Graph parameters and invariants of the orthogonal group

Regts, G.

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List of symbols

| $\alpha$ | sum of the $\alpha_i$, 47 |
| Aut$(a, B)$ | automorphism group of the weighed graph $G(a, B)$, 51 |
| $\mathcal{A}$ | algebra of all fragments, 54 |
| $(\cdot, \cdot)_w$ | bilinear form: $(e_i, e_j)_w := w_i \delta_{i,j}$, 61 |
| $\mathcal{B}$(H) | closed unit ball in $\mathcal{H}$, 86 |
| $C_{i,j}$ | contraction operator for tensors, 20 |
| $C_1^*$ | labeled loop, 2 |
| $C_n$ | $n$-th Catalan number, 41 |
| $C$ | field of complex numbers, 7 |
| $\bigcirc$ | circle; the graph with one edge and no vertices, 8 |
| $C_{i,j}^l$ | contraction operator for fragments, 56 |
| $F_1 \cdot F_2$ | gluing product of $2l$-fragments $F_1$ and $F_2$, 10 |
| $\delta(v)$ | set of edges incident with the vertex $v$, 8 |
| $\delta_{s_1,s_2}$ | the delta function (equal to 1 if $s_1 = s_2$ and 0 otherwise), 7 |
| $d(V)$ | degree of the vertex $v$, 8 |
| $E(F)$ | edge set of the fragment $F$, 55 |
| $E(H)$ | edge set of the graph $H$, 8 |
| $E_s$ | edges associated to the map $s$, 36 |
| $\text{ev}_u$ | evaluation map, 69 |
| $\text{End}(V)$ | linear maps from $V$ to itself, 8 |
| $e_\phi$ | $e_{\phi(1)} \otimes \cdots \otimes e_{\phi(n)}$, 54 |
| $\mathcal{F}_l$ | set of all $l$-fragments, 10 |
| $\mathcal{F}_l$ | space of $l$-quantum fragments, 54 |
LIST OF SYMBOLS

\( \mathbb{F} \)  field of characteristic zero
\( \mathbb{F}^* \)  nonzero elements of the field \( \mathbb{F} \)
\( \mathbb{F} \)  algebraic closure of \( \mathbb{F} \)
FFT  First Fundamental Theorem

\( G(a, B) \)  weighted graph with vertex weights \( a \) and edge weights \( B \)
\( G' \)  set of all graphs including \( \bigcirc \)
\( G \)  set of all graphs
\( G_l \)  set of all \( l \)-labeled graphs
\( G_n \)  set of graphs with vertex set \([n]\)
\( G_{\text{sim}} \)  set of all simple graphs
\( \mathbb{F}G_l \)  semigroup algebra of \( G_l \)
\( \text{GL}(W) \)  group of invertible linear maps from \( W \) to itself

\( H/s \)  graph obtained from \( H_s \) by contracting the edges in \( E_s \)
\( H_1 H_2 \)  product of the labeled graphs \( H_1 \) and \( H_2 \)
\( H_s \)  graph obtained from \( H \) by adding the edges in \( E_s \)
\( \mathcal{H} \)  Hilbert space
\( \mathcal{H}^k_k \)  space of \( S_k \)-invariants in \( \mathcal{H}_k \)
\( \mathcal{H}_k \)  the Hilbert space \( l^2(C^k) \)
\( \text{hom}(H, G) \)  number of homomorphisms from \( H \) to \( G \)
\( h_l \)  restriction of \( h \) to the space of homogenous polynomials of degree \( l \)

\( I_V \)  identity map in \( \text{End}(V) \)
\( I_1(f) \)  ideal in \( \mathbb{F}G_l \) generated by the kernel of \( f \)
\( I_1(h) \)  kernel of \( M_{ph,l} \)

\( K_{i,j}^l \)  labeled contraction operator for tensors
\( K^*_l \)  labeled vertex
\( K^{**}_l \)  2-labeled edge
\( K^l_{i,j} \)  labeled contraction operator for labeled graphs

\( M_h \)  moment matrix of \( h \)
\( M_{f,l} \)  \( l \)-th edge connection matrix of \( f \)
\( M_m \)  set of perfect matchings on \([2m]\)

\( N_{f,l} \)  \( l \)-th vertex connection matrix of \( f \)
\( [n] \)  the set \( \{0, 1, \ldots, n\} \)
\( \mathbb{N} \)  the natural numbers including 0
\( \mathbb{N}^k \)  set of those \( \alpha \in \mathbb{N}^k \) with \( |\alpha| \leq d \)
\( \| x \|_R \)  seminorm associated to \( R \)
LIST OF SYMBOLS

\( \mathcal{O}(V) \) algebra generated by the dual of \( V \), 22
\( \overline{h}(p) \) complex conjugate of \( h(p) \), 71
\( \overline{A} \) Zariski closure of \( A \), 29
\( O(\mathcal{H}) \) orthogonal group of the real Hilbert space \( l^2(C, \mathbb{R}) \), 89
\( O_k(\mathbb{F}) \) orthogonal group over \( \mathbb{F} \), 21
\( h(p) \) complex conjugate of \( h(p) \), 71
\( \overline{h}(p) \) complex conjugate of \( h(p) \), 71
\( A \) Zariski closure of \( A \), 29
\( pr_d \) projection from \( \mathbb{N}_k^{\leq d} \) onto \( \mathbb{N}_k^{\leq d} \), 47
\( p \) map from \( G \) to \( T \), 43
\( p(A) \) image of \( A \) in the tensor algebra under the map \( p_h \), 57
\( p_n \) restriction of \( p \) to the set of graphs with \( n \) vertices, 43
\( p_{a,B} \) partition function of \( (a, B) \), 18
\( Q_l(f) \) quotient algebra \( F G_l / I_l(f) \), 13
\( R(\mathbb{F}) \) polynomial ring \( \mathbb{F}[x_1, \ldots, x_k] \), 18
\( R \) polynomial ring \( \mathbb{F}[x_1, \ldots, x_k] \), 18
\( R_k \) \( \{ r_1 \otimes \ldots \otimes r_k \mid r_1, \ldots, r_k \in B(H_1) \} \), 89
\( \mathbb{R} \) field of real numbers, 7
\( \text{rk}(M) \) rank of the matrix \( M \), 8

\( (C \circ D) \) Schur product of \( C \) and \( D \), 62
\( C \ast D \) operation on 2-tensors, 62
\( F_1 \ast F_2 \) gluing operation of \( F_1 \) and \( F_2 \), 10
\( S\mathbb{F}^{n \times n} \) space of symmetric \( n \times n \) matrices in \( \mathbb{F}^{n \times n} \), 28
\( S_n \) symmetric group, 30
\( \text{Stab}(A) \) pointwise stabilizer of \( A \), 57
\( \text{Stab}(h) \) stabilizer of the edge-coloring model \( h \), 52
\( \text{SFT} \) Second Fundamental Theorem, 27

\( F_1 \otimes F_2 \) tensor product of the fragments \( F_1 \) and \( F_2 \), 54
\( M^* \) conjugate transpose of the matrix \( M \), 8
\( M^T \) transpose of the matrix \( M \), 8
\( T(V)^{\text{Stab}(h)} \) algebra of tensors invariant under the stabilizer of \( h \), 58
\( T \) polynomial ring in the variables \( y_\alpha, \alpha \in \mathbb{N}_k \), 42
\( T_n \) homogeneous polynomials in \( T \) of degree \( n \), 43
\( \text{tr} \) trace, 77
\( t_M \) tensor associated to the perfect matching \( M \), 27

\( U_i \) unlabeling operator for tensors, 62
\( [H] \) underlying graph of the labeled graph \( H \), 9
\( U_i^l \) unlabeling operator for labeled graphs, 62
LIST OF SYMBOLS

\((V \otimes 2^m)^O_k\) space of \(O_k\)-invariant \(2m\)-tensors, \(27\)

\(V(F)\) vertex set of the fragment \(F\), \(55\)

\(V(H)\) vertex set of the graph \(H\), \(8\)

\(V^*\) dual vectorspace of the vectorspace \(V\), \(8\)

\(W^G\) subspace of \(G\)-invariants in \(W\), \(26\)

\(X/G\) orbit space of \(G\) acting on \(X\), \(86\)

\(Y_d\) the common zeros of the polynomials \(p(H) - f(H)\), with \(H \in \mathcal{G}\) of max. degree \(d\), \(47\)