Graph parameters and invariants of the orthogonal group
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<td>sum of the $\alpha_i$, 47</td>
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<td>$\text{Aut}(a, B)$</td>
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<td>$C_n$</td>
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<td>$\mathbb{C}$</td>
<td>field of complex numbers, 7</td>
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<td>$\circ$</td>
<td>circle; the graph with one edge and no vertices, 8</td>
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<td>$C_{i,j}^l$</td>
<td>contraction operator for fragments, 56</td>
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<td>$F_1 \cdot F_2$</td>
<td>gluing product of 2l-fragments $F_1$ and $F_2$, 10</td>
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<td>$\delta(v)$</td>
<td>set of edges incident with the vertex $v$, 8</td>
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<tr>
<td>$\delta_{s_1,s_2}$</td>
<td>the delta function (equal to 1 if $s_1 = s_2$ and 0 otherwise), 7</td>
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<td>$d(V)$</td>
<td>degree of the vertex $v$, 8</td>
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<td>$E(F)$</td>
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<td>$\text{ev}_u$</td>
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<td>$\mathcal{F}_l$</td>
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<td>$\mathcal{F}_l^l$</td>
<td>space of $l$-quantum fragments, 54</td>
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LIST OF SYMBOLS

$\mathbb{F}$ field of characteristic zero, 7
$\mathbb{F}^*$ nonzero elements of the field $\mathbb{F}$, 8
$\mathbb{F}$ algebraic closure of $\mathbb{F}$, 7
FFT First Fundamental Theorem, 27

$G(a,B)$ weighted graph with vertex weights $a$ and edge weights $B$, 18
$G'$ set of all graphs including $\bigcirc$, 8
$G$ set of all graphs, 8
$G_l$ set of all $l$-labeled graphs, 9
$G_n$ set of graphs with vertex set $[n]$, 43
$G_{\text{sim}}$ set of all simple graphs, 87
$\mathbb{FG}_l$ semigroup algebra of $G_l$, 12
$\text{GL}(W)$ group of invertible linear maps from $W$ to itself, 25

$H / s$ graph obtained from $H_s$ by contracting the edges in $E_s$, 36
$H_1H_2$ product of the labeled graphs $H_1$ and $H_2$, 9
$H_s$ graph obtained from $H$ by adding the edges in $E_s$, 36
$\mathcal{H}$ Hilbert space, 86
$\mathcal{H}_k$ space of $S_k$-invariants in $\mathcal{H}_k$, 88
$\mathcal{H}_k^l$ the Hilbert space $l^2(C)^l$, 88
$\text{hom}(H,G)$ number of homomorphisms from $H$ to $G$, 18
$h_l$ restriction of $h$ to the space of homogenous polynomials of degree $l$, 57

$I_V(I)$ identity map in $\text{End}(V)$, 8
$I_l(f)$ ideal in $\mathbb{FG}_l$ generated by the kernel of $f$, 13
$I_l(h)$ kernel of $M_{ph,l}$, 54

$K_{i,j}$ labeled contraction operator for tensors, 62
$K^*$ labeled vertex, 9
$K^*$ labeled edge, 9
$K^*_{i,j}$ labeled contraction operator for labeled graphs, 62

$M_h$ moment matrix of $h$, 36
$M_{f,l}$ $l$-th edge connection matrix of $f$, 11
$M_m$ set of perfect matchings on $[2m]$, 27

$N_{f,l}$ $l$-th vertex connection matrix of $f$, 11
$[n]$ the set $\{0,1,\ldots,n\}$, 7
$\mathbb{N}$ the natural numbers including 0, 7
$\mathbb{N}^k_{\leq d}$ set of those $\alpha \in \mathbb{N}^k$ with $|\alpha| \leq d$, 47
$|| x ||_R$ seminorm associated to $R$, 86
**LIST OF SYMBOLS**

- $\mathcal{O}(V)$ algebra generated by the dual of $V$, 22
- $\overline{h(p)}$ complex conjugate of $h(p)$, 71
- $A$ Zariski closure of $A$, 29
- $O(H)$ orthogonal group of the real Hilbert space $l^2(C, \mathbb{R})$, 89
- $O_k(\mathbb{F})$ orthogonal group over $\mathbb{F}$, 21
- $pr_d$ projection from $\mathbb{N}_k^{d} \to \mathbb{N}_k^{d'}$, 47
- $p$ map from $G$ to $T$, 43
- $p(A)$ image of $A$ in the tensor algebra under the map $p_h$, 57
- $p_n$ restriction of $p$ to the set of graphs with $n$ vertices, 43
- $p_{a,B}$ partition function of $(a, B)$, 18
- $Q_l(f)$ quotient algebra $\mathbb{F}_l G_l / I_l(f)$, 13
- $R(\mathbb{F})$ polynomial ring $\mathbb{F}[x_1, \ldots, x_k]$, 18
- $R$ polynomial ring $\mathbb{F}[x_1, \ldots, x_k]$, 18
- $R_k \{ r_1 \otimes \ldots \otimes r_k \mid r_1, \ldots, r_k \in B(H_1) \}$, 89
- $\mathbb{R}$ field of real numbers, 7
- $rk(M)$ rank of the matrix $M$, 8
- $(C \circ D)$ Schur product of $C$ and $D$, 62
- $C \ast D$ operation on 2-tensors, 62
- $F_1 \ast F_2$ gluing operation of $F_1$ and $F_2$, 10
- $\mathbb{S}F^n$ space of symmetric $n \times n$ matrices in $\mathbb{F}^n \times n$, 28
- $S_n$ symmetric group, 30
- $\text{Stab}(A)$ pointwise stabilizer of $A$, 57
- $\text{Stab}(\mathcal{h})$ stabilizer of the edge-coloring model $\mathcal{h}$, 52
- SFT Second Fundamental Theorem, 27
- $F_1 \otimes F_2$ tensor product of the fragments $F_1$ and $F_2$, 54
- $M^*$ conjugate transpose of the matrix $M$, 8
- $M^T$ transpose of the matrix $M$, 8
- $T(V)^{\text{Stab}(\mathcal{h})}$ algebra of tensors invariant under the stabilizer of $\mathcal{h}$, 58
- $T$ polynomial ring in the variables $y_\alpha$, $\alpha \in \mathbb{N}_k$, 42
- $T_n$ homogeneous polynomials in $T$ of degree $n$, 43
- $\text{tr}$ trace, 77
- $t_M$ tensor associated to the perfect matching $M$, 27
- $U_i$ unlabeling operator for tensors, 62
- $[H]$ underlying graph of the labeled graph $H$, 9
- $U^l_i$ unlabeling operator for labeled graphs, 62

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LIST OF SYMBOLS

$(V^\otimes 2m)^O_k$ space of $O_k$-invariant $2m$-tensors, $27$
$V(F)$ vertex set of the fragment $F$, $55$
$V(H)$ vertex set of the graph $H$, $8$
$V^*$ dual vectorspace of the vectorspace $V$, $8$
$W^G$ subspace of $G$-invariants in $W$, $26$
$X/G$ orbit space of $G$ acting on $X$, $86$
$\gamma_d$ the common zeros of the polynomials $p(H) - f(H)$, with $H \in G$ of max. degree $d$, $47$