After finishing my master’s thesis at the University of Amsterdam in 2009, I started a PhD-project at the Centrum Wiskunde & Informatica (CWI) under supervision of Lex Schrijver, who was supported by a Spinoza grant. The aim of this project was to apply and develop algebraic techniques to and for combinatorial optimization. Despite the aim of the project, Lex gave me a lot of freedom to work on almost anything in the field of discrete mathematics. So in the beginning I tried to work on various topics. In particular, together with Dion Gijswijt, we showed that matroid base polytopes have integer Carathéodory-like properties. However, about nine months after I started at the CWI, Lex gave a talk about a question of Balazs Szegedy, which got me interested in so-called partition functions of edge-coloring models. This eventually resulted in this thesis entitled ‘Graph Parameters and Invariants of the Orthogonal Group’, in which these partition functions are the main characters. Incidentally, this thesis even fits within the original aim of the project, as it contains a significant interaction between combinatorics (in the form of graph theory and graph parameters) and algebra (in the form of invariant theory and some basic algebraic geometry).

Needless to say that without the help, support and advice of Lex, this thesis would not exist. It has been a great honor and pleasure to have been supervised by Lex. The way Lex approaches mathematics has been really inspiring. Not only is he open to learn new things in mathematics (and in life probably), he also always seeks ways to simplify proofs, always wondering whether things can be made more insightful. Thanks for the help support, advice and inspiration Lex!

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I have met a number of great, friendly, interesting and inspiring people the past four years. It was a privilege to have had this opportunity.

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Guus Regts
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