they are also directly connected by means of the difference equation. A disadvantage of this approach may be that students may feel insecure because Sc and Sd are so much alike.

Sub question Se, on formula-based construction of structures of connectors and auxiliary variables

Sub question Se addresses formula-based construction abilities of the students with respect to direct relations. Students are asked to extend the diagram of the test to incorporate a new direct relation into the graphical model. This should be done by adding one auxiliary icon and one connector, which links this new icon to an existing icon. In test version 1 (Figure 7.10), this new direct relation is

\[ \text{Advertising\_investment} = \text{part\_for\_advertising} \times \text{Revenue}. \]

For \text{part\_for\_advertising}, a new icon should be added. This new icon must be linked to \text{Advertising\_investment}. Students are allowed to draw only the essential parts of the model (see Figure 7.11).

![Figure 7.11: correct answer to sub question Se from test version 1. Only essential parts are shown. Elements that must be added are labelled.](image_url)
Comparison with students’ understanding of the key feature of connectors

In our relation approach, both the understanding of dependencies and the construction abilities are based on the understanding of the key feature of connectors as assessed in sub questions Sa2 and Sb2. Therefore, all students’ answers to Sc, Sd, and Se are analysed by comparing them with the answers to Sa2 and Sb2.

7.10 Outcomes of the test questions of the module Sound

7.10.1 Sa and Sb, on the key feature of connectors

Some of the students had problems with the term quantity, thinking that quantity should be interpreted in the sense of measurable quantity (or measurable property), and therefore suggesting ‘money’ as quantity for Revenue. This actually is correct, but not what was intended. This shows both the importance and the difficulty of finding and using terminology that is unambiguous and clear to students.

In Table 7.3 is shown how many students have correctly answered the sub questions Sa1, Sa2, Sb1, and Sb2. Each is correctly answered by a majority of students, but only half of all students have correctly answered all four. As is stated in Section 7.9, a student is considered to understand the key feature of connectors if this student correctly answers both Sa2 and Sb2. Only 37 students (55%) have answered these two sub questions completely correct. Four incorrect answers probably are the result of a reading or writing error (these students have written Revenue instead of Revenue_per_customer, but they have answered Sc, in which these same model variables take part, correctly). Therefore we conclude that 61% of all students fully understand how connectors indicate the defining variables of a model variable. Without being asked, twelve students even have given a formula as an answer. Eight of these formulas are correct.

Some students have clearly based their answers on the diagram (by statements such as “because there are two auxiliary variable arrows pointing to the small sphere for Revenue”), but for many answers it is not possible to distinguish whether the students have based their answer on the diagram or on the realistic context situation. We found eight indications (12% of all students) of the relation inversion conception described in Section 7.2.3. These students probably thought that a defined variable can also appear in the definition of one of its defining variables; some students even have answered that a defined variable should be subtracted from a defining variable. To a certain extent, this error mirrors the relation as it exists between a stock variable and an outflow. The relation inversion conception does not dominate students’ thinking,
however: part of these students expressed some doubt and no student made this error in both questions. Almost no students have treated a defining stock differently than they have treated a defining auxiliary variable.

### Table 7.3

<table>
<thead>
<tr>
<th>Sub question and focus of the question</th>
<th>Correct answer</th>
<th>Number of correct answers</th>
<th>Both sub questions correctly answered</th>
<th>All 4 sub questions correctly answered</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type of the definition: ‘formula’ or ‘only a number’</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>So1 ‘formula’</td>
<td>51 (76%)</td>
<td>48 (72%)</td>
<td>33 (49%)</td>
<td></td>
</tr>
<tr>
<td>Sb1 ‘number’</td>
<td>59 (88%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Variables that are part of the definition</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>So2 The 2 correct variables are both mentioned</td>
<td>43 (64%)</td>
<td>41 (61%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sb2 No variables</td>
<td>55 (82%)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The cell corresponding to an understanding of the key feature of connectors is shaded grey.

### Table 7.4

<table>
<thead>
<tr>
<th>Students’ answers to sub question Sc compared to their answers to Sa2 and Sb2.</th>
<th>All students</th>
<th>Students answering Sa2 and Sb2:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Correct</td>
<td>Not correct</td>
</tr>
<tr>
<td><strong>Correct (“yes”)</strong></td>
<td>50 (75%)</td>
<td>34 (51%)</td>
</tr>
<tr>
<td><strong>Not correct (“no”)</strong></td>
<td>13 (19%)</td>
<td>7 (10%)</td>
</tr>
<tr>
<td><strong>No answer</strong></td>
<td>4 (6%)</td>
<td>4 (6%)</td>
</tr>
</tbody>
</table>

In Table 7.4, students’ answers to Sc are compared with their understanding of the key feature of connectors, probed by Sa2 and Sb2. The conclusions of three quarters of all students with respect to dependencies are correct. This is promising, especially because students had practised determining dependencies only once. Some students spontaneously have detected the feedback-loop: “It is a cycle”. But Table 7.4 also shows that the answers of an important minority of 16 students cannot be based on a fully correct conception of the relation
between connectors and formulas. Closer inspection reveals that among these
16 students, there are 6 students showing signs of a relation inversion concep-
tion, and there are three students who apparently have thought that variables in
graphical computer models can be defined by means of numbers only. Both
these incorrect conceptions do not contradict dependency in question Sc (at
least not from the students’ point of view).

With respect to the explanations given by the students, part of the stu-
dents clearly have pointed out how the model variables are connected by means
of the structure of connectors and auxiliary variables, but other explanations
seem to be based on an interpretation of the realistic context:

“Because of the revenue per customer, they earn more and can do more
advertising”.

This possibility to base an answer on an understanding of the realistic context
can be useful for finding a correct answer but it can also be disadvantageous
because it may conceal students’ misconceptions with respect to the model
structure.

The conception that secondarily linked variables do not depend on each
other explains the answers of 7 students (10%). The answers of these students
to the next sub question (Sd) are in accordance with this conception.

In Table 7.5, students’ answers to sub question Sd are compared with
their answers to Sc. From this table it can be inferred that, with respect to
dependencies through secondarily linked variables, the majority of students
have not treated the stock variable in Sd different from the auxiliary variable in
Sc. Almost all explanations to affirmative answers are correct, although in Sd

<table>
<thead>
<tr>
<th>Students’ answers to Sd</th>
<th>All students</th>
<th>Students answering Sc:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Correct: “yes”, argument also correct</td>
<td>Correct: “yes”</td>
</tr>
<tr>
<td></td>
<td>38 (57%)</td>
<td>33 (49%)</td>
</tr>
<tr>
<td>“yes”, argument based on stock-flow diagram instead of connectors</td>
<td>4 (6%)</td>
<td>2 (3%)</td>
</tr>
<tr>
<td>Not correct: “no”</td>
<td>22 (33%)</td>
<td>14 (21%)</td>
</tr>
<tr>
<td>No answer</td>
<td>3 (4%)</td>
<td>1 (1%)</td>
</tr>
</tbody>
</table>

Sub questions Sc and Sd are both about dependency between secondarily
linked variables. The only difference is the starting point of the chain of
connectors; in Sd this is a stock variable whereas in Sc it is an auxiliary variable.
Students’ answers and argumentations show that the presence of a stock-flow diagram in the test question has been confusing for approximately a quarter of the students. Two ways can be distinguished in which the stock-flow diagram may have influenced the students:

1. The stock-flow diagram, or the corresponding realistic situation, may have dominated the students’ perception. Even in a few correct explanations of correct affirmative answers, some doubt is expressed:

   “Half: it’s the same as Sc, but it is more the other way round. Customers depends on Customers_growth.”

   The perception that the stock depends on the flow and not vice-versa can clearly be found in the explanations of 4 out of the 14 students who have considered the flow as depending on the auxiliary variable (in Sc), but not on the stock variable (in Sd):

   “No, Customers_growth goes to Customers”.

   It is possible that more of these 14 students have held this opinion; the other 10 students have not given clear explanations to their negative answers on question Sd. From the answers of these students to question Sa2 it follows that this perception is not caused by an incorrect understanding of how connectors indicate direct relations: nine of these 14 students have answered question Sa2, in which the stock is a defining variable, completely correct. Apparently, these students have perceived that the stock is influenced by the flow, and they have not looked any further.

2. Four affirmative answers on question Sd have clearly been based on an inversion of the stock-flow relationship.

7.10.4 Se, on formula-based construction of structures of connectors and auxiliary variables

In Table 7.6, outcomes of sub question Se are shown in relation to the outcomes of questions Sa2 and Sb2. Question Se has been answered completely correct by almost half of all students, but from the answers on questions Sa2 and Sb2 it follows that a quarter of these correct answers have probably been based on an incorrect understanding of connectors. The answers of most students from this minority show signs of the relation inversion conception or of the conception that in graphical models only numbers are used to define variables.
Other students have correctly translated the direct relation into a model structure, but have added an extra connector. The most occurring addition (Figure 7.12) may have been based on additional students’ logic: it makes sense to calculate part_for_advertising from Revenue. Apparently, these students have understood how connectors correspond to direct relations, but they also have added a logic-based element. In Figure 7.13 an incorrect answer is shown that has been given by 9% of all students. Here, the diagram seems to be viewed as a flow chart for the money.

<table>
<thead>
<tr>
<th>Students’ answers to Se</th>
<th>All students</th>
<th>Students having answered Sa2 and Sb2:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct (see Figure 7.11)</td>
<td>31 (46%)</td>
<td>23 (34%) 8 (12%)</td>
</tr>
<tr>
<td>Correct + extra connector</td>
<td>12 (18%)</td>
<td>9 (13%) 3 (4%)</td>
</tr>
<tr>
<td>Not correct</td>
<td>13 (19%)</td>
<td>5 (7%) 8 (12%)</td>
</tr>
<tr>
<td>No answer</td>
<td>11 (16%)</td>
<td>4 (6%) 7 (10%)</td>
</tr>
</tbody>
</table>

Table 7.6

Figure 7.12: example of an often occurring answer to sub question Se from test version 1. This diagram is in accordance with the direct relation that must be added, but also an extra connector is drawn.

Figure 7.13: example of an incorrect answer to sub question Se from test version 1.
10.5 Summary of the outcomes of the test questions from the module Sound

Approximately half of all students have correctly answered all questions about structures formed by connectors. Per item, the scores are somewhat better. Apparently, many students have a basic understanding of the role of connectors in showing which are the defining variables of a defined model variable. Also, many students have correctly understood dependencies. Only 10% of all students have considered secondarily linked variables as not dependent on each other. Only a minority of students has been troubled by the presence of a stock-flow diagram in a chain of connectors. Constructing part of a model is the most difficult.

But the outcomes of these test questions also show that students easily arrive at correct conclusions with respect to model structure without having a complete understanding of the underlying concepts, using alternative conceptions. Especially, this holds for dependencies:

• students have used their understanding of the realistic context instead of interpreting the diagram;
• students holding a relation inversion conception have answered almost all questions correctly, apart from questions Sa or Sb;
• the conception of students that all model variables may be only defined by numbers does not lead them to incorrect conclusions regarding dependencies.

7.11 Design of the module Force and Movement

The physics domain on force and movement is useful for the fifth and sixth phase of the partial learning path. It contains a system of coupled difference equations, and several direct relations are available as definitions for different forces. This module is very large compared to other modules and therefore has been split into two parts. In the first part, basic physics and mathematics concepts are introduced. Among these is Newton’s second law, written as

$$\Delta v = \frac{F_{\text{net}}}{m} \Delta t$$

where \(\Delta v\) is the change in velocity of a mass \(m\) over an interval of time \(\Delta t\) as a result of a net force \(F_{\text{net}}\). The second part is directed towards modelling. The main goal of this part with respect to diagrams is the integration of all elements and the combination of the two subsystems used in the diagrams (see Section 7.2.3). Falling with air resistance has been chosen to be modelled. \(F_{\text{air}} = k \cdot v^2\) is used as candidate for air resistance. In this formula, \(k\) is a constant, to be determined for the falling object. As limiting cases, free fall and falling in case of equilibrium between gravity and air resistance are studied first, both experimentally and theoretically. Thereafter, the general case is studied. The formula for air resistance complicates the mathematics so much that a computer model is needed. At this point in the module, a switch is made towards
graphical modelling. All elements of graphical models are summarized, and remaining elements are introduced. The exercise shown in Figure 7.7 is explicitly used to introduce the difference between the two graphical subsystems: the stock-flow diagrams and the relation structures formed by connectors. Students get seven pen-and-paper exercises in which they practise all aspects discussed in Section 7.2.3. Subsequently, students design the model for falling with air resistance step by step, guided by the instructional materials. In order to learn to use the required features of the computer modelling environment, first an elementary model consisting of one stock-flow diagram is built, followed by the system of coupled difference equations from Figure 7.6, through which the relation between stock-flow diagrams and difference equations is clarified. The structure of a stock-flow diagram forces it user to rewrite Newton’s second law, originally written as \( \Delta v = \frac{F_{\text{net}}}{m} \). This is done by the introduction of a ‘dummy variable’ \( a = \frac{F_{\text{net}}}{m} \). Finally, students construct the model for falling with air resistance (Figure 7.14), following the general construction plan of phase 6.

\[ \begin{align*}
\Delta v &= \frac{F_{\text{net}}}{m} \\
\end{align*} \]

Figure 7.14: Diagram for the model for falling with air resistance.

7.12. Test questions of the module Force and Movement

In this section, the design is described of the test questions probing students’ understanding of all aspects of the diagrams: not only the subsystem of direct relations indicated by connectors, but also the subsystem of difference equations represented by stock-flow diagrams, and the combination of these two subsystems. Three test questions have been designed, labelled \( F_1 \), \( F_2 \), and \( F_3 \), each addressing a feature of students’ understanding of the diagrams. Each test question consists of a few sub questions, labelled with lower case letters. In question \( F_1 \), the student ability to derive information from the diagram about the definitions of model variables is probed in a way similar to the test question from Sound: in sub questions \( F_{1a} \) to \( F_{1c} \), students must derive information
about the definitions of three model variables from a diagram (Figure 7.15). Students’ understanding of dependencies is investigated in $F1d$.

In question $F2$, the student ability to construct a diagram for a given system of equations is investigated. In $F3$, the focus is on the abilities to construct formulas and subsequently construct the corresponding diagram for a new situation.

![Figure 7.15: Diagram from test question $F1$. Students are questioned about the definitions of the three encircled model variables.](image)

### 7.12.1 Test question $F1$, on the interpretation of diagrams

Sub questions $F1a$, $F1b$, and $F1c$, on the definitions of model variables. Sub questions $F1a$, $F1b$, and $F1c$, have been split in a similar way as sub questions $S_a$ and $S_b$ from Sound. In sub questions $F1a1$, $F1b1$, and $F1c1$, students are asked to name the type of relation defining a model variable. In case they think that a variable is defined by one of the two types of formulas, in the subsequent conditional sub questions $F1a2$, $F1b2$, and $F1c2$, students must also give the model variables that appear in this formula. Focus is on the effect of mixing stock-flow diagrams and structures of connectors.

In question $F1a$, the defined variable is an outflow of a stock-flow diagram. This outflow must be defined by a direct relation in which an auxiliary variable, a stock variable from a second stock-flow diagram, and the stock-variable from its own stock-flow diagram appear. This combination has been chosen to find out to what extent students are confused by the stock-flow diagram which the outflow is part of.

In question $F1b$, the defined variable is an auxiliary variable that is defined by a number, analogous to question $S_b$ from Sound.

In question $F1c$, the defined variable is the stock from the same stock-flow diagram as used in question $F1a$. The question for research is whether or not students realise that in the formula defining the change of the stock only both its flows and the time step can appear. This stock is also directly linked to
three other model variables. A question for research is to what extent students are confused by the presence of these three directly linked other variables.

**Sub question F1d, on the dependency of a stock on its outflow.** Sub question F1d consists of two parts, F1d1 and F1d2. In F1d1, students are asked whether the stock Foxes in Figure 7.15 depends on the constant auxiliary variable \( F_{\text{mortality factor}} \). These two variables are secondarily linked, with the outflow of the stock as an intermediate variable. In F1d2, the students are asked to explain their answers. F1d has been designed to investigate:

1. whether students consider a stock variable as dependent on its outflow(s).
   The difference between F1d and F1c is, that in F1c students are asked about the definition of the stock, whereas F1d is about dependency. In F1d, the relation between stock and flow is one of two relations linking the auxiliary variable \( F_{\text{mortality factor}} \) secondarily to the stock Foxes. Students’ explanations need also to be analysed with respect to the misconception that variables do not depend on each other in case they are not directly linked. In Section 7.10.2, this misconception was held by 10% of the students;
2. to what extent students base their answer on the graphical model, and to what extent their understanding of the realistic context is involved.

**7.12.2 Test question F2, on formula-based construction of diagrams**

In this question, students’ ability is investigated to construct a simple diagram based on two given formulas. The formulas are a difference equation containing one flow variable, and a direct relation defining this flow variable.

**7.12.3 Test question F3, on the construction of a graphical model for a realistic situation**

This question is designed to probe to what extent students are able to construct simple versions of both types of formulas for a given context situation, and to investigate how their ability to construct these formulas relates to their ability to construct the diagram for this situation. The model is about the number of trees in the forest. Each year, a forester cuts 20% of the trees and plants a constant number of new trees. Thus, the difference equation contains two flow variables, of which the inflow is a constant and the outflow depends on the stock variable. Students are subsequently asked to construct:

- **F3a.** the difference equation for the number of trees in the forest,
- **F3b.** the formula that can be used to calculate the number of trees to be cut each year, and
- **F3c.** the diagram.
7.13 Outcomes of the test questions of the module

*Force and Movement*

7.13.1 *F1*, on the interpretation of diagrams

Outcomes of sub questions *F1a* to *F1c* are presented together in Section 7.13.1.1, followed by the presentation of outcomes of *F1d* in Section 7.13.1.2.

**7.13.1.1 *F1a* to *F1c*, on the definitions of model variables.** To question *F1b*, about the model variable that must be defined by a number only, 89% of all students’ answers are correct. This number equals the number for the similar test question of *Sound*. The answers of 2 out of 27 students show signs of a relation inversion conception.

Results on questions *F1a* and *F1c* are shown in Tables 7.7 and 7.8, respectively. In these tables can be seen that less than 11% of all students have not understood that the model variables in these questions must be defined by formulas, but many students have had problems naming the types of these formulas. The formula defining the flow variable is more often incorrectly named than the formula defining the stock variable. Apparently, the fact that a variable is part of a stock-flow diagram tempts students to call the defining formula a difference equation (‘Δ-formula’). But Table 7.7 also reveals that 70% of all students have correctly derived from the diagram which model variables appear in the definition of the flow variable. This suggests that many students understand the relation structure as formed by connectors, in spite of the incorrect names they have used.

<table>
<thead>
<tr>
<th>Table 7.7</th>
</tr>
</thead>
</table>

**Outcomes of test question *F1a***

<table>
<thead>
<tr>
<th><em>F1a1</em>: type of definition</th>
<th><em>F1a2</em>: model variables that appear in the definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>Correct</td>
</tr>
<tr>
<td>Number</td>
<td>3 (11%)</td>
</tr>
<tr>
<td>Direct relation</td>
<td>11 (41%)</td>
</tr>
<tr>
<td>Difference equation</td>
<td>8 (30%)</td>
</tr>
<tr>
<td>Total</td>
<td>19 (70%)</td>
</tr>
</tbody>
</table>

In *F1a1*, students have to name the type of definition of a flow variable (*H_decrease* in Figure 7.15). In *F1a2*, they must give defining variables. These are the stocks *Hares’* and *Foxes*, and the constant auxiliary *H_decrease_factor*. The cell corresponding to the completely correct answer is shaded.

**Table 7.7**
The understanding of the relation structure of the stock-flow diagrams is more problematic: in Table 7.8 is shown that only 41% of all students in test question F1c have correctly mentioned both flow variables as being part of the definition for the stock variable; an important minority of nine students (33%) have mentioned the inflow but have not mentioned the outflow. There may be two reasons for this conception that a stock is not influenced by its outflow(s). The first reason is diagram-based: students may be confused by the similarity between flow icons and connectors, because both are represented by arrows. A second reason may be that students really think that an outflow does not have an influence on a stock. In answers to other questions, students often only discuss increases of variables and rarely mention decreases. If students really think so, this should appear from answers on test questions F1d and F3, but we have found just one indication for this second reason (see Sections 7.13.1.2 and 7.13.3.1). Furthermore, no students have mentioned time or a time step as part of the formula defining the stock. Other student errors in test question F1c are the reversion of the connectors linked to the stock variable (2 students), and confusion of ‘dependency’ with ‘being part of a formula’ for constants that are directly connected to some part of the stock-flow diagram (2 students).

Table 7.8

<table>
<thead>
<tr>
<th>F1c1: type of definition</th>
<th>only both flows</th>
<th>inflow, but no outflow</th>
<th>otherwise incorrect</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>2 (7%)</td>
<td>4 (15%)</td>
<td>2 (7%)</td>
<td>8 (30%)</td>
</tr>
<tr>
<td>Direct relation</td>
<td>9 (33%)</td>
<td>5 (19%)</td>
<td>3 (11%)</td>
<td>17 (63%)</td>
</tr>
<tr>
<td>Difference equation</td>
<td>11 (41%)</td>
<td>9 (33%)</td>
<td>7 (26%)</td>
<td>27 (100%)</td>
</tr>
</tbody>
</table>

In F1c1, students are asked to name the type of definition of a stock variable (Hares in Figure 7.15). In F1c2, they are asked for the defining variables. These are the flows H_decrease and H_increase. The cell corresponding to the completely correct answer is shaded.

Table 7.8

The understanding of the relation structure of the stock-flow diagrams is more problematic: in Table 7.8 is shown that only 41% of all students in test question F1c have correctly mentioned both flow variables as being part of the definition for the stock variable; an important minority of nine students (33%) have mentioned the inflow but have not mentioned the outflow. There may be two reasons for this conception that a stock is not influenced by its outflow(s). The first reason is diagram-based: students may be confused by the similarity between flow icons and connectors, because both are represented by arrows. A second reason may be that students really think that an outflow does not have an influence on a stock. In answers to other questions, students often only discuss increases of variables and rarely mention decreases. If students really think so, this should appear from answers on test questions F1d and F3, but we have found just one indication for this second reason (see Sections 7.13.1.2 and 7.13.3.1). Furthermore, no students have mentioned time or a time step as part of the formula defining the stock. Other student errors in test question F1c are the reversion of the connectors linked to the stock variable (2 students), and confusion of ‘dependency’ with ‘being part of a formula’ for constants that are directly connected to some part of the stock-flow diagram (2 students).

F1d, on the dependency of a stock on its outflow. In Table 7.9, an overview of student answers and explanations to sub question F1d is presented. Less than half of all students have come to the correct conclusion that the stock variable (Foxes) does depend on the auxiliary variable to which it is secondarily linked (F_mortality_factor).
Understanding of relation structures of graphical models

Students’ interpretations of the relation between stock and outflow.

Four relations may be discussed in the explanation of an answer to test question F1d. These relations, labelled 1 to 4, are indicated in Figure 7.16. In the explanations of the students in both categories, in general the focus is on the relation between the stock and the outflow (relations 2 and 3), and not so much on the relation between the auxiliary variable and the outflow (relation 1). The way the relation between the stock and the outflow is understood by the students turns out to be crucial. All but one of the students who have answered ‘yes’ to sub question F1d1 have noted that the stock depends on the outflow, whereas a large group of students who have answered ‘no’ have only mentioned that the outflow is determined by the stock (as is shown by connector 3 in

Table 7.9

Students’ answers and students’ explanations of their answers to test question F1d

<table>
<thead>
<tr>
<th>Answer on the question F1d1 whether the stock depends on a auxiliary variable connected to the outflow of the stock</th>
<th>Correct: yes</th>
<th>Incorrect: no</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students giving this answer</td>
<td>12 (44%)</td>
<td>15 (56%)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>In the explanation (question F1d2), attention is paid ...</th>
<th>to the relation from auxiliary variable to outflow (relation 1 in Figure 7.16)</th>
<th>7</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>to the relation from outflow to stock (relation 2 in Figure 7.16)</td>
<td>11</td>
<td>5*</td>
</tr>
<tr>
<td></td>
<td>to the relation from stock to outflow (relation 3 in Figure 7.16)</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>to the relation from auxiliary variable to stock (the not existing relation 4 in Figure 7.16)</td>
<td>7*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>to the fact that there are no connectors pointing towards the stock</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nature of explanation</th>
<th>The model is explained referring to the realistic context</th>
<th>6</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Elements of the structures of the diagram are used as arguments</td>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>In the explanation, references are made to calculations of formulas</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>No explanation</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

* These students mention this relation in a negative sense: according to them, this relation is missing in the diagram.

In this table, there are two categories: students answering ‘yes’ to the question whether the stock variable Foxes depends on the auxiliary variable F_mortality_factor, and students answering ‘no’. Their explanations are analysed with respect to the four possible relations in the stock-flow diagrams discussed in these explanations, and with respect to the nature of the explanation.

Table 7.9
Figure 7.16) or have even explicitly indicated that the stock is not influenced by the outflow (denying relation 2). As could be expected, this group contains almost all nine students who, as noted in Section 7.13.1.1, hold the opinion that an outflow does not influence a stock. Only two of these nine students have answered $F1d1$ correctly, and one of these two probably has made a double mistake. The conclusion is that at least a quarter of all students do not understand the relation between an outflow and a stock correctly. It is possible that the connector from the stock to the flow has confused some of these students.

The lack of a direct link between the auxiliary variable and the stock (the not existing relation 4 in Figure 7.16) has been used as an argument by at least 7 students for the stock to be not dependent on the auxiliary variable. From three of these students, this clearly has been the only argument. Apparently, these three students (11% of all students) have been of opinion that variables only depend on each other in case they are directly linked. This outcome is in agreement with our findings in Section 7.10.2.

![Figure 7.16: The four possible relations that may be discussed by the students in $F1d$ are indicated by numbers in rectangles.](image)

- Relations 1 and 3 are clearly indicated by connectors.
- Relation 2 is the dependency of the stock Foxes on the outflow $F_{\text{mortality}}$ (because $F_{\text{mortality}}$ is part of the $\Delta$-formula defining the change of Foxes).
- Relation 4 does not exist and is therefore represented by a dotted arrow. Students may use the absence of this direct relation as an argument.

**Nature of students’ explanations**

In Table 7.9 can be seen that 6 out of 7 students referring to the realistic situation in their explanations have arrived at the correct conclusion about the dependency of the variables, whereas the majority of the students who have based their explanation only on their understanding of the diagram have arrived at the incorrect conclusion. Half of all students who have affirmatively answered question $F1d1$ have involve the realistic context in their explanations:

“When foxes die, the mortality factor is important. If it is very bad, more foxes will die and the stock variable ‘Foxes’ will decrease”.

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As this example shows, these students have not necessarily ignored the diagram, but they have interpreted the diagram in terms connected to reality. In contrast, almost all students who think that the stock does not depend on the auxiliary variable, have only referred to elements from the diagrams, and not to reality:

“There is an arrow from Foxes to mortality, but not from mortality to Foxes. And the mortality factor goes only to mortality”.

Almost all students who have held the conception that a stock does not depend on an outflow have used such diagram based arguments. Only one of these students has clearly discussed a link with reality too. This student appears to understand reality differently:

“It is not necessary for foxes to die in order to get foxes, that does not make sense to me. Of course, first there must be foxes before they can die. In the graphical model, this is shown by the connectors: there is 1 from foxes to \( F_{\text{mortality}} \), but not the other way round.”

In this answer, the diagram is to a certain extent considered as indicating some sort of sequence in time, or some sort of flow chart for individual foxes: first, foxes are born, then they live, and finally they die. The explanation of one other student also indicates such a view. The diagram seems not to have been considered by these two students as describing the (more abstract) population.

These students’ explanations show that there are two ways for students to understand a model for a realistic situation. One way is based on the model structure, the other way is based on the realistic situation. This makes it unclear to distinguish what students actually understand when they answer questions concerning the graphical model correctly: the diagram, or reality. In the case we are studying here, students may well have used their knowledge of reality to correct their interpretation of the diagram.

Finally, just three students have explicitly referred to calculations in their explanations: “Foxes is calculated from \( F_{\text{mortality}} \). \( F_{\text{mortality}} \) is calculated from \( F_{\text{mortality\_factor}} \).” The answers of these students are completely correct. For them, a mathematical approach is successful. A question for further research is, whether more students can learn to use such an approach.

7.13.2 \( F_2 \), on formula-based construction of diagrams

As shown in Table 7.10, the majority of students have correctly constructed the simple diagram shown in Figure 7.17, based on a difference equation and a direct relation that must define the flow. All students who have answered \( F_{1d} \) correctly, and most students who in \( F_{1d} \) have only mentioned the inflow, have correctly answered question \( F_2 \). Apparently, for some of the students the role of the outflow is a special problem and this problem is irrelevant for this question.

A number of students initially misunderstood this question, thinking that two separate models had to be drawn, one for the difference equation and one
for the direct relation. According to these students, this must be regarded as merely a misinterpretation of the text of the question.

If students have problems with the construction of the diagram, this generally is with respect to the stock-flow diagram, and not with respect to connectors, as can be derived from Table 7.10. Finally, in all answers that are not completely correct, an icon for time or for the time step $\Delta t$ has been added.

### Table 7.10:

**Students’ results on test question F2**

<table>
<thead>
<tr>
<th>Quality of the drawn model diagram</th>
<th>Number of students*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Completely correct</td>
<td>19 (70%)</td>
</tr>
<tr>
<td>Direct relation correctly repre-</td>
<td>24 (89%)</td>
</tr>
<tr>
<td>sented</td>
<td></td>
</tr>
<tr>
<td>Correct stock-flow diagram for the difference equation.</td>
<td>20 (74%)</td>
</tr>
<tr>
<td>Completely incorrect answers</td>
<td>2 (7%)</td>
</tr>
<tr>
<td>Answers containing an icon for time or time step</td>
<td>8 (30%)</td>
</tr>
</tbody>
</table>

* 100% ↔ 27 students.

Students were asked to draw a graphical diagram based on two given formulas: a difference equation and a direct relation.

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#### 7.13.3 F3, on the construction of a graphical model for a realistic situation

In the context of forestry, in this section the symbol $B$ is used for the stock variable (the number of trees in the forest), the symbol $A$ for the constant inflow (the number trees planted each year), and the symbol $K$ for the outflow (the number of trees that are cut each year).

In Section 7.13.3.1, students’ answers on question F3a are analysed in coherence with the quality of the stock-flow diagram constructed by them as part of question F3c. In Section 7.13.3.2, students’ answers on question F3b are analysed in coherence with the quality of the structure of connectors in the diagrams they have drawn as answer to question F3c.
7.13.3.1 Construction of difference equations and corresponding stock-flow diagrams. Main students’ results regarding the construction of the difference equation in question F3a and the related stock-flow diagram in question F3c are shown in Table 7.11. We distinguish three cases:

<table>
<thead>
<tr>
<th>F3a: Constructed difference equation</th>
<th>F3c: Constructed stock-flow diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct</td>
<td>Correct (or correct alternative diagram)</td>
</tr>
<tr>
<td>Correct</td>
<td>9 (33%)</td>
</tr>
<tr>
<td>Incorrect</td>
<td>11 (41%)</td>
</tr>
</tbody>
</table>

Table 7.11

**Quality of the constructed difference equation in question F3a versus the quality of the constructed stock-flow diagram in question F3c.**

**Case 1: both difference equation and stock-flow diagram correct.**

A minority of 33% of the students have constructed a completely correct difference equation; all of these students have also constructed a correct stock-flow diagram, but some of them have made an error concerning connectors. Apparently, if students who have followed the learning path are able to construct difference equations, they can also construct stock-flow diagrams.

**Case 2: incorrect difference equation and correct stock-flow diagram**

Of all students who have not constructed a correct difference equation, 11 students nevertheless have constructed a correct stock-flow diagram. In order to find out how this is possible, we must study their diagrams, difference equations, and their answers to other questions in detail.

As a first lead, it is noticed that none of these 11 students have drawn a completely correct standard diagram, consisting of an inflow, an outflow, and one correct connector. Most of these students have made errors with respect to connectors; two students have drawn an alternative correct solution.

A second lead is that none of the 9 students who, as noted in Section 7.13.1.1, were of opinion that a stock does not automatically depend on its outflow have drawn a completely correct diagram; 7 of these students belong to the group of 11 students we are studying here. Apparently, this alternative conception has influenced the construction of the diagrams.

Further leads follow from analysis of the errors in the difference equations; these errors can easily be classified. Some of these errors do not necessarily hinder the construction of the stock-flow diagram. Such errors are the exclusion of $\Delta t$ from the difference equation (4 students) and the incorporation of an initial value into the difference equation (4 students). But the question remains how to explain why these students have also made errors with respect...
to connectors. Another question is how to explain that students leave out a variable for the outflow or the inflow from the difference equation, but can still construct a correct stock-flow diagram or alternative diagram.

Plausible explanations are found by considering consequences of the conception that the stock does not depend on the outflow. In case the difference equation in question $F3a$ contains the variable for the outflow, from this point of view there is a conflict that can be solved in two ways:

1. by constructing an alternative solution without using an icon for the outflow (see Figure 7.18);
2. incorrectly, by adding a connector from the outflow to the stock, which should indicate that the stock depends on the outflow (Figure 7.19).

We have found several examples of both approaches in the students’ work.

Figure 7.18: Answer to question $F3c$ of a student who in Section 7.13.1.1 was of opinion that a stock does not depend on an outflow. The only flow in this diagram is an inflow, and what should have been an outflow ("$K$") is represented by an auxiliary icon which is linked to this inflow. Apart from the fact that the connectors are not represented as arrows, this diagram is correct.

Figure 7.19: Diagram to question $F3c$ of a student who in Section 7.13.1.1 was of opinion that a stock does not automatically depend on an outflow. Erroneously, an extra connector is drawn from the outflow ("$K$") to the stock ("$B$") to indicate that in this case the stock depends on the outflow.

Another consequence of the conception that the stock does not depend on the outflow may be, that the variable for the outflow should not appear in the difference equation. We have found examples of such difference equations created by students. These students have drawn an outflow but have also added a connector to the diagram in a way similar to Figure 7.19, or have not drawn a diagram at all (1 student).
Eventually, all answers of these 11 students can be explained using one or more of the above conceptions. These misconceptions have not prevented these students from constructing a correct stock-flow diagram, but have hindered them in the construction of the difference equation.

**Case 3: both difference equations and stock-flow diagram not correct**

Of the 18 students who have not constructed a correct difference equation, 7 students have also not constructed a correct stock-flow diagram. Five of these students have not constructed a correct stock-flow diagram in question F2 also. One error that these 5 students have in common, is the addition of an icon to the diagram for the time step in question F2. This indicates that the relation between difference equations and stock-flow diagrams has not been understood by these students and that the special role of the independent variable is unclear for them too.

**Role of the time step**

Van Buuren et al. (2013b) note that part of the students have problems with the role of the time step $\Delta t$ when asked to construct a difference equation for a realistic situation (as in question F3a) or to construct a stock-flow diagram to a given difference equation (as in question F2). This is probably a consequence of the fact that the integration time step $\Delta t$ is not clearly visible in the stock-flow diagrams. In question F3c students’ ability to construct a stock-flow diagram for a realistic situation is investigated, although students first, in question F3a, have been asked to construct the corresponding difference equation; part of them therefore may have used this equation. With respect to the time step, results of questions F1c, F2, F3a, and F3c are summarized in Figure 7.20.

**Figure 7.20:** The way students deal with the time-step $\Delta t$ when asked to translate between a realistic situation, a difference equation, and a stock-flow diagram. Data from three different questions are incorporated in this figure. Each question is labeled. The question mark indicates that we do not precisely know to what extent students’ answers on question F3c have been influenced by their answers on question F3a.
This figure shows that, as an answer to question F3a, 33% of the students have (incorrectly) left out the time step $\Delta t$ when constructing the difference equation for a realistic situation, and, as an answer to question F2, 30% of the students (incorrectly) have added an icon for the time step $\Delta t$ when constructing a diagram for a given difference equation. Results of question F3c are more successful: 85% of all students have constructed a diagram without an icon for time or time step for the realistic situation; only 7% of all students have made the error to add such an icon. So, students more successfully construct a stock-flow diagram directly from reality than they construct a difference equation from reality or a stock-flow diagram to a given difference equation. Indeed, one of the arguments in favour of graphical modelling in education is that students would not need to understand difference equations. But these results also indicate that such a reality based construction of a stock-flow diagram may well conceal an incorrect understanding of the role of time and time step. In addition: the fact that in question F1c no student has mentioned time as one of the variables that appear in the formula defining a stock-variable, may mean that students correctly do not consider time as a causal agent, as is suggested by Kurtz dos Santos and Ogborn (1994). But it may as well mean that some of these students have not thought of the role of time at all. This raises the question whether the way in which stock-flow diagrams are used in graphical modelling, without reference to the time step, must be considered an advantage (because difference equations are circumvented), or as a disadvantage (because this essential element of the model is concealed and therefore the model can easily be misunderstood).

7.13.3.2 Construction of direct relations and corresponding use of connectors. Main outcomes with respect to the construction of the direct relation in question F3b and the related structure of connectors in the diagram in question F3c are shown in Table 7.12. Half of all students have constructed a completely correct direct relation for the outflow $K$; if minor errors, such as calculation errors, and errors concerning a less adequate conception of variable and formula are ignored, we can state that 78% of all students realises that $K$ is a function of $B$. Approximately one quarter of all students have constructed a completely correct diagram, and approximately one third of the students have drawn a diagram in which the structure of connectors can be considered as consistent with their answer to question F3b. Almost all students’ errors can be divided into three categories:

1. five students (19%) have not drawn connectors at all. Common misconceptions have not been found for these five students;
2. five students have drawn both a connector from the stock to the outflow and from the stock to the inflow, as in Figure 7.19. The connector to the inflow is incorrect because the inflow is given to be a constant. Students may have added this extra connector because in reality, the inflow (the number of trees planted each year) may well depend on the stock (the number of trees
Another explanation is that these students have been merely imitating the diagram shown to them in question \( F1 \);
3. at least five students have drawn a connector from the outflow to the stock;
   two other students have drawn lines instead of connectors between the outflow and the stock. At least five of these seven students have held the opinion that a stock does not automatically depend on an outflow.

Table 7.12

<table>
<thead>
<tr>
<th>Constructed direct relation as definition for ( K (F3b) )</th>
<th>Consistency of the diagram as answer to ( F3c ) with the direct relation of ( F3b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality</td>
<td>Number of students</td>
</tr>
<tr>
<td>correct function of ( B )</td>
<td>14 (52%)</td>
</tr>
<tr>
<td>Function of ( B ), containing minor errors</td>
<td>4 (15%)</td>
</tr>
<tr>
<td>function of the initial value of ( B ) instead of ( B )</td>
<td>3 (11%)</td>
</tr>
<tr>
<td>function of another variable or more variables</td>
<td>5 (19%)</td>
</tr>
<tr>
<td>no answer</td>
<td>1 (4%)</td>
</tr>
<tr>
<td>Total</td>
<td>27 (100%)</td>
</tr>
</tbody>
</table>

The number of consistent diagrams (with respect to the connectors) in question \( F3c \) is far less than in question \( F2 \), in which the direct relations are represented correctly by means of connectors by almost 90% of all students. Apparently, question \( F3 \) is far more difficult for students than question \( F2 \). One reason for this is the presence of the outflow in question \( F3 \). Another reason is the realistic situation, which is not part of the more mathematical question \( F2 \), but which is an important part of question \( F3c \). Students may have doubted the formulas that they have constructed themselves in question \( F3 \), or students may have been influenced by the realistic situation. Finally, question \( F3 \) is more difficult because in question \( F3 \) the stock-flow diagram and the structure of connectors are more closely intertwined.
7.13.4 Summary of the outcomes of the test questions from the module *Force and Movement*

Test question *F1* (Section 7.13.1.1) reveals that approximately 90% of the students can establish from a diagram which variables are defined by numbers only or by means of formulas. Only 7% of the students’ answers show signs of relation inversion. But part of the students have problems naming the type of formula. If a variable is a flow variable in a stock-flow diagram, part of the students are tempted to call the defining formula a difference equation. To a certain extent, this may also be considered as a case of relation inversion conception.

A majority of the students understand structures of connectors; the defining variables for a flow variable have been correctly identified in the diagram of *F1a* by 70% of the students. Just approximately 10% of the students has confused ‘not being part of a formula’ with ‘not being dependent on’.

But with respect to the relation between stock-flow diagrams and difference equations, most students have problems. None of the students has mentioned the time step as being part of the difference equation. Both in their answers to question *F1c* and *F1d*, an important minority of the students hold the opinion that a stock does not depend on its outflow. This accords with a remark of Lane (2000, p.244) that many people find it troublesome to run their gaze backwards along an outflow. The main reason for this opinion is the fact that flows and connectors both are represented as arrows. Only a few students probably have other reasons for this opinion.

From students’ explanations to their answers on question *F1d1*, we conclude that students who involve the realistic context in their explanations more often correctly identify dependences between variables than students who base their answers only on the structure of the diagram without yet having understood these structures exactly. This makes it difficult to distinguish what students are understanding when they correctly answer questions concerning a model: the diagram or reality. Students who use mathematics-based explanations arrive all at correct answers, but this type of explanation is used by only a few students.

Student’s answers to test question *F2* (Section 7.13.2) show that a majority of students can correctly construct a diagram for a simple system consisting of one difference equation and one direct relation in case the difference equation contains only an inflow. The subsystem of connectors has been correctly constructed by almost all students; construction of the stock-flow diagram is somewhat more difficult. The most important error is the addition of some icon for time or time step to the diagram.

In test question *F3* (Section 7.13.3), only one third of all students have constructed a correct difference equation containing two flow variables; subsequently, all of these students also have constructed correct stock-flow diagrams. Other students have constructed correct stock-flow diagrams too, but
these correct stock-flow diagrams conceal an incorrect understanding of the formulas or of the relation structure of the diagrams, in the following ways:

- as a consequence of the misconception that a stock does not depend on its outflow, students incorrectly use additional connectors or avoid the use of outflow icons;
- students construct correct diagrams without realising that the time step is part of the difference equation and maybe even without realising that time is involved at all;
- students have difficulties with the role of initial values.

Results of the construction of the direct relation in question $F3b$ are better than results of construction of the difference equations in question $F3a$: more than half of the students can construct the correct or almost correct direct relation. Students have much more difficulties correctly using connectors that represent their direct relations in this more complicated situation than in the more mathematics-oriented question $F2$. Plausible reasons for this are difficulties that result from mixing the subsystems of stock-flow diagrams with the subsystem of connectors (the misconception with respect to the outflow can be considered as such a difficulty), and deviations in the structure of connectors that may be the result of students’ interpretation of the realistic context.

### 7.14 Discussion, conclusions, and recommendations

The main research question is how students understand the relation structures used in graphical system dynamics based modelling after a relation based instruction. The term understanding has been operationalized as:

1. being able to derive the information provided by the diagrams as described in Section 7.2.3, and
2. being able to construct graphical models or parts of models based on given formulas.

With respect to the first point, a distinction has been made between occurrence of variables in definitions of other variables and dependency. Another distinction has been made between the first subsystem, of connectors concerning direct relations, the second subsystem of stock-flow diagrams concerning difference equations, and the combination of these two subsystems.

#### 7.14.1 First subsystem: structures formed by connectors and direct relations

Results show that a majority of the third year lower secondary students involved in this project can learn to interpret the structures of graphical models formed by connectors and are able to construct such structures using a relation approach. More than 60% of the students have correctly derived from the structure of connectors which are the defining variables of defined model variables (Sections 7.10.1 and 7.13.1).
In the course of the learning path, the number of students who can translate a direct relation into the correct part of a diagram has increased from 46% (Section 7.10.4) to 89% (Section 7.13.2), although this latter figure probably contains a bias towards hard working students.

Dependency as shown by the structures of connectors seems to be understood by many students, but part of these students hold alternative conceptions, such as the relation inversion conception and the conception that in a model numbers are required only, or use reality-based reasoning instead of (or together with) diagram-based reasoning (Section 7.9.3). A positive result of our relation approach is that just 10% of all students still think that there is no dependency between secondarily linked variables.

Effects of mixing the two subsystems on students’ understanding of the structure of connectors are limited. In Question Sd, some students have incorrectly ignored such a structure in favour of the one-step relation between a flow and corresponding stock, and in question F1a a relation defining a flow variable has incorrectly been called a difference equation (‘Δ-formula’) by a minority of the students. In spite of this, 70% of the students have been able to correctly determine the variables defining this flow variable. In Section 7.13.3 many students have not used connectors completely correct when constructing diagrams, but in many cases the incorrect use is the result of students’ attempts to overcome difficulties with the use of the subsystem of difference equations and stock-flow diagrams. Some of these students have creatively used a structure of connectors, auxiliary variables, and/or inflows to overcome such difficulties.

7.14.2 Second subsystem: stock-flow diagrams and difference equations

With respect to the subsystem of stock-flow diagrams and difference equations, most students have difficulties. Two of these difficulties can be considered a consequence of the fact that the difference equations are not clearly visualised in the stock-flow diagrams. The first difficulty is that part of the students, in spite of our relation approach, are not fully aware of the presence of the time step $\Delta t$ in the difference equation (Section 7.13.3.1). This lack of understanding can be left unnoticed in case students are asked to construct a stock-flow diagram for a realistic situation without creating a difference equation, as is usually the case when students construct diagrams using a stock-flow metaphor or causal

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4 Although it is plausible that a relation inversion conception as described in Section 7.2.3 is the cause of a number of coherent student errors, it must be noted that other causes for these errors cannot be excluded yet.

5 The conception that in graphical models only numbers are required can be successfully counteracted by providing students with proper operational definitions of variable and formula. Students must learn to understand that, for defining a variable, a formula is required (Van Buuren, Heck, & Ellermeijer, 2013b).
relations. The second difficulty concerns the role of the initial value with respect to the difference equation (Section 7.13.3.1). Initial values for stocks must be entered in a graphical computer model, but their role is not made clear by the diagrams or the dialog window for defining a stock. The fact that for a stock only a number for the initial value must be entered may even contribute to the misconception that a stock is not defined by a formula at all (Van Buuren et al., 2012).

One goal of the diagrams is communicating the model to people with less mathematics education. Understanding of difference equations would not be required. This may be true in case these people only have to interpret the graphical models, or listen to a modeller explaining the model (although it can be questioned whether these people really understand the models if they are not sufficiently aware of the role of time), but the diagrams do not sufficiently communicate the equations for (part of the) novice modellers who are expected to learn to construct models themselves.

A third difficulty is the misconception that a stock does not depend on its outflows. Only incidentally, this misconception is the consequence of incorrect understanding the realistic situation or of misinterpretation of the diagram as some sort of flow chart (Sections 7.13.1.2 and 7.10.4). The main reason for this misconception is the fact that both flows and connectors are represented as arrows (Sections 7.4.1, 7.13.1.1, 7.13.1.2, and 7.13.3). For inflows this is not a problem, because the convention that the defining variables of a defined quantity can be determined from the direction of the ‘arrows’ also works for stocks and inflows, but this convention does not work for outflows. This conception, of considering the arrows of the flows in the same way as the connectors, has been explicitly addressed in the instructional materials (Sections 7.4.1 and 7.4.2) but, apparently, is not easily changed.

In case this outflow-problem can be avoided, students’ understanding of the relation between stock-flow diagrams and difference equations can be considered promising. This follows from the results in Section 7.13.2, where 74% of the students construct a correct stock-flow diagram based on a difference equation. It also follows from the results discussed in Section 7.13.3, where students who did not hold this misconception have constructed correct stock-flow diagrams, and some of the other students created correct alternative solutions.

7.14.3 Role of the realistic situation

In Sections 7.10.1, 7.10.2, and 7.10.3 part of the explanations of the students of their answers are based on reality instead of on the diagram. In Section 7.13.1.2, it is shown that students referring to reality in their explanations more often have arrive at the correct conclusion about the dependencies of variables than students who have based their explanation only on their (incorrect) under-
standing of the diagram. These explanations show that there are two ways for students to understand a model for a real situation: one is based on the diagram structure, the other on the real situation. This makes it unclear to distinguish what students actually understand when they answer questions about the model correctly: the model or the real situation. In the cases we have investigated, we suspect that the students have used their knowledge of reality and some logic to correct their interpretation of the diagrams. In Section 7.13.3.1 part of the students have constructed a stock-flow diagram directly from reality more successfully than they have constructed a difference equation from reality or a stock-flow diagram from a difference equation. Yet, this does not mean that novice modellers do not need to understand difference equations: this correct reality based construction of a stock-flow diagram may conceal an incorrect understanding of graphical models. This phenomenon may be part of an explanation to findings of research hitherto, that modelling activities seem to be successful only until students are asked to construct parts of graphical models without assistance (Westra, 2008; Van Borkulo, 2009). Communication of the causal assumptions and the main features of the mathematical model are supposed to be the main purposes of the diagrams (Section 7.2.2). The diagrams apparently are helpful for students to organise their thoughts on reality in a way more or less similar to mind-maps. The diagrams may help students to reason about reality. But this strength of the diagrams is at the same time a weakness. The icons can be interpreted in several ways. An ‘arrow’ has at least three different meanings: (1) indicating relationship; (2) representing a flow variable, in which case the arrow gives information on the sign of the flow variable in the difference equation for the stock; or, (3) incorrectly, suggesting a direction on a flow chart. Misconceptions are concealed by the diagrams. Students do not need to fully understand the diagrams for interpreting them. Only when students are asked to construct diagrams, or to debug an existing incorrect diagram, it becomes clear whether students really understand the graphical models. Quoting one of the students of Westra (p.178):

“I understand a model when it is explained, but I am not able to build it myself”.

It must be emphasised that this holds for novice modellers. For more experienced modellers, who already have some understanding of the equations, the difficulties we have established in this paper can easily be overcome.

It must be noticed that some of the difficulties we have found using a relation approach not necessarily will be present when another approach is used. With another approach, other misconceptions may arise. But it is plausible that these other misconceptions also can stay concealed because students have the powerful alternative strategy at their disposal of using their knowledge of reality to interpret the diagrams, until they are asked to construct graphical models.

In summary, three categories of difficulties with graphical models can be distinguished:

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1. The difficulty of the alternative of reality-based interpretation: students can use the diagrams to interpret reality in a way similar to the way they can use mind maps or analogies. Up to a point, this can be an advantage, but if students understand reality well enough, an exact understanding of the diagrams is not required for the interpretation of reality. As a consequence, an incorrect understanding of the diagrams by the students will not be noticed until students are asked to construct diagrams;

2. Difficulties that arise because some (mathematical) aspects are not clearly communicated or even hidden by the diagrams. Examples are the role of the time step and the conception that in graphical models numbers are required only;

3. Difficulties that arise because the diagrams can be misleading. The misconception that a stock does not depend on its outflow is one example, the interpretation of flows and connectors as some flow chart is another.

7.14.4 Recommendations

An interesting question is whether or not we should dispose of graphical modelling in favour of text-oriented modelling. In this we would also throw away certain advantages:

- the graphical structure can help students to interpret the model afterwards;
- the diagrams do provide a clear overview of the model structure in case of medium-sized models;
- more sophisticated algorithms for numerical integration can be used, such as Runge-Kutta, without changing the diagram; a detailed knowledge of these algorithms is not required for novice modellers.

An alternative may be to use another graphical modelling tool, such as Simulation Insight (see, for example, Rogers, 2008). In this modelling environment, all variables are represented by rectangular icons, all connected by connector-like arrows, showing the defining variables of a defined variable. The modeller must consciously choose whether a variable is defined by means of a direct relation or by a difference equation. But in this approach, the structure of the diagram does not clearly show the difference between a direct relation and a difference equation, and the use of a metaphor such as the stock-flow metaphor to clarify the meaning of the diagram, is not possible. In the modelling environment Modus, arrows for both the inflows and outflows are oriented from the flow variable to the stock (Klieme & Maichle, 1991). This may prevent the conception that an outflow does not influence the stock, but does not yet clarify the relation between difference equations and stock-flow diagrams.

Another way to clarify the meaning of the diagrams is to show how the diagram can be translated into a text-based model at an early stage of the modelling learning path. In Coach 6, an option is available for this translation. But, although this approach probably helps clarifying the mathematical meaning
of the diagrams, it may not create a real need for understanding the relation between stock-flow diagrams and difference equations.

It must be realised that we are dealing with young novice modellers, who have just started to learn about modelling. Probably, they can overcome the aforementioned problems with some extra effort. But then, the question is whether the diagrams can be made easier to interpret for novice modellers. With respect to the second category of difficulties mentioned in Section 7.14.3, we recommend that the relation between stock-flow diagrams and difference equations should be made more clear. This may be done by showing the difference equation in the dialog window for the stock. Also, an option could be implemented into the modelling software to permanently show all equations to the relevant icons. This option needs only to be used in the initial phases of a modelling learning trajectory. With respect to the third category of difficulties, we recommend adaptation of the icons for the flow variables. Actually, there exist several implementations of system dynamics based graphical modelling. According to Lane (2000, p. 243), there are at least three or four widely used symbols for flow variables, all taking the form of valve or regulator symbols on the flows, which are indicated by double or thick arrows. In our opinion, a suitable flow icon should fulfill two conditions:

1. The only difference between an auxiliary variable and a flow variable is that the latter is part of a difference equation. In all other respects, flow variables and auxiliary variables are treated the same in the diagrams. Therefore we recommend to use icons that show this similarity better. In some implementations, this similarity already is more clear. See for example the stock-flow diagram in Figure 7.21 from STELLA.

![Figure 7.21: Stock-flow diagram from STELLA.](image)

2. Icons for flows, which are part of difference equations and in which the arrow merely is an indication of addition or subtraction, should differ more from the icons for connectors, that show the defining variables of a model variable. A question is whether flow icons should be arrow-like. From the perspective of a relation approach, a tube-like symbol in which flow directions are indicated by means of a + or – sign, as sketched in Figure 7.22, or by means of a word (in or out), might be an alternative.

![Figure 7.22: Sketch for a stock-flow diagram using alternative icons for the flows.](image)
References


