Graph parameters and invariants of the orthogonal group
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<td>$\circ$</td>
<td>circle; the graph with one edge and no vertices, 8</td>
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<td>gluing product of 2l-fragments $F_1$ and $F_2$, 10</td>
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<td>$\delta(v)$</td>
<td>set of edges incident with the vertex $v$, 8</td>
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<td>$\delta_{s_1,s_2}$</td>
<td>the delta function (equal to 1 if $s_1 = s_2$ and 0 otherwise), 7</td>
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<td>$\mathcal{F}_l^l$</td>
<td>space of $l$-quantum fragments, 54</td>
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### LIST OF SYMBOLS

- $\mathbb{F}$: field of characteristic zero, 7
- $\mathbb{F}^*$: nonzero elements of the field $\mathbb{F}$, 8
- $\overline{\mathbb{F}}$: algebraic closure of $\mathbb{F}$, 7
- FFT: First Fundamental Theorem, 27

- $G(a,B)$: weighted graph with vertex weights $a$ and edge weights $B$, 18
- $\mathcal{G}'$: set of all graphs including $\bigcirc$, 8
- $\mathcal{G}$: set of all graphs, 8
- $\mathcal{G}_l$: set of all $l$-labeled graphs, 9
- $\mathcal{G}_n$: set of graphs with vertex set $[n]$, 43
- $\mathcal{G}_{\text{sim}}$: set of all simple graphs, 87
- $\mathbb{F}G_l$: semigroup algebra of $\mathcal{G}_l$, 12
- $\text{GL}(W)$: group of invertible linear maps from $W$ to itself, 25

- $H / s$: graph obtained from $H_s$ by contracting the edges in $E_s$, 36
- $H_1H_2$: product of the labeled graphs $H_1$ and $H_2$, 9
- $H_s$: graph obtained from $H$ by adding the edges in $E_s$, 36
- $\mathcal{H}$: Hilbert space, 86
- $\mathcal{H}_k$: space of $S_k$-invariants in $H_k$, 88
- $\mathcal{H}_k^l$: the Hilbert space $l^2(C^k)$, 88
- $\text{hom}(H,G)$: number of homomorphisms from $H$ to $G$, 18
- $h_1$: restriction of $h$ to the space of homogenous polynomials of degree 1, 57

- $\mathcal{I}_V(I)$: identity map in $\text{End}(V)$, 8
- $\mathcal{I}_1(f)$: ideal in $\mathbb{F}G_l$ generated by the kernel of $f$, 13
- $\mathcal{I}_1(h)$: kernel of $M_{ph,l}$, 54

- $K^l_{i,j}$: labeled contraction operator for tensors, 62
- $K^\bullet_1$: labeled vertex, 9
- $K^\text{**}_2$: 2-labeled edge, 9
- $K^l_{i,j}$: labeled contraction operator for labeled graphs, 62

- $M_h$: moment matrix of $h$, 36
- $M_{f,l}$: $l$-th edge connection matrix of $f$, 11
- $\mathcal{M}_m$: set of perfect matchings on $[2m]$, 27

- $N_{f,l}$: $l$-th vertex connection matrix of $f$, 11
- $[n]$: the set $\{0,1,\ldots,n\}$, 7
- $\mathbb{N}$: the natural numbers including 0, 7
- $\mathbb{N}_d^k$: set of those $\alpha \in \mathbb{N}^k$ with $|\alpha| \leq d$, 47
- $|| \bar{x} ||_R$: seminorm associated to $R$, 86

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\( \mathcal{O}(V) \) algebra generated by the dual of \( V \), 22

\( \overline{h(p)} \) complex conjugate of \( h(p) \), 71

\( \mathcal{A} \) Zariski closure of \( \mathcal{A} \), 29

\( \text{O}(\mathcal{H}) \) orthogonal group of the real Hilbert space \( l^2(C, \mathbb{R}) \), 89

\( \text{O}_k(\mathbb{F}) \) orthogonal group over \( \mathbb{F} \), 21

\( \text{pr}_d \) projection from \( \mathbb{N}^k \leq d' \) onto \( \mathbb{N}^k \leq d \), 47

\( p \) map from \( G \) to \( T \), 43

\( p_{h(A)} \) image of \( A \) in the tensor algebra under the map \( p_h \), 57

\( p_{n} \) restriction of \( p \) to the set of graphs with \( n \) vertices, 43

\( p_{a,B} \) partition function of \((a,B)\), 18

\( \mathcal{Q}_l(f) \) quotient algebra \( F G_l / I_l(f) \), 13

\( R(\mathbb{F}) \) polynomial ring \( \mathbb{F}[x_1, \ldots, x_k] \), 18

\( R \) polynomial ring \( \mathbb{F}[x_1, \ldots, x_k] \), 18

\( R_k \) \{ \( r_1 \otimes \cdots \otimes r_k \mid r_1, \ldots, r_k \in B(\mathcal{H}_1) \} \), 89

\( \mathbb{R} \) field of real numbers, 7

\( \text{rk}(M) \) rank of the matrix \( M \), 8

\( (C \circ D) \) Schur product of \( C \) and \( D \), 62

\( C \ast D \) operation on 2-tensors, 62

\( F_1 \ast F_2 \) gluing operation of \( F_1 \) and \( F_2 \), 10

\( \text{SF}^{n \times n} \) space of symmetric \( n \times n \) matrices in \( \mathbb{F}^{n \times n} \), 28

\( S_n \) symmetric group, 30

\( \text{Stab}(A) \) pointwise stabilizer of \( A \), 57

\( \text{Stab}(h) \) stabilizer of the edge-coloring model \( h \), 52

\( \text{SFT} \) Second Fundamental Theorem, 27

\( F_1 \otimes F_2 \) tensor product of the fragments \( F_1 \) and \( F_2 \), 54

\( M^* \) conjugate transpose of the matrix \( M \), 8

\( M^T \) transpose of the matrix \( M \), 8

\( T(V)^{\text{Stab}(h)} \) algebra of tensors invariant under the stabilizer of \( h \), 58

\( T \) polynomial ring in the variables \( y_\alpha, \alpha \in \mathbb{N}^k \), 42

\( T_n \) homogeneous polynomials in \( T \) of degree \( n \), 43

\( \text{tr} \) trace, 77

\( t_M \) tensor associated to the perfect matching \( M \), 27

\( \mathcal{U}_i \) unlabeling operator for tensors, 62

\( [H] \) underlying graph of the labeled graph \( H \), 9

\( \mathcal{U}_i \) unlabeling operator for labeled graphs, 62
LIST OF SYMBOLS

\((V \otimes 2^m)^O_k\) space of \(O_k\)-invariant \(2m\)-tensors, 27

\(V(F)\) vertex set of the fragment \(F\), 55

\(V(H)\) vertex set of the graph \(H\), 8

\(V^*\) dual vectorspace of the vectorspace \(V\), 8

\(W^G\) subspace of \(G\)-invariants in \(W\), 26

\(X/G\) orbit space of \(G\) acting on \(X\), 86

\(Y_d\) the common zeros of the polynomials \(p(H) - f(H)\), with \(H \in G\) of max. degree \(d\), 47