Graph parameters and invariants of the orthogonal group
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List of symbols

| α | sum of the $\alpha_i$, 47 |
| Aut($a, B$) | automorphism group of the weighed graph $G(a, B)$, 51 |
| A | algebra of all fragments, 54 |
| $(\cdot, \cdot)_w$ | bilinear form: $(e_i, e_j)_w := w_i \delta_{i,j}$, 61 |
| $B(\mathcal{H})$ | closed unit ball in $\mathcal{H}$, 86 |
| $C_{i,j}$ | contraction operator for tensors, 20 |
| $C^1$ | labeled loop, 2 |
| $C_n$ | $n$-th Catalan number, 11 |
| $C$ | field of complex numbers, 7 |
| $\bigcirc$ | circle; the graph with one edge and no vertices, 8 |
| $C^l_{i,j}$ | contraction operator for fragments, 56 |
| $F_1 \cdot F_2$ | gluing product of 2l-fragments $F_1$ and $F_2$, 10 |
| $\delta(v)$ | set of edges incident with the vertex $v$, 8 |
| $\delta_{s_1,s_2}$ | the delta function (equal to 1 if $s_1 = s_2$ and 0 otherwise), 7 |
| $d(V)$ | degree of the vertex $v$, 8 |
| $E(F)$ | edge set of the fragment $F$, 55 |
| $E(H)$ | edge set of the graph $H$, 8 |
| $E_s$ | edges associated to the map $s$, 36 |
| $ev_{\mu}$ | evaluation map, 69 |
| $\text{End}(V)$ | linear maps from $V$ to itself, 8 |
| $e_{\phi} \otimes \cdots \otimes e_{\phi(n)}$ | 54 |
| $\mathcal{F}_l$ | set of all $l$-fragments, 10 |
| $\mathcal{F}$ | space of $l$-quantum fragments, 54 |
LIST OF SYMBOLS

\( \mathbb{F} \) field of characteristic zero, 7
\( \mathbb{F}^* \) nonzero elements of the field \( \mathbb{F} \), 8
\( \overline{\mathbb{F}} \) algebraic closure of \( \mathbb{F} \), 7
FFT First Fundamental Theorem, 27

\( G(a, B) \) weighted graph with vertex weights \( a \) and edge weights \( B \), 18
\( \mathcal{G}' \) set of all graphs including \( \bigcirc \), 8
\( \mathcal{G} \) set of all graphs, 8
\( \mathcal{G}_l \) set of all \( l \)-labeled graphs, 9
\( \mathcal{G}_n \) set of graphs with vertex set \([n]\), 43
\( \mathcal{G}_{\text{sim}} \) set of all simple graphs, 87
\( \mathbb{F} \mathcal{G}_l \) semigroup algebra of \( \mathcal{G}_l \), 12

\( \mathcal{H} \) Hilbert space, 86
\( \mathcal{H}^S_k \) space of \( S_k \)-invariants in \( \mathcal{H}_k \), 88
\( \mathcal{H}_k \) the Hilbert space \( l^2(C^k) \), 88
hom(\( H, G \)) number of homomorphisms from \( H \) to \( G \), 18
\( h_l \) restriction of \( h \) to the space of homogenous polynomials of degree \( l \), 57

\( I_{V(I)} \) identity map in \( \text{End}(V) \), 8
\( \mathcal{I}_l(f) \) ideal in \( \mathbb{F} \mathcal{G}_l \) generated by the kernel of \( f \), 13
\( \mathcal{I}_l(h) \) kernel of \( M_{ph,l} \), 54

\( K_{i,j}^l \) labeled contraction operator for tensors, 62
\( K_{i,j}^\bullet \) labeled vertex, 9
\( K_{1,2}^\bullet \) 2-labeled edge, 9
\( K_{i,j}^\bigodot \) labeled contraction operator for labeled graphs, 62

\( M_h \) moment matrix of \( h \), 36
\( M_{f,i} \) \( i \)-th edge connection matrix of \( f \), 11
\( \mathcal{M}_m \) set of perfect matchings on \([2m]\), 27

\( N_{f,i} \) \( i \)-th vertex connection matrix of \( f \), 11
\( [n] \) the set \( \{0, 1, \ldots, n\} \), 7
\( \mathbb{N} \) the natural numbers including 0, 7
\( \mathbb{N}_d^k \) set of those \( \alpha \in \mathbb{N}^k \) with \( |\alpha| \leq d \), 47
\( |x|_R \) seminorm associated to \( R \), 86
LIST OF SYMBOLS

\( \mathcal{O}(V) \) algebra generated by the dual of \( V \), 22
\( \overline{h(p)} \) complex conjugate of \( h(p) \), 71
\( \overline{A} \) Zariski closure of \( A \), 29
\( \text{O}(\mathcal{H}) \) orthogonal group of the real Hilbert space \( l^2(C, \mathbb{R}) \), 89
\( \text{O}(\mathcal{H}) \) orthogonal group over \( \mathbb{F} \), 21
\( h(p) \) complex conjugate of \( h(p) \), 71
\( \text{pr}_d \) projection from \( \mathbb{N}^k \leq d' \) onto \( \mathbb{N}^k \leq d' \), 47
\( p \) map from \( G \) to \( T \), 43
\( p_n \) restriction of \( p \) to the set of graphs with \( n \) vertices, 43
\( p_{a,B} \) partition function of \( (a, B) \), 18
\( \mathcal{Q}_l(f) \) quotient algebra \( \mathcal{F}G_l/I_l(f) \), 13
\( \mathcal{R}(\mathcal{F}) \) polynomial ring \( \mathcal{F}[x_1, \ldots, x_k] \), 18
\( \mathcal{R} \) polynomial ring \( \mathcal{F}[x_1, \ldots, x_k] \), 18
\( \mathcal{R}_k \) \( \{ r_1 \otimes \ldots \otimes r_k \mid r_1, \ldots, r_k \in B(\mathcal{H}) \} \), 89
\( \mathbb{R} \) field of real numbers, 7
\( \text{rk}(M) \) rank of the matrix \( M \), 8
\( (C \circ D) \) Schur product of \( C \) and \( D \), 62
\( C \ast D \) operation on 2-tensors, 62
\( F_1 \ast F_2 \) gluing operation of \( F_1 \) and \( F_2 \), 10
\( \text{SF}^{n \times n} \) space of symmetric \( n \times n \) matrices in \( \mathbb{F}^{n \times n} \), 28
\( S_n \) symmetric group, 30
\( \text{Stab}(A) \) pointwise stabilizer of \( A \), 57
\( \text{Stab}(h) \) stabilizer of the edge-coloring model \( h \), 52
\( \text{SFT} \) Second Fundamental Theorem, 27
\( F_1 \otimes F_2 \) tensor product of the fragments \( F_1 \) and \( F_2 \), 54
\( M^* \) conjugate transpose of the matrix \( M \), 8
\( M^T \) transpose of the matrix \( M \), 8
\( T(V)^{\text{Stab}(h)} \) algebra of tensors invariant under the stabilizer of \( h \), 58
\( T \) polynomial ring in the variables \( y_\alpha, \alpha \in \mathbb{N}^k \), 42
\( T_n \) homogeneous polynomials in \( T \) of degree \( n \), 43
\( \text{tr} \) trace, 77
\( t_M \) tensor associated to the perfect matching \( M \), 27

\( U_i \) unlabeling operator for tensors, 62
\( [H] \) underlying graph of the labeled graph \( H \), 9
\( U_i^l \) unlabeling operator for labeled graphs, 62
LIST OF SYMBOLS

\((V \otimes^2 m)^O_k\)  space of \(O_k\)-invariant \(2m\)-tensors, \(27\)

\(V(F)\)  vertex set of the fragment \(F\), \(55\)

\(V(H)\)  vertex set of the graph \(H\), \(8\)

\(V^*\)  dual vectorspace of the vectorspace \(V\), \(8\)

\(W^G\)  subspace of \(G\)-invariants in \(W\), \(26\)

\(X/G\)  orbit space of \(G\) acting on \(X\), \(86\)

\(Y_d\)  the common zeros of the polynomials \(p(H) - f(H)\), with \(H \in \mathcal{G}\) of max. degree \(d\), \(47\)